CEE 6460
Theoretical Geomechanics
Georgia Institute of Technology

Analytical Solution for One-Dimensional Consolidation

Saturated and Unsaturated

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1.Motivation –

The consolidation problem of soil has always been one of the key problems of geomechanics, it is important as it has close relation with the deformation and strength of soils. Though Terzaghi proposed solution for one-dimensional consolidation, in practice we deal with unsaturated soils. Studying the behavior of unsaturated soils are equally important.

There have been a significant amount of research going on consolidation theory of unsaturated soils in the field of geomechanics namely Blight's consolidation theory, Barden's consolidation theory, Fredlund's consolidation theory, which is adopted for this study.

Unsaturated soil consists of three phases: solid, water, and air. The unsaturated soils can be classified into three different types based on the continuity of the pore fluid phases as follows,

- Water phase is continuous, air phase is discontinuous- High degree of saturation
- Both water and air phase are continuous- Low degree of saturation
- Air phase is continuous, water phase is discontinuous – Lower degree of saturation

A general formulation in which both air and water is continuous is discussed in detail in this project.

An analytical solution for one dimensional consolidation for both saturated and unsaturated conditions are derived in this study. The derived solution for unsaturated condition is verified buy degenerating to Terzaghi's consolidation.

2. Literature Review –

<u>Terzaghi's One-Dimensional Consolidation for saturated soils –</u>

Sun et al (2018) established the analytical solution for one-dimensional solution of soils under a ramp load and verified the solution with existing analytical solutions. They studied the consolidation behavior different interface parameters or loading scheme and found that the exponentially time growing drainage boundary reflects the phenomenon that the excess pore water pressure at the drainage boundaries dissipates smoothly rather than abruptly from its initial value to value of zero. They also degenerated their solution to Schiffman's solution by adjusting the values of interface parameters. It is said that use of ETGD boundary is that it can be utilized to describe the asymmetric drainage characteristics of the top and bottom drainage surfaces of actual soil layer. The paper presented solution for homogenous soils and not for layered soils.

Wu et al. (2018) discussed the analytical solution for one-dimensional consolidation of double layered soil with exponentially time-growing drainage boundary to obtain excess pore water pressure and average consolidation. They degraded their solution to Xie's solution by adjusting the interface parameters and validated their results by comparing it with existing analytical solutions.

Ndiaye et al. (2014) derived the analytical solution for different drainage conditions and compared the results with numerical solutions. They found that analytical solution gives more accuracy due to the fact that the numerical resolution gives an approximate solution while the finite difference method some specific conditions that lead to stable resolution.

Fredlund's One-Dimensional Consolidation for Unsaturated Soils –

Fredlund and Hasan (1979) presented a theoretical framework for one dimensional consolidation. The assumptions made are in keeping with those used in the conventional theory for saturated soils, with additional assumption that air phase is continuous. They used the experimentally verified constitutive equations proposed by Fredlund and Morgenstren (1976) from which they derived the governing equations for air and water phase. They derived two partial equations to depict the transient process taking place as the application of the load to an unsaturated soil. The simultaneous solution of the two partial differential equations gives the pore-air and pore-water pressure at any time and any depth throughout the soil. They also proposed two equations to predict the initial pore-air and pore-water pressure boundary conditions.

Qin et al (2008) proposed an analytical solution of the one-dimensional consolidation in unsaturated soil under vertical loading and confinements in lateral directions with a finite thickness for single drainage condition. They used Laplace transform and Cayley-Hamilton mathematical methods to governing equations of water and air. By inverse Laplace transform the analytical solution was obtained in time domain. They illustrated the consolidation characteristics of unsaturated soil from analytical solution with a help of an example and compared the results with finite difference method.

Zhou et al. presented a simple analytical solution to Fredlund and Hasan's (1979) one-dimensional consolidation of unsaturated soils. The solution presented in this paper is much simpler compared to previous study and is adopted and explained in very detail in this study.

3. Proposed approach –

Consider an unsaturated soil layer with infinite horizontal extend and thickness H. A representative soil element of volume $V_0 = 1 \times 1 \times dz$ with one-dimensional water and air flow in the z direction as shown in the Figure. The top layer is a free drainage boundary and the bottom layer is impermeable to both air and water.

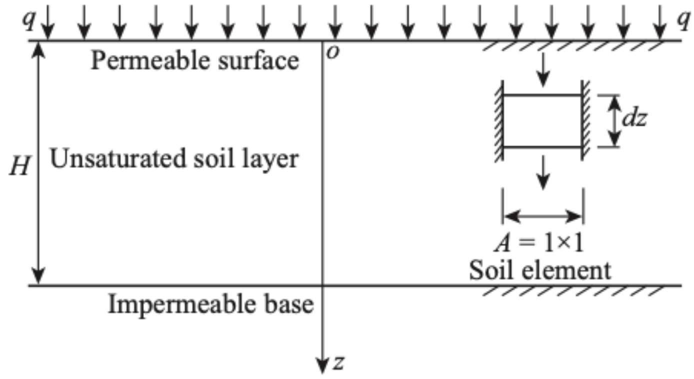

Figure 1- A model for one-dimensional consolidation in unsaturated soil with free drainage boundary at the top and an impermeable boundary at the bottom. (Qin et al. (2008))

In this study, a simple analytical solution to Fredlund and Hasan's consolidation model for unsaturated soils. Two new variables, $\phi_1 and\ \phi_2$ are introduced to transform the two coupled governing equations of pore-water and pore-air pressure into two conventional diffusion equations so that the analytical solutions are simply obtained. The solution is obtained for instantaneous loading and is degenerated for Terzhagi's solution by changing the parameters. The analytical solution for Terzhagi's one dimensional consolidation for saturated soils is derived using method of separation of variables. The solution is for the model described above with vertical loading and K_0 loading conditions without deformation.

4. Theoretical Framework –

<u>Terzaghi's One-Dimensional Consolidation for saturated soils –</u>

The compressible soil layer is assumed to be both homogenous and completely saturated with water, the mineral grains in the soil and the water pores are assumed to be incompressible. Darcy's law is considered to govern the egress of water from the soil pores, and both drainage and compression are one dimensional. The Terzaghi theory is a small strain theory in that the applied stress increment is assumed to produce only small strains in the soil; therefore, both the coefficient of compressibility a_v and Darcy's coefficient of permeability, k, remains constant during consolidation process.

Governing Equations –

The governing equations are derived considering water mass conservation and moment balance equation (i.e.) fluid must satisfy the condition of continuity, or mass balance equation which implies the net mass of flow of fluid into or out of a given volume in a specified time interval must be equal to the storage or loss of storage of fluid in the volume in the interval.

Water mass conservation-

$$\left(\frac{\alpha - \phi}{K_s} + \frac{\phi}{K_w}\right) \frac{D^2 p_w}{Dt} + \alpha div \left(\frac{\partial u}{\partial t}\right) = \frac{k_w}{\mu_w} \nabla^2 p_w$$

This can be rewritten as,

$$s\ \frac{\partial p_w}{\partial t} + \alpha\ \frac{\partial \epsilon}{\partial t} = \frac{k_w}{\mu}\ \ \nabla^2 p_w$$

where,
$s = \left(\frac{\alpha-\phi}{K_s} + \frac{\phi}{K_w}\right);\quad \epsilon = \frac{\partial u}{\partial t}$

Momentum balance equation-

$$D_e \vdots \nabla^2 u - \alpha \nabla p_w = 0$$

$$D_e \vdots \nabla \epsilon - \alpha \nabla p_w = 0$$

From linear elasticity,

$$\epsilon_{zz} = -m_v \sigma'_{zz}$$

$$\frac{\partial \epsilon}{\partial t} = -m_v\ \frac{\partial}{\partial t}(\sigma'_{zz})$$

$$\sigma' = \sigma - \alpha p$$

$$\frac{\partial \epsilon}{\partial t} = -m_v\ \frac{\partial(\sigma_{zz} - \alpha p)}{\partial t}$$

During consolidation the vertical total stress is unchanged,

$$\therefore \sigma = 0$$

$$\frac{\partial \epsilon}{\partial t} = -m_v\ \frac{\partial(-\alpha p)}{\partial t}$$

$$\frac{\partial \epsilon}{\partial t} = \alpha m_v \frac{\partial p}{\partial t}$$

From the equation above,

$$s\ \frac{\partial p}{\partial t} + \alpha^2 m_v \frac{\partial p}{\partial t} = \frac{k}{\mu}\ \nabla^2 p_w$$

$$\frac{\partial p}{\partial t}\ [s + \alpha^2 m_v] = \frac{k}{\mu}\ \nabla^2 p_w$$

$$\frac{\partial p}{\partial t} = \frac{k}{\mu[s + \alpha^2 m_v]}\ \nabla^2 p_w$$

$$\Rightarrow \frac{\partial p}{\partial t} = C_v\ \frac{\partial^2 p}{\partial z^2}$$

<u>Fredlund's One-Dimensional Consolidation for unsaturated soils –</u>

To obtain a closed-form solution, the main assumptions are summarized as follows:

1. Solid particles and water phase are incompressible.
2. Both water and air flows are governed by Darcy-type law.
3. The effects of temperature change, air dissolved in water, air diffusion, and generation and diffusion of vapor are disregarded.
4. The loading and deformation take place in only in one-dimension.

Continuity of unsaturated soil element requires that the overall volume change of the element must equal the sum of the volume changes associated with the component phases. The continuity requirement can be written:

$$\frac{\Delta V}{V} = \frac{\Delta V_w}{V} + \frac{\Delta V_s}{V}$$

where,

V = overall volume of the soil element;
V_w= volume of water in the soil element; and
V_a = volume of air in the soil element

Fredlund and Morgenstern proposed and tested constitutive relations to link stress and deformation state variables. The proposed constitutive relationship for the soil and water phase is given by,

$$\frac{\Delta V}{V} = m_1^s d(\sigma - u_a) + m_2^s d(u_a - u_w)$$

$$\frac{\Delta V_w}{V} = m_1^w d(\sigma - u_a) + m_2^w d(u_a - u_w)$$

Where,
m_1^s is compressibility of soil structure when d$(u_a - u_w)$ is zero
m_2^s is compressibility of soil structure when d$(\sigma - u_a)$ is zero
m_1^w is compressibility of water structure when d$(u_a - u_w)$ is zero
m_2^w is compressibility of water structure when d$(\sigma - u_a)$ is zero

The constitutive relationship for air is found by the difference,

$$\frac{\Delta V_a}{V} = m_1^a d(\sigma - u_a) + m_2^a d(u_a - u_w)$$

Flow of the water phase is governed by Darcy's law,

$$v = \left(\frac{k_w}{\gamma_w}\right)\left(\frac{\partial u_w}{\partial y}\right)$$

where,
K_w = coefficient of permeability

Flow of air phase is described by Fick's law,

$$v_a = k_a\left(\frac{\partial p}{\partial y}\right)$$

where,
k_a = transmission constant

Governing Equation for water phase –

The water phase is assumed to be incompressible. During consolidation water flows out of the element with time. The volume of water entering and leaving the element is described by Darcy's law as,

$$volume\ entering = \left(\frac{k_w}{\gamma_w}\right)\left(\frac{\partial u_w}{\partial y}\right)dxdy$$

The net flux of water in the element is given by,

$$\frac{\partial\left(\frac{\Delta V_w}{V}\right)}{\partial t} = \left(\frac{k_w}{\gamma_w}\right)\left(\frac{\partial^2 u_w}{\partial y^2}\right)$$

From the net flux and constitutive relation of water phase, we get,

$$\frac{m_1^w\ \partial(\sigma - u_a)}{\partial t} + \frac{m_2^w \partial(u_a - u_w)}{\partial t} = \left(\frac{k_w}{\gamma_w}\right)\left(\frac{\partial^2 u_w}{\partial y^2}\right)$$

In consolidation process, the change in total stress does not change with respect to time. So, the above equation becomes,

$$\frac{\partial u_w}{\partial t} = -c_v\left(\frac{\partial u_a}{\partial t}\right) + c_v^w\left(\frac{\partial^2 u_w}{\partial y^2}\right)$$

where,
$C_w = \frac{\left(\frac{1-m_2^w}{m_1^w}\right)}{\left(\frac{m_2^w}{m_1^w}\right)}$ is called the interactive constant associated with water phase

$C_v^w = \left(\frac{1}{R_w}\right)\left(\frac{k_w}{\gamma_w m_1^w}\right)$ is the coefficient of consolidation with respect to water phase

$R_w = m_2^w / m_1^w$; it is zero when soil is completely saturated

Governing Equation for Air phase –

The air phase is compressible and flows independent of water phase when subjected to an air pressure gradient. According to Fick's law the mass of air entering the element is,

$$mass\ entering = k_a \left(\frac{\partial p}{\partial y}\right) dxdz$$

The net mass flux of air in the element is,

$$\frac{\partial m}{\partial t} = k_a \left(\frac{\partial p}{\partial y^2}\right)$$

where m is mass of air in the element.

The mass rate of change in terms of volume rate of change by differentiating the relationship between mass and volume,

$$\frac{\partial \left(\frac{V_a}{V}\right)}{\partial t} = \frac{\partial \left(\frac{m}{\gamma_a}\right)}{\partial t}$$

For isothermal conditions the density of air, γ_a, is,

$$\gamma_a = \left(\frac{w}{RT}\right) p$$

where,
$w = molecular\ weight\ of\ the\ mass\ of\ air$
$R = universal\ gas\ constant$
$T = absolute\ temperature$

The mass of air is written in terms of the density of air, the degree of saturation, S, and the porosity of the soil, n,

$$m = (1 - S) m \gamma_a$$

From the above equations, we can write mass rate of change as,

$$\frac{\partial \left(\frac{V_a}{V}\right)}{\partial t} = \frac{k_a RT}{g \bar{u}_a^0 M} \frac{\partial^2 u_a}{\partial y^2} + \frac{u_{atm}(1 - s)n}{u_a} \frac{\partial u_a}{\partial t}$$

Equating the above equation to the constitutive relationship of air phase,

$$\frac{\partial\left(\frac{V_a}{V}\right)}{\partial t} = \frac{m_1^w\,\partial(\sigma - u_a)}{\partial t} + \frac{m_2^w\partial(u_a - u_w)}{\partial t}$$

$$\frac{m_1^w\,\partial(\sigma - u_a)}{\partial t} + \frac{m_2^w\partial(u_a - u_w)}{\partial t} = \frac{k_a RT}{g\overline{u}_a^0 M}\,\frac{\partial^2 u_a}{\partial y^2} + \frac{u_{atm}(1-s)n}{u_a}\,\frac{\partial u_a}{\partial t}$$

In consolidation process, the change in total stress does not change with respect to time. So, the above equation becomes,

$$\frac{\partial u_a}{\partial t} = -C_a\left(\frac{\partial u_w}{\partial t}\right) + C_v^a\left(\frac{\partial^2 u_a}{\partial y^2}\right)$$

Where,

$C_a = \frac{m_2^a}{m_1^a - m_2^a - \frac{u_{atm}n(1-S)}{u_a^0}}$; $C_v^a = \frac{k_a RT}{g u_a M\left(m_1^a - m_2^a - \frac{u_{atm}\,n(1-s)}{\overline{u}_a^0}\right)}$

$\overline{u}_a^0$ = initial excess air pressure

$$\overline{u}_a^0 = u_a^0 + u_{atm}$$

5. Analytical Solution

<u>Terzaghi's One-Dimensional Consolidation for Saturated Soils –</u>

Governing equations –

$$\boldsymbol{\frac{\partial u}{\partial t} = C_v\,\frac{\partial^2 u}{\partial z^2}}$$

Using method of separation variables,

$$u = T(t)Z(z)$$

$$T'(t)Z(z) = C_v T(t)Z''(z)$$

Separating the variables,

$$\frac{T'(t)}{C_v T(t)} = \frac{Z''(z)}{Z(z)} = k^2$$

Considering only the term dependent on time, the solution can be written as,

$$T(t) = Ae^{k^2 t C_v}$$

Considering the term dependent on space, the solution can be written as,

$$Z(z) = \alpha \sin(kz) + \beta \cos(kz)$$

Combining the solution of both the terms,

$$u = Ae^{k^2 t C_v}(\alpha \sin(kz) + \beta \cos(kz))$$

Boundary and initial conditions –

Considering the sample is permeable to water on top and impermeable at the bottom (single drainage),

The pore pressure at the top layer is zero,

$$u(0, t) = 0$$

The bottom layer is impermeable to water,

$$\frac{\partial u(H, t)}{\partial z} = 0$$

The initial pore pressure is given as u_0,

$$u = u_0 \ at \ t = 0$$

Substituting the boundary conditions and matching with the Fourier series, the analytical solution for the partial differential equation can be given as,

$$\boldsymbol{u = \frac{4u_0}{\pi} \sum_{m=1}^{m=\infty} \frac{(-1)^{m-1}}{2m-1} \cos\left[\frac{(2m-1)\pi z}{2h}\right] \exp\left[-\frac{(2k-1)^2 \pi^2 C_v t}{4h^2}\right]}$$

<u>Fredlund's One-Dimensional Consolidation for Unsaturated soils –</u>

Governing equations for water and air phase –

$$\boldsymbol{\frac{\partial u_w}{\partial t} = -c_v \left(\frac{\partial u_a}{\partial t}\right) + c_v^w \left(\frac{\partial^2 u_w}{\partial y^2}\right)}$$

$$\boldsymbol{\frac{\partial u_a}{\partial t} = -C_a \left(\frac{\partial u_w}{\partial t}\right) + C_v^a \left(\frac{\partial^2 u_a}{\partial y^2}\right)}$$

Boundary and Initial Conditions –

Considering the sample is permeable to water and air on top and impermeable at the bottom (single drainage),

The pore water and air pressure at the top layer is zero,

$$u_a(0,t) = 0, \qquad u_w(0,t) = 0$$

For single drainage, the lower boundary is impermeable to both air and water,

$$\frac{\partial u_a(h,t)}{\partial z} = 0; \; \frac{\partial u_w(h,t)}{\partial z} = 0$$

The initial pore- water and pore air pressure distributions along the depth can be expressed as:

$$u_a(z,0) = u_a^i(z); \;\; u_w(z,0) = u_w^i(z)$$

From the governing equations of water and air phases, we get,

$$\frac{\partial u_w}{\partial t} = C_w C_a \frac{\partial u_w}{\partial t} + C_w C_v^a \frac{\partial^2 u_a}{\partial z^2} - \; C_v^w \frac{\partial^2 u_w}{\partial z^2}$$

$$\frac{\partial u_w}{\partial t} - C_w C_a \frac{\partial u_w}{\partial t} = \; C_w C_v^a \frac{\partial^2 u_a}{\partial z^2} - \; C_v^w \frac{\partial^2 u_w}{\partial z^2}$$

Multiply by $\gamma_w m_1^s h^2 / k_w$ on both sides, and defining,

$$T = \frac{\gamma_w m_1^s h^2}{k_w}; \;\; \overline{z} = \frac{z}{h}; \; A_a = \; - \frac{C_v^a \gamma_w m_1^s}{(1 - C_w C_a) k_w}; \;\; A_w = \; \frac{C_a \gamma_w m_1^s}{(1 - C_w C_a) k_w};$$

$$W_a = \frac{C_w C_v^a \gamma_w m_1^s}{(1 - C_w C_a) k_w}; \;\; W_w = \frac{-C_v^w \gamma_w m_1^s}{(1 - C_w C_a) k_w}$$

We can rewrite the above equation,

$$\frac{\partial u_a}{\partial T} = A_a \frac{\partial^2 u_a}{\partial \overline{z}^2} + A_w \; \frac{\partial^2 u_w}{\partial \overline{z}^2}$$

$$\frac{\partial u_w}{\partial T} = W_a \frac{\partial^2 u_a}{\partial \overline{z}^2} + W_w \; \frac{\partial^2 u_w}{\partial \overline{z}^2}$$

Multiply the above equations with c1 and c2 and adding them we get,

$$\frac{\partial(u_a c_1 + u_w c_2)}{\partial T} = (A_a C_1 + W_a C_2)\left(\frac{\partial^2 u_a}{\partial \overline{z}^2}\right) + (A_w C_1 + W_w C_2)\left(\frac{\partial^2 u_w}{\partial \overline{z}^2}\right)$$

This equation can be transformed into conventional heat equation with $\phi = u_a c_1 + u_w c_2$ and by introducing a constant Q,

Q should satisfy,

$$QC_1 = A_a C_1 + W_a C_2$$

$$QC_2 = A_w C_1 + W_w C_2$$

And

$$(Q - A_a)(Q - W_w) - A_w W_a = 0$$

The above equation is a quadratic equation in which Q has two equations $Q_1 \; and \; Q_2$,

$$Q_{1,2} = \frac{1}{2}\left[A_a + W_a \pm \sqrt{\sqrt{((A_a - W_w)^2} + 4A_w W_a}\right]$$

From the solutions of Q, the constants can be evaluated. Assuming $C_{11} = C_{22} = 1$, $C_{12} and \; C_{21}$ can be expressed as,

$$c_{12} = \frac{W_a}{Q_2 - A_a} = \frac{Q_2 - W_w}{A_w}$$

$$c_{12} = \frac{A_w}{Q_1 - W_w} = \frac{Q_1 - A_a}{W_a}$$

With the solutions, the equation above can be rewritten in the following form,

$$\frac{\partial \phi_1}{\partial T} = Q_1 \frac{\partial^2 \phi_1}{\partial \overline{z}^2}$$

$$\frac{\partial \phi_2}{\partial T} = Q_2 \frac{\partial^2 \phi_2}{\partial \overline{z}^2}$$

Where $\phi_1 = \overline{u_a} + c_{21}\overline{u}_w, \; \phi_2 = c_{21}\overline{u}_a + \overline{u}_w$

Therefore, the transformed upper boundary conditions are,

$$\phi_1(0,T) = \overline{u}_a(0,T) + c_{21}\overline{u}_w(0,T) = 0$$

$$\phi_2(0,T) = c_{12}\,\overline{u}_a(0,T) + \overline{u}_w(0,T) = 0$$

The transformed lower boundary conditions $\phi_1(1,T) and \; \phi_2(1,T)$ are,

$$\frac{\partial \phi_1(1,T)}{\partial z} = \frac{\partial \overline{u}_a(1,T)}{\partial z} + c_{21}\frac{\partial \overline{u}_w(1,T)}{\partial z} = 0$$

$$\frac{\partial \phi_2(1,T)}{\partial z} = c_{12}\frac{\partial \overline{u}_a(1,T)}{\partial z} + \frac{\partial \overline{u}_w(1,T)}{\partial z} = 0$$

The transformed initial conditions $\phi_1(\overline{z},0)$ and $\phi_2(\overline{z},0)$ are

$$\phi_1(\overline{z},0) = \phi_1'(\overline{z}) = \overline{u_a^i} + c_{21}\overline{u}_w^i$$

$$\phi_2(\overline{z},0) = \phi_2'(\overline{z}) = c_{12}\overline{u_a}^i + \overline{u}_w^i$$

Using variable separation method, the solution for the above equation can be obtained by,

$$\phi = (A_1 \cos(B\overline{z}) + A_2 \sin(B\overline{z})) \exp(-B^2 QT)$$

where, A_1, A_2 and B are constants which ca be found by substituting boundary conditions.

For single drainage conditions, the analytical solution is given by,

$$\phi = \sum_{m=1}^{m=\infty} \left(2\int_0^1 \phi^i(\overline{z}) \sin(K\overline{z})\, d\overline{z}\right) \sin(K\overline{z}) \exp(-K^2TQ)\,, K = (2m+1)\pi/2$$

Further simplifying the solution, we get,

$$\boldsymbol{\phi} = \sum_{\boldsymbol{m=1}}^{\boldsymbol{m=\infty}} \frac{\boldsymbol{2\phi_0}}{\boldsymbol{K}} \;\; \mathbf{sin}(\boldsymbol{K\overline{z}})\, \mathbf{exp}(-\boldsymbol{K^2TQ}), \quad \boldsymbol{\phi_0} = \boldsymbol{\phi^i}(\overline{\boldsymbol{z}})$$

6. Example and verification –

The analytical solution of one-dimensional for unsaturated soil, an example is computed by implementing the solution in python and the results are plotted.

Consider a soil layer is unlimited in the horizontal direction, with free drainage boundary at the top surface and impermeable boundary at the bottom, with the following parameters,

$$h = 10\ m$$

$$n = 0.5, S_o = 0.8$$

$$k_w = 10^{-10}\frac{m}{s}$$

$$m_1^w = -0.5\ \times 10^{-4}\ \ kPa^{-1}$$

$$m_2^w = -2.0 \times 10^{-4}\ kPa^{-1}$$

$$m_1^s = -2.5 \times 10^{-4}\ kPa^{-1};\quad m_2^s = -1.0 \times 10^{-4}\ kPa^{-1}$$

$$q_0 = 100\ kPa;\quad u_a^i = 40\ kPa,\quad u_w^i = 40\ kPa$$

$$T = 0.001 - 0.1$$

The solution is also verified by degenerating to Terzaghi's one-dimensional consolidation equation for saturated soils, For a fully saturated soil,

$$m_1^w = m_2^w = m_v, Coefficient\ of\ volume\ change$$

$$m_1^a = m_2^a$$

$$k_w = k_s, coefficient\ of\ permeability$$

$$c_w = c_a = c_v^a = 0$$

$$W_a = A_a = A_w = 0$$

$$W_w = 1$$

$$u^i = 100\ kPa$$

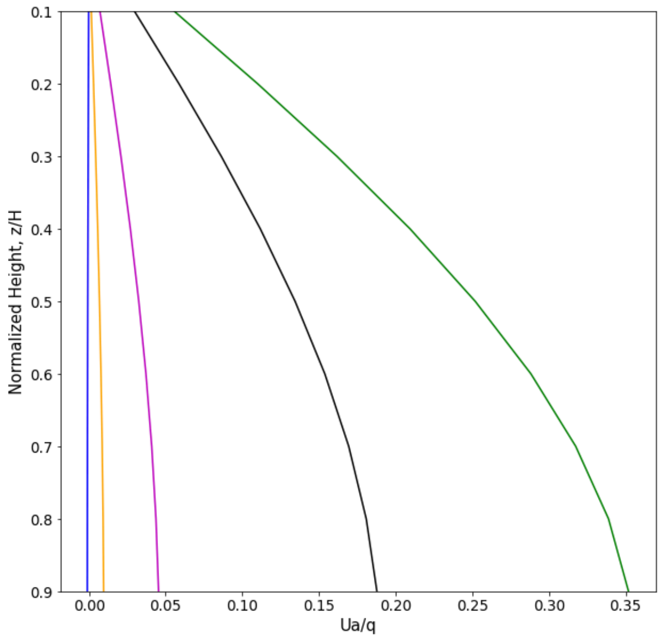

Figure 2 - Pore air pressure isochrones (Unsaturated soils)

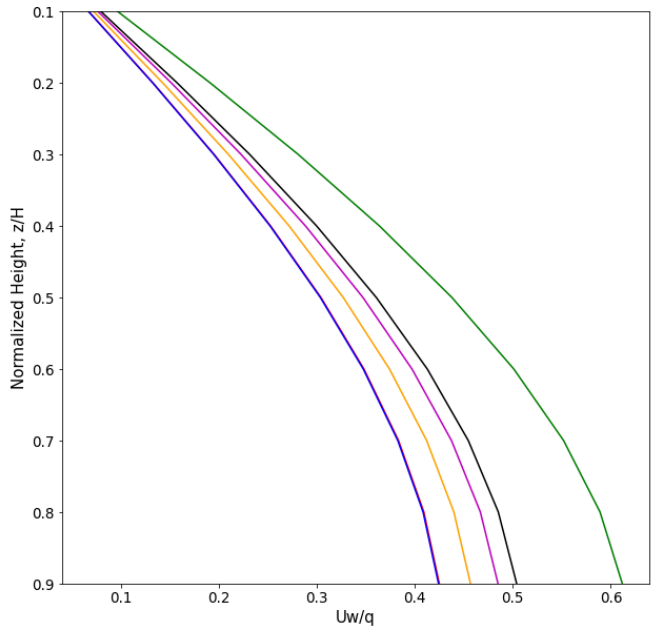

Figure 3- Pore water pressure isochrones (unsaturated soil)

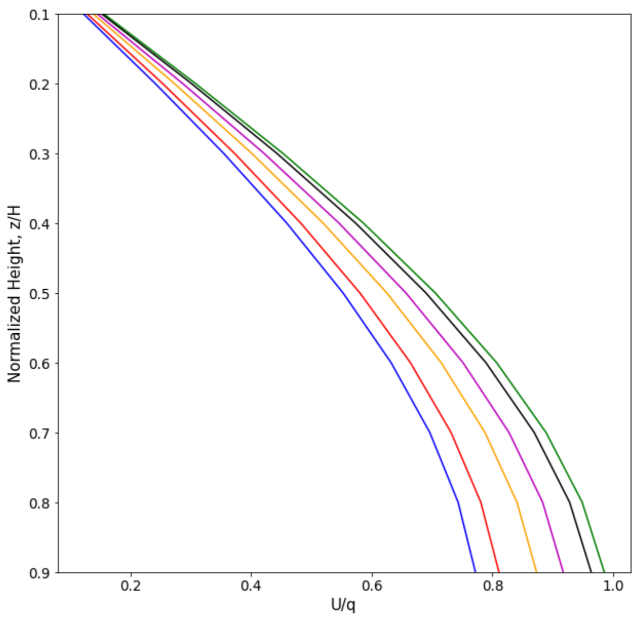

Figure 4- Pore water pressure isochrones (Saturated soils)

7. Conclusion –

The study presented an analytical solution for Fredlund and Hasan's one-dimensional consolidation. The solution is simple and can be easily implemented and analyzed by practice engineers. This can be developed with future investigation. The solution can be expanded for different loading patterns and drainage conditions. The study is limited to one-dimension only owing to the given situation, but in future can be expanded for two and three dimensions. Further, the sensitivity of the model can be studied by changing the parameters and can be improved based on the results.

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