Pipes & Cistern

PIPE & CISTERN

A pipe is used to fill or empty a tank or cistern. If the pipe empties the tank or cistern, it is called an Outlet pipe and if the pipe fills the cistern it is termed as Inlet Pipe.
The approach to solve Pipes & Cisterns questions is same as the questions on time and work. In this case, the filling rate or capacity of the pipes are given and then questions are asked about the time it will take to fill the tank completely or partially or about the capacity of the tank.

Basic Concepts:

1. All the pipes are either inlet pipes or outlet pipes:

Case 1: If an Inlet pipe fills the tank in "a" hours then the part of work done by Inlet pipe in one hour will be $\frac{1}{a}$. Another Inlet pipe can fill the tank in "b" hours then the part of work done by Inlet pipe in one hour will be $\frac{1}{b}$. Thus, if both the inlet pipes are opened simultaneously then the part of work done by both the pipes in one hour be $\left(\frac{1}{a}+\frac{1}{b}\right)=\frac{a+b}{ab}$
And time taken by both the Inlet pipes to fill the tank = $\frac{ab}{a+b}$

Example 1: If an Inlet pipe can fill a tank in 20 hr. and another inlet pipe can fill the tank in 30 hr. then find the find the total time in which both the pipes together fill the tank.

Solution:
Since, Inlet pipe can fill the tank in 20 hrs.
So, work done by Inlet pipe in 1 hour = $\frac{1}{20}\ unit$
Another Inlet pipe can fill the tank in 30 hrs.
So, work done by another Inlet pipe in 1 hour = $\frac{1}{30}\ unit$
Thus, Total work done by bot the pipes in one hour = $\frac{1}{20}+\frac{1}{30}=\frac{30+20}{20\times30}=\frac{50}{600}=\frac{1}{12}\ unit$
So, Total time taken by both the pipes to fill the tank = $\frac{1}{\frac{1}{12}}=12\ hours.$

Case 2: If an Outlet pipe empties a tank in "a" hours and another Outlet pipe empties the same tank in "b" hours. Thus, if both the outlet pipes are opened simultaneously then the part of work done by both the pipes in one hour be $\left(\frac{1}{a}+\frac{1}{b}\right)=\left(\frac{a+b}{ab}\right)$ (Here "-' sign means emptying of the tank only)
Total time taken by both the pipes to empty the tank = $\frac{ab}{a+b}$

Example 2: If an Outlet pipe can empty a tank in 24 hr. and another Outlet pipe can fill the tank in 36 hr. then find the total time in which both the pipes together empty the tank.

Solution:
Part of work done by 1st Outlet pipe in one hour = $\frac{1}{24}\ unit$
Also, Part of work done by 2nd Outlet pipe in one hour = $\frac{1}{36}\ unit$
Total work done by both the Outlet pipes in one hour = $\frac{1}{24}+\frac{1}{30}=\frac{24+30}{24\times30}=\frac{54}{24\times30}=\frac{5}{72}\ unit$
Thus, Time taken by both the Outlet pipes to empty the tank = $\frac{72}{5}hr.$

Note: We use "+" sign if pipes are filling the tank and "-" sign when pipes are emptying the tank.

2. Some pipes are Inlet pipes and some are Outlet Pipes:

If an Inlet pipe fills the tank in "a" hours then the part of work done by Inlet pipe in one hour will be $\frac{1}{a}$. Similarly, if an Outlet pipe fills the tank in "b" hours then the part of work done by Outlet pipe in one hour will be $\frac{1}{b}$. Thus, If the Inlet pipe fills the tank in "a" hours and other Outlet pipe empties it "b" hours then the part of work done by both the pipes in one hour when open together will be $\left(\frac{1}{a}-\frac{1}{b}\right)$.
So, total time taken by both the pipes to fill/empty the tank = $\frac{ab}{a-b}$

Note: Whether the inlet pipes and outlet pipes are filling the tank or emptying the tank, depends upon the values of time taken by the pipes individually. In above given info, if a < b then pipes will empty the tank and if a > b then pipes will fill the tank.

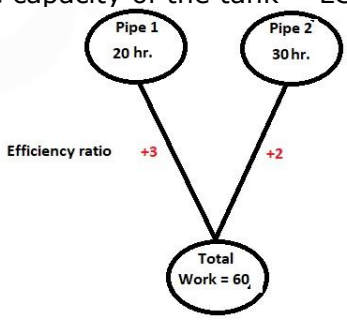
Example 3: If an Inlet pipe can fill a tank in 20 hr. and an Outlet pipe can empty the tank in 30 hr. then find the total time in which both the pipes together fill/empty the tank.

Solution:
Since, Inlet pipe can fill the tank in 20 hrs.
So, work done by Inlet pipe in 1 hour = $\frac{1}{20}\ unit$
Similarly, Outlet pipe can empty the tank in 30 hrs.
So, work done by Outlet pipe in 1 hour = $(-\frac{1}{30})\ unit$ (Negative sign as it is an outlet pipe)
Thus, Total work done by both the pipes in one hour = $\frac{1}{20}-\frac{1}{30}=\frac{30-20}{20\times30}=\frac{10}{600}=\frac{1}{60}\ unit$
Since, a < b:
So, Total time taken by both the pipes to fill the tank = $\frac{1}{\frac{1}{60}}=60\ hours.$

Example 4: If two Inlet pipes can fill a tank in 20 hr. and 25 hr. respectively and an Outlet pipe can empty the same tank in 30 hr. then find the total time in which all the pipes together fill/empty the tank.

Solution:
Work done by 1st Inlet pipe in one hour = $+\frac{1}{20}\ unit$
Work done by 2nd Inlet pipe in one hour = $+\frac{1}{25}\ unit$
And, Work done by 3rd Outlet pipe in one hour = $-\frac{1}{30}\ unit$
("+" sign as first two pipes are inlet pipes and "-" sign as the third pipe is outlet pipe)
Total work done by all the three pipes in one hour = $\frac{1}{20}+\frac{1}{25}-\frac{1}{30}$
$=\frac{1}{20}+\frac{1}{25}-\frac{1}{30}=\frac{25\times30+20\times30-20\times25}{20\times25\times30}=\frac{37}{300}$ unit
Thus, Total time taken by all three pipes to fill the tank = $\frac{1}{\frac{37}{300}}=\frac{300}{37}\ hr.$

Here, same questions on Pipes and Cisterns can be solved by LCM Method also. The Method is as following;
Let's take Example 1,
Given that, The Inlet pipe can fill the tank in 20 hr. and another inlet pipe can fill the tank in 30 hr.
The total amount of work or the capacity of the tank = LCM (20, 30) = 60 unit



Efficiency or filling rate of 1st Inlet pipe = $\frac{60}{20}=+3$
Efficiency or filling rate of 2nd Inlet pipe = $\frac{60}{30}=+2$
Thus, Total time taken by both the inlet pipes to fill the tank = $\frac{60}{3+2}=12\ hr.$
Similarly, Example 3,
Given that, The Inlet pipe can fill the tank in 20 hr. and another Outlet pipe can empty the tank in 30 hr.
The total amount of work or the capacity of the tank = LCM (20, 30) = 60 unit
Efficiency or filling rate of 1st Inlet pipe = $\frac{60}{20}=3$
Efficiency or emptying rate of 2nd outlet pipe = $\frac{60}{30}=-2$
Thus, Total time taken by both the pipes to fill the tank = $\frac{60}{3-2}=60\ hr.$