Machine Learning Homework - 1

TASK 1

[LP 1] Show that $Y(t) W^T(t) X(t) < 0$

Suppose $X(t)$ be a misclassified point by weights (wt)

i.e $w^T(t) X(t)$ is -ve and our target value $Y(t)$ is +ve and vice-versa

Let $w^T(t) X(t) = -ve$ [misclassified Point $X(t)$ by $w(t)$]

$Y(t) = +ve$ [Data point in given data]

$$y^{(t)} w^T(t) X(t) < 0$$

in the similar way in the other case i.e $w^T(t) X(t) = +ve$ and $Y(t) = -ve$ when we multiply we always get -ve values i.e less than zero. And also as its misclassified $X(t)$ and $w^T(t) X(t)$ will be in opposite direction, angle between them will be obtuse. The dot product will be negative if two vectors have obtuse angle between them.

[LP 2]

TO Prove $Y_n w^T_{final} X_n > 0$ for all values of n

As our w^T_{final} is linear seperable we will never have an misclassified, Samples.

It classifies all the data points correctly i.e positives as positive and all negatives as negative.

If $w^T_{final} x_t = +ve$

$x_{(t)} = +ve$

and

$y(t)\ w^T_{(final)}\ x_t = +ve$

and similary if its negative sample $y(t) = -ve$ and $w^T_{(final)}\ x(t)$ will also be -ve as its correctly classified

Then always $y(t)\ w^T_{final}\ x_{(t)} \geq 0$

TASK 2 :

[LP 1] $h(x) = \text{Sign}(w^T x)$, where $w = [w_0 w_1 w_2]^T$
$x = [1\ x_1\ x_2]^T$.

Traditional line equation $x_2 = mx_1 + c$ where
$m \rightarrow$ slope and $c \rightarrow$ intercept

The traditional equation can also be written as

$$x_2 - mx_1 - c = 0 \rightarrow (1)$$

Comparing eq (1) with PLA's linear seperator,

$$w_0 = -c, \quad w_1 = -m, \quad w_2 = 1$$

from this we can say

$$m = -w_1, \quad c = -w_0$$

In terms of w_0, w_1, w_2 the slope of line will be $-w_1$ and intercept will $-w_0$

[LP 2]

(a) $w = [1\ 1\ -1]^T$ (b) $w = -[1\ 1\ -1]^T$

(a) As we proved above in a line equation that slope (m) is $-w_1$ and intercept $c = -w_0$ in this case

slope $= -1$, intercept -1

line eqv $\Rightarrow x_2 = -1 x_1 - 1$

$\Rightarrow x_2 = -x_1 - 1$