

# Applications of Hash Functions

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- Message Integrity
- Message and Message Digest
- Cryptographic Hash function Criteria
- Hashes are "Digests", not "Encryption"

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One way to preserve the integrity of the document is through use of **fingerprints**.

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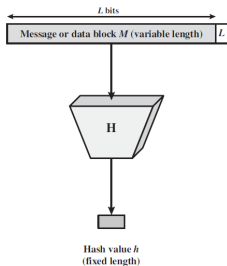
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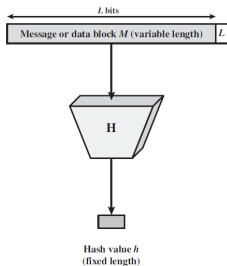
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- 3) **Collision resistance:** A hash function  $H$  is collision resistant if it is hard to find two inputs that hash to the same output; that is, two inputs  $x$  and  $y$  such that  $H(x) = H(y)$ , and  $x \neq y$ .

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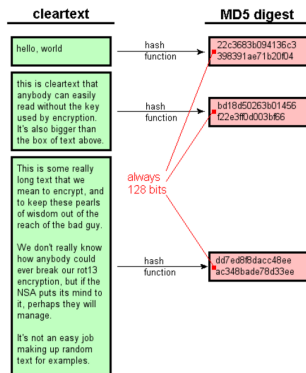


Figure: Hashing - a one-way operation

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### Example

Here is an example which shows the Message and Message Digest listed in Hexadecimal.

Message	Message Digest
4523AB1352CDEF45126	13AB
723BAE38F2AB3457AC	02CA
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- We call this scheme as **iterated cryptographic hash function**.



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Merkle-Damgard Scheme is an iterated hash function that is **collision resistant** if the compression function is collision resistant. The scheme uses the following steps.

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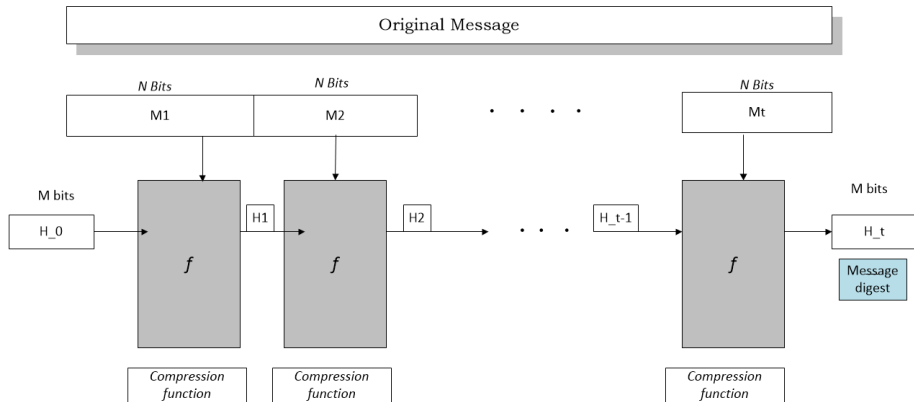


Figure: Merkle-Damgard scheme

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- Now to provide such a proof, Alice uses a hash function to create a **Message Authentication code (MAC)** from the concatenation of the key and the message,  $h(K||M)$ .



# Message Authentication - Implementation Steps

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- If two MAC's match, then the message is authentic.

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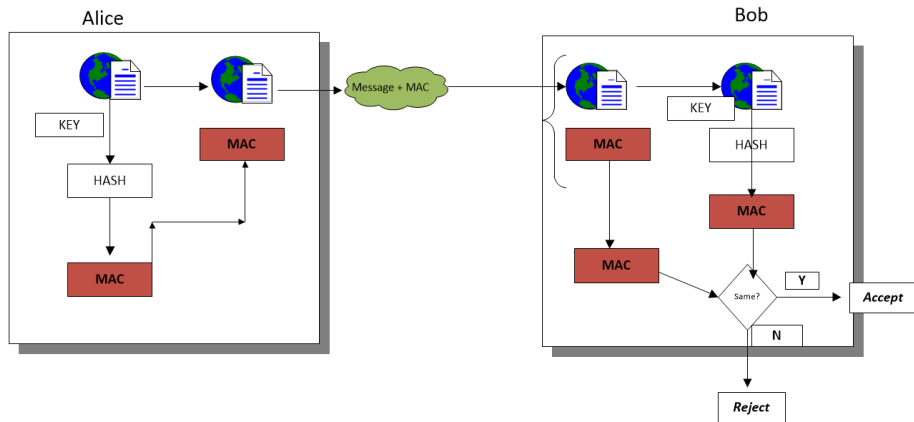


Figure: Message authentication code (MAC)

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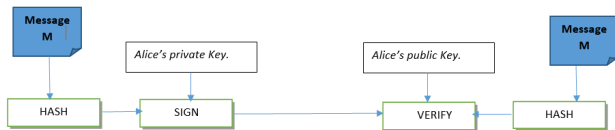


Figure: Signing the message digest

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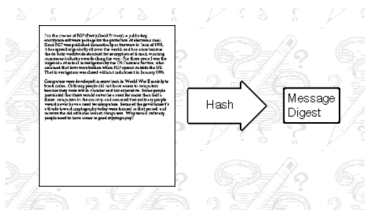


Figure: Converting message to message digest

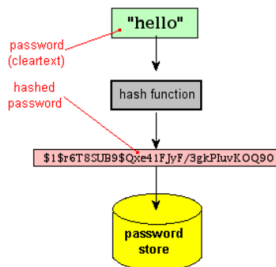


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- In simple terms, when a user enters a password, the hash of that password is compared to the stored hash value for verification. This approach to **password protection** is used by most operating systems.



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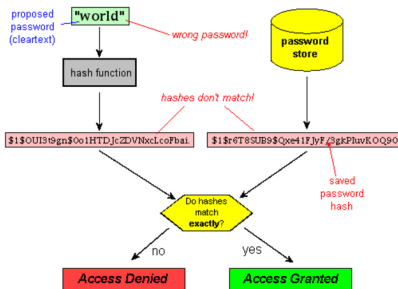
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Thank you for your attention.