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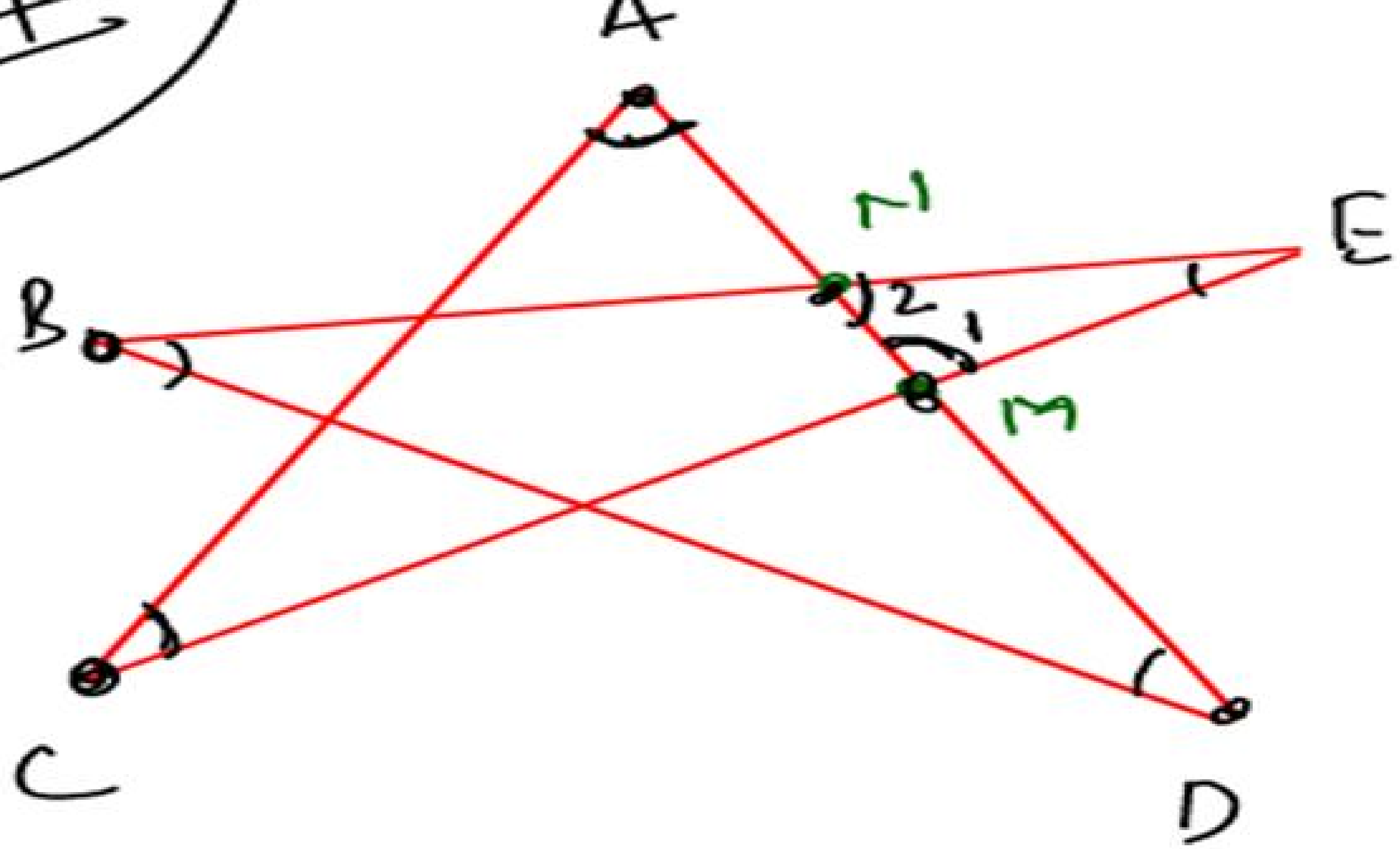
Sahi Prep Hai Toh Life Set Hai

TRIANGLE-1

* SSC

Concepts
+
Calculation

v. imp



ΔCAM

$\angle 1 = \angle C + \angle A$

ΔBDN

$\angle 2 = \angle B + \angle D$

ΔMNE

$\angle 1 + \angle 2 + \angle E = 180$

$\angle C + \angle A + \angle B + \angle D + \angle E = 180^\circ$

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Agenda

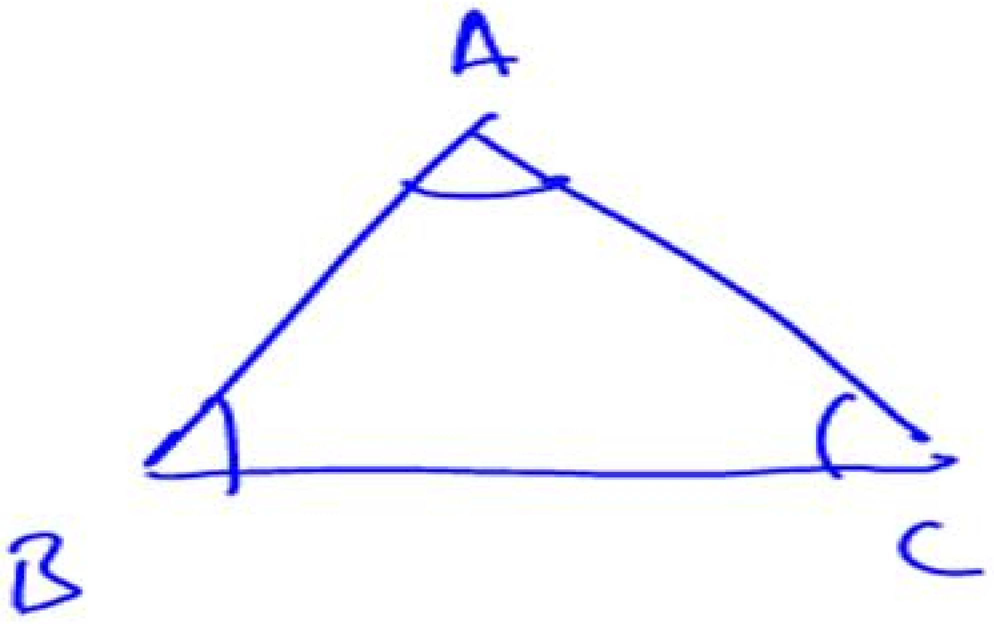
Triangles → " Basics of Δ "

BASICS OF TRIANGLE

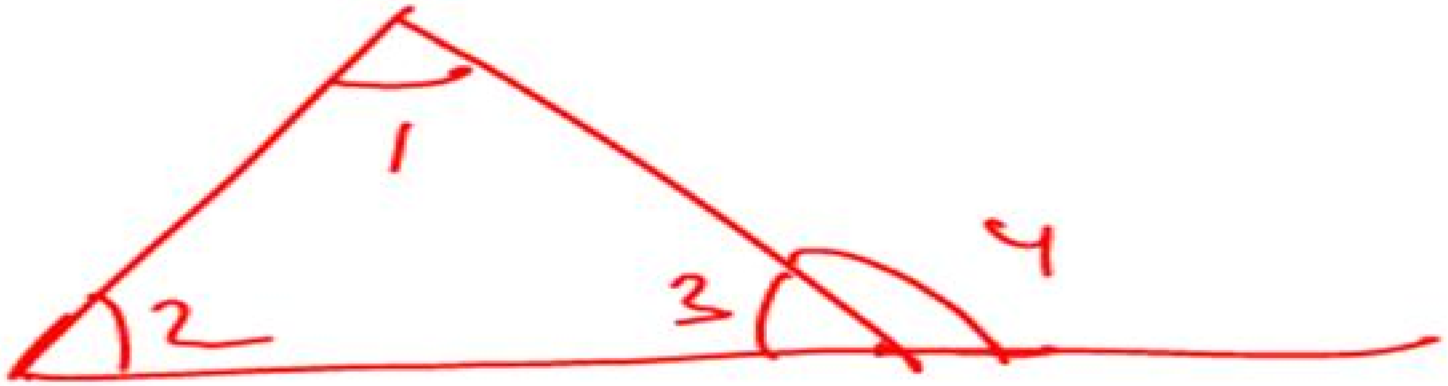
Points which we have discussed :

(1) Sum of all angles of a △ = 180°

(2) Exterior angle of a △ is equal to sum of its interior opposite angle.



$\angle A + \angle B + \angle C = 180^\circ$



$\angle 4 = \angle 1 + \angle 2$

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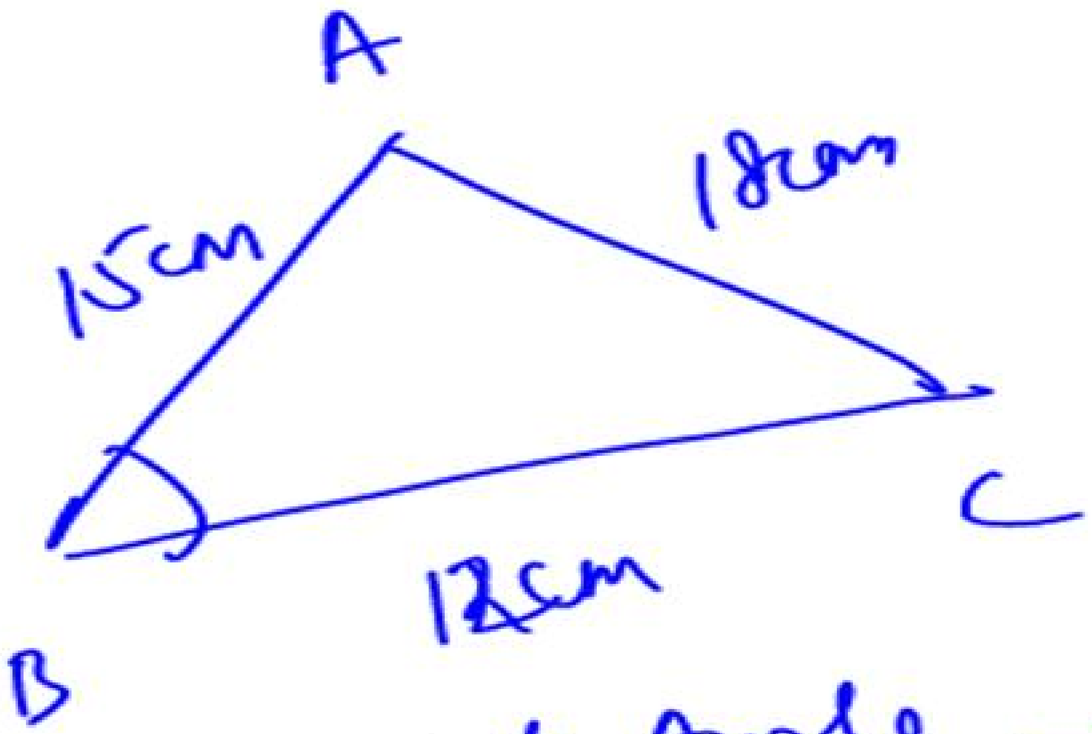
(3) Side opposite to largest angle is largest.

$a \le b \le c$

(4) Scalene
Isosceles
Equilateral

Acute → $a^2+b^2 > c^2$
Right → $a^2+b^2 = c^2$
Obtuse → $a^2+b^2 < c^2$



Largest Angle → ∠B
Smallest Angle → ∠A

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V. imp **Sum of any two sides of a triangle is greater than the third side.**

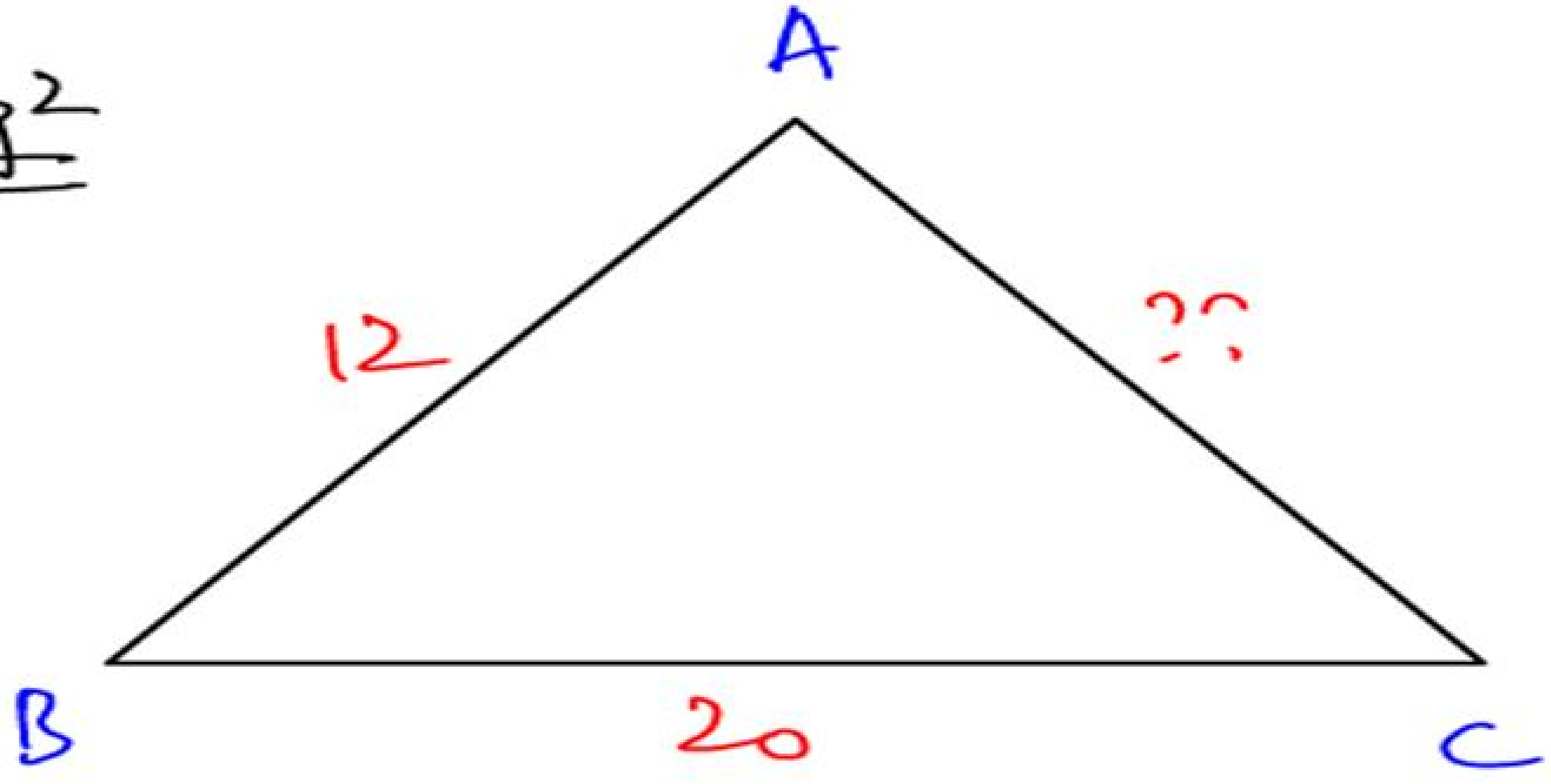
$a \leq b \leq c$

$\boxed{a+b > c}$

eg 1 which of the following can make Δ

(i)	5, 8, 11	$5+8 > 11$	✓
(ii)	3, 7, 10	$3+7$ 10	✗
(iii)	9, 6, 12	$9+6 > 12$	✓
(iv)	15, 7, 11	$7+11 > 15$	✓

Eg 2



$AB = 12\,cm$

$BC = 20\,cm$

How many values of CA are possible If length of CA is a natural no??

$12 + 20 > CA$

$32 > CA$

$\boxed{CA < 32}$

$12 + CA > 20$

$CA > 8$

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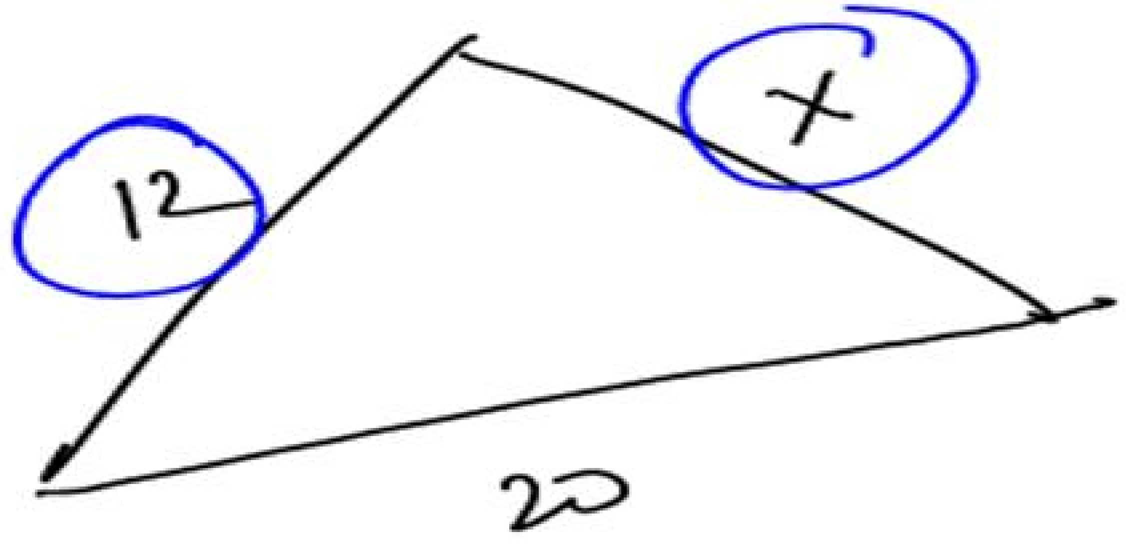
$$8 < CA < 32$$

$9, 10, 11, - - - - 31$

$31 - 9 + 1 = 23$ values are possible

Shortcut

$$\boxed{\text{Diff of 2 sides} < 3^{rd} \text{ side} < \text{Sum of 2 sides}}$$



$8 < x < 32$

$9, 10, \ldots 31$

$31 - 9 + 1 = \underline{23}$ values

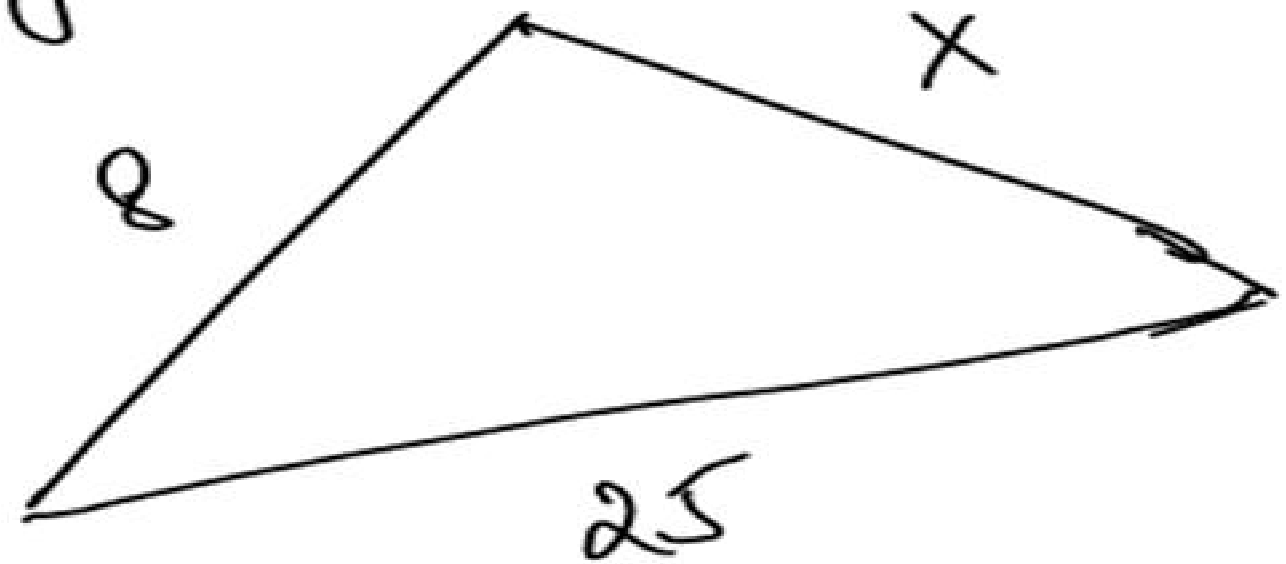
Better shortcut

$2(\text{smallest side}) - 1$

$2 \cdot 12 - 1$

$= \underline{23}$ values

eg



(i) How many values of x are possible

$\longrightarrow$ Infinite

(ii) How many values of x are possible if x is a natural no

$\longrightarrow$ $2 \cdot 8 - 1$

$= 15$

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Eg1. Out of given options what can be the sides of Δ?

(a) 5, 12, 8

(b) 9, 6, 15

(c) 4, 8, 11

(d) 3, 4, 5

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Eg2. By using line segments of length 2 cm, 3 cm, 5 cm and 7 cm. How many triangles can be formed?

2 3 5 7

2 3 5 ✗

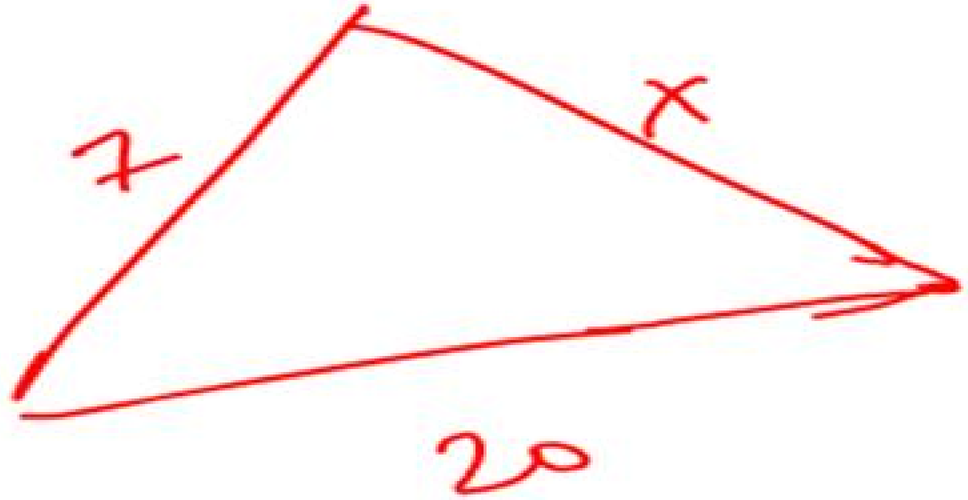
2 3 7 ✗

2 5 7 ✗

3 5 7 ✓

One Δ will be formed

Eg3. If two sides of a triangle are 7 cm and 20 cm. How many values of third side are possible where length of third side is a natural number?



$x \rightarrow 2 \cdot 7 - 1$

13 values

If sides of Δ $a \leq b \leq c$

for obtuse angle Δ

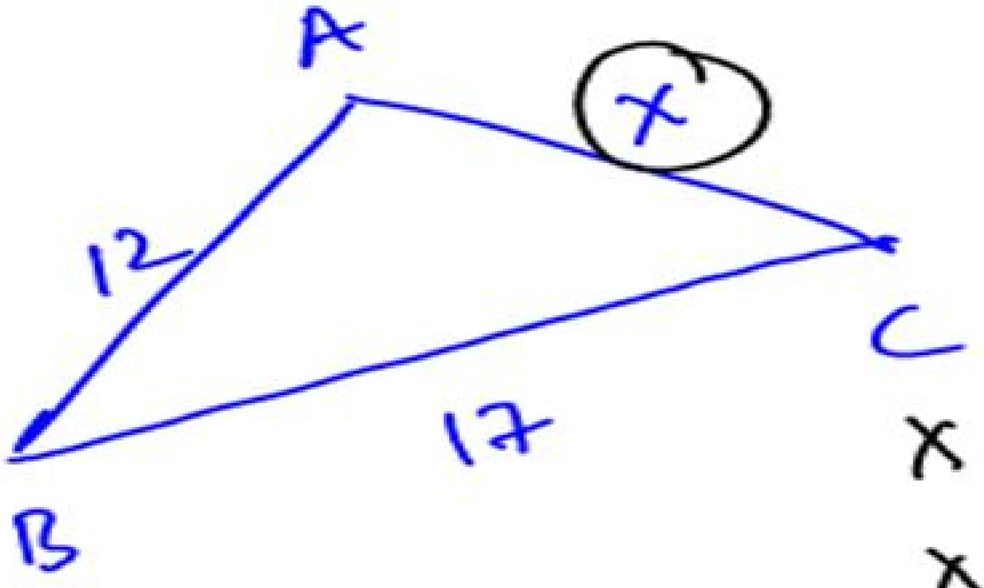
$$\boxed{a^2 + b^2 < c^2}$$

Eg4. If triangle ABC is an obtuse angle triangle.

AB = 12 cm, BC = 17 cm

How many values of AC are possible where length of AC is a natural number?

v.ang



I when x is the largest side

$$12^2 + 17^2 < x^2$$
$$144 + 289 < x^2$$
$$433 < x^2$$

$x_{min} \rightarrow 21$

$x_{max} \rightarrow 28$

8 values are possible

II when 17 is the largest side

$$12^2 + x^2 < 17^2$$
$$x^2 < 145$$

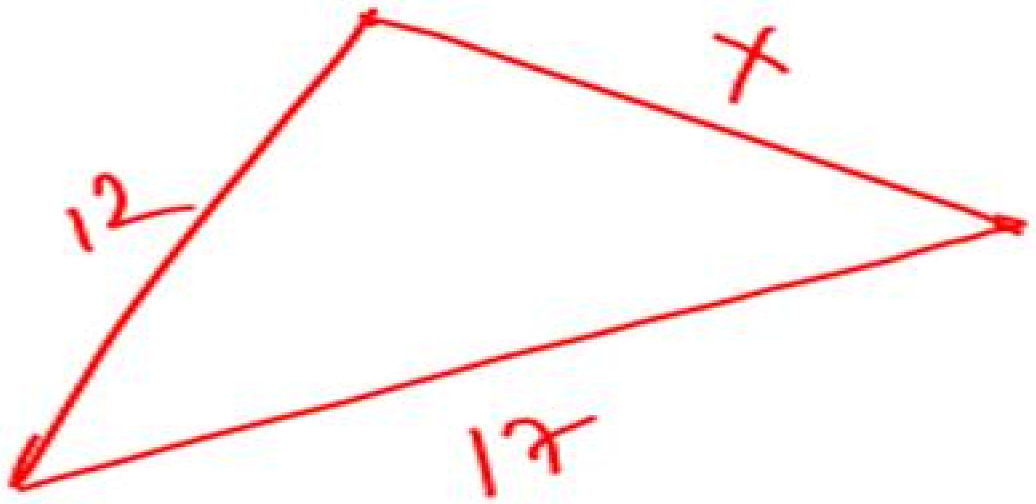
$x_{min} \rightarrow 6$

$x_{max} \rightarrow 12$

7 values are possible

15 values are possible

Ans. 15 values of AC are possible.



I

$$12^2 + 17^2 < x^2$$

$$433 < x^2$$

$x_{min} \to 21$

$x_{max} \to 28$

$\to$ 8 values are poss

II

$$12^2 + x^2 < 17^2$$

$$x^2 < 145$$

$x_{min} \to 6$

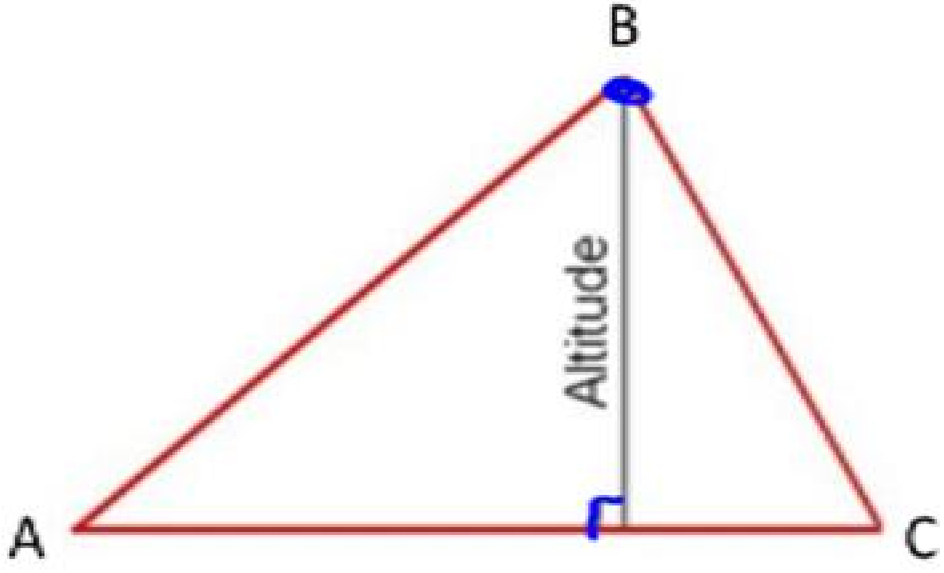
$x_{max} \to 12$

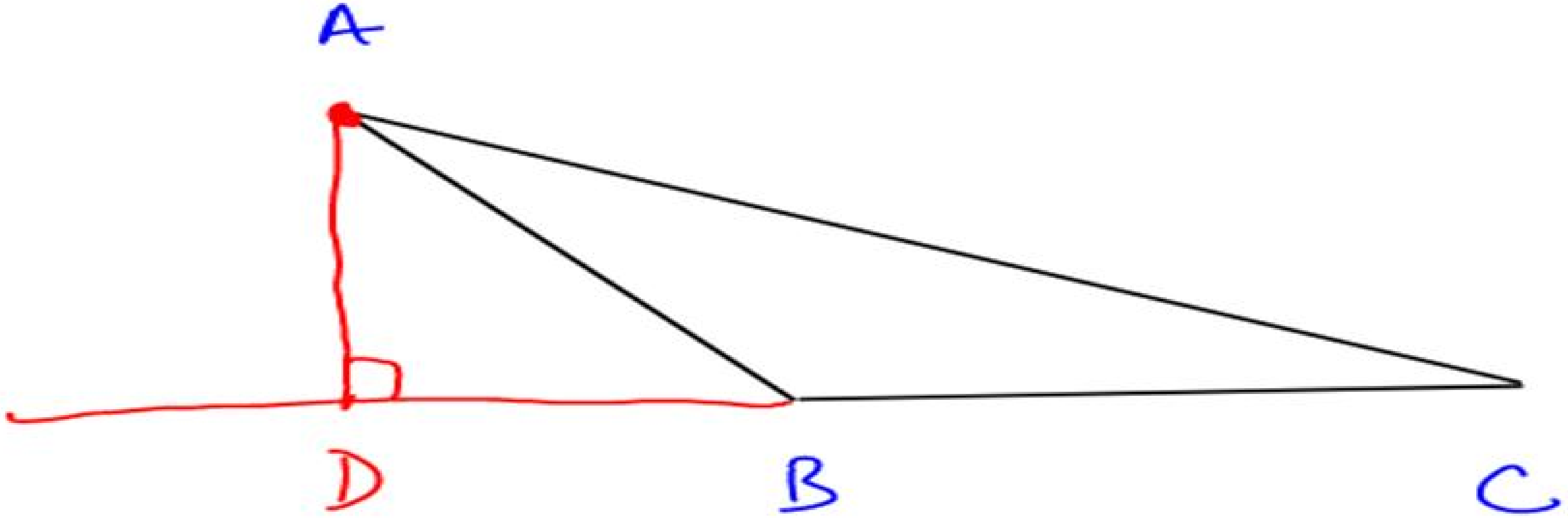
7 values are possible

BASIC TERMS USED IN TRIANGLES

(Height)

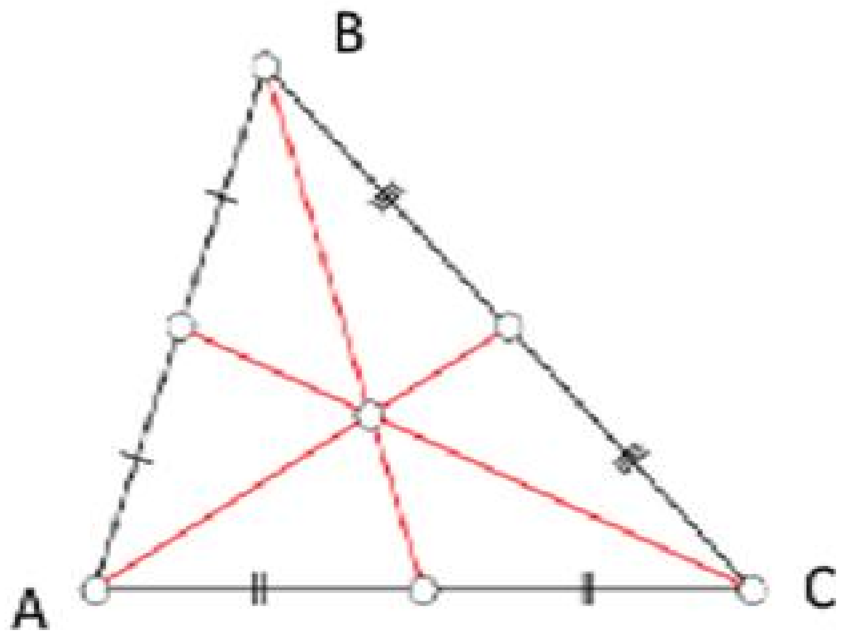
(1) Altitude : Altitude of a triangle is the perpendicular drawn from the vertex of the triangle to the opposite side. Also, known as the height of the triangle, the altitude makes a right angle triangle with the base.

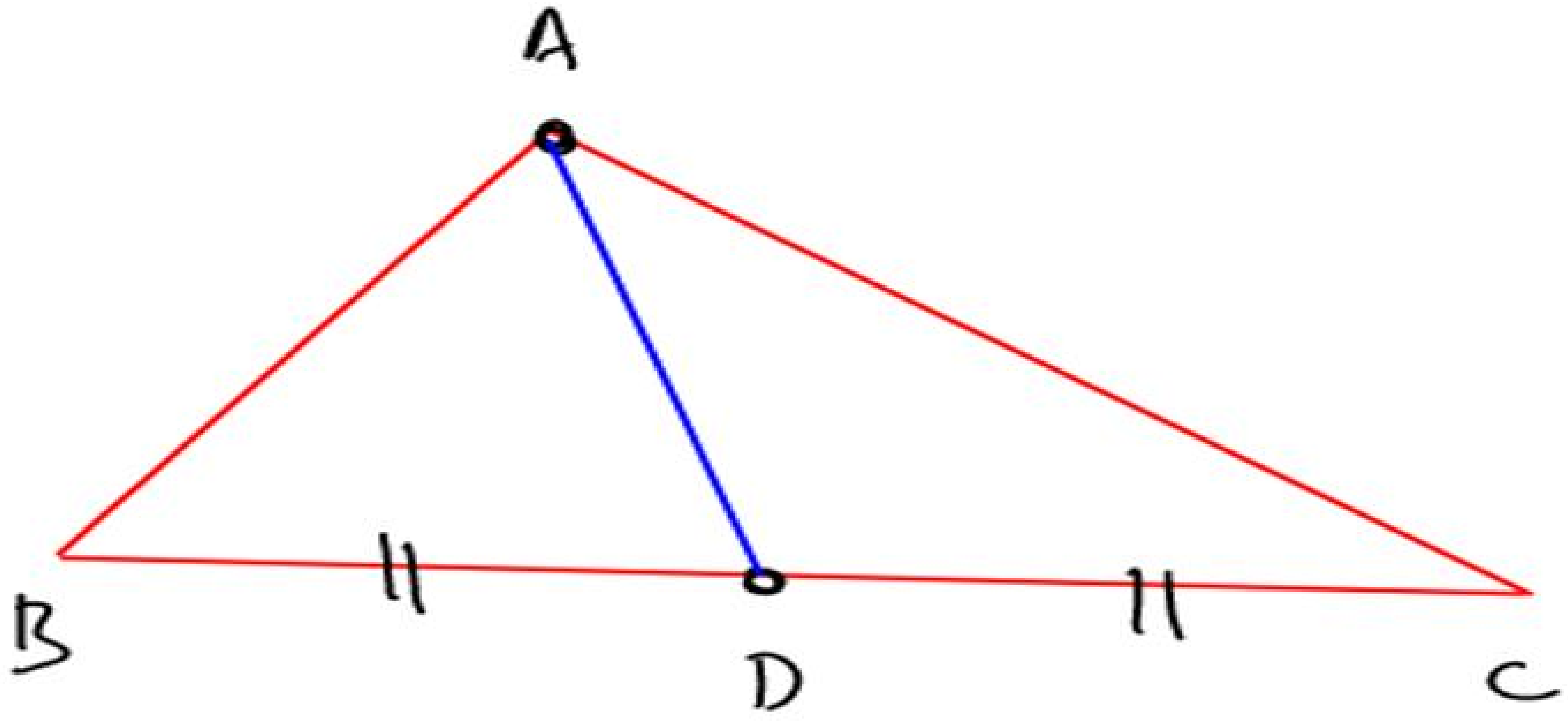




AD is altitude of $\triangle ABC$

(2) Median : A median of a triangle is a line segment joining a vertex to the midpoint of the opposite side, thus bisecting that side. Every triangle has exactly three medians, one from each vertex, and they all intersect each other at the triangle's centroid.

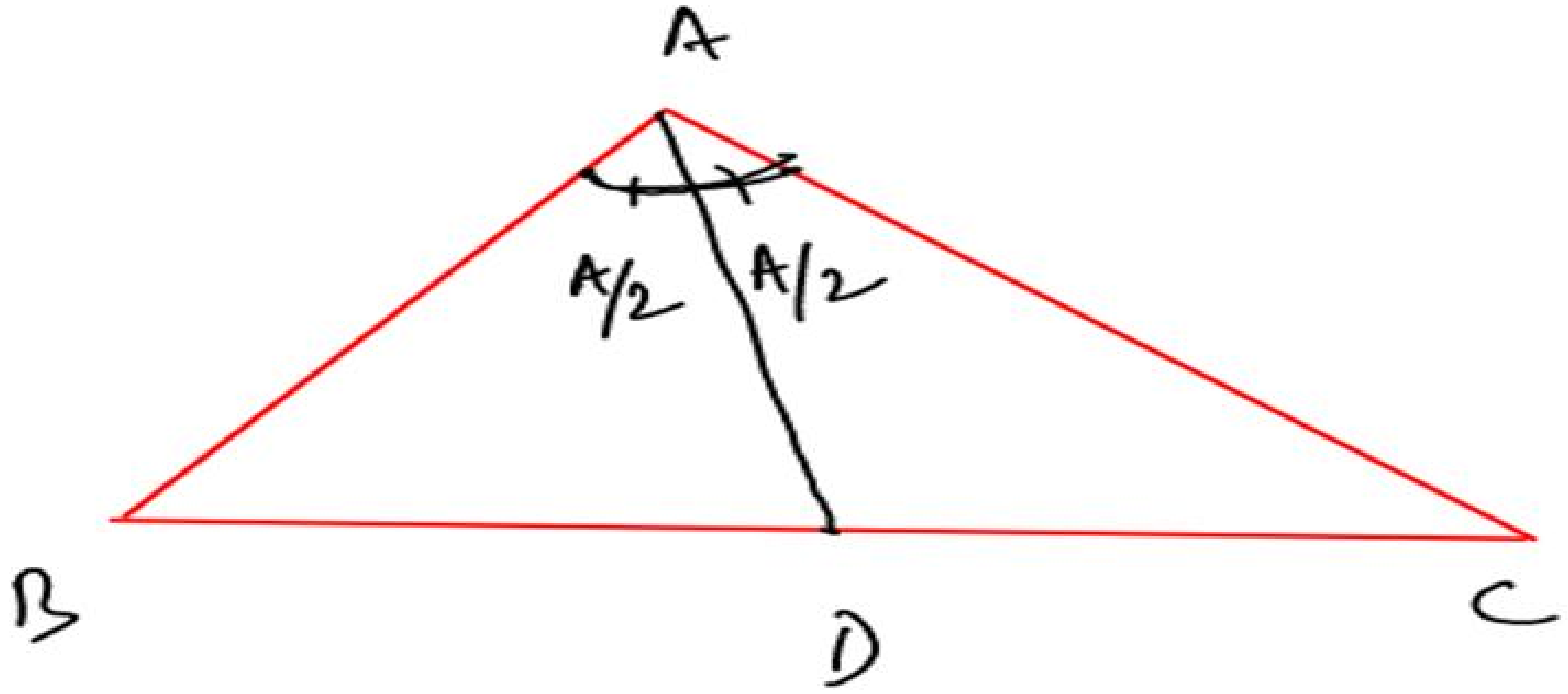




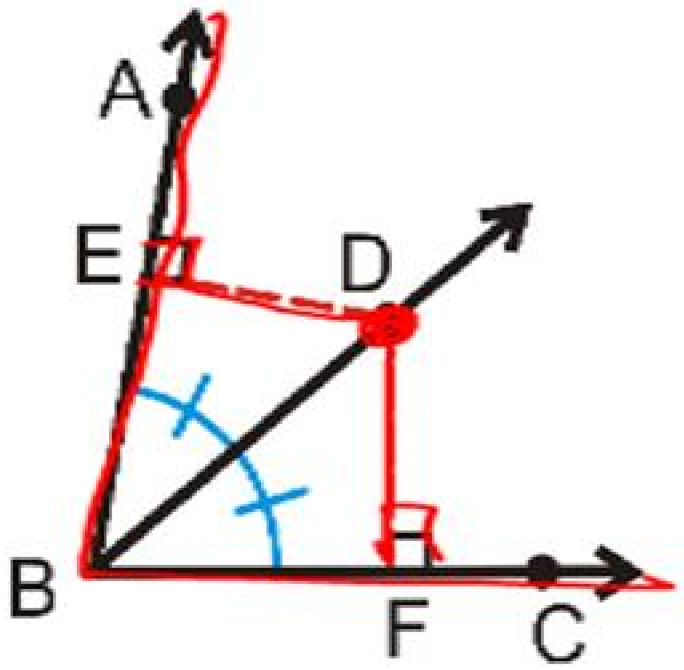
If AD → median

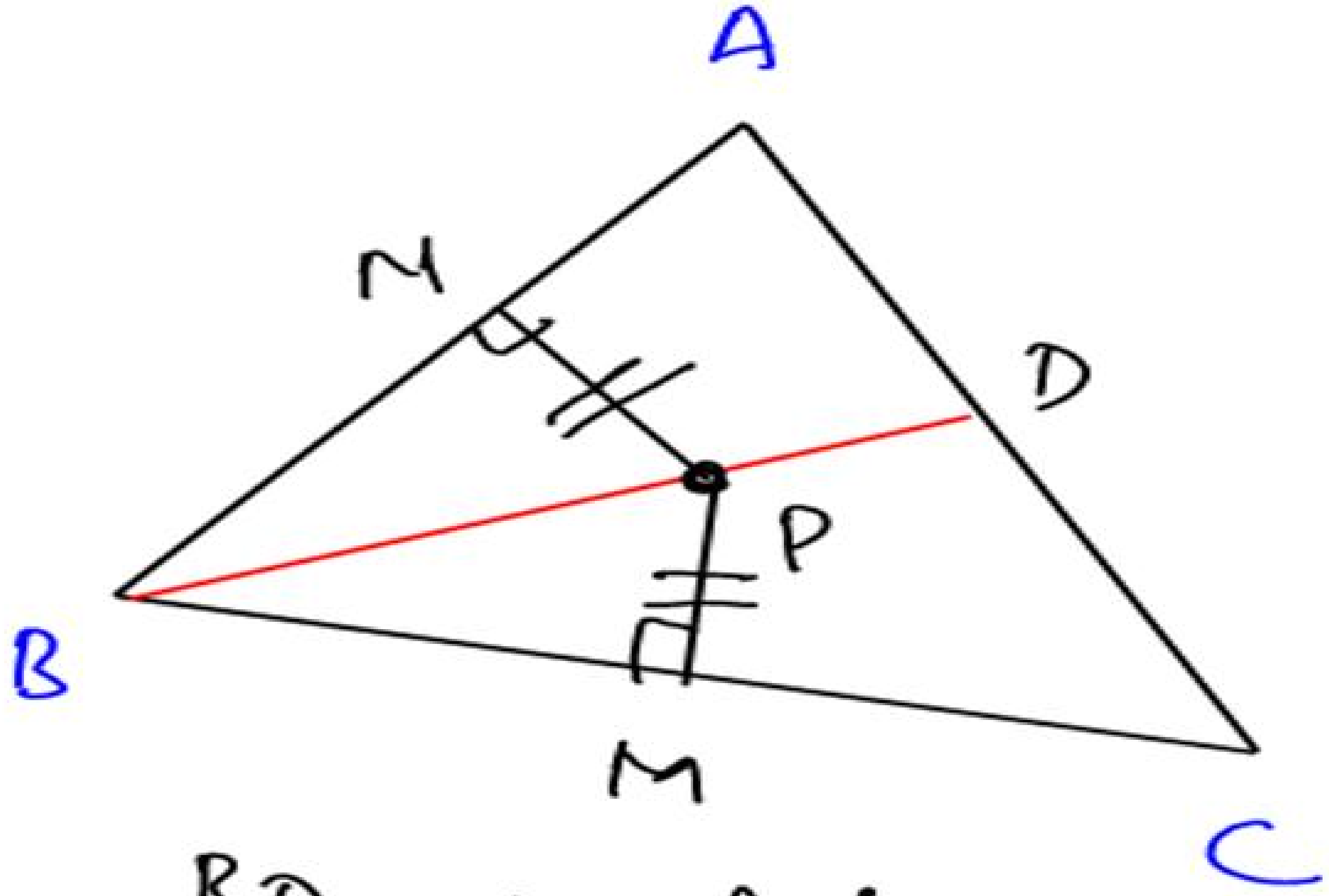
$\underline{BD = DC}$

(3) Angle Bisector : The angle bisector of a triangle is a line segment that bisects one of the vertex angles of a triangle.



Angle bisector is equidistant from the sides containing the angle.

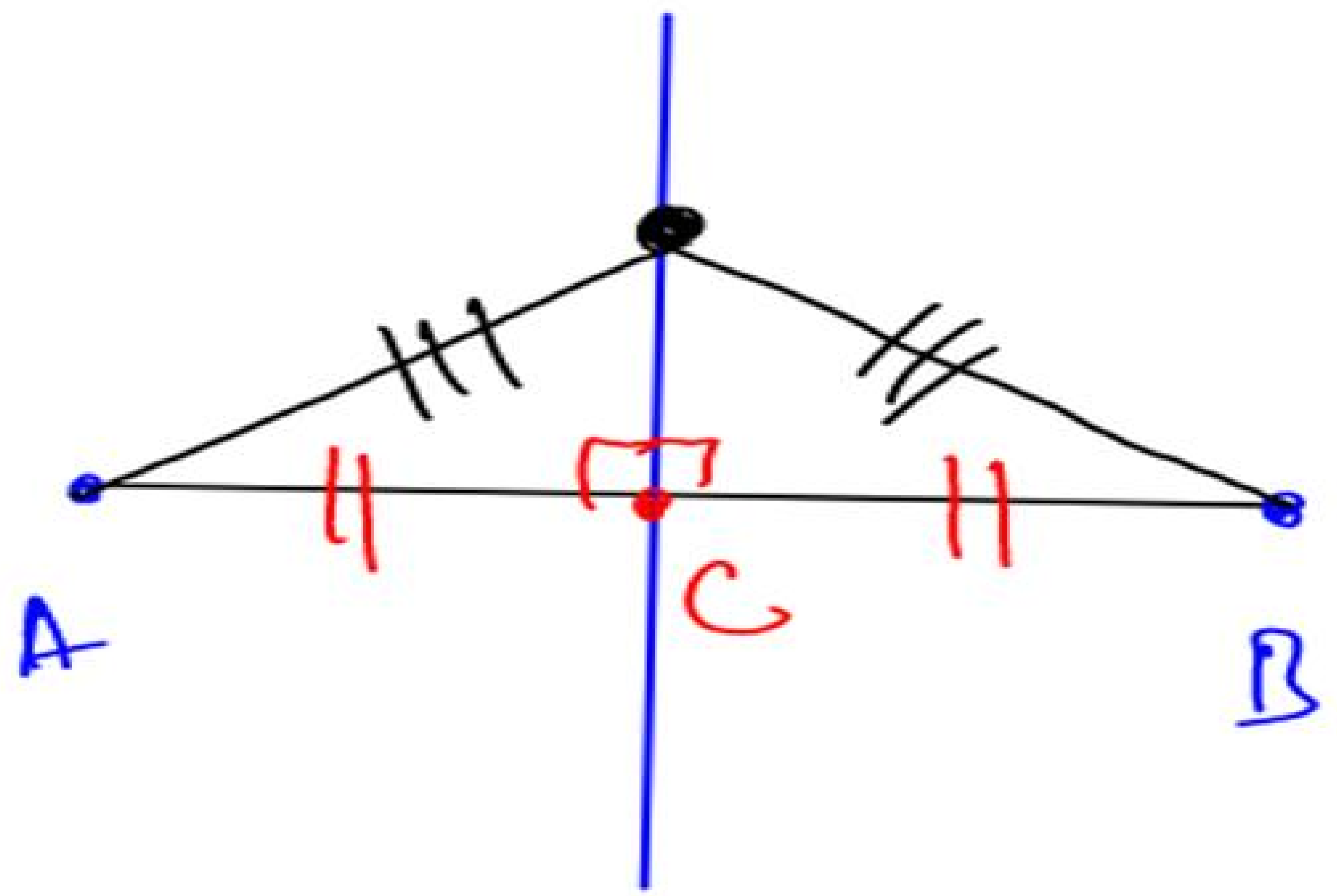


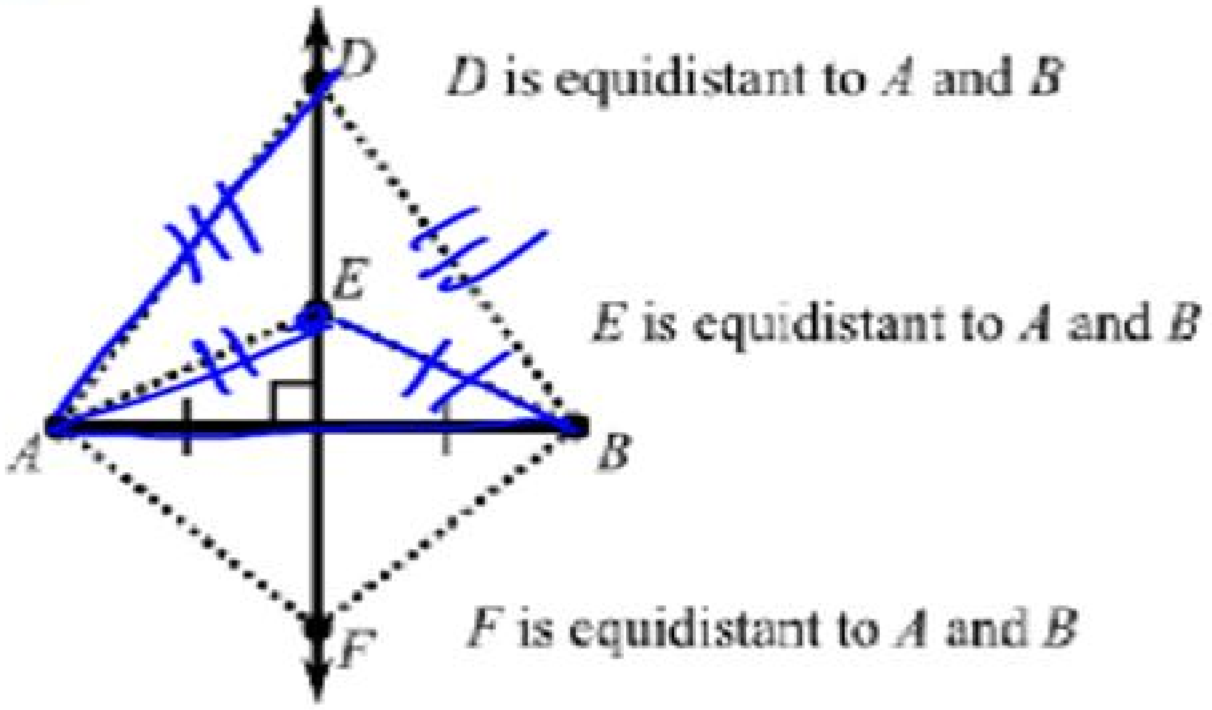


BD → Angle Bisector of ∠B

PM = PN

(4) Perpendicular Bisector : The perpendicular bisector of a side of a triangle is a line segment that is both perpendicular to a side of a triangle and passes through its midpoint.

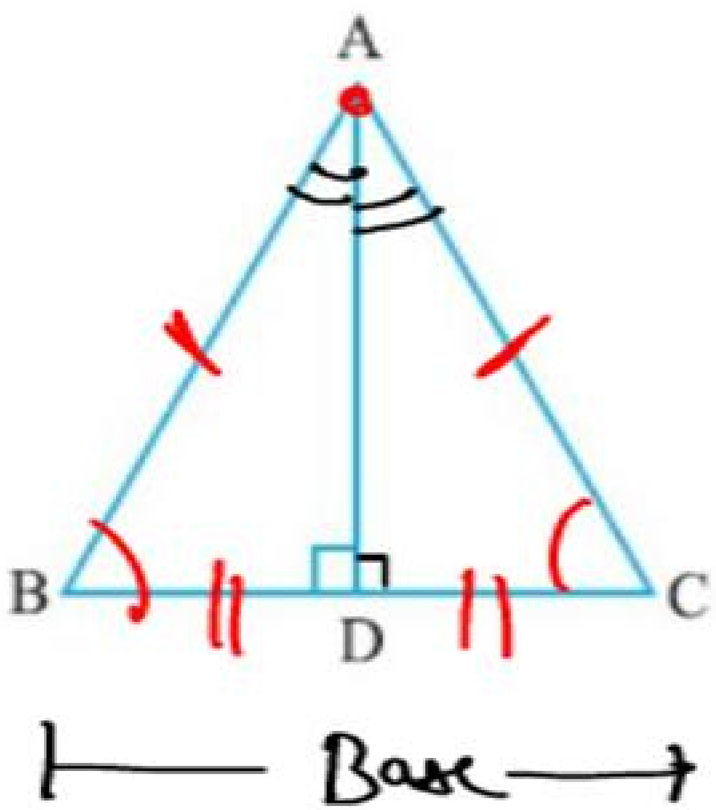




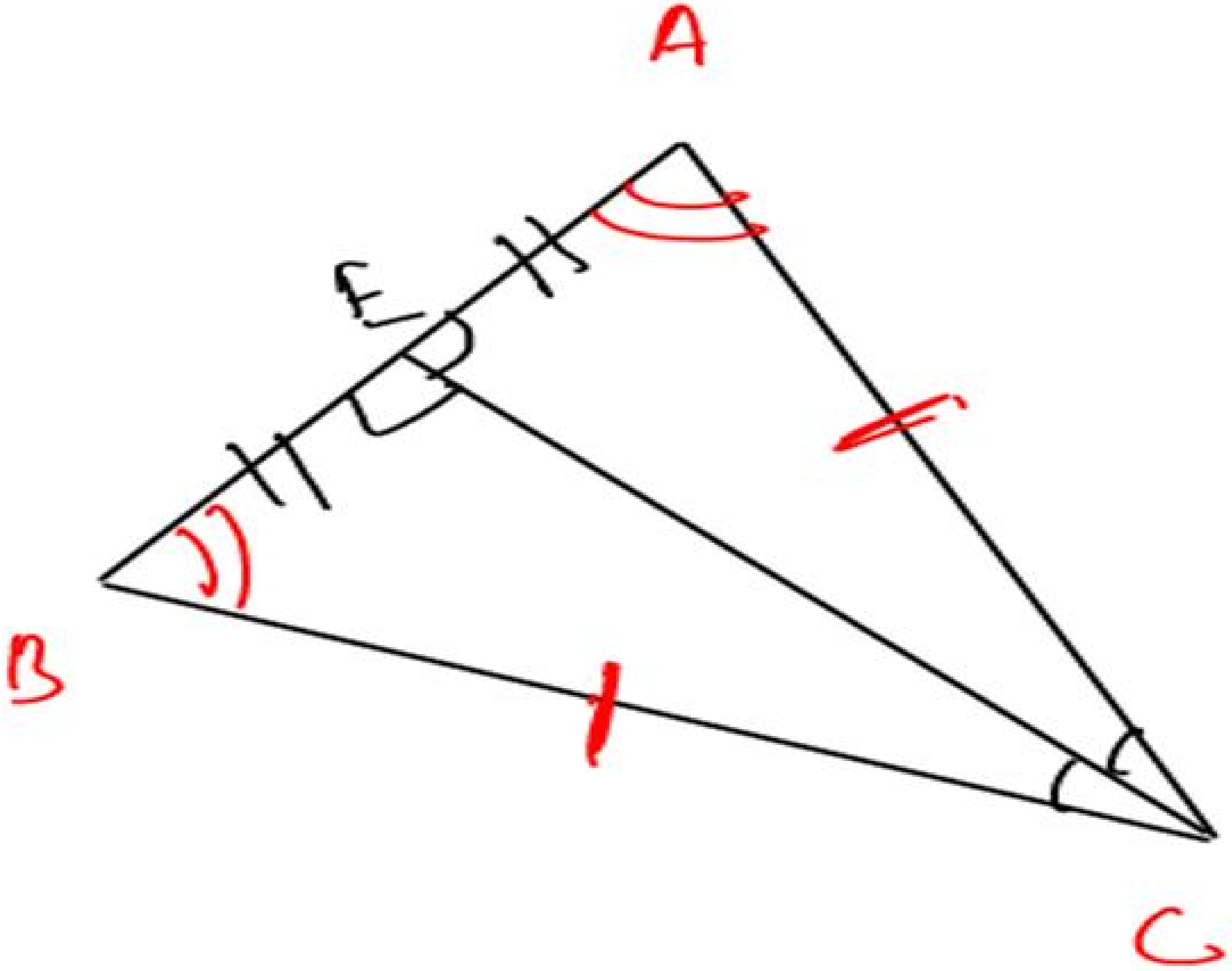
Perpendicular bisector of a line segment is equidistant from the end points of that line segment.

Isosceles Triangle

In an isosceles triangle (where base is the side which is not equal to any other side):
– the altitude drawn to the base is the median and the angle bisector;
– the median drawn to the base is the altitude and the angle bisector;
– the bisector of the angle opposite to the base is the altitude and the median.



Altitude = Median = Angle Bisector

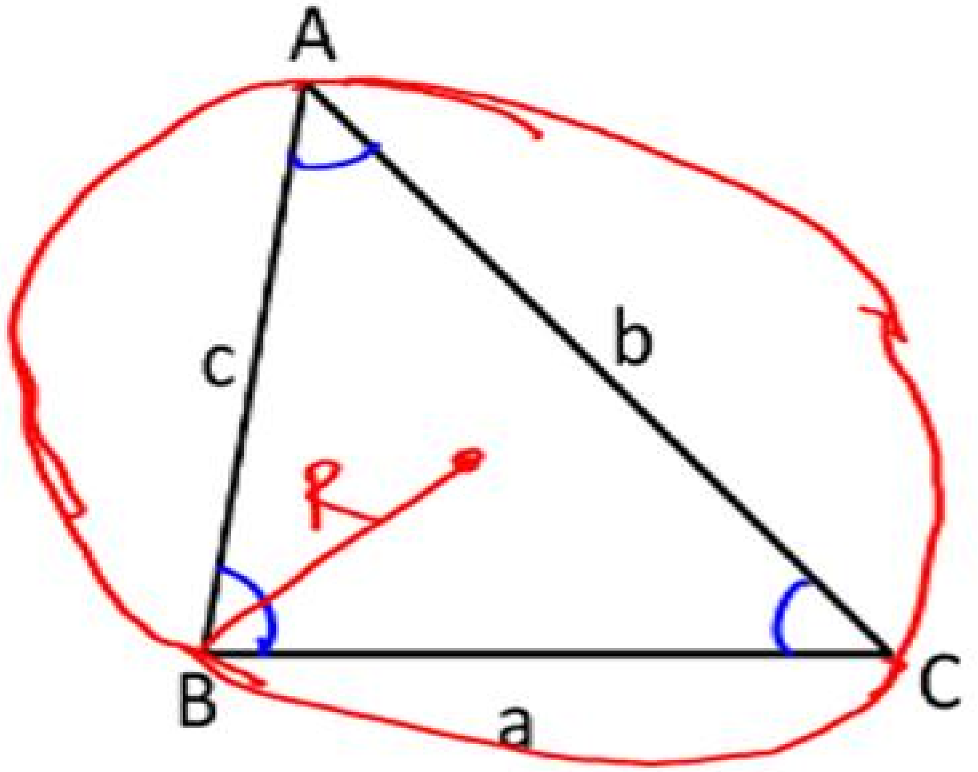


BC = AC (Given)

CE $\perp^{nd}$ AB

Sine formula

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$



R → Circumradius

Proof → we will discuss later

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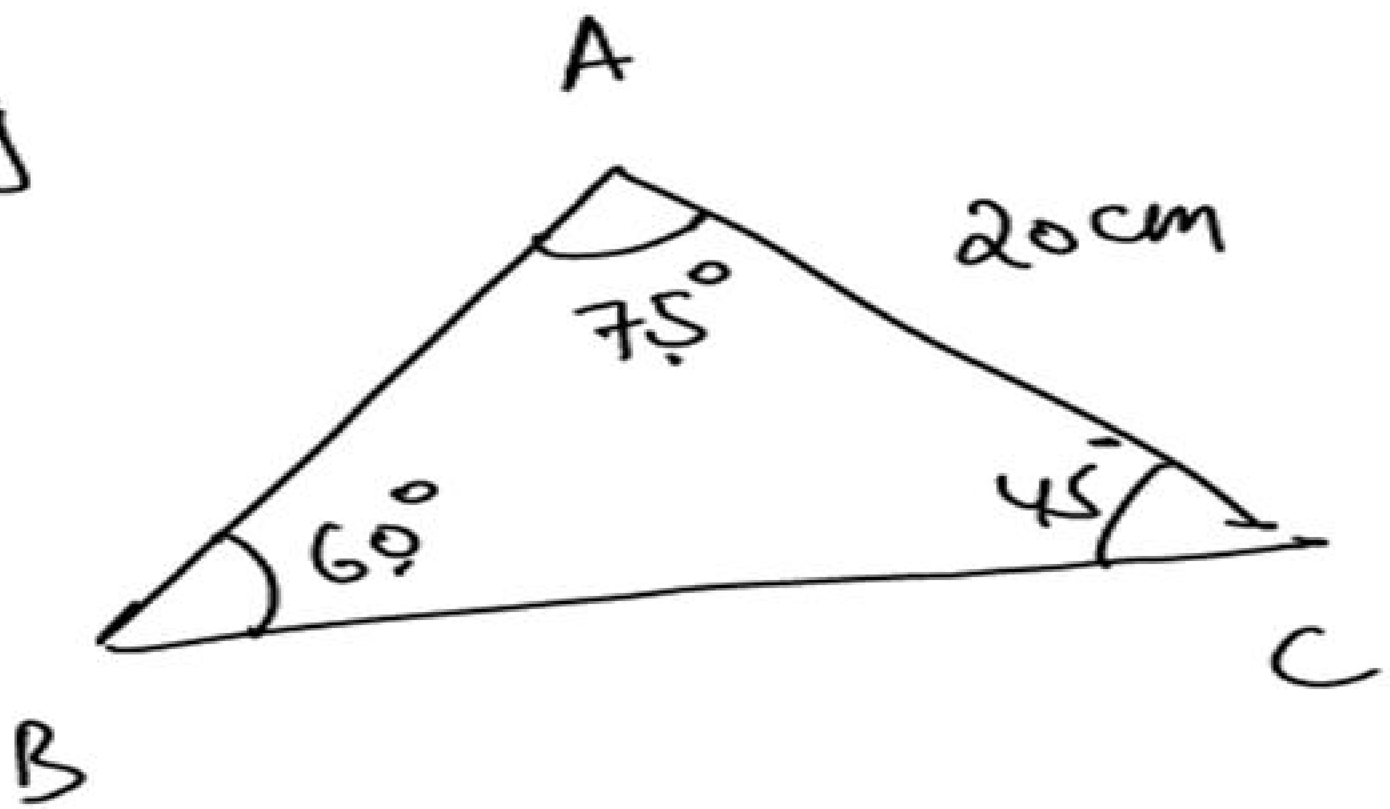
eg



Find AB = ??

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\sin 45 = \frac{1}{\sqrt{2}}$

$$\frac{20}{\sin 60} = \frac{AB}{\sin 45}$$

$$\frac{20 \cdot \cancel{2}^{\sqrt{2}}}{\sqrt{3}} = \frac{AB}{1} \cdot \cancel{\sqrt{2}}$$

$$AB = \frac{20\sqrt{2}}{\sqrt{3}}$$

Eg. If the angles of a triangle are 90°, 60° and 30°, then what is the ratio of the sides opposite to these angles?

a $\sqrt{3}:\sqrt{2}:1$ b $1:\sqrt{2}:\sqrt{2}$

c $2:\sqrt{3}:1$ d $3:2:1$

$\sin 30 = \frac{1}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\sin 90 = 1$

90 60 30

$1 : \frac{\sqrt{3}}{2} : \frac{1}{2}$

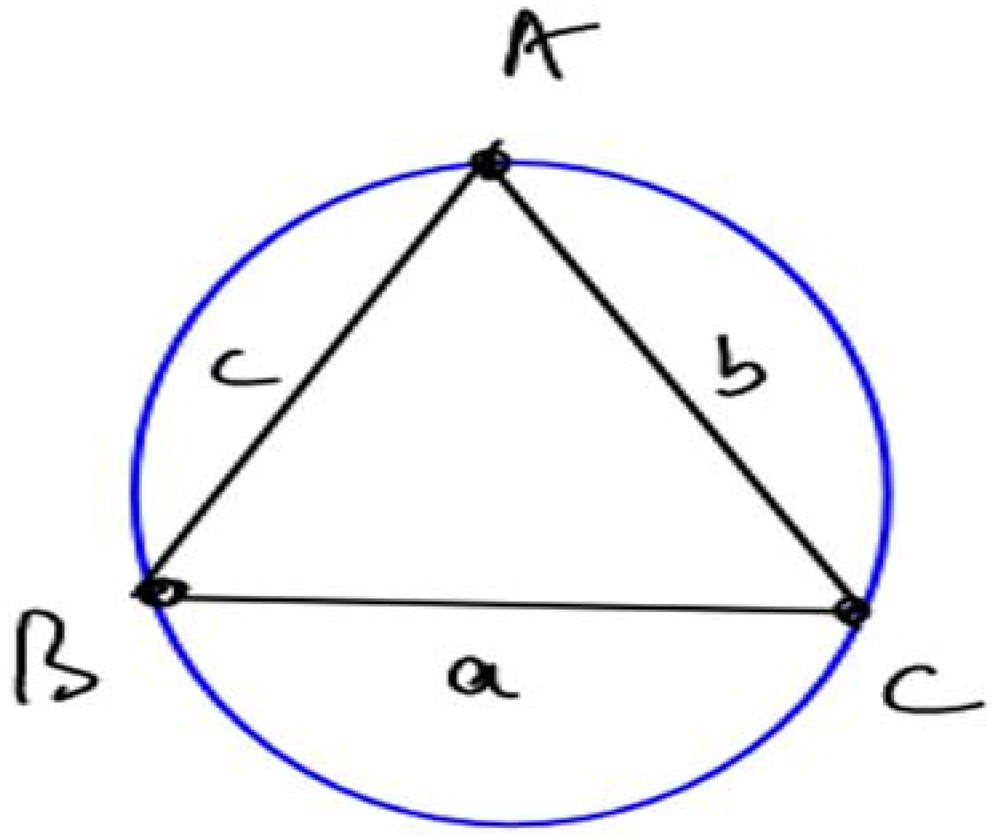
$2 : \sqrt{3} : 1$

Ans. (c)

PYQ of SSC
imp

Eg. ΔABC is inscribed in a circle. If sum of the squares of sides of the triangle is equal to twice the square of the diameter, then $\sin^2 A + \sin^2$ of $B + \sin^2 C$ is equal to:

(a) 2 (b) 3 (c) 4 (d) None of these



$$a^2+b^2+c^2 = 2(2R)^2$$

$$a^2+b^2+c^2 = 8R^2$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$(2R\sin A)^2 + (2R\sin B)^2 + (2R\sin C)^2 = 8R^2$$

$$\cancel{4R^2}(\sin^2 A + \sin^2 B + \sin^2 C) = \cancel{8R^2}\,2$$

Ans. (a)

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Eg. In ΔABC, $\angle B = \frac{\pi}{3}$, $\angle C = \frac{\pi}{4}$ and D cuts BC internally in ratio 1 : 3, then

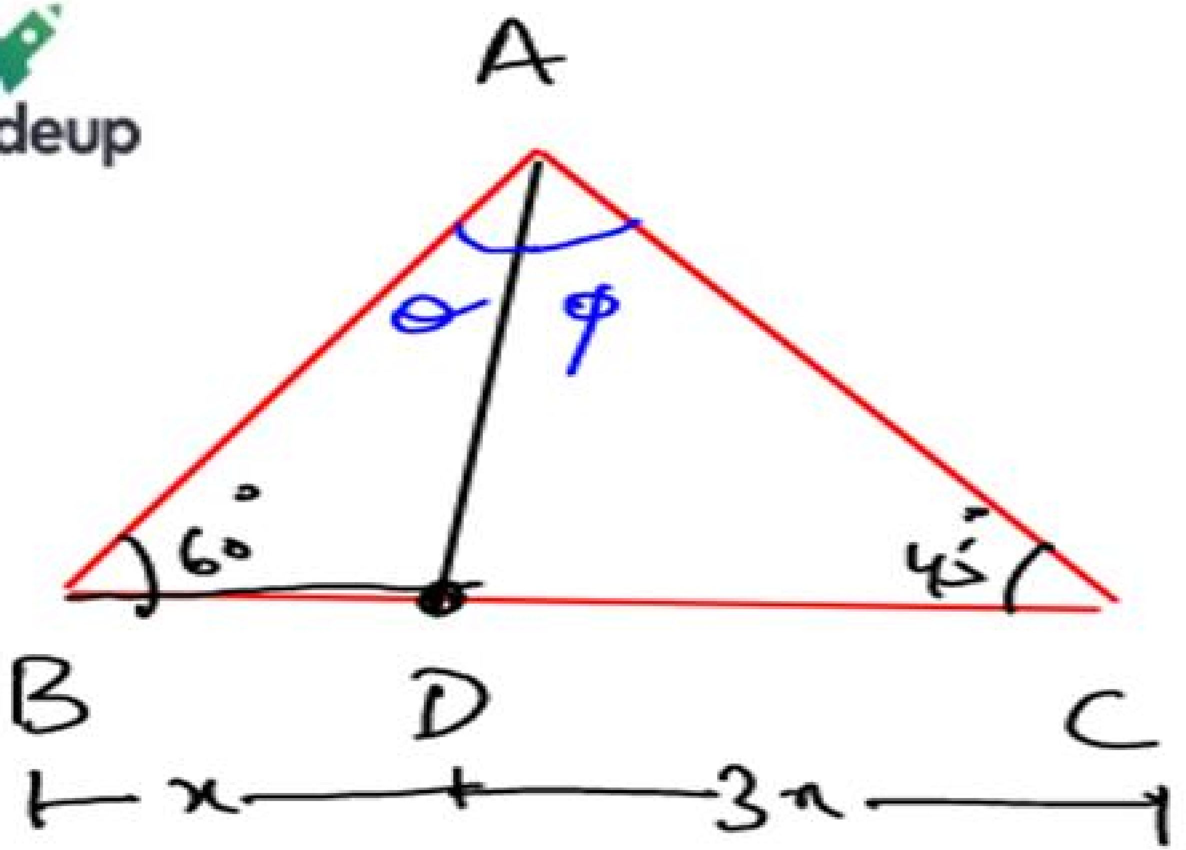
$$\frac{\sin \angle BAD}{\sin \angle CAD} = ?$$

$\angle B = 60°$

$\angle C = 45°$

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{6}$



$$\frac{\sin\theta}{\sin\phi} = ??$$

Sol^n ΔABD

$$\frac{x}{\sin\theta} = \frac{AD}{\sin 60} \quad —(1)$$

ΔACD

$$\frac{3x}{\sin\phi} = \frac{AD}{\sin 45} \quad —(2)$$

(2) ÷ (1)

$$\frac{3\cancel{x}}{\sin\phi} \cdot \frac{\sin\theta}{\cancel{x}} = \frac{\cancel{AD}}{\sin 45} \cdot \frac{\sin 60}{\cancel{AD}}$$

$$\frac{\sin\theta}{\sin\phi} = \frac{1}{3} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{1}$$

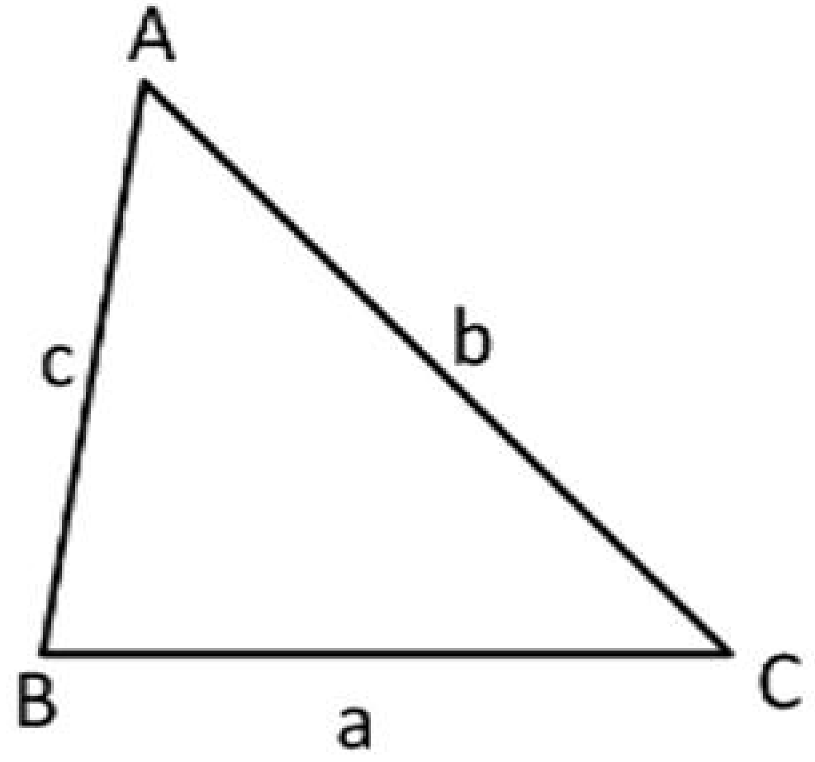
$$= \frac{1}{\sqrt{6}}$$

$\sqrt{3}\sqrt{2}$

Ans. (c)

Cosine formula

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

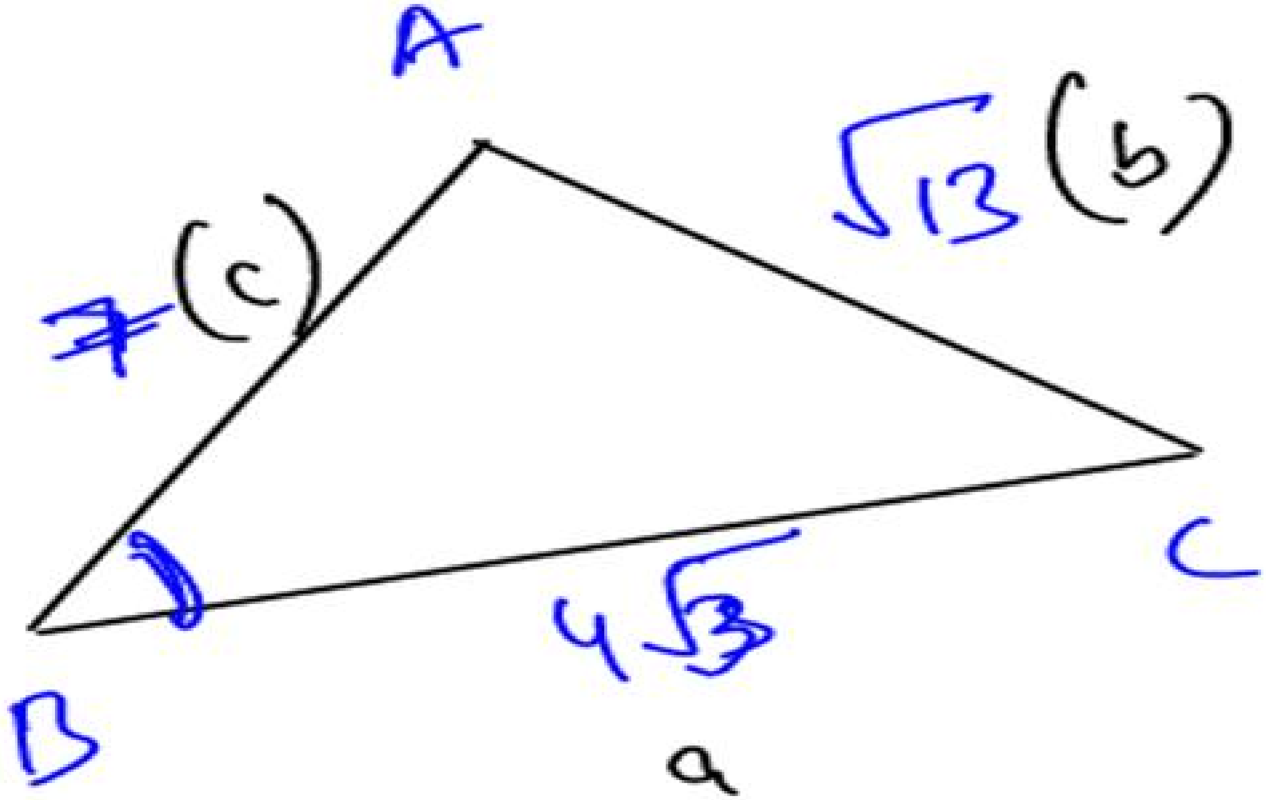
$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\cos 120^\circ = \frac{-1}{2}$$

Eg. Three sides of a triangle are 7 cm, $4\sqrt{3}$ cm and $\sqrt{13}$ cm then smallest angle is:

(a) 15° (b) 30° (c) 45° 45 (d) 60° 60°



AC is the smallest side

B is smallest angle

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos B = \frac{48+49-13}{2\cdot 7\cdot 4\sqrt{3}} = \frac{84}{2\cdot 7\cdot 4\sqrt{3}}$$

$$\boxed{B = 30^\circ}$$

Ans. (b)

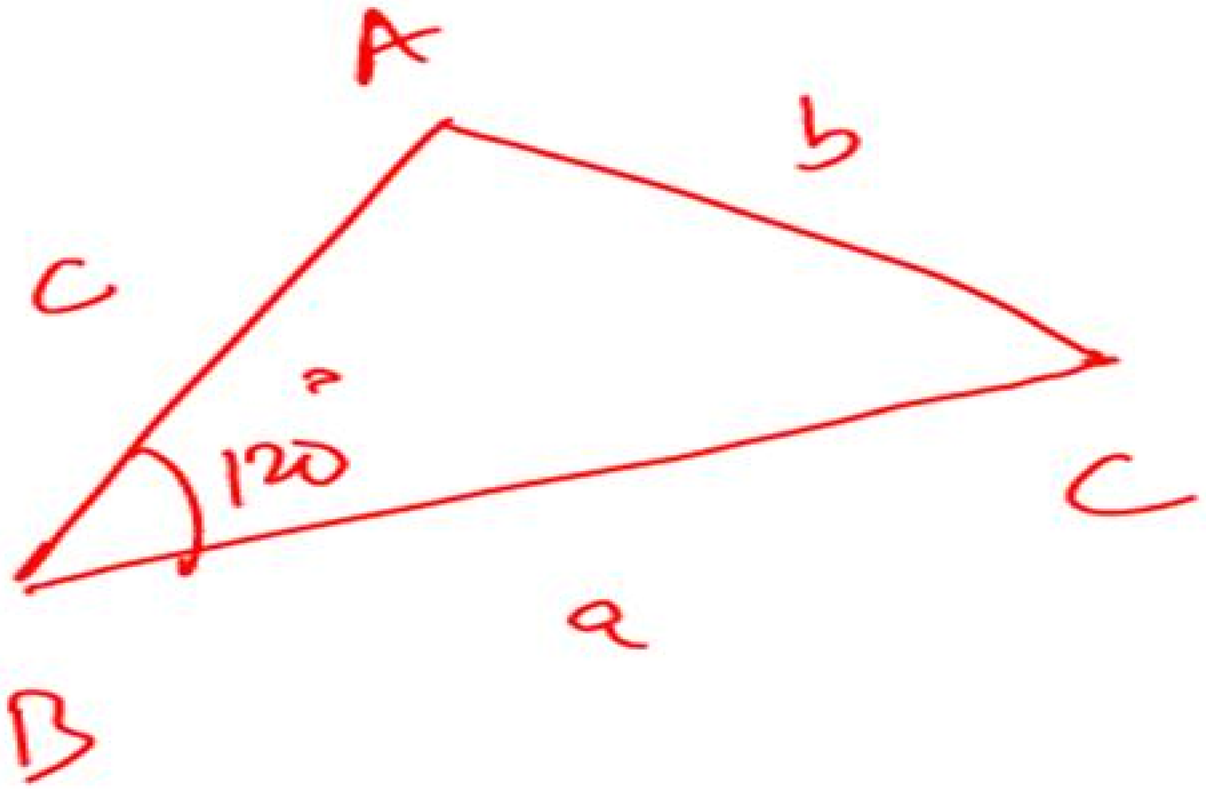
Eg. In ΔABC, $\angle ABC = 120°$, then relation between sides is:

(a) $b^2 = a^2 + c^2 + ac$ ✓

(b) $b^2 = a^2 + c^2 - ac$

(c) $b^2 = a^2 + c^2 - 2ac$

(d) $b^2 = a^2 + c^2 + 2ac$



$$\cos 120 = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\frac{-1}{\cancel{2}} = \frac{a^2 + c^2 - b^2}{\cancel{2}ac}$$

$$-ac = a^2 + c^2 - b^2$$

$$b^2 = a^2 + c^2 + ac$$

Ans. (a)

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If a, b & c are sides of Δ

$a \le b \le c$

$a^2+b^2 > c^2$ Acute

$a^2+b^2 = c^2$ Right

$a^2+b^2 < c^2$ Obtuse

Reason

If $\angle C$ is Acute

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$+ve = \frac{a^2+b^2-c^2}{+ve}$$

$$a^2+b^2-c^2 > 0$$

$$a^2+b^2 > c^2$$

If $\angle C$ is Right angle

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$0 = a^2+b^2-c^2 \qquad a^2+b^2=c^2$$

If $\angle C$ is Obtuse

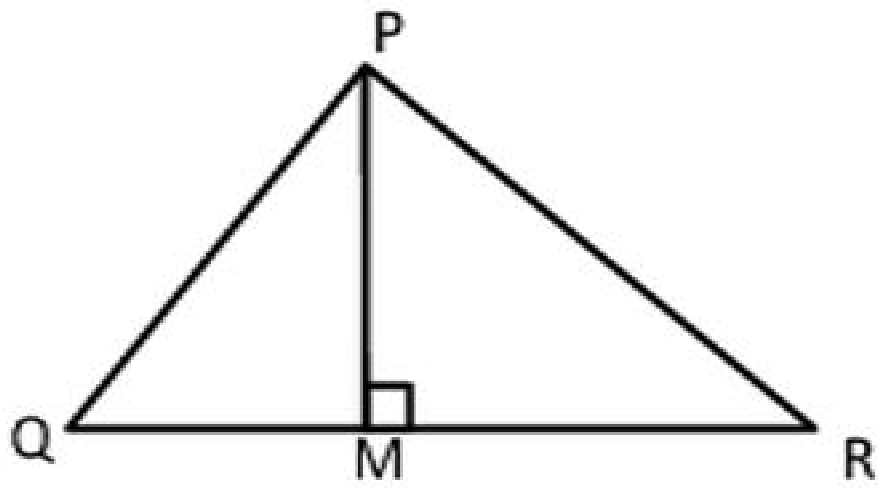
$$-ve = \frac{a^2+b^2-c^2}{+ve}$$

$$a^2+b^2-c^2<0 \qquad a^2+b^2<c^2$$

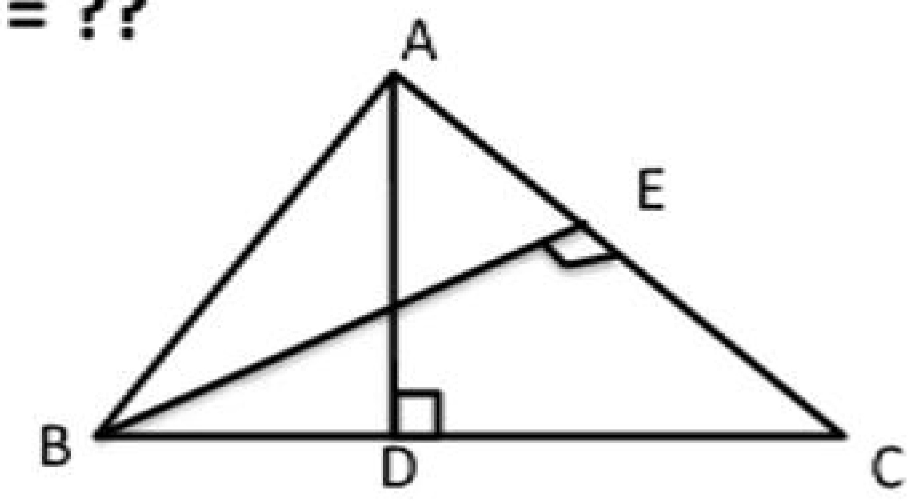
AREAS OF TRIANGLE

(1) $$Area = \frac{1}{2} \times Base \times Height$$

Eg1. If QR = 20 cm
PM = 12 cm
Find area of triangle.



Q1. If AD = 10 cm, BC = 24 cm, AC = 20 cm, BE = ??



Q2. Area of 2 triangles are in the ratio 16 : 25 and their altitudes are in the ratio 5 : 4. Find the ratio of their corresponding base?

Eg2. Find the area of given triangle.

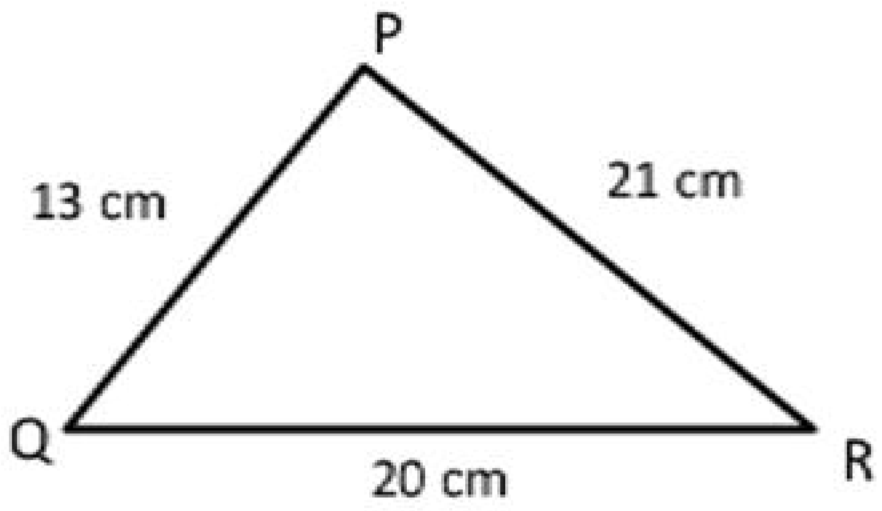
(2)

$$Area\ of\ \Delta = \sqrt{s\ s-a\ \ s-b\ \ s-c}$$

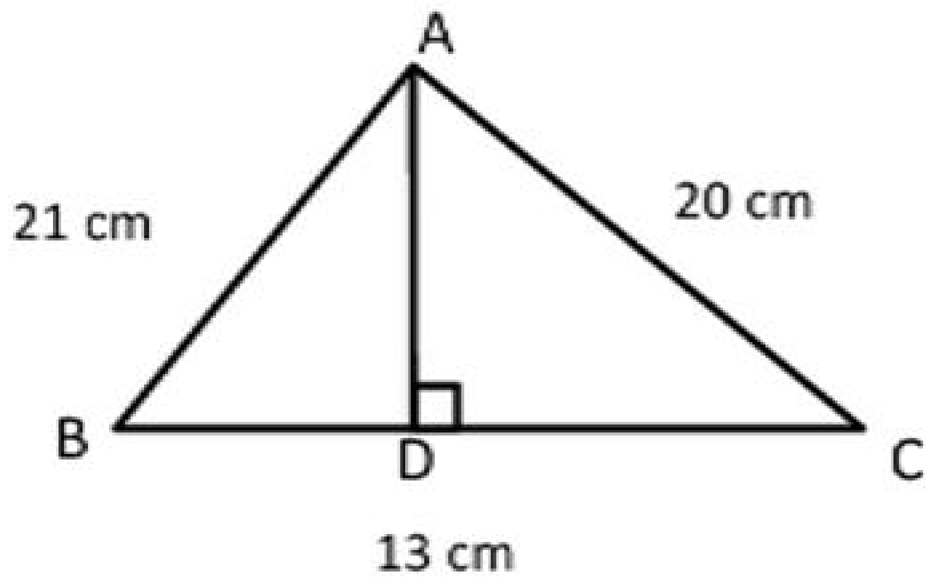
$s \rightarrow semi-perimeter$

$$s = \frac{a+b+c}{2}$$

$a, b \,\&\, c$ are sides of Δ.

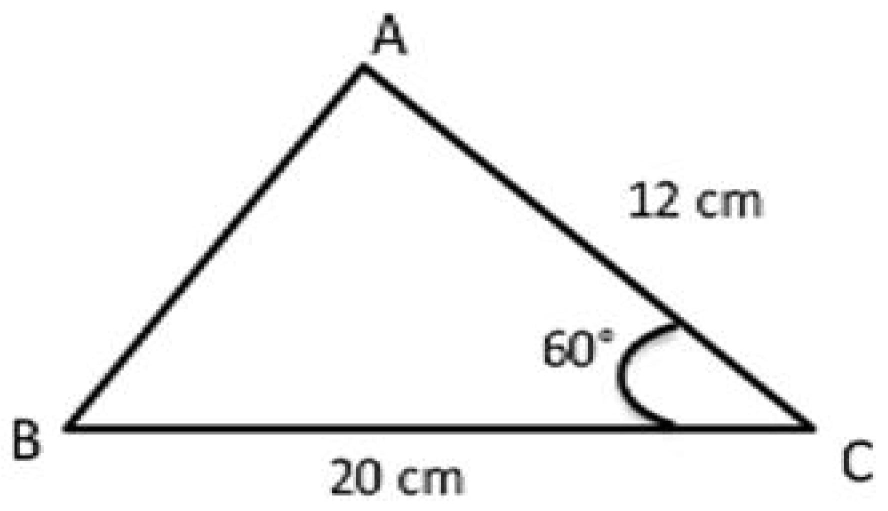


Q3. Find AD = ??



(3) $$Area\ of\ \Delta = \frac{1}{2}ab\sin C$$

Eg3. Find area of triangle.



Q4. If 2 sides of a triangle are 12 cm and 20 cm, what can be the maximum area of triangle?

If 2 sides of Δ are given then maximum area is always of a Right Angled Triangle.

If a, b are 2 sides of a Δ :

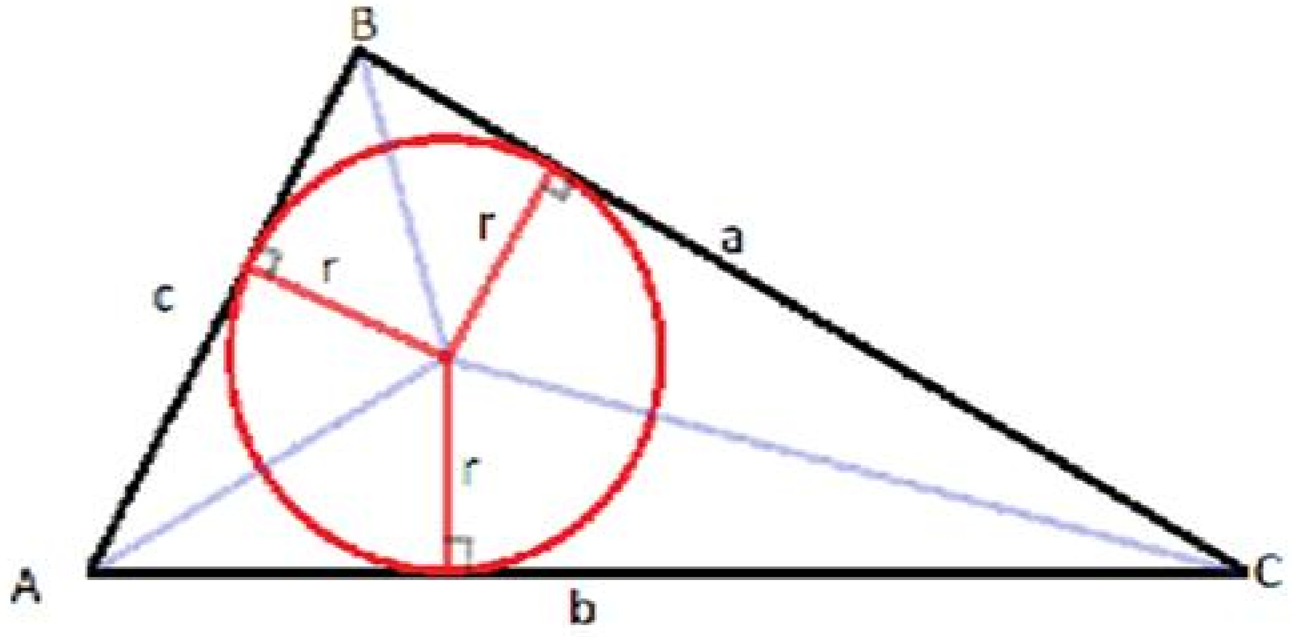
$$Max\ Area = \frac{1}{2}ab$$

(4) $\mathbf{Area = r \cdot s}$

where :

r - inradius

s - semi-perimeter



Q5. Find the in-radius of triangle whose sides are 13 cm, 21 cm and 20 cm.

$r \cdot s = \text{Area}$

$$r = \frac{Area}{s}$$

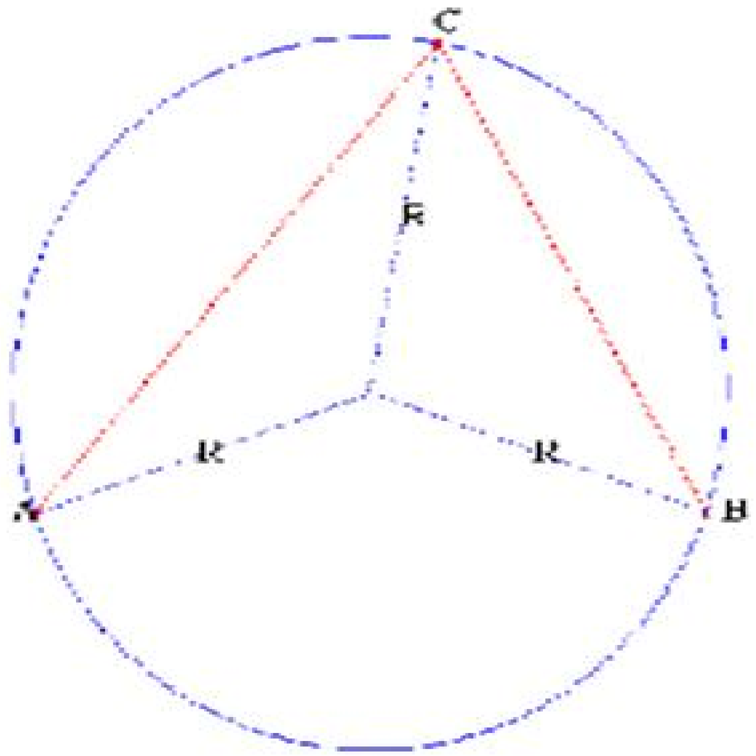
Inradius (r)

For any Δ $= \dfrac{Area}{s}$

Equilateral Δ $= \dfrac{Side}{2\sqrt{3}}$

Right angle Δ $= \dfrac{Base + Perpendicular - Hypotenuse}{2}$

CIRCUMRADIUS



The circumradius is the radius of the circumscribed circle of that polygon.

(5) $$Area\ of\ \Delta = \frac{a \cdot b \cdot c}{4R}$$

where, a, b, c are sides of triangle.

R → Circum-radius

Eg4. Find the circum-radius of triangle whose sides are 13 cm, 21 cm and 20 cm.

$$Area\ of\ \Delta = \frac{a \cdot b \cdot c}{4R}$$

$$R = \frac{a \cdot b \cdot c}{Area\ of\ \Delta}$$

Circumradius (R)

For any Δ $= \dfrac{a \cdot b \cdot c}{Area\ of\ \Delta}$

Equilateral Δ $= \dfrac{Side}{\sqrt{3}}$

Right angle Δ $= \dfrac{Hypotenuse}{2}$

AREA OF TRIANGLE (For any Δ)

(1) $Area = \frac{1}{2} \times Base \times Height$

(2) $Area\ of\ \Delta = \sqrt{s\ s-a\ \ s-b\ \ s-c}$

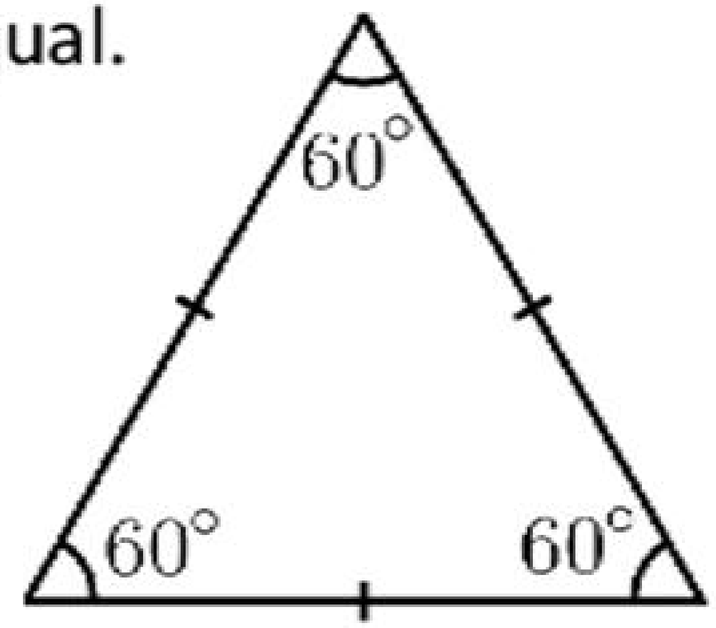
(3) $Area\ of\ \Delta = \frac{1}{2} ab\sin C$

(4) $Area\ of\ \Delta = r \cdot s$

(5) $Area\ of\ \Delta = \frac{a \cdot b \cdot c}{4R}$

EQUILATERAL TRIANGLE

An **equilateral triangle** is a **triangle** in which all three sides are equal.



$$Height\ of\ equilateral\ \Delta = \frac{\sqrt{3}}{2} \times S$$

$$Area\ of\ equilateral\ \Delta = \frac{\sqrt{3}}{4} \times S^2$$

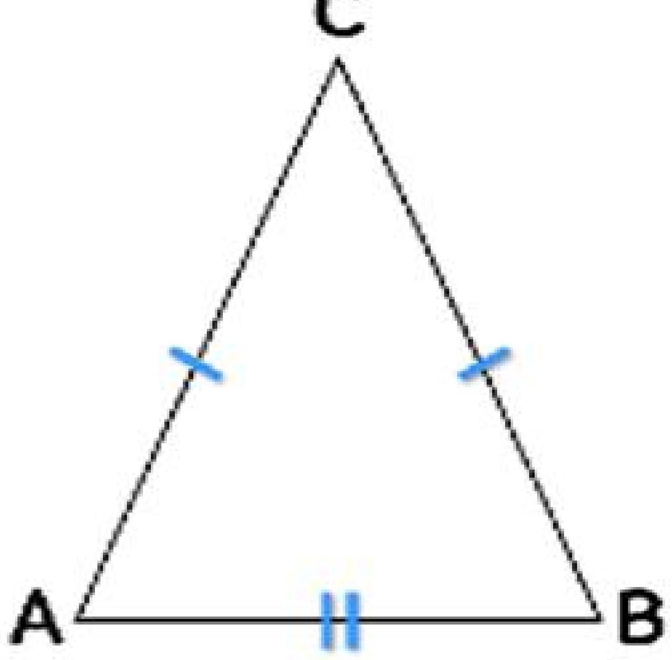
Eg. If height of equilateral triangle = 12 cm.
Find area of equilateral triangle.

ISOSCELES TRIANGLE

An **isosceles triangle** is a **triangle** that has two sides of equal length.

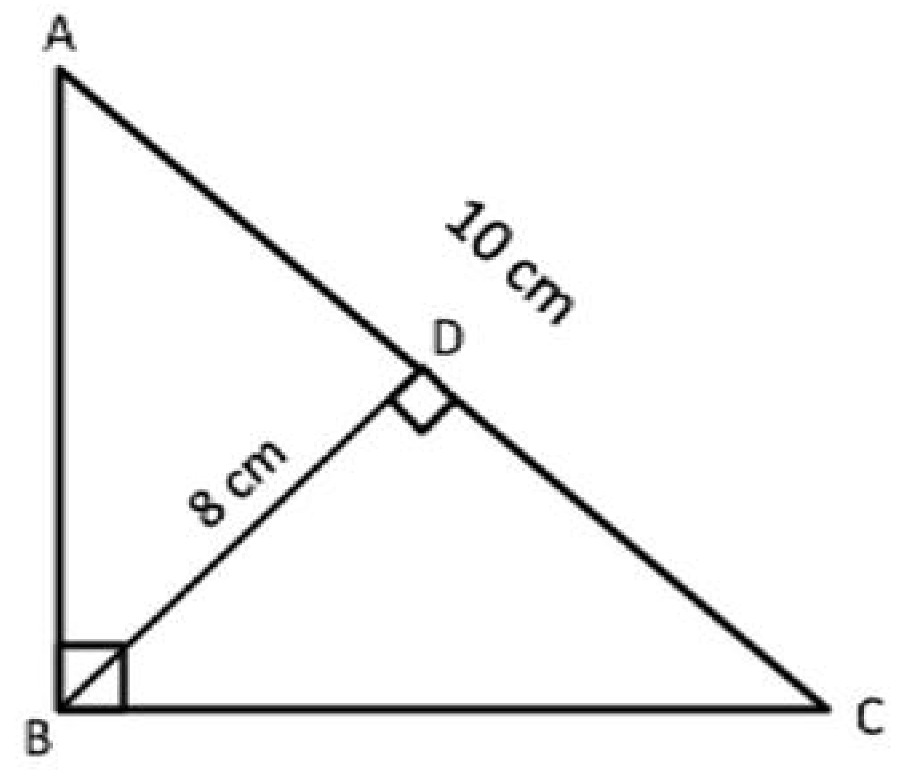
$$Area\ of\ isosceles\ \Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$$

Where, b is base of isosceles Δ.
and a is length of equal sides.



Eg. In a ΔABC, AC = 10 cm ; BD = 8 cm

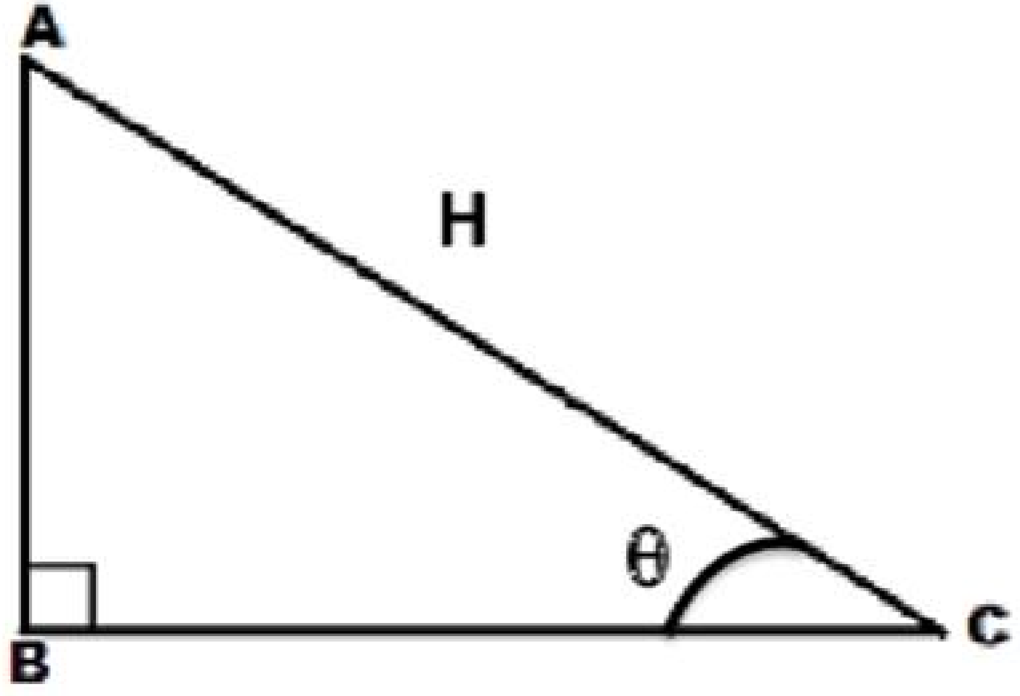
Find area of ΔABC.



RIGHT ANGLE TRIANGLE

$$\text{Area of right angle } \Delta = \frac{H^2}{4}\sin 2\theta$$

Where, H → Hypotenuse
and, θ → one of the acute angle of right angle triangle.



Eg. If hypotenuse of a right angle Δ is 10 cm. What can be its maximum area?

PRACTICE QUESTIONS

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Q1. If ΔFGH is isosceles and $FG < 3$ cm, $GH = 8$ cm, then of the following, the true relation is :

(a) $GH = FH$ (b) $GF = GH$

(c) $FH > GH$ (d) $GH < GF$

Ans. (a)

Q2. If in ΔABC, $\angle C$ is obtuse and length of sides BC and AC are respectively 9 cm. and 7 cm., the minimum possible length of AB is (where length of AB is an integer)

(a) 11 cm (b) 12 cm

(c) 14 cm (d) 16 cm

Ans. (b)

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Q3. $\angle A$, $\angle B$, $\angle C$ are three angle of a traingle. If $\angle A - \angle B = 15°$, $\angle B - \angle C = 30°$, then $\angle A$, $\angle B$ and $\angle C$ are :

(a) $80°, 60°, 40°$ (b) $70°, 50°, 60°$
(c) $80°, 65°, 35°$ (d) $80°, 55°, 45°$

Ans. (c)

Q4. In a ΔPQR, the sum of the exterior angles of Q and R will be equal to:

(a) $180^\circ - \angle QPR$
(b) $180^\circ + \angle QPR$
(c) $180^\circ - 2\angle QPR$
(d) $180^\circ + 2\angle QPR$

Ans. (b)

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Q5. By decreasing $15°$ of each angle of a triangle, the ratio of their angles is $2 : 3 : 5$, the measure of greatest angle is :

(a) $\frac{11}{24}\pi$

(b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$

(d) $\frac{5}{24}\pi$

Ans. (a)

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Q6. In an Isosceles ΔABC, $AB = AC = 17$ cm. D is a point on side BC such that $CD = 4$cm and AD 15 cm, then find length of $BD = ?$

(a) 16 (b) 8

(c) 4 (d) 15

Ans. (a)

Q7. Let ABC be an equilateral triangle. If the side BC is produced to the point D so that BC = 2CD, then AD^2 is equal to :

(a) $3CD^2$ (b) $4CD^2$

(c) $5CD^2$ (d) $7CD^2$

Ans. (d)

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Q8. In a ΔABC, AC = 20 cm and BC = 10 cm. If area of triangle is 80 cm^2, then find the length of AB :

(a) $3\sqrt{39}$ (b) $2\sqrt{78}$

(c) $2\sqrt{52}$ (d) $2\sqrt{65}$

Ans. (d)

Q9. In ΔABC, $AB = AC$, D is any point on side BC, find $AB^2 - AD^2$.

(a) $CD \times AB$ (b) $BD \times CD$

(c) $BD \times AB$ (d) None of these

Ans. (b)

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Q10. If A is the area of a right angled triangle and b is one of the sides containing the right angle, then what is the length of the altitude on the hypotenuse ?

(a) $\dfrac{2Ab}{\sqrt{b^4 + 4A^2}}$

(b) $\dfrac{2A^2b}{\sqrt{b^4 + 4A^2}}$

(c) $\dfrac{2Ab^2}{\sqrt{b^4 + 4A^2}}$

(d) $\dfrac{2A^2b^2}{\sqrt{b^4 + A^2}}$

Ans. (a)

Q11. In a right angled triangle, the product of two sides is equal to half of the square of the third side i.e. hypotenuse. One of the acute angle must be :

(a) 60° (b) 30°
(c) 45° (d) 15°

Ans. (c)

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Q12. A point D is taken from the side BC of a right-angled triangle ABC, where AB is hypotenuse then

(a) $AB^2 + CD^2 = BC^2 + AD^2$
(b) $CD^2 + BD^2 = 2AD^2$
(c) $AB^2 + AC^2 = 2AD^2$
(d) $AB^2 = AD^2 + BD^2$

Ans. (a)

Q13. In a right angled ΔABC, $\angle B = 90°$ if P and Q are two points on sides AB and BC respectively then–

(a) $AQ^2 + CP^2 = AC^2 + PQ^2$

(b) $AQ^2 + CP^2 = \frac{1}{2}(AC^2 + PQ^2)$

(c) $AQ^2 + CP^2 = 2(AC^2 + PQ^2)$

(d) $AQ^2 + AC^2 = CP^2 + PQ^2$

Ans. (a)

Q14. The length of radius of a circumcircle of a triangle having sides 3 cm, 4 cm and 5 cm is :

(a) 2 cm
(b) 2.5 cm
(c) 3 cm
(d) 1.5 cm

Ans. (b)

Q15. If the length of the sides of a triangle are in the ratio 4 : 5 : 6 and the inradius of the triangle is 3 cm, then the altitude of the triangle corresponding to the largest side as base is :

(a) 7.5 cm (b) 6 cm

(c) 10 cm (d) 8 cm

Ans. (a)

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Q16. The three sides of a triangle are 15, 25, x units which one of the following is correct.

(a) $10 < x < 40$ (b) $20 < x < 40$

(c) $30 < x < 40$ (d) $10 < x < 30$

Ans. (a)

Q17. Two sides of a Δ are 13 cm and 5 cm. How many different values of 3rd side are possible where the length of 3rd side is integer.

(a) 8 (b) 9

(c) 10 (d) 10

Ans. (b)

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Q18. Perimeter of a Δ is 12 cm. How many different Δ (traingle) can be formed.

(a) 6 (b) 5

(c) 4 (d) 3

Ans. (d)

Q19. If ΔFGH is isosceles and $FG < 3$ cm, $GH = 8$ cm, then of the following, the true relation is :

(a) $GH = FH$

(b) $GF = GH$

(c) $FH > GH$

(d) $GH < GF$

Ans. (a)

Q20. If the measure of the sides of triangle are $(x^2 - 1)$, $(x^2 + 1)$ & $2x$ cm, then the triangle will be :

(a) Equilateral (b) Acute-Angled

(c) Isosceles (d) Right angle

Ans. (d)

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Q21. If the sides of a triangle are in the ratio $3 : 1\frac{1}{4} : 3\frac{1}{4}$, then the triangle is

(a) Right angle triangle
(b) Obtuse angle triangle
(c) Equilateral triangle
(d) Acute angle triangle

Ans. (a)

Q22. The sides of a triangle are 14 cm, 12 cm, 8 cm respectively the triangle is

(a) Right angle triangle
(b) Obtuse angle triangle
(c) Equilateral triangle
(d) Acute angle triangle

Ans. (d)

Q23. If the three angles of a triangle are : $(k + 15)^\circ$, $\left(\frac{2k}{3}+30\right)^\circ$ and $\left(\frac{6k}{5}+6\right)^\circ$, then the triangle is :

(a) Scalene
(b) Equilateral
(c) Right angle
(d) Isosceles

Ans. (b)

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Q24. In a ΔABC, Median AD is perpendicular to side AB. Find the value of $\dfrac{\tan A}{\tan B}$

(a) 1 (b) -1

(c) 2 (d) -2

Ans. (d)

Q25. If an isoscles ΔPQR has sides $PR = QR$ and $PQ^2 = 2PR^2$ then $\angle R = ?$

(a) 60° (b) 30°

(c) 45° (d) 90°

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Ans. (d)