Project 5: One week worth of learning activities

This document lists the details of the learning activities and the lesson plan for a particular week in the semester for the course on Introduction to Probability and Statistics in Civil Engineering. The learning objectives of my class state that at the end of the course students will be able to:

1. Interpret, formulate, and solve problems involving elementary combinatorics, probability distribution, and inferential statistics
2. Conduct basic descriptive and inferential statistical analysis of data using a software program such as R
3. Identify and demonstrate the use of probability and statistical methods for applications in civil engineering

This week's learning activities focus on confidence intervals which are a part of inferential statistics component of the course. The week comprises of two lectures each of 75 minutes duration and one lab of 50 minutes duration. The following section provides an overview of the learning activities for the week and the details for each lecture and lab component is then described in the following sections.

1 Overview

For each lecture/lab, I formulate sub-learning objectives that I want students to take away at the end of the lecture. The lesson plan for each lecture is framed using a six-step procedure proposed in Milkova (2012)[1]. The six steps include the following:

1. Outline learning objectives for the particular lecture
2. Develop the introduction
3. Plan the specific learning activities
4. Plan to check the understanding
5. Develop a conclusion and a preview
6. Create a realistic timeline

Each lecture and lab contents are explained to include each step of the proposed procedure. For presentation purposes, a minor change is made to the order of the steps, where the timeline in the sixth step is presented before details on the other steps. In addition to that, each lecture/lab content is mapped to the primary learning outcome of the course and the coordination of each lecture/lab with the planned assessments for this course is shown.

[1] Milkova, S. (2012). Strategies for effective lesson planning. *Center for Research on learning and Teaching*, 1-4.

2 Lecture 1 (75 minutes)

2.1 Timeline

The timeline of the lecture is shown in Table 1:

Table 1 Timeline lecture 1

Activity/Content	Time allotted	Learning outcome covered
Introduction • Correlation with course content flowchart • Introduce confidence interval: what are they? Why need an interval? • Types of confidence intervals: one-sided interval vs two-sided interval • Constructing confidence interval: width of interval vs level of confidence	20 minutes	1
Activity 1: constructing confidence intervals for example cases	10 minutes	1
How to interpret confidence intervals? • Correct and incorrect interpretation • Activity 2: Individual response to examples on interpretation and discussion	5-10 minutes	1
Concept map: Preview of the math for constructing a confidence interval	5 minutes	1
Derive confidence interval for the case of normal distribution with population mean as the parameter of interest • Central limit theorem • Motivate normal distribution and derive formula for one-sided and two-sided confidence intervals	15 minutes	1
Example 1: show an example of the use of formula • Activity 3: Group discussion for solving an extension of the example	10 minutes	1 and 3
Conclusion, next class preview, and end-of-class assignments	5 minutes	1
	Total time= 70-75 minutes	

2.2 Sub-learning objectives and mapping to primary learning outcomes

By the end of this lecture, students will be able to:

a) Interpret the meaning of confidence interval
b) Identify whether the problem asks for a one-sided or two-sided confidence interval
c) Use the correct formula to construct the confidence interval around population mean for an asked problem

These sub-learning objectives map to the first and third learning outcome of the course.

2.3 Introduction

The 20 minute introduction planned in the beginning of the lecture will set the stage for confidence interval in the domain of inferential statistics. The lecture will start with a big picture setting of the course and where the content of this week fit as shown in Figure 1. Students will be asked what do they think when they hear the name confidence interval, which will help me assess the prior knowledge of the class. Two examples of confidence interval will then be shown to motivate how intervals help infer information about population parameters.

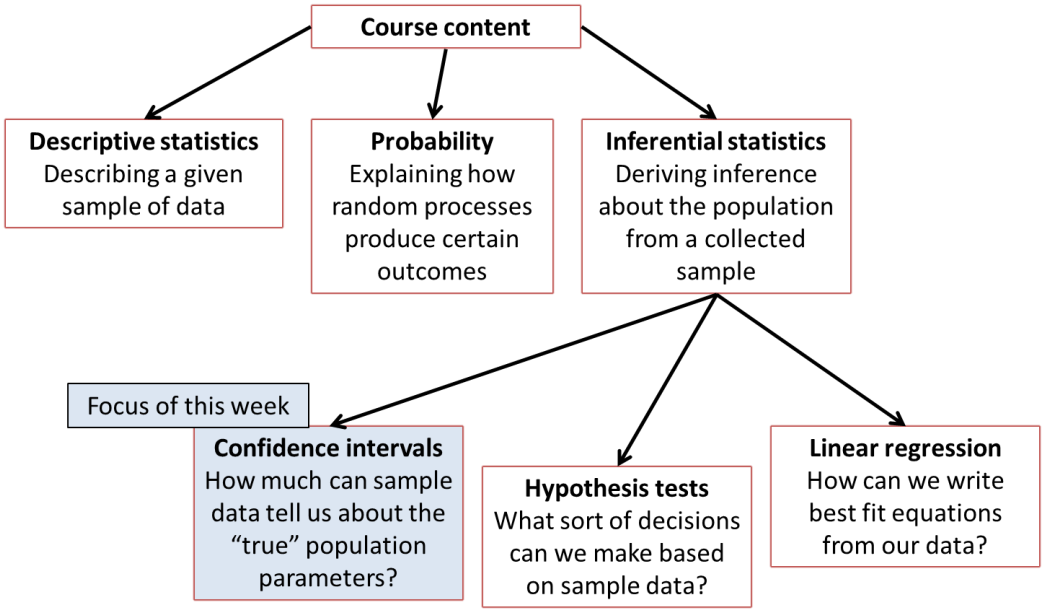

Figure 2.1 Overview of the course and the concept covered this week

2.4 Specific learning activities

2.4.1 Lecture: Introduction to confidence intervals

Confidence intervals will be introduced in a lecture-style where following things will be discussed:

- Definition of confidence intervals
- Why need intervals rather than point estimates?
- Types of confidence interval and ways to identify which one is of relevance for the current problem
- Inputs and outputs to the black box which constructs confidence intervals
- Tradeoff between width of the interval and the confidence level

2.4.2 Activity 1: Constructing confidence intervals for example cases

This activity will be a fun activity where students will be asked to work in a group of two. Five examples will be shown on the slide and students will be asked to construct a 90% confidence interval for the asked parameter in the example. Students will be given 3 minutes to discuss and write down their answers. Then, the next 7 minutes will be devoted to summarizing what each group wrote down. The objective will be to use the results of all groups to reinforce idea about tradeoff between width and confidence level of an interval and identifying whether the interval should be a one-sided or two-sided interval. For example, a group which doesn't know the exact detail of the fact will provide an interval of larger width for 90% confidence.

A sample of examples is listed below:

- Interval for population of the United States on January 1, 2000
- Interval stating an upper bound on the maximum takeoff weight of a Boeing 747
- Interval stating the number of bones in an adult human body
- Interval on minutes for a space shuttle to orbit the earth
- Interval on number of stop signs on forty acres of the campus

2.4.3 Lecture and Activity 2: interpretation of confidence interval

The next 5-10 minutes will be devoted to learning about the interpretation of confidence interval. A particular example of incorrect and correct interpretation will be explained along with the reasoning behind the decision.

Following this, a quick activity on evaluating the interpretation will be conducted. Each student will be asked to take out their green and red cards (color is only on one side of the card; the other side is white) which have been provided to them earlier in the semester (I'll carry extra cards if someone missed those). Different interpretation of confidence intervals will be shown on the slides and students will be asked to raise a green card if they think the interpretation is correct and red card if the interpretation is wrong. I will then discuss the examples where many students make a mistake. This will serve as an informal assessment of whether students understood how to interpret a confidence interval.

2.4.4 Concept map: Preview of the Math for constructing a confidence interval

At this point in the lecture students will be introduced to the math behind constructing a confidence interval. First, a 5-minute overview of what we are expecting to develop will be explained. A concept map will be used for that purpose. A sample concept map is shown in Figure 2.

The map shows the "levers" we play around with and for a certain combination of levers we seek to derive a formula for the confidence interval. Then, based on which formula is being derived the particular choice of "levers" will be highlighted in blue.

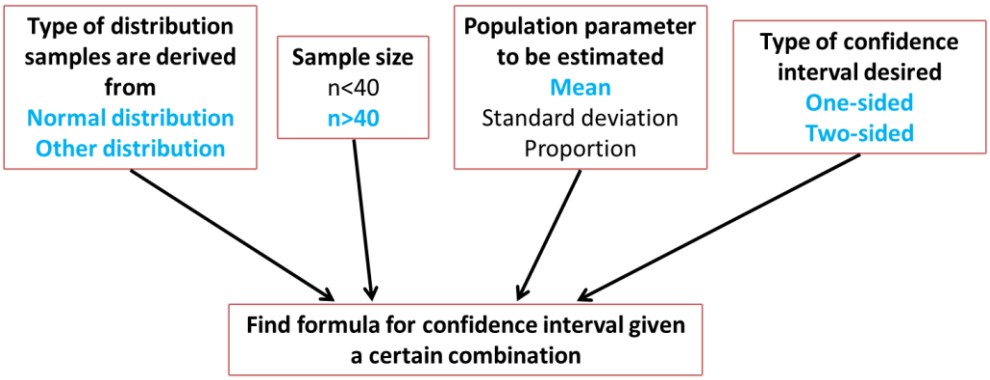

Figure 2 Concept map for preview of the math

2.4.5 Lecture: Deriving confidence interval

For the next 15 minutes, the formula for confidence interval on population mean will be derived assuming normal distribution of the underlying population. The use of z-tables to determine the confidence interval will be explained. The formula for estimating the sample size will be derived for a certain width of confidence interval. The students will be introduced gradually to the math behind the problem and each step will be clarified using the concepts from before.

2.4.6 Example and Activity 3

The next 10 minutes will be devoted to solving an example using the derived formula. I will solve the first part of the problem on board. An extension to the original problem will then be given to the students and they will be asked to identify what type of interval to use and which formula to apply. Students will be given 3 minutes to solve the problem during which time I will go around the class to observe the challenges faced by the students. After 3 minutes, some groups will be asked to report back their findings and then I will solve the problem based on the classroom discussion.

2.4.7 Conclusion, next class preview, and end-of-class assignments

To wrap the discussion in the last 5 minutes, students will be given a preview of the next class where the confidence intervals for other cases will be derived. Students will also be informed about the following:

- Existing online resources uploaded on Canvas or the textbook for more didactic approach to deriving confidence intervals
- Homework 5 that shall be uploaded few hours after the lecture. The homework will be due the week following the current week and will cover confidence intervals
- Ungraded practice questions quiz based on confidence intervals will be unlocked on Canvas for students to practice the examples
- A food-for-thought question will be assigned to students asking whether normal distribution is a reasonable distribution to model population parameter given that it can take values till negative infinity.

2.5 Checking the understanding and coordination with assessments

Activities 2 and 3 will help in checking the understanding of students against the sub-learning objectives of the lecture. Based on the responses of students, the pace of the lecture can be controlled or a certain concept can be re-explained.

This lecture prepares the student for solving homework 5 which is a part of graded assessment for the class. Additionally, it will form the foundation of data analysis for their course project which is due four weeks after the current week of the lecture.

3 Lab (50 minutes)

3.1 Timeline

The timeline of the lab is shown in Table 2:

Table 2 Timeline for the lab

Activity/Content	**Time allotted**	**Learning outcome covered**
Introduction : Review of previous lecture components	5 minutes	1
Activity 1: Seeing theory website for confidence interval interpretation	5-7 minutes	1
Example 1 on two sided intervals (Activity 2) • Find the interval • Determine sample size for a certain width • Interpret the interval	10 minutes	1 and 3
Example 2 on one sided intervals (Activity 3) • Find the upper bound and lower bound interval	10 minutes	1 and 3
Lab learning: practicing commands in R to find the confidence interval	10 minutes	2
Questions from students	8-10 minutes	1
	Total time= 50 minutes	

3.2 Sub-learning objectives and mapping to primary learning outcomes

By the end of this lab, students will be able to:

a) Use R to evaluate the confidence interval
b) Use the correct formula to construct the confidence interval around population mean for an asked problem

These sub-learning objectives map to the first and second learning outcome of the course.

3.3 Introduction

The 5 minute introduction in the beginning of the lab will review the concepts discussed in class. The students will be divided into four groups based on their seating location. The slides will have missing blanks and students from each group will be asked in turn to provide the filling word/term/formula completing the blank.

3.4 Specific learning activities

3.4.1 Activity 1: Seeing theory website for confidence interval interpretation

In this activity, students will be asked to visit the seeing theory website on confidence intervals. The website explains the interpretation of confidence interval and is available at http://students.brown.edu/seeing-theory/statistical-inference/index.html#second. A sample demonstration of running the experiment will be given to the students. Students will then be asked to run experiments for 10sec, 20 sec, and 30 sec duration and make a note if the interval contained the population parameter the desired number of times. The activity will take about 5-7 minutes.

3.4.2 Example 1 on two sided intervals

For next 10 minutes, the class will work on another example. The TA will solve the example on two sided intervals for a certain level of significance with more challenging components that the one discussed during class. The students will then be asked to repeat the same process for another level of significance and the TA will go around the class to ensure that students follow the example through. The latter half of the class will be a group discussion activity on solving the problem for another level of significance in a group of two.

3.4.3 Example 2 on one sided intervals

Similar to the previous case, this activity will solve a challenging example on one-sided intervals along with a group activity component solving the same problem for a different set of parameters.

3.4.4 Practicing commands in R

The R commands to evaluate the confidence interval will be shown on the screen. Then, the students will be asked to practice those commands and derive the same answer for examples 1 and 2 as obtained earlier. This will be a lab session activity where the TA will go around to every student ensuring that they have an understanding of how to use the command.

3.4.5 Questions from students

The lab will conclude with open ended questions that the students might have for the TA or if they are confused with any concept (which is the objective of the lab: to encourage more classroom participation). In my past experience, I have noticed that students rarely ask questions. In such cases, the TA can present them another example (combining the concepts covered in the course so far) and ask them to solve it on their own.

3.5 Checking the understanding and coordination with assessments

The lab session offers the environment where more group problem solving can happen. This ensures that the TA gets to learn the understanding of each student as he/she goes around the lab.

Almost all the activities work towards checking how well the students are scaling against the learning objectives of the lab.

Similar to the lecture, this lab prepares the student for solving homework 5 and forms a foundation of data analysis for their course project.

4 Lecture 2 (75 minutes)

4.1 Timeline

The timeline of the lecture is shown in Table 3:

Table 3 Timeline for Lecture 2

Activity/Content	Time allotted	Learning outcome covered
Introduction • Review of last class • Review of concept map previewing the math involved for constructing confidence intervals	5-10 minutes	1
Lecture: Deriving formula for not large sample but normal distribution • Activity 1: Identify whether to use z-distribution or t-distribution tables	15 minutes	1
Lecture: Deriving formula for the confidence interval on population standard deviation	10 minutes	1
Activity 2: Group discussion on problem solving	20 minutes	1 and 3
Lecture: Prediction and tolerance interval • Definition • Types: one sided and two sided • Activity 3: Identify the type of interval asked in a problem	15 minutes	1
Conclusion, next class preview, and end-of-class assignments	5 minutes	1
	Total time= 70-75 minutes	

4.2 Sub-learning objectives and mapping to primary learning outcomes

By the end of this lecture, students will be able to:

1. Use the correct formula to construct the confidence interval around population mean and standard deviation for an asked problem
2. Identify given the application in the problem whether the asked interval is a confidence, prediction, or tolerance interval

These sub-learning objectives map to the first and third learning outcome of the course.

4.3 Introduction

A 5 to 10-minute introduction planned in the beginning of this lecture will review the contents from the last class. The review will start with covering basics like definitions and concept map from the last class while pausing appropriately with specific questions to let students speak up from what they remember from the activities last time.

Specific learning activities

1.1.1 Lecture and Activity 1: Deriving formula for not large sample but normal population

The concept of confidence interval for population mean for not large samples will be taught in a lecture based style by starting off with an overview using the concept map of the math involved in deriving confidence interval. The t-distribution will be introduced in contrast to the z-distribution and methods on reading the t-table will be shown. The updated confidence interval formula will be shown in relation to the formula already known for better knowledge organization.

In addition to the lecture, a green/red card activity will be conducted before deriving the formula where students will be asked to raise an appropriate card based on whether they feel the current problem involves t-distribution or z-distribution. Based on the response, the examples will be discussed and concepts will be clarified if there is any confusion. The activity will constitute 3-5 minutes where the rest 10-12 minutes will be devoted to deriving the formula for this case.

An example for this case will be shown together with an example for the concept following this for better contrast and comparison.

1.1.2 Lecture: Deriving formula for confidence interval on population standard deviation

The concepts of confidence interval on population standard deviation will be introduced similarly. The notation of chi-squared distribution will be introduced and the plots will be shown. The way to read the chi-squared table will be taught. The updated confidence interval formula for both variance and standard deviation will be shown in relation to the formula described so far.

At the end of this portion of lecture, a summary table listing the formula for different cases studied so far will be shown together for better knowledge organization. A sample table with incomplete columns is shown in Table 4.

Table 4 Sample table to assist with knowledge organization

Assumption	X-I's are from a **normal distribution** with unknown mean and known variance	X-I's are from **any distribution** with unknown mean and unknown variance and N>40	X-I's are from **normal distribution** with unknown mean and unknown variance and N<40	X-I's are from a **normal distribution** with unknown mean and unknown variance

Population parameter to be estimated	Mean	Mean	Mean	Variance
$(\mathbf{1-\alpha})\mathbf{100}\%$ **two-sided confidence interval**				
Sample size for CI of width $\boldsymbol{w}$	$\left(\frac{2z_{\alpha/2}\sigma}{w}\right)^2$			
$(\mathbf{1-\alpha})\mathbf{100}\%$ **one-sided upper bound CI**				
$(\mathbf{1-\alpha})\mathbf{100}\%$ **one-sided lower bound CI**				

1.1.3 Activity 2: group discussion on problem solving

The next 20 minutes will be devoted to solving the example for the two formula just introduced. Instead of me solving problems for both cases separately, the students will be asked to think through how to use their prior knowledge about solving problems from last lecture to approach the shown problems.

A single problem with two parts will be presented to the class. The students will be divided into groups of four and will be asked to brainstorm the solution to the problem. The activity setup time will be 2 minutes. Each group will be given 10 minutes to solve the problem which includes the time where I will go around to check progress with the groups. The next five minutes will then be devoted to wrapping up the discussion by me asking two of the groups to share what they solved. The last three minutes will be used to summarize the results and show correct answers for each problem.

The objective of this activity is to engage students through peer-learning and by challenging them to apply the theory to an example without having seen any example before.

1.1.4 Lecture and Activity 3: prediction and tolerance intervals

This activity will introduce the two other types of intervals: prediction and tolerance interval. A 10 minutes lecture in the beginning will motivate the need for these types of intervals, will provide their exact definitions, and will compare it with the definition of confidence intervals. The one-sided and two-sided types of prediction and tolerance interval will also be explained.

Following this lecture, the class will be asked to participate in a 5-minute activity where different example problems will be shown on the slide and the task of students will be to classify the asked interval as confidence, prediction, or tolerance interval. This will be an individual activity

where every student will be given a handout and asked to mark the type of interval on the handout. The students will then be asked to exchange their responses with the partner next to them who will evaluate the correctness of the response. Such an activity will encourage students to get feedback on their performance from their peers and will serve as a self-reflection assessment tool.

The formula for these types of intervals will be derived in the class following this lecture.

1.1.5 Conclusion, next class preview, and end-of-class assignments

To wrap the discussion in the last 5 minutes, students will be given a preview of the next class where the formula for prediction and tolerance intervals will be derived and hypothesis testing concepts will be introduced. Students will also be informed about the following:

- Existing online resources uploaded on Canvas or textbook for more didactic approach to deriving confidence intervals
- All questions in homework 5 can be solved using the lecture content till today. Students will be encouraged to visit during office hours if they have questions
- Given the mid-term progress report for course project will be due in a week, students will be asked to frame interval based questions from their selected dataset using the examples we saw in class.
- Ungraded practice questions quiz based on confidence intervals on standard deviation, t-distribution, and basics of prediction and tolerance interval will be unlocked on Canvas for students to practice the examples

1.2 Checking the understanding and coordination with assessments

Activities 1, 2, and 3 will help in checking the understanding of students against the sub-learning objectives of the lecture.

Like the previous lecture, this lecture prepares the student for solving homework 5 which is a part of graded assessment for the class. Additionally, this lecture provides examples for the type of questions that can be asked using the dataset relevant for the group project. Given the group project progress reports are due in a week, this lecture will serve as a valuable tool in framing interval related questions from the dataset.