Inventory Management

— Problem Description

You are a store manager at a retail store and one of your tasks is to manage the inventory of P products that you sell. You accomplish this by deciding how much inventory of each product to procure from the warehouse on each day. You have shelves to stock the products. You have an exact estimate of how much demand each of your products will have over the next D days. Your products are perishables; hence some part of the products remaining on your shelf get wasted. Each of your products gets wasted in different proportions. As you sell different products, you will also need to replenish those products vis-a-vis the demand forecast. To make replenishments daily, you have a fixed fleet of vehicles. You cannot replenish more products than the capacity of your fleet. Consider further simplification by assuming that products can be replenished only once a day.

The above paragraph provides a general outline of the problem statement. The specifics are as follows:

- Number of shelves is same as P i.e. number of products. You cannot store different products in the same shelf. P will be provided as input
- The capacity of your fleet, in terms of maximum quantity across all products which it can replenish, will be provided as input
- It is not permitted to store more quantity of a product than the maximum capacity of the shelf at any point in the day
- Daily demand for all P products for next D days will be provided as input
- Wastage fraction of all products will be provided as input. Wastage fraction will apply only to those quantities of products which remain unsold at the end of each day. So by the end of the day, if there is N quantity of a given product remaining on the shelf after meeting the demand, and the wastage fraction is *w* for that given product, then the product wastage for that day will be **Round(N - (N * w))** for that product. Wastage for ith day is summation of quantities of all products that get wasted i.e. **Σ Round(N - (N * w)).** Final wastage will be summation of all products wasted across all days
- You are allowed to place your replenishment requests in a manner that also leads to a shortfall in the availability of a product (zero inventory)
- Any order which you cannot satisfy due to the lack of inventory, will be counted as shortfall. Final shortfall will be the summation of the shortfall of all products across all days.
- To better manage the wastage cost, your task is to order replenishments for your products such that
  - Wastage across all P products across all D days is less than a certain number which will be provided as input
  - Total shortfall across all P products across all D days is less than a certain number which will be provided as input
  - While meeting both the above constraints, the sum of wastage and shortfall across all products and days is minimized

It is clear from the above that there can be multiple correct answers that can satisfy the constraints on wastage and shortfall. Also, the way to correctly understand W and S is that they are desired ballparks of losses. Some solutions which slightly exceed W and/or S may also be acceptable in real life.

Hence, if your answer is strictly less than the W and S constraints, it will pass all the test cases. However, if your answer slightly exceeds these limits, then also there is a chance that you may pass some test cases. Hence the private test cases will progressively get tougher to pass. Your goal should be to minimize wastage (W) and shortfall (S)

Note: - Answers which have wastage <= (W * accepting factor) and shortfall <= (S * accepting factor), are acceptable. **Accepting factor can vary for different test cases and it will not be given in the input specification.**

For better understanding refer the *Examples* section.

Constraints

1 < D <= 100

1 < P <= 100

50 <= I <= 100

50 <= C < 200

R <= 5000

0 <= individual demand for a particular product <= C

0 < wastage fraction < 0.5

1 < Accepting factor for exceeding wastage and shortfall target < 2 (For public test cases)

1 < Accepting factor for exceeding wastage and shortfall target < 1.5 (For private test cases)

Input

First line contains an integer P which is the number of products

Second line contains an integer D which is the number of days for which the demand forecast is available

Third line contains an integer I which is the initial inventory i.e. same initial quantity for all products

Fourth line contains an integer C which is the shelf capacity for each product

Fifth line contains an integer R which is the maximum daily replenishment across all the products (one unit of each product is assumed to consume one unit of capacity)

Sixth line contains P space separated fractions depicting wastage fraction after every day for all products remaining on the shelf (round off to nearest integer)

Next D lines contains P space separated integers depicting daily individual demand of all products

Next line contains an integer W which is the maximum product wastage allowed across all products across all D days

Next line contains an integer S which is the maximum product shortfall allowed across all products across all D days

Output

D lines containing P space separated integers each, depicting the daily replenishment quantity ordered for each product such that all constraints are met

Time Limit (secs)

1

Examples

Example 1

Input

3

3

10

20

19

0.035 0.099 0.086

11 7 0

11 7 14

2 9 15

4

2

Output

10 7 0

7 4 8

3 6 10

Explanation:

Given that:

Number of Products P: 3

Number of days D: 3

Initial inventory of all products I: 10

Shelf capacity for each product C: 20

Maximum daily replenishment across all the products R: 19

Wastage fraction for all products: 0.035 0.099 0.086

Demand for Day1: 11 7 0

Demand for Day2: 11 7 14

Demand for Day3: 2 9 15

Wastage allowed W: 4

Shortfall allowed S: 2

For this input we can have one of the possible outputs as

Decision for Day1: 10 7 0

Decision for Day2: 7 4 8

Decision for Day3: 3 6 10

Now let's see how this output is getting accepted.

For Day 1

Current inventory: [10, 10, 10]

Current demand: [11, 7, 0] {From input matrix}

Current decision: [10, 7, 0] {From output matrix}

Total replenishment: 17 {Based on current decision}

Current decision is valid because total replenishment <= 19

After taking decision for day1, shelf will look like

Current preshelf: [20, 17, 10]

Current decision is valid because shelf capacity <= 20

Current shortfall: [0, 0, 0]

total shortfall(S) till now = 0 + 0 + 0 = 0

After full filling the current day's demand, inventory will look like this

Inv pre wastage: [9, 10, 10]

Now we have to remove the wastage products from the inventory, which is (product - (wastage fraction * product))

For product 1 = 9 - Round (9 * 0.035) = 9 {Wastage = 0}

For product 2 = 10 - Round (10 * 0.099) = 9 {Wastage = 1}

For product 3 = 10 - Round (10 * 0.086) = 9 {Wastage = 1}

Wasted quantity: [0, 1, 1]

Total wastage(W) till now = 0 + 1 + 1 = 2

Final inventory: [9, 9, 9]

Same goes for other days:

For Day 2

Current inventory: [9, 9, 9]

Current demand: [11, 7, 14]

Current decision: [7, 4, 8]

Total replenishment: 19

Current decision is valid because total replenishment <= 19

Current preshelf: [16, 13, 17]

Current decision is valid because shelf capacity <= 20

Current shortfall: [0, 0, 0]

total shortfall(S) till now = (0) + 0 + 0 + 0 = 0

Inv pre wastage: [5, 6, 3]

Wasted quantity: [0, 1, 0]

Total wastage(W) till now = (2) + 0 + 1 + 0 = 3

Final inventory: [5, 5, 3]

Day 3

Current inventory: [5, 5, 3]

Current demand: [2, 9, 15]

Current decision: [3, 6, 10]

Total replenishment: 19

Current decision is valid because total replenishment <= 19

Current preshelf: [8, 11, 13]

Current decision is valid because shelf capacity <= 20

Current shortfall: [0, 0, 2]

total shortfall(S) till now = (0) + 0 + 0 + 2 = 2

Inv pre wastage: [6, 2, 0]

Wasted quantity: [0, 0, 0]

Total wastage(W) till now = (3) + 0 + 0 + 0 = 3

Final inventory: [6, 2, 0]

So here total wastage is 3 which is <= W (4)

Shortfall is 2 which is <= S (2)

Hence, this answer is acceptable.

Here, since we have final Wastage and Shortfall less than input-provided constraints on W and S, the solution would always pass this test case. However, if Wastage and / or Shortage would have slightly exceeded 4 and 2 respectively, the acceptance factor would come into play.

Example 2

Input

4

5

23

46

60

0.188 0.423 0.387 0.025

1 17 23 28

5 35 1 23

9 33 4 17

11 22 6 14

10 23 22 10

55

3

Output

2 15 20 23

6 28 8 18

9 28 8 15

14 21 10 15

11 21 23 5

Explanation:

Given that:

Number of Products P: 4

Number of days D: 5

Initial inventory of all products I: 23

Shelf capacity for each product C: 46

Maximum daily replenishment across all the products R: 60

Wastage fraction for all products: 0.188 0.423 0.387 0.025

Demand for Day1: 1 17 23 28

Demand for Day2: 5 35 1 23

Demand for Day3: 9 33 4 17

Demand for Day4: 11 22 6 14

Demand for Day5: 0 23 22 10

Wastage allowed W: 55

Shortfall allowed S: 3

One of the possible outputs for this input can be:

2 15 20 23

6 28 8 18

9 28 8 15

14 21 10 15

11 21 23 5

Let's say here the Acceptance fraction is 1.66. Note that Acceptance Ratio will not be known to you.

The total wastage will be 59 which is <= (55 * 1.66) and total shortfall will be 5 which is also <= (3 * 1.66). Hence, this answer is acceptable.

Trivia

This question is inspired by a real-life problem statement that Retailers are trying to solve along with TCS. In real-life, there are thousands of products and hundreds of constraints. Interested participants can this link for TCS's approach to the solution.