DEPARTMENT OF INFORMATION TECHNOLOGY, NITK SURATHKAL

MID SEMESTER EXAMINATION, FEBRUARY 2018

IT252: DESIGN AND ANALYSIS OF ALGORITHMS

Class: IV SEM B.TECH (IT)

Date: 08/02/2018

Time: 1½ Hrs.

Marks: 26

Register No. 16IT201

NOTE: 1. Both sides of this paper contain questions.

2. Use Pseudo-code to describe algorithms, unless asked otherwise.

Problem 1: [5 x 1 = 5 marks] State if the following statements are True/False. Briefly justify your answer. A correct answer with no or incorrect justification will only get partial credit.

a) *In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.*

b) *Consider an instance of the Stable Matching Problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.*

c) *If* $f(n)=O(g(n))$ *then* $\lim_{n\to\infty} (f(n) / g(n)) = 0$.

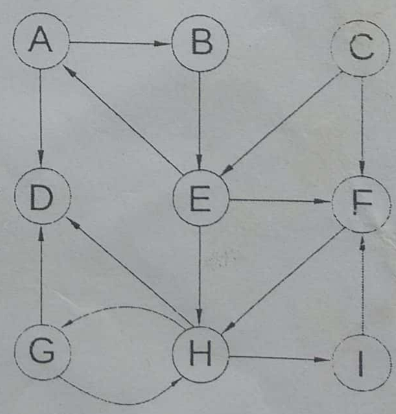
d) *If the recurrence relation for the runtime of algorithm A is* $T_A(n) = 6T_A(n/3)+O(n^2)$ *and that of algorithm B is* $T_B(n) = 5T_B(n/2)+O(n^2)$, *then algorithm A is asymptotically faster than algorithm B.*

e) *The tightest asymptotic runtime for the following piece of code is* $O(n^2)$:

```
i=1
while (i < n)
  i = i*2
  for (j = 1 ; j < n ; j=j+2)
    print ("something")
```

Problem 2: [6 marks]

a) Run Kosaraju's strongly connected components algorithm on the directed graph shown on the right . Whenever there is a choice of vertices to explore in the DFS, use the lexicographic order.



i) Show the time-stamps of each vertex after the DFS on the reverse graph.
ii) What are the vertices in each SCC?
iii) In what order are these SCCs discovered?
iv) Draw the SCC meta-graph of the graph.
v) If a directed edge from vertex G to F is added to the graph, draw the resulting SCC meta-graph.
vi) What is the minimum number of edges you must add to this graph to make it strongly connected? specify why?

Problem 3: [6 marks]

You are consulting for a bank that has a fraud detection problem: they have confiscated n bank cards which they suspect to be fraudulent. Each card contains a magnetic stripe with some encrypted data. A given card corresponds to a unique bank account; a given bank account can however have more than one card corresponding to it.

It's impossible to read the account number directly from a bank card, but the bank has a sophisticated equivalence testing machine that takes two bank cards, and after performing some computations, can detect if they correspond to the same account or not.

Answer the following question: given a set of n cards, is there a set of more than n/2 cards, all of which correspond to the same bank account? The only feasible operation you are allowed to is to test for equivalence using the machine (i.e. take two cards and put them in the equivalence tester). Design a way to answer this question with only O(nlogn) invocations of the equivalence tester. Argue briefly why your approach is indeed correct.

Problem 4: [2+7=9 marks]

The Department of I.T. at NITK has **n** courses all of which are mandatory (i.e. non-electives). Each course might have one or more courses listed as pre-requisites. A student can register for a course only after having completed all its pre-requisite courses.

a) Model this scenario with a suitable data structure. State clearly how your data structure captures the information in the input.

b) We want to find out the minimum number of semesters needed to complete the curriculum. (Assume that a student is allowed to take any number of courses in a given semester). Design a linear time algorithm to solve this problem. Give a proof that your algorithm is indeed correct.

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MID SEMESTER EXAMINATION, FEBRUARY 2017

IT252: DESIGN AND ANALYSIS OF ALGORITHMS

Class: IV SEM B.TECH (IT)
Date: 13/02/2017

Time: 1½ Hrs.
Marks: 24

Register No.

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NOTE: 1. Answer all questions and to the point.

Problem 1: [1+1+2 = 4 marks]

a) Give the tightest *Big-O* representation for the following expressions:
i) $1+2+3+...+n$ ii) $\log(n^2) + \sqrt{n}$

b) Solve the following recurrences using Master's Theorem:
i) $T(n) = 5T(n/4)+O(n)$ ii) $T(n) = 3T(n/2)+2n^2$

c) What is the complexity of the following program? Show steps to calculate.

```
fun()
{       n= 2^(2^k)
        for(i=1;i<=n; i++)
        {       j=2
                while(j<=n)
                {       j=j^2
                        printf("NITK");
                }
        }
}
```

Problem 2: [2+2 = 4 marks]

a) Suppose Kruskal's algorithm is run on the graph as shown in the figure 1. In what order are the edges added to the MST? For any two edges in this sequence, give a cut that justifies its addition.

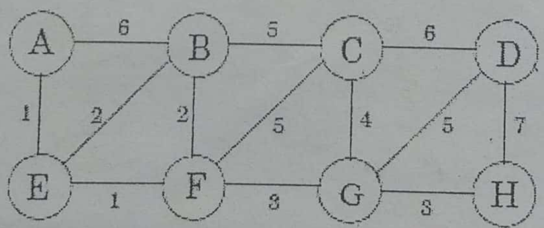

Figure 1.

b) A ***majority element*** in an array of size n is an element that appears more than $n/2$ times in the array. Design a $O(n \log n)$ algorithm to find the majority element in an input array A of size n. Return NIL if the array does not contain a majority element. The elements of the array A are NOT comparable (they could be image, audio or video files for instance, so your algorithm can not check if $A[i] > A[j]$), so we can not sort the array. We can however check for equality (i.e. if $A[i]==A[j]$).

Problem 3 [2+2 = 4 marks]

a) Under the *union-by-rank* and *path-compression* heuristics, perform the following operations *sequentially* on the Disjoint Set Data Structure in figure 2:

findSet(g); findSet(h); union(e,l).

(The second operation should be executed on the forest resulting from the first operation, and so on.) Draw the resulting forest after each operation, clearly showing parent pointers and node ranks.

b) *Write the pseudo-code for the findSet(x)* operation that implements Path-Compression.

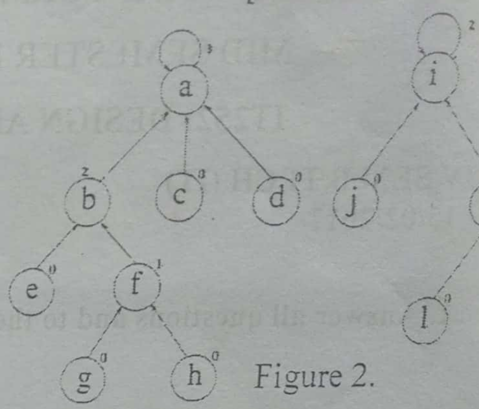

Figure 2.

Problem 4: [1+3 = 4 marks]

a) In a sequence of n operations, the i^{th} operation takes i units of time to execute. What is the amortized cost for each individual operation in the sequence?

b) For an array A, a pair of indices (i,j) is a *significant inversion* if $i<j$ and $A[i] > 2A[j]$. Give a $O(n \log n)$ algorithm to compute the number of significant inversions in an input array of distinct integers.

Problem 5 [2+2= 4 marks]

a) Give two conditions to find articulation point in a graph? Give a proof.

b) Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counter example.
"In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m".

Problem 6 [1+1+1+1=4 marks]

Run the strongly connected components algorithm on the directed graph G as shown in figure 3. When doing DFS on G^R: whenever there is a choice of vertices to explore, always pick the one that is alphabetically first.

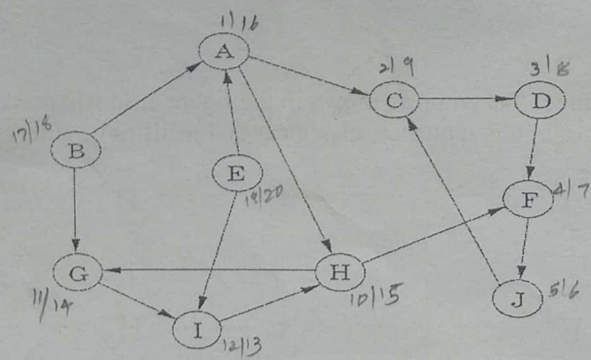

Figure 3.

In the above case, answer the following questions.
(a) In what order are the strongly connected components (SCCs) found?
(b) Which are source SCCs and which are sink SCCs?
(c) Draw the meta graph /component graph
(d) What is the minimum number of edges you must add to this graph to make it strongly connected?

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