magnetic material in volume V, Calculate Demag field using FEM/BEM ①

① $\rho_m = \operatorname{div} \vec{M}$ (including surface divergences)

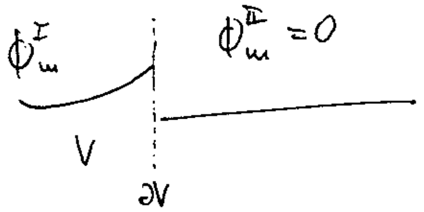
② Solve $\Delta \phi_m^I = \rho_m$ for ϕ_m^I (with gauge fixing)
(and no Dirichlet boundary conditions)

③ Extract boundary values $\phi_m^I \big|_{\partial V}$ ("short vector")

$$\frac{\partial \phi_m^I}{\partial \vec{n}} = \operatorname{div} \vec{M}$$ (von Neumann boundary condition)
(i.e. surface charges on ∂V)

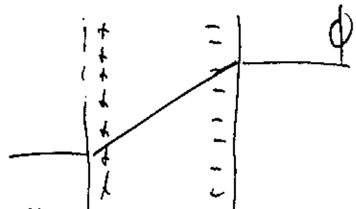
Now we need a continuation of ϕ_m^I in outer space.

At this stage:



i.e. step in ϕ.

A step in ϕ corresponds to a dipole layer:



By not solving ② in V and outside, we introduce the step in ϕ (a "virtual dipole layer"). Now we use the boundary element method to introduce an "anti-dipole layer":

④ using BEM, compute $\phi_m^{II}\big|_{\partial V} = B \cdot \phi_m^I \big|_{\partial V}$

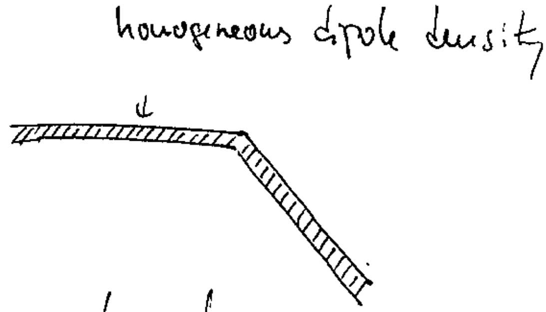
⑤ Continue $\phi_m^{II}\big|_{\partial V}$ in V by solving $\Delta \phi_m^{II} = 0$ with Dirichlet BC given by $\phi_m^{II}\big|_{\partial V}$

⑥ $\phi_m = \phi_m^I + \phi_m^{II}$

In mathematics: $\phi_m^{II} = B^0 \phi_m^{I} + \left(\frac{\Omega(x)}{4\pi} - 1\right) \phi_m^{I}$

where $\Omega(x)$ is the surface angle of the material (V) at point x.

This only matters at the surface ∂V; at other points in $V \setminus \partial V$ we have $\Omega(x) = 4\pi$.



potential of a surface of homogeneous dipole density is proportial to the surface angle under which the surface is seen.

We absorb $\left(\frac{\Omega(x)}{4\pi} - 1\right)$ into the calculation of our matrix B.

7 Compute demag field (strictly: co-field)

$$\vec{H}_D^c = -\vec{\nabla}\phi_m$$

8 Convert back to field using "Box method".

$$\vec{H}_D^c \rightarrow \vec{H}_D$$

Fangohr/Fischbacher 21/03/07, Southampton

Lin Alg Machine script for FEM/BEM Demag

Param: M-field (m), $\qquad$ KSP: $KSP_{\Delta NBC}$

H_d-field (H_d)

Matrices: div_M = "$\langle \rho \,||\, \frac{\partial}{\partial x_j}$ m_mat(j)$\rangle$" + surface, grad_ϕ = "$\langle H\_d(j) \,||\, \frac{\partial}{\partial x_j}\phi\rangle$"

Δ_{NBC} = "$-\langle \frac{\partial}{\partial x_j}\phi \,||\, \frac{\partial}{\partial x_j}\rho\rangle$", B (demag),

$-\Delta^{N\times n}_{nb}$ (N - # nodes, n - # surface nodes on boundary), Δ_{nn} ((N-n)x(N-n) matrix)

Buffer: parallel m, parallel ρ: par_m, par_ρ

parallel ϕ: par_ϕ1, ϕ on ∂V: par_$\phi 1^B$

par_$\phi 2B$, par_ρ2, par_ϕ2, par_ϕ, par_Hd, par_Hd_invvols

Script:

distribute:

1. m → par_m

2. Mxv : div_M, par_m, par_ρ

Matrix times Vector $\qquad$ target

3. Solve : $KSP_{\Delta NBC}$, par_ρ, par_ϕ1

4. Extract : par$\phi 1$ → par$\phi 1^B$

5. Mxv : B, par$\phi 1^B$, par_$\phi 2^B$

6. Mxv : $-\Delta_{nb}$, par_$\phi 2^B$, par_ρ2

7. Solve : $KSP_{\Delta_{nn}}$, par_ρ2, par_ϕ2

8. AXPBY : 1, par_ϕ1, 0, par_ϕ $\qquad$ (par_ϕ1 → par_ϕ)

9. AXPBY : 1, par_ϕ2, 1, par_ϕ $\qquad$ (par_ϕ2 + par_ϕ → par_ϕ)

10. Mxv : grad_ϕ, par_ϕ, par_Hd

11. VxV : par_Hd, par_Hd_invvol, → par_Hd

12. collect : par_Hd → Hd