EX-05 - 2D Transformations in C++ using OpenGL

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AIM

To implement 2d-Transformations in C++.

SPECIFICATION

To apply the following 2D transformations on objects and to render the final output along with the original object.

- 1) Translation
- 2) Rotation
 - a) about origin
 - b) with respect to a fixed point (x_r, y_r)
- 3) Scaling with respect to
 - a) origin Uniform Vs Differential Scaling
 - b) fixed point (x_f, y_f) Uniform Vs Differential Scaling
- 4) Reflection with respect to
 - a) x-axis
 - b) y-axis
 - c) origin
 - d) the line x = y
- 5) Shearing
 - a) x-direction shear
 - b) y-direction shear

Note: Use Homogeneous coordinate representations and matrix multiplication to perform transformations. Divide the output window into four quadrants. (Use LINES primitive to draw x and y axis.

Traslation

```
1 // To apply the following 2D transformations on objects and to render the \hookleftarrow
       final output along with the original object.
 2
   // 1) Translation
 3
 4
 5 #include<bits/stdc++.h>
 6 #include<GL/glut.h>
7
8 using namespace std;
9 using ld = long double;
10 using ll = long long;
11
12 #define X
                   first
13 #define Y
                   second
14
15 const int WINDOW_WIDTH = 900;
16 const int WINDOW_HEIGHT = 900;
17
18 const int X_MIN = -300;
19 const int X_MAX = 300;
20 const int Y_MIN = -300;
21 const int Y_MAX = 300;
22
23 void myInit();
24 void myDisplay();
25
26 void printAxes();
27 ll multiply(vector<ll> a, vector<ll> b);
28 vector<ll> multiply(vector<vector<ll>> a, vector<ll> b);
29
30 vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h=1);
31 vector<ll> translate(vector<ll> &point_matrix, ll tx=0, ll ty=0);
32 pair<11,11> getPoint(vector<11> point_matrix);
33 void translateShape();
34
35 const ld PADDING = 0;
36 const ld STEP = 10;
37 const ld SCALE = 1;
38
39
   int main(int argc,char* argv[]) {
40
       glutInit(&argc,argv);
       glutInitDisplayMode(GLUT_SINGLE|GLUT_RGB);
41
       glutInitWindowSize(WINDOW_WIDTH,WINDOW_HEIGHT);
42
```

```
43
        glutCreateWindow("2D - Translation");
44
        glutDisplayFunc(myDisplay);
45
        myInit();
46
        glutMainLoop();
47
        return 1;
48 }
49
50
   void myInit() {
51
        glClearColor(1.0,1.0,1.0,0.0);
52
        glColor3f(0.0f,0.0f,0.0f);
53
        glPointSize(2.0);
        glMatrixMode(GL_PROJECTION);
54
        glBlendFunc(GL_SRC_ALPHA, GL_ONE_MINUS_SRC_ALPHA);
55
56
        glEnable( GL_BLEND );
57
        glLoadIdentity();
58
        gluOrtho2D(X_MIN,X_MAX,Y_MIN,Y_MAX);
59 }
60
   void myDisplay() {
61
62
        glClear(GL_COLOR_BUFFER_BIT);
63
64
        printAxes();
65
        translateShape();
66
67
        glFlush();
68 }
69
70 void printAxes() {
71
        glBegin(GL_LINES);
72
73
        glColor3f(1.0f,0.0f,0.0f);
74
        glVertex2d(X_MIN,0);
75
        glVertex2d(X_MAX,0);
76
77
        glColor3f(1.0f,0.0f,0.0f);
78
        glVertex2d(0,Y_MIN);
79
        glVertex2d(0,Y_MAX);
80
        for(ll i=X_MIN;i<X_MAX;i+=STEP) {</pre>
81
82
            glVertex2d(i,-0.3*STEP);
83
            glVertex2d(i,0.3*STEP);
84
        }
85
        for(ll i=Y_MIN;i<Y_MAX;i+=STEP) {</pre>
86
87
            glVertex2d(-0.3*STEP,i);
88
            glVertex2d(0.3*STEP,i);
89
        }
```

```
90
91
        glEnd();
92 }
93
 94
    void translateShape() {
 95
        //Plot original shape;
 96
97
        vector<pair<11,11>> shape = {{20,20}, {20,50}, {50,50}, {50,20}};
98
99
        glBegin(GL_POLYGON);
        glColor4f(0.7f,0.0f,1.0f,1.0f);
100
101
102
        for(auto point : shape) {
103
             glVertex2d(point.X,point.Y);
104
        }
105
106
        glEnd();
107
108
        //Translate shape;
109
110
        glBegin(GL_POLYGON);
        glColor4f(0.7f,0.0f,1.0f,0.4f);
111
112
        for(auto point : shape) {
113
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
114
115
             pair<ll,ll> translated_point = getPoint(translate(point_matrix←)
                ,50,50));
116
117
             glVertex2d(translated_point.X,translated_point.Y);
118
        }
119
120
        glEnd();
121 }
122
    pair<ll,ll> getPoint(vector<ll> point_matrix) {
123
124
        11 h = point_matrix[2];
125
        11 x = point_matrix[0];
126
        11 y = point_matrix[1];
127
128
        return {x/h,y/h};
129 }
130
131
132
    vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h) {
133
        vector<ll> point_matrix;
134
        point_matrix.push_back(h*point.first);
135
        point_matrix.push_back(h*point.second);
```

```
136
        point_matrix.push_back(h);
137
         return point_matrix;
138 }
139
140 vector<ll> translate(vector<ll> &point_matrix, ll tx, ll ty) {
        vector<vector<ll>>> translate_matrix = {
141
142
                                                   {1,0,tx},
143
                                                   {0,1,ty},
144
                                                   {0,0,1}
145
                                               };
146
         return multiply(translate_matrix, point_matrix);
147 }
148
149
    vector<ll> multiply(vector<vector<ll>> a, vector<ll> b) {
150
        vector<ll> result;
151
         for(int i=0;i<a.size();i++) {</pre>
             11 temp = multiply(a[i],b);
152
             result.push_back(temp);
153
154
        }
155
        return result;
156 }
157
158
    ll multiply(vector<ll> a, vector<ll> b) {
159
         11 result=0;
         for(int i=0;i<a.size();i++) {</pre>
160
161
             result+=(a[i]*b[i]);
162
         }
163
         return result;
164 }
```

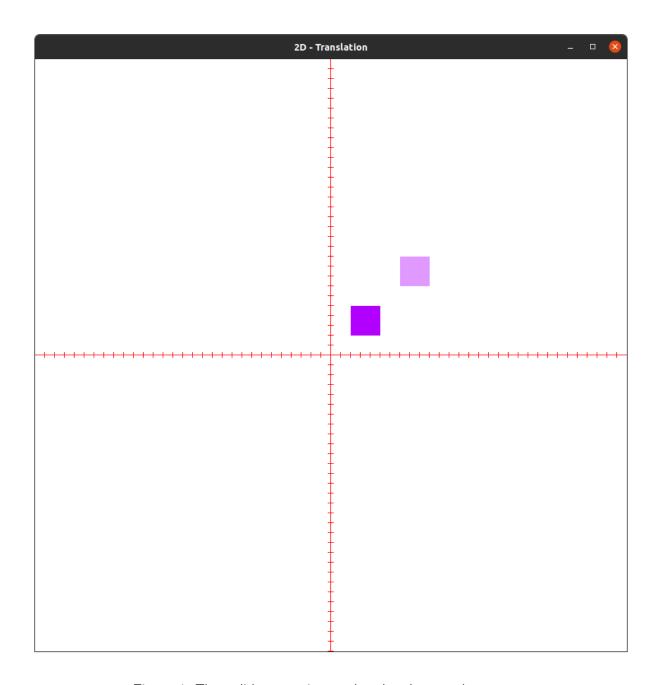


Figure 1: The solid square is translated to the translucent one

Rotation

```
1 // To apply the following 2D transformations on objects and to render the \hookleftarrow
       final output along with the original object.
 2
 3 // 2) Rotation
       a) about origin
 4 //
5 //
          b) with respect to a fixed point (xr,yr)
 6
 7 #include<bits/stdc++.h>
8 #include<GL/glut.h>
9
10 using namespace std;
11 using ld = long double;
12 using ll = long long;
13
14 #define X
                  first
15 #define Y
                   second
16
17 const int WINDOW_WIDTH = 900;
18 const int WINDOW_HEIGHT = 900;
19
20 const int X_MIN = -300;
21 const int X_MAX = 300;
22 const int Y_MIN = -300;
23 const int Y_MAX = 300;
24
25 const ld PADDING = 0;
26 const ld STEP = 10;
27 const ld SCALE = 1;
28 const ld PI = 3.14159265358979323846264338327950288419716939937510582;
29
30 void myInit();
31 void myDisplay();
32
33 void printAxes();
34 ld getRadian(ld degree);
35 ld multiply(vector<ld> a, vector<ll> b);
36 vector<ld> multiply(vector<vector<ld>> a, vector<ll> b);
37
38 vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h=1);
39 vector<ld> rotate(vector<ll> &point_matrix, ld angle=0, pair<ll,ll> pivot=←
       make_pair(0,0));
40 pair<ld,ld> getPoint(vector<ld> point_matrix);
41 void rotateShape();
```

```
42
43
44
   int main(int argc,char* argv[]) {
45
        glutInit(&argc,argv);
        glutInitDisplayMode(GLUT_SINGLE|GLUT_RGB);
46
47
        glutInitWindowSize(WINDOW_WIDTH, WINDOW_HEIGHT);
48
        glutCreateWindow("2D - Rotation");
49
        glutDisplayFunc(myDisplay);
50
        myInit();
51
        glutMainLoop();
52
        return 1;
53 }
54
   void myInit() {
55
56
        glClearColor(1.0,1.0,1.0,0.0);
57
        glColor3f(0.0f,0.0f,0.0f);
        glPointSize(5.0);
58
        glMatrixMode(GL_PROJECTION);
59
60
        glBlendFunc(GL_SRC_ALPHA, GL_ONE_MINUS_SRC_ALPHA);
61
        glEnable( GL_BLEND );
62
        glLoadIdentity();
63
        gluOrtho2D(X_MIN,X_MAX,Y_MIN,Y_MAX);
64 }
65
   void myDisplay() {
66
67
        glClear(GL_COLOR_BUFFER_BIT);
68
69
        printAxes();
70
        rotateShape();
71
72
        glFlush();
73 }
74
75
   void printAxes() {
76
        glBegin(GL_LINES);
77
78
        glColor3f(1.0f,0.0f,0.0f);
79
        glVertex2d(X_MIN,0);
80
        glVertex2d(X_MAX,0);
81
        glColor3f(1.0f,0.0f,0.0f);
82
83
        glVertex2d(0,Y_MIN);
        glVertex2d(0,Y_MAX);
84
85
86
        for(ll i=X_MIN;i<X_MAX;i+=STEP) {</pre>
87
            glVertex2d(i,-0.3*STEP);
88
            glVertex2d(i,0.3*STEP);
```

```
89
         }
90
91
         for(ll i=Y_MIN;i<Y_MAX;i+=STEP) {</pre>
             glVertex2d(-0.3*STEP,i);
92
 93
             glVertex2d(0.3*STEP,i);
94
         }
 95
96
        glEnd();
97 }
98
99
    void rotateShape() {
100
         //Plot original shape;
101
102
        vector<pair<11,11>> shape = {{20,20}, {60,20}, {40,60}};
103
104
        glBegin(GL_POLYGON);
         glColor4f(0.7f,0.0f,1.0f,1.0f);
105
106
107
         for(auto point : shape) {
108
             glVertex2d(point.X,point.Y);
109
         }
110
111
        glEnd();
112
         //Rotate shape with respect to origin;
113
114
115
        glBegin(GL_POLYGON);
116
         glColor4f(0.7f,0.0f,1.0f,0.4f);
117
118
         for(auto point : shape) {
119
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
120
             pair<ld,ld> rotated_point = getPoint(rotate(point_matrix,90));
121
122
             glVertex2d(rotated_point.X,rotated_point.Y);
123
         }
124
        glEnd();
125
126
127
         //Rotate shape with respect to a fixed point;
128
129
        pair<11,11> pivot = {40,70};
130
        glBegin(GL_POINTS);
131
         glColor4f(0.8f,0.0f,0.5f,0.9f);
132
         glVertex2d(pivot.X, pivot.Y);
133
134
         glEnd();
135
```

```
136
        glBegin(GL_POLYGON);
137
        glColor4f(0.7f,0.0f,1.0f,0.4f);
138
        for(auto point : shape) {
139
140
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
141
             pair<ld,ld> rotated_point = getPoint(rotate(point_matrix,180,pivot←)
                ));
142
143
            glVertex2d(rotated_point.X,rotated_point.Y);
144
        }
145
146
        glEnd();
147 }
148
149
    pair<ld,ld> getPoint(vector<ld> point_matrix) {
        11 h = point_matrix[2];
150
        ld x = point_matrix[0];
151
152
        ld y = point_matrix[1];
153
154
        return {x/h,y/h};
155 }
156
157
    ld getRadian(ld degree) {
        return degree*PI/180;
158
159 }
160
161
    vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h) {
162
        vector<ll> point_matrix;
        point_matrix.push_back(h*point.first);
163
        point_matrix.push_back(h*point.second);
164
165
        point_matrix.push_back(h);
166
        return point_matrix;
167 }
168
    vector<ld> rotate(vector<ll> &point_matrix, ld angle, pair<ll,ll> pivot) {
169
170
        angle = getRadian(angle);
        11 xr = pivot.X;
171
        11 yr = pivot.Y;
172
173
174
        vector<vector<ld>> rotate_matrix = {
175
                                      \{\cos(angle), -\sin(angle), xr*(1-\cos(angle) \leftarrow
                                           + yr*sin(angle))},
176
                                      {sin(angle), cos(angle), yr*(1-cos(angle)←
                                           - xr*sin(angle))},
177
                                      {0
                                                  , 0
                                                               ,1}
178
                                  };
         return multiply(rotate_matrix, point_matrix);
179
```

```
180 }
181
182 vector<ld> multiply(vector<vector<ld>> a, vector<ll> b) {
         vector<ld> result;
183
         for(int i=0;i<a.size();i++) {</pre>
184
             11 temp = multiply(a[i],b);
185
             result.push_back(temp);
186
187
188
         return result;
189 }
190
191
    ld multiply(vector<ld> a, vector<ll> b) {
192
         ld result=0;
         for(int i=0;i<a.size();i++) {</pre>
193
194
             result+=(a[i]*b[i]);
195
         }
196
         return result;
197 }
```

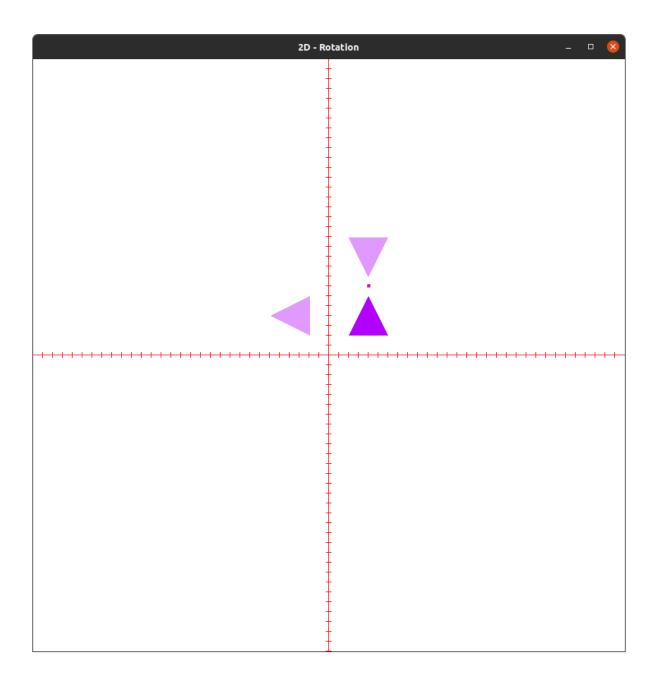


Figure 2: (a) The solid triangle is rotated 90° w.r.t origin to obtain triangle at 2^{nd} quadrant. (b) The solid triangle is rotated 180° with respect to the point shown to obtain the triangle above

it.

Scaling

```
1 // To apply the following 2D transformations on objects and to render the \hookleftarrow
       final output along with the original object.
 2
 3 // 3) Scaling with respect to
           a) origin - Uniform Vs Differential Scaling
5 //
          b) fixed point (xf,yf)
 6
 7
8 #include<bits/stdc++.h>
9 #include<GL/glut.h>
10
11 using namespace std;
12 using ld = long double;
13 using ll = long long;
14
15 #define X
                   first
16 #define Y
                  second
17
18 const int WINDOW_WIDTH = 900;
19 const int WINDOW_HEIGHT = 900;
20
21 const int X_MIN = -300;
22 const int X_MAX = 300;
23 const int Y_MIN = -300;
24 const int Y_MAX = 300;
25
26 const ld PADDING = 0;
27 const ld STEP = 10;
28 const ld SCALE = 1;
29
30 void myInit();
31 void myDisplay();
32
33 void printAxes();
34 ld multiply(vector<ld> a, vector<ll> b);
35 vector<ld> multiply(vector<vector<ld>> a, vector<ll> b);
36
37 vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h=1);
38 vector<ld> scale(vector<ll> &point_matrix, ld sx=1, ld sy=2, pair<ll,ll> \leftarrow
       pivot=make_pair(0,0));
39 pair<ld,ld> getPoint(vector<ld> point_matrix);
40 void scaleShape();
41
```

```
42
43
   int main(int argc,char* argv[]) {
44
        glutInit(&argc,argv);
        glutInitDisplayMode(GLUT_SINGLE|GLUT_RGB);
45
46
        glutInitWindowSize(WINDOW_WIDTH, WINDOW_HEIGHT);
        glutCreateWindow("2D - Scaling");
47
48
        glutDisplayFunc(myDisplay);
49
        myInit();
50
        glutMainLoop();
51
        return 1;
52 }
53
54
   void myInit() {
        glClearColor(1.0,1.0,1.0,0.0);
55
56
        glColor3f(0.0f,0.0f,0.0f);
57
        glPointSize(5.0);
        glMatrixMode(GL_PROJECTION);
58
        glBlendFunc(GL_SRC_ALPHA, GL_ONE_MINUS_SRC_ALPHA);
59
60
        glEnable( GL_BLEND );
61
        glLoadIdentity();
62
        gluOrtho2D(X_MIN,X_MAX,Y_MIN,Y_MAX);
63 }
64
65
   void myDisplay() {
66
        glClear(GL_COLOR_BUFFER_BIT);
67
68
        printAxes();
69
        scaleShape();
70
71
        glFlush();
72 }
73
74
   void printAxes() {
75
        glBegin(GL_LINES);
76
77
        glColor3f(1.0f,0.0f,0.0f);
78
        glVertex2d(X_MIN,0);
79
        glVertex2d(X_MAX,0);
80
81
        glColor3f(1.0f,0.0f,0.0f);
82
        glVertex2d(0,Y_MIN);
83
        glVertex2d(0,Y_MAX);
84
        for(ll i=X_MIN;i<X_MAX;i+=STEP) {</pre>
85
            glVertex2d(i,-0.3*STEP);
86
87
            glVertex2d(i,0.3*STEP);
88
        }
```

```
89
90
         for(ll i=Y_MIN;i<Y_MAX;i+=STEP) {</pre>
91
             glVertex2d(-0.3*STEP,i);
92
             glVertex2d(0.3*STEP,i);
 93
         }
94
 95
        glEnd();
96 }
97
98
    void scaleShape() {
99
100
         vector<pair<11,11>> shape;
101
102
         //Plot original shape;
103
104
         shape = \{\{20,40\}, \{60,40\}, \{40,80\}\};
105
106
         glBegin(GL_POLYGON);
107
         glColor4f(0.7f,0.0f,1.0f,1.0f);
108
109
         for(auto point : shape) {
110
             glVertex2d(point.X,point.Y);
111
         }
112
         glEnd();
113
114
115
         //Scale shape w.r.t. origin - Uniform Scaling;
116
117
         glBegin(GL_POLYGON);
         glColor4f(0.7f,0.0f,1.0f,0.4f);
118
119
120
         for(auto point : shape) {
121
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
122
             pair<ld,ld> scaled_point = getPoint(scale(point_matrix, 2, 2));
123
124
             glVertex2d(scaled_point.X,scaled_point.Y);
125
         }
126
127
         glEnd();
128
129
130
131
132
         //Plot original shape;
133
134
         shape = \{\{-20,40\}, \{-60,40\}, \{-40,80\}\};
135
```

```
136
        glBegin(GL_POLYGON);
137
         glColor4f(0.0f,1.0f,0.7f,1.0f);
138
139
         for(auto point : shape) {
140
             glVertex2d(point.X,point.Y);
141
         }
142
143
        glEnd();
144
145
         //Scale shape w.r.t. origin - Differential Scaling;
146
147
         glBegin(GL_POLYGON);
         glColor4f(0.0f,1.0f,0.7f,0.4f);
148
149
150
         for(auto point : shape) {
151
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> scaled_point = getPoint(scale(point_matrix, 0.5, 2));
152
153
154
             glVertex2d(scaled_point.X,scaled_point.Y);
155
         }
156
        glEnd();
157
158
159
160
161
162
         //Plot original shape;
163
         shape = \{\{20, -40\}, \{60, -40\}, \{40, -80\}\};
164
165
166
        glBegin(GL_POLYGON);
         glColor4f(0.7f,1.0f,0.0f,0.7f);
167
168
         for(auto point : shape) {
169
170
             glVertex2d(point.X,point.Y);
171
         }
172
173
        glEnd();
174
175
         //Scale shape w.r.t. fixed point - Uniform Scaling;
176
177
        pair<11,11> pivot = {40, -60};
178
179
        glBegin(GL_POINTS);
180
         glColor4f(0.0f,0.0f,1.0f,0.9f);
181
         glVertex2d(pivot.X, pivot.Y);
182
         glEnd();
```

```
183
184
        glBegin(GL_POLYGON);
         glColor4f(0.7f,1.0f,0.0f,0.6f);
185
186
187
         for(auto point : shape) {
188
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
189
             pair<ld,ld> scaled_point = getPoint(scale(point_matrix, 1.5, 1.5, ←
                pivot));
190
191
             glVertex2d(scaled_point.X,scaled_point.Y);
192
         }
193
194
        glEnd();
195
196
197
198
199
         //Plot original shape;
200
201
         shape = \{\{-20, -40\}, \{-60, -40\}, \{-40, -80\}\};
202
203
        glBegin(GL_POLYGON);
204
         glColor4f(1.0f,0.7f,0.0f,0.7f);
205
206
         for(auto point : shape) {
207
             glVertex2d(point.X,point.Y);
208
         }
209
210
        glEnd();
211
212
         //Scale shape w.r.t. fixed point - Differential Scaling;
213
214
        pivot = \{-40, -60\};
215
216
        glBegin(GL_POINTS);
217
         glColor4f(1.0f,0.0f,0.0f,0.9f);
218
         glVertex2d(pivot.X, pivot.Y);
219
         glEnd();
220
        glBegin(GL_POLYGON);
221
         glColor4f(1.0f,0.7f,0.0f,0.5f);
222
223
224
         for(auto point : shape) {
225
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
226
             pair<ld,ld> scaled_point = getPoint(scale(point_matrix, 0.5, 1.5, ←
                pivot));
227
```

```
228
             glVertex2d(scaled_point.X,scaled_point.Y);
229
        }
230
231
        glEnd();
232
233 }
234
235
    pair<ld,ld> getPoint(vector<ld> point_matrix) {
236
         11 h = point_matrix[2];
237
         ld x = point matrix[0];
238
         ld y = point_matrix[1];
239
240
         return {x/h,y/h};
241 }
242
243
    vector<1l> getHomogeneousPointCoords(pair<1l,1l> point, 1l h) {
244
         vector<ll> point_matrix;
         point_matrix.push_back(h*point.first);
245
246
         point_matrix.push_back(h*point.second);
247
         point_matrix.push_back(h);
248
         return point_matrix;
249 }
250
    vector<ld> scale(vector<ll> &point_matrix, ld sx, ld sy, pair<ll,ll> pivot←
251
        ) {
252
253
        11 xf = pivot.X;
254
         11 yf = pivot.Y;
255
        vector<vector<ld>>> scale_matrix = {
256
257
                                      \{sx, 0, xf*(1-sx)\},\
258
                                      \{0, sy, yf*(1-sy)\},\
259
                                      \{0, 0, 1\}
260
261
         return multiply(scale_matrix,point_matrix);
262 }
263
264
    vector<ld> multiply(vector<vector<ld>> a, vector<ll> b) {
        vector<ld> result;
265
266
         for(int i=0;i<a.size();i++) {</pre>
267
             11 temp = multiply(a[i],b);
             result.push_back(temp);
268
269
         }
270
         return result;
271 }
272
    ld multiply(vector<ld> a, vector<ll> b) {
273
```

```
274    ld result=0;
275    for(int i=0;i<a.size();i++) {
276        result+=(a[i]*b[i]);
277    }
278    return result;
279 }</pre>
```

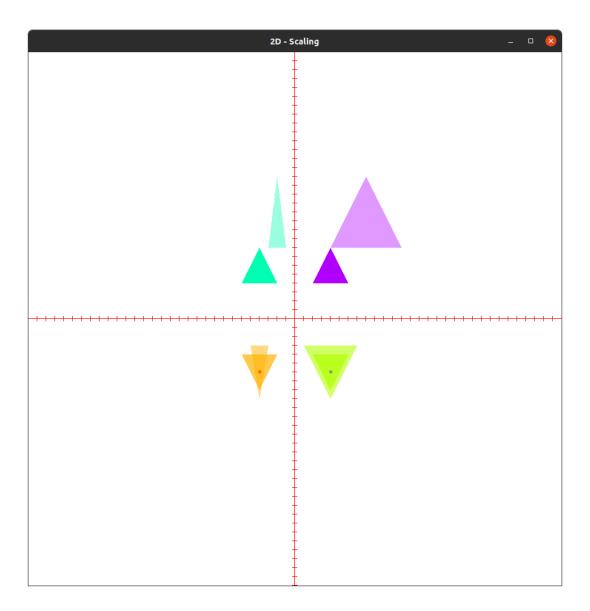


Figure 3: (a) The solid triangle in $\mathbf{1}^{st}$ quadrant is scaled uniformly w.r.t origin to its translucent version.

- (b) The solid triangle in 2^{nd} quadrant is scaled deferentially w.r.t origin to its translucent version.
- (c) The solid triangle in 3^{rd} quadrant is scaled uniformly w.r.t a fixed point shown to its translucent version.
- (d) The solid triangle in 4^{th} quadrant is scaled deferentially w.r.t fixed point shown to its translucent version.

Reflection

```
1 // To apply the following 2D transformations on objects and to render the \hookleftarrow
       final output along with the original object.
 2
 3 // 4) Reflection with respect to
 4 //
          a) x-axis
5 //
          b) y-axis
 6 //
        c) origin
7 //
          d) the line x=y
8
9 #include<bits/stdc++.h>
10 #include<GL/glut.h>
11
12 using namespace std;
13 using ld = long double;
14 using ll = long long;
15
16 #define X
                   first
17 #define Y
                   second
18
19 const int WINDOW WIDTH = 900;
20 const int WINDOW_HEIGHT = 900;
21
22 const int X_MIN = -300;
23 const int X_MAX = 300;
24 const int Y_MIN = -300;
25 const int Y_MAX = 300;
26
27 const ld PADDING = 0;
28 const ld STEP = 10;
29 const ld SCALE = 1;
30
31 void myInit();
32 void myDisplay();
33
34 void printAxes();
35 ld multiply(vector<ld> a, vector<ll> b);
36 vector<ld> multiply(vector<vector<ld>> a, vector<ll> b);
37
38 enum ReflectionType {X_AXIS, Y_AXIS, ORIGIN, X_EQUALS_Y_LINE};
39
40 vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h=1);
41 vector<ld> reflect(vector<ll> &point_matrix, ReflectionType type=ORIGIN);
42 pair<ld,ld> getPoint(vector<ld> point_matrix);
```

```
43
   void reflectShape();
44
45
46
   int main(int argc,char* argv[]) {
47
       glutInit(&argc,argv);
        glutInitDisplayMode(GLUT_SINGLE|GLUT_RGB);
48
49
        glutInitWindowSize(WINDOW_WIDTH,WINDOW_HEIGHT);
50
        glutCreateWindow("2D - Reflection");
51
       glutDisplayFunc(myDisplay);
52
       myInit();
53
       glutMainLoop();
        return 1;
54
55 }
56
57
   void myInit() {
58
       glClearColor(1.0,1.0,1.0,0.0);
        glColor3f(0.0f,0.0f,0.0f);
59
       glPointSize(2.0);
60
61
        glMatrixMode(GL_PROJECTION);
62
       glBlendFunc(GL_SRC_ALPHA, GL_ONE_MINUS_SRC_ALPHA);
63
       glEnable( GL_BLEND );
64
       glLoadIdentity();
       gluOrtho2D(X_MIN,X_MAX,Y_MIN,Y_MAX);
65
66 }
67
68
   void myDisplay() {
69
       glClear(GL_COLOR_BUFFER_BIT);
70
71
       printAxes();
72
        reflectShape();
73
74
       glFlush();
75 }
76
77
   void printAxes() {
78
       glBegin(GL_LINES);
79
80
        glColor3f(1.0f,0.0f,0.0f);
81
        glVertex2d(X_MIN,0);
82
       glVertex2d(X_MAX,0);
83
84
       glColor3f(1.0f,0.0f,0.0f);
       glVertex2d(0,Y_MIN);
85
86
       glVertex2d(0,Y_MAX);
87
88
        for(ll i=X_MIN;i<X_MAX;i+=STEP) {</pre>
89
            glVertex2d(i,-0.3*STEP);
```

```
90
             glVertex2d(i,0.3*STEP);
91
         }
92
93
         for(ll i=Y_MIN;i<Y_MAX;i+=STEP) {</pre>
 94
             glVertex2d(-0.3*STEP,i);
95
             glVertex2d(0.3*STEP,i);
 96
         }
97
        glEnd();
98
99 }
100
101
    void reflectShape() {
102
103
         vector<pair<ll,ll>> shape;
104
105
         //Plot original shape;
106
107
         shape = \{\{20,100\}, \{40,80\}, \{60,100\}, \{40,120\}\};
108
109
         glBegin(GL_POLYGON);
110
         glColor4f(0.0f,1.0f,0.7f,1.0f);
111
112
         for(auto point : shape) {
113
             glVertex2d(point.X,point.Y);
114
         }
115
116
        glEnd();
117
118
         //Reflect shape along x-axis;
119
120
         glBegin(GL_POLYGON);
         glColor4f(0.0f,0.1f,0.7f,0.4f);
121
122
123
         for(auto point : shape) {
124
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
125
             pair<ld,ld> reflected_point = getPoint(reflect(point_matrix, ←
                X_AXIS));
126
127
             glVertex2d(reflected_point.X,reflected_point.Y);
128
         }
129
130
         glEnd();
131
132
         //Reflect shape along y-axis;
133
134
         glBegin(GL_POLYGON);
135
         glColor4f(0.0f,0.1f,0.7f,0.4f);
```

```
136
137
        for(auto point : shape) {
138
            vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> reflected_point = getPoint(reflect(point_matrix, ←
139
                Y_AXIS));
140
141
            glVertex2d(reflected_point.X,reflected_point.Y);
        }
142
143
144
        glEnd();
145
146
        //Reflect shape along origin;
147
        glBegin(GL_POLYGON);
148
149
        glColor4f(0.0f,0.1f,0.7f,0.4f);
150
151
        for(auto point : shape) {
152
            vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> reflected_point = getPoint(reflect(point_matrix, ←
153
                ORIGIN));
154
            glVertex2d(reflected_point.X,reflected_point.Y);
155
156
        }
157
158
        glEnd();
159
160
        //Reflect shape along x=y line;
161
162
        glBegin(GL_LINES);
        glColor4f(0.0f,1.0f,0.0f,0.4f);
163
164
        glVertex2d(min(X_MIN,Y_MIN),min(X_MIN,Y_MIN));
165
166
        glVertex2d(max(X_MAX,Y_MAX),max(X_MAX,Y_MAX));
167
168
        glEnd();
169
        glBegin(GL_POLYGON);
170
171
        glColor4f(0.0f,0.1f,0.7f,0.4f);
172
173
        for(auto point : shape) {
174
            vector<ll> point_matrix = getHomogeneousPointCoords(point);
175
             pair<ld,ld> reflected_point = getPoint(reflect(point_matrix, ←
                X_EQUALS_Y_LINE));
176
177
            glVertex2d(reflected_point.X,reflected_point.Y);
178
        }
179
```

```
180
         glEnd();
181 }
182
    pair<ld,ld> getPoint(vector<ld> point_matrix) {
183
184
         11 h = point_matrix[2];
185
         ld x = point_matrix[0];
186
         ld y = point_matrix[1];
187
188
         return {x/h,y/h};
189 }
190
191
    vector<1l> getHomogeneousPointCoords(pair<1l,1l> point, 1l h) {
         vector<ll> point_matrix;
192
193
         point_matrix.push_back(h*point.first);
194
         point_matrix.push_back(h*point.second);
195
         point_matrix.push_back(h);
         return point_matrix;
196
197 }
198
199
    vector<ld> reflect(vector<ll> &point_matrix, ReflectionType type) {
200
201
         vector<vector<ld>> reflection_matrix;
202
203
         switch (type)
204
205
             case X_AXIS:
206
                 reflection_matrix = {
207
                                           {1,0,0},
208
                                           \{0,-1,0\},\
209
                                           {0,0,1}
210
                                       };
211
                 break;
212
             case Y_AXIS:
213
214
                 reflection_matrix = {
215
                                           \{-1,0,0\},
216
                                           {0,1,0},
217
                                           {0,0,1}
218
                                       };
219
                 break;
220
221
             case ORIGIN:
222
                 reflection matrix = {
223
                                           \{-1,0,0\},
224
                                           \{0,-1,0\},\
225
                                           {0,0,1}
226
                                       };
```

```
227
                  break;
228
229
             case X_EQUALS_Y_LINE:
230
                  reflection_matrix = {
231
                                            {0,1,0},
232
                                            {1,0,0},
233
                                            {0,0,1}
234
                                       };
235
                  break;
236
237
             default:
                  reflection_matrix = {
238
239
                                            {1,0,0},
240
                                            {0,1,0},
241
                                            {0,0,1}
242
                                       };
243
                 break;
244
         }
245
246
         return multiply(reflection_matrix, point_matrix);
247 }
248
249
    vector<ld> multiply(vector<vector<ld>> a, vector<ll> b) {
250
         vector<ld> result;
251
         for(int i=0;i<a.size();i++) {</pre>
252
             11 temp = multiply(a[i],b);
253
             result.push_back(temp);
254
         }
255
         return result;
256 }
257
258
    ld multiply(vector<ld> a, vector<ll> b) {
         ld result=0;
259
260
         for(int i=0;i<a.size();i++) {</pre>
261
             result+=(a[i]*b[i]);
262
         }
263
         return result;
264
    }
```

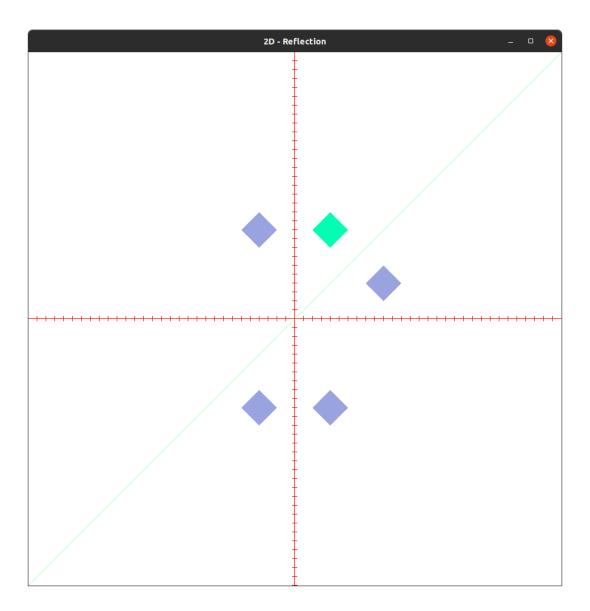


Figure 4: (a) The green diamond is reflected w.r.t x-axis to the blue one in 4^{th} quadrant.

- (b) The green diamond is reflected w.r.t y-axis to the blue one in 2^{nd} quadrant.
- (c) The green diamond is reflected w.r.t origin to the blue one in 3^{rd} quadrant.
- (d) The green diamond is reflected w.r.t green line x=y to the blue one in 1^{st} quadrant.

Shearing

```
1 // To apply the following 2D transformations on objects and to render the \hookleftarrow
       final output along with the original object.
 2
 3 // 5) Shearing
 4 //
          a) x-direction shear
5 //
          b) y-direction shear
 6
 7 #include<bits/stdc++.h>
8 #include<GL/glut.h>
9
10 using namespace std;
11 using ld = long double;
12 using ll = long long;
13
14 #define X
                   first
15 #define Y
                    second
16
17 const int WINDOW_WIDTH = 900;
18 const int WINDOW_HEIGHT = 900;
19
20 const int X_MIN = -300;
21 const int X_MAX = 300;
22 const int Y_MIN = -300;
23 const int Y_MAX = 300;
24
25 const ld PADDING = 0;
26 const ld STEP = 10;
27 const ld SCALE = 1;
28
29 void myInit();
30 void myDisplay();
31
32 void printAxes();
33 ld multiply(vector<ld> a, vector<ll> b);
34 vector<ld> multiply(vector<vector<ld>> a, vector<ll> b);
35
36 enum ShearType {X_SHEAR, Y_SHEAR};
37
38 vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h=1);
39 vector<ld> shear(vector<ll> &point_matrix, ShearType type=X_SHEAR, ld sh\leftarrow
       =1, ld refLine=0);
40 pair<ld,ld> getPoint(vector<ld> point_matrix);
41 void shearShape();
```

```
42
43
44
   int main(int argc,char* argv[]) {
45
        glutInit(&argc,argv);
        glutInitDisplayMode(GLUT_SINGLE|GLUT_RGB);
46
47
        glutInitWindowSize(WINDOW_WIDTH, WINDOW_HEIGHT);
48
        glutCreateWindow("2D - Shearing");
49
        glutDisplayFunc(myDisplay);
50
        myInit();
51
        glutMainLoop();
52
        return 1;
53 }
54
   void myInit() {
55
56
        glClearColor(1.0,1.0,1.0,0.0);
57
        glColor3f(0.0f,0.0f,0.0f);
        glPointSize(2.0);
58
        glMatrixMode(GL_PROJECTION);
59
60
        glBlendFunc(GL_SRC_ALPHA, GL_ONE_MINUS_SRC_ALPHA);
61
        glEnable( GL_BLEND );
62
        glLoadIdentity();
        gluOrtho2D(X_MIN,X_MAX,Y_MIN,Y_MAX);
63
64 }
65
   void myDisplay() {
66
67
        glClear(GL_COLOR_BUFFER_BIT);
68
69
        printAxes();
70
        shearShape();
71
72
        glFlush();
73 }
74
75
   void printAxes() {
76
        glBegin(GL_LINES);
77
78
        glColor3f(1.0f,0.0f,0.0f);
79
        glVertex2d(X_MIN,0);
80
        glVertex2d(X_MAX,0);
81
82
        glColor3f(1.0f,0.0f,0.0f);
83
        glVertex2d(0,Y_MIN);
        glVertex2d(0,Y_MAX);
84
85
        for(ll i=X_MIN;i<X_MAX;i+=STEP) {</pre>
86
87
            glVertex2d(i,-0.3*STEP);
88
            glVertex2d(i,0.3*STEP);
```

```
89
         }
90
91
         for(ll i=Y_MIN;i<Y_MAX;i+=STEP) {</pre>
             glVertex2d(-0.3*STEP,i);
92
 93
             glVertex2d(0.3*STEP,i);
94
         }
95
96
         glEnd();
97 }
98
99 void shearShape() {
100
101
         vector<pair<11,11>> shape;
102
103
         //QUAD - 1
104
105
         //Plot original shape;
106
107
         shape = \{\{0,0\}, \{40,0\}, \{40,40\}, \{0,40\}\};
108
109
         glBegin(GL_POLYGON);
110
         glColor4f(0.7f,0.5f,1.0f,0.6f);
111
112
         for(auto point : shape) {
113
             glVertex2d(point.X,point.Y);
114
         }
115
116
        glEnd();
117
118
         //Shear the shape w.r.t x-axis;
119
120
         glBegin(GL_POLYGON);
         glColor4f(0.0f,0.1f,0.7f,0.4f);
121
122
123
         for(auto point : shape) {
124
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
125
                3));
126
127
             glVertex2d(sheared_point.X,sheared_point.Y);
128
         }
129
130
        glEnd();
131
132
         //Shear the shape w.r.t x-axis with y_ref;
133
134
         glBegin(GL_POLYGON);
```

```
135
        glColor4f(0.0f,0.1f,0.7f,0.4f);
136
137
        for(auto point : shape) {
138
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
139
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
                3, -30));
140
141
            glVertex2d(sheared_point.X,sheared_point.Y);
142
        }
143
        glEnd();
144
145
146
        //Shear the shape w.r.t y-axis;
147
148
        glBegin(GL_POLYGON);
        glColor4f(0.502f,0.0f,0.502f,0.6f);
149
150
151
        for(auto point : shape) {
152
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
153
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
                3));
154
155
             glVertex2d(sheared_point.X, sheared_point.Y);
156
        }
157
158
        glEnd();
159
160
        //Shear the shape w.r.t y-axis with x_ref;
161
        glBegin(GL_POLYGON);
162
        glColor4f(0.502f,0.0f,0.502f,0.6f);
163
164
165
        for(auto point : shape) {
166
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
167
                3, -30));
168
             glVertex2d(sheared_point.X, sheared_point.Y);
169
170
        }
171
172
        glEnd();
173
        //QUAD - 2
174
175
176
        //Plot original shape;
177
178
         shape = \{\{0,0\}, \{-40,0\}, \{-40,40\}, \{0,40\}\};
```

```
179
180
        glBegin(GL_POLYGON);
        glColor4f(0.486f,0.988f,0.000f,0.4f);
181
182
183
        for(auto point : shape) {
184
             glVertex2d(point.X,point.Y);
185
        }
186
187
        glEnd();
188
189
        //Shear the shape w.r.t x-axis;
190
191
        glBegin(GL_POLYGON);
        glColor4f(0.596f,0.984f,0.596f,0.7f);
192
193
194
        for(auto point : shape) {
195
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
196
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
                -3));
197
198
             glVertex2d(sheared_point.X, sheared_point.Y);
        }
199
200
201
        glEnd();
202
203
        //Shear the shape w.r.t x-axis with y_ref;
204
205
        glBegin(GL_POLYGON);
        glColor4f(0.596f,0.984f,0.596f,0.7f);
206
207
208
        for(auto point : shape) {
209
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
210
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
                -3, -30));
211
212
             glVertex2d(sheared_point.X, sheared_point.Y);
213
        }
214
215
        glEnd();
216
217
        //Shear the shape w.r.t y-axis;
218
        glBegin(GL_POLYGON);
219
        glColor4f(0.678f,1.0f,0.184f,0.7f);
220
221
222
        for(auto point : shape) {
223
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
```

```
224
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
                -3));
225
226
             glVertex2d(sheared_point.X, sheared_point.Y);
227
         }
228
229
        glEnd();
230
231
         //Shear the shape w.r.t y-axis with x_ref;
232
233
         glBegin(GL_POLYGON);
234
         glColor4f(0.678f,1.0f,0.184f,0.7f);
235
236
         for(auto point : shape) {
237
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
238
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
                -3, 30));
239
240
             glVertex2d(sheared_point.X, sheared_point.Y);
241
         }
242
243
        glEnd();
244
245
         //QUAD - 3
246
247
         //Plot original shape;
248
249
         shape = \{\{0,0\}, \{-40,0\}, \{-40,-40\}, \{0,-40\}\};
250
         glBegin(GL_POLYGON);
251
        glColor4f(0.824f,0.412f,0.118f,0.6f);
252
253
254
         for(auto point : shape) {
255
             glVertex2d(point.X,point.Y);
256
         }
257
258
        glEnd();
259
260
         //Shear the shape w.r.t x-axis;
261
262
         glBegin(GL_POLYGON);
         glColor4f(0.722f,0.525f,0.043f,0.7f);
263
264
265
         for(auto point : shape) {
266
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
267
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
                3));
```

```
268
269
            glVertex2d(sheared_point.X,sheared_point.Y);
270
        }
271
272
        glEnd();
273
274
         //Shear the shape w.r.t x-axis with y_ref;
275
276
        glBegin(GL_POLYGON);
        glColor4f(0.722f,0.525f,0.043f,0.7f);
277
278
279
         for(auto point : shape) {
280
            vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
281
                3, 30));
282
283
            glVertex2d(sheared_point.X,sheared_point.Y);
284
        }
285
286
        glEnd();
287
288
        //Shear the shape w.r.t y-axis;
289
290
        glBegin(GL_POLYGON);
291
        glColor4f(0.627f,0.322f,0.176f,0.6f);
292
293
         for(auto point : shape) {
294
            vector<ll> point_matrix = getHomogeneousPointCoords(point);
295
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
                3));
296
297
            glVertex2d(sheared_point.X,sheared_point.Y);
298
        }
299
300
        glEnd();
301
302
        //Shear the shape w.r.t y-axis with x_ref;
303
304
        glBegin(GL_POLYGON);
        glColor4f(0.627f,0.322f,0.176f,0.6f);
305
306
307
        for(auto point : shape) {
308
            vector<ll> point_matrix = getHomogeneousPointCoords(point);
309
            pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
                3, 30));
310
311
             glVertex2d(sheared_point.X,sheared_point.Y);
```

```
312
         }
313
314
        glEnd();
315
316
         //QUAD - 4
317
318
         //Plot original shape;
319
320
         shape = \{\{0,0\}, \{40,0\}, \{40,-40\}, \{0,-40\}\};
321
322
         glBegin(GL_POLYGON);
323
         glColor4f(0.498f,1.0f,0.831f,0.6f);
324
325
         for(auto point : shape) {
326
             glVertex2d(point.X,point.Y);
327
         }
328
329
        glEnd();
330
331
         //Shear the shape w.r.t x-axis;
332
333
        glBegin(GL_POLYGON);
         glColor4f(0.0f,0.545f,0.545f,0.6f);
334
335
336
         for(auto point : shape) {
337
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
338
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
                -3));
339
340
             glVertex2d(sheared_point.X, sheared_point.Y);
341
         }
342
343
        glEnd();
344
345
         //Shear the shape w.r.t x-axis with y_ref;
346
347
         glBegin(GL_POLYGON);
         glColor4f(0.0f,0.545f,0.545f,0.6f);
348
349
350
         for(auto point : shape) {
351
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
352
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, X_SHEAR, ←
                -3, 30));
353
354
             glVertex2d(sheared_point.X,sheared_point.Y);
355
         }
356
```

```
357
        glEnd();
358
359
        //Shear the shape w.r.t y-axis;
360
361
        glBegin(GL_POLYGON);
362
        glColor4f(0.282f,0.820f,0.800f,0.6f);
363
364
        for(auto point : shape) {
365
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
366
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
                -3));
367
368
             glVertex2d(sheared_point.X,sheared_point.Y);
369
        }
370
371
        glEnd();
372
373
        //Shear the shape w.r.t y-axis with x_ref;
374
375
        glBegin(GL_POLYGON);
376
        glColor4f(0.282f,0.820f,0.800f,0.6f);
377
378
         for(auto point : shape) {
379
             vector<ll> point_matrix = getHomogeneousPointCoords(point);
             pair<ld,ld> sheared_point = getPoint(shear(point_matrix, Y_SHEAR, ←
380
                -3, -30));
381
382
             glVertex2d(sheared_point.X,sheared_point.Y);
383
        }
384
385
        glEnd();
386
387
    }
388
    pair<ld,ld> getPoint(vector<ld> point_matrix) {
389
390
        11 h = point matrix[2];
391
        ld x = point_matrix[0];
392
        ld y = point_matrix[1];
393
394
        return {x/h,y/h};
395
    }
396
397
    vector<ll> getHomogeneousPointCoords(pair<ll,ll> point, ll h) {
398
        vector<ll> point_matrix;
399
        point_matrix.push_back(h*point.first);
400
        point_matrix.push_back(h*point.second);
401
        point_matrix.push_back(h);
```

```
402
         return point_matrix;
403 }
404
405 vector<ld> shear(vector<ll> &point_matrix, ShearType type, ld sh, ld ←
        refLine) {
406
407
         vector<vector<ld>> shear_matrix;
408
409
         switch (type)
410
         {
411
             case X_SHEAR:
                  shear_matrix = {
412
                                           {1,sh,-sh*refLine},
413
414
                                           {0,1,0},
415
                                           {0,0,1}
416
                                       };
417
                 break;
418
419
             case Y_SHEAR:
                  shear_matrix = {
420
421
                                           {1,0,0},
422
                                           {sh,1,-sh*refLine},
423
                                           {0,0,1}
424
                                       };
425
                 break;
426
427
             default:
428
                 shear_matrix = {
429
                                           {1,0,0},
430
                                           {0,1,0},
                                           {0,0,1}
431
432
                                       };
433
                 break;
434
         }
435
436
         return multiply(shear_matrix, point_matrix);
437 }
438
439
    vector<ld> multiply(vector<vector<ld>> a, vector<ll> b) {
440
         vector<ld> result;
441
         for(int i=0;i<a.size();i++) {</pre>
442
             11 temp = multiply(a[i],b);
443
             result.push_back(temp);
444
         }
445
         return result;
446 }
447
```

```
448 ld multiply(vector<ld> a, vector<ll> b) {
449         ld result=0;
450         for(int i=0;i<a.size();i++) {
451             result+=(a[i]*b[i]);
452         }
453         return result;
454 }</pre>
```

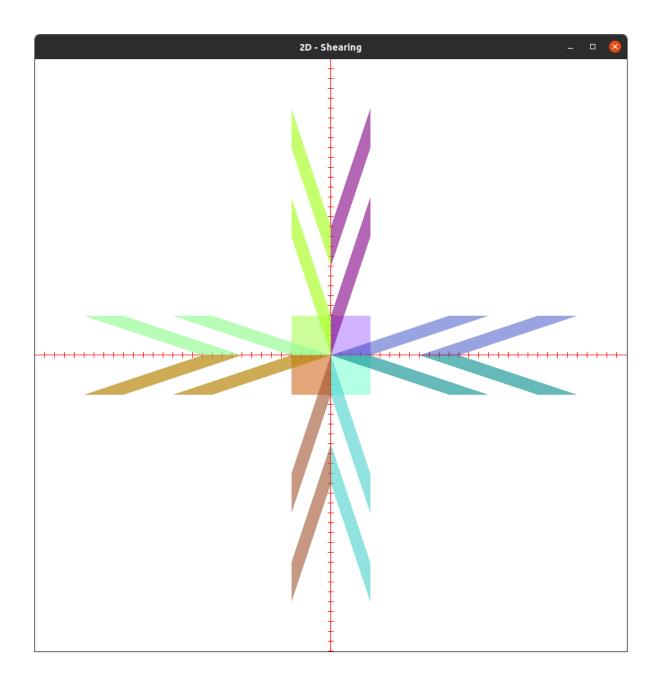


Figure 5: In each quadrant the sqaure is (a) x-sheared. (b) x-sheared w.r.t a reference line. (c) y-sheared. (d) y-sheared w.r.t a reference line.

The code to implement 2d transformations are written and output is verified.

RESULT