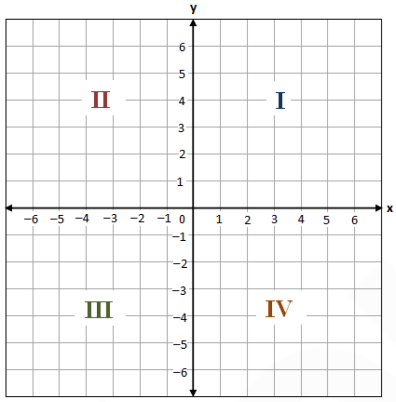
Coordinate Geometry

Co-ordinate Geometry

Co-ordinate Plane: A coordinate plane is a 2-D plane formed by the intersection of a vertical line called y-axis and a horizontal line called x-axis. These are perpendicular lines that intersect each other at zero, and this point is called the origin O (0, 0). The axes cut the coordinate plane into four equal sections, and each section is known as quadrant.



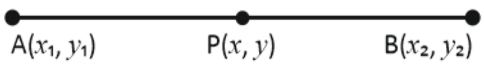
The two-dimensional plane is called the Cartesian plane, or the coordinate plane and the axes are called the coordinate axes or x-axis and y-axis. The given plane has four equal divisions by origin called quadrants.

- The horizontal line towards the right of the origin (denoted by O) is positive x-axis.
- The horizontal line towards the left of the origin is negative x-axis.
- The vertical line above the origin is positive y-axis.
- The vertical line below the origin is negative y-axis.
- The x-coordinate or abscissa of a point is its perpendicular distance from the y-axis measured along the x-axis.
- The y-coordinate or ordinate of a point is its perpendicular distance from the x-axis measured along the y-axis.
- In stating the coordinates of a point in the coordinate plane, the x-coordinate comes first, and then comes the y-coordinate. We place the coordinates in brackets as (x, y).

Distance between two points (x_1, y_1), (x_2, y_2):

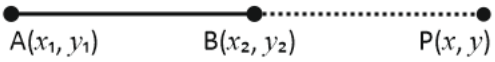
$$\text{Distance} = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Section Formula: The co-ordinates of a point P(x ,y), dividing the line segment joining the two points $A(x_1, y_1)$ and B (x_2, y_2) internally in the ratio m : n are given by



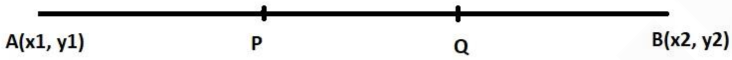
$$x = \frac{m.x_2 + n.x_1}{m+n}, y = \frac{m.y_2 + n.y_1}{m+n}$$

The co-ordinate of the point P(x, y), dividing the line segment joining the two points A(x_1, y_1) and B (x_2, y_2) externally in the ratio m : n are given by



$$x = \frac{m.x_2 - n.x_1}{m - n}, y = \frac{m.y_2 - n.y_1}{m - n}$$

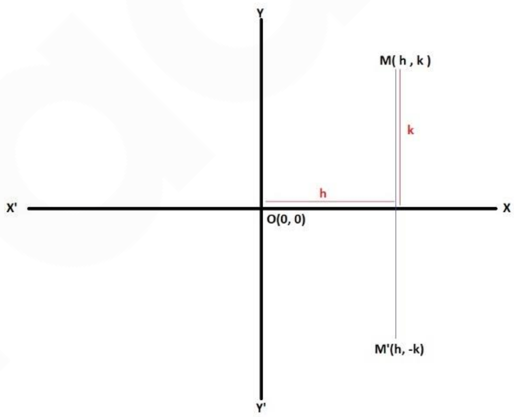
Trisection Formula of a line segment: If points P and Q which lie on line segment AB divide it into three equal parts that means, if **AP = PQ = QB** then the points P and Q are called **Points of Trisection** of AB.



Here, *P* divides AB in the ratio 2 : 1 and Q divides AB in the ratio 1 : 2. Now use the section formula for finding the coordinates of P and Q.

Reflection of a point in the axes and origin:

1. **Reflection in the X-axis:** Here, x-axis represents the plain mirror. When point M is reflected in x-axis, the image M’ is formed in the horizontally opposite quadrant whose co-ordinates are (h, -k). Thus, when a point is reflected in x-axis, then the x-co-ordinate remains same, but the y co-ordinate becomes negative.



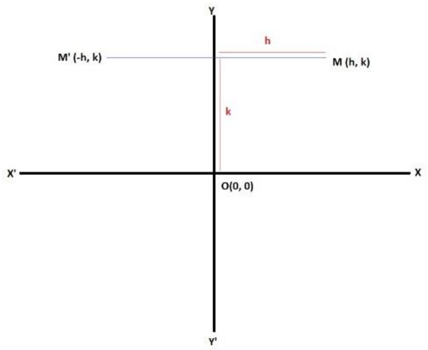
Thus, the image of point M (h, k) is M'(h, -k).

Rule:

(i) Retain the abscissa i.e., x-coordinate.

(ii) Change the sign of ordinate i.e., y-coordinate.

2. **Reflection in the Y-axis:** Here, y-axis represents the plane mirror. when point M is reflected in y-axis, the image M' is formed in the vertically opposite quadrant whose co-ordinates are (-h, k). Thus, when a point is reflected in y-axis, then the y-co-ordinate remains same and then x-co-ordinate become negative.



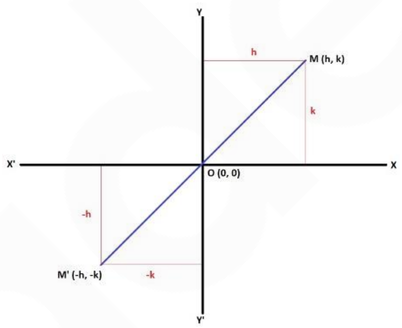
Thus, the image of M (h, k) is M'(-h, k).

Rule:

(i) Change the sign of abscissa i.e., x-coordinate.

(ii) Retain the ordinate i.e., y-coordinate.

3. **Reflection through Origin:** When a point is reflected in origin, both x-co-ordinate and y-co-ordinate change. Thus, the reflection of M (h, k) is M′ (-h, -k) in the origin.



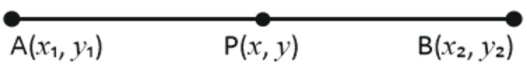
Rule:

(i) Change the sign of abscissa i.e., x-coordinate.

(ii) Change the sign of ordinate i.e., y-coordinate.

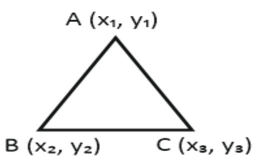
The co-ordinates of midpoint of the line formed by A(x_1, y_1), B(x_2, y_2):

Here, P point divides the line segment AB into ratio 1:1. Thus, m = n = 1.



$$\mathbf{P(x, y)} = \left(\frac{\mathbf{x_1 + x_2}}{\mathbf{2}}, \frac{\mathbf{y_1 + y_2}}{\mathbf{2}}\right)$$

Area of triangle whose coordinates are A(x_1, y_1), B(x_2, y_2), C(x_3, y_3):



$$\text{Area of the Triangle ABC} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Collinear points: Three or more points that lie on a same straight line are called collinear points. There are two methods to find if three points are collinear:

(i) **Slope formula method:** Three or more points are collinear, if slope of any two pairs of points is same. Let three points be A, B and C, three pairs of points can be formed as AB, BC and AC.

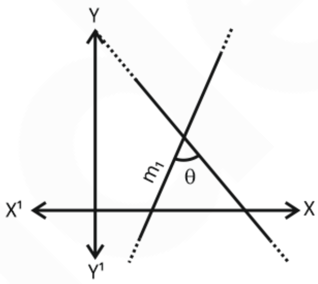
If slope of AB = slope of BC = slope of AC, then A, B and C are collinear points.

(ii) **Area of triangle method**: Three points are collinear if the value of area of triangle formed by the three points is zero.

Slope of a line: If a line joining two points A(x_1, y_1) and B (x_2, y_2) then the slope of the line joining the two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = -\frac{\text{coefficient of x}}{\text{coefficient of y}}$$

Angle between two lines: If two lines having slopes m_1 and m_2 then angle between the two lines is given by:



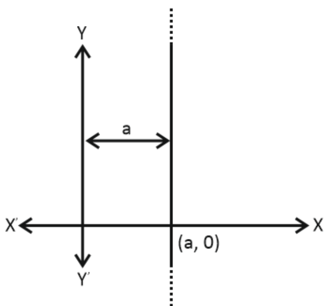
$\tan\theta = \pm\frac{m_2 - m_1}{1 + m_1 m_2}$ where, m_1, m_2 = slope of the lines

$\Rightarrow \theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$

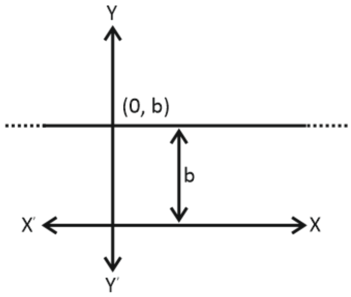
Note: If lines are parallel to each other then tanθ = 0°

If lines are perpendicular to each other then cotθ = 0°

Equation of line parallel to y-axis: The equation of a straight line to the x -axis and at a distance a from it, is given by **X = a.**



Equation of line parallel to x-axis: The equation of a straight line parallel to the y-axis and at a distance a from is given by **Y = b**.



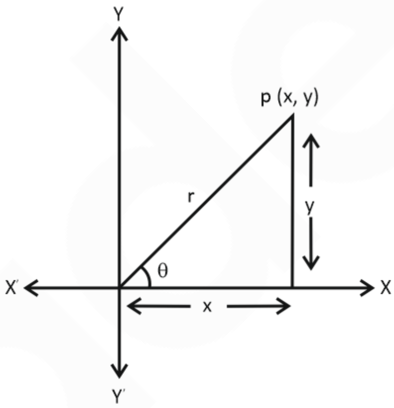
Different types of Equations of line:

1. Normal equation of the line:

ax + by + c = 0

Note: Area of the triangle formed by co-ordinate axes and the line ax + by + c = 0 is given by $\frac{c^2}{2ab}$.

2. Polar Form of an equation:



$$r = \sqrt{x^2 + y^2}$$

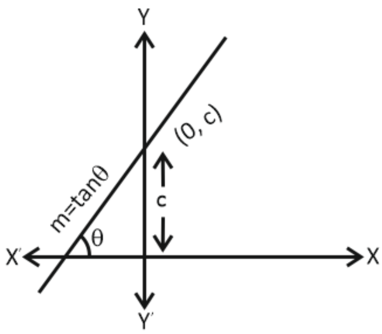
$$\sin\theta = \frac{y}{r} \Rightarrow y = r.\sin\theta$$

$$\cos\theta = \frac{x}{r} \Rightarrow x = r.\cos\theta$$

Co-ordinates of points in Polar Form: $(\mathbf{rSin}\,\boldsymbol{\theta}, \mathbf{rCos}\,\boldsymbol{\theta})$

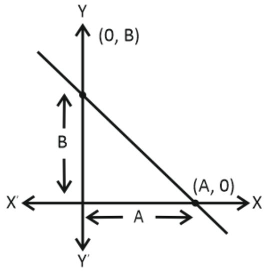
3. Slope – Intercept Form:

y = mx + c Where, m = slope of the line & c = intercept on Y-axis

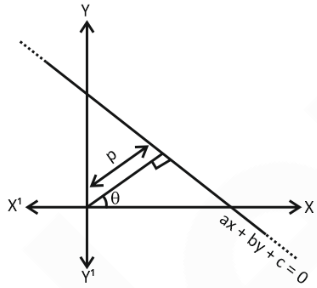


4. Intercept Form:

$\frac{x}{A} + \frac{y}{B} = 1$, Where, A & B are x-intercept & y-intercept respectively.



5. Trigonometric form of equation of line, ax + by + c = 0

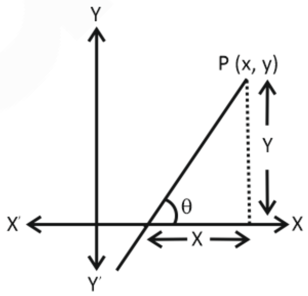


x cosθ + y sinθ = p,

Where, $\cos\theta = -\frac{a}{\sqrt{a^2+b^2}}$, $\sin\theta = -\frac{b}{\left(\sqrt{a^2+b^2}\right)}$ & $p = \frac{c}{\sqrt{a^2+b^2}}$

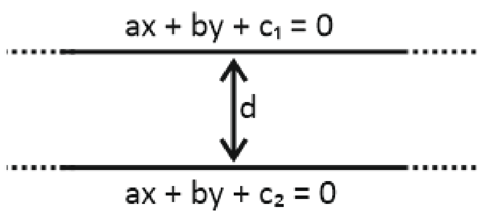
6. Equation of line passing through point (x_1, y_1) & has a slope "m":

$$y - y_1 = m(x - x_1)$$



7. Equation of two lines parallel to each other:

Here, $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ represent the equations of two lines parallel to each other. "d" represent the distance between the two parallel lines.



Note: Here, coefficient of x & y will be same.

8. Equation of two lines perpendicular to each other:

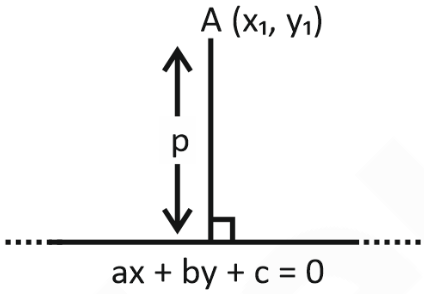
$ax + by + c_1 = 0$

$bx - ay + c_2 = 0$

Note: Here, coefficient of x & y are opposite & in one equation there is negative sign.

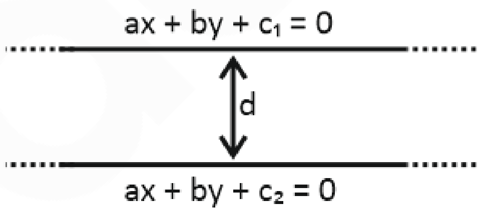
Note: If $\mathbf{m_1, m_2}$ are slopes of two perpendicular lines then $\mathbf{m_1.m_2 = -1.}$

The Distance of a Point from a Line: The length of perpendicular from a point $A(x_1, y_1)$ to a line with equation $ax + by + c = 0$ is:



$$\mathbf{p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}}$$

The Distance between two parallel lines: When two parallel straight lines with equations $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$, then the distance between them is given by:



$$\mathbf{d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}}$$