Ratio & Proportion

Prep Smart. Score Better. Go **gradeup**

Ratio & Proportion

Definition of Ratio:
Comparison of two different quantities having same units.

Types of Ratio:
Let us assume that, two numbers are 'a' and 'b'. Then the ratio is a : b. Therefore,
1. Duplicate ratio: **a²: b²**

2. Sub duplicate ratio: $\sqrt{\frac{a}{b}}$

3. Triplicate ratio: **a³: b³**
4. Sub triplicate ratio: $\sqrt[3]{a} : \sqrt[3]{b}$
5. Inverse ratio: $\frac{1}{a} : \frac{1}{b}$
6. If three different ratios are a : b, c : d and d : e
Compounded ratio: $\frac{acd}{bde}$

Some important properties of ratio:
1. If in the ratio a/b, the numerator and the denominator are multiplied or divided by the same number then the value of the ratio remains same.
Case 1: Multiplying numerator and denominator by same number x:
Ratio $= \frac{a}{b} = \frac{xa}{xb}$
Thus, cancelling out x further results in same ratio a/b.
Case 2: Dividing numerator and denominator by same number y:
Ratio $= \frac{a}{b} = \frac{\frac{a}{y}}{\frac{b}{y}}$
Thus, cancelling out y further results in same ratio a/b.
2. If p/q = r/s = t/u = v/w = m then
m $= \frac{p+r+t+v}{q+s+u+w}$

Comparison of two ratios:
Suppose we have to compare two different ratios 12/17 and 13/11.
Here to find which ratio is greater or lesser than other, we use cross multiplication method.
Simply cross multiply the denominator to the numerator of another ratio.
$$\frac{12}{17} \quad \frac{13}{11}$$
(12x11) (13x17)
= 132 221
Comparing we get 132 < 221 thus, $\frac{12}{17} < \frac{13}{11}$.

Proportion: If two ratios are equal then the 4 terms are called proportion.
For example: $\frac{a}{b} = \frac{c}{d}$
It can also be written as:
a : b :: c : d
Here terms a and d are called extremes and terms c and d are called means.

Types of Proportion:
If the ratio is a : b
1. Mean proportion: $\sqrt{ab}$

2. Third proportion: $\frac{b^2}{a}$

3. If three numbers a, b and c are given then
Fourth proportion: $\frac{bc}{a}$

Note:
1. If a : b = 2 : 3 and b : c = 4 : 5
Then

a : b : c = 2 : 3

4 : 5

a : b : c =   8   :   12   :   15

(2 x 4)   (3 x 4)   (3 x 5)

2. If a : b = 1 : 2, b : c = 3 : 4 and c : d = 2 : 3

a : b : c : d = 1 : 2

3 : 4

2 : 3

⇒ a : b : c : d = 6 : 12 : 16 : 24

Example 1: If 2a = 3b = 4c = 5d then find a : b : c : d.
Solution:
2a = 3b = 4c = 5d = (60) = LCM of (2,3,4,5)
$$\frac{60}{2} : \frac{60}{3} : \frac{60}{4} : \frac{60}{5}$$
⇒ a : b : c : d = 30 : 20 : 15 : 12

Example 2: If a : (b + c) = 1 : 2 and b : (c + a) = 3 : 4 then find c : (a + b).
Solution:
In first case: a + (b + c) = 1 + 2 = 3
In second case: b + (c + a) = 3 + 4 = 7
Now multiplying first equation with 7 and Second equation with 3:
a : (b + c) = 7 : 14 and b : (c + a) = 9 : 12
Thus, by comparison:
a = 7
b = 9
So, c = 21 – (7 + 9) = 5
Thus, c : (a + b) = 5 : 16