AC→Controlled DC
Phase-controlled Rectifiers

Jishnu Sankar VC
Asst. Professor
EEE Dept.
ASE Amritapuri

Introduction to Rectifiers

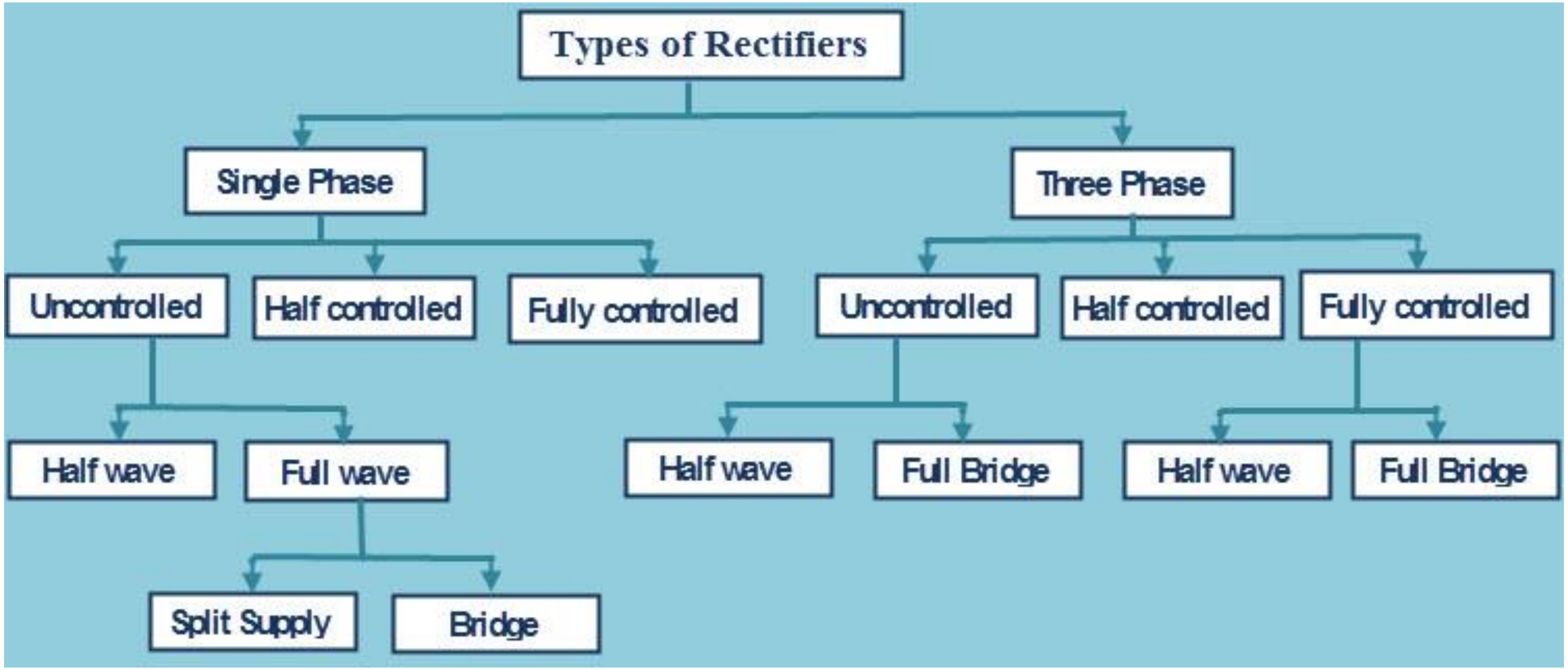
- One of the first and most widely used application of power electronic devices have been in rectification.
- ***Rectification refers to the process of converting an ac voltage or current source to dc voltage and current***.
- Rectifiers specially refer to power electronic converters where the electrical power flows from the ac side to the dc side.
- In many situations the same converter circuit may carry electrical power from the dc side to the ac side where upon **they are referred to as inverters**.
- In most power electronic applications, the power input is in the form of a 50- or 60-Hz sine wave ac voltage provided by the electric utility hence name line-frequency

Uncontrolled rectification and Phase controlled rectification

- The diode rectifiers are termed as uncontrolled rectifiers.
  - In Diode rectifiers, regulating the output voltage is not possible and depend on the input voltage.
- Unlike diode rectifiers, phase controlled rectifiers has an advantage of regulating the output voltage.
  - The o/p voltage can be regulated by changing the firing angle of the Thyristors.
- The main application of these Phase controlled rectifiers is involved in speed control of DC motor.
- A phase control Thyristor is activated by applying a short pulse to its gate terminal and it is deactivated due to line commutation or natural.
  - In case of heavy inductive load, it is deactivated by firing another Thyristor of the rectifier during the negative half cycle of i/p voltage.

Types of Rectifiers

- The rectifiers is classified into two types based on the type of input power supply.



Single-phase Controlled Rectifier

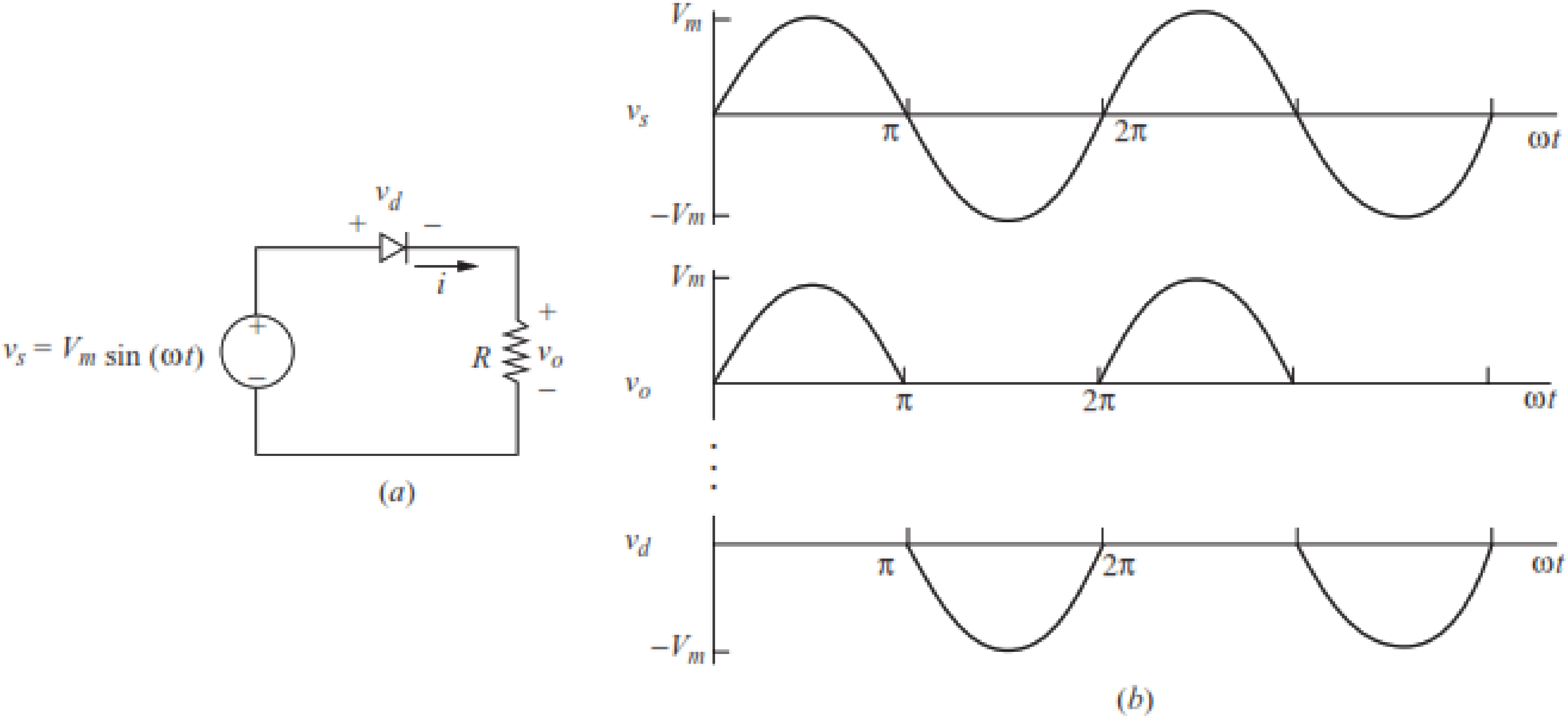
- Single-phase Controlled Rectifier
  - This type of rectifier which works from single phase AC i/p power supply.
- Single Phase Controlled Rectifiers are classified into different types
  - Half wave Controlled Rectifier: This type of rectifier uses a single Thyristor device to provide o/p control only in one half cycle of input AC supply, and it offers low DC output.
  - Full wave Controlled Rectifier: This type of rectifier provides higher DC output
    - Two types of Full wave
      - Single-Phase Semi-Converter: Half controlled
      - **Single-Phase Full converter: Fully controlled**
- half-wave rectifier is used most often in low-power applications
  - because the average current in the supply will not be zero

Reference

Uncontrolled half wave rectifier

half wave rectifier with R load

Reference



(a) Half-wave rectifier with resistive load; (b) Voltage waveforms.

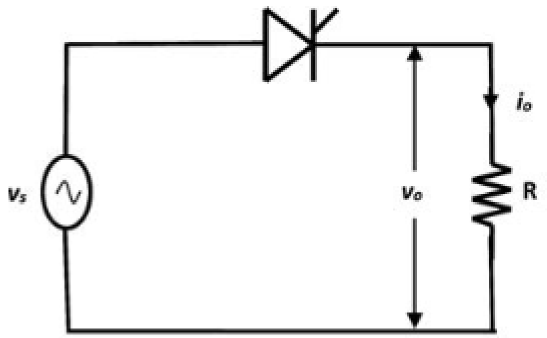
Reference

Half wave : Controlled rectifier

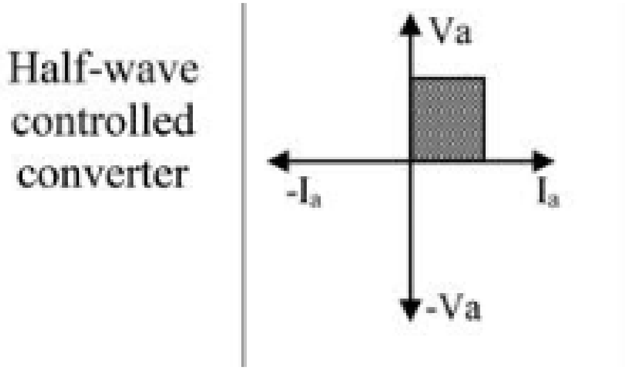
Firing Angle of SCR is defined as **the angle between the instant SCR would conduct if it were a diode and the instant it is triggered**

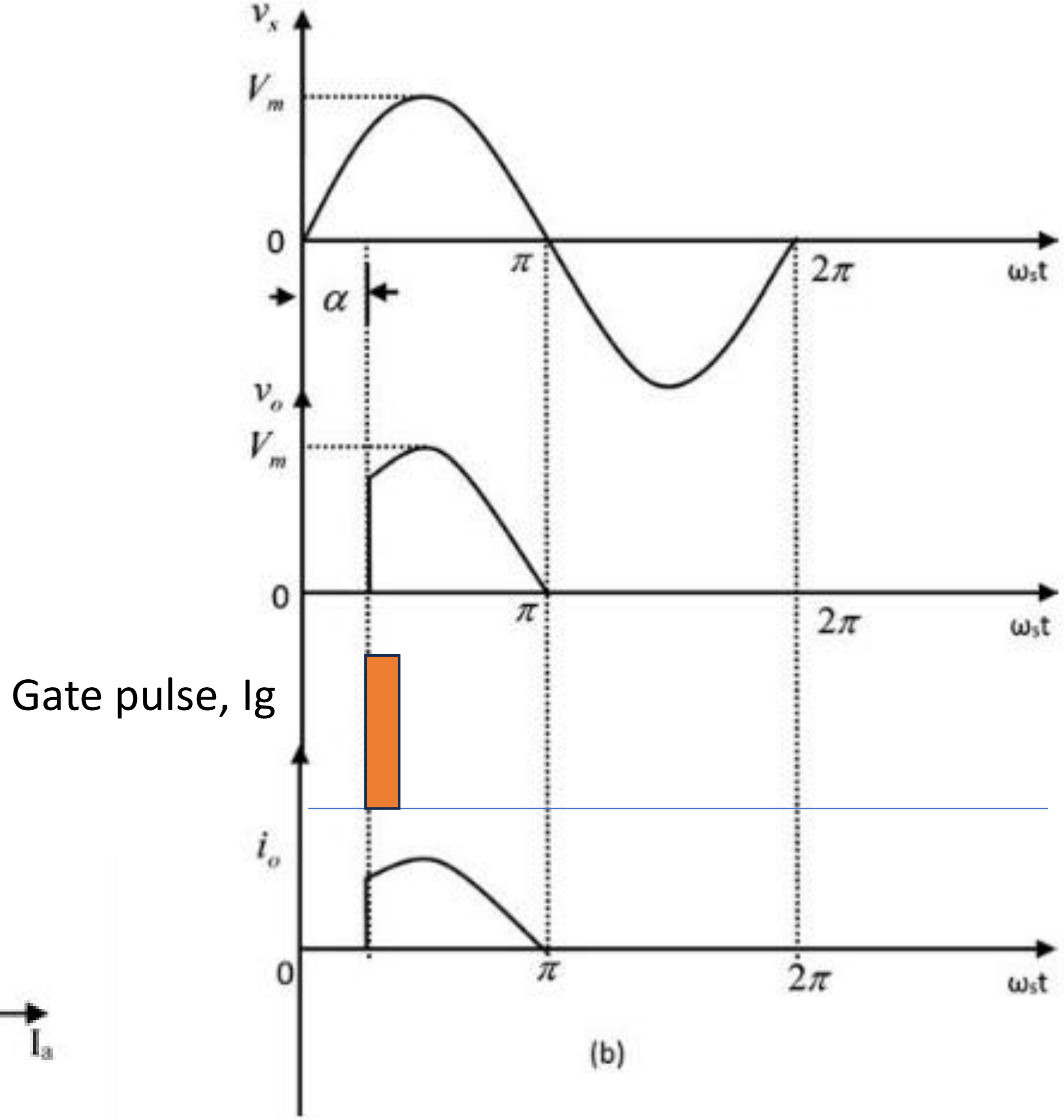
Half wave : Controlled rectifier with R Load

Reference



(a)





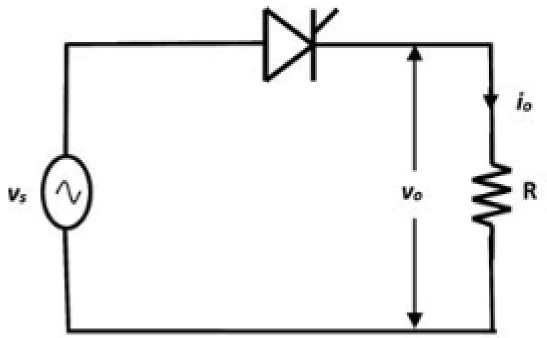
(b)

Firing Angle of SCR

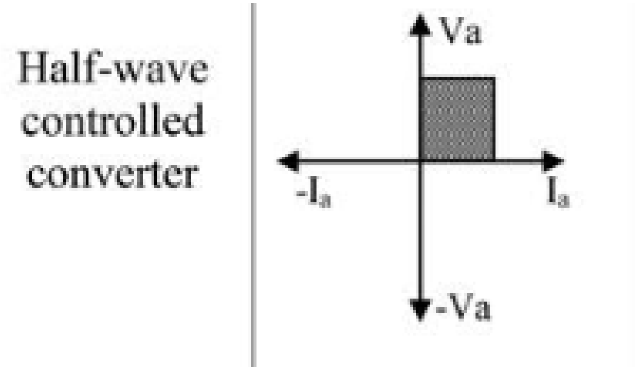
- The **firing angle (α)** of an SCR is the **delay angle at which the SCR is triggered (turned ON) in an AC circuit**. It is measured in **degrees** from the beginning of the AC cycle (zero crossing of the input voltage).
- **Explanation:**
- SCRs **do not turn on automatically** when forward biased; they require a triggering pulse at the **gate terminal**.
- The angle at which the **gate trigger pulse is applied** with respect to the zero crossing of the AC supply voltage is called the **firing angle (α)**.
- By adjusting the firing angle, we can **control the output voltage** and power delivered to the load.
- **Effect of Firing Angle (α):**
- **Small Firing Angle (α ≈ 0°):** SCR turns ON early in the cycle → **Maximum output voltage**.
- **Larger Firing Angle (α > 0°):** SCR turns ON later → **Reduced output voltage**.
- **Firing Angle Close to 180°:** Almost no output voltage → **SCR remains off for most of the cycle**.

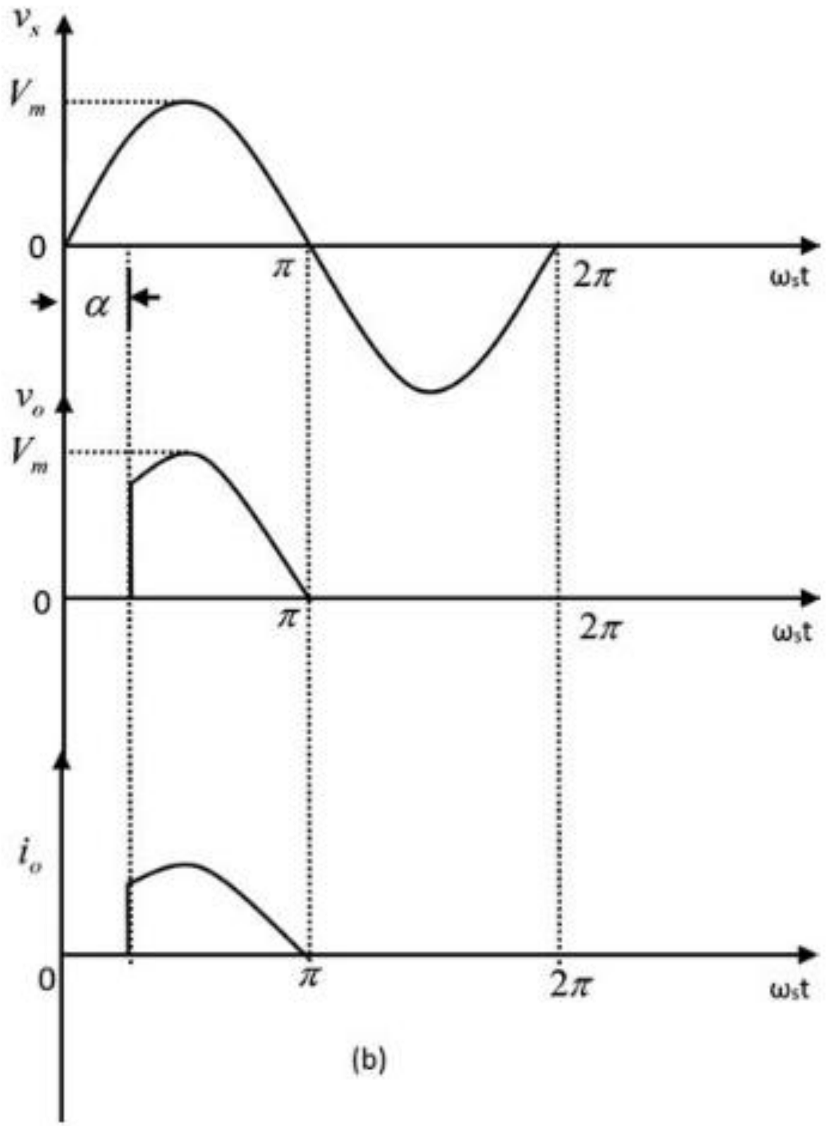
Half wave : Controlled rectifier with R Load

Reference



(a)



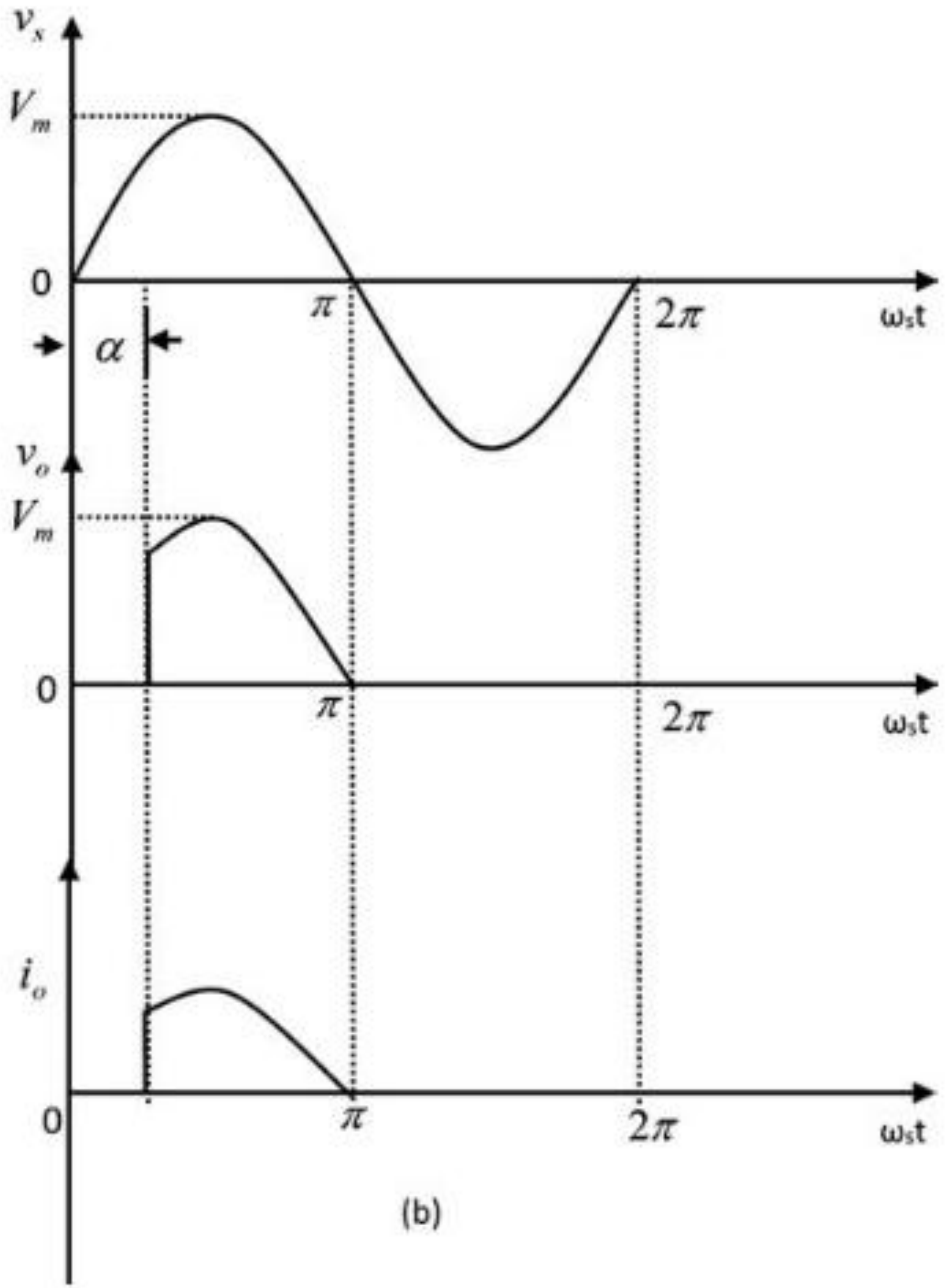


(b)

Working

- **Operation:**
- During Forward Biased Condition:
- In the positive half-cycle of the AC input, the anode is positive with respect to the cathode, making the SCR forward biased.
- However, the SCR does not conduct immediately like a diode. It remains OFF until a gate trigger pulse is applied at a controlled angle (α).
- When the firing angle (α) is reached, the SCR turns ON, allowing current to flow through the load.
- The SCR remains in conduction mode until the AC voltage crosses zero.
- During Reverse Biased Condition:
- In the negative half-cycle, the anode becomes negative, making the SCR reverse biased.
- Since an SCR cannot conduct in reverse bias, it automatically turns OFF.
- No current flows through the load during this period, leading to a gap in the output waveform.

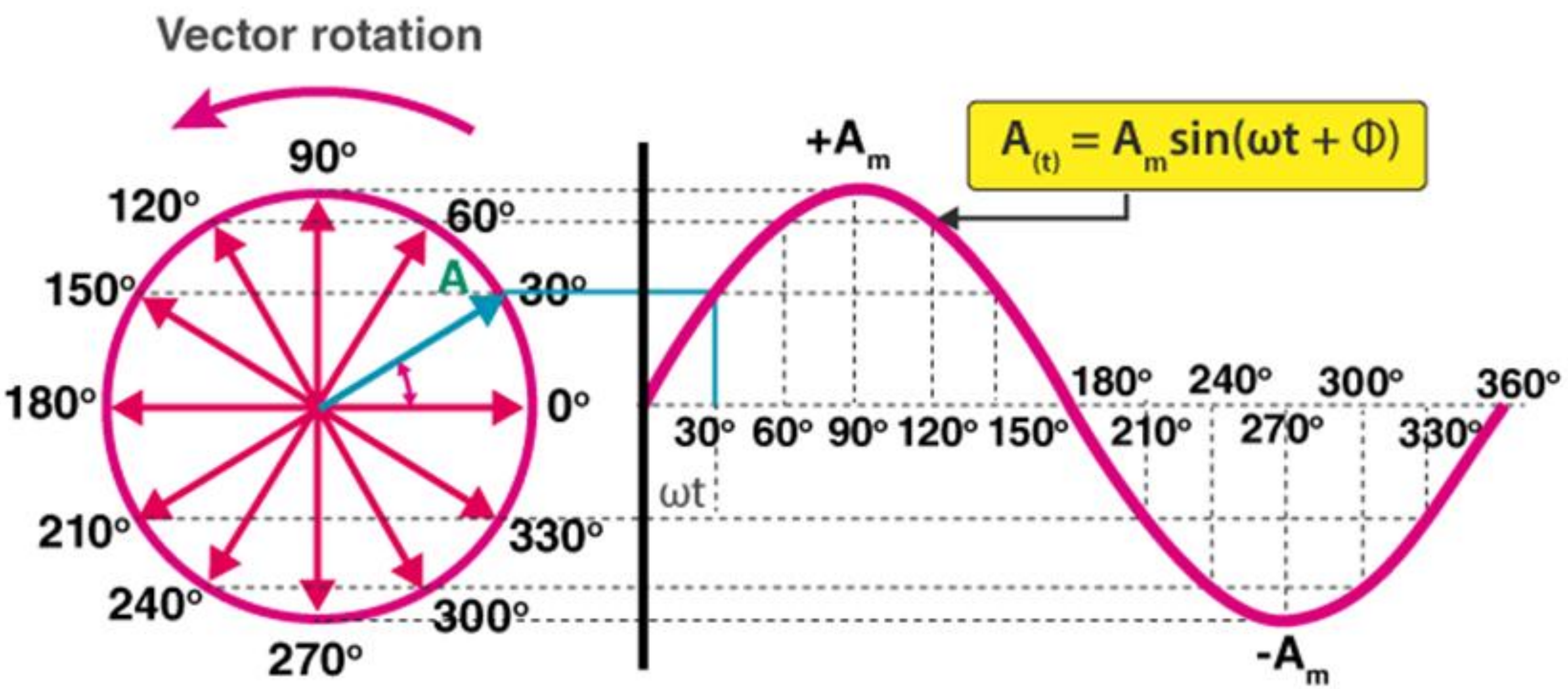
Reference



v_s
V_m
0
π
2π
$\omega_s t$
α
v_o
V_m
0
π
2π
$\omega_s t$
i_o
0
π
2π
$\omega_s t$

(b)

Reference



Rotating Phasor

Sinusoidal waveform in the time domain

Parameter	Average Voltage (V_{avg})	RMS Voltage (V_{rms})
Definition	Represents the **DC component** of output	Represents the **effective voltage** (heating effect)
Application	Determines **DC power** delivered to load	Determines **total power dissipation**
Control in SCR rectifiers	Controlled by **firing angle (α)**	Less affected by α but influences heating
Effect on ripple	Higher V_{avg} → Lower ripple	Higher V_{rms} → More ripple
Load Design Impact	Important for **battery charging, DC motors**	Important for **thermal management and insulation**

Analysis

The circuit diagram is shown in Fig. 2.24(a). The thyristor is turned on at $\omega_s t = \alpha$, and due to the resistive nature of load, it is turned off at $\omega_s t = \pi$. The KVL can be written as

$$R \times i_o = V_m \sin\omega_s t,\ \alpha < \omega_s t < \pi$$
$$i_o = \frac{V_m \sin\omega_s t}{R},\ \alpha < \omega_s t < \pi \qquad (2.31)$$

The variation of v_o, v_s, and i_o is shown in Fig. 2.24(b). From Fig. 2.24(b), the average output voltage and rms value of output current are derived below.

$$\text{Average output voltage } V_{o(av)} = \frac{1}{2\pi}\int_\alpha^\pi V_m \sin\omega_s t\, d(\omega_s t)$$
$$= \frac{V_m}{2\pi}(-\cos\omega_s t)_\alpha^\pi$$
$$= \frac{V_m}{2\pi}(-\cos\pi + \cos\alpha)$$
$$= \frac{V_m}{2\pi}(1+\cos\alpha) \qquad (2.31A)$$

The rms value of the output voltage is

$$i_{o(rms)}^2 = \frac{1}{2\pi}\int_\alpha^\pi \frac{V_m^2 \sin^2\omega_s t}{R^2} d(\omega_s t)$$
$$= \frac{V_m^2}{2\pi R^2}\int_\alpha^\pi \frac{1-\cos 2\omega_s t}{2} d(\omega_s t)$$
$$= \frac{V_m^2}{4\pi R^2}\int_\alpha^\pi (1-\cos 2\omega_s t)\, d(\omega_s t)$$
$$= \frac{V_m^2}{4\pi R^2}\left[\omega_s t - \frac{\sin 2\omega_s t}{2}\right]_\alpha^\pi$$
$$= \frac{V_m^2}{4\pi R^2}\left[\pi - \alpha - \left[\frac{\sin 2\pi}{2} - \frac{\sin 2\alpha}{2}\right]\right]$$
$$= \frac{V_m^2}{4\pi R^2}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]$$
$$i_{o(rms)} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]\right]^{\frac{1}{2}}$$

Analysis

The circuit diagram is shown in Fig. 2.24(a). The thyristor is turned on at $\omega_s t = \alpha$, and due to the resistive nature of load, it is turned off at $\omega_s t = \pi$. The KVL can be written as

$$R \times i_o = V_m \sin \omega_s t,\ \alpha < \omega_s t < \pi$$
$$i_o = \frac{V_m \sin \omega_s t}{R},\ \alpha < \omega_s t < \pi \tag{2.31}$$

The variation of v_o, v_s, and i_o is shown in Fig. 2.24(b). From Fig. 2.24(b), the average output voltage and rms value of output current are derived below.

$$\text{Average output voltage } V_{o(av)} = \frac{1}{2\pi}\int_\alpha^\pi V_m \sin \omega_s t\, d(\omega_s t)$$
$$= \frac{V_m}{2\pi}(-\cos \omega_s t)_\alpha^\pi$$
$$= \frac{V_m}{2\pi}(-\cos \pi + \cos \alpha)$$
$$= \frac{V_m}{2\pi}(1 + \cos \alpha) \tag{2.31A}$$

$$V_{rms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]}$$

$V_{o\,rms}$

$$V_{o\,Rms}^2 = \sqrt{\frac{1}{T}\int_0^T v_o^2(\omega t)\cdot d\omega t}$$
$$= \frac{1}{2\pi}\int_\alpha^\pi V_m^2 \sin^2 \omega t\, d\omega t$$
$$= V_m \times \frac{}{2\pi}$$

The rms value of the output voltage is

$$i_{o(rms)}^2 = \frac{1}{2\pi}\int_\alpha^\pi \frac{V_m^2 \sin^2 \omega_s t}{R^2} d(\omega_s t)$$
$$= \frac{V_m^2}{2\pi R^2}\int_\alpha^\pi \frac{1 - \cos 2\omega_s t}{2} d(\omega_s t)$$
$$= \frac{V_m^2}{4\pi R^2}\int_\alpha^\pi (1 - \cos 2\omega_s t)\, d(\omega_s t)$$
$$= \frac{V_m^2}{4\pi R^2}\left[\omega_s t - \frac{\sin 2\omega_s t}{2}\right]_\alpha^\pi$$
$$= \frac{V_m^2}{4\pi R^2}\left[\pi - \alpha - \left[\frac{\sin 2\pi}{2} - \frac{\sin 2\alpha}{2}\right]\right]$$
$$= \frac{V_m^2}{4\pi R^2}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]$$
$$i_{o(rms)} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]\right]^{\frac{1}{2}}$$

Example

A resistor of 100 Ω is connected to $230\sqrt{2}\sin(314t)$ through a single thyristor fired at $\alpha = 40°$. Compute the supply power factor.

$$\alpha = 40° = 40° \times \frac{\pi}{180°} = 0.698 \text{ rad}$$

$$i_{o(rms)} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]\right]^{\frac{1}{2}}$$

$$i_{o(rms)} = \frac{230\sqrt{2}}{2 \times 100}\left[\frac{1}{\pi}\left[\pi - 0.698 + \frac{\sin(2 \times 0.698)}{2}\right]\right]^{\frac{1}{2}} = 1.575 \text{ A}$$

$$\text{Supply power factor} = \frac{\text{Output power}}{V_{rms} \times i_{rms}}$$

$$\text{Because } i_{rms} = i_{o(rms)}$$

$$\text{Supply power factor} = \frac{i_{o(rms)}^2 \times R}{V_{rms} \times i_{o(rms)}} = \frac{1.575^2 \times 100}{230 \times 1.575} = 0.685 \text{(lagging)}$$

$$2\pi^c = 360^0$$

$$\pi^c = 180^0$$

$$1^0 = \left(\frac{\pi}{180}\right)^c$$

$$40^0 = \left(\frac{\pi}{180} \times 40^0\right)^c$$

Reference

Example

$v(\omega t) = V_m \sin(\omega t)$

$V_m = 230\sqrt{2}$

230 V supply

$230 \rightarrow RMS$

$\boxed{V_m = 230 * \sqrt{2}}$

A resistor of 100 Ω is connected to $230\sqrt{2}\sin(314t)$ through a single thyristor fired at $\alpha = 40°$. Compute the supply power factor.

$\alpha = 40°$

$2\pi^c = 360°$

$\pi^c = 180°$

$1° = \left(\frac{\pi}{180}\right)^c$

$40° = \left(\frac{\pi}{180} \times 40°\right)^c$

$$\alpha = 40° = 40° \times \frac{\pi}{180°} = 0.698 \text{ rad}$$

$$i_{o(rms)} = \frac{V_m}{2R}\left[\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin 2\alpha}{2}\right]\right]^{\frac{1}{2}}$$

$P_{out,avg} = I_{o,Rms}^2 R$

$$i_{o(rms)} = \frac{230\sqrt{2}}{2 \times 100}\left[\frac{1}{\pi}\left[\pi - 0.698 + \frac{\sin(2 \times 0.698)}{2}\right]\right]^{\frac{1}{2}} = 1.575 \text{ A}$$

$\pi \rightarrow 3.14$

$P_{in} = P_{out,avg}$

$$\text{Supply power factor} = \frac{\text{Output power}}{V_{rms} \times i_{rms}} = \frac{P_{in}}{S} = \frac{P_{out}}{S}$$

$$Pf = \frac{P_{in}}{S_{in}} = \frac{P_{in}}{V_{i,Rms} * I_{i,Rms}}$$

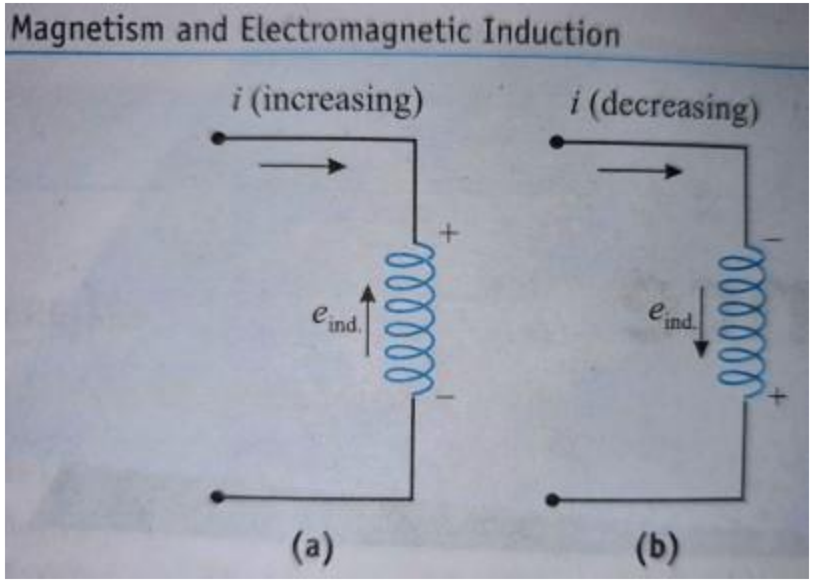
Because $i_{rms} = i_{o(rms)}$

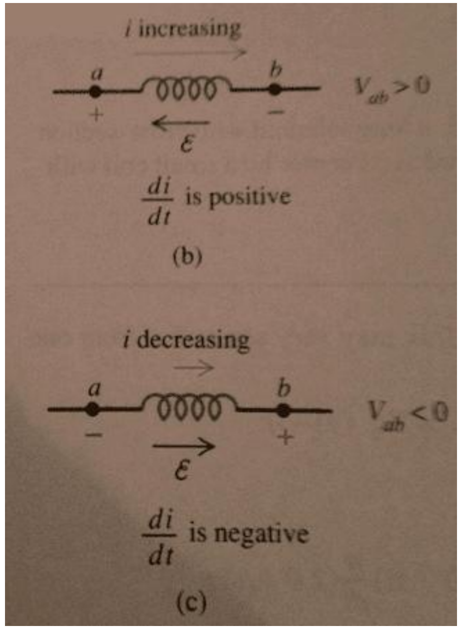
$$\text{Supply power factor} = \frac{i_{o(rms)}^2 \times R}{V_{rms} \times i_{o(rms)}} = \frac{1.575^2 \times 100}{230 \times 1.575} = 0.685 \text{(lagging)}$$

$I_{i,Rms} = I_{o,Rms}$

Inductor behavior

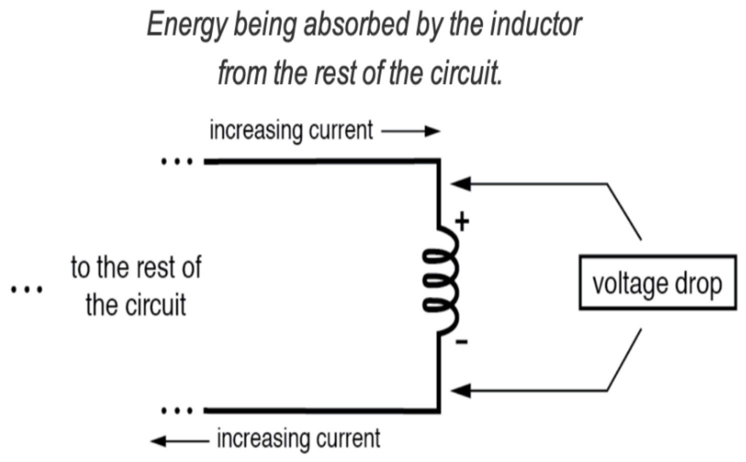
- (a) Increasing Current
- When the current flowing through the inductor is increasing, the inductor opposes this change by generating an induced EMF.
- According to Lenz's Law, the induced EMF will act in a direction that opposes the increase in current.
- The inductor tries to oppose the rise in current by developing a voltage polarity where the upper terminal is positive and the lower terminal is negative.
- The direction of the induced EMF is opposite to the direction of the applied current increase
- (b) Decreasing Current
- When the current is decreasing, the inductor again opposes this change by generating an induced EMF in the opposite direction.
- Now, the inductor tries to oppose the fall in current by acting as a temporary voltage source.
- The polarity of the induced EMF reverses, making the upper terminal negative and the lower terminal positive.
- The direction of induced EMF is now in the same direction as the original current flow, trying to sustain the decreasing current.





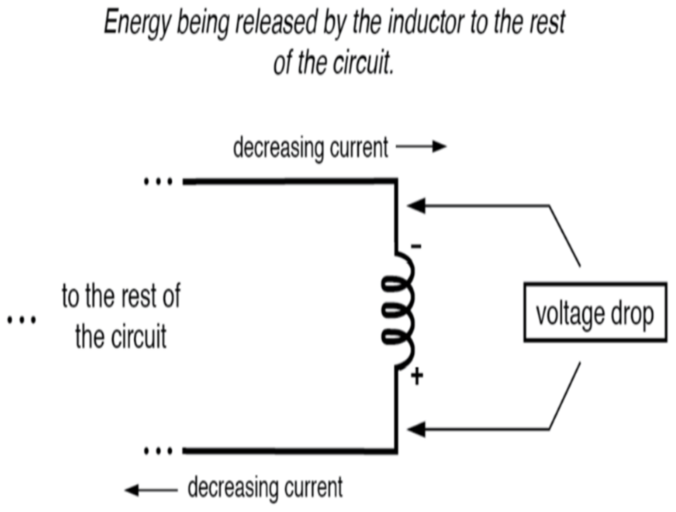
Inductor behavior

- When the current through an inductor is increased, it drops a voltage opposing the direction of current flow, acting as a power load.
- In this condition, the inductor is said to be *charging*, because there is an increasing amount of energy being stored in its magnetic field.
- Note the polarity of the voltage with regard to the direction of current:



The inductor acts as a LOAD

- Conversely, when the current through the inductor is decreased, it drops a voltage aiding the direction of current flow, acting as a power source.
- In this condition, the inductor is said to be *discharging*, because its store of energy is decreasing as it releases energy from its magnetic field to the rest of the circuit.
- Note the polarity of the voltage with regard to the direction of current.



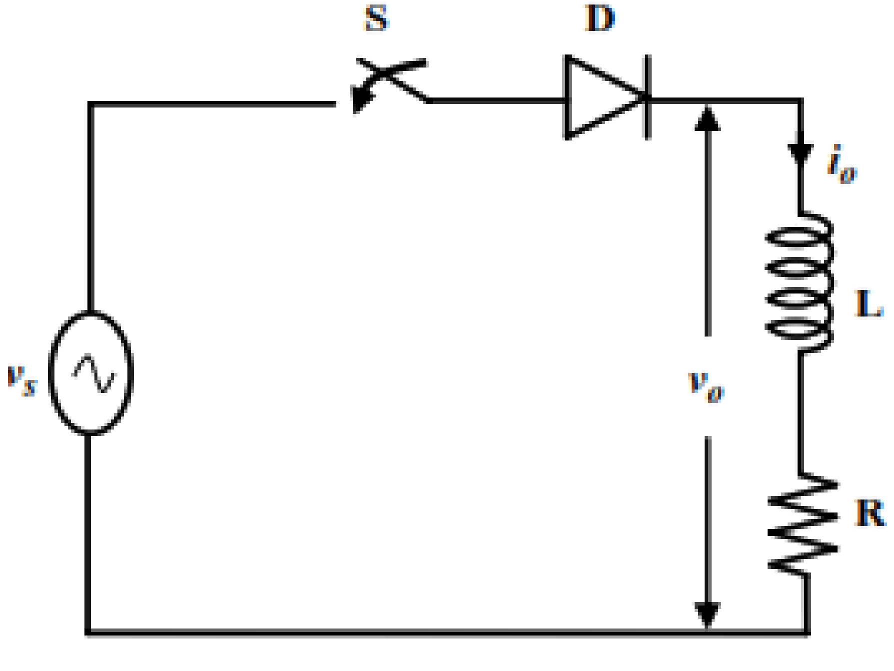
The inductor acts as a SOURCE

Reference

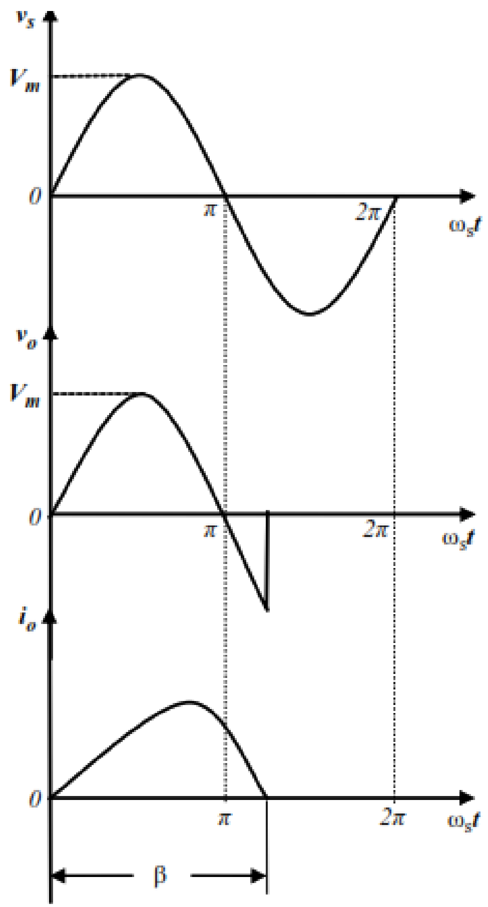
Uncontrolled half wave rectifier with RL load

Half-wave Rectifier with Resistive-inductive(RL) Load: Uncontrolled

Reference



(a)



R — L

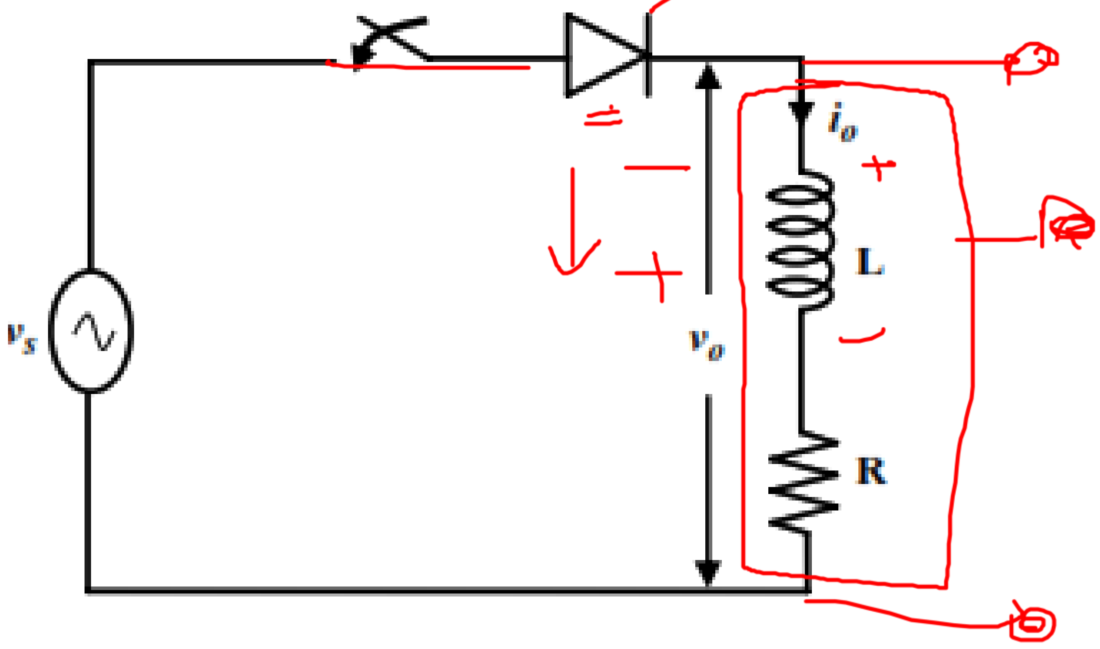
Half-wave Rectifier with Resistive-inductive Load: Uncontrolled

Reference

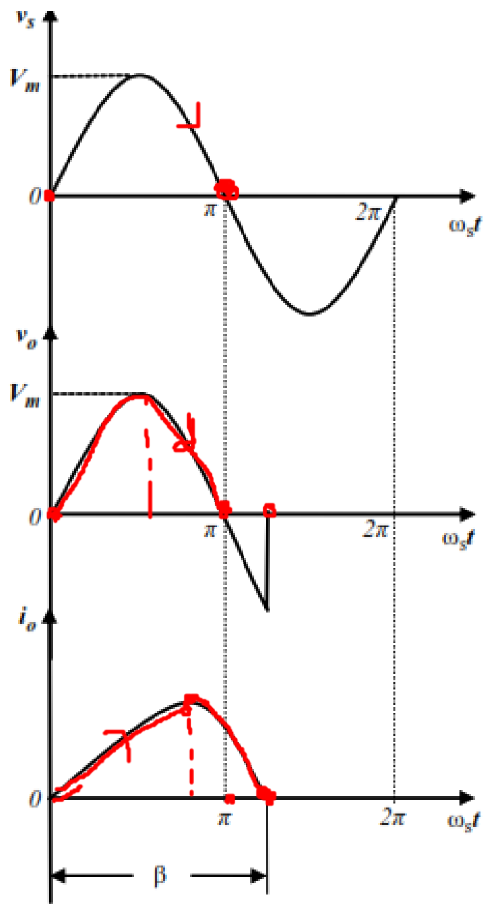
t=0

Diode will rectify

i = 0



(a)



charging

discharge

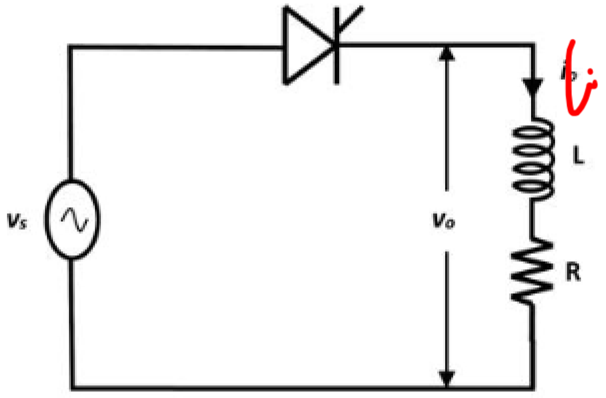
Reference

Half wave : Controlled rectifier with R-L Load

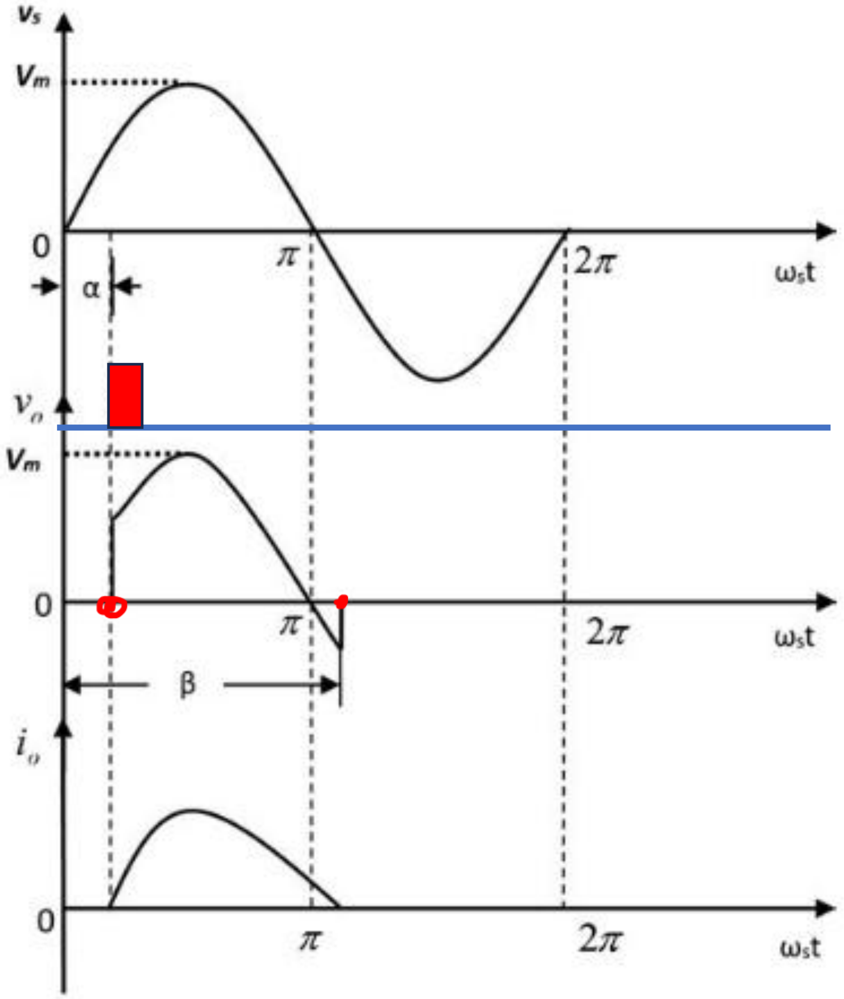
Reference

Half wave : Controlled rectifier with R-L Load

$V_s = L\frac{di_o}{dt} + i_o R$ α to β



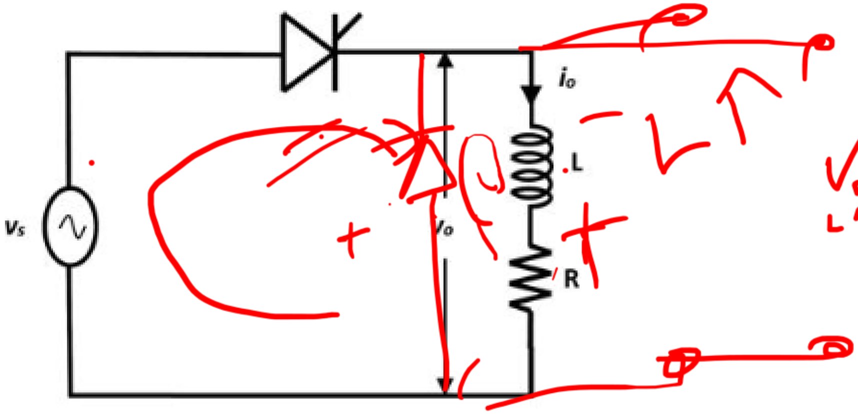
(a)



Reference

Half wave : Controlled rectifier with R-L Load

$i_{out} = i_{in}$ below



(a)

50 Hz

$T = \frac{1}{f} = \frac{1}{50} = 0.02\,s = 20\,ms$

$\alpha = 90° \mid 5\,ms$

$20\,ms \rightarrow 360°$

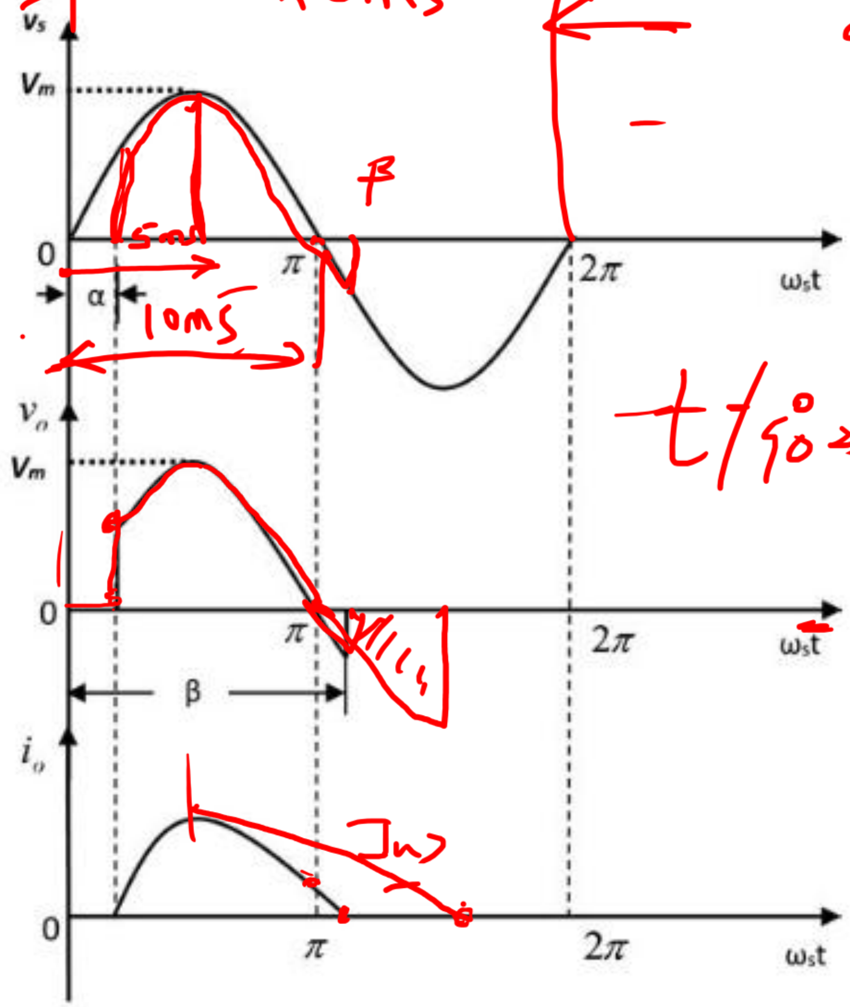
$360° \rightarrow 20\,ms$

$45° \rightarrow \frac{20 \times 45}{360} = 2.5\,ms$

$\alpha = 180° \rightarrow 10\,ms$

$\cancel{\alpha = \frac{20}{360} \times \alpha}$

20ms



$t / 90° \rightarrow \frac{20}{360} \times 90$

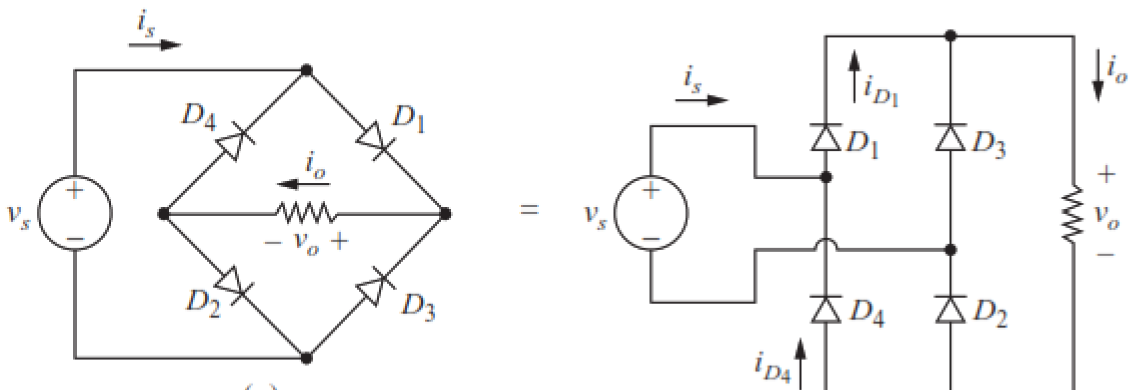
5ms

Reference

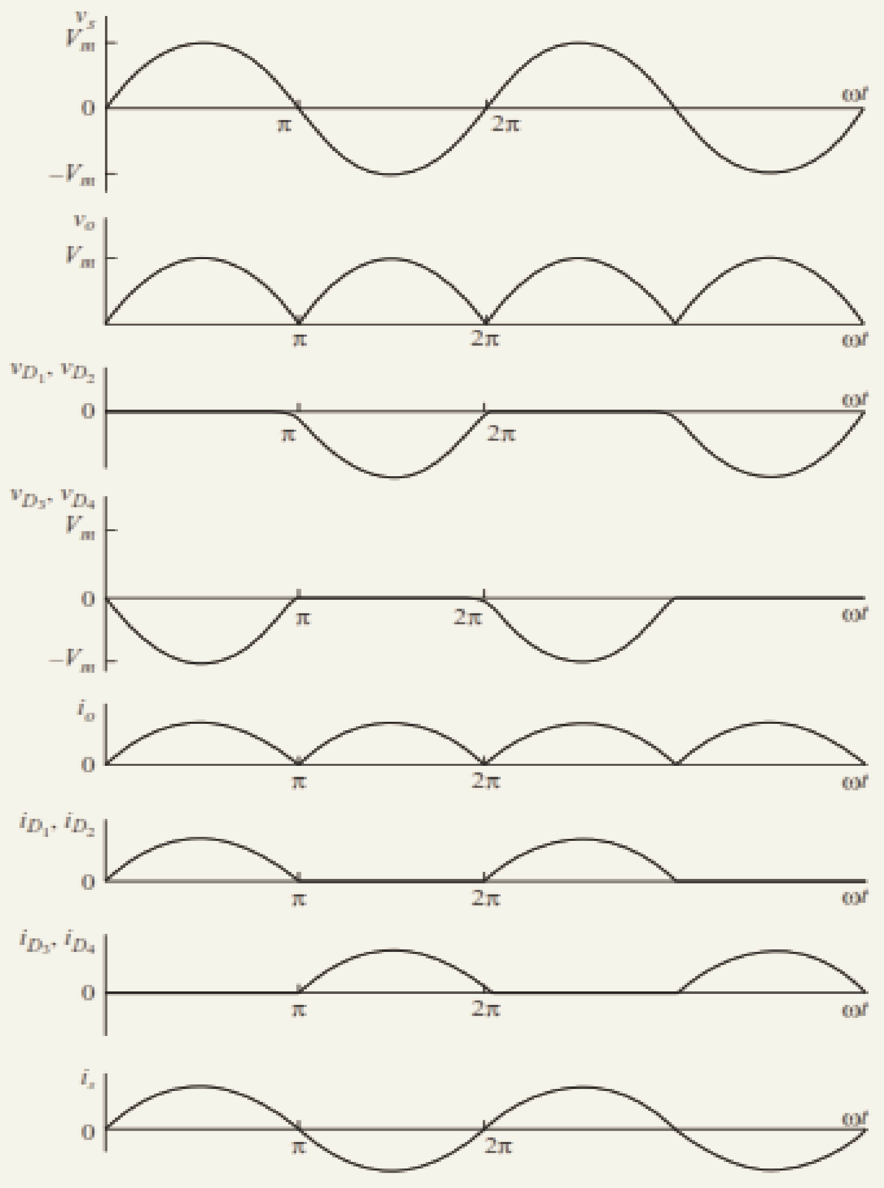
As in the case of a resistive load, the thyristor T becomes forward biased when the supply voltage becomes positive at $\omega t = 0$. However, it does not start conduction until a gate pulse is applied at $\omega t = \alpha$. As the thyristor turns ON at $\omega t = \alpha$ the input voltage appears across the load and the load current starts building up. However, unlike a resistive load, the load current does not become zero at $\omega t = \pi$, instead it continues to flow through the thyristor and the negative supply voltage appears across the load forcing the load current to decrease. Finally, at $\omega t = \beta$ ($\beta > \pi$) the load current becomes zero and the thyristor undergoes reverse recovery. From this point onwards the thyristor starts blocking the supply voltage and the load voltage remains zero until the thyristor is turned ON again in the next cycle. It is to be noted that the value of β depends on the load parameters. Therefore, unlike the resistive load the average and RMS output voltage depends on the load parameters. Since the thyristors does not conduct over the entire input supply cycle this mode of operation is called the "discontinuous conduction mode".

Single phase Full wave Uncontrolled : Diode bridge

Reference

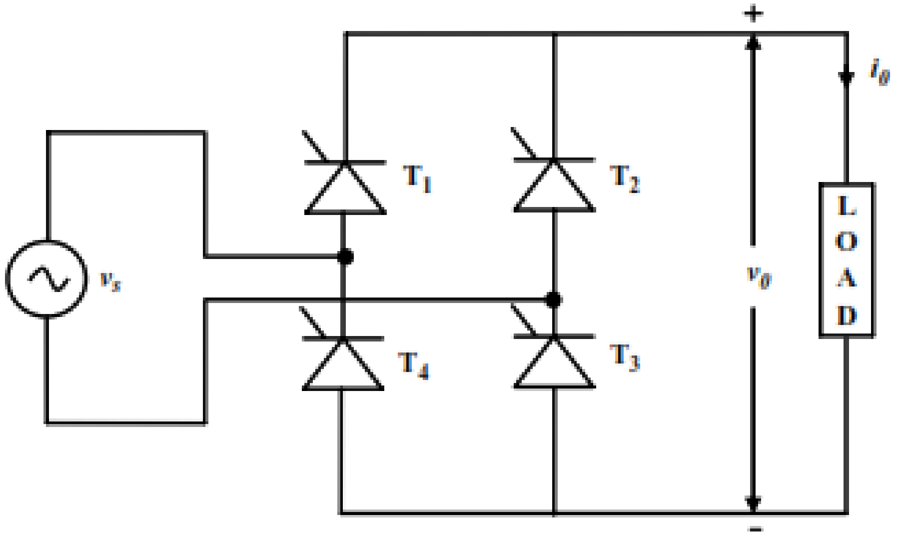


(a)
(b)



(c)

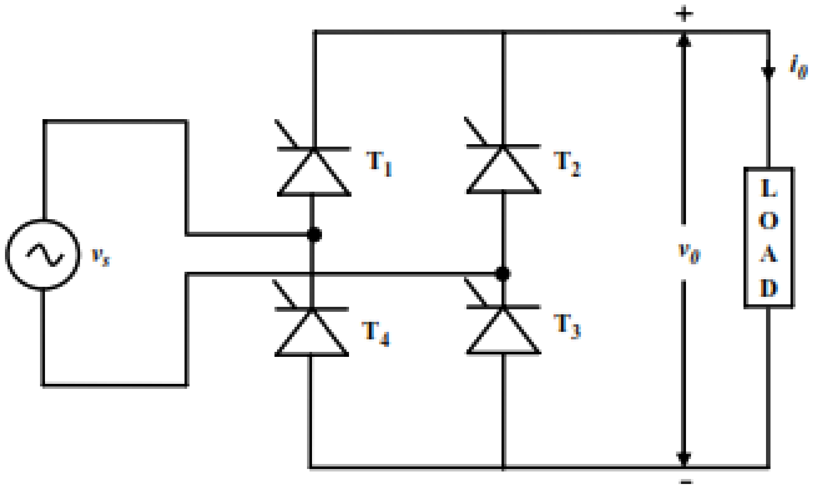
CONTROLLED FULL-WAVE RECTIFIERS

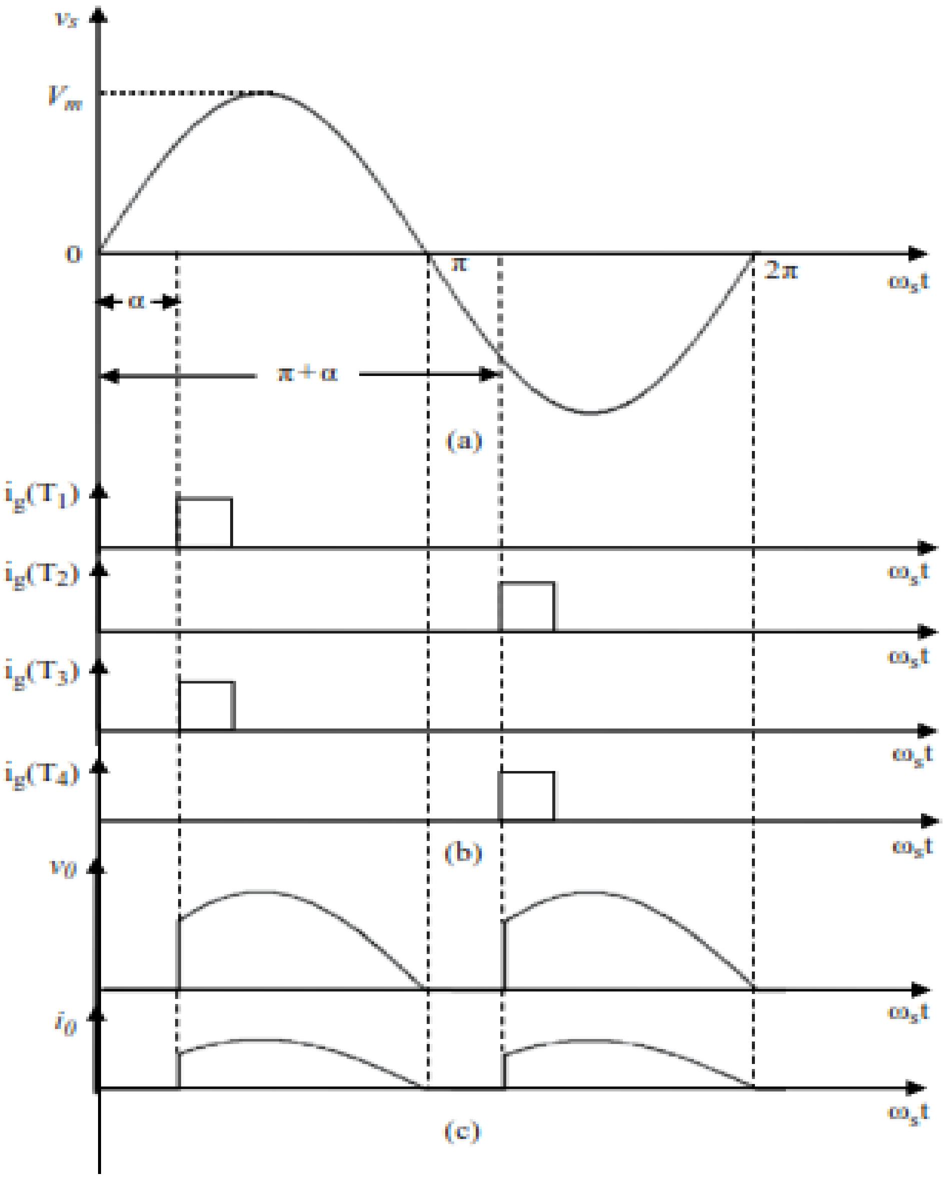


The power circuit of a single-phase full converter is shown in Fig. 3.8. Here, there are four SCRs marked as T_1, T_2, T_3, and T_4. The thyristors are triggered in sequence in such a way that, at any given time, any two thyristors—one from the top and the other from bottom—will be conducting. From the power circuit, it is evident that thyristors T_1 and T_3 will be conducting during the positive half-cycle of the supply voltage; similarly, thyristors T_2 and T_4 will be conducting during the negative half-cycle of the supply voltage. The firing angle α is measured from the zero crossing of the supply voltage; hence T_1 and T_3 are triggered at $\omega_s t = \alpha$, and T_2 and T_4 are triggered at $\omega_s t = \pi + \alpha$. The supply voltage and firing pulses are shown in Fig. 3.9(a) and 3.9(b), respectively. Thus, at $\omega_s t = \alpha$, thyristors T_1 and T_3 are turned ON and the current starts from the source and passes through T_1-load-T_3 back to the supply, and the load voltage is now the supply voltage. Similarly, at $\omega_s t = \pi + \alpha$, T_2 and T_4 are turned ON and the load current flows from the source through T_2-load-T_4. The nature of the load current depends on the characteristics of the load. Three typical loads are discussed below.

CONTROLLED FULL-WAVE RECTIFIERS

R - Load





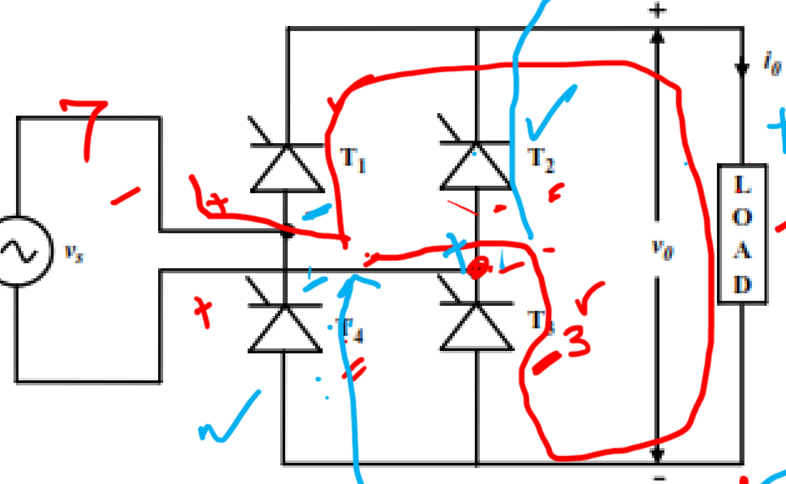
The power circuit of a single-phase full converter is shown in Fig. 3.8. Here, there are four SCRs marked as T_1, T_2, T_3, and T_4. The thyristors are triggered in sequence in such a way that, at any given time, any two thyristors—one from the top and the other from bottom—will be conducting. From the power circuit, it is evident that thyristors T_1 and T_3 will be conducting during the positive half-cycle of the supply voltage; similarly, thyristors T_2 and T_4 will be conducting during the negative half-cycle of the supply voltage. The firing angle α is measured from the zero crossing of the supply voltage; hence T_1 and T_3 are triggered at $\omega_s t = \alpha$, and T_2 and T_4 are triggered at $\omega_s t = \pi + \alpha$. The supply voltage and firing pulses are shown in Fig. 3.9(a) and 3.9(b), respectively. Thus, at $\omega_s t = \alpha$, thyristors T_1 and T_3 are turned ON and the current starts from the source and passes through T_1-load-T_3 back to the supply, and the load voltage is now the supply voltage. Similarly, at $\omega_s t = \pi + \alpha$, T_2 and T_4 are turned ON and the load current flows from the source through T_2-load-T_4. The nature of the load current depends on the characteristics of the load. Three typical loads are discussed below.

CONTROLLED FULL-WAVE RECTIFIERS

$V_{RMS} = \sqrt{\frac{1}{T_0}\int^{T} V^2_{(\omega t)}\, d(\omega t)}$

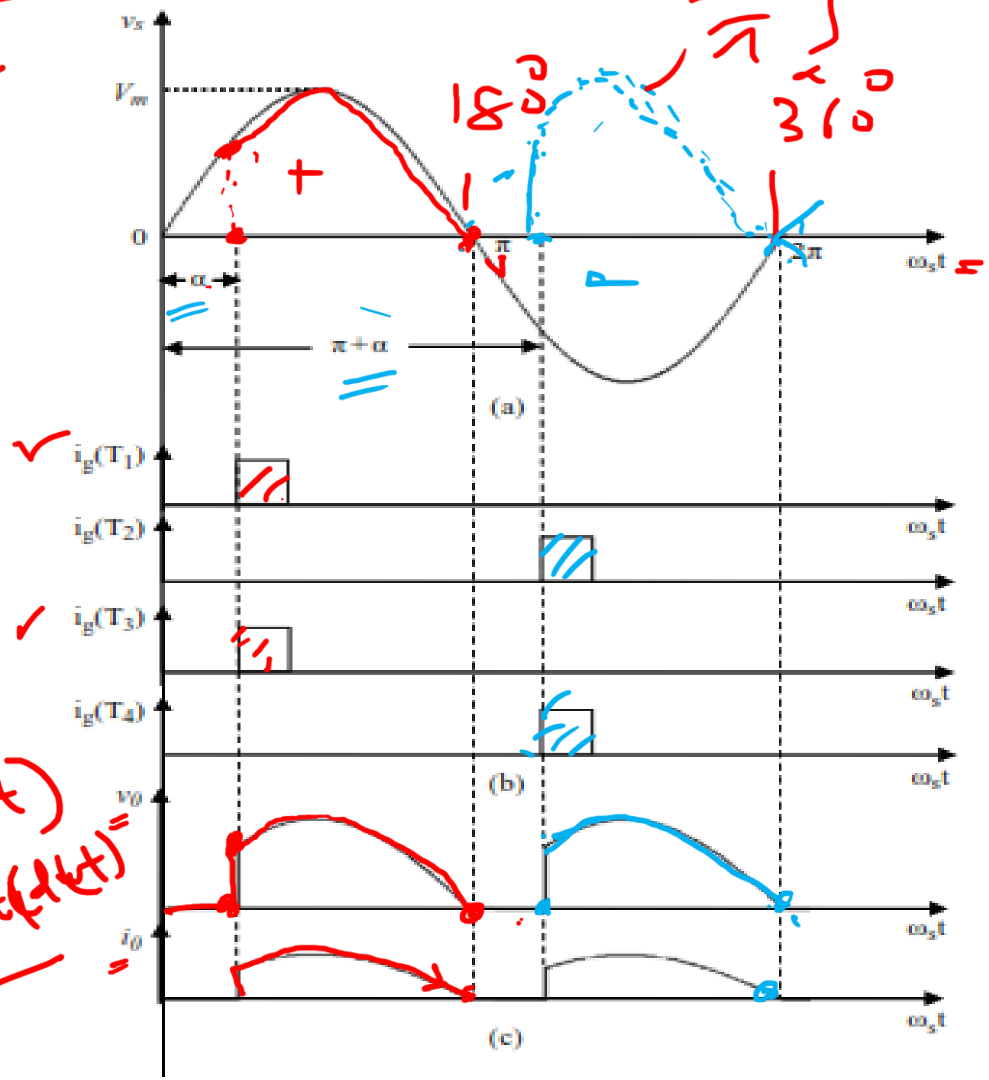
R - Load

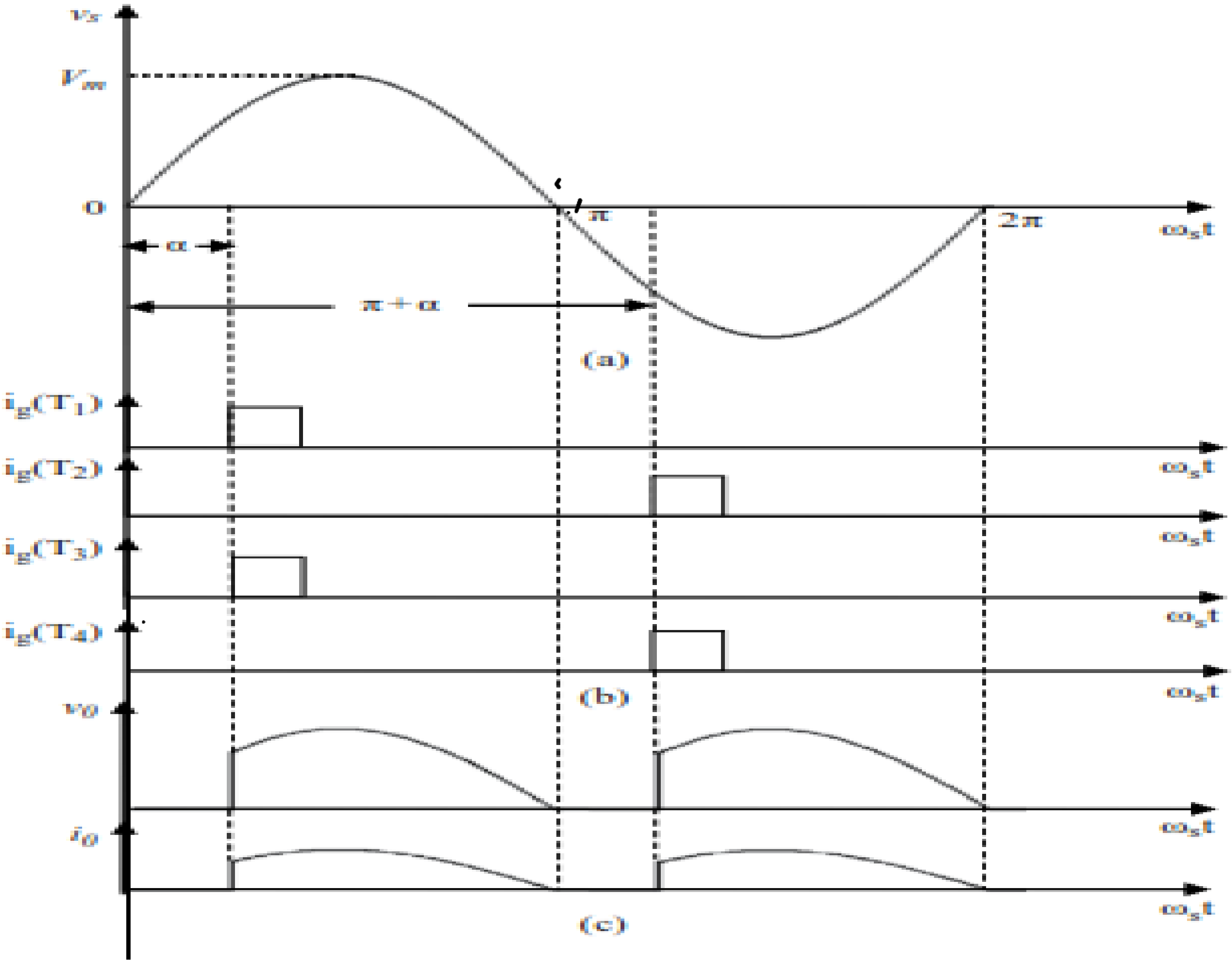
$- I_{o\,avg} = \frac{V_{o\,avg}}{R}$



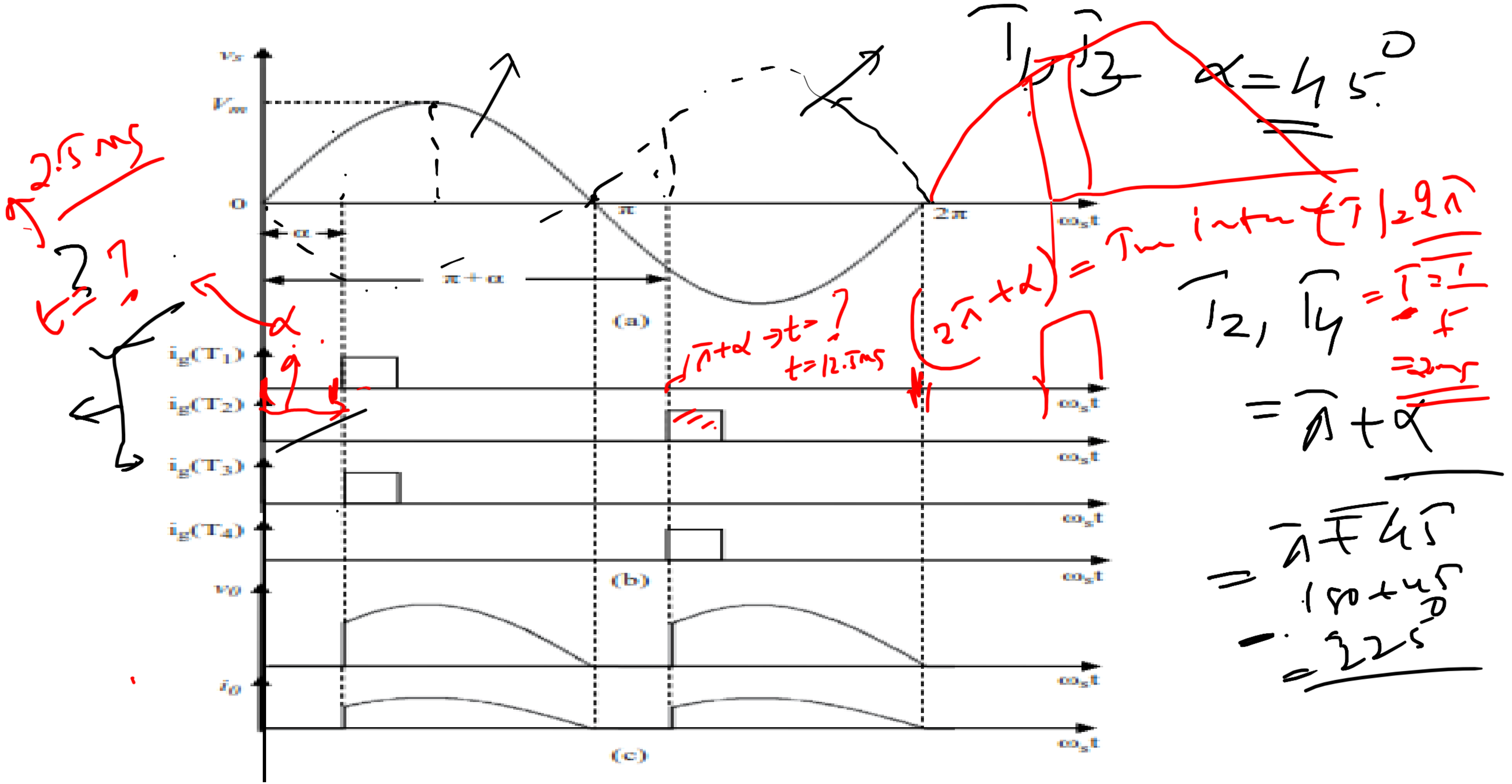
$I = \frac{V}{R}$

$V_{o\,avg} = \frac{1}{T}\int_0^{T} v(\omega t)\, d(\omega t) = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega t\, d(\omega t) =$





v_s
V_m
0
α
$\pi+\alpha$
π
2π
$\omega_s t$
(a)
$i_g(T_1)$
$i_g(T_2)$
$i_g(T_3)$
$i_g(T_4)$
(b)
v_0
i_0
(c)



v_s
V_m
0
π
2π
α
π+α
(a)
$\omega_s t$
$i_g(T_1)$
$i_g(T_2)$
$i_g(T_3)$
$i_g(T_4)$
(b)
v_0
i_0
(c)
2.5 ms
t=?
π+α ⇒ t=?
t=12.5ms
(2π+α) = Turn in to T_1, 2π
T_1, T_2
α=45°
T_2, T_4
T = 1/f = 20ms
= π+α
= π+45
180+45°
= 225°

Calculate the time delay / firing angle

$\xrightarrow{\omega_0 t}$

$(20\,ms \rightarrow 360^\circ)$

$\omega_0 t = 45^\circ$

(rad) $\omega_0 t = \left(45^\circ * \frac{\pi}{180}\right)^c$

$2\pi f * t = \left(45^\circ * \frac{\pi}{180}\right)^c$

$t = \frac{1}{2f} * \left(\frac{\alpha}{180}\right)$

f = 50 Hz

$= \frac{1}{2*50} * \left(\frac{45^\circ}{180}\right)$

$= 2.5\,ms$

$\omega_0 t = 180 + \alpha$

$= 180 + 45^\circ$

$= 225^\circ$

$t = \frac{1}{2*50} * \left(\frac{225^\circ}{180}\right)$

$= 12.5\,ms$

Controlled Full-wave Rectifier

R - Load

At $\omega_s t = \alpha$, T_1 and T_3 start conducting and the load voltage and current are as indicated in Fig. 3.9(c). With a purely resistive load, the load current i_0 is $\frac{v_o}{R}$, and it therefore has a shape similar to that of the output voltage. At $\omega_s t = \pi$, v_o goes to zero and hence i_0 also goes to zero value. Thyristors T_1 and T_3 are turned OFF by natural commutation, and the load is disconnected from the supply leading to zero output voltage. At $\omega_s t = \pi + \alpha$, T_2 and T_4 are triggered, making the output voltage once again positive. At $\omega_s t = 2\pi$, load voltage and current become zero, thereby turning OFF T_2 and T_4 by natural commutation. An expression for the average value of output voltage is derived below.

$$V_{o(av)} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin\omega_s t \, d(\omega_s t)$$

$$V_{o(av)} = \frac{V_m}{\pi}\left[-\cos(\omega_s t)\right]_{\alpha}^{\pi} \tag{3.11}$$

$$V_{o(av)} = \frac{V_m}{\pi}[1+\cos\alpha]$$

Average output current is then

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha) \tag{4-24}$$

The power delivered to the load is a function of the input voltage, the delay angle, and the load components; $P = I_{rms}^2 R$ is used to determine the power in a resistive load, where

$$\begin{aligned} I_{rms} &= \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)} \\ &= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}} \end{aligned} \tag{4-25}$$

The rms current in the source is the same as the rms current in the load.

The full-wave controlled bridge rectifier of Fig. 4-10*a* has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

■ Solution

The average output voltage is determined from Eq. (4-23).

$$V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40°\right) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

Power absorbed by the load is determined from the rms current from Eq. (4-24), remembering to use α in radians.

$$I_{\text{rms}} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{\text{rms}}^2 R = (5.80)^2 (20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

$$S = V_{\text{rms}} I_{\text{rms}} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

firing (α)

The full-wave controlled bridge rectifier of Fig. 4-10a has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

, pf

■ Solution

The average output voltage is determined from Eq. (4-23).

$V_m = V_{peak}$

$V_m = V_{rms} * \sqrt{2}$

$V_m = 120 * \sqrt{2}$

$$V_o = \frac{V_m}{\pi}(1 + \cos\alpha) = \frac{\sqrt{2}\,(120)}{\pi}(1 + \cos 40°) = 95.4 \text{ V}$$

Average load current is

$$I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77 \text{ A}$$

$P = I_{rms}^2 * R$

Power absorbed by the load is determined from the rms current from Eq. (4-24), remembering to use α in radians.

$$I_{rms} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80 \text{ A}$$

$$P = I_{rms}^2 R = (5.80)^2(20) = 673 \text{ W}$$

The rms current in the source is also 5.80 A, and the apparent power of the source is

Source VA

$V_{rms\,source} * I_{rms\,source}$

$= 120 * 5.8$

$$S = V_{rms} I_{rms} = (120)(5.80) = 696 \text{ VA}$$

Power factor is

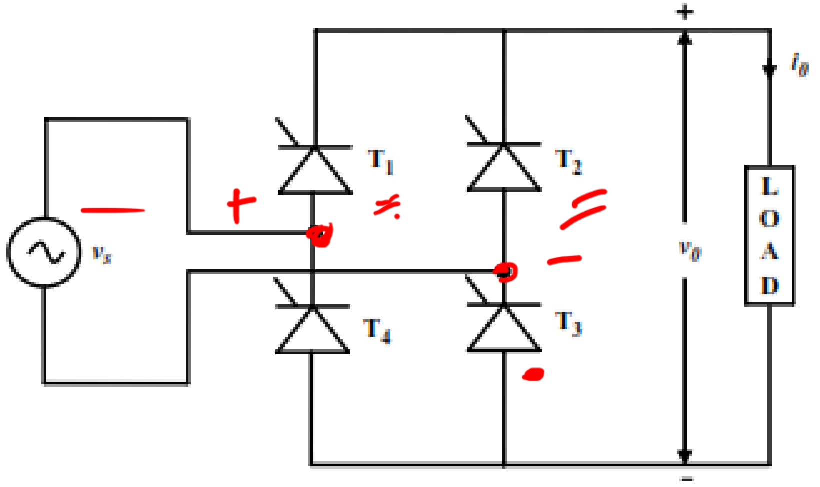
$$\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967$$

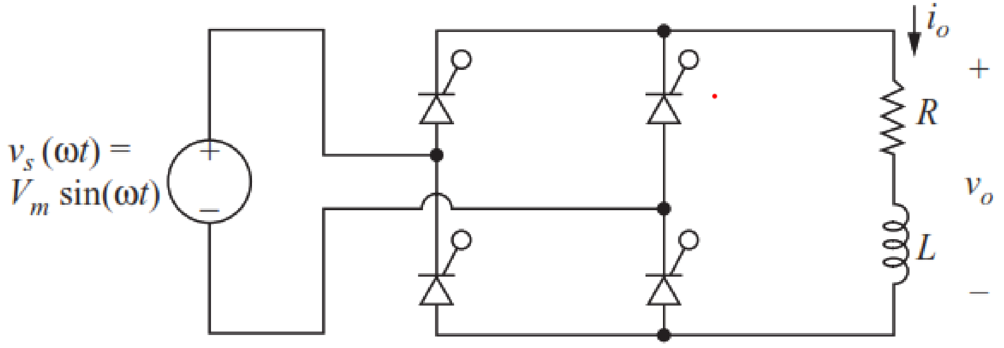
pf = 673 / 696

RL

Single Phase Controlled Full-wave Rectifier

R-L Load





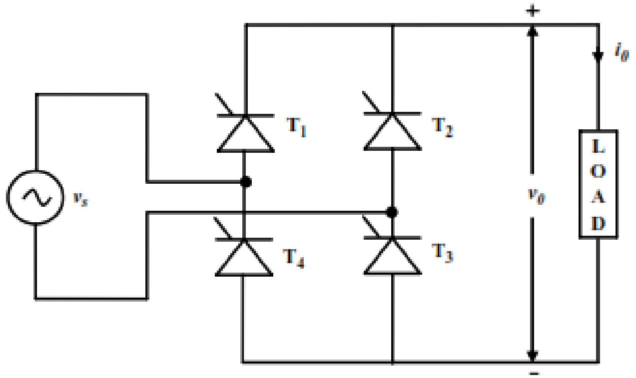
(*a*)

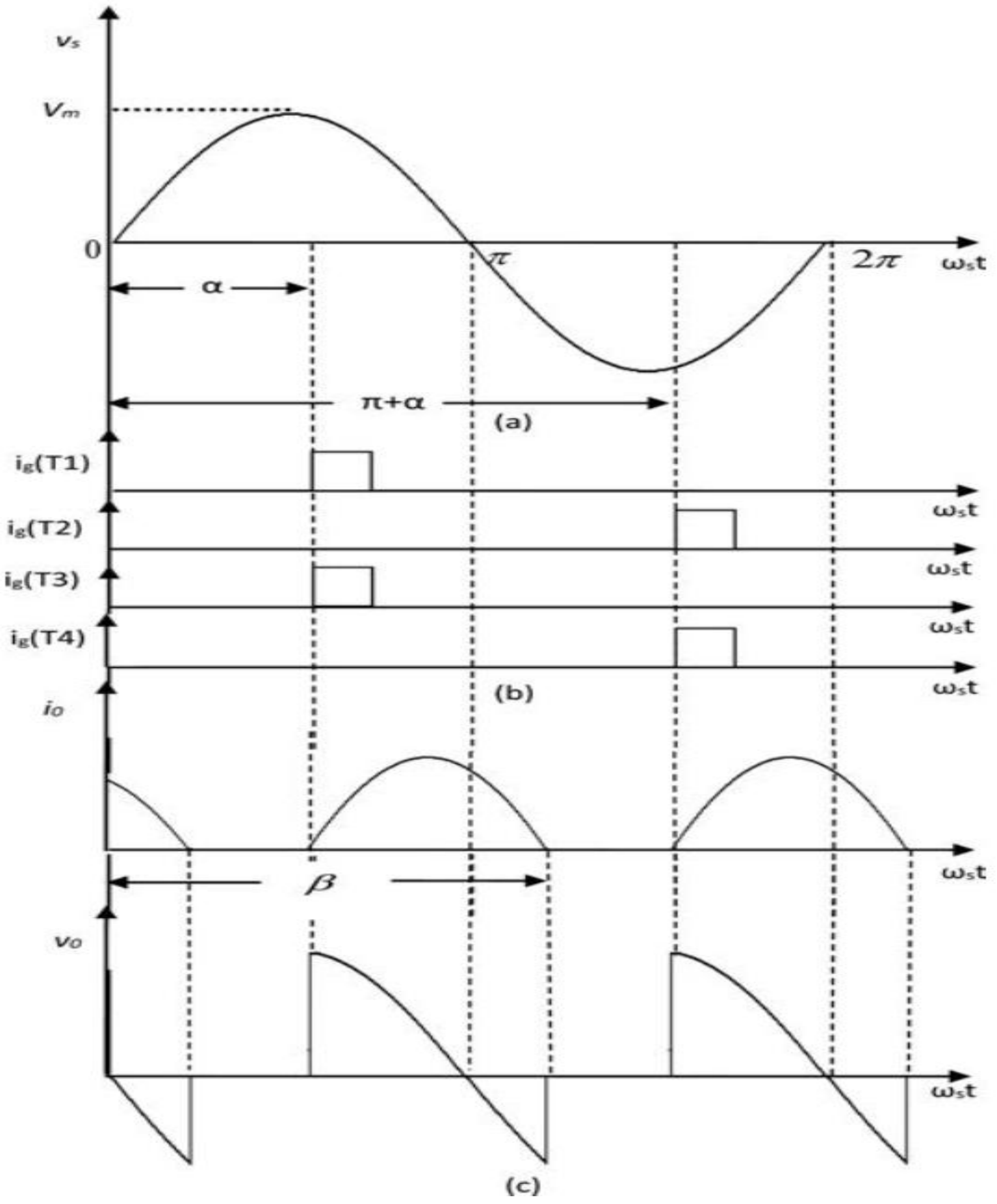
Now consider the operation of the full converter with a series resistance-inductance load. Here the major difference of operation compared to a purely resistance type load is the variation of the load current and the output voltage. The load current may be either continuous or discontinuous depending on the value of the ratio of load inductance to resistance and the firing angle α.

Considering a discontinuous mode of operation, at $\omega_s t = \alpha$, when T_1 and T_3 are triggered, current starts increasing through the load. At $\omega_s t = \pi$, the output voltage goes to zero value, but because it is a lagging power factor load, the load current has not yet become zero. This is indicated in Fig. 3.11(c). Hence, T_1 and T_3 continue to conduct until the load current becomes zero at $\omega_s t = \beta$. At this instant, T_1 and T_3 are turned OFF by natural commutation. During the negative half-cycle of the supply voltage, T_2 and T_4 are triggered at $\omega_s t = \pi + \alpha$ and a similar operation takes place.

Single Phase Controlled Full-wave Rectifier

RL Load, Discontinuous Current





Now consider the operation of the full converter with a series resistance-inductance lo
Here the major difference of operation compared to a purely resistance type load is t
variation of the load current and the output voltage. The load current may be either c
tinuous or discontinuous depending on the value of the ratio of load inductance to res
tance and the firing angle α.

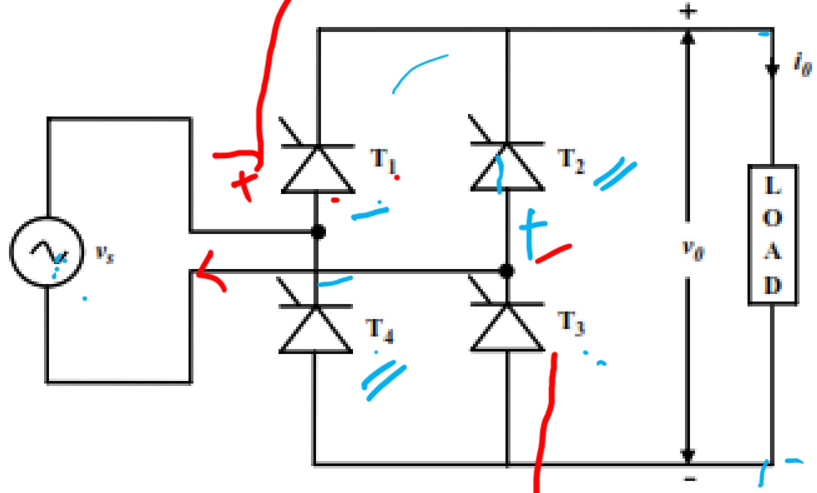
Considering a discontinuous mode of operation, at $\omega_s t = \alpha$, when T_1 and T_3 are trigger
current starts increasing through the load. At $\omega_s t = \pi$, the output voltage goes to zero val
but because it is a lagging power factor load, the load current has not yet become ze
This is indicated in Fig. 3.11(c). Hence, T_1 and T_3 continue to conduct until the load curr
becomes zero at $\omega_s t = \beta$. At this instant, T_1 and T_3 are turned OFF by natural commutati
During the negative half-cycle of the supply voltage, T_2 and T_4 are triggered at $\omega_s t = \pi + \alpha$
and a similar operation takes place.

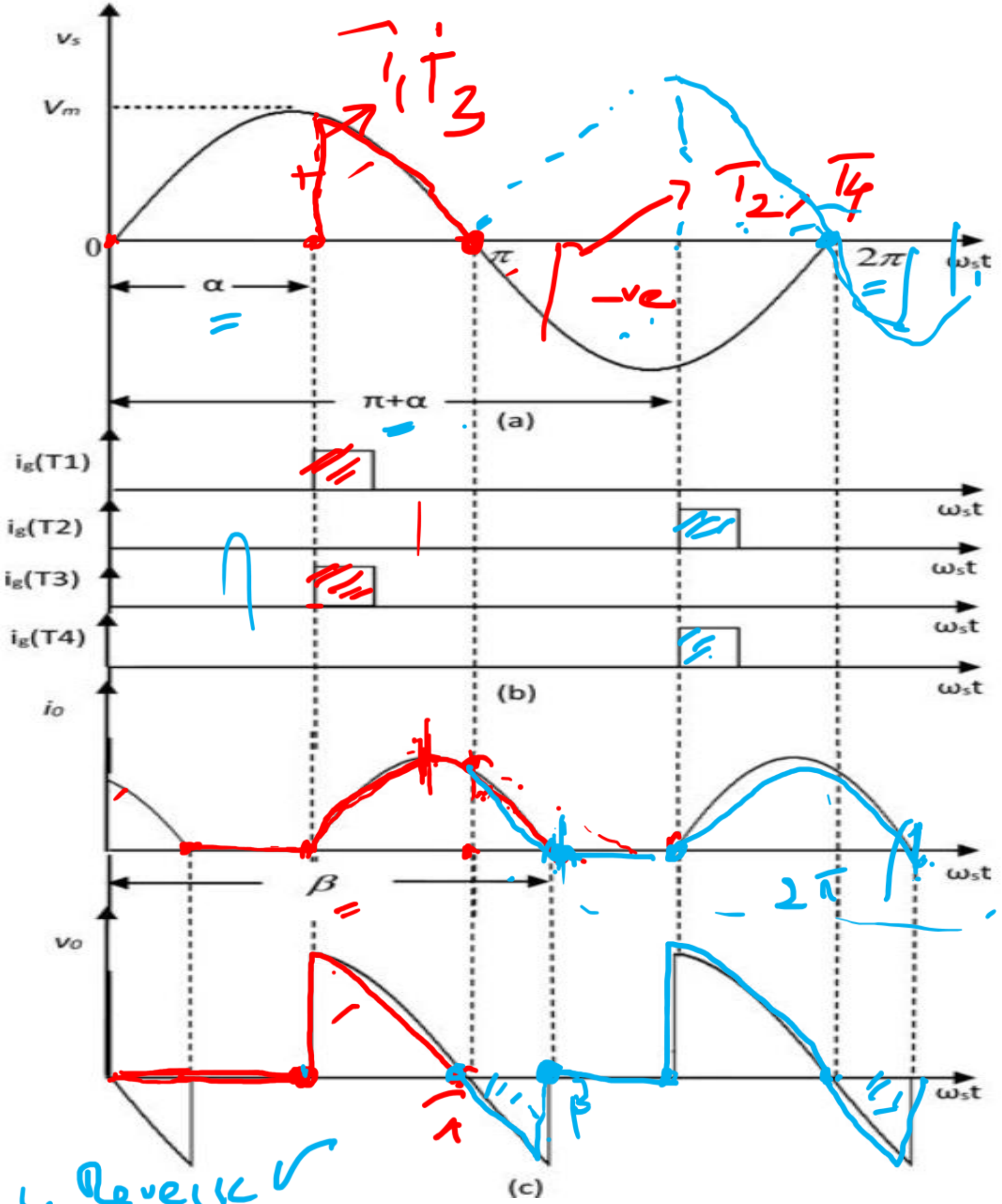
Single Phase Controlled Full-wave Rectifier

Continue ✓

Discontinu

RL Load, Discontinuous Current





Now consider the operation of the full converter with a series resistance-inductance load. Here the major difference of operation compared to a purely resistance type load is the variation of the load current and the output voltage. The load current may be either continuous or discontinuous depending on the value of the ratio of load inductance to resistance and the firing angle α.

Considering a discontinuous mode of operation, at $\omega_s t = \alpha$, when T_1 and T_3 are triggered, current starts increasing through the load. At $\omega_s t = \pi$, the output voltage goes to zero value, but because it is a lagging power factor load, the load current has not yet become zero. This is indicated in Fig. 3.11(c). Hence, T_1 and T_3 continue to conduct until the load current becomes zero at $\omega_s t = \beta$. At this instant, T_1 and T_3 are turned OFF by natural commutation. During the negative half-cycle of the supply voltage, T_2 and T_4 are triggered at $\omega_s t = \pi + \alpha$ and a similar operation takes place.

1. Reverse ✓

2. $I_L \downarrow I_h$ ✗

$V_m = V_{rms}\ m \times \sqrt{2}$
source

$i_{out}(\omega t)$

$\omega t = \beta = i_{out} = 0$

. The average output voltage is computed below:

$$V_{o(av)} = \frac{1}{\pi}\left[\int_{\alpha}^{\beta} V_m \sin(\omega_s t)\, d(\omega_s t)\right]$$

$$V_{o(av)} = \frac{1}{\pi}\left[V_m\left(-\cos(\omega_s t)\right)_{\alpha}^{\beta}\right]$$

$$V_{o(av)} = \frac{1}{\pi}\left[V_m(\cos\alpha - \cos\beta)\right]$$

V_m = peak value of the source pattern

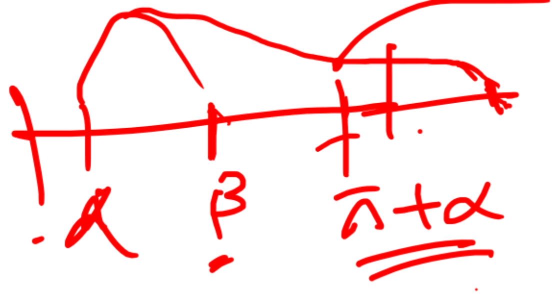
sinusoidal

β!

RL discontinuous

$$I_{o\,avg} = \frac{V_{avg}}{R}$$

$$V_{RMS} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} (V_m \sin\omega t)^2\, d(\omega t)}$$



β > π + α

Reference

α

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

β

where (4-26)

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

π+α

The above current function becomes zero at $\omega t = \beta$. If $\beta < \pi + \alpha$, the current remains at zero until $\omega t = \pi + \alpha$ when gate signals are applied to S_3 and S_4 which are then forward-biased and begin to conduct. This mode of operation is called *discontinuous current*, which is illustrated in Fig. 4-11*b*.

h

$$\beta < \alpha + \pi \quad \rightarrow \quad \text{discontinuous current} \tag{4-27}$$

Analysis of the controlled full-wave rectifier operating in the discontinuous-current mode is identical to that of the controlled half-wave rectifier except that the period for the output current is π rather than 2π rad.

Controlled Full-Wave Rectifier, Discontinuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, $R = 10\ \Omega$, $L = 20$ mH, and $\alpha = 60°$. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

■ Solution

From the parameters given,

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

(*a*) Substituting into Eq. (4-26),

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \text{ A} \qquad \text{for } \alpha \le \omega t \le \beta$$

Solving $i_o(\beta) = 0$ numerically for β, $\beta = 3.78$ rad (216°). Since $\pi + \alpha = 4.19 > \beta$, the current is discontinuous, and the above expression for current is valid.

(*b*) Average load current is determined from the numerical integration of

$$I_o = \frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t) = 7.05 \text{ A}$$

(*c*) Power absorbed by the load occurs in the resistor and is computed from $I_{\text{rms}}^2 R$, where

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t)} = 8.35 \text{ A}$$

$$P = (8.35)^2(10) = 697 \text{ W}$$

Reference

Controlled Full-Wave Rectifier, Discontinuous Current

A controlled full-wave bridge rectifier of Fig. 4-11*a* has a source of 120 V rms at 60 Hz, $R = 10\ \Omega$, $L = 20$ mH, and $\alpha = 60°$. Determine (*a*) an expression for load current, (*b*) the average load current, and (*c*) the power absorbed by the load.

■ **Solution**

From the parameters given,

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

(*a*) Substituting into Eq. (4-26),

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \text{ A} \qquad \text{for } \alpha \le \omega t \le \beta$$

Solving $i_o(\beta) = 0$ numerically for β, $\beta = 3.78$ rad (216°). Since $\pi + \alpha = 4.19 > \beta$, the current is discontinuous, and the above expression for current is valid.

(*b*) Average load current is determined from the numerical integration of

$$I_o = \frac{1}{\pi}\int_\alpha^\beta i_o(\omega t)\, d(\omega t) = 7.05 \text{ A}$$

(*c*) Power absorbed by the load occurs in the resistor and is computed from $I_{rms}^2 R$, where

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_\alpha^\beta i_o^2(\omega t)\, d(\omega t)} = 8.35 \text{ A}$$

$$P = (8.35)^2(10) = 697 \text{ W}$$

1Φ

Sinusoidal

V_{source}

$V_m = V_{rms\ source} * \sqrt{2} = 120 * \sqrt{2}$

$\omega = 2\pi f$

$\alpha < \omega t < \beta$

KVL

$i_{ss} + i_{transient}$

$V_{source} = 0$

$+ K \cdot e^{-\frac{R}{L} t}$

$\omega t = \beta \quad i = 0$

$\beta < \pi + \alpha$ discont

$I_{avg} = \frac{V_{avg}}{R}$

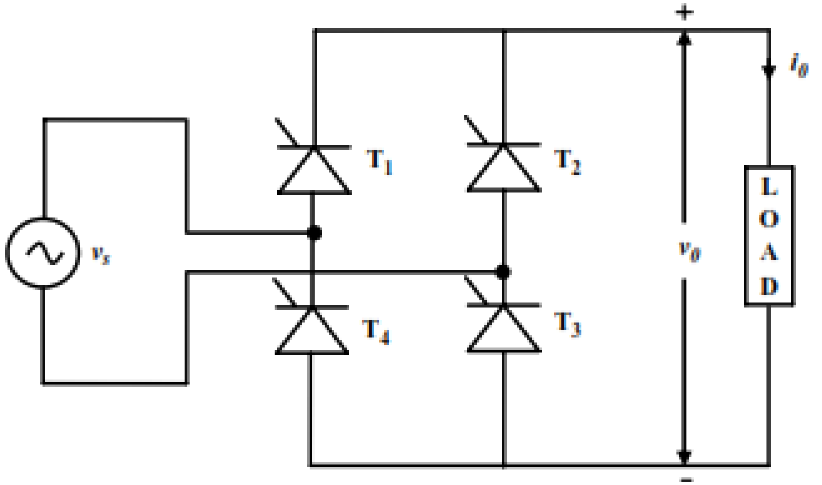
output nonsinusoidal

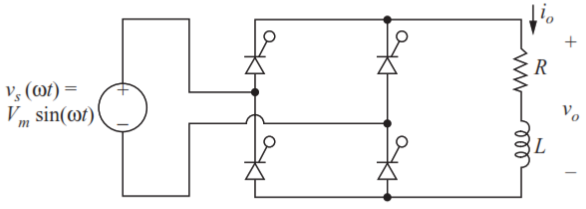
$I_{avg} = I_{dc}$

I_{rms}

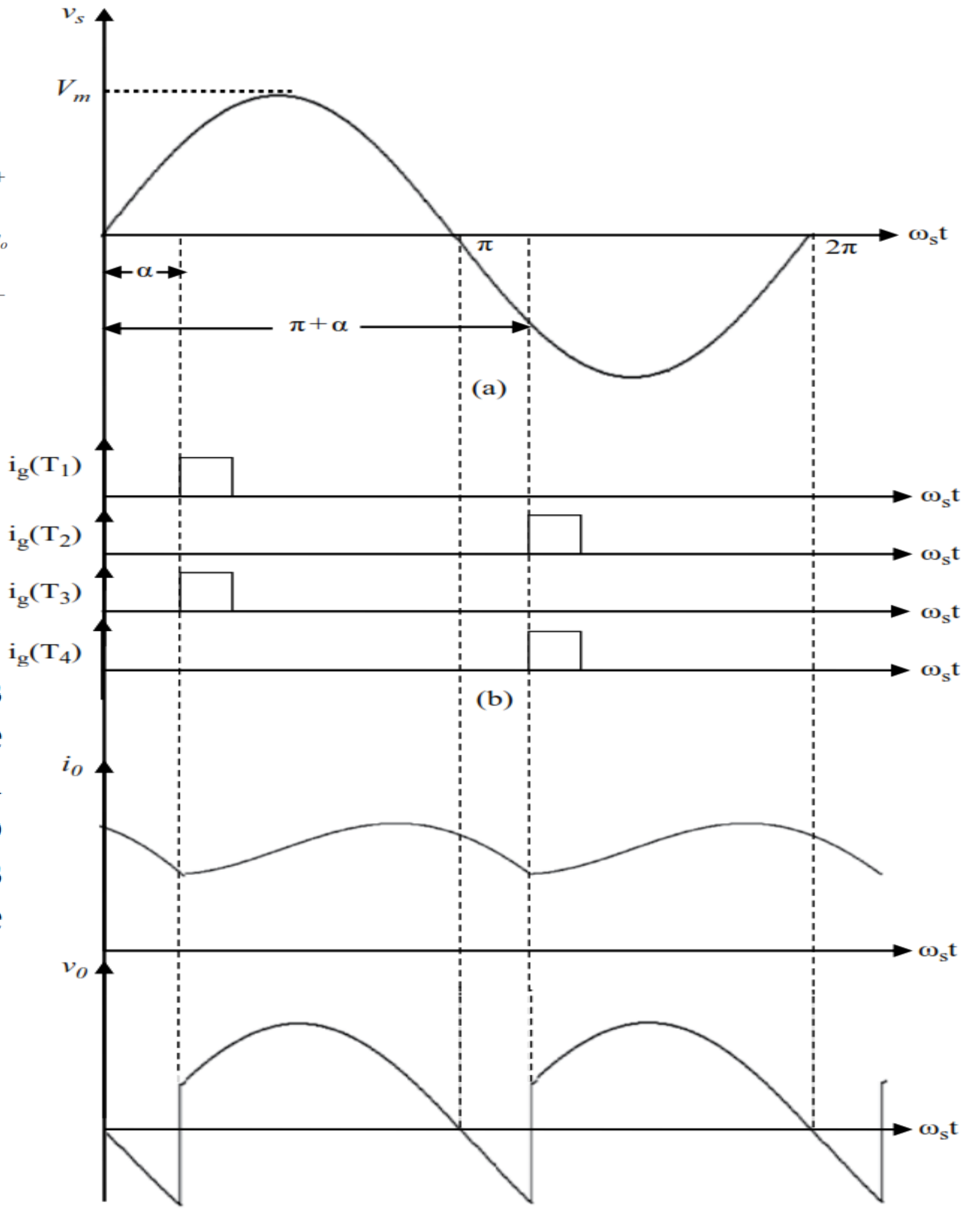
Single Phase Controlled Full-wave Rectifier

RL Load, Continuous Current





(a)

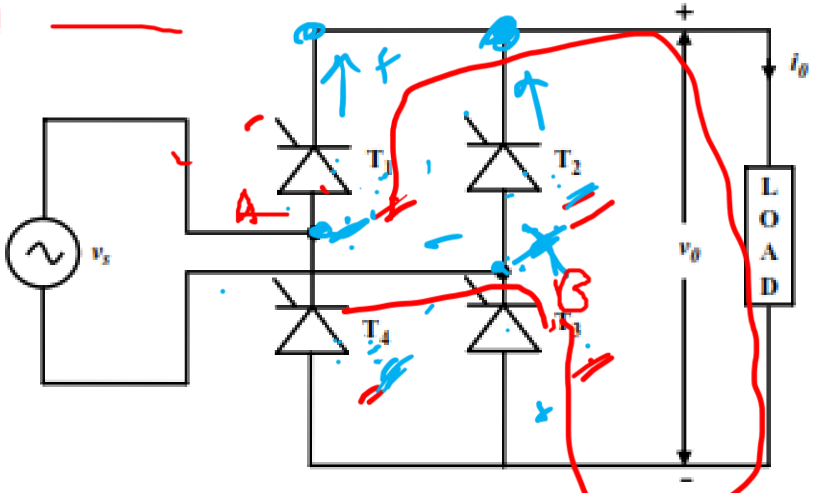


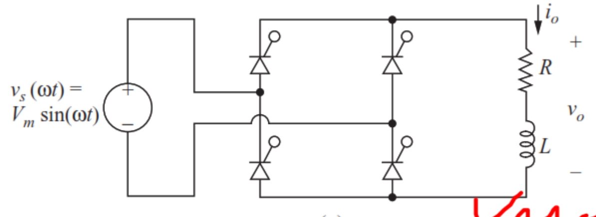
(c)

For reduced values of α and increased inductance to resistance ratios, the load current is continuous as shown in Fig. 3.12(c). Here T_1 and T_3 are turned ON at $\omega_s t = \alpha$ and carry the load current. At $\omega_s t = \pi + \alpha$, when T_2 is triggered, the supply voltage appears across T_1 and reverse-biases it. T_1 is turned OFF instantaneously, and the load current is transferred to T_2. Here, T_1 is turned OFF with the use of the line voltage, and such a turning OFF process is called line commutation. Similarly, when T_4 is triggered, T_3 is also turned OFF by line commutation and the load current is transferred to T_4.

Single Phase Controlled Full-wave Rectifier

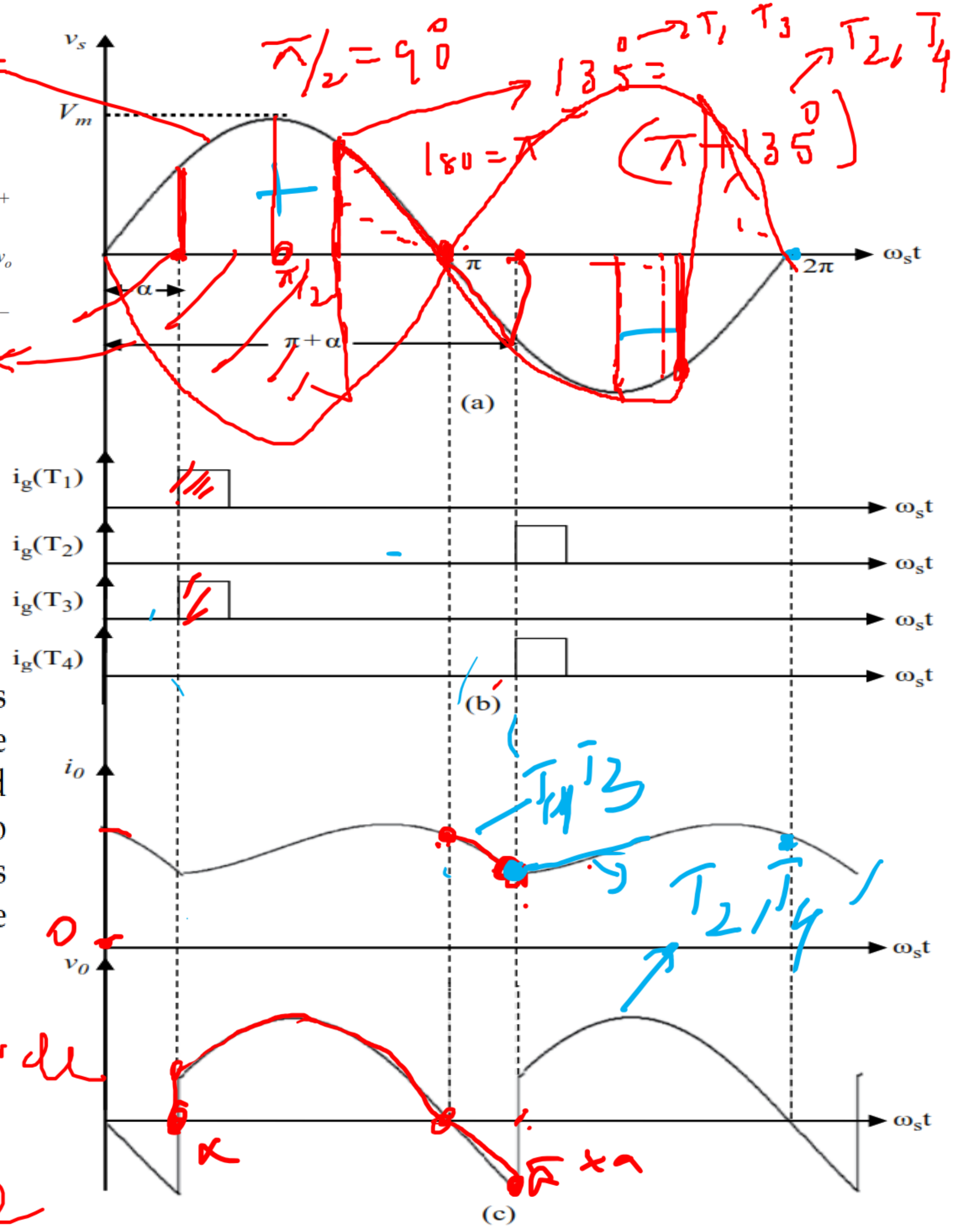
RL Load, Continuous Current





(a)

Reverse biase



For reduced values of α and increased inductance to resistance ratios, the load current is continuous as shown in Fig. 3.12(c). Here T_1 and T_3 are turned ON at $\omega_s t = \alpha$ and carry the load current. At $\omega_s t = \pi + \alpha$, when T_2 is triggered, the supply voltage appears across T_1 and reverse-biases it. T_1 is turned OFF instantaneously, and the load current is transferred to T_2. Here, T_1 is turned OFF with the use of the line voltage, and such a turning OFF process is called line commutation. Similarly, when T_4 is triggered, T_3 is also turned OFF by line commutation and the load current is transferred to T_4.

P = VI/2

V → -ve

I → +ve

α > π/2 inverter mode

α > π/2

$V_{in\,RMS} = V_{source\,RMS}$

$V_{o\,RMS} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m^2 \sin^2(\omega t) \, d\omega t}$

The expressions for average output voltage is derived below:

$$V_{o(av)} = \frac{1}{\pi}\left[\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega_s t) d\omega_s t\right]$$

$$V_{o(av)} = \frac{1}{\pi}\left[V_m \left(-\cos(\omega_s t)\right)_{\alpha}^{\pi+\alpha}\right]$$

$$V_{o(av)} = \frac{1}{\pi}\left[V_m \left(-\cos(\pi+\alpha)+\cos\alpha\right)\right]$$

$$V_{o(av)} = \frac{2V_m}{\pi}\cos\alpha$$

$V_{out\,RMS} =$ → formula

I_{RMS} not possible

generate analysis

$I_{avg} = \frac{V_{o\,avg}}{R}$