CS6360 - Computer Graphics

Tutorial 1; Total time: 50 mins

Question 1:

Consider a raster monitor of resolution 640*480 pixels. A scanning is used with horizontal retrace time of 4 micro seconds and vertical retrace time of 20 micro seconds respectively. Calculate the time available to display a pixel for both cases of (i) non-interlaced and (ii) interlaced. Assume a scan rate of 50 frames per seconds

Answer:
(A)

(i) Non-interlaced:

Time per scan = (1000/50) = 20 m sec

Retrace/ scan = (20 + 480x4) micro sec = 1.94 ms

Total time to scan pixels = 20 – 1.94 = 18.06 ms

Therefore, time per pixel = $\frac{18.06}{640.480}$ = 58.8 nano sec

(ii) Interlaced:

Time per fields = 100 ms

Retrace/ field = (20 + 240 x 4) = 0.98 ms

Total time to scan pixels/ field = 10 – 0.98 = 9.02 ms

Therefore, time per pixel = $\frac{9.02}{640.240}$= 58.7 nano sec

(B) No. of pixels displayed per second = 640*480*50.

No. of horizontal retraces per second = 480*50

No. of vertical retrace per second for non-interlaced case = 50

No. of vertical retrace per second for interlaced case = 50*2 = 100

Total horizontal retrace time per second = 480*50*4 micro-secs ≈ 96 milli-secs

Total vertical retrace time per second for non-interlaced case = 50*20 micro-secs ≈ 1 milli-sec

Total vertical retrace time per second for interlaced case = 100*20 micro-secs ≈ 2 milli-secs.

(i) Non-interlaced total retrace time in a second for non-interlaced case is 97 milli-secs. So, time available for all pixel displays during a particular scan is: (1000-97)milli-secs = 903 milli-secs.

Therefore, time available to display a single pixel for non-interlaced case is :
903/(640*480*50) milli-sec = 58.8 nano-secs.

(ii) Interlaced total retrace time in a second for interlaced case is 98 milli-secs.
So, time available for pixel display during a particular scan is: (1000-98) = 902 milli-secs.

Therefore, time available to display a single pixel for interlaced case is :
902/(640*480*50) milli-sec = 58.7 nano-secs.

Question 2:

Write the pixels to be considered to draw a line between (10,15) and (17,20) using mid-point line drawing algorithm. Mention the initial values of the required parameters as well as the change in values of the necessary parameters.

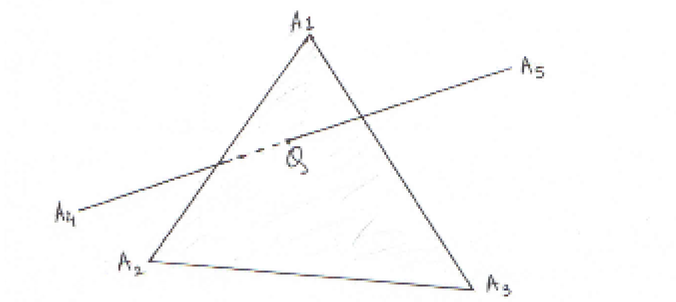
Answer:

$x_1 = 17$, $y_1 = 20$; $x_0 = 10$, $y_0 = 15$; dx = 7, dy = 5, $\Delta d_E = 10, \Delta d_{NE} = -4$. Dstart = 3

Sl. No.	d	x	y	change
0	d = 3	10	15	-
1	d = 3+(-4) = -1	11	16	NE
2	d = -1+10 = 9	12	16	E
3	d = 9-4 = 5	13	17	NE
4	d = 5-4 = 1	14	18	NE
5	d = 1-4 = -3	15	19	NE
6	d = -3+10 = 7	16	19	E
7	d = 7-4 = 3	17	20	NE

Question 3:

A plane determined by the points $A_1, A_2$and $A_3$and a line passing through the points $A_4$and A_5intersect the plane at a point, Q. Find the coordinates of Q.



Answer:

The point Q satisfies eqn of both line and plane,

Parametric eqn of plane,

$$\vec{q} = \overrightarrow{a_1} + (\overrightarrow{a_2} - \overrightarrow{a_1})u + (\overrightarrow{a_3} - \overrightarrow{a_1})v$$

Parametric eqn of line,

$$\vec{q} = \overrightarrow{a_4} + (\overrightarrow{a_5} - \overrightarrow{a_4})t$$

$$\overrightarrow{a_4} + (\overrightarrow{a_5} - \overrightarrow{a_4})t = \overrightarrow{a_1} + (\overrightarrow{a_2} - \overrightarrow{a_1})u + (\overrightarrow{a_3} - \overrightarrow{a_1})v$$
$$(\overrightarrow{a_2} - \overrightarrow{a_1})u + (\overrightarrow{a_3} - \overrightarrow{a_1})v + (\overrightarrow{a_4} - \overrightarrow{a_5})t = \overrightarrow{a_4} - \overrightarrow{a_1}$$

t can be solved as,

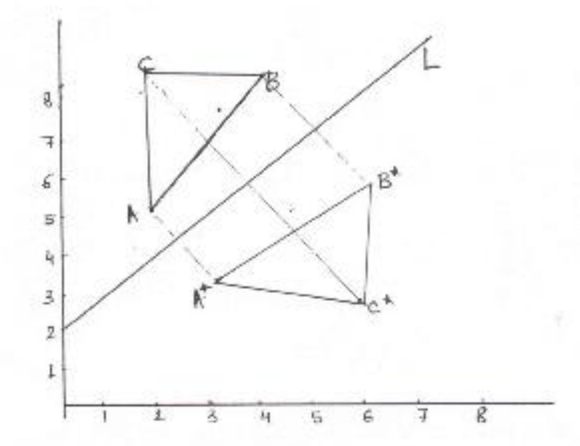
$$t = \frac{det\begin{bmatrix} x_2 - x_1 & x_3 - x_1 & x_1 - x_4 \\ y_2 - y_1 & y_3 - y_1 & y_1 - y_4 \\ z_2 - z_1 & z_3 - x_1 & z_1 - z_4 \end{bmatrix}}{det\begin{bmatrix} x_2 - x_1 & x_3 - x_1 & x_4 - x_5 \\ y_2 - y_1 & y_3 - y_1 & y_4 - y_5 \\ z_2 - z_1 & z_3 - x_1 & z_4 - z_5 \end{bmatrix}}$$

This value can then be plugged back in parametric eqn of line to give the point of intersection.

Question 4:

Consider the line L and the triangle ABC as shown in the figure given below. The equation of the line L is y=x+2. The coordinates of the points A, B and C are (2,5,1), (2,8,1) and (4,8,1) respectively. A*B*C* represents the reflection of the original triangle ABC through the line L. Find the co-ordinates of A*, B* and C*.

Answer:



The line will pass through the origin by translating it -2 units in the y direction. The resultant line can be made coincident with the x axis by rotating it by $-\tan^{-1}(1)$ = -45 degree about the origin. After that, the triangle ABC can be reflected through the line L, which now coincides with the x axis. The transformed position vectors of the triangle are then rotated and translated back to the original orientation. The combined transformation is:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\sqrt{2} & -\sqrt{2} & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ -\sqrt{2} & \sqrt{2} & 1 \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix}$$

Hence, the transformed position vectors for the triangle A*B*C* are:

$$\begin{bmatrix} 2 & 5 & 1 \\ 2 & 8 & 1 \\ 4 & 8 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & 1 \\ 6 & 4 & 1 \\ 6 & 6 & 1 \end{bmatrix}$$

Question 5:

Let a point P moves with at an uniform velocity v on a linear path in 3D. Find the velocity of point Q, which is the perspective projection of P on a 2D plane. Consider the focal length of the camera be f.

Answer:

Let the velocity of P along x, y and z coordinates be v_1, v_2 and v_3 respectively ($\vec{V}$ = [v_1 v_2 v_3]). Let after one unit time, the point P moves to the location P'. Hence, in the projection plane, the point Q moves to the new location Q'. If the projection matrix is given by the matrix T, where:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1/f \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, displacement of P' from P is $[v_1 \quad v_2 \quad v_3 \quad 1]$

Displacement of Q' from Q = P'T-PT = $[v_1 \quad v_2 \quad v_3 \quad 1]$*T = $\left[\frac{v_1 f}{f-v_3} \quad \frac{v_2 f}{f-v_3} \quad \frac{v_3 f}{f-v_3} \quad 1\right]$

Hence the velocity in the projection plane is $\left[\frac{v_1 f}{f-v_3} \quad \frac{v_2 f}{f-v_3}\right]$