MA 200 - Assignment 2018

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1. How many automorphisms do the following (labeled) graphs have?
   (a) The complete graph on n vertices.
   (b) The cycle on n vertices, with $n \geq 3$.
   (c) The path on n vertices.

2. Give an example of each of the following or explain why no such example exists.
   (a) A tree of order 4 whose complement is not a tree.
   (b) A tree of order 6 containing four vertices of degree 1 and two vertices of degree 3.
   (c) tree of order 8 containing six vertices of degree 1 and two vertices of degree 4.

3. The 20 members of a local tennis club have scheduled exactly 14 two-person games among themselves, with each member playing in at least one game. Prove that within this schedule there must be a set of 6 games with 12 distinct players.

4. There are k people in a room. Assume each person's birthday is equally likely to be any of the 365 days of the year (we exclude February 29), and that people's birthdays are independent (we assume there are no twins in the room). What is the probability that two or more people in the group have the same birthday? Also, state the minimum value of k for which this probability exceeds 0.5.

5. Consider two probability spaces, $(S_1, \mathcal{F}_1, P_1)$ and $(S_2, \mathcal{F}_2, P_2)$. Here, S_1 and S_2 are finite sets, and $\mathcal{F}_1$ and $\mathcal{F}_2$ are their power sets. Let $S_1 = \{x_1, \ldots, x_m\}$ and $S_2 = \{y_1, \ldots, x_n\}$. Let $S := S_1 \times S_2$, and let $\mathcal{F}$ be the power set of S. Define an additive function $P : \mathcal{F} \to \mathbb{R}$ such that
$$P((x_i, y_j)) = P_1(x_i)P_2(y_j)$$
for each $(x_i, y_j) \in S$. Use additivity to define P for each element of $\mathcal{F}$. Show that P is a probability function.