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Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Examination: Mid Semester — Maximum Marks: 50
Odd Semester (2017-18) — Course Code: MA 200
Course Name: Mathematical Foundations of Information Technology
Date: 15/9/2017 — Time: 8.30 a.m to 10 a.m

INSTRUCTIONS:

1. Answer ALL the eleven questions.
2. Rough work should NOT be done anywhere on the Question Paper.
3. Do the indexing properly. You will be penalized if you do not do the indexing.
4. There are questions on the other side of this paper also.

Q.1. Prove or disprove: If an edge appears exactly once in a closed walk W, then W contains a cycle. [3]

Q.2. A graph has 12 edges and 6 vertices, each of which has degree 2 or 5. If possible, find out the number of vertices of each degree. [3]

Q.3. Let G be a connected graph with $V(G) = \{v_1, ..., v_n\}$ with $n \geq 2$. Show that at least two of the subgraphs $G - v_1, ..., G - v_n$ are connected. [4]

Q.4. Suppose that a simple graph G has at least 11 vertices. Prove or disprove: Both G and its complement cannot be planar graphs. [4]

Q.5. Does there exist a simple graph of seven vertices $v_1, v_2, \ldots, v_7$, such that deg $(v_1) = 6$, deg (v_2) = deg $(v_3) = 5$, deg $(v_4) = 4$, deg $(v_5) = 3$, deg $(v_6) = 2$, deg $(v_7) = 1$? Justify your answer. [4]

Q.6. Does there exist a complete bipartite graph $K_{r,s}$ with 100 regions? Justify your answer by giving an example of such a graph if it exists, or otherwise explain why such a graph does not exist. [4]

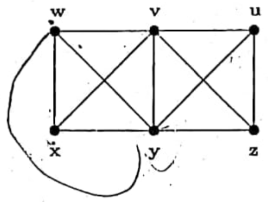
Q.7. Consider a simple graph G in which each vertex has degree at least k. Is it necessary that G has a path of length k? Justify your answer. [5]

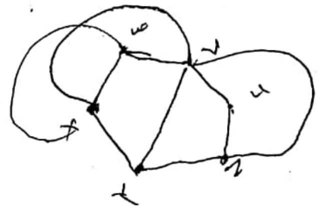
Q.8. The set of natural numbers from 1 to 5 can be arranged in a circle so that each number is adjacent to every other number exactly once by putting them in the order 1,2,3,4,5,3,1,4,2,5. Is a similar arrangement possible for the set of natural numbers from 1 to 55? Justify your answer. [5]

Q.9. Prove that a graph with n vertices is a tree if and only if it is connected and has $n-1$ edges. [6]

2k. = 2k.

Q.10. Consider the following graph G:





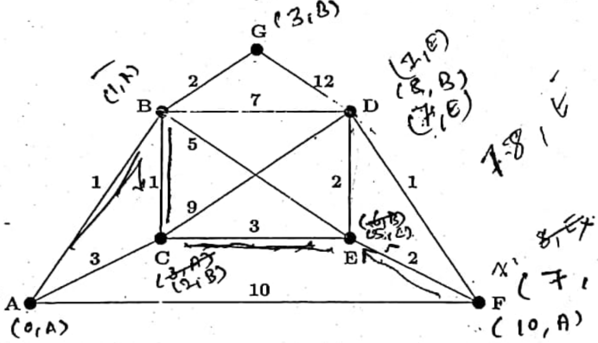
Answer the following with justification:

(a) Is G Eulerian? [2]

(b) Is G bipartite? [2]

(c) If G is planar then draw a plane graph representing G and compute the number of regions of that plane graph. Otherwise prove that G is not planar. [2]

Q.11. Use Dijkstra's Algorithm to find the cost of a cheapest path between A and F in the following weighted graph. Also find a path of the cheapest weight. [6]



(*This question paper contains* 2 *page(s) and* 11 *Questions.*)

A → B → C → E → F.

15+8 = (23)

Roll No:.16IT202..........

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
End Semester Examination Part A
Course Name: Mathematical Foundations of Information Technology

Odd Semester (2017-18) Course Code: MA 200

Date: 17/11/2017 Time: 2 p.m to 5 p.m Maximum Marks: 60

1. Answer ALL the 12 questions.
2. Rough work should NOT be done anywhere on the Question Paper.
3. Do the indexing properly.
4. There are questions on the other side of this paper also.

1. Suppose that l lines are drawn through a circle and these lines form p points of intersection (involving exactly two lines at each intersection). How many regions r are formed inside the circle by these lines? Assume that the lines end at the edge of the circle at $2l$ distinct points. (6)

2. G is a simple graph with p vertices and each vertex has degree at least $(p-1)/2$. Is it necessary that G is connected? Explain. (5)

3. Let G be a bipartite graph. Prove that G contains a perfect matching if and only if $|S| \leq |N(S)|$ for all $S \subset V(G)$. (5)

4. Let G be a connected bipartite graph with bipartition (X, Y). Also, $deg(x) \leq 6$ for each x in X. What is the best upper bound for $|Y|$ in terms of $|X|$? (5)

5. Suppose we know that an agency gets telephone calls at an average rate of 1.8 calls per hour. Find the probability that we observe at least 5 calls in a given 3 hour interval. (3)

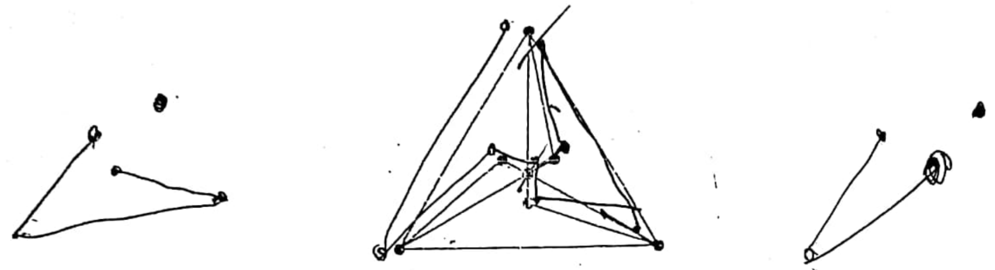
6. P and Q are considering to apply for job. The probability that P applies for job is 1/4. The probability that P applies for job given that Q applies for the job 1/2 and The probability that Q applies for job given that P applies for the job 1/3.Find the probability that P does not apply for job given that Q does not apply for the job. (5)

7. Joan and Jim agree to meet at the library after school between 3 p.m. and 4 p.m. Each agrees to wait no longer than 15 min for the other. What is the probability that they will meet? (6)

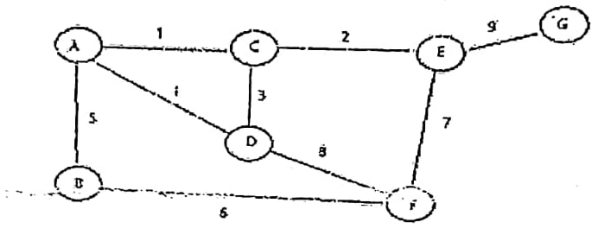
8. Consider the experiment of choosing a point at random from the disk of radius R in the plane centered at the origin. Let X be the random variable denoting the distance of the point chosen from the origin. Find the distribution function and the probability density function of the random variable X. (5)

9. Suppose 2% of the items made by a factory are defective. Find the probability p that there are 3 defective items in a sample of 100 items, assuming binomial distribution. Also find an approximation of p by using Poisson distrbution. (6)

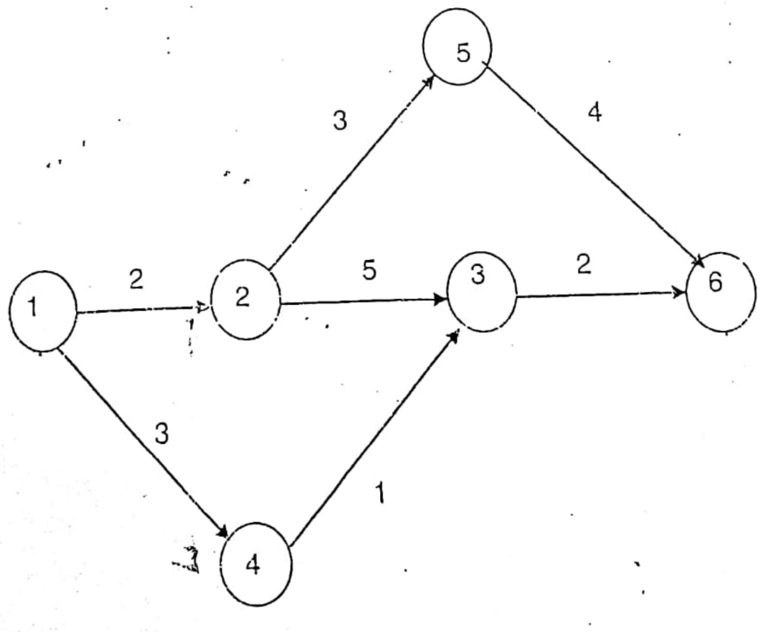
10. Is the following graph Hamiltonian? Justify your answer. (4)



11. Find a minimum spanning tree using (i) Kruskal's Algorithm, (ii) Prim's Algorithm. In both cases clearly indicate the order in which you are choosing the edges. (5)



12. Find a maximum flow from vertex 1 to vertex 6 in the graph network given below. Also find a corresponding minimum cut. (5)



INSTRUCTIONS:

1. Answer ALL the seven questions.
2. Rough work should NOT be done anywhere on the Question Paper.
3. Do the indexing properly. You will be penalized if you do not do the indexing.
4. There are questions on the other side of this paper also.

Q.1. Prove or disprove: If every vertex of a graph has even degree, then G has no cut-edge. [5]

Q.2. Prove or disprove: Every closed odd walk contains an odd cycle. [5]

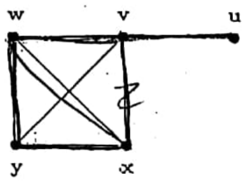
Q.3. Prove or disprove: Every tree with at least two vertices has at least two vertices of degree one. [4]

Q.4. Prove or disprove: If $m > \frac{1}{2}(n-1)(n-2)$, then a simple graph with n vertices and m edges is connected. [4]

Q.5. (a) At a committee meeting of 20 persons, every member of the committee has previously sat next to at most 3 other members. Show that the members may be seated round a circular table in such a way that no one is next to some one they have previously sat beside. [3]

(b) In the situation given above where every member of the committee has previously sat next to at most 3 other members, what is the minmum number of possible arrangements where no one is seated next to some one they have previously sat beside? Justify your answer. [4]

Q.6. Consider the following graph G:



Answer the following with justification:

(a) Is G Eulerian? [3]

(b) Is G bipartite? [3]

(c) If G is planar then draw a plane graph representing G and compute the number of regions of that plane graph. Otherwise prove that G is not planar. [3]

(d) Is G maximal non-Hamiltonian? [4]

13

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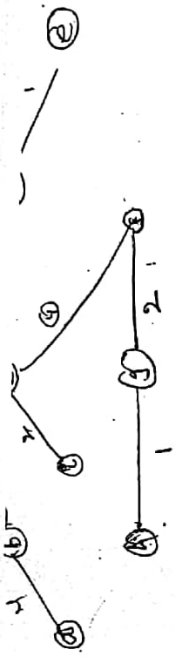
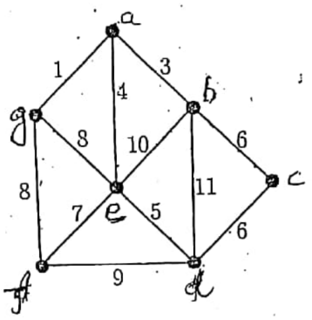
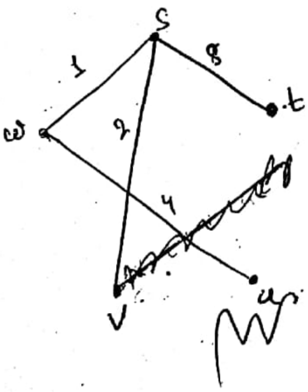
Q.7. (a) For the weighted graph below, the weight of each edge represents the length of a direct route between its two endpoints. Apply Dijkstra's Algorithm to find the minimum distance between s and t. Also find a path corresponding to the minmum distance. (4)



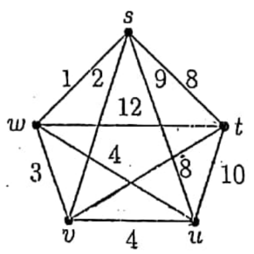


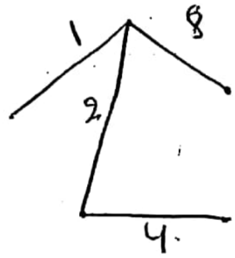
12

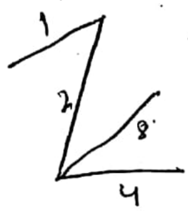
(b) For the weighted graph below, use Kruskal's Algorithm to find a minimal spanning tree. Clearly indicate the edges which you are selecting at various stages. Also find the weight of the minimal spanning tree. [4]



1+2+8+4 => 15







Registration No.: | 1 | 5 | 1 | T | 2 | 4 | 1 |

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
Examination: Mid Semester
Odd Semester (2016-17)
Course Name: Mathematical Foundations of Information Technology
Date: 6/9/2016

Maximum Marks: 50
Course Code: MA 200
Time: 8.30 a.m to 10 a.m

INSTRUCTIONS:

1. Answer ALL the ten questions.
2. Rough work should NOT be done anywhere on the Question Paper.
3. Do the indexing properly. You will be penalized if you do not do the indexing.
4. There are questions on the other side of this paper also.

Q.1. Prove or disprove: Every tree with at least two vertices has at least two vertices of degree one. [4

Q.2. Ten players participate at a chess tournament. Eleven games have already been played. Is it true that there must be a player who has played at least three games? Justify your answer. [5

Q.3. Give an example of a graph having 5 vertices, which has the property that the addition of *any* edge produces an Eulerian graph. [4

Q.4. Let G be a connected graph having 19 vertices and 23 edges.

(a) What is the minimum number of cycles G must have? Explain clearly. In case you have read some formula regarding minimum number of cycles, and are using it, you should derive that formula. [

(b) What is the maximum number of cycles G can have? Explain. [

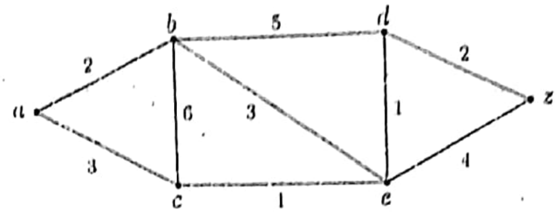
Q.5. Let G be a simple graph. Prove or disprove: G or its complement is connected. [

Q.6. A graph is called regular if all its vertices have the same degree. Let G be a connected, regular, simple graph with 22 edges. Suppose it has x number of vertices. What is/are the possible value(s) of x? Justify your answer.

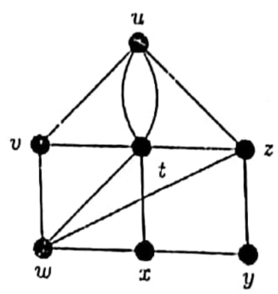
Q.7. Does there exist a simple graph of eight vertices $v_1, v_2, \ldots, v_8$, such that deg $(v_1) = 2$, deg $(v_2) =$ deg $(v_3) =$ deg $(v_4) =$ deg $(v_5) =$ deg $(v_6) = 3$, deg $(v_7) = 4$, deg $(v_8) = 5$? Justify your answer.

Q.8. A simple graph G of n vertices and m edges does not contain any cycle of length 3, and also $m \leq 2n - 4$. Is G necessarily planar? Justify your answer.

Q.9. Use Dijkstra's algorithm to find the cost of a cheapest path between a and z in the following weighted graph. Also find a path of the cheapest weight. [6]



Q.10. Is the following graph Eulerian? Can this graph be drawn without lifting one's pen from the paper or covering an edge more than once? Is it a bipartite graph? Is it a planar graph? Give reasons for your answers. [6]



(*This question paper contains 2 page(s) and 10 Questions.*)

Examination: End Semester - Part A | **Maximum Marks: 65**
Odd Semester (2015-16) | **Course Code: MA 200**
Course Name: Mathematical Foundations of Information Technology
Date: 24/11/2015 | **Time: 9.00 a.m to 12 p.m**

INSTRUCTIONS:

1. Answer ALL the ten questions.
2. Rough work should NOT be done anywhere on the Question Paper.
3. Do the indexing properly. You will be penalized if you do not do the indexing.

Q.1. Prove that a graph of n vertices is a tree if and only if it is connected and has $n-1$ edges. [6]

Q.2. Show that there is no simple graph having 12 vertices and 26 edges in which the degree of each vertex is either 3 or 6. [6]

Q.3. G is a 5-regular simple planar graph. In a plane representation of G, each region is bounded by 3 edges. Find the number of edges in G. [5]

Q.4. Suppose there are 10 persons (referred to by 1, 2, ..., 10), who are part of the following seven committees: $c_1 = \{1,2,3\}, c_2 = \{1,3,4,5\}, c_3 = \{2,5,6,7\}, c_4 = \{4,7,8,9\}, c_5 = \{2,6,7\}$, $c_6 = \{8,9,10\}, c_7 = \{1,3,9,10\}$. A minimum of how many different time periods on a particular day are needed so that meetings of all these seven committees can be arranged in such a manner that no two committees meet during the same period if they have a member in common? Suggest such an arrangement. [7]

7

Q.5. Three boys b1, b2, b3 and four girls g1, g2, g3, g4 are such that (i) b1 likes g1, g3, g4. (ii) b2 likes g2 and g4 (iii) b3 likes g2 and g3. Can every one of the boys marry a girl whom he likes? State some result to justify your answer. If possible, find one such arrangement where each boy marries a girl whom he likes. [5]

Q.6. Suppose that a coin is tossed twice so that the sample space is $S = \{HH, HT, TH, TT\}$. Let X represent the number of heads that can come up.

(a) Write out the probability mass function corresponding to the random variable X. [3]

(b) Write out the cumulative distribution function for X. [3]

6

Q.7. Suppose a dart is thrown at a circular target with radius 6 metres. Assume that whenever a dart is thrown, it always hits the target, never misses it. If it lands at a point having distance less than or equal to 1 metre from the center, you win Rs. 5. If it lands at a point having a distance greater than 1 but less than or equal to 4 metres from the center, you win Rs. 10. Otherwise, you lose Rs. 8.

(a) If the dart is thrown once, what is the probability that you win Rs.10? [3]

(b) If the dart is thrown a large number of times, then are you expected to win some money, or lose some money? Explain. [5]

8?

Q.8. The probability of a man hitting a target is 1/4.

(a) If he fires 7 times, what is the probability of his hitting the target at least twice? [3]

6

(b) How many times must he fire so that the probability of his hitting the target at least once is greater than 2/3? [3]

Q.9. If a bank receives on an average 6 bad cheques daily, what is the probability that it will receive 4 bad cheques on a given day? What is the probability that it will receive at least 5 bad cheques from Monday to Wednesday in a given week? [8]

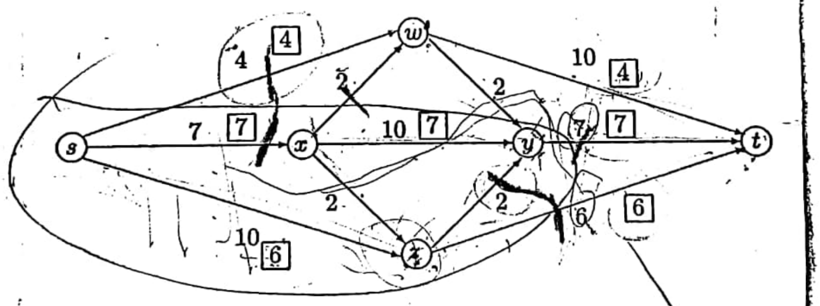
Q.10. Consider the following problem: [8]

The figure below shows a flow network on which an s-t flow is shown. The capacity of each edge appears as a label next to the edge, and the numbers in boxes give the amount of flow sent on each edge. (Edges without boxed numbers have no flow being sent on them.)

(a) What is the value of this flow?

(b) Is this a maximum s-t flow in this graph? If not, find a maximum s-t flow.

(c) Find a minimum s-t cut. (Specify which vertices belong to the sets of the cut.)



28

$\frac{28}{6.5}$

30

8

48

x, y, t, w W

4+2+ 20

20 + 8 => 25

(*This question paper contains* 2 *page(s) and* 10 *Questions.*)

Q.1. On the real line, points a and b are selected at random such that $-2 \leq b \leq 0$ and $0 \leq a \leq 3$. Find the probability that the distance d between a and b is greater than 3. [5

Q.2. The probability that A hits a target is $1/4$ and the probability that B hits it is $2/5$. What is the probability that the target will be hit if A and B each shoot at the target? [4 $11/20$.

$P(A)P(B') + P(A')\cdot P(B) + P(A \cap B)$

Q.3. Three machines A, B and C produce respectively 30%, 40% and 30% of the total number of items of a factory. The percentages of defective output of these machines A, B and C are 4%, 3% and 2%.

(a) If an item is selected at random, find the probability that the item is defective.

(b) Now consider the case when an item is selected at random and found to be defective. Find the probability that the item was produced by machine A. Give proper explanation. There is no need to compute the final answer in decimals.

Q.4. The probability of producing a defective screw is $p = 0.01$.

(a) Find the probability that a collection of 100 screws will contain more than 2 defectives using binomial distribution. There is no need to compute the final answer in decimals.

(b) Also find an approximation of the above probability using Poisson distribution. There is no need to compute the final answer in decimals.

Q.5. A random variable X has normal distribution with mean 6. It is given that $P\{X \geq 7\} = 0.2$ If you feel that the given data is enough to evaluate $P\{5 \leq X \leq 6\}$, do so. Otherwise explain why the data is incomplete.

Q.6. The 20 members of a local tennis club have scheduled exactly 14 two-person games among themselves, with each member playing in at least one game. Prove that within this schedule there must be a set of 6 games with 12 distinct players.

Q.7. In a group of 12 people, each of them knows at least 6 other people. Prove or disprove: They can sit at a round table in such a way that everyone knows the two people sitting next to them.

$P(A \cup B) = \frac{1}{4} + \frac{2}{5} - \frac{1}{10} = \frac{5+4-2}{20} = \frac{7}{20}$

$\frac{1}{4} + \frac{2}{5} - \frac{1}{4} \times \frac{2}{5} \Rightarrow \frac{5+8}{20} - \frac{2}{20}$

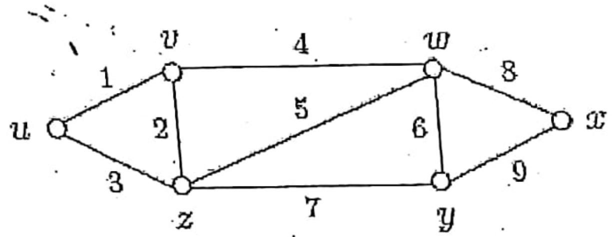
$11/20$

Q.8. Suppose that four applicants a_1, a_2, a_3 and a_4 are available to fill six vacant positions p_1, p_2, p_3, p_4, p_5 and p_6. Applicant a_1 is qualified to fill position p_2 or p_5. Applicant a_2 can fill p_2 or p_5. Applicant a_3 is qualified for p_1, p_2, p_3, p_4 or p_6. Applicant a_4 can fill jobs p_2 or p_5.

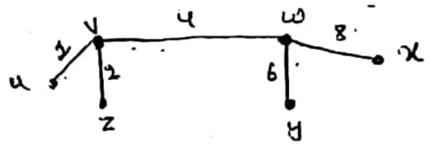
(a) Is it possible to hire all the applicants and assign each a position for which he is suitable? Justify your answer using Hall's theorem. [4]

(b) Use Ford-Fulkerson Algorithm to find the maximum number of positions that can be filled from the given set of applicants. [4]

Q.9. For the weighted graph below, use Kruskal's Algorithm to find a minimal spanning tree. Clearly indicate the edges which you are selecting at various stages. Also find the weight of the minimal spanning tree. [5]



8+5+9
14
+8
22



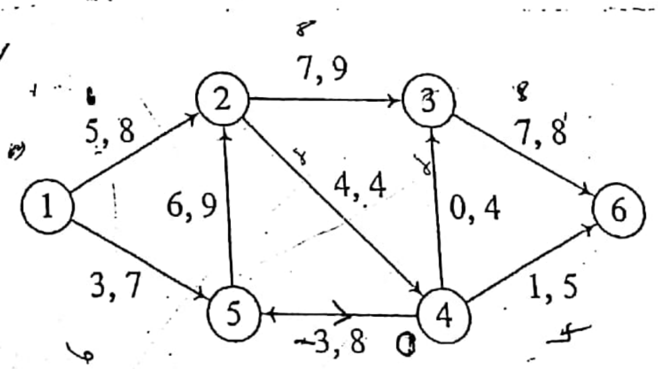
1+2+4+6+8
13+8 = 21
21

Q.10. The figure below shows a flow network on which a flow from vertex 1 to vertex 6 is shown.

(a) What is the value of this flow? [2]

(b) Is this a maximum flow from vertex 1 to vertex 6 in this graph? If not, find a maximum flow from vertex 1 to vertex 6. [4]

(c) Find a corresponding minimum cut. (Specify which vertices belong to the sets of the cut.) [3]



(This question paper contains 2 page(s) and 10 Questions.)