gradeup

LCM-HCF

HCF and LCM

Highest Common Factor (HCF): The highest common factor of two or more numbers is the greatest common divisor, which divides each of those numbers an exact number of times. The process to find the HCF is:

a. Express the numbers given as a product of prime numbers separately i.e. find factors of numbers.

b. Take the product of prime numbers common to all the given numbers.

Example 1: Find HCF of 540 and 1024.

Solution:

Step 1: Express the numbers given as a product of prime numbers.

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^3 \times 5^1$

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{10}$

Step 2: Take the product of prime numbers common to all the given numbers.

We can see that only 2^2 is common to both the given numbers. Thus, H.C.F. = 2 x 2 = 4

Example 2: Find the HCF of 27, 81, 165, 360.

Solution:

Step 1:

$27 = 3 \times 3 \times 3 = 3^3$

$81 = 3 \times 3 \times 3 \times 3 = 3^4$

$165 = 3 \times 5 \times 11$

$360 = 2^3 \times 3^2 \times 5$

Step 2:

We can see that only 3 is common to all the given numbers. Thus, HCF = 3.

Sometimes finding HCF becomes very calculative and time consuming as the given numbers can be many and their respective values are also large. In that case,

Property: Let us suppose two numbers N_1 and N_2 are given. So,

HCF of (N_1, N_2) = HCF of difference between N_1, N_2 or any factor of the difference between the given numbers.

Example 3: Find the HCF of 36 and 54.

Solution:

By ordinary method: $36 = 2^2 \times 3^2$ and $54 = 2^1 \times 3^3$. Thus, HCF = $2^1 \times 3^2 = 18$

Or the Difference between 36 and 54 = 18

Check whether 18 divides both 36 and 54 or not. Here, 18 divides 36 and 54 completely. Thus, HCF of 36 and 54 will be 18. So, both the cases have the same answer.

Example 4: Find the HCF of 210, 360 and 540.

Solution:

HCF of (210, 360 and 540) = HCF (360 - 210 and 540 - 360) = HCF (150 and 180) = 30

Check whether 30 divides all the three numbers or not. Here, 30 divides 210, 360 and 540 completely. Thus, HCF of 210, 360 and 540 will be 30.

Note: Try to find difference between given numbers which is as minimum as possible.

Example 5: Find the HCF of 2190, 1800, 1890 and 2520.

Solution:

Here, Finding the HCF of these four numbers will be hectic as these numbers are large and their factorization will be time consuming. So, try to find shortest possible difference between the given numbers. Here, shortest possible difference will be between 1890 and 1800 which is 90.

Now, Check whether 90 divides all the four numbers or not. Here, 90 divides 1800, 1890 and 2520 completely but not 2190. So, HCF will not be 90 but a factor of 90.

Factors of 90 = 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90. Out of all the twelve factors of 90, highest factor that divides all the given numbers is 30. Thus, HCF will be 30.

Least Common Multiple (LCM): The least common multiple (LCM) of two or more numbers is the smallest of the numbers, which is exactly divisible by each of them, e.g. consider two numbers 18 and 24.
The multiples of 18 are: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216, ...
The multiples of 24 are: 24, 48, 72, 96, 120, 144, 168, 192, 216, ...
The common multiples of both 18 and 24 are 72, 144, 216, ...
The least common multiple is 72.
Here again try to break the words in reverse order and understand the concept. Firstly, find the multiples of the numbers. Secondly, the common multiples of the numbers and finally the least out of those will be the LCM. The process to find the LCM is:
a. Express the numbers given as a product of prime numbers separately i.e. find factors of numbers
b. Take the product of prime factors of the given numbers after eliminating repetition of the common factors.
Example 6: Find LCM of 36 and 54.
Solution:
Step 1: Express the numbers given as a product of prime numbers.
$36 = 2^2 \times 3^2$
$54 = 2^1 \times 3^3$
Step 2: Take the product of prime factors of the given numbers after eliminating repetition of the common factors.
Here, eliminating the common factors 2^1 and 3^2 and multiplying the remaining factors i.e. 2^2 and 3^3 .
So, LCM = $2^2 \times 3^3 = 108$
Example 7: Find the LCM of 210, 360 and 540.
Solution:
Step 1: Express the numbers given as a product of prime numbers.
$210 = 2 \times 3 \times 5 \times 7$
$360 = 2^3 \times 3^2 \times 5$
$540 = 2^2 \times 3^3 \times 5$
Step 2: Take the product of prime factors of the given numbers after eliminating repetition of the common factors.
Here, eliminating the common factors 2^1, 3^1 and 5^1 and multiplying the remaining factors i.e. 2^3 and 3^3, 5 and 7.
So, LCM = $2^3 \times 3^3 \times 5 \times 7 = 7560$
Relationship between HCF and LCM:
The relationship between any two numbers x and y and their HCF and LCM: $\boldsymbol{x \times y = LCM \times HCF}$
Proof: Let us take any two numbers such as 14 and 78.
Factorization of 14 = 2 x 7 and 78 = 2 x 3 x 13
Product of 14 and 78 = 14 x 78 = 1092
HCF of 14 and 78 = 2
LCM of 14 and 78 = 2 x 3 x 7 x 13 = 546
Product of HCF and LCM = 2 x 546 = 1092
Thus, Product of 14 and 78 = HCF x LCM of 14 and 78.
Special case of finding HCF: There are some cases when HCF is asked in question but two numbers (N_1 and N_2) are not given instead their sum and LCM is given. So, in that case:
HCF of (N_1 and N_2) = HCF of (Sum of N_1 and N_2 , LCM of N_1 and N_2)
Proof: Find the HCF of 36 and 54.
$36 = 2^2 \times 3^2$
$54 = 2^1 \times 3^3$
HCF = $2^1 \times 3^2 = 18$

Here, Sum of 36 and 54 = 90
And Sum of 36 and 54 = 2^2 x 3^3 = 108
HCF of (Sum of 36 and 54, LCM of 36 and 54) = HCF of (90 and 108) = 18
So, in both the cases HCF is same.
HCF of fraction values: To calculate the HCF of fraction values, we calculate the ratio of HCF of all the numerators to LCM of all the denominators.
Example 8: Find the HCF of $\frac{1}{4}, \frac{3}{8}, \frac{5}{6}, \frac{13}{12}$.
Solution:
HCF of numerators = HCF (1, 3, 5, 13) = 1
LCM of denominators = LCM (4, 8, 6, 12) = 24
Thus, HCF of given fractions = $\frac{1}{24}$
LCM of fraction values: To calculate the LCM of fraction values, we calculate the ratio of LCM of all the numerators to HCF of all the denominators.
Example 9: Find the LCM of $\frac{1}{4}, \frac{3}{8}, \frac{5}{6}, \frac{13}{12}$.
Solution:
LCM of numerators = LCM (1, 3, 5, 13) = 195
HCF of denominators = HCF (4, 8, 6, 12) = 2
Thus, LCM of given fractions = $\frac{195}{2}$