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Sahi Prep Hai Toh Life Set Hai

Ratio & Proportion

Part-3

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Agenda

* Direct & Inverse Proportion } (70-75min)

* All Partnership Homework Questions → 30min

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BASIC TERMS RELATED TO RATIO

If A : B is a ratio, then

Duplicate Ratio	→ $A^2 : B^2$
Triplicate Ratio	→ $A^3 : B^3$
Sub-Duplicate Ratio	→ $\sqrt{A} : \sqrt{B}$
Sub-Triplicate Ratio	→ $\sqrt[3]{A} : \sqrt[3]{B}$

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$\frac{a}{b}$ → a → antecedent, b → consequent

Compounded Ratio:

For two or more ratios, if we take antecedent as product of antecedents of the ratios and consequent as product of consequents of the ratios, then the ratio thus formed is called mixed or compound ratio. As, compound ratio of m : n and p : q is mp : nq.

Compound Ratio of a : b & c : d

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

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Eg. Find the compound ratio:

(i) 2 : 3, 9 : 14 and 14 : 27

$$\frac{2}{3} \times \frac{9}{14} \times \frac{14}{27_3} = \frac{2}{9}$$

(ii) $(x^2 - y^2) : (x^2 + y^2)$ and $(x^4 - y^4) : (x + y)^4$

$$\frac{x^2 - y^2}{(x^2+y^2)} \times \frac{x^4 - y^4}{(x+y)^4} \; x^2 - y^2 \Rightarrow \frac{(x+y)^2 (x-y)^2}{(x+y)^4}$$

$$\boxed{\frac{(x-y)^2}{(x+y)^2}}$$

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Eg. Find the duplicate ratio:

(i) 3 : 4 → 9 : 16

(ii) 3a : 4b → $9a^2 : 16b^2$

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Eg. Find triplicate ratio:

(i) $1:3$ $\quad 1:27$

(ii) $\frac{m}{2}:\frac{n}{3}$ $\quad \frac{m^3}{8}:\frac{n^3}{27}$

$27m^3:8n^3$

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Eg. Find the sub-duplicate ratio:

(i) 9 : 16 $\longrightarrow$ 3 : 4

(ii) $(x - y)^4 : (x + y)^6$

$$\frac{(x-y)^2}{(x+y)^3}$$

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Eg. Find the sub-triplicate ratio:

(i) $64 : 27$ $\longrightarrow$ $4 : 3$

(ii) $x^3 : 125y^3$ $\longrightarrow$ $x : 5y$

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PROPORTION

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PROPORTION

"Equality of 2 ratios is called as Proportion."

eg $\frac{2}{3} = \frac{12}{18}$

2, 3, 12 & 18 are in proportion

If $\frac{a}{b} = \frac{c}{d}$ then a, b, c & d are in proportion

a : b : : c : d

: : → Symbol of Proportion

Eg. 5, 8, x & 36 are in proportion. Find x.

$\frac{5}{8} = \frac{x}{36}$

$x = \frac{5 \cdot 36^{9}}{8_{2}} = \frac{45}{2} = 225$

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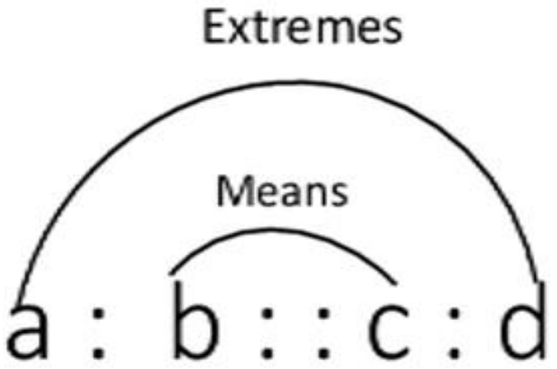

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Product of extremes = Product of Means

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If a, b & c are in proportion.

Then $\frac{a}{b} = \frac{b}{c}$

$b^2 = ac$

Here, b is called as Mean Proportional.

eg 5, x & 20 are in proportion

Find x

$\frac{5}{x} = \frac{x}{20}$

$x^2 = 100$

$x = 10$

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Eg1. If 5, 8, 20, x are in proportion. Find the value of x.

$$\frac{5}{8} = \frac{20}{x}$$

$$x = 32$$

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Eg2. 23, 31, 15, 21

What should be added to all the 4 numbers, so that they become in proportion?

$(23+x)$, $(31+x)$, $(15+x)$, $(21+x)$

$$\frac{23+x}{31+x} = \frac{15+x}{21+x}$$

$$483 + 44x + x^2 = 465 + 46x + x^2$$

$$2x = 18$$

$$\boxed{x = 9}$$

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Eg3. Find the Mean Proportional of 8 & 18.

$$\frac{8}{x} = \frac{x}{18}$$

$$x^2 = 144$$

$$\boxed{x = 12}$$

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Q1. What number should be added to each of 6, 14, 18, and 38, so that the resulting numbers make a proportion?

(a) 1 (b) 2

(c) 3 (d) 4

II nd

$$\frac{6+x}{14+x} = \frac{18+x}{38+x}$$

$$228 + 44x + x^2 = 252 + 32x + x^2$$

$$12x = 24$$

$$x = 21$$

I st with help of option

7, 15, 19, 39 ×

8, 16, 20, 40

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Ans. (b)

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a, b, c & d

are in proportio

a → I^st proportion

b → II^nd "

c → III^rd "

d → IV^th "

a, b, c
I II III

Q2. The third proportional to 0.8 and 0.2.

(a) 0.05 (b) 0.8
(c) 0.4 (d) 0.032

0.8, 0.2, x

$\frac{0.8}{0.2} = \frac{0.2}{x}$

$x = \frac{0.2}{4}$

$= 0.05$

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Ans. (a)

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Q3. The mean proportional between $(3+\sqrt{2})$ *and* $(12-\sqrt{32})$ is:

(a) $\sqrt{7}$ (b) $2\sqrt{7}$

(c) 7 (d) $\frac{15-3\sqrt{2}}{2}$

$(3+\sqrt{2})$; x , $(12-\sqrt{32})$

$$\frac{3+\sqrt{2}}{x} = \frac{x}{12-\sqrt{32}}$$

$$x^2 = 36 - 12\sqrt{2} + 12\sqrt{2} - 8$$

$$x^2 = 28$$

$$x = 2\sqrt{7}$$

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Ans. (b)

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Q4. If a : b = b : c the $a^4 : b^4$ is equal to:

(a) $ac : b^2$ ✗
(b) $a^2 : c^2$ ✓
(c) $c^2 : a^2$ ✗
(d) $b^2 : ac$ ✗

II nd Detailed App

$$\frac{a}{b} = \frac{b}{c}$$

$$\boxed{b^2 = ac}$$

$$\frac{a^4}{b^4} = \frac{a^4}{(b^2)^2} = \frac{a^4}{a^2 c^2}$$

I st Putting values

let a = 1, b = 2, c = 4 ∴

$$\frac{a^4}{b^4} = \frac{1}{2^4} = \frac{1}{16}$$

(a) → 4 : 4 ✗
(b) 1 : 16 ✓

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Ans. (b)

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II nd Detailed App

$$\frac{a}{b} = \frac{b}{c} = k$$

$$\frac{a-b}{b-c} = k$$

$$\frac{(a-b)^3}{(b-c)^3} = k^3$$

$$\frac{a^3}{b^3} = k^2$$

Q5. If b is the mean proportional of a and c, then $(a-b)^3 : (b-c)^3$ equals to:

(a) $a^3:c^3$
(b) $b^2:c^2$
(c) $a^2:c^2$
(d) $a^3:b^3$

I Value App

$a = 1, \quad b = 2, \quad c = 4$

$(1-2)^3 = -1$

$(2-4)^3 = -8$

$-1 : -8$

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Ans. (d)

If $\frac{a}{b} = \frac{b}{c} = k$

$$\begin{cases} k & \frac{a+b}{b+c} = k \\ k & \frac{a-b}{b-c} = k \\ k & \frac{2a+3b}{2b+3c} = k \end{cases}$$

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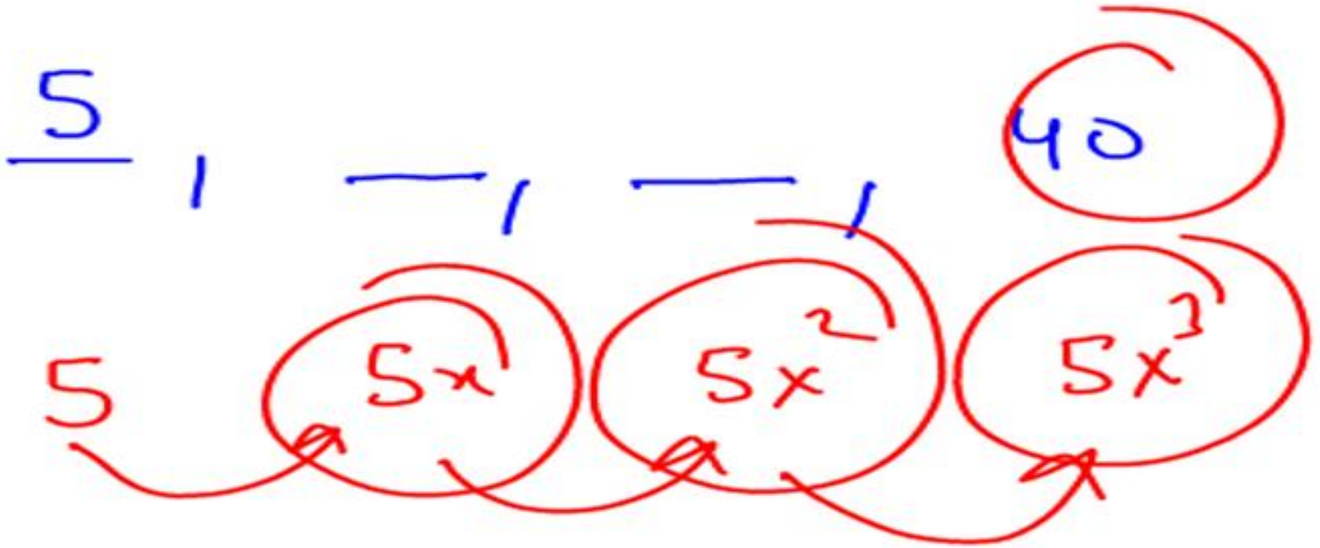
$5 \cdot 2 = 10$

$5 \cdot 2^2 = 20$

Eg. Find two Mean Proportional between 5 and 40.



$5x^3 = 40$

$x^3 = 8$

$x = 2$

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Q6. If b is the Mean Proportional of a & c, then find the mean proportional $(a^2 - b^2)$ and $(b^2 - c^2)$

(a) 0

(b) $b(a^2 - c^2)$

(c) $b(a - c)$

(d) 1

$$\frac{a}{b} = \frac{b}{c}$$

$$b^2 = ac$$

$$(a^2 - ac)\ (ac - c^2)$$

$$\text{Mean} = \sqrt{(a^2 - ac)(ac - c^2)}$$

$$= \sqrt{a(a-c)c(a-c)}$$

$$= b(a-c)$$

I Value Approach

Let $a = 4$, $b = 2$ & $c = 1$

$a^2 - b^2 \Rightarrow 12$ $\quad b^2 - c^2 = 3$

Mean proportional of 12 & 3

$= 6$

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Ans. (c)

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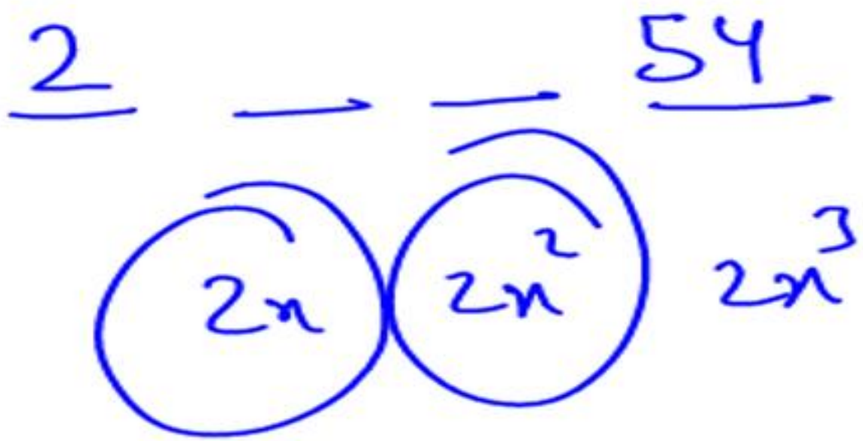
Q7. Find 2 Mean Proportional between 2 and 54.

(a) 6 and 18
(b) 6 and 12
(c) 12 and 18
(d) 6 and 9



$2x^3 = 54$

$x = 3$

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Ans. (a)

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CONTINUED PROPORTION

A proportion in which the consequent of each ratio is the antecedent of the next.

As,

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e}$$

Hence, a, b, c, d & e are in Continued Proportion.

Eg. 3, 6, 12, 24, 48, 96.....

eg 5, 20, 80, 320, x

are in continued proportion

And the value of

2x = ??

x → 1280

2560

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* Continued Proportion is same as Geometric Progression

Eg. If 8, x, 50 are in Continued Proportion, then the value of x is:

$$\frac{8}{x} = \frac{x}{50}$$

$$x = 20$$

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Ans. 20

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DIRECT PROPORTION

$A \propto B$

$A = kB$

Direct proportion is the relation between two quantities where the ratio of the two is equal to a constant value.

It is represented by the proportional symbol, $\propto$.

$A \propto B$

$A = kB$, where **k is a proportionality constant.**

$\frac{A}{B} = k$ (constant)

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Examples of Direct Proportion

Amount ∝ Consumption

eg1

Price/Unit × No. of units = Amount

↑ ↑

eg2

$D \propto T$ (If speed is constant)

eg

$D \propto S$ (If Time is constant)

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Eg. $A \propto B^2$, if A = 10 then B = 8
Find the value of B, if A = 40

$$\frac{A}{B^2} \rightarrow \text{constant}$$

$$\frac{10}{8^2} = \frac{40}{x^2}$$

$$x^2 = 256$$

$$\boxed{x = 16}$$

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Ans. 16

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Eg. $A \propto B^3$, if A = 2 then B = 5
Find the value of A, if B = 10

$$\frac{2}{5^3} = \frac{x}{10^3}$$

$$\frac{2}{125} = \frac{x}{8}$$

$$x = 16$$

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Ans. 16

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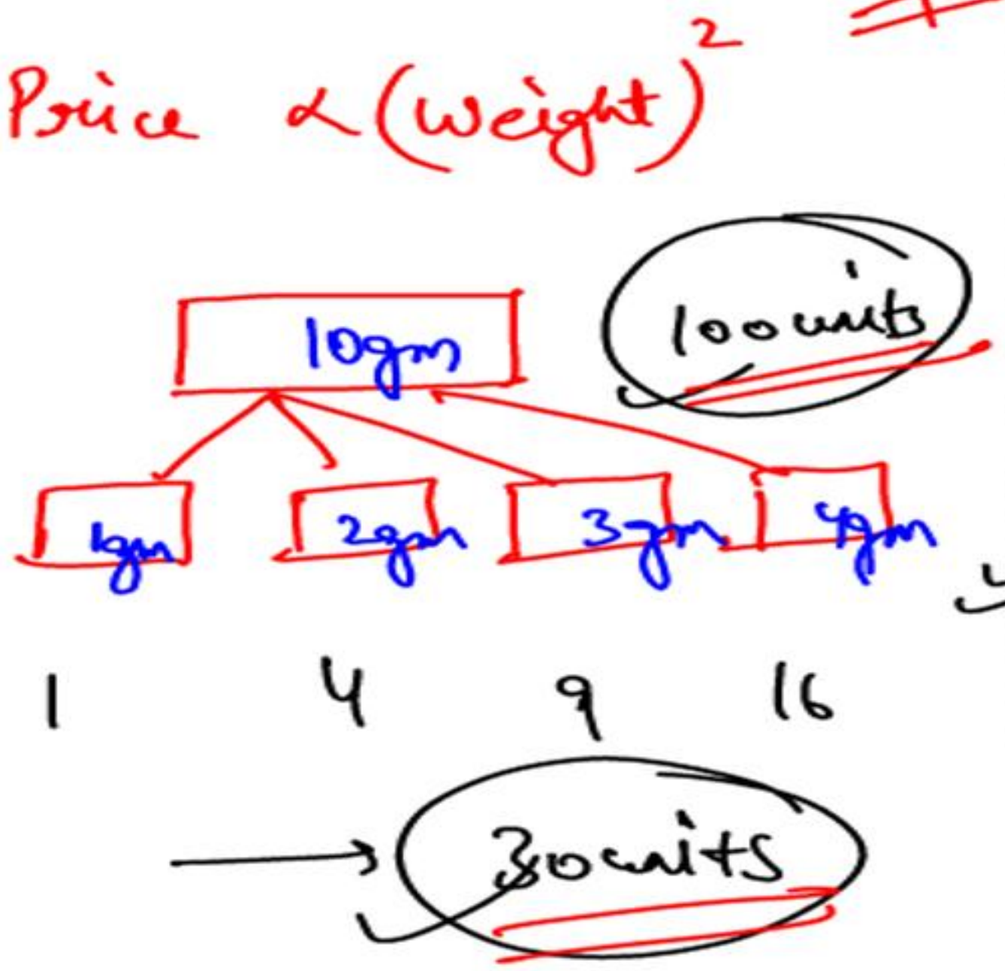
Eg. Price of a diamond is directly proportional to the square of its weight. If the diamond break into 4 pieces by mistake, the ratio of their weight becomes 1:2:3:4, because of which their occurs a loss of Rs.1,40,000. Find the original price of the diamond.

(a) 240000
(b) 200000
(c) 220000
(d) 180000



70 → 1,40,000

1 → 2000

100 → 2,00,000

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Ans. (b)

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Eg. The reduction in the speed of an engine is directly proportional to the square of the number of bogies attached to it. The speed of the train is 250 km/hr when there were no bogies attached to the train. If there was 1 bogie attached to the train, then the speed of the train was 240 km/hr. What is the maximum number of bogies that can be attached to the train so that it can move?

Reduction in speed $\propto$ (No. of Bogies)^2

$S = 250$ km/h

If $B = 1$ speed $= 240$ km/h

Sol^n $\quad Rd^n = K \cdot B^2$

Speed $= 250 - KB^2$

If $B = 1$ $\quad 250 - K \cdot 1^2 = 240$

$K = 10$

Max $B \to ?$

$250 - 10B^2 > 0$

$25 > B^2$

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Reduction in speed $= KB^2$

Speed $= 250 - KB^2$

If $B = 1$ $S = 240$ $250 - K \cdot 1^2 = 240$

$\boxed{K = 10}$

$250 - 10 \cdot B^2 > 0$

$25 > B^2$

Max value of $B \rightarrow (4)$

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Ans. Max. 4 bogies

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INVERSE PROPORTION

$A \propto \frac{1}{B}$

$A = \frac{k}{B}$

$AB = K$

Two quantities a and b are said to be in inverse proportion if, the product of their corresponding values is constant.

$A \propto \frac{1}{B}$

A x B = k,
where **k is a proportionality constant.**

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Eg. $A \propto \frac{1}{B^2}$, if A = 5 then B = 4

Find the value of B, if A = 2

$A \cdot B^2 = \text{constant}$

$5 \cdot 4^2 = 2 \cdot B^2$

$B^2 = 40$

$B = \underline{2\sqrt{10}}$

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Ans. $2\sqrt{10}$

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Resistance ⟶ R

radius ⟶ r

length ⟶ l

$R \propto \frac{l}{r^2}$

$\propto \frac{1}{r^2}$

Eg. The resistance of wire is directly proportional to its length and inversely proportional to the square of its radius. Two wires of same material having the same resistance and their radii are in the ratio 9:8. If the length of the first wire is 162 cm, then find the length of the other wire.

(a) 128 cm (b) 81 cm

(c) 108 cm (d) 168 cm

$\frac{R \cdot r^2}{l}$ = constant

$r_1 : r_2 = 9 : 8$ $l_1 = 162\text{cm}$

$\frac{r_1^2}{2l_2} = \frac{8^2}{l_2}$

$l_2 = 128\text{cm}$.

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Ans. (a)

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PARTNERSHIP

PRACTICE QUESTIONS

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Q7. Two partners invest Rs. 17000 and Rs. 13000 respectively in a business and agree that 75% of the profit should be divided equally between them and the remaining profit is to be treated as interest on capital. If one partner gets Rs. 532 more than the other, find the total profit made in the business.

(a) Rs. 16960 **(b) Rs. 14960**

(c) Rs. 16950 **(d) Rs. 15960**

4 units → 532

1 → $\frac{532}{4}$

30 → $\frac{532 \cdot 30}{4}$

A B

17000 13000

17 : 13

$\frac{1}{4}P = \frac{532 \cdot 30}{4}$

$P = 15960$ Rs

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Ans. (d)

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Q8. A, B and C are partners in a business. A whose money has been used for 4 months claims 1/8 of profit, B whose money has been used for 6 months, claims 1/3 of the profit. C had invested Rs. 1560 for 8 months. What is the difference between investment of A and B.

(a) Rs. 720 (b) Rs. 560

(c) Rs. 420 (d) Rs. 500

$P = I \cdot T$

	A	B	C
Time →	~~4~~ 2	~~6~~ 3	~~8~~ 4
Profit →	~~1/8~~ 3	~~1/3~~ 8	13

24

Invest $\frac{3}{2} : \frac{8}{3} : \frac{13}{4}$

18 : 32 : 39

1560

1 → 40

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Ans. (b)

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Q9. A puts Rs. 375 more in a business than B, but B has invested his capital for 4 months while A has invested his capital for 8 months. If the share of A is Rs. 75 more than that of B out of the total profit of Rs. 125, find the capital contributed by B ?

(a) Rs. 750
(b) Rs. 375
(c) Rs. 735
(d) Rs. 573

A B Profit = 125

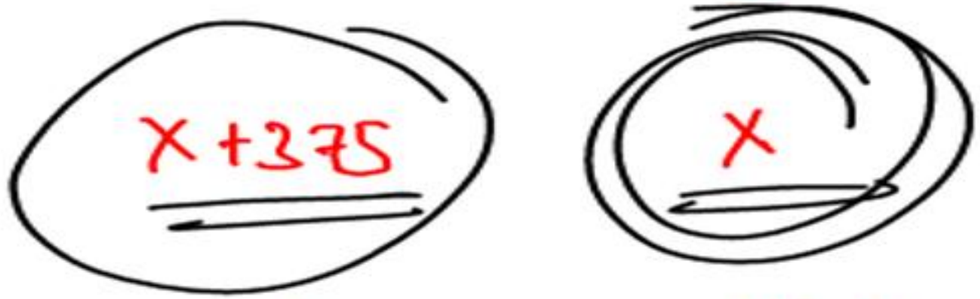
75+25 25

100 25

Invest

Time

Profit



8 2 4 1

100 4 25 1

$\frac{4}{2} : \frac{1}{1}$

2 : 1

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Ans. (b)

Q10. A, B and C have respectively invested Rs. 20, Rs. 18 and Rs. 12 (all in thousands) jointly in a business. A and B receive respectively, 12% and 8% of the annual profit for services, and remaining profit being divided among A, B and C in proportion to their capitals. If at the end of the year A receives altogether Rs. 648 more than that of B then C's share is :

(a) Rs. 3960 (b) Rs. 3312

(c) Rs. 1728 (d) Rs. 2400

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Ans. (c)

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Q11. A and B enter into partnership. A supplies whole of the capital amounting to Rs. 45000 with the condition that the profit should be divided equally and that B pays A interest on half of the capital at 10% per annum, but receives, Rs. 120 per month for carrying on the concern. When B's income is half of A's income then their total yearly profit is :

(a) Rs. 9180 (b) Rs. 7150

(c) Rs. 3060 (d) Rs. 6300

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Ans. (d)

Q12. A, B and C are partners. A receives 5/8 of the profit, B and C share the remaining profit equally. A's income is increased by Rs. 450 when the profit rises from 4% to 9%. Find the capital invested by B.

(a) Rs. 3366
(b) Rs. 1687.5
(c) Rs. 3475
(d) Rs. 2700

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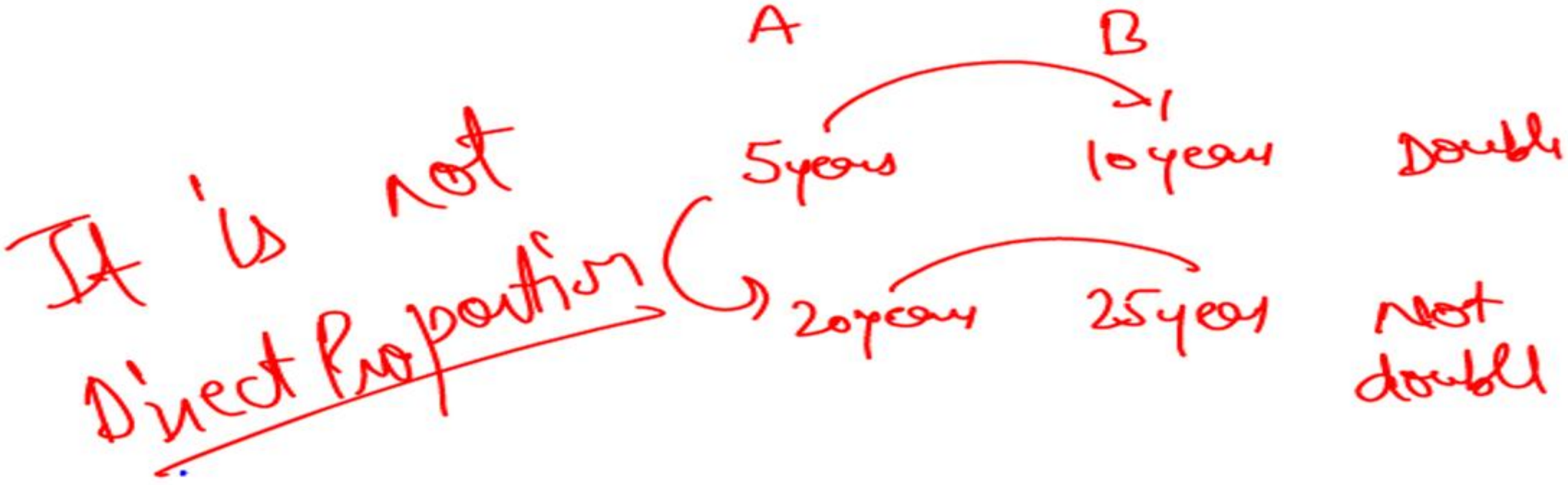
Ans. (d)

Age of 2 Brothers

→ This is NOT an eg of Direct Proportion



If 2 things are in Direct Proportion their ratio is a constant

If 2 things are in Inverse Proportion their product is a constant