Assignment

January 5, 2024

CBSE 2019 Mathematics Questions

1 Matrix

1. If A is a square matrix of order 3 with $|A| = 4$, then write the value of $|-2A|$.

2. If $\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $k\mathbf{A} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k, a and b.

3. Using properties of determinants, prove that

$$\begin{bmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{bmatrix} = 4abc.$$

4. Find the inverse of the following matrix, using elementary transformations:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}.$$

5. If $\mathbf{A} = \begin{bmatrix} -3 & 6 \\ -2 & 4 \end{bmatrix}$, then show that $A^3 = A$.

6. Using properties of determinants, prove that

$$\begin{bmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{bmatrix} = 1 + a^2 + b^2 + c^2.$$

7. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$,find A^2 and show that $A^2 = A^{-1}$.

8. Using matrix method, solve the following system of equations :

$$\begin{aligned} 2x - 3y + 5z &= 13 \\ 3x + 2y - 4z &= -2 \\ x + y - 2z &= -2. \end{aligned}$$

9. If $\left[A\right] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$,find A^{-1}. Hence, solve the system of equations :

$$\begin{aligned} x + y + z &= 6, \\ y + 3z &= 11, \\ \text{and} \quad x - 2y + z &= 0 \end{aligned}$$

10. If $A = \begin{bmatrix} 3 & 9 & 0 \\ 1 & 8 & -2 \\ 7 & 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 & 2 \\ 7 & 1 & 4 \\ 2 & 2 & 6 \end{bmatrix}$, then find the matrix $B'A'$.

2 Differentiation

11. If

$$y = \sin^{-1} x + \cos^{-1} x,$$

find $\dfrac{dy}{dx}$.

12. Write the order and the degree of the differential equation

$$\left(\frac{d^4y}{dx^4}\right)^2 = \left(x + \left(\frac{dy}{dx}\right)^2\right)^3.$$

13. If $\sin y = x\sin(a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

14. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$.

15. If $y = \left(\sec^{-1} x^2\right), x > 0$, show that

$$x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$$

16. Solve the differential equations

$$\frac{dy}{dx} = \frac{x + y}{x - y}$$

17. Solve the differential equations

$$(1 + x)^2\, dy + 2xy dx = \cot x \;\; dx.$$

18. Find the integrating factor of the differential equation

$$x \;\frac{dy}{dx} - 2y = 2x^2$$

19. Find $\frac{dy}{dx}, \;\; if \;\; xy^2 - x^2 = 4$

20. If $y = \left(\cot^{-1} x\right)^2$, show that

$$\left(x^2 + 1\right)^2 \frac{d^2y}{dx^2} + 2x\left(x^2 + 1\right)\frac{dy}{dx} = 2.$$

21. y=$\sin^{-1} + \cos^{-1}$, find $\dfrac{dy}{dx}$.

22. If $e^y(x+1) = 1$, then show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$.

23. Find $\dfrac{dy}{dx}$, if $y = \sin^{-1}\left(\dfrac{2^{x+1}}{1+4^x}\right)$.

24. Solve the differential equation :

$$\frac{dy}{dx} = \frac{x+y}{x-y}.$$

25. Solve the differential equation :

$$\left(1+x^2\right)dy + 2xydx = \cot x \;\; dx.$$

26. Form the differential equation representing the family of curves $y^2 = m\left(a^2 - x^2\right)$ by eliminating the arbitrary constants $'m'$ and $'a'$.

3 Probability

27. Let X be a random variable which assumes values $x1, x2, x3, x4$ such that

$$2P(X = x1) = 3P(X = x2) = P(X = x3) = 5P(X = x4).$$

Find the probability distribution of X.

28. Mother, father and son line up at random for a family photo. If A and B are two events given by A = Son on one end, B = Father in the middle, find $P(B \mid A)$.

29. A coin is tossed 5 times. Find the probability of getting

   i. at least 4 heads

   ii. at most 4 heads.

30. There are two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and $'n'$ black balls. One of the two boxes, box I and box I is selected at random and a ball is drawn at random. The ball drawn is found to be red. If the probability that this red ball comes out from box I is $\frac{3}{5}$,find the value of $'n'$.

31. Find the mean and variance of the random variable X which denotes the number of doublets in four throws of a pair of dice.

32. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling. The profit is ₹50 each for type A and ₹60 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize profit ? Formulate the above LPP and solve it graphically and also find the maximum profit .

4 Function's

33. Show that the relation R on the set Z of all integers, given by $R = \{(a, b) : 2\text{divides } (a - b)\}$ is an equivalence relation.

34. If $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right), x > 0$,find the value of x and hence find the value of $\sec^{-1}\left(\frac{2}{x}\right)$.

35. Let $*$ be a binary operation on $R - \{-1\}$ defined by $a * b = \frac{a}{b+1}$,for all $a, b \in R - \{-1\}$. Show that $*$ is neither commutative nor associative in $R - \{-1\}$.

36. Find the intervals in which the function f given by $f(x) = 4x^3$ - $6x^2 - 72x + 30$ is

    (a) strictly increasing,

    (b) strictly decreasing .

37. Find the local maxima and local minima, if any, of the following function. Also find the local maximum and the local minimum values, as the case may be :

$$f(x) = \sin x + \frac{1}{2}\cos 2x, 0 \le x \le \frac{\pi}{2}.$$

38. If $*$ is defined on the set **R** of all real numbers by :$a * b = \sqrt{a^2 + b^2}$, find the identity element,if it exists in **R**with respect to $*$.

39. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $f(x) = x$ for all $x \neq \frac{2}{3}$.Also, find the inverse of f.

5 Integration

40. Find

$$\int \frac{\sin 2x}{(\sin^2 x + 1)(\sin^2 x + 3)} dx$$

41. Prove that

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

and hence evaluate

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}.$$

42. Find

$$\int \frac{\sin x - \cos x}{\sqrt{1+2x}} dx, 0 < x < \pi/2.$$

43. Find

$$\int \frac{\sin(x-a)}{\sin(x+a)}dx.$$

44. Find

$$\int (\log x)^2\,dx$$

45. Find

$$\int_a^b \frac{\log x}{x}dx.$$

46. Evaluate

$$\int_1^4 \left(1 + x + e^2 x\right) dx$$

as limit of sums.

6 Algebra

47. Find the value of λ for which the following lines are perpendicular to each other :

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1};\ \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}.$$

Hence, find whether the lines intersect or not.

48. Find the value of p for which the following lines are perpendicular :

$$\frac{1-x}{3} = \frac{2y-14}{2p} = \frac{z-3}{2};\ \frac{1-x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}.$$

7 Cricle's

49. Show that the height of a cylinder, which is open at the top, having a given surface area and greatest volume, is equal to the radius of its base.

8 Vectors

50. Find the vector and Cartesian equations of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$, and $(5, 3, -3)$.Also, find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

51. If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines ?

52. Find the equation of the plane passing through the intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 1$ and $\vec{r}.2\hat{i}+3\hat{j}-\hat{k}+4 = 0$ and parallel to $x - axis$. Hence, find the distance of the plane from $x - axis$.

53. Find the Cartesian equation of the line which passes through the point $(-2, 4, -5)$ and is parallel to the line

$$\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}.$$

54. Find a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a} = \hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b} = 3\hat{i}-2\hat{j}+2\hat{k}$.

55. Show that the vectors $\hat{i}-2\hat{j}+3\hat{k}, -2\hat{i}+3\hat{j}-4\hat{k}$ and $\hat{i}-3\hat{j}+5\hat{k}$ are coplanar.

56. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 2$ and $|\vec{c}| = 3$ .If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$; and $\vec{b}, \vec{c}$ are perpendicular to each other, then find $\left|3\vec{a}-2\vec{b}+2\vec{c}\right|$.

57. Find the direction cosines of the line joining the points $P(4, 3, -5)$ and $Q(-2, 1, -8)$.

9 Trigonometry

58. If

$$\sin^{-1}\left(\frac{3}{x}\right) + \sin^{-1}\left(\frac{4}{x}\right) = \frac{\pi}{2}$$

then find the value of x.

59. Find the value of x, if $\tan\left(\sec^{-1}\left(\frac{1}{x}\right)\right) = \sin\left(\tan^{-1} 2\right), x > 0$.

10 Intersection Of Conics

60. Find the area of the region bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ using integration.

61. Find the area of the region

$$\left\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x + 2, -1 \leq x \leq 3\right\}.$$

62. Find the area of the region bounded by the curves

$$(x-1)^2 + y^2 = 1$$
$$x^2 + y^2 = 1$$

using integration.

63. Find the equations of the tangent and the normal to the curve

$$y = \frac{x-7}{(x-2)(x-3)}$$

at the point where it cuts the x-axis.

64. Find the area of the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$,using integration.