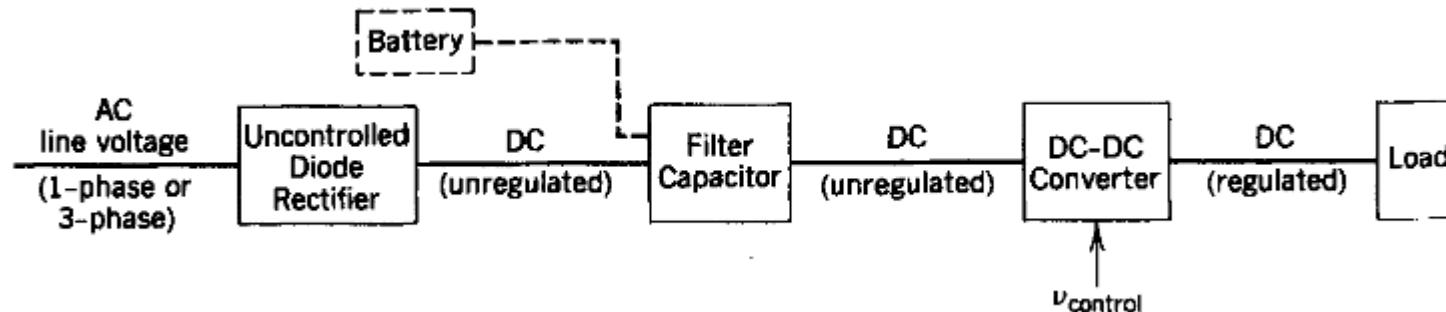


DC- DC SWITCH-MODE CONVERTERS

dc-dc converters/Choppers

- The dc-dc converters are widely used in regulated switch-mode dc power supplies and in dc motor drive applications.
- As shown in Fig. , often the input to these converters is an unregulated dc voltage, which is obtained by rectifying the line voltage, and therefore it will fluctuate due to changes in the line-voltage magnitude.
- Switch-mode dc-to-dc converters are used to convert the unregulated dc input into a controlled dc output at a desired voltage level.



LINEAR VOLTAGE REGULATORS

Before we discuss switched-mode converters, it is useful to review the motivation for an alternative to linear dc-dc converters that was introduced in Chapt. 1. One method of converting a dc voltage to a lower dc voltage is a simple circuit as shown in Fig. 6-1. The output voltage is

$$V_o = I_L R_L$$

where the load current is controlled by the transistor. By adjusting the transistor base current, the output voltage may be controlled over a range of 0 to roughly V_s . The base current can be adjusted to compensate for variations in the supply voltage or the load, thus regulating the output. This type of circuit is called a linear dc-dc converter or a linear regulator because the transistor operates in the linear region, rather than in the saturation or cutoff regions. The transistor in effect operates as a variable resistance.

While this may be a simple way of converting a dc supply voltage to a lower dc voltage and regulating the output, the low efficiency of this circuit is a serious drawback for power applications. The power absorbed by the load is $V_o I_L$, and

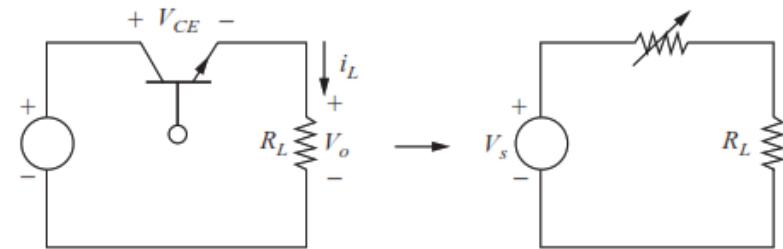
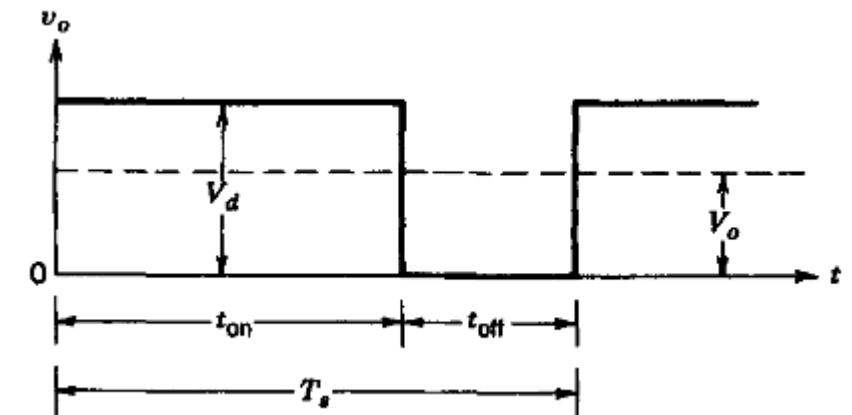
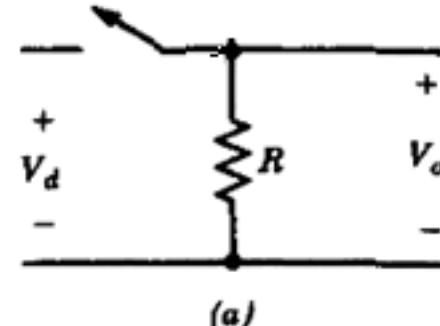


Figure 6-1 A basic linear regulator.

the power absorbed by the transistor is $V_{CE} I_L$, assuming a small base current. The power loss in the transistor makes this circuit inefficient. For example, if the output voltage is one-quarter of the input voltage, the load resistor absorbs one-quarter of the source power, which is an efficiency of 25 percent. The transistor absorbs the other 75 percent of the power supplied by the source. Lower output voltages result in even lower efficiencies. Therefore, the linear voltage regulator is suitable only for low-power applications.

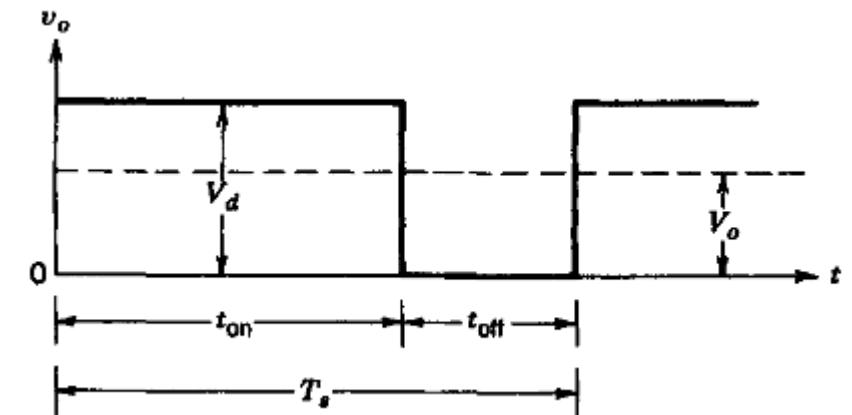
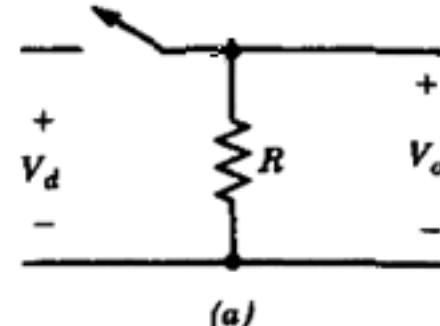
CONTROL OF dc-dc CONVERTERS

- In dc-dc converters, the average dc output voltage must be controlled to a desired level, though the input voltage and the output load may fluctuate.
- Switch-mode dc-dc converters utilize one or more switches to transform dc from one level to another.
- In a dc-dc converter with a given input voltage, the average output voltage is controlled by controlling the switch on and off durations (t_{on} and t_{off})
- To illustrate the switch-mode conversion concept, consider a basic dc-dc converter shown in Fig..
- The average value V_d of the output voltage v_o , in Fig. 7-26 depends on t_{on} and t_{off} .
- One of the methods for controlling the output voltage employs switching at a constant frequency (hence, a constant switching time period $T_s = t_{on} + t_{off}$) and adjusting the on duration of the switch to control the average output voltage.
- In this method, called pulse-width modulation (PWM) switching,
- The switch **duty ratio D**, which is defined as the ratio of the on duration to the switching time period, is varied.



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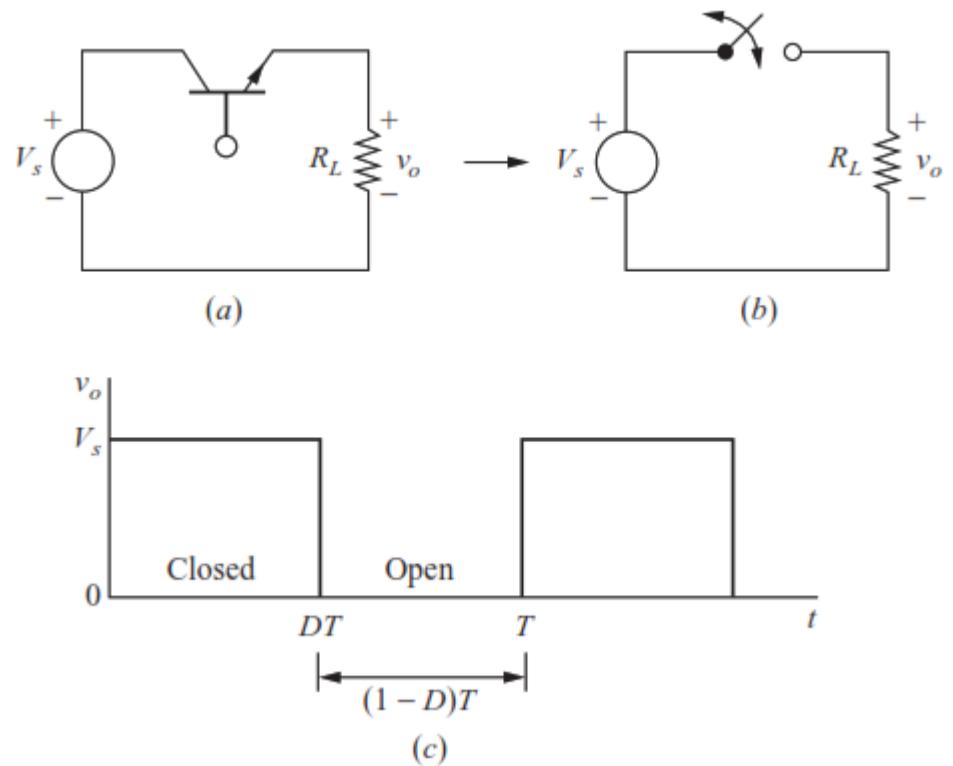


The average or dc component of the output voltage is

$$V_o = \frac{1}{T} \int_0^T v_o(t) dt = \frac{1}{T} \int_0^{DT} V_s dt = V_s D$$

The dc component of the output voltage is controlled by adjusting the duty ratio D , which is the fraction of the switching period that the switch is closed

$$D \equiv \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} = \frac{t_{\text{on}}}{T} = t_{\text{on}} f$$



CONTROL OF dc-dc CONVERTERS

The duty cycle D can be varied from 0 to 1 by varying t_1 , T , or f . Therefore, the output voltage V_o can be varied from 0 to V_s by controlling D , and the power flow can be controlled.

1. *Constant-frequency operation*: The converter, or switching, frequency f (or chopping period T) is kept constant and the on-time t_1 is varied. The width of the pulse is varied and this type of control is known as *pulse-width-modulation* (PWM) control.
2. *Variable-frequency operation*: The chopping, or switching, frequency f is varied. Either on-time t_1 or off-time t_2 is kept constant. This is called *frequency modulation*. The frequency has to be varied over a wide range to obtain the full output voltage range. This type of control would generate harmonics at unpredictable frequencies and the filter design would be difficult.

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CONTROL OF dc-dc CONVERTERS

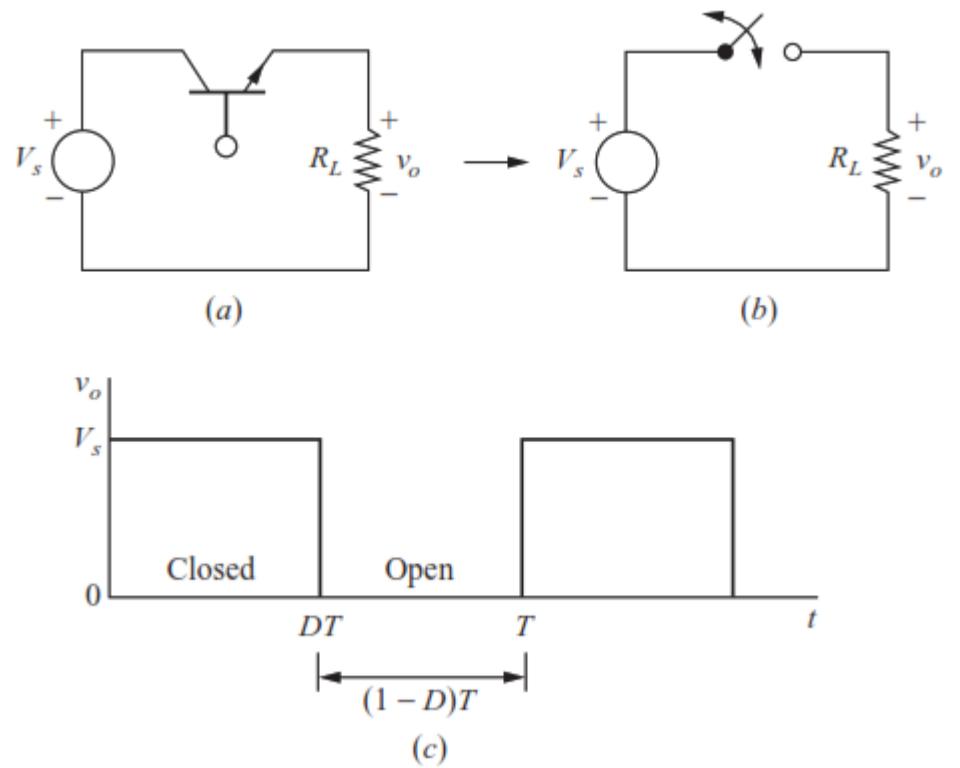
- The other control method is more general, where both the switching frequency (and hence the time period) and the on duration of the switch are varied.
- This method is used only in dc-dc converters utilizing forced-commutated thyristors
- ***Variation in the switching frequency makes it difficult to filter the ripple components in the input and the output waveforms of the converter.***

The average or dc component of the output voltage is

$$V_o = \frac{1}{T} \int_0^T v_o(t) dt = \frac{1}{T} \int_0^{DT} V_s dt = V_s D$$

The dc component of the output voltage is controlled by adjusting the duty ratio D , which is the fraction of the switching period that the switch is closed

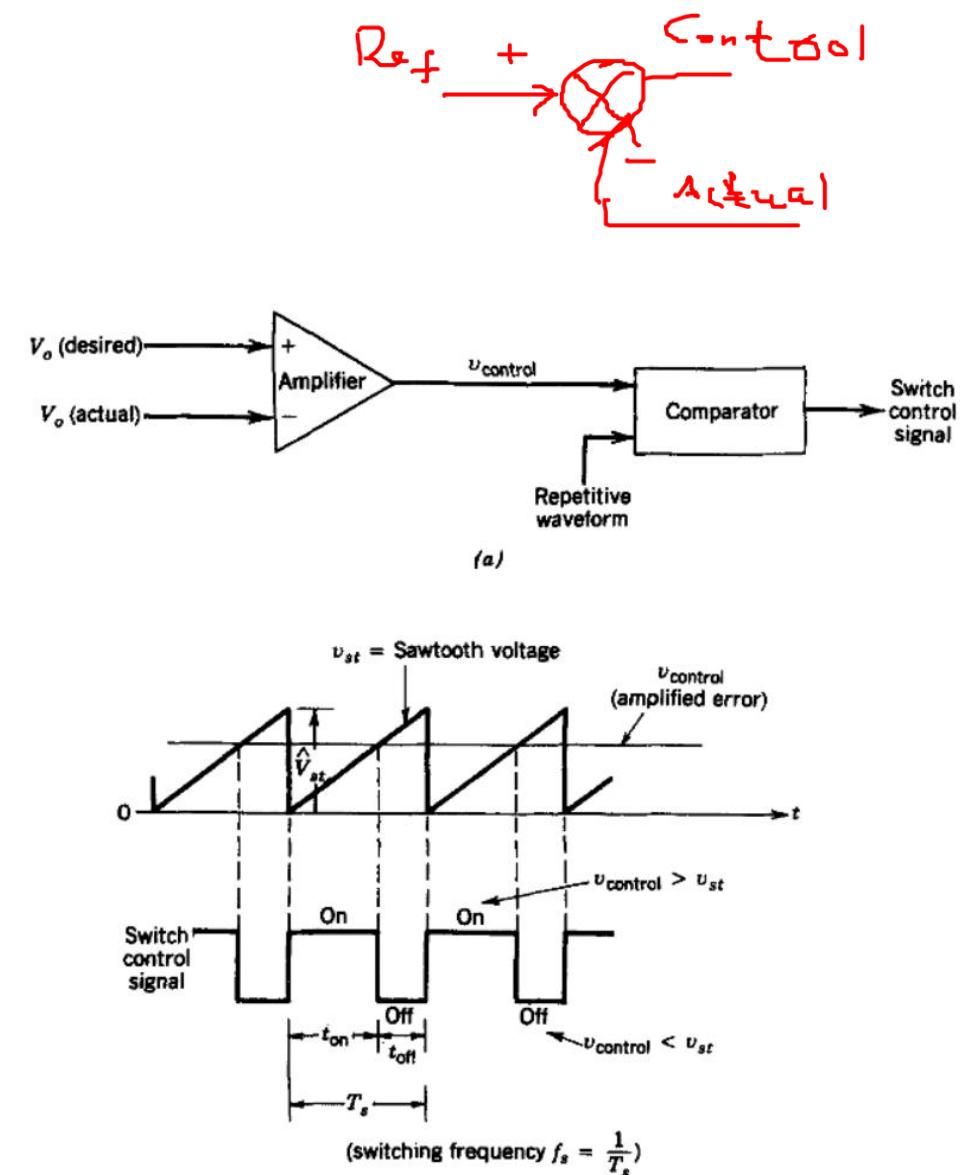
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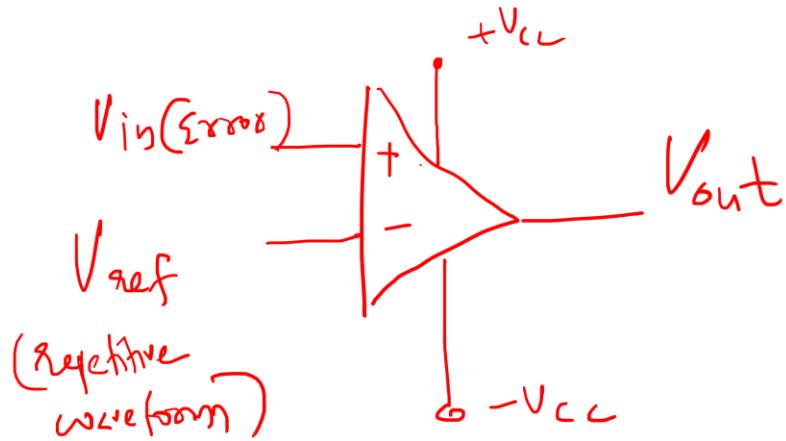


Pulse-width modulator:

- In the PWM switching at a constant switching frequency, the switch control signal, which controls the state (on or off) of the switch, is generated by comparing a signal-level control voltage v_{con} , with a repetitive waveform as shown in Figs. a and b.
- The control voltage signal generally is obtained by amplifying the error, or the difference between the actual output voltage and its desired value.
- The frequency of the repetitive waveform with a constant peak, which is shown to be a sawtooth, establishes the switching frequency.
- This frequency is kept constant in a PWM control and is chosen to be in a few kilohertz to a few hundred kilohertz range.
- When the amplified error signal, which varies very slowly with time relative to the switching frequency, is greater than the sawtooth waveform, the switch control signal becomes high, causing the switch to turn on.
 - Otherwise, the switch is off.
- In terms of v_{control} and peak voltage of sawtooth, and the peak of the sawtooth waveform v_{st} in Fig. the switch duty ratio can be expressed as

$$D = \frac{t_{\text{on}}}{T_s} = \frac{v_{\text{control}}}{\hat{V}_{st}}$$





(0.5 V comp)

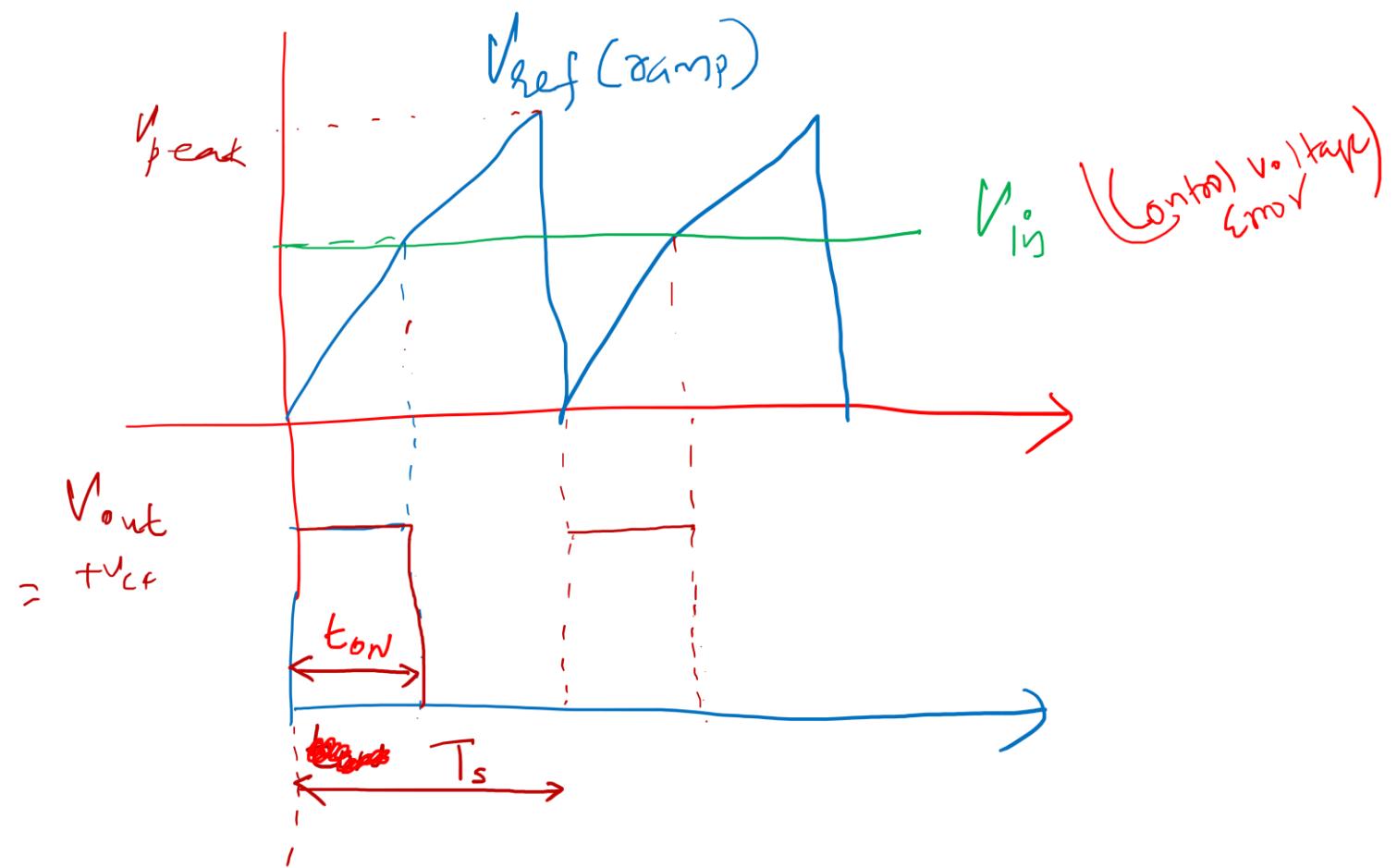
$$\frac{V_{in}}{V_{peak}} = \frac{t_{on}}{T_s} = \frac{D T_s}{T_s}$$

$$D = \frac{t_{on}}{T_s} = \frac{V_{in}}{V_{peak}}$$

$= \frac{V_{control}}{V_{peak}}$

$$V_{in} > V_{ref} \Rightarrow V_{out} = +V_{cc}$$

$$V_{in} < V_{ref} \Rightarrow V_{out} = -V_{cc}$$

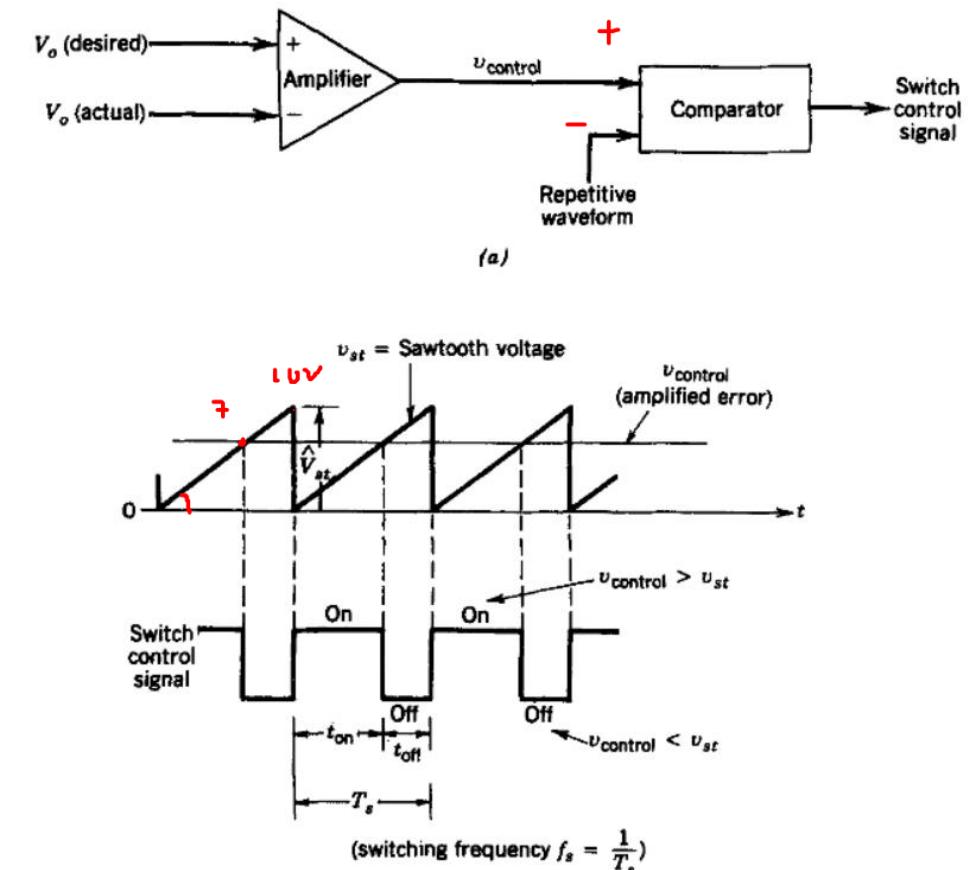


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$\tan \theta =$



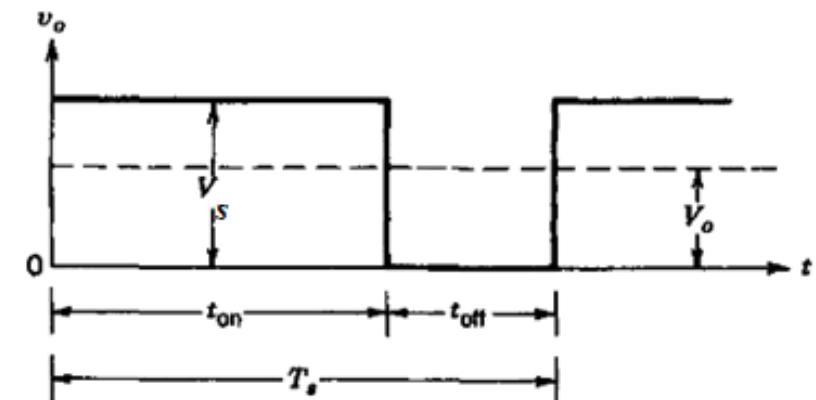
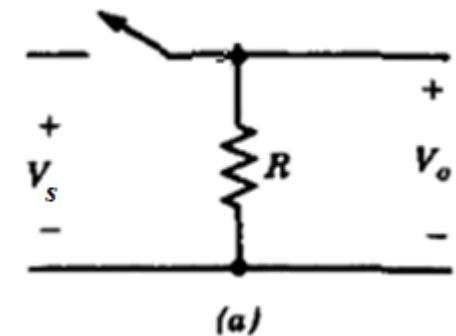
STEP-DOWN (BUCK) CONVERTER

- As the name implies, a step-down converter produces a lower average output voltage than the dc input voltage v_d . Its main application is in regulated dc power supplies and dc motor speed control.
- Conceptually, the basic circuit of Fig. a constitutes a step-down converter for a purely resistive load.
- Assuming an ideal switch, a constant instantaneous input voltage v_d , and a purely resistive load, the instantaneous output voltage waveform is shown in Fig.b as a function of the switch position.
- The average output voltage can be calculated in terms of the switch duty ratio

$$V_o = \frac{1}{T_s} \int_0^{T_s} v_o(t) dt = \frac{1}{T_s} \left(\int_0^{t_{on}} V_s dt + \int_{t_{on}}^{T_s} 0 dt \right) = \frac{t_{on}}{T_s} V_s = D V_s$$

$$V_o = \frac{V_d}{\hat{V}_{st}} v_{control} = k v_{control}$$

$$k = \frac{V_d}{\hat{V}_{st}} = \text{constant}$$

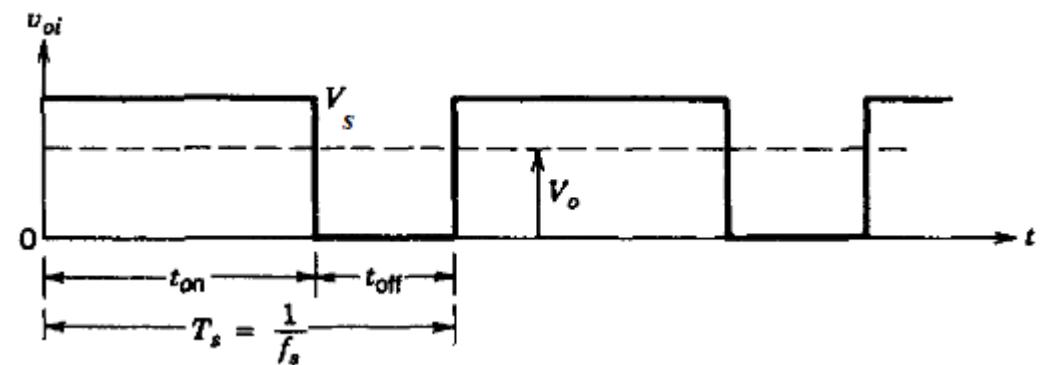
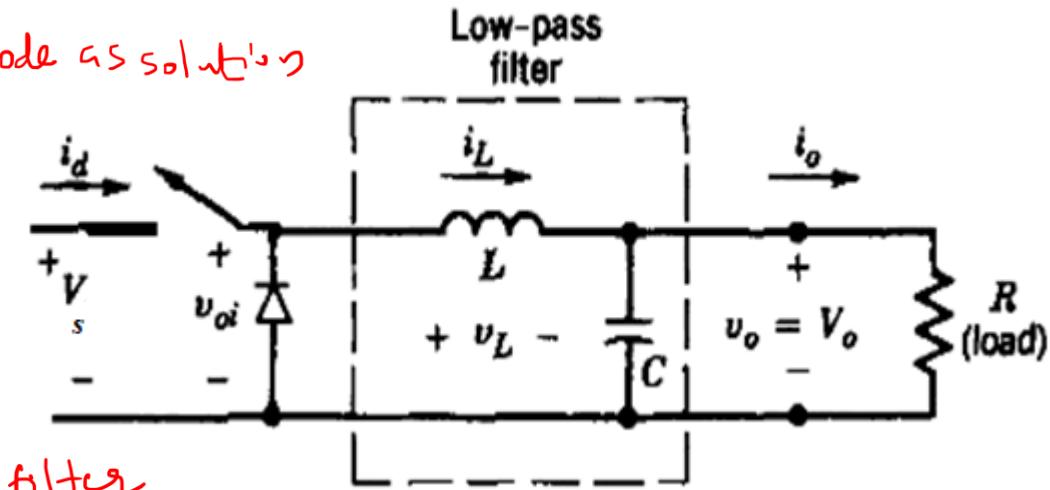
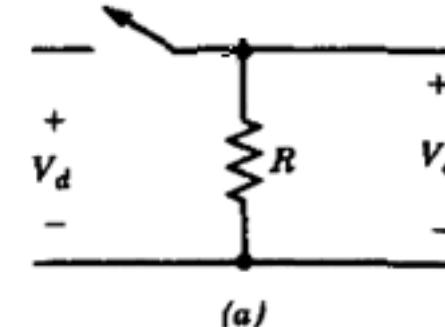


By varying the duty ratio t_{on}/T_s of the switch, V , can be controlled.

Another important observation is that the average output voltage V , varies linearly with the control voltage, as is the case in linear amplifiers

- In actual applications, the foregoing circuit has two drawbacks:
- (1) In practice the load would be inductive.
 - Even with a resistive load, there would always be certain associated stray inductance.
 - This means that the switch would have to absorb (or dissipate) the inductive energy and therefore it may be destroyed.
- (2) The output voltage fluctuates between zero and v_d , which is not acceptable in most applications.
- The problem of stored inductive energy is overcome by using a diode as shown in Fig.
- The output voltage fluctuations are very much diminished by using a low-pass filter, consisting of an inductor and a capacitor.

$$V_L = L \frac{di}{dt}$$

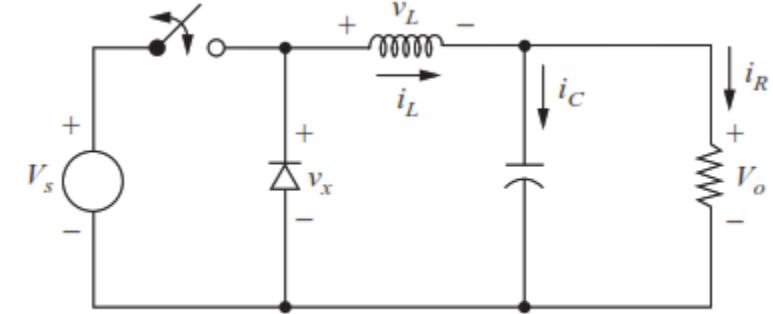


Assumptions in the Analysis of DC-DC Converters

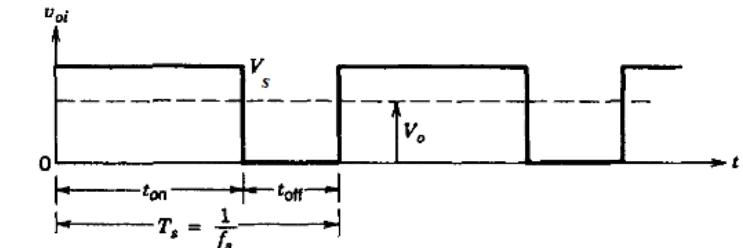
- the converters are analyzed in steady state.
- The switches are treated as being ideal, and the losses in the inductive and the capacitive elements are neglected.
- The dc input voltage to the converters is assumed to have zero internal impedance.
 - It could be a battery source; however, in most cases, the input is a diode rectified ac line voltage with a large filter capacitance, to provide a low internal impedance and a low-ripple dc voltage source.
- In the output stage of the converter, a small filter is treated as an integral part of the dc-to-dc converter.
- The output is assumed to supply a load that can be represented by an equivalent resistance, as is usually the case in switch-mode dc power supplies.
- A dc motor load (the other application of these converters) can be represented by a dc voltage in series with the motor winding resistance and inductance.

THE BUCK (STEP-DOWN) CONVERTER

- Controlling the dc component of a pulsed output voltage may be sufficient for some applications, such as controlling the speed of a dc motor,
- Often the objective is to produce an output that is **purely dc**.
- One way of obtaining a dc output from the circuit of Fig. a is to insert a low-pass filter after the switch.
- Figure a shows an LC low-pass filter added to the basic converter.
- The diode provides a path for the inductor current when the switch is opened and is reverse-biased when the switch is closed.
- This circuit is called a buck converter or a step-down converter because the output voltage is less than the input.
- If the low-pass filter is ideal, the output voltage is the average of the input voltage to the filter.
- The input to the filter, v_x in Fig. a, is V_s when the switch is closed
 - and is zero when the switch is open, provided that the inductor current remains positive, keeping the diode on.
- If the switch is closed periodically at a duty ratio D , the average voltage at the filter input is $V_s D$.
- An **inductor current that remains positive throughout the switching period is known as continuous current**.
- Conversely, **discontinuous current is characterized by the inductor current's returning to zero during each period**.



(a)



$$\begin{aligned}
 V_o &= \frac{1}{T_s} \int_0^{T_s} v_o(t) dt \\
 &= \frac{1}{T_s} \left(\int_0^{t_{on}} V_s dt + \int_{t_{on}}^{T_s} 0 dt \right) \\
 &= \frac{t_{on}}{T_s} V_s = DV_s
 \end{aligned}$$

- Another way of analyzing the operation of the buck converter is to examine the inductor voltage and current.
- This analysis method will prove useful for designing the filter and for analyzing circuits
- Buck converters and dc-dc converters in general, have the following properties when operating in the steady state:

general properties of dc-dc converters operating in the steady state:

1. The inductor current is periodic.

$$i_L(t+T) = i_L(t)$$

2. The average inductor voltage is zero (see Sec. 2.3).

$$V_L = \frac{1}{T} \int_t^{t+T} v_L(\lambda) d\lambda = 0$$

3. The average capacitor current is zero (see Sec. 2.3).

$$I_C = \frac{1}{T} \int_t^{t+T} i_C(\lambda) d\lambda = 0$$

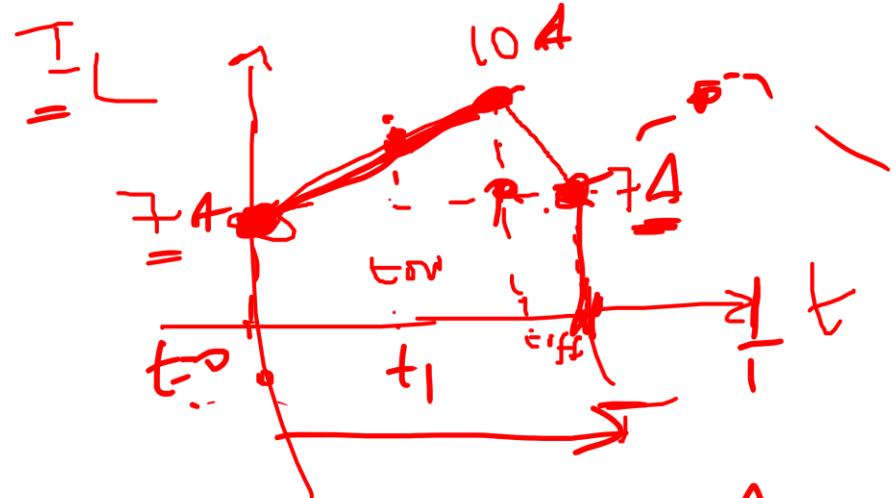
4. The power supplied by the source is the same as the power delivered to the load. For nonideal components, the source also supplies the losses.

$$\checkmark P_s = P_o$$

ideal

$$P_s = P_o + \text{losses}$$

nonideal



$$\Delta I = \frac{(0 - 7)}{t_{on}} = 3 \text{ A}$$

$$\Delta T = \sum_2 T_j$$

$$t_{off} = 7 - 4 = 3 \text{ s}$$

$$= 3 \text{ s}$$

net charge

$$\Delta I_{on}^2 \cdot \Delta t_{on} + \Delta I_{off}^2 \cdot \Delta t_{off} = \Delta I_{on} \cdot P_o / L$$

$$= 3 + 3 - 0 = 0$$

Analysis of the buck converter

C C M of buck converter

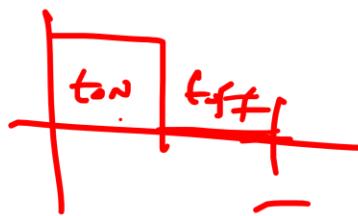
- Analysis of the buck converter of Fig. 6-3a begins by making these assumptions:

1. The circuit is operating in the steady state.
2. The inductor current is continuous (always positive).
3. The capacitor is very large, and the output voltage is held constant at voltage V_o . This restriction will be relaxed later to show the effects of finite capacitance.
4. The switching period is T ; the switch is closed for time DT and open for time $(1-D)T$.
5. The components are ideal.



The key to the analysis for determining the output V_o is to examine the inductor current and inductor voltage first for the switch closed and then for the switch open. The net change in inductor current over one period must be zero for steady-state operation. The average inductor voltage is zero.

$$V_o =$$



$$t_{on} + t_{off} = T$$

$$D = \frac{t_{on}}{T}$$

$$t_{on} = D \cdot T$$
$$t_{off} = (1-D) \cdot T$$

Analysis for the Switch Closed When the switch is closed in the buck converter circuit of Fig. 6-3a, the diode is reverse-biased and Fig. 6-3b is an equivalent circuit. The voltage across the inductor is

$$v_L = V_s - V_o = L \frac{di_L}{dt}$$

Rearranging,

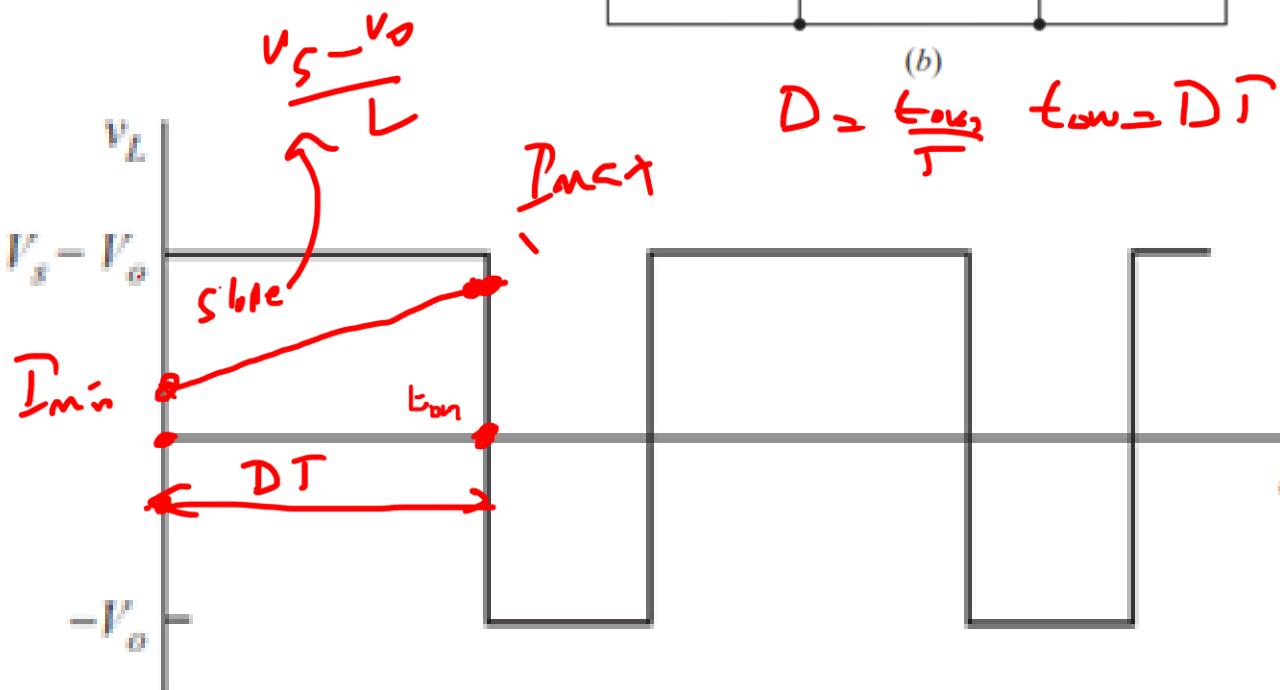
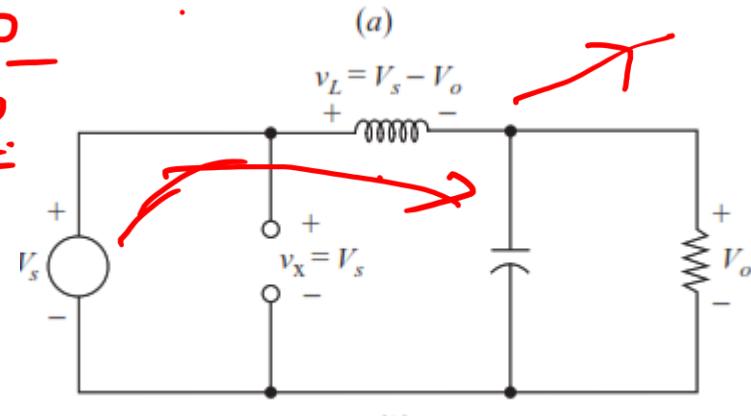
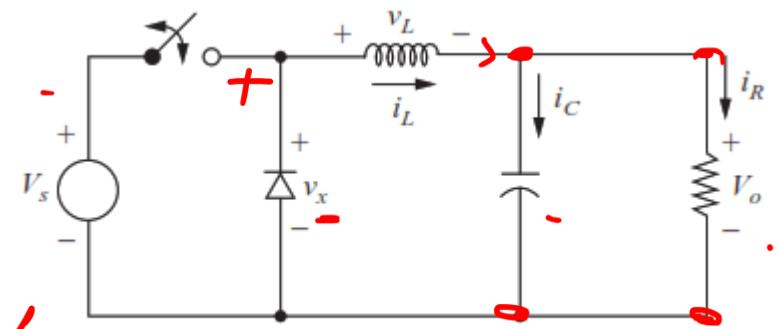
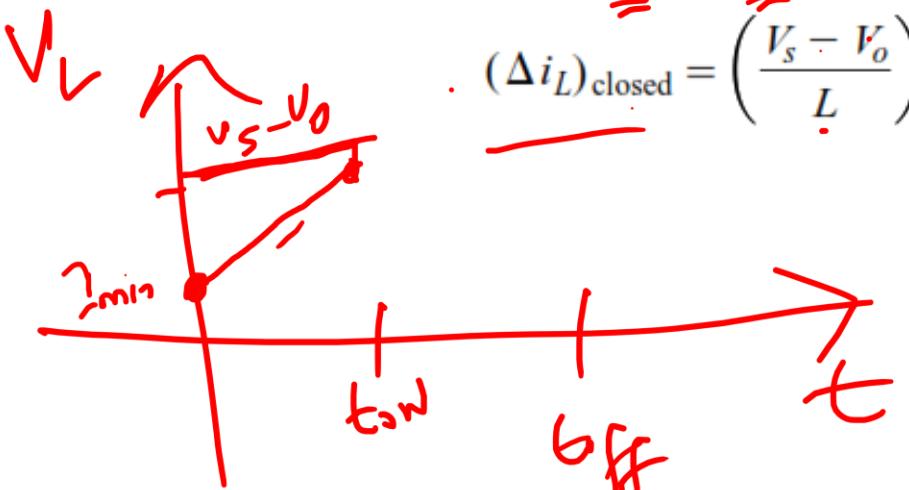
$$i_L = \frac{V_s - V_o}{L} t \quad \text{switch closed}$$

$$\begin{aligned} & \text{ON} \quad v_L = V_s - V_o \\ & \text{OFF} \quad v_L = V_s + V_o \\ \text{Simplifying: } & V_s = \frac{V_L + V_o}{2} \\ & V_L = \frac{V_s - V_o}{2} \end{aligned}$$

Since the derivative of the current is a positive constant, the current increases linearly as shown in Fig. 6-4b. The change in current while the switch is closed is computed by modifying the preceding equation.

$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L}$$

$$(\Delta i_L)_{closed} = \left(\frac{V_s - V_o}{L} \right) DT$$



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$$v_L = V_s - V_o = L \frac{di_L}{dt}$$

Rearranging,

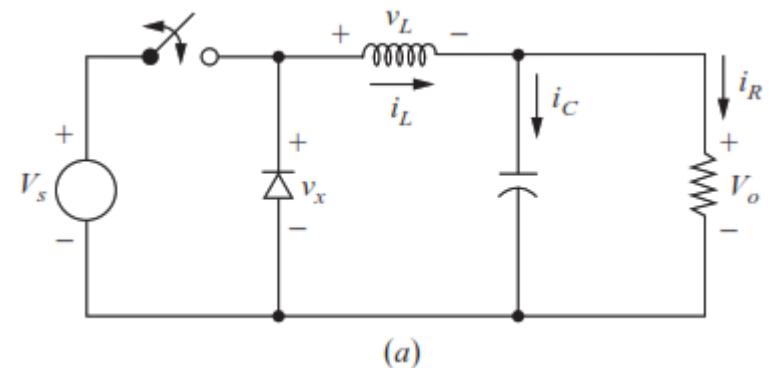
$$\frac{di_L}{dt} = \frac{V_s - V_o}{L} \quad \text{switch closed}$$

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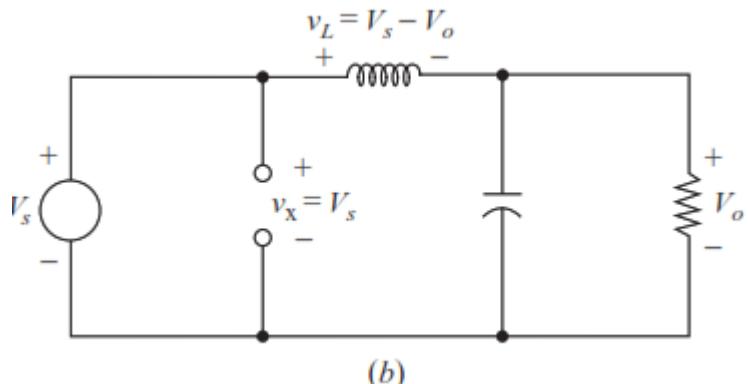
$$\frac{di_L}{dt} = \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s - V_o}{L} \quad (6-7)$$

$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L} \right) DT$$

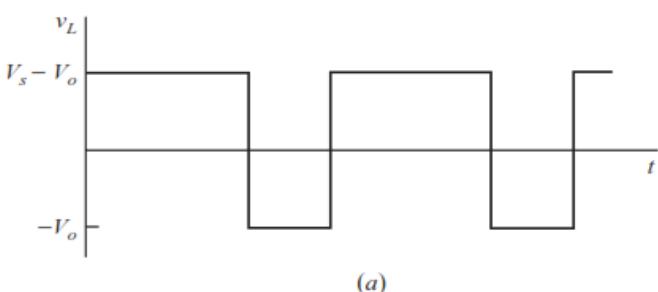
V_S



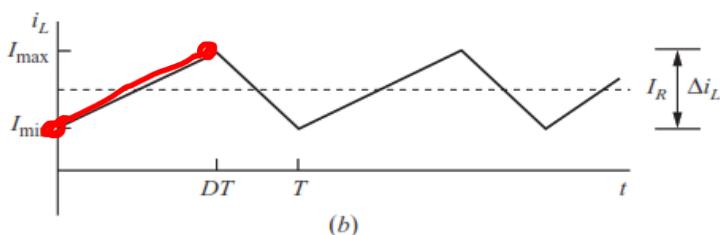
(a)



(b)



(a)



(b)

Analysis for the Switch Open When the switch is open, the diode becomes forward-biased to carry the inductor current and the equivalent circuit of Fig. 6-3c applies. The voltage across the inductor when the switch is open is

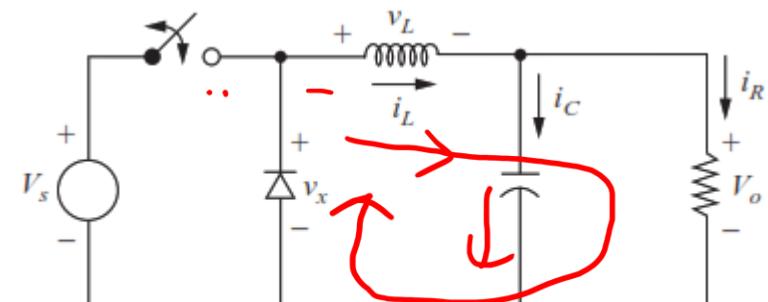
$$v_L = -V_o = L \frac{di_L}{dt}$$

Rearranging,

$$\frac{di_L}{dt} = \frac{-V_o}{L} \quad \text{switch open}$$

on
- v_L -
 Δi_L -
 $\Delta i_L = V_o / L + V_o$

- $m\alpha$ -



(a)

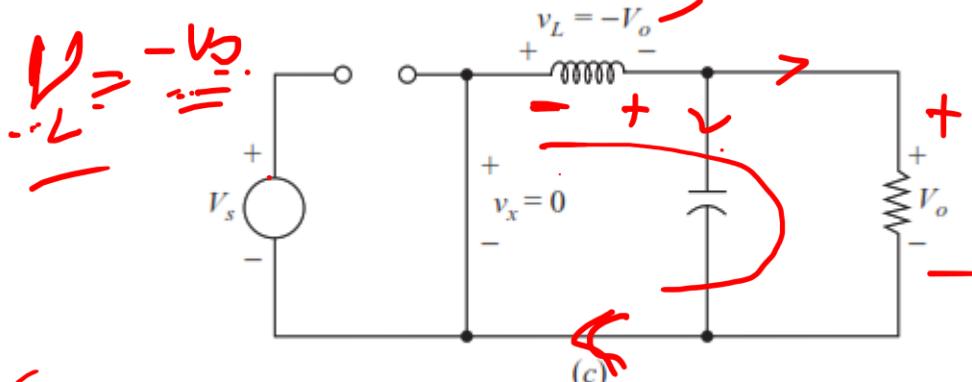
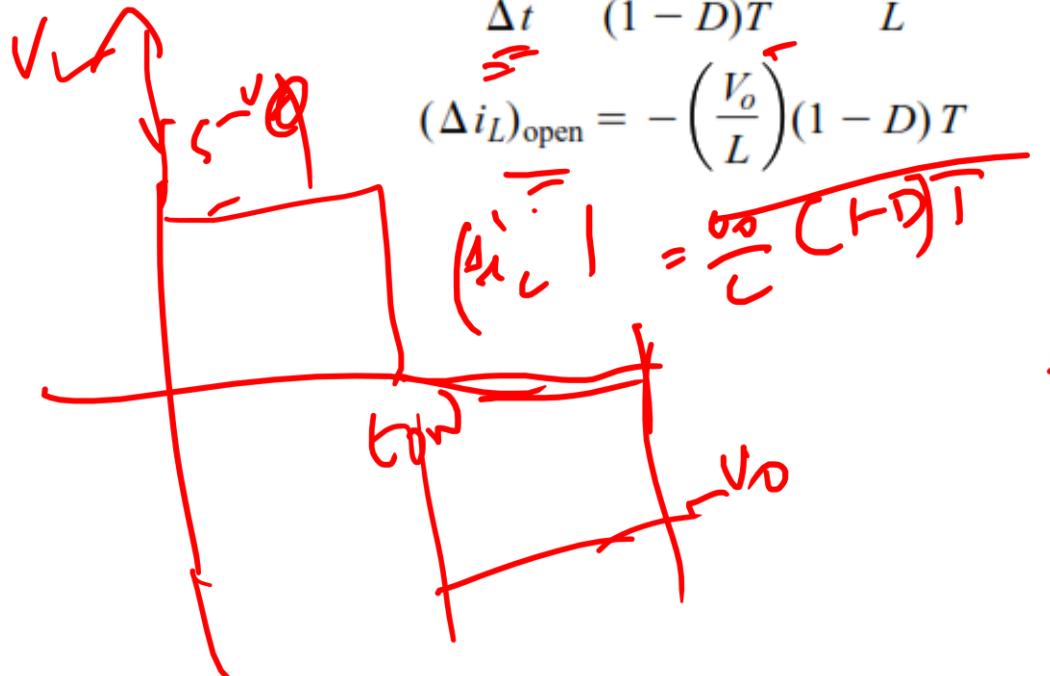
The derivative of current in the inductor is a negative constant, and the current decreases linearly as shown in Fig. 6-4b. The change in inductor current when the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L}$$

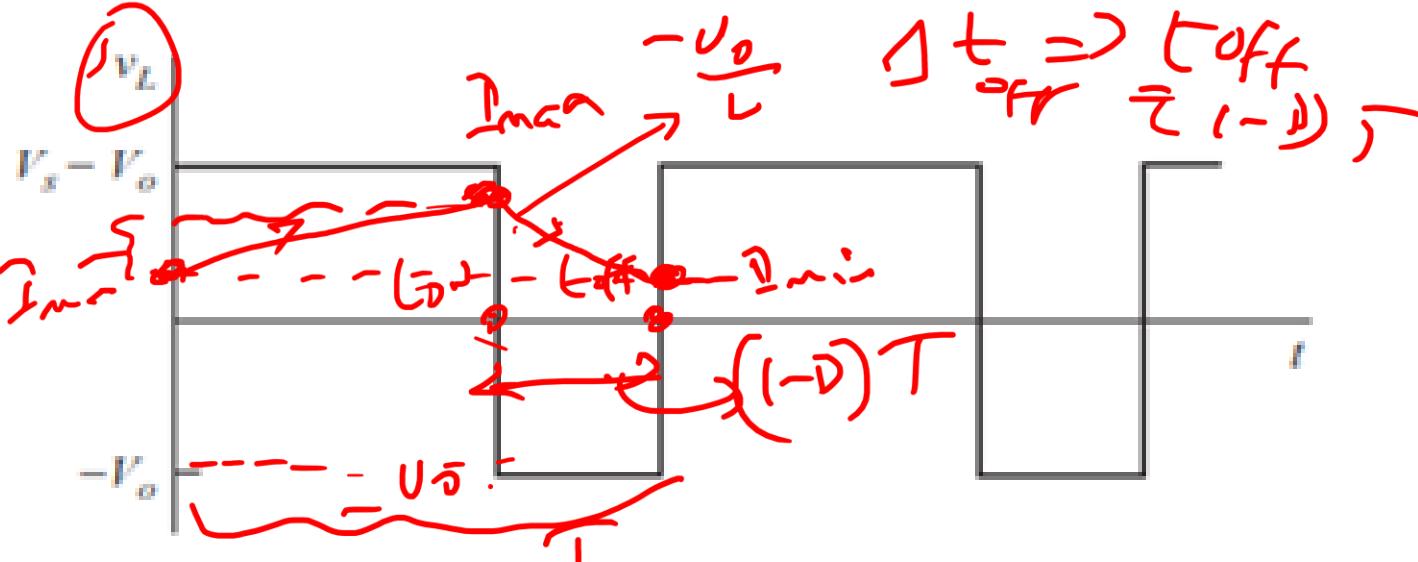
$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T$$



(6-8)



(c)



Analysis for the Switch Open When the switch is open, the diode becomes forward-biased to carry the inductor current and the equivalent circuit of Fig. 6-3c applies. The voltage across the inductor when the switch is open is

$$v_L = -V_o = L \frac{di_L}{dt}$$

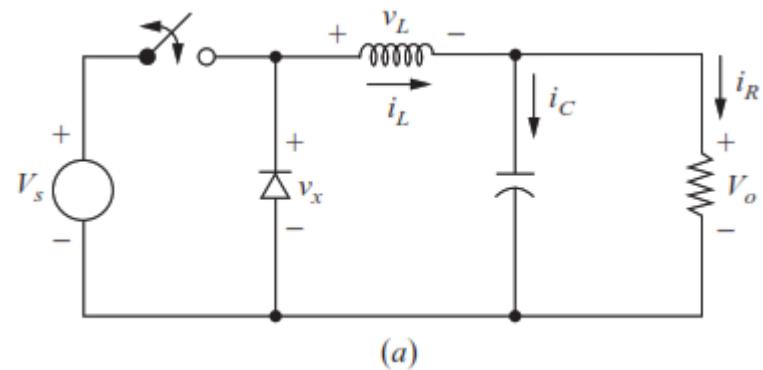
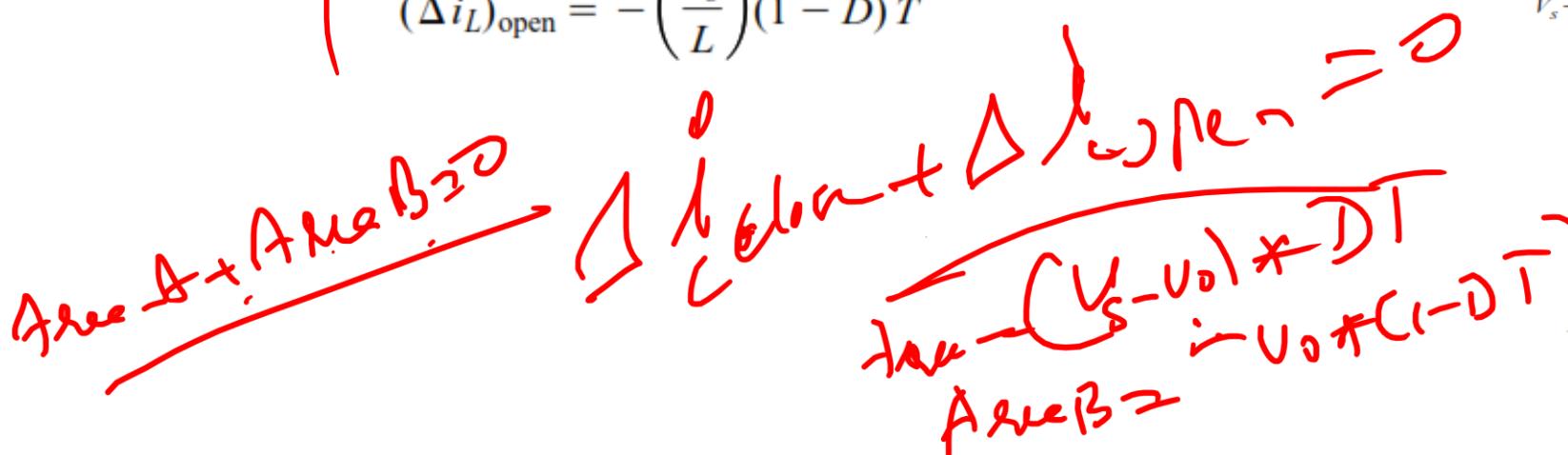
Rearranging,

$$\frac{di_L}{dt} = \frac{-V_o}{L} \quad \text{switch open}$$

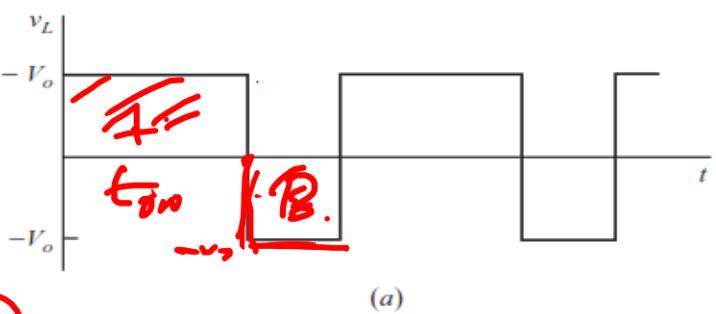
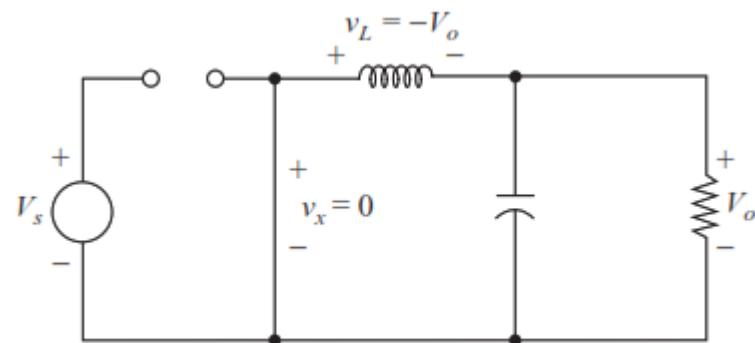
The derivative of current in the inductor is a negative constant, and the current decreases linearly as shown in Fig. 6-4b. The change in inductor current when the switch is open is

$$\frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = -\frac{V_o}{L} \quad (6-8)$$

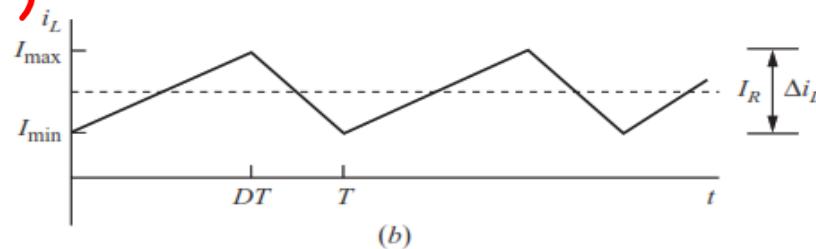
$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T$$



(a)

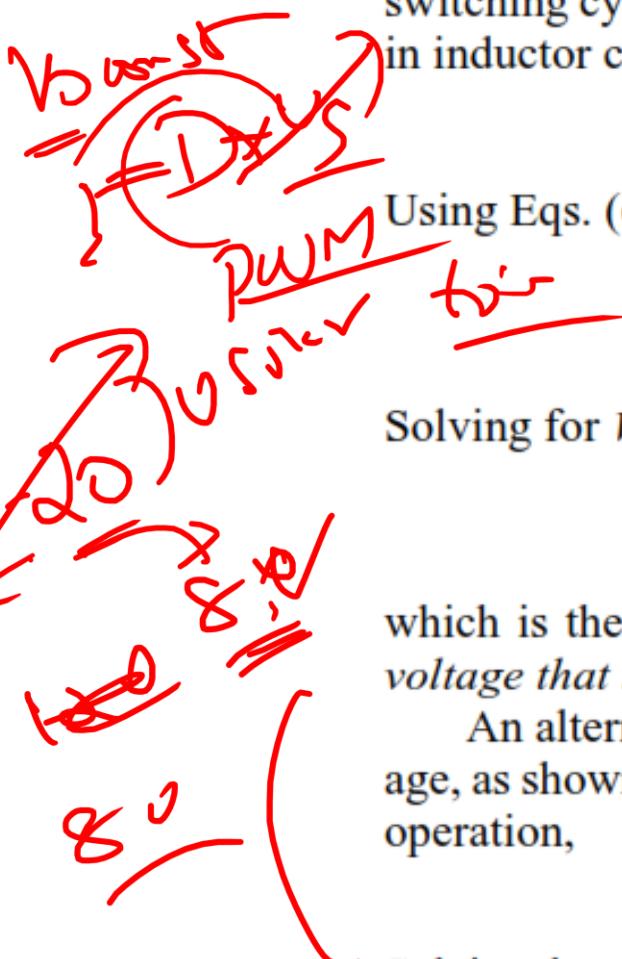


(a)



(b)

Steady-state operation requires that the inductor current at the end of the switching cycle be the same as that at the beginning, meaning that the net change in inductor current over one period is zero. This requires



Using Eqs. (6-7) and (6-8),

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0$$

$$\left(\frac{V_s - V_o}{L}\right)(DT) - \left(\frac{V_o}{L}\right)(1 - D)T = 0$$

Solving for V_o ,

$$V_o = V_s D$$

which is the same result as Eq. (6-1). *The buck converter produces an output voltage that is less than or equal to the input.*

An alternative derivation of the output voltage is based on the inductor voltage, as shown in Fig. 6-4a. Since the average inductor voltage is zero for periodic operation,

$$V_L = (V_s - V_o)DT + (-V_o)(1 - D)T = 0$$

Solving the preceding equation for V_o yields the same result as Eq. (6-9), $V_o = V_s D$.

Note that the output voltage depends on only the input and the duty ratio D . If the input voltage fluctuates, the output voltage can be regulated by adjusting the duty ratio appropriately. A feedback loop is required to sample the output voltage, compare it to a reference, and set the duty ratio of the switch accordingly.

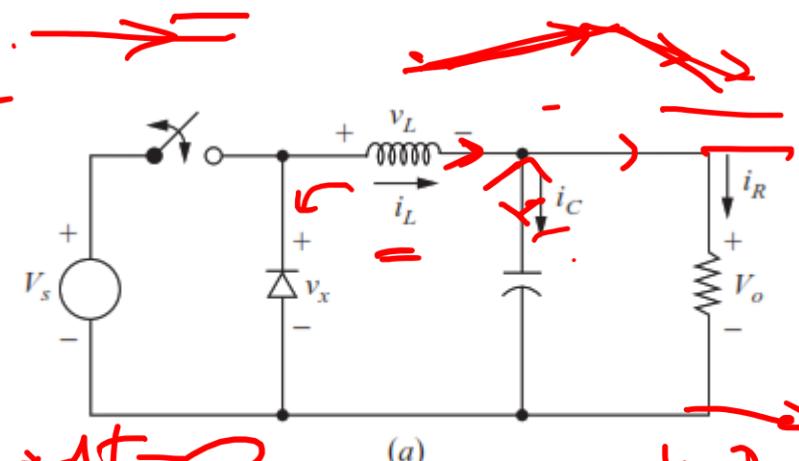
$D < 1$

$$\begin{aligned} \text{const} \\ \cancel{D} \\ M_o &= V_s D \\ \cancel{V_o} &= V_s D \\ (6-9) \quad &= V_s D \\ &= 10 \\ &< 10 \\ V_o &\approx V_s + D \\ V_o \text{ const!} \end{aligned}$$

$$\underline{I} \underline{L}_{ans} + \cancel{\underline{I}_{sys}} = \underline{I}_{fr}$$

The average inductor current must be the same as the average current in the load resistor, since the average capacitor current must be zero for steady-state operation:

$$I_L = I_R = \frac{V_o}{R} \quad \text{Pron Sk} \quad (6-10)$$



Since the change in inductor current is known from Eqs. (6-7) and (6-8), the maximum and minimum values of the inductor current are computed as

$$I_{\max} = I_L + \frac{\Delta i_L}{2}$$

magnitude
peak

$$f = \frac{1}{T}$$

i_L avg

$$= \frac{V_o}{R} + \frac{1}{2} \left[\frac{V_o}{L} (1 - D) T \right] = V_o \left(\frac{1}{R} + \frac{1 - D}{2L} T \right)$$

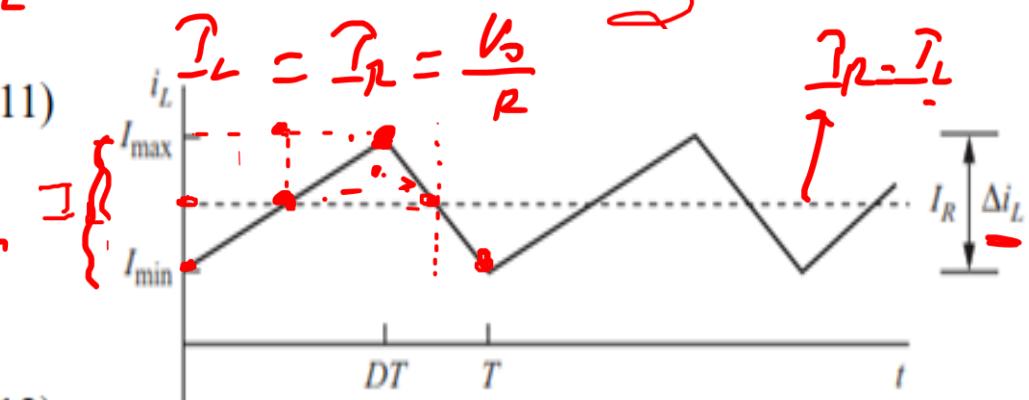
$$I_{\min} = I_L - \frac{\Delta i_L}{2} \quad \text{à l'origine}$$

$$= \frac{V_o}{R} - \frac{1}{2} \left[\frac{V_o}{L} (1 - D) T \right] = V_o \left(\frac{1}{R} - \frac{1 - D}{2Lf} \right)$$

where $f = 1/T$ is the switching frequency.

$$i_L = i_C + i_R \quad I_L = \frac{I_{max} - I_{min}}{2}$$

$$I_L = \frac{i_C + i_R}{2} \quad (6-11)$$



$$T_{\text{cycle}} = \frac{T_R}{n}$$

For the preceding analysis to be valid, continuous current in the inductor must be verified. An easy check for continuous current is to calculate the minimum inductor current from Eq. (6-12). Since the minimum value of inductor current must be positive for continuous current, a negative minimum calculated from Eq. (6-12) is not allowed due to the diode and indicates discontinuous current. The circuit will operate for discontinuous inductor current, but the preceding analysis is not valid.

Equation (6-12) can be used to determine the combination of L and f that will result in continuous current. Since $I_{\min} = 0$ is the boundary between continuous and discontinuous current,

$$I_{\min} = 0 = V_o \left(\frac{1}{R} - \frac{1-D}{2Lf} \right)$$

$$(Lf)_{\min} = \frac{(1-D)R}{2}$$

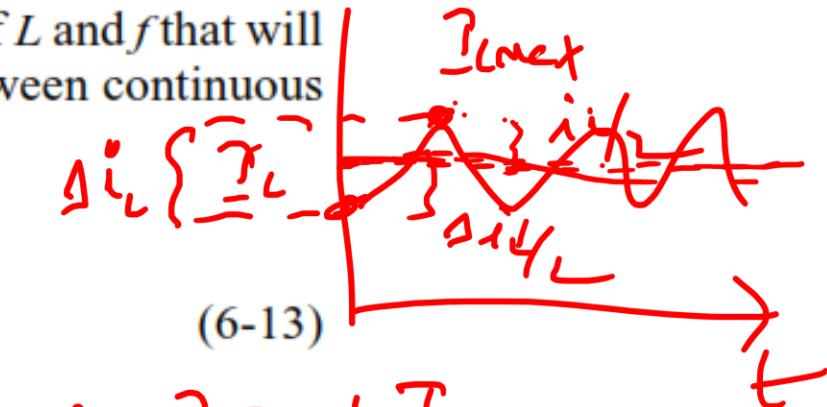
If the desired switching frequency is established,

$$L_{\min} = \frac{(1-D)R}{2f}$$

for continuous current

where L_{\min} is the minimum inductance required for continuous current. In practice, a value of inductance greater than L_{\min} is desirable to ensure continuous current.

$$I_{\min} = -N_o$$



$$I_{\min} = \frac{P_o - I_{Lmax}}{2} = P_L + \frac{\Delta i}{2}$$

(6-14)

$\cdot 2f L_{\min}$

In the design of a buck converter, the peak-to-peak variation in the inductor current is often used as a design criterion. Equation (6-7) can be combined with Eq. (6-9) to determine the value of inductance for a specified peak-to-peak inductor current for continuous-current operation:

$$V_o = D \cdot V_s \quad \left| \Delta i_L \right| = \left(\frac{V_s - V_o}{L} \right) DT = \left(\frac{V_s - V_o}{L_f} \right) D = \frac{V_o(1 - D)}{L_f} \quad (6-15)$$

$$\text{or } V_s = \frac{V_o}{D} \quad L = \left(\frac{V_s - V_o}{\Delta i_L f} \right) D = \frac{V_o(1 - D)}{\Delta i_L f} \quad (6-16)$$

Since the converter components are assumed to be ideal, the power supplied by the source must be the same as the power absorbed by the load resistor.

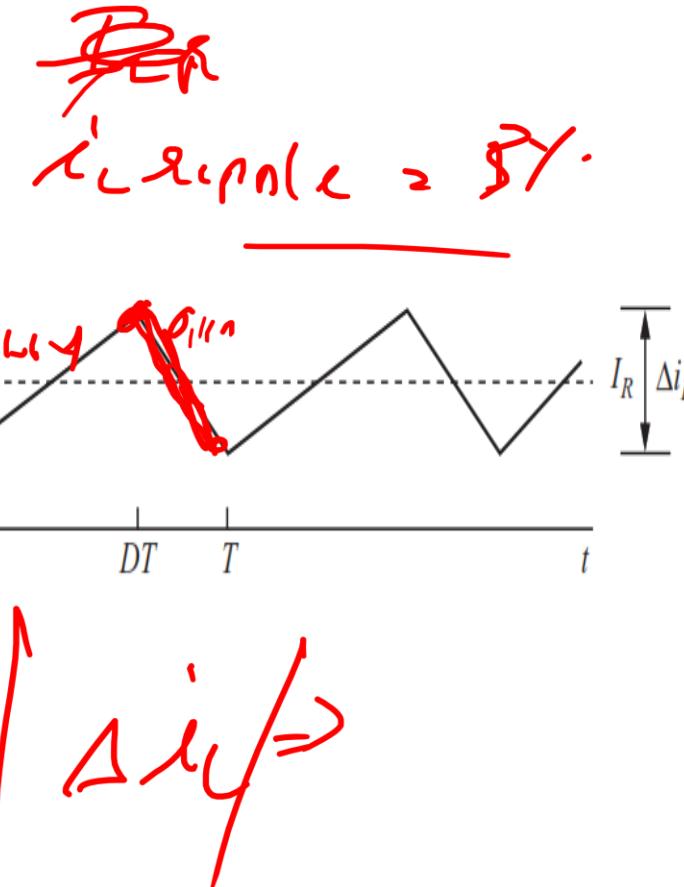
$$P_s = P_o$$

$$V_s I_s = V_o I_o$$

$$\frac{V_o}{V_s} = \frac{I_s}{I_o}$$

$$\text{or } \frac{V_o}{V_s} = D \quad (6-17)$$

Note that the preceding relationship is similar to the voltage-current relationship for a transformer in ac applications. Therefore, the buck converter circuit is equivalent to a dc transformer.



Output Voltage Ripple

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$

shown in Fig. 6-5a.

While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,

$$Q = CV_o$$

$$\Delta Q = C \Delta V_o$$

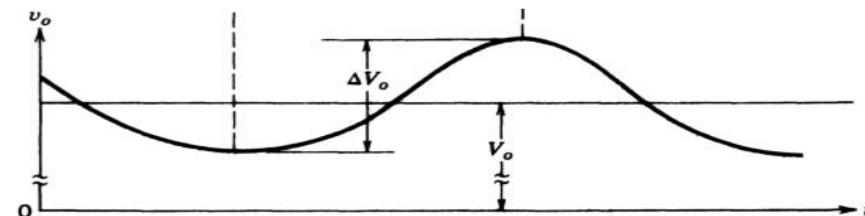
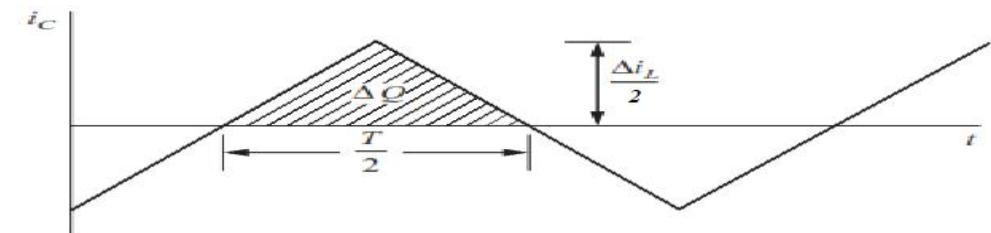
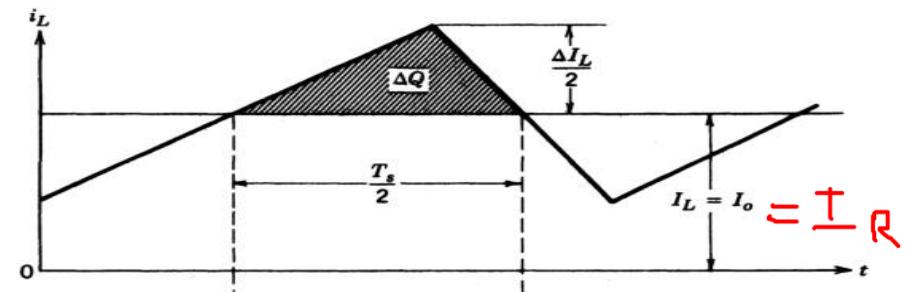
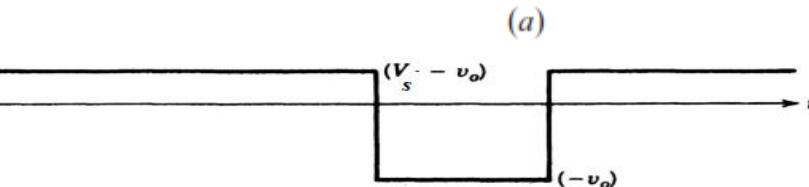
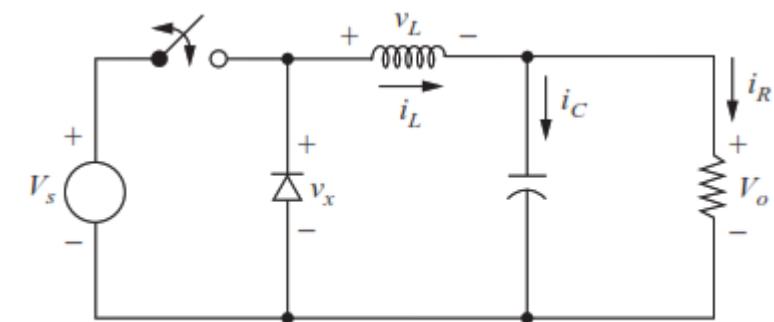
$$\Delta V_o = \frac{\Delta Q}{C}$$

The change in charge ΔQ is the area of the triangle above the time axis

$$\Delta Q = \frac{1}{2} \left(\frac{T}{2} \right) \left(\frac{\Delta i_L}{2} \right) = \frac{T \Delta i_L}{8}$$

resulting in

$$\Delta V_o = \frac{T \Delta i_L}{8C}$$



Output Voltage Ripple

Using Eq. (6-8) for Δi_L ,

$$\Delta V_o = \frac{T V_o}{8CL} (1 - D)T = \frac{V_o(1 - D)}{8LCf^2} \quad (6-18)$$

In this equation, ΔV_o is the peak-to-peak ripple voltage at the output, as shown in Fig. 6-5b. It is also useful to express the ripple as a fraction of the output voltage,

$$\frac{\Delta V_o}{V_o} = \frac{1 - D}{8LCf^2} \quad (6-19)$$

In design, it is useful to rearrange the preceding equation to express required capacitance in terms of specified voltage ripple:

$$C = \frac{1 - D}{8L(\Delta V_o/V_o)f^2} \quad (6-20)$$

If the ripple is not large, the assumption of a constant output voltage is reasonable and the preceding analysis is essentially valid.

Output Voltage Ripple

In the preceding analysis, the capacitor was assumed to be very large to keep the output voltage constant. In practice, the output voltage cannot be kept perfectly constant with a finite capacitance. The variation in output voltage, or ripple, is computed from the voltage-current relationship of the capacitor. The current in the capacitor is

$$i_C = i_L - i_R$$

shown in Fig. 6-5a.

While the capacitor current is positive, the capacitor is charging. From the definition of capacitance,

$$\Delta Q \approx C * \Delta V_o$$

$$Q = CV_o$$

$$\Delta Q = C \Delta V_o$$

$$\Delta V_o = \frac{\Delta Q}{C}$$

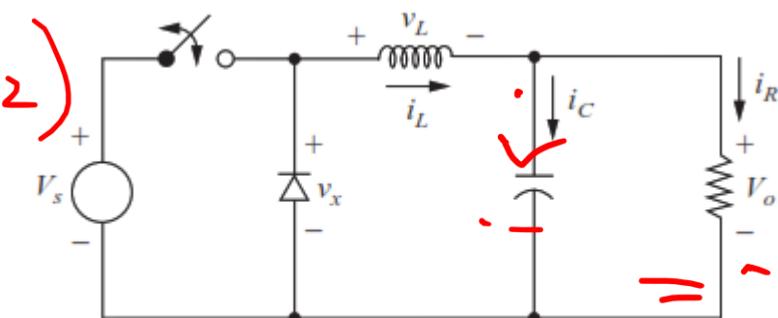
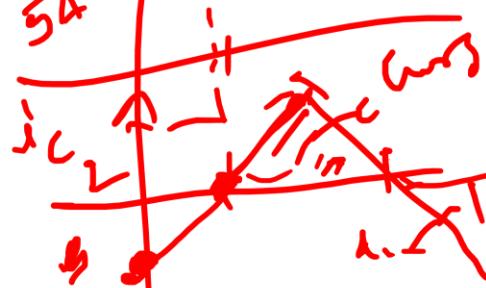
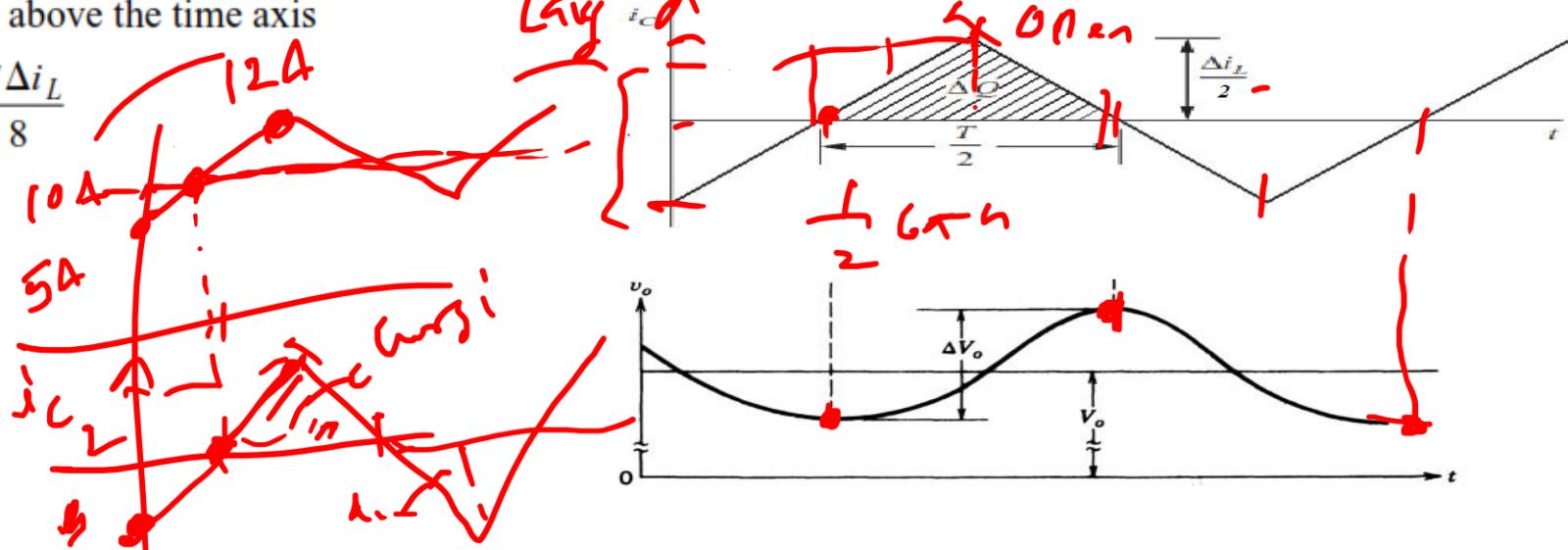
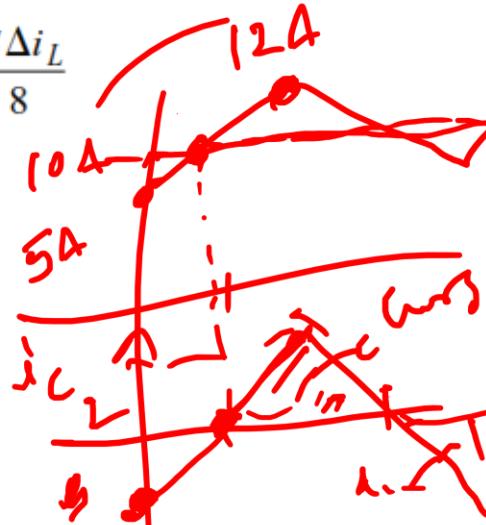
The change in charge ΔQ is the area of the triangle above the time axis

$$\Delta Q = \frac{1}{2} \left(\frac{T}{2} \right) \left(\frac{\Delta i_L}{2} \right) = \frac{T \Delta i_L}{8}$$

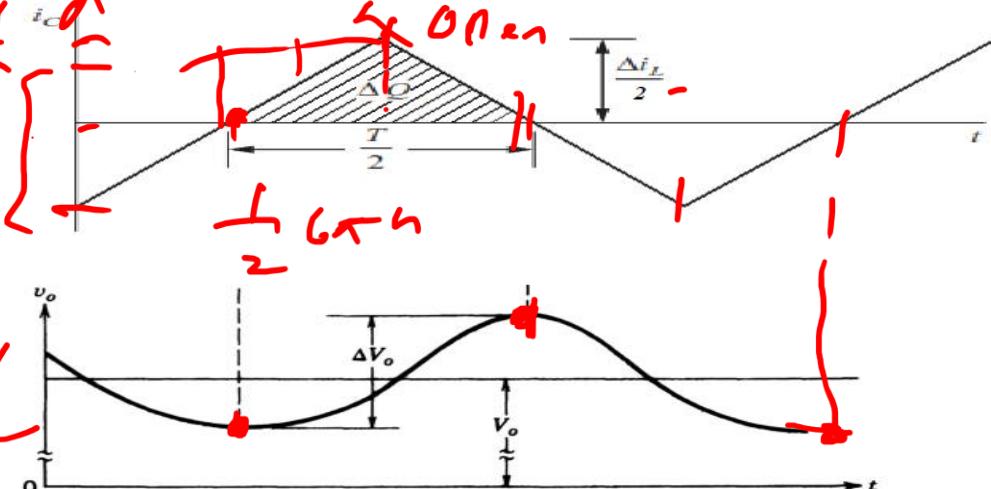
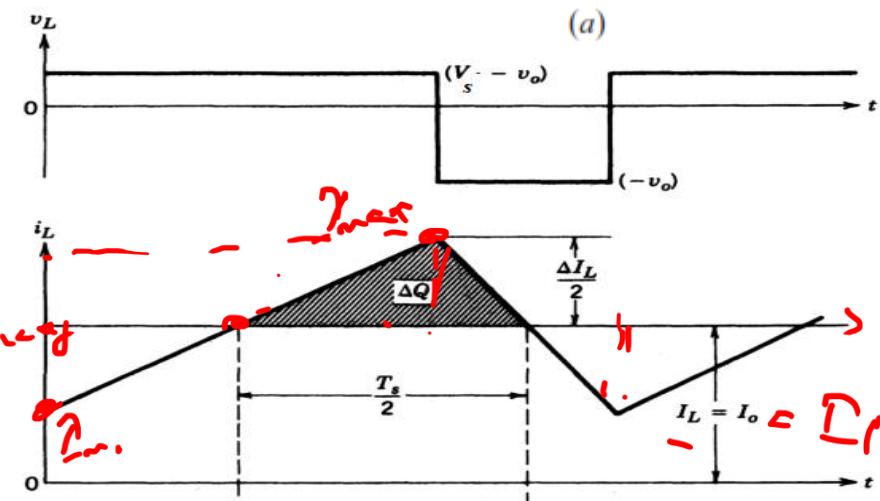
resulting in

$$\Delta V_o = \frac{T \Delta i_L}{8C}$$

$i_C = 0$



(a)



Output Voltage Ripple

Using Eq. (6-8) for Δi_L ,

$$\Delta V_o = \frac{T V_o}{8CL} (1 - D) T = \frac{V_o (1 - D)}{8LCf^2} \quad (6-18)$$

In this equation, ΔV_o is the peak-to-peak ripple voltage at the output, as shown in Fig. 6-5b. It is also useful to express the ripple as a fraction of the output voltage,

$$\frac{\Delta V_o}{V_o} = \frac{1 - D}{8LCf^2} \quad (6-19)$$

In design, it is useful to rearrange the preceding equation to express required capacitance in terms of specified voltage ripple:

$$C = \frac{1 - D}{8L(\Delta V_o/V_o)f^2} \quad (6-20)$$

If the ripple is not large, the assumption of a constant output voltage is reasonable and the preceding analysis is essentially valid.

The buck dc-dc converter of Fig. 6-3a has the following parameters:

$$\begin{aligned}V_s &= 50 \text{ V} \\D &= 0.4 \\L &= 400 \mu\text{H} \\C &= 100 \mu\text{F} \\f &= 20 \text{ kHz} \\R &= 20 \Omega\end{aligned}$$

$$I_{L_{avg}} = \frac{V_o}{R} = \frac{20 \text{ V}}{20 \Omega} = \underline{\underline{1 \text{ A}}}$$

Assuming ideal components, calculate (a) the output voltage V_o , (b) the maximum and minimum inductor current, and (c) the output voltage ripple.

■ Solution

- (a) The inductor current is assumed to be continuous, and the output voltage is computed from Eq. (6-9),

$$V_o = V_s D = (50)(0.4) = 20 \text{ V}$$

- (b) Maximum and minimum inductor currents are computed from Eqs. (6-11) and (6-12).

$$\begin{aligned}I_{max} &= V_o \left(\frac{1}{R} + \frac{1-D}{2Lf} \right) \\&= 20 \left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3} \right] \\&= 1 + \frac{1.5}{2} = 1.75 \text{ A}\end{aligned}$$

$$I_{max} = I_{L_{avg}} + \left| \frac{\Delta i_L}{2} \right|$$

$$\left| \Delta i_L \right| = \frac{V_o}{L} (1-D) T$$

$$\frac{1}{f} = T = \frac{20 \times 10^{-6}}{400 \times 10^3} = \frac{1-0.4}{20 \times 10^3}$$

$$I_{max} = 1 + \frac{1.5}{2} = \underline{\underline{1.75 \text{ A}}} = 1.54$$

$$I_{\min} = V_o \left(\frac{1}{R} - \frac{1-D}{2Lf} \right)$$

$$= 1 - \frac{1.5}{2} = \underline{\underline{0.25 \text{ A}}}$$

C_0

$\underline{\underline{D_{\min} > 0}} \Rightarrow \text{continuous}$

The average inductor current is 1 A, and $\underline{\Delta i_L = 1.5 \text{ A}}$. Note that the minimum inductor current is positive, verifying that the assumption of continuous current was valid.

- (c) The output voltage ripple is computed from Eq. (6-19).

$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20,000)^2}$$

$$= 0.00469 = 0.469\%$$

Since the output ripple is sufficiently small, the assumption of a constant output voltage was reasonable.

Buck Converter Design 1

Design a buck converter to produce an output voltage of 18 V across a $10\text{-}\Omega$ load resistor. The output voltage ripple must not exceed 0.5 percent. The dc supply is 48 V. Design for continuous inductor current. Specify the duty ratio, the switching frequency, the values of the inductor and capacitor, the peak voltage rating of each device, and the rms current in the inductor and capacitor. Assume ideal components.

■ Solution

Using the buck converter circuit in Fig. 6-3a, the duty ratio for continuous-current operation is determined from Eq. (6-9):

$$D = \frac{V_o}{V_s} = \frac{18}{48} = 0.375$$

The switching frequency and inductor size must be selected for continuous-current operation. Let the switching frequency arbitrarily be 40 kHz, which is well above the audio range and is low enough to keep switching losses small. The minimum inductor size is determined from Eq. (6-14).

$$L_{\min} = \frac{(1 - D)(R)}{2f} = \frac{(1 - 0.375)(10)}{2(40,000)} = 78 \mu\text{H}$$

Let the inductor be 25 percent larger than the minimum to ensure that inductor current is continuous.

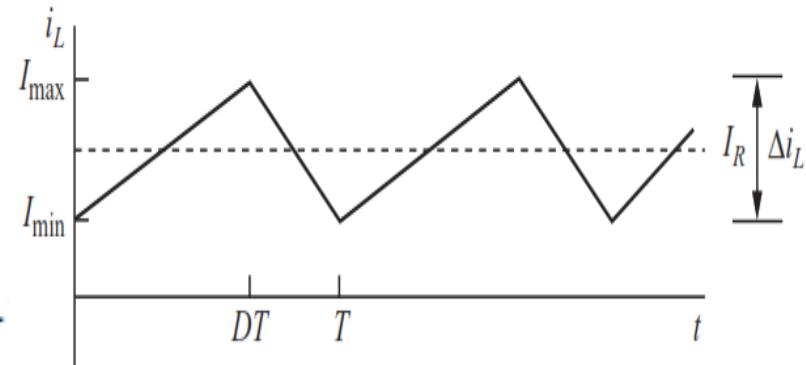
$$L = 1.25L_{\min} = (1.25)(78 \mu\text{H}) = 97.5 \mu\text{H}$$

Average inductor current and the change in current are determined from Eqs. (6-10) and (6-17).

$$I_L = \frac{V_o}{R} = \frac{18}{10} = 1.8 \text{ A}$$

$$\Delta i_L = \left(\frac{V_s - V_o}{L} \right) DT = \frac{48 - 18}{97.5(10)^{-6}} (0.375) \left(\frac{1}{40,000} \right) = 2.88 \text{ A}$$

The maximum and minimum inductor currents are determined from Eqs. (6-11) and (6-



$$I_{\max} = I_L + \frac{\Delta i_L}{2} = 1.8 + 1.44 = 3.24 \text{ A}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = 1.8 - 1.44 = 0.36 \text{ A}$$

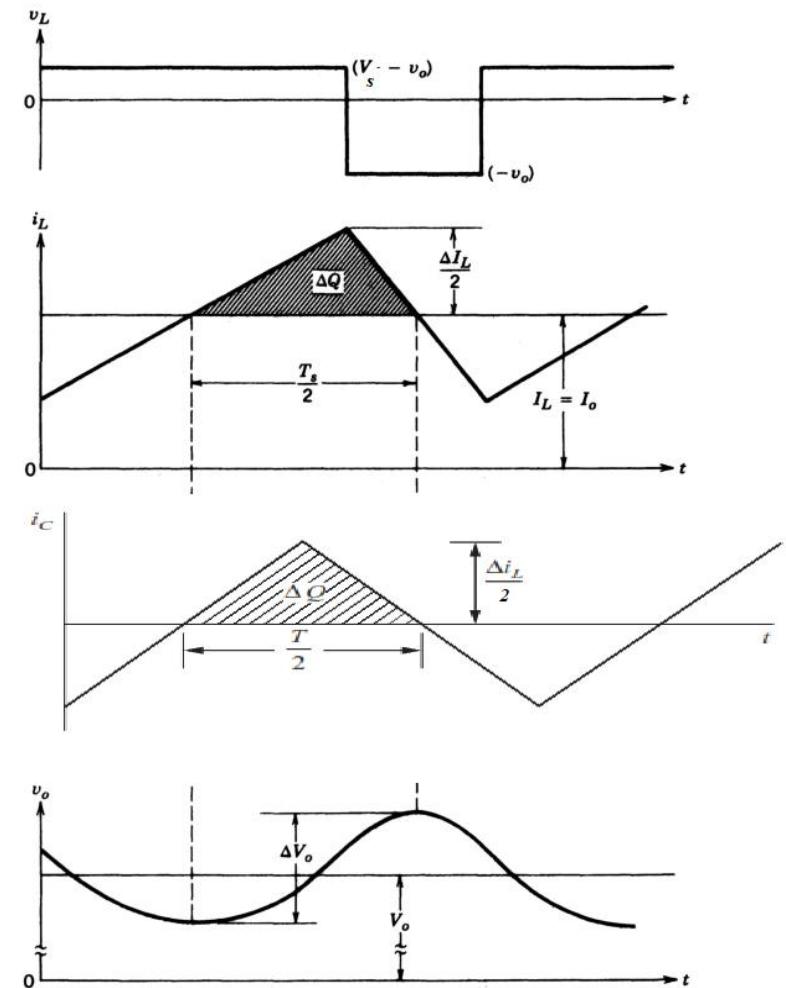
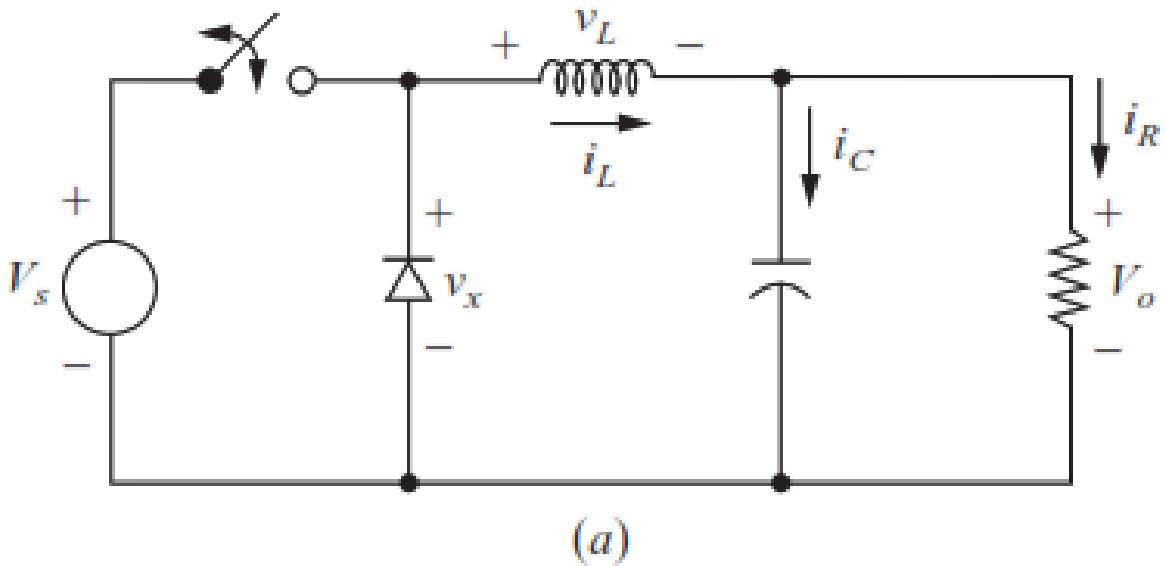
The inductor must be rated for rms current, which is computed as in Chap. 2 (see Example 2-8). For the offset triangular wave,

$$I_{L,\text{rms}} = \sqrt{I_L^2 + \left(\frac{\Delta i_L/2}{\sqrt{3}} \right)^2} = \sqrt{(1.8)^2 + \left(\frac{1.44}{\sqrt{3}} \right)^2} = 1.98 \text{ A}$$

The capacitor is selected using Eq. (6-20).

$$C = \frac{1 - D}{8L(\Delta V_o/V_o)f^2} = \frac{1 - 0.375}{8(97.5)(10)^{-6}(0.005)(40,000)^2} = 100 \mu\text{F}$$

Peak capacitor current is $\Delta i_L/2 = 1.44 \text{ A}$, and rms capacitor current for the triangular waveform is $1.44/\sqrt{3} = 0.83 \text{ A}$. The maximum voltage across the switch and diode is V_s , or 48 V. The inductor voltage when the switch is closed is $V_s - V_o = 48 - 18 = 30 \text{ V}$. The inductor voltage when the switch is open is $V_o = 18 \text{ V}$. Therefore, the inductor must withstand 30 V. The capacitor must be rated for the 18-V output.



Buck Converter Design 1

Design a buck converter to produce an output voltage of 18 V across a $10\text{-}\Omega$ load resistor. The output voltage ripple must not exceed 0.5 percent. The dc supply is 48 V. Design for continuous inductor current. Specify the duty ratio, the switching frequency, the values of the inductor and capacitor, the peak voltage rating of each device, and the rms current in the inductor and capacitor. Assume ideal components.

Solution

Using the buck converter circuit in Fig. 6-3a, the duty ratio for continuous-current operation is determined from Eq. (6-9):

$$D = \frac{V_o}{V_s} = \frac{18}{48} = 0.375$$

The switching frequency and inductor size must be selected for continuous-current operation. Let the switching frequency arbitrarily be 40 kHz, which is well above the audio range and is low enough to keep switching losses small. The minimum inductor size is determined from Eq. (6-14).

$$L_{\min} = \frac{(1 - D)(R)}{2f} = \frac{(1 - 0.375)(10)}{2(40,000)} = 78 \mu\text{H}$$

Let the inductor be 25 percent larger than the minimum to ensure that inductor current is continuous.

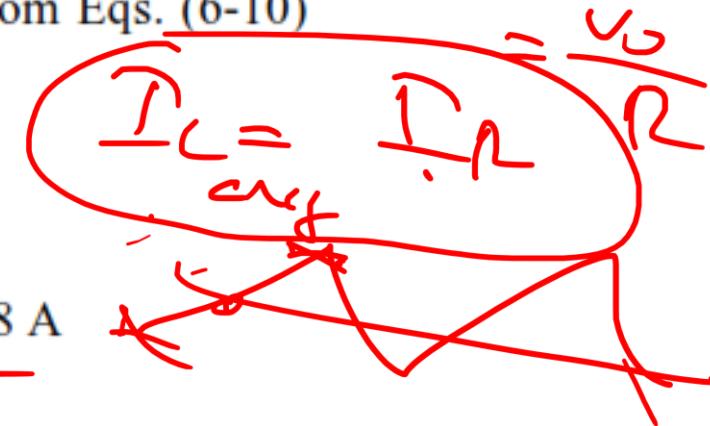
$$L = 1.25L_{\min} = (1.25)(78 \mu\text{H}) = 97.5 \mu\text{H}$$

$$\begin{aligned} D &= \frac{V_o}{V_s} + D \\ D &= \frac{V_o}{V_s} \\ f &= \frac{1}{T} \\ T &= \frac{1}{f} \\ L &= \frac{V_o}{f} \\ C &= \frac{1}{f^2 L} \\ \frac{1}{f^2 L} &= 0.5 \times \frac{1}{10000} \\ f &= 20\text{kHz} \end{aligned}$$

Average inductor current and the change in current are determined from Eqs. (6-10) and (6-17).

$$I_L = \frac{V_o}{R} = \frac{18}{10} = 1.8 \text{ A}$$

$$\Delta i_L = \left(\frac{V_s - V_o}{L} \right) DT = \frac{48 - 18}{97.5(10)^{-6}} (0.375) \left(\frac{1}{40,000} \right) = 2.88 \text{ A}$$



The maximum and minimum inductor currents are determined from Eqs. (6-11) and (6-12).

$$\underline{\underline{I_{\max}}} = \underline{\underline{I_L}} + \frac{\Delta i_L}{2} = 1.8 + 1.44 = 3.24 \text{ A}$$

$$\underline{\underline{I_{\min}}} = \underline{\underline{I_L}} - \frac{\Delta i_L}{2} = 1.8 - 1.44 = 0.36 \text{ A}$$

The inductor must be rated for rms current, which is computed as in Chap. 2 (see Example 2-8). For the offset triangular wave,

$$\underline{\underline{I_{L,\text{rms}}}} = \sqrt{I_L^2 + \left(\frac{\Delta i_L/2}{\sqrt{3}} \right)^2} = \sqrt{(1.8)^2 + \left(\frac{1.44}{\sqrt{3}} \right)^2} = 1.98 \text{ A}$$

The capacitor is selected using Eq. (6-20).

$$C = \frac{1 - D}{8L(\Delta V_o/V_o)f^2} = \frac{1 - 0.375}{8(97.5)(10)^{-6}(0.005)(40,000)^2} = 100 \mu\text{F}$$

Peak capacitor current is $\Delta i_L/2 = 1.44 \text{ A}$, and rms capacitor current for the triangular waveform is $1.44/\sqrt{3} = 0.83 \text{ A}$. The maximum voltage across the switch and diode is V_s , or 48 V. The inductor voltage when the switch is closed is $V_s - V_o = 48 - 18 = 30 \text{ V}$. The inductor voltage when the switch is open is $V_o = 18 \text{ V}$. Therefore, the inductor must withstand 30 V. The capacitor must be rated for the 18-V output.

RMS Value of Triangular Waveforms

- (a) A triangular current waveform like that shown in Fig. 2-9a is commonly encountered in dc power supply circuits. Determine the rms value of this current.
- (b) Determine the rms value of the offset triangular waveform in Fig. 2-9b.

Solution

(a) The current is expressed as

$$i(t) = \begin{cases} \frac{2I_m}{t_1}t - I_m & 0 < t < t_1 \\ \frac{-2I_m}{T-t_1}t + \frac{I_m(T+t_1)}{T-t_1} & t_1 < t < T \end{cases}$$

The rms value is determined from Eq. (2-38).

$$I_{\text{rms}}^2 = \frac{1}{T} \left[\int_0^{t_1} \left(\frac{2I_m}{t_1}t - I_m \right)^2 dt + \int_{t_1}^T \left(\frac{-2I_m}{T-t_1}t + \frac{I_m(T+t_1)}{T-t_1} \right)^2 dt \right]$$

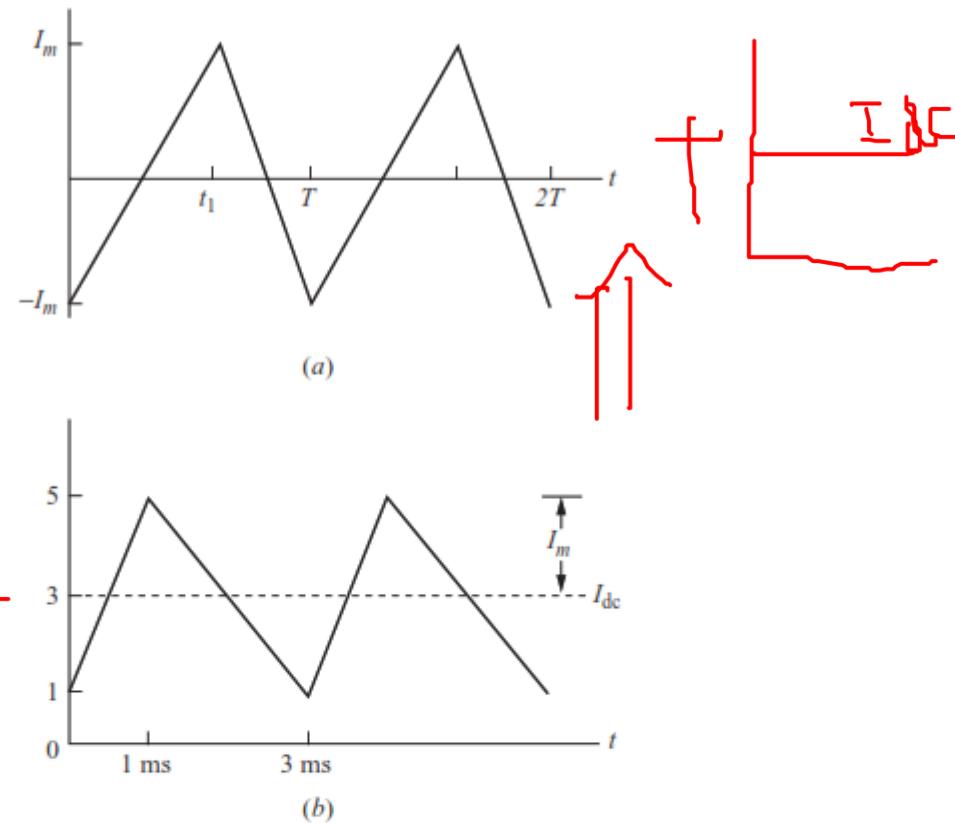


Figure 2-9 (a) Triangular waveform for Example 2-8; (b) offset triangular waveform.

The details of the integration are quite long, but the result is simple: The rms value of a triangular current waveform is

$$I_{\text{rms}} = \frac{I_m}{\sqrt{3}}$$

- (b) The rms value of the offset triangular waveform can be determined by using the result of part (a). Since the triangular waveform of part (a) contains no dc component, the dc signal and the triangular waveform are orthogonal, and Eq. (2-40) applies.

$$I_{\text{rms}} = \sqrt{I_{1,\text{rms}}^2 + I_{2,\text{rms}}^2} = \sqrt{\left(\frac{I_m}{\sqrt{3}}\right)^2 + I_{\text{dc}}^2} = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 + 3^2} = 3.22 \text{ A}$$