Assignment

January 4, 2024

Questions

1. If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.
2. If
$$y = \sin^{-1} x + \cos^{-1} x, \tag{1}$$
find $\dfrac{dy}{dx}$
3. Write the order and the degree of the differential equation
$$\left(\frac{d^4y}{dx^4}\right)^2 = \left(x + \left(\frac{dy}{dx}\right)^2\right)^3 \tag{2}$$
4. If a line has the direction ratios-18, 12, -4, then what are its direction cosines ?
5. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line
$$\left(\frac{X+3}{3}\right) = \left(\frac{4-Y}{5}\right) = \left(\frac{Z+8}{6}\right) \tag{3}$$
6. If * is defined on the set of all real numbers by * : a * b = $\sqrt{a^2 + b^2}$ find the identity element, if it exists in with respect to *

7. If $\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $\mathbf{kA} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b.

8.
$$\int \frac{sinx - cosx}{\sqrt{1 + sin2x}} dx, 0 < x < \frac{\pi}{2} \tag{4}$$

9.
$$\int \frac{sin(x-a)}{sin(x+a)} dx \tag{5}$$

10.
$$\int (logx)^2 dx \tag{6}$$

11. Form the differential equation representing the family of curves $y^2 = m(a^2 - x^2)$ by eliminating the arbitrary constants 'm' and 'a' .

12. Find a unit vector perpendicular to both the vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, where $\overrightarrow{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\overrightarrow{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

13. Show that the vectors $\hat{i} - 2\hat{j} + 3\hat{k}, -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\hat{i} - 3\hat{j} + 5\hat{k}$

14. Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P(B/A).

15. Let X be a random variable which assumes values $x1, x2, x3, x4$ such that $2P(X = x1) = 3P(X = x2) = P(X = x3) = 5P(X = x4)$. Find the probability distribution of X.

16. A coin is tossed 5 times. Find the probability of getting (i) at least 4 heads, and (ii) at most 4 heads.

17. Show that the relation R on the set Z of all integers, given by $R = (a, b) : 2divides(a - b)$is an equivalence relation.

18. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $fof(x) = xforallx \neq \frac{2}{3}$. Also, find the inverse of f.

19. If $\tan^{-1}x - cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$, $x > 0$,find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

20. Using properties of determinants, prove that $\begin{bmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{bmatrix}$=4abc

21. If sin y = x sin (a + y), prove that

$$\frac{dy}{dx} = \frac{sin^2(a+y)}{sina} \tag{7}$$

22. If $(sinx)^y = x + y$, find $\frac{dy}{dx}$.

23. If y=$(sec^{-1}x)^2$,$x > 0$, show that

$$x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0 \tag{8}$$

24. Find the equations of the tangent and the normal to the curve

$$y = \frac{x-7}{(x-2)(x-3)} \tag{9}$$

at the point where it cuts the x-axis.

25.

$$\int \frac{sin2x}{(sin^2x + 1(sin^2 + 3)} \tag{10}$$

26.

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx \tag{11}$$

and hence evaluate

27.

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{tanx}}. \tag{12}$$

28. Slove the differential equations

$$\frac{dy}{dx} = \frac{x + y}{x - y} \tag{13}$$

29. Slove the differential equations

$$(1 + x^2)dy + 2xydx = cotxdx \tag{14}$$

30. Let $\vec{a}, \vec{b}\, and\, \vec{c}$ be three vectors such that $|\vec{a}| = 1$

31. Find the value of λ for which the following lines are perpendicular to each other :

$$\frac{x - 5}{5\lambda + 2} = \frac{2 - y}{5} = \frac{1 - z}{-1}; \frac{x}{1} = \frac{y + \frac{1}{2}}{2\lambda} = \frac{z - 1}{3} \tag{15}$$

Hence, find whether the lines intersect or not.

32. $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$,find A^{-1}

33. Hence, solve the following system of equations :

$$x + y + z = 6, \tag{16}$$
$$y + 3z = 11 \tag{17}$$
$$x - 2y + z = 0 \tag{18}$$

34. Find the inverse of the following matrix, using elementary transformations: $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

35. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

36. Find the area of the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$,using integration.

37. Find the area of the region bounded by the curves

$$(x-1)^2 + y^2 = 1 \tag{19}$$
$$x^2 + y^2 = 1 \tag{20}$$

using integration.

38. Find the vector and cartesian equations of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$, and $(5, 3, -3)$.Also, find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

39. Find the equation of the plane passing through the intersection of the planes $\overrightarrow{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\overrightarrow{r}.2\hat{i} + 3\hat{j} - \hat{k} + 4 = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis .

40. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$, find the value of 'n'.

41. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ¡ 50 each for type A and ¡ 60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit.

42. Find the integrating factor of the differential equation $x\frac{dy}{dx} - 2y = 2x^2$

43. Find $\frac{dy}{dx}$,if $xy^2 - x^2 = 4$

44. If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

45. If a line has the direction ratios -18, 12, -4, then what are its direction cosines ?

46. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line

$$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}. \tag{21}$$

47. Let * be a binary operation on $R - \{-1\}$defined by a * b = $\frac{a}{b+1}$,for all a, b $\in R - \{-1\}$. Show that * is neither commutative nor associative in R - $\{-1\}$.

48. If A = $\begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}$, then show that A^3 = A .

49. Form the differential equation representing the family of curves y^2 = m $\left(a^2 - x^2\right)$ by eliminating the arbitrary constants 'm' and 'a'.

50.

$$\int \frac{\sin x - \cos x}{\sqrt{1+2x}} dx, 0 < x < \pi/2 \tag{22}$$

51.

$$\int \frac{\sin(x-a)}{\sin(x+a)} dx \tag{23}$$

52.

$$\int (logx)^2 \, dx \tag{24}$$

53. Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find P$\left(\frac{B}{A}\right)$.

54. Let X be a random variable which assumes values x1, x2, x3, x4 such that

$$2P(X = x1) = 3P(X = x2) = P(X = x3) = 5P(X = x4). \tag{25}$$

Find the probability distribution of X.

55. A coin is tossed 5 times. Find the probability of getting (*i*) at least 4 heads, and (*ii*) at most 4 heads.

56. Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i}$ -7 $\hat{j}$ +7$\hat{k}$ and $\vec{b}$= 3 $\hat{i}$ -2 $\hat{j}$+ 2 $\hat{k}$.

57. Show that the vectors $\hat{i}$ -2 $\hat{j}$+ 3 $\hat{k}$,-2 $\hat{i}$+ 3 $\hat{j}$ -4 $\hat{k}$ and $\hat{i}$ -3 $\hat{j}$+ 5 $\hat{k}$ are coplanar.

58. Show that the relation R on the set Z of all integers, given by R = $\{(a, b) : 2divides(a - b)\}$ is an equivalence relation.

59. If f (x) = $\frac{4x + 3}{6x - 4}$,x $\neq$ $\frac{2}{3}$,show that fof(x) = x for all x $\neq$ $\frac{2}{3}$.Also, find the inverse of f.

60. sin y = x sin $(a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a} \tag{26}$$

61. If (sin $x)^y$ = x + y, find $\frac{dy}{dx}$.

62.

$$If \sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}, \tag{27}$$

then find the value of x.

63. Using properties of determinants, prove that

$$\begin{bmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ac & bc & c^2 + 1 \end{bmatrix} = 1 + a^2 + b^2 + c^2 \tag{28}$$

64. If y = $(\cot^{-} 1x)^2$, show that

$$\left(x^2+1\right)^2 \frac{d^2y}{dx^2} + 2x\left(x^2+1\right)\frac{dy}{dx} = 2 \tag{29}$$

65. Find

$$\int \frac{\sin 2x}{\left(\sin^2 x + 1\right) + (\sin)^2 x + 3} dx \tag{30}$$

66.

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-c)\,dx \tag{31}$$

67.

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}. \tag{32}$$

68. Solve the differential equation:

$$\frac{dy}{dx} = \frac{x+y}{x-y} \tag{33}$$

69. Solve the differential equation:

$$\left(1+x^2\right)dy + 2xydx = \cot xdx \tag{34}$$

70. Let $\overrightarrow{a}$,$\overrightarrow{b}$ and $\overrightarrow{c}$ be three vectors such that $|\overrightarrow{a}| = 1$,$|\overrightarrow{b}|= 2$ and $|\overrightarrow{c}| =3$ .If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$; and $\overrightarrow{b}$,*overrightarrowc* are perpendicular to each other, then find $|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}|$

71. Find the equations of the tangent and normal to the curve

$$y = \frac{x-7}{(x-2)(x-3)} \tag{35}$$

at the point where it cuts the x-axis.

72. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x + \frac{1}{2}\cos 2x, 0 \leq x \leq \frac{\pi}{2} \tag{36}$$

73. If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$,find A^2 and show that $A^2 = A^{-1}$.

74. Using matrix method, solve the following system of equations :

$$2x - 3y + 5z = 13 \tag{37}$$

$$3x + 2y - 4z = -2 \tag{38}$$

$$x + y - 2z = -2 \tag{39}$$

75. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 'n' black balls. One of the two boxes, box I and box II is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box II is $\frac{3}{5}$,find the value of 'n'.

76. Find the vector and cartesian equations of the plane passing through the points $(2, 5, -3)$,$(-2, -3, 5)$ and $(5, 3, -3)$. Also, find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

77. Find the equation of the plane passing through the intersection of the planes $\vec{r}\,.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\vec{r}.\left(2\hat{i} + 3\hat{j} - \hat{k} + 4\right) = 0$ and parallel to x-axis. Hence, find the distance of the plane from x-axis.

78. Find the area of the triangle whose vertices are $(-1, 1)$, $(0, 5)$, and $(3, 2)$, using integration.

79. Find the area of the region bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ using integration.

80. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each

for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹50 each for type A and ₹60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit .