Number of Zeroes

Number of zeros
In this topic we are going to go through the following concepts:

- Number of trailing zeros in a Product or an Expression
- Number of trailing zeros in Power of an expression
- Number of trailing zeros in a factorial (n!)

Number of zeros is nothing else but the number of zeros at the end i.e. Number of trailing zeros.
Just to make you clear**, 170130000** has **5 zeros** but **4 trailing / ending zeros**.
Note: In questions based on these ideas, you should assume that the examiner is asking about trailing zeros unless specified otherwise.

Number of trailing zeros in a Product or an Expression:
If we look at a number N, such that N = 2580000 = 258 x 10^4
Number of trailing zeros is the Power of 10 in the expression or in other words, the number of times N is divisible by 10.
We know 10 = 2 x 5
For a number to be divisible by 10, it should be divisible by 2 & 5, since making a pair of 2 and 5 will give us 10.
So, **Number of trailing zeros is going to be the power of 2 or 5, whichever is less.**
Example 1: Find the number of trailing zeros in N = 1x2x3x4x5x…..x10
Solution:
For finding the powers of 2:
We know 2^1 = 2(This means that 2 gives us a single power of 2)
2^2 = 4(This means that 4 gives us a two powers of 2)
2^3 = 8(This means that 8 gives us a three powers of 2)
Also all even numbers consist of at least one power of 2
Here 6 = 2x3 Thus giving us a single power of 2
Also 10 = 2x5 giving us three powers of 2
Thus in total there are eight times 2 appearing in the expression
For finding the powers of 5:
We have only 5 and 10 in the expression that gives us powers of 5
5 = 5 x 1
10 = 5 x 2
Thus there are two times 5 appearing in the expression
Since Power of 5 is lesser than power of 2
Thus number of zeros = Power of 5 in the expression = 2 (Answer)
Example 2: Find the number of zeros in the following expression:
N = 5x25x50x75x100
Solution: Breaking each term of the product in prime factors
5 = 5x1
25 = 5^2
50 = 2x5^2
75 = 3x5^2
100 = 2^2 x 5^2
Thus in the product
Powers of 2 = 3
Powers of 5 = 9
Thus number of zeros = 3.(Answer)
Number of trailing zeros in Power of an expression:
We know that
10^1 = 10 …….. one trailing zero
10^2 = 100 …….. two trailing zeros
10^3 = 1000 …….. three trailing zeros

$100^2 = 10000$ …… four trailing zeros
Thus for finding the number of zeros in N^p
If N has t trailing zeros
Then N^p has t x p trailing zeros
Example 3: Find the number of zeros in the expression $(23200)^{2\times5}$
Solution: $(23200)^{10}$
Number of zeros = 2 x 10 = 20(Answer)
Number of trailing zeros in a factorial (n!):
Factorial of a number n is given by n! = n x (n−1) x (n−2) x (n−3) x ….. x 2 x 1
Number of trailing zeros in n! = Number of times n! is divisible by 10 = Highest power of 10 which divides n! = Highest power of 5 in n! (Since in a factorial number of occurrences of 5 is always less than the number of occurrences of 2)
Calculating Highest power of a number in a factorial.
If p is prime number, then the highest power of p in a factorial n is given by

$$\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$$

Where [x] represents the greatest integer function of x i.e. the greatest integer that is less than or equal to x.
Example 4: Find the highest power of 4 in 18!
Solution: Powers of 4 =

$$\left[\frac{18}{4}\right] + \left[\frac{18}{4^2}\right] + \cdots$$

On dividing by further powers of 4 will give integer value as zero
 = 4 + 1 = 5 (Answer)
Example 5: Find the number of zeros in 26!
Solution: To find the number of zeros we basically need to find the power of 5 in the given factorial
Powers of 5 =

$$\left[\frac{26}{5}\right] + \left[\frac{26}{5^2}\right] + \left[\frac{26}{5^3}\right] + \cdots$$

 = 5 + 1 + 0
 = 6 (Answer)
Important Trick: Instead of dividing by 25, 125, etc. (higher powers of 5); it would be much faster if we divide the number or expression by 5 recursively.
i.e. [26/5] = 5
[5/5] = 1
[1/5] = 0
Answer = 5+1 = 6