uc3m | Universidad Carlos III de Madrid
Departamento de Matemáticas

Year 2024/2025

Further topics in numerical methods

Bachelor in Applied Mathematics and Computing

Extra-Assignment 2

Exercise 1. For any r-step linear multistep method, a general expression for the local truncation error is

$$\begin{aligned}
\tau(t_{n+r}) &= \frac{1}{k}\left[\sum_{j=0}^{r}\alpha_j u(t_{n+j}) - k\sum_{j=0}^{r}\beta_j u'(t_{n+j})\right] \\
&= \frac{1}{k}\left[\sum_{j=0}^{r}\alpha_j\right]u(t_n) + \left[\sum_{j=0}^{r}(j\alpha_j-\beta_j)\right]u'(t_n) + k\left[\sum_{j=0}^{r}(\frac{1}{2}j^2\alpha_j - j\beta_j)\right]u''(t_n)+\dots \\
&\qquad \dots + k^{q-1}\left[\sum_{j=0}^{r}\left(\frac{1}{q!}j^q\alpha_j - \frac{1}{(q-1)!}j^{q-1}\beta_j\right)\right]u^{(q)}(t_n)+\dots
\end{aligned}$$

Write a function in Python that given the number of steps r and the type of method (either Adams-Bashforth, Adams-Moulton or Backward Differentiation), returns the corresponding α_j and β_j coefficients. Call this function to demonstrate that it works for $r = 1, 2, 3, 4$ (compare against your classnotes).