RESEARCH PROPOSAL

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My research is in the area of algebraic combinatorics. I am interested in (a) applying representation theory and finite group theory to compute some numerical invariants of certain combinatorial objects; and (b) to investigate group theoretic/linear algebraic analogues of some classical results in combinatorics.

My current focus is on the permutation groups analogue of the classical Erdős-Ko-Rado (EKR) theorem from extremal set theory. In particular, I am interested in the sizes of intersecting sets in permutation groups. I am invested in the many open problems arising from my work in [8] and from related work in [20]. The PIMS fellowship would give me the opportunity to work on these problems with Prof. Meagher, who is a prolific researcher in this area. I am also interested in investigating the EKR-module property defined in [13]. In §2, I describe the main results of [8] and [20], and in §2.1, I describe the related problems I wish to solve in the near future.

My doctoral dissertation was on computing the Smith normal forms of some families of combinatorial incidence matrices. I am interested in working on problems related to computing various numerical invariants of combinatorial matrices.

While my main focus is on intersecting sets in permutation groups, I also have plans to conduct some open-ended investigations related to few of my other works. Before delving into the specifics of my project on permutation groups, I will give a brief description of the projects I undertook.

1. Brief Summary of recent work.

Intersecting sets in Permutation groups. During my first year as a postdoc at SUSTech, I started working on intersecting sets in permutation groups. A subset S of a permutation group G is called an intersecting set, if for every $s, t \in S$, the permutation st^{-1} fixes a point. Cosets of point stabilizers are natural examples of intersecting sets, such intersecting sets are called the *canonical* intersecting sets. An intersecting set of maximum possible size is called a maximum intersecting set. The intersection density $\rho(G)$ of G is the ratio of size of a maximum intersecting set and the size of a point stabilizer. In [8], we investigated upper bounds on intersection densities. This was also the the theme in [20].

I am also interested in the EKR module property of permutation groups. In [13], the authors show that every 2-transitive group satisfies the EKR module property, that is, the characteristic vectors of any maximum intersecting set is a linear combination of the characteristic vectors of its canonical intersecting sets. I found another family of groups satisfying this property. I am interested in investigating groups with this property.

Questions for future research: My main goal is to investigate the upper bounds on intersection densities of different classes of groups. In §2.1, I give a list of specific problems I wish to solve.

Borewein Conjectures. I worked on problems related to signs of coefficients of polynomials appearing in the study of restricted partitions in number theory. In [6], we worked on coefficients of polynomials related to the famous Borewein conjecture. Let p be a prime, n, s be positive integers, and D be the set of non-multiples of p less than $N = p(n+1)$. We consider polynomials of the form $T_{n,p,s}(q) = \prod_{j \in D}(1 - q^j)^s$. Borwein's conjectures are as follows:

(1) when $p = 3$, $s = 1$, the coefficient q^b is (a) positive when $3 \mid b$; and (b) negative when $3 \nmid b$.
(2) when $p = 3$, $s = 2$, the coefficient q^b is (a) positive when $3 \mid b$; and (b) negative when $3 \nmid b$.
(3) when $p = 5$, $s = 1$, the coefficient q^b is (a) positive when $5 \mid b$; and (b) negative when $5 \nmid b$.

The first conjecture was proved by Chen Wang, using some subtle analytic techniques. As far as elementary elementary approaches to these conjectures are concerned, a few authors ([22], [9]) attempted to find an asymptotic formula for certain partial sums of coefficients of $T_{n,p,s}$. The coefficients in these partial sums are indexed by an arithmetic progression, all of whose terms are congruent mod p. The aim is to have a big common difference so our sum has fewer terms/ coefficients. In the case $p = 3, s = 1$, the common difference in Zaharescu's work [22] was improved upon by J.Li in [9]. He used a sieving technique given in [10]. In our work, we extended the techniques used by Jiyou Li to the general case. We found asymptotic estimates of partial sums of coefficients of $T_{n,p,s}$, the coefficients

of these partial sums are indexed by an AP whose common difference is N and all of whose terms are congruent to $mod p$. These estimates were compatible with the Borwein conjectures mentioned above. In [7], using similar techniques, we found asymptotic estimates of partial sums of coefficients of $R_{n,s} = \prod_{j=1}^{n}(1-q^j)^s$. These are truncated versions of some interesting infinite products appearing in number theory. For instance, the product $q\prod_{j=1}^{\infty}(1-q^j)^{24}$ is the Ramanujan Tau function.
Questions for future research: Find the asymptotic estimate of partial sum of coefficients (of $T_{n,p,s}$) indexed by an AP whose common difference d is such that $d \mid p$ and $d > N$.

Smith Normal Froms of Incidence matrices. I obtained my PhD in 2019. My doctoral thesis at University of Florida was on computing the Smith normal forms(SNF) of combinatorial incidence matrices. The SNF of an incidence matrix is a powerful invariant that may help distinguish the underlying incidence structure. I computed the SNF's of the adjacency and Laplacian matrices of graphs. The *critical* (respectively Smith) group of a graph is the finite part of abelian group whose invariant factors are given by the SNF of its Laplacian (respectively adjacency) matrix. The critical groups of various graphs arise in combinatorics in the context of chip firing games (cf. [3]), as the abelian sandpile group in statistical mechanics (cf. [4]), and also in arithmetic geometry. One may refer to [12] for a discussion on these connections. In [19] and [17], I computed the Smith and critical groups of certain families strongly regular graphs that admit some rank 3 permutation groups as automorphisms. A skew-Hadamard matrix of order n is an $n\times n$ matrix of 1's and -1's satisfying $HH^\intercal = nI$ and $H+H^\intercal = 2I$. Two skew-Hadamard matrices are considered equivalent if one can be obtained from the other by negating rows or columns, or by interchanging rows or columns. It was shown in [21] that the existence of a Doubly Regular Tournament (DRT) on n vertices is equivalent to the existence of a skew Hadamard Matrix of size $n+1$. It was shown in [16] that the Smith normal forms of all skew-Hadamard matrices of a certain size have the same Smith normal forms. So SNFs do not distinguish nonequivalent skew-Hadamard matrices. Given a skew-Hadamard matrix H, let $T(H)$ be the corresponding DRT. The *critical* group of $T(H)$ is also an invariant of H. In [18], I used this invariant to show the non-equivalence of some families of skew-Hadamard matrices. The key observation in all the three problems is as follows:
Let $\Gamma = (V, E)$ be a graph. The adjacency and Laplacian matrices can be treated as linear transformations on End($\mathbb{Z}V$), where $\mathbb{Z}V$ is the free Z-module with V as a basis. If G is a group of automorphisms of Γ, then $\mathbb{Z}V$ is a $\mathbb{Z}G$-module, and since G preserves the adjacency, the adjacency and Laplacian matrices are G-module maps. This property of the matrices enables us to use representation theory of G to compute the Smith normal form of the adjacency and Laplacian matrices.
Questions for future research: I am interested in finding the Smith normal forms of incidence matrices associated with a few other families of strongly regular graphs, for example the family of strongly regular graphs associated with Latin squares. In [1], the authors investigated the behavior of 2-ranks of adjacency matrices of graphs under the Siedel and Godsel-Mackay switching procedure. These "switches" are procedures on a graph that produce new graphs with the same spectrum. I would like to do a similar investigation on the behavior of Smith normal forms of incidence matrices of graphs under these switching procedures.

Current work. Apart from working on intersecting sets in permutation groups, I am currently investigating the existence of m-spreads in certain point-line geometries. Let q be an even power of a prime and let $\mathbb{F}_q$ be a field of order q. Let $V = \mathbb{F}_q^5$ be endowed with a Hermitian form H. A subspace $W < V$ is said to be isotropic if for any $w_1, w_2 \in W$, we have $H(w_1, w_2) = 0$. By $\mathcal{H}(4, q^2)$, we denote the point-line geometry whose points are the isotropic one dimensional subspaces of V, and whose lines are the isotropic two dimensional subspaces of V. An m-spread is a collection M of lines such that each point is on exactly m lines of M. We are currently investigating the existence of m-spreads in $\mathcal{H}(4, q^2)$.

2. Intersecting sets of permutation groups.

The Erdős-Ko-Rado (EKR) theorem is a classical result of Erdős-Ko-Rado in extremal set theory, which states that, for a collection $\mathfrak{S}$ of k-subsets of a set of size n with $k < n/2$, if any two members of $\mathfrak{S}$ intersect, then $|\mathfrak{S}| \leqslant \binom{n-1}{k-1}$, and equality holds if and only if $\mathfrak{S}$ consists of all k-subsets which contains a common element. Given a permutation group $G < S_n$ acting on $[n] := \{1, 2, \dots, n\}$, we say two permutations $g, h \in G$ intersect if there exists $i \in [n]$ such that $i^g = i^h$. In view of the classical EKR theorem, we wish to determine the size and structure of the largest collections of pairwise intersecting permutations in G. A subset $S \subset G$ is said to be an *intersecting* subset if any two permutations in S are intersecting. We say that S is a *maximum* intersecting subset (MIS) if it is an intersecting subset of the largest possible size. We note that the stabilizer G_i of any $i \in [n]$ is an intersecting subset, so for any MIS $S \subset G$, we must have $|S| \geq |G_i|$. We say that G satisfies the EKR property, if G_i is an MIS, that is, every MIS in G has the same size as a stabilizer. Now the cosets of stabilizer subgroups in G are intersecting sets. We say that G satisfies the strict-EKR property if cosets of stabilizer subgroups are the only maximum intersecting subsets. A few classes of permutation groups satisfying the strict-EKR property are as follows: (a)Alternating groups of degree n (c.f [2]); (b)the groups $PGL(2, q)$ (c.f [14]) and $PSL(2, q)$ (c.f [11]) acting on 1-spaces . It was shown in [15] that all 2-transitive permutation groups have the EKR property. Not all 2-transitive permutation groups have the strict-EKR property, for example the affine group $AGL(1, p^d) = \mathbb{Z}_p^d : \mathbb{Z}_{p^d-1}$ is 2-transitive but does not have the strict-EKR property unless $(p, d) = (3, 1)$. A recent result [13], shows that the characteristic vector of any maximum intersecting set of 2-transitive groups is a linear combination of characteristic vectors of cosets of point stabilizers. Groups with this property are said to have the *EKR-Module* property.

Intersecting sets can be studied as co-cliques/independent sets of certain Cayley graphs. Let D denote the set of fixed-point free permutations in G. Then the intersecting sets in G are exactly the co-cliques of the Cayley graph $\Gamma_G := Cay(G, D)$ on the group G with "connection" set D. This Cayley graph is known as the derangement graph of G. As Γ_G is a vertex-transitive graph, we may use the well known clique-co-clique bound ([5, 2.1.1]) which states that given a clique C and a co-clique S, we have $|C||S| \leq |G|$. Another commonly used result is the Hoffman bound (([5, 2.4.1])) on the sizes of co-cliques in regular graphs. For any co-clique S in Γ_G, the Hoffman bound implies that $|S| \leq \frac{|G|}{1-\frac{|D|}{\tau}}$, where τ is the smallest eigenvalue for the adjacency matrix of Γ_G. Given $X \subset G$, let v_X denote the vector $\sum_{x \in X} x \in \mathbb{C}G$. It is also known that Hoffman bound is tight if and only if $v_S - \frac{|S|}{|G|} v_G$ is an eigenvector with eigenvalue τ. The eigenvalues of this Cayley graph can be expressed in terms of the characters of G, and the corresponding eigenspaces are certain two-sided ideals of the group ring $\mathbb{C}G$. These results have been used extensively in the study ([15, 13]) of intersecting sets of 2-transitive groups.

One direction of research is to investigate groups that satisfy the various EKR properties we defined in the previous paragraph. Given a permutation group G acting on $[n]$, by $\rho(G)$, we denote the ratio $|S|/|G_i|$, where S is a maximum intersecting subset of G, and G_i is the stabilizer of some $i \in [n]$. We note that G satisfies the EKR property if and only if $\rho(G) = 1$. It is natural to wonder about an upper bound for $\rho(G)$. In [8], C.Li, S.Song, and I showed:

Theorem (Li-Song-Pantangi)**.** *Given $M > 0$, $\epsilon \in (0, 1)$,*
(i) There exists a permutation group $G \leq S_n$ such that $\rho(G) > M$ and $(1-\epsilon)\sqrt{n} < \rho(G) < \sqrt{n}$, and
(ii) there exists a primitive permutation group $G \leq S_n$ such that $\rho(G) > M$ and $(1-\epsilon)\frac{\sqrt{n}}{\sqrt{2}} < \rho(G) < \frac{\sqrt{n}}{\sqrt{2}}$.

This shows that for a general permutation groups there is no absolute upper bound on intersection densities. We conjectured that for any permutation group G acting on n points, $\rho(G) \leq \sqrt{n}$. However, this was disproved by Razafimahatratra, Meagher, and Spiga in [20]. The authors of [20] observed that by Jordan's theorem, D cannot be an empty set and thus there is a clique C of size 2. Now application of the clique-co-clique bound shows that $|C|\rho(G) = 2|S|/|G_i| \leq |G|/|G_i| = n$. Thus we must have $\rho(G) \leq \frac{n}{2}$. The main result of [20] is:

Theorem (Razafimahatratra-Meagher-Spiga)**.** *Let $G < S_n$ be a permutation group, then the following hold.*
(a) If the derangement graph Γ_G is bipartite, then $n \leq 2$; and
(b) If $n \geq 3$, then Γ_G has a triangle.

Now by clique-co-clique bound, we get:

Corollary. *If G is a transitive group on n points with $n > 3$, then $\rho(G) = \frac{n}{3}$.*

With the help of a computer, the authors then went on to show that this bound is tight. If $G < S_n$ is a group with a complete-tripartite derangement graph, then it follows that it has a co-clique of size $n/3$ and thus $\rho(G) = n/3$. The following result followed via an exhaustive computer search.

Theorem. [20, Theorem 5.1] *Upto the degree 48, there are four permutation groups whose derangement graphs are complete tripartite graphs.*

2.1. **Future research.** Three among the four groups in the above theorem also serve as counter examples to the conjecture in [8] which states that for all $G < S_n$, $\rho(G) \leq \sqrt{n}$. I am currently interested in the following open problems arising from [20] and [8].

Problem 1. *(*[20, Problem 6.1]*) Given $n \in \mathbb{N}$, let $\mathcal{I}_n = \{\rho(G) | G$ is a transitive group of degree $n\}$. Determine $I(n) := max(\mathcal{I}_n)$ as a function of n.*

Any 2-transitive group T satisfies the EKR property, and hence $\rho(T) = 1$. However the main results of [8] shows that this is not true for quasi-primitive groups in general. This now leads to the following question.

Problem 2. *Given $n \in \mathbb{N}$, let $\mathcal{Q}_n = \{\rho(G) |\ G$ is a quasi-primitive transitive group of degree $n\}$. Determine $Q(n) := max(\mathcal{Q}_n)$ as a function of n.*

We note that for a prime p, the Sylow-p subgroup P of any transitive group of degree p is a regular subgroup and thus a clique in the derangement graph. Now the clique-coclique bound may be used to show that $I(p) = 1$ for all primes p (c.f [20, Lemma 2.2]). Based on computational evidence on the transitive groups of order 48, the authors conjecture the following.

Conjecture 1. *(*[20, Conjecture 6.6]*)*

1. *If n is either a prime power or a product of two odd primes, then $I(n) = 1$.*
2. *If q is a prime, then $I(2q) = 2$.*

Given $G \leq S_n$, [20, Theorem 1.4] states that if Γ_G is bipartite, then $n \leq 2$. Although we have seen that $\rho(G) \leq \frac{n}{3}$, and that this bound is tight, there is no evidence to see that there are infinitely many G with $\rho(G) = \frac{n}{3}$.

Problem 3. *(*[20, Question 6.1]*) Let G be a transitive group of degree n such that Γ_G is a k-partite graph. Is there an upper bound on n as a function of k only?*

The following conjectures about the structure of derangement graphs is based on computational evidence.

Conjecture 2. *(*[20, Conjecture 6.2]*) If G is a transitive group of degree n with intersection density $n/3$, then its derangement graph Γ_G is complete tripartite.*

Conjecture 3. *(*[20, Conjecture 6.6]*) If n is an even non-power of 2, then there is a transitive group G of degree n, whose derangement graph Γ_G is a complete $n/2$-partite graph.*

Let G be a transitive group of degree n. We say a subgroup $H < G$ is an *intersecting* subgroup if it is an intersecting set. It is easy to see that $H < G$ is an intersecting subgroup if and only if every element of H is conjugate to an element of a stabilizer subgroup of G. In many cases, there is always a maximum intersecting set which is also a subgroup. We note that there are maximum intersecting sets that are not cosets of subgroups. This leads to the following question.

Problem 4. *(*[8, Problem 1.8]*) Given a transitive group G, is there a subgroup in the set of maximum intersecting sets in G?*

This question lead us to investigate the weaker problem of finding all intersecting subgroups of groups. As a starting point, in [8, Theorem 1.7], we characterized all the intersecting subgroups of primitive permutation groups isomorphic to $G := PSL(2, p)$. In other words, given a maximal subgroup G_ω in G, we found all the subgroups H which are intersecting sets with respect to the action of G on the coset space $\Omega = G/G_\omega$. Among other things, we found that for $(p, G_\omega) \notin \{(29, D_{30}),\ (31, D_{30})\}$, all the maximum intersecting subgroups are conjugates of G_ω. Here, by D_n, we denote a subgroup of G isomorphic to the dihedral group of order n. In the case $(p, G_\omega) \in \{(29, D_{30}),\ (31, D_{30})\}$, all the maximum intersecting subgroups are isomorphic to A_5. If the answer to the question in the previous problem is true, then almost all primitive actions of $PSL(2, p)$ will satisfy the EKR property.

Let $p \geqslant 5$ be a prime and $G = PSL(2, p)$ be a primitive permutation group on Ω, with $G_\omega \cong D_{p+1}$ (with $\omega \in \Omega$). We show in [8, Example 2.4] that G also satisfies the EKR property in the case $p \equiv 3 \pmod 4$. Now by [8, Theorem 1.7], if $p \neq 29$, all maximum intersecting subgroups are stabilizer subgroups. So, if the answer to the above problem is positive, then the action of G on Ω satisfies the EKR property whenever $p \neq 29$. The group $\tilde{G} = PGL(2, p)$ has a natural action on Ω. We show in [8, Example 2.4] that $\tilde{G}$ also satisfies the EKR property. This leads to the following question.

Problem 5. *Let $p \geqslant 5$ be a prime with $p \equiv 1 \pmod 4$ and $p \neq 29$. Consider the primitive permutation action of $G = PGL(2, p)$ whose stabilizers are isomorphic to D_{p+1} (with $\omega \in \Omega$). Does G satisfy the EKR property?*

There was common theme in all the known examples of groups with large intersecting density. Given a permutation group G with $H < G$ as a stabilizer, a subgroup $H < K < G$ is an intersecting set if and only if $K \subset \cup_{g \in G} g^{-1}Hg$. A subgroup K satisfying this property is a maximum intersecting set if and only if the action of G on G/K satisfies the EKR property. It is now natural to ask the following question.

Problem 6. *Let G be a group and $H < G$ be such that the action of G on G/H satisfies the EKR property. What can be said about the EKR property of G with respect to the action of G on G/E, where $H < E < G$? What can be said about the EKR property of G with respect to the action of G on G/E, where $H < E < G$?*

In [8], we found some almost simple primitive permutation groups with large intersecting subgroups. Our proof was based on the observation described before the previous problem. This now leads to the following question, which asks to classify almost simple primitive permutation groups without large intersection subgroups.

Problem 7. *(*[8, Problem 1.6]*) Classify all almost simple primitive permutation groups $G \leqslant Sym(\Omega)$ such that $|H| \leqslant |G|/|\Omega|$, for any subgroup $H < G$ with $H \subset \bigcup_{\omega\in\Omega} G_\omega$.*

I am also interested in other problems about intersecting sets in permutation groups. I conducted some investigations on the EKR module property defined in [13]. Consider a permutation group G acting on a set Ω. Given $S \subset G$, let $\mathbf{1}_S$ denote the element $\sum_{s\in S} s$ of the group algebra $\mathbb{C}G$. For $\alpha, \beta \in \Omega$, define $S_{\alpha,\beta} := \{g \in G | g.\alpha = \beta\}$ and $\mathbf{1}_{\alpha,\beta} := \mathbf{1}_{S_{\alpha,\beta}}$. Note that $S_{\alpha,\beta}$ is a coset of a stabilizer subgroup of G. We say that G satisfies the EKR module property if for every maximum intersecting set M, the vector $\mathbf{1}_M$ is a linear combination of $\mathbf{1}_{\alpha,\beta}$'s. The main result of [13] states that every 2-transitive group satisfies the EKR module property. I was able to generalize an argument in that paper to show the following.

Theorem 4. *(Unpublished) Any permutation group with an abelian regular normal subgroup has the EKR module property.*

Permutation groups with abelian normal subgroups may not be 2-transitive or even primitive. I wish to find more examples of groups that satisfy the EKR module property.

Problem 8. *Find examples of families of groups satisfying the EKR module property and of those not satisfying the EKR module property.*

In [13], the authors were interested in investigating the existence of maximum intersecting sets which are not cosets of a subgroup. Such sets exist in 2-transitive Frobenius groups which are not generalized dihedral groups. The derangement set of such a 2-transitive Frobenius group is disconnected. They asked the following question in the case of connected derangement graphs.

Problem 9. [13, Problem 8.1] *Are there 2-transitive groups G, with connected derangement graphs, that have maximum intersecting sets are neither subgroups nor cosets of a subgroup.*

We could ask a similar question about groups with EKR module property.

Problem 10. *If G is a permutation group with EKR module property, are all maximum intersecting sets cosets of some subgroups?*

In [20], the authors constructed examples of permutation groups of degree n with intersection densities of $n/3$. All these groups satisfy the EKR module property.

Problem 11.
(1) *Given $n \in \mathbb{N}$, let $\mathcal{M}_n = \{\rho(G) |\ G \leq S_n$ and has the EKR module property$\}$. Determine $M(n) := max(\mathcal{M}_n)$ as a function of n.*
(2) *Given $n \in \mathbb{N}$, let $\mathcal{N}_n = \{\rho(G) |\ G \leq S_n$ and doesn't satsify the EKR module property$\}$. Determine $N(n) := max(\mathcal{N}_n)$ as a function of n.*

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