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Sahi Prep Hai Toh Life Set Hai

TRIGONOMETRIC IDENTITIES

Trigonometric Identities

18 Question

19 - 20 Homework

Today (21 - 40)

Eg19. If $\tan^2\theta = 1 - a^2$

Prove that : $\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (2 - a^2)^{3/2}$

Eg20. If $\operatorname{cosec}\theta - \sin\theta = l$ & $\sec\theta - \cos\theta = m$

Find the value of $l^2m^2\,(l^2 + m^2 + 3)$

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Eg21. If $\sin\theta + \sin^2\theta = 1$

$\sin^1\theta = \cos^2\theta$

Find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta$

$(\sin^6\theta + 3\sin^5\theta + 3\sin^4\theta + \sin^3\theta) + 2\sin^2\theta + 2\sin\theta$

$(\sin^2\theta + \sin\theta)^3 + 2(\sin^2\theta + \sin\theta)$

$1 + 2\cdot 1$

$= \underline{\underline{3}}$ ✓

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$= 1 \quad 3 \quad 3 \quad 1$$

→ You get a 'hint', a cube formula is used

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Eg22. If $\dfrac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}=2\dfrac{51}{79}$

Find $\sin\theta$.

$$\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}=\frac{209}{79}$$

$$79\sec\theta+79\tan\theta=209\sec\theta-209\tan\theta$$

$$288\tan\theta=130\sec\theta$$

$$288\frac{\sin\theta}{\cancel{\cos\theta}}=130\cdot\frac{1}{\cancel{\cos\theta}}$$

$$\sin\theta=\frac{\cancel{130}\ 65}{\cancel{288}\ 144}$$

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Eg23. Let $0 < \theta < 90°$ and $100\theta = 90°$. If $\alpha = \Pi_{n-1}^{99} \cot n\theta$

then, which one of the following is correct?

(a) $\alpha = 1$ (b) $\alpha = 0$

(c) $\alpha > 1$ (d) $0 < \alpha < 1$

$\cot\theta \cot 2\theta$ ----- $\cot 50\theta$ -- -- $\cot 98\theta \cot 99\theta$

$\cot\theta \cot 2\theta$ ---- $\cot 50\theta$ ----- $\cot(100\theta - 2\theta) \cot(100\theta - \theta)$

$\cancel{\cot\theta} \cancel{\cot 2\theta}$ --- $\cot 50\theta$ --- $\cancel{\tan 2\theta} \cancel{\tan\theta}$

$\cot 45 = 1$

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Ans. (a)

$\sum \rightarrow$ Sum $\quad \sum_{i=1}^{10} \sin i \rightarrow \sin 1 + \sin 2 + \sin 3 + \cdots \sin 10$

$\prod \rightarrow$ Product $\quad \prod_{i=1}^{10} \sin i \rightarrow (\sin 1)(\sin 2)(\sin 3)\cdots \sin 10$

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Eg24. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$

Find the value of $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta \longrightarrow 4$

$$\sin\theta + \sin^3\theta = 1 - \sin^2\theta$$

$$\sin\theta(1+\sin^2\theta) = \cos^2\theta$$

Squaring both sides

$$\sin^2\theta\left[1+1-\cos^2\theta\right]^2 = \cos^4\theta$$

$$(1-\cos^2\theta)(2-\cos^2\theta)^2 = \cos^4\theta$$

$$(1-\cos^2\theta)(4+\cos^4\theta-4\cos^2\theta) = \cos^4\theta$$

$$4+\cancel{\cos^4\theta}-4\cos^2\theta-4\cos^2\theta-\cos^6\theta+4\cos^4\theta = \cancel{\cos^4\theta}$$

$$\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$$

Eg25. If $\tan\alpha = n\tan\beta$ and $\sin\alpha = m\sin\beta$ Then $\cos^2\alpha$ is

(a) $\frac{m^2}{n^2+1}$ (b) $\frac{m^2}{n^2}$

(c) $\frac{m^2-1}{n^2-1}$ (d) $\frac{m^2+1}{n^2+1}$

Ist Putting values

$\alpha = 60^\circ \quad \beta = 30$

$\sqrt{3} = \frac{n}{\sqrt{3}}$ $\quad \frac{\sqrt{3}}{2} = \frac{m}{2}$

$n = 3$ $\quad m = \sqrt{3}$

$\cos^2 60^\circ \rightarrow 1/4$

(a) $\rightarrow 3/10$ ✗

(b) $\rightarrow 3/9$ ✗

$\boxed{(c)} \rightarrow 2/8 \rightarrow 1/4$ ✓

(d) $\frac{4}{10}$ ✗

IInd Sol^{n}

$\tan\alpha = n\tan\beta$ $\qquad$ $\sin\alpha = m\sin\beta$ $\qquad$ $\cos^2\alpha$

$$\frac{1}{\cot\alpha} = \frac{n}{\cot\beta} \qquad \frac{1}{\operatorname{cosec}\alpha} = \frac{m}{\operatorname{cosec}\beta}$$

$$\cot\beta = n\cot\alpha \qquad \operatorname{cosec}\beta = m\operatorname{cosec}\alpha$$

$$\operatorname{cosec}^2\beta - \cot^2\beta = 1$$

$$m^2\operatorname{cosec}^2\alpha - n^2\cot^2\alpha = 1$$

$$\frac{m^2}{\sin^2\alpha} - \frac{n^2\cos^2\alpha}{\sin^2\alpha} = 1 \longrightarrow m^2 - n^2\cos^2\alpha = \sin^2\alpha$$

$$m^2 - n^2\cos^2\alpha = 1 - \cos^2\alpha$$

$$m^2 - 1 = (n^2 - 1)\cos^2\alpha$$

$$\boxed{\cos^2\alpha = \frac{m^2-1}{n^2-1}}$$

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$\tan\alpha = n\tan\beta$ $\sin\alpha = m\sin\beta$ $\cos^2\alpha$

[11]

$$\frac{\sin\alpha}{\cos\alpha} = n\frac{\sin\beta}{\cos\beta}$$

$$\frac{m\cancel{\sin\beta}}{\cos\alpha} = \frac{n\cancel{\sin\beta}}{\cos\beta}$$

$$\cos\beta = \frac{n\cos\alpha}{m}$$

$$\sin^2\beta + \cos^2\beta = 1$$

$$\frac{\sin^2\alpha}{m^2} + \frac{n^2}{m^2}\cos^2\alpha = 1$$

$$\sin^2\alpha + n^2\cos^2\alpha = m^2$$

$$1 - \cos^2\alpha + n^2\cos^2\alpha = m^2$$

$$\cos^2\alpha(n^2-1) = m^2-1$$

$$\boxed{\cos^2\alpha = \frac{m^2-1}{n^2-1}}$$

Ans. (c)

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Eg26. If $\frac{\cos\alpha}{\cos\beta} = a$ and $\frac{\sin\alpha}{\sin\beta} = b$ then the value of $\sin^2\beta$ terms of a & b

6asec

(a) $\frac{a^2+1}{a^2-b^2}$

(b) $\frac{a^2-b^2}{a^2+b^2}$

(c) $\frac{a^2-1}{a^2-b^2}$

(d) $\frac{a^2-1}{a^2+b^2}$

$\cos\alpha = a\cos\beta$

$\sin\alpha = b\sin\beta$

$\cos^2\alpha + \sin^2\alpha = 1$

$a^2\cos^2\beta + b^2\sin^2\beta = 1$

$a^2(1-\sin^2\beta) + b^2\sin^2\beta = 1$

$a^2 - a^2\sin^2\beta + b^2\sin^2\beta = 1$

$\sin^2\beta(b^2-a^2) = 1-a^2$

$\sin^2\beta = \frac{1-a^2}{b^2-a^2} \Rightarrow \frac{a^2-1}{a^2-b^2}$

Ans. (c)

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Eg27. If $\frac{\cos\alpha}{\sin\beta} = n$ & $\frac{\cos\alpha}{\cos\beta} = m$ Find the value of $\cos^2\beta$

45 sec

(a) $\frac{m^2}{m^2+n^2}$

(b) $\frac{1}{m^2+n^2}$

(c) $\frac{n^2}{m^2+n^2}$

(d) 0

$$\boxed{\cos^2\beta = \frac{n^2}{m^2+n^2}}$$

$$n\sin\beta = m\cos\beta$$

$$n^2\sin^2\beta = m^2\cos^2\beta$$

$$n^2(1-\cos^2\beta) = m^2\cos^2\beta$$

$$n^2 = m^2\cos^2\beta + n^2\cos^2\beta$$

For Doubts

(i) Telegram →

(ii) Grade up → Doubt Section

(iii) Doubt class after every Module

(iv) Personally → "Last Option"

Ans. (c)

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Eg28. The equation $\cos^2\theta = \frac{(x+y)^2}{4xy}$ is only possible when

(a) $x = -y$

(b) $x > y$

(c) $x = y$

(d) $x < y$

SSC

$0 \le \cos^2\theta \le 1$

$0 \le \frac{(x+y)^2}{4xy} \le 1$

$0 \le \frac{(x+y)^2}{4xy}$

$0 \le (x+y)^2$

$x+y=0$

$x=-y$

$(x+y)^2 \le 4xy$

$(x+y)^2 - 4xy \le 0$

$(x-y)^2 \le 0$

$\boxed{x=y}$

Ans. (a & c)

Priority

+ve

0

-ve

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Eg29. If $\tan^2\alpha = 1+2\tan^2\beta$ $(0 < \alpha, \beta < 90)$ find $\sqrt{2}\cos\alpha - \cos\beta$

(a) 0

(b) $\sqrt{2}$

(c) 1

(d) -1

$$1+\tan^2\alpha = 2(1+\tan^2\beta)$$

$$\sec^2\alpha = 2\sec^2\beta$$

$$\frac{1}{\cos^2\alpha} = \frac{2}{\cos^2\beta}$$

$$\frac{1}{\cos\alpha} = \frac{\sqrt{2}}{\cos\beta}$$

$$\sqrt{2}\cos\alpha = \cos\beta$$

Ans. (a)

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Eg30. If $\sin\alpha + \cos\beta = 2$ $(0^\circ \le \beta \le \alpha \le 90)$ find $\sin\left(\frac{2\alpha+\beta}{3}\right)$

(a) $\sin\frac{\alpha}{2}$ ✗

(b) $\cos\frac{\alpha}{3}$ ✓

(c) $\sin\frac{\alpha}{3}$

(d) $\cos^2\frac{\alpha}{3}$

$\sin\alpha + \cos\beta = 2$

$\downarrow \qquad \downarrow$

$1 \qquad 1$

$\underline{\underline{\alpha = 90}} \qquad \underline{\underline{\beta = 0}}$

$\underline{\underline{\sin 60}} \longrightarrow \cos 30$

Ans. (b)

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Eg31. If $2\sin\left(\frac{\pi x}{2}\right) = x^2 + \frac{1}{x^2}$ find $x - \frac{1}{x}$

(a) –1

(b) 2

(c) 1

(d) 0

L.H.S Max $\longrightarrow$ 2

R.H.S Min $\longrightarrow$ 2

L.H.S ≤ 2

R.H.S ≥ 2

$\sin\left(\frac{\pi}{2}x\right) = 1$

$\sin\left(\frac{\pi}{2}x\right) = \sin\left(\frac{\pi}{2}\right)$

$x = 1$

min value of $\left(x^2+\frac{1}{x^2}\right) = 2$

when $\underline{\underline{x^2 = 1}}$

Ans. (d)

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Eg32. Find $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

(a) 14 (b) 11

(c) 12 (d) 13

I Putting values $x = 90$

$$3(1-0)^4 + 6(1+0)^2 + 4(1+0)$$

$$3 + 6 + 4 = 13$$

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Ans. (d)

Detailed $3(\sin x-\cos x)^4+6(\sin x+\cos x)^2+4(\sin^6 x+\cos^6 x)$

Sol^n $3\left[(\sin x-\cos x)^2\right]^2+6\left[\sin^2 x+\cos^2 x+2\sin x\cos x\right]+4(1-3\sin^2 x\cos^2 x)$

$\rightarrow 3\left[1-2\sin x\cos x\right]^2+6\left[1+2\sin x\cos x\right]+4(1-3\sin^2 x\cos^2 x)$

$\rightarrow 3\left[1+4\sin^2 x\cos^2 x-4\sin x\cos x\right]+6\left[1+2\sin x\cos x\right]$
$+4(1-3\sin^2 x\cos^2 x)$

$3+6+4 \rightarrow 13$

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Eg33. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ Find $\cos\theta - \sin\theta$

90sec

(a) $\sqrt{2}\tan\theta$

(b) $-\sqrt{2}\cos\theta$

(c) $-\sqrt{2}\sin\theta$

(d) $\sqrt{2}\sin\theta$

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$$

$$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$2\sin\theta\cos\theta = \cos^2\theta - \sin^2\theta$$

$$2\sin\theta\cos\theta = (\cos\theta - \sin\theta)(\cos\theta + \sin\theta)$$

$$\frac{\sqrt{2}\,\cancel{2}\sin\theta\,\cancel{\cos\theta}}{\sqrt{2}\,\cancel{\cos\theta}} \qquad \cos\theta - \sin\theta$$

Ans. (d)

Eg34. If $5\cos\theta + 12\sin\theta = 13$, $0^\circ < \theta < 90^\circ$, then the value of $\sin\theta$ is

(a) $\frac{5}{13}$

(b) $-\frac{12}{13}$

(c) $\frac{6}{13}$

(d) $\frac{12}{13}$

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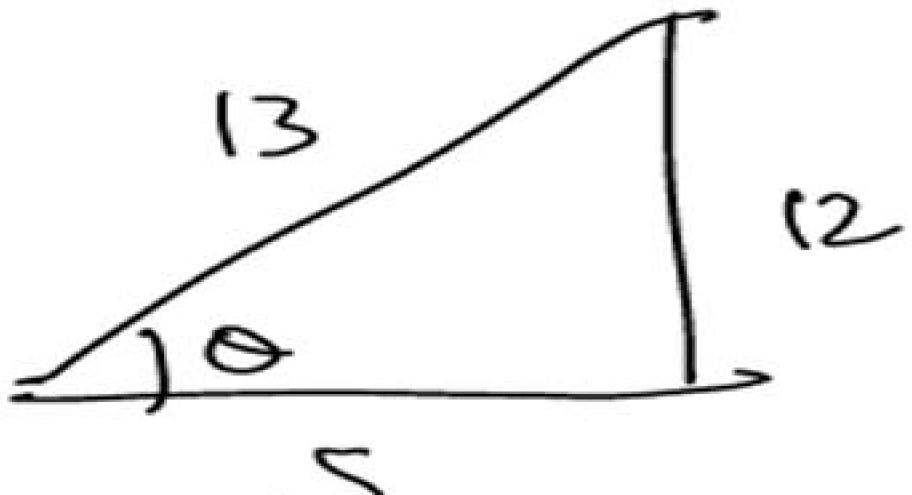
$$5\cos\theta + 12\sin\theta = 13$$

$$12\sin\theta = 13 - 5\cos\theta$$

$$144\sin^2\theta = 169 + 25\cos^2\theta - 130\cos\theta$$

$$144(1-\cos^2\theta) = 169 + 25\cos^2\theta - 130\cos\theta$$

$$169\cos^2\theta - 130\cos\theta + 25 = 0$$



$$(13\cos\theta - 5)^2 = 0$$

$$\cos\theta = \frac{5}{13}$$

$$\sin\theta = \frac{12}{13}$$

Ans. (d)

$If\ \boxed{a\sin\theta + b\cos\theta = c}\ \&\ c = \sqrt{a^2 + b^2}$

$then,$

$$\frac{a}{c}\sin\theta + \frac{b}{c}\cos\theta = 1$$

$$\boxed{\sin\theta = \frac{a}{c}} \qquad \boxed{\cos\theta = \frac{b}{c}}$$

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eg1

$$5\sin\theta + 12\cos\theta = 13$$

$$\frac{5}{13}\sin\theta + \frac{12}{13}\cos\theta = 1$$

$$\sin\theta = 5/13 \qquad \cos\theta = 12/13$$

eg2

$$7\sin\theta + 24\cos\theta = 25$$

$\sec\theta = ??$

$$\frac{7}{25}\sin\theta + \frac{24}{25}\cos\theta = 1$$

$$\frac{25}{24}$$

eg3

$$5\sin\theta - 12\cos\theta = 13$$

$$\frac{5}{13}\sin\theta \left(-\frac{12}{13}\right)\cos\theta = 1$$

Find $\sin\theta - \cos\theta = ??$

$$\frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

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Eg35. If $x\cos\theta - y\sin\theta = \sqrt{x^2+y^2}$ and $\dfrac{\cos^2\theta}{a^2}+\dfrac{\sin^2\theta}{b^2}=\dfrac{1}{x^2+y^2}$ Then the correct relation is

PYQ of SSC

(a) $\dfrac{x^2}{b^2}-\dfrac{y^2}{a^2}=1$

(b) $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

(c) $\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$

(d) $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

$$\frac{x}{\sqrt{x^2+y^2}}\cos\theta - \frac{y}{\sqrt{x^2+y^2}}\sin\theta = 1$$

$$\frac{x^2}{\cancel{(x^2+y^2)}a^2}+\frac{y^2}{\cancel{(x^2+y^2)}b^2}=\frac{1}{\cancel{(x^2+y^2)}}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Ans. (b)

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Eg36. If $\theta = 60°$ then $\frac{1}{2}\sqrt{1+\sin\theta}+\frac{1}{2}\sqrt{1-\sin\theta}$

(a) $\cot\frac{\theta}{2}$

(b) $\sec\frac{\theta}{2}$

(c) $\sin\frac{\theta}{2}$

(d) $\cos\frac{\theta}{2}$

Ans. (d)

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(1) $\sin 2\theta = 2\sin\theta\cos\theta$

(2) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

Eg37. If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$ find $\sin\alpha + \cos\alpha$

(a) $\pm\sqrt{2}\sin\theta$

(b) $\pm\sqrt{2}\cos\theta$

(c) $\pm\dfrac{1}{\sqrt{2}}\sin\theta$

(d) $\pm\dfrac{1}{\sqrt{2}}\cos\theta$

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Ans. (b)

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Eg38. If $(a^2 - b^2)\sin\theta + 2ab\cos\theta = a^2 + b^2$ Find $\tan\theta$ = ??

(a) $\frac{2ab}{a^2 - b^2}$

(b) $\frac{a^2 - b^2}{2ab}$

(c) $\frac{ab}{a^2 - b^2}$

(d) $\frac{a^2 - b^2}{ab}$

$$\frac{a^2-b^2}{a^2+b^2}\sin\theta + \frac{2ab}{a^2+b^2}\cos\theta = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta = 1$$

$$\tan\theta = \frac{a^2-b^2}{2ab}$$

Ans. (b)

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Eg39. $8 \cos 10^\circ \cos 20^\circ \cos 40^\circ$

(a) $\tan 80^\circ$

(b) $\cot 10^\circ$

(c) $\tan 80^\circ$ or $\cot 10^\circ$

(d) None of these

Ans. (c)

Eg40. Find $\tan\theta\,(1 + \sec 2\theta)\,(1 + \sec 4\theta)\,(1 + \sec 8\theta)$

(a) $\tan 4\theta$

(b) $\tan 8\theta$

(c) $2\tan 8\theta$

(d) $2\tan 4\theta$

Ans. (b)

Eg41. Find $\cot\theta - \tan\theta - 2\tan 2\theta$

(a) $4\cot 4\theta$ (b) 0

(c) $2\cot 4\theta$ (d) $\cot 4\theta$

Ans. (a)

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Eg42. For any real values of θ, $\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}=?$

(a) $\cot\theta - \operatorname{cosec}\theta$

(b) $\sec\theta - \tan\theta$

(c) $\operatorname{cosec}\theta - \cot\theta$

(d) $\tan\theta - \sec\theta$

Ans. (c)

Eg43. The value of the expression:

$\sin^2 1° + \sin^2 11° + \sin^2 21° + \sin^2 31° + \sin^2 41° + \sin^2 45° + \sin^2 49° + \sin^2 59° + \sin^2 69° + \sin^2 79° + \sin^2 89°$ is

(a) 0

(b) $5\frac{1}{2}$

(c) 5

(d) $4\frac{1}{2}$

Ans. (b)

gradeup **Eg44.** The value of the expression $(1 + \sec 22^\circ + \cot 68^\circ)(1 - \operatorname{cosec} 22^\circ + \tan 68^\circ)$ is

(a) 0 (b) 1

(c) −1 (d) 2

Ans. (d)

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Eg45. For how many integral values of 'x', $\sin\phi = \frac{(3x-2)}{4}$, where $0° \le \phi \le 90°$

(a) 2

(b) 3

(c) 0

(d) 1

Ans. (a)

Eg46. The expression $\frac{\tan 57^\circ + \cot 37^\circ}{\tan 33^\circ + \cot 53^\circ}$ is equal to:

(a) $\tan 33^\circ \cot 57^\circ$

(b) $\tan 57^\circ \cot 37^\circ$

(c) $\tan 33^\circ \cot 53^\circ$

(d) $\tan 53^\circ \cot 37^\circ$

Ans. (b)

Eg47. The value of :

$152\,(\sin 30^\circ + 2\cos^2 45^\circ + 3\sin 30^\circ + 4\cos^2 45^\circ + \ldots\ldots\ldots\ldots\ldots\ 17\sin 30^\circ + 18\cos^2 45^\circ)$ is:

(a) an integer but not a perfect square

(b) a rational number but not an integer

(c) a perfect square

(d) irrational

Ans. (c)

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Eg48. If 29 $\tan\theta = 31$.
Find the value of $\frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$

(a) 810 (b) 900 (c) 540 (d) 490

Ans. (b)

Eg49. If $x \cos\theta - \sin\theta = 1$

Find $x^2 - (1 + x^2)\sin\theta$.

(a) 2 (b) 1 (c) –1 (d) 0

Ans. (b)

Eg50. If $0 < \theta < 90°$

$\operatorname{cosec}\theta = \cot^2\theta$

then $\operatorname{cosec}^4\theta - 2\operatorname{cosec}^3\theta + \cot^2\theta = ??$

(a) 0 (b) 1 (c) 2 (d) 3

Ans. (a)

Eg51. If $\sin\theta = a\cos\phi$; $\cos\theta = b\sin\phi$

Find the value of : $(a^2 - 1)\cot^2\phi + (1 - b^2)\cot^2\theta$

a $\dfrac{a^2+b^2}{a^2}$

b $\dfrac{a^2+b^2}{b^2}$

c $\dfrac{a^2-b^2}{b^2}$

d $\dfrac{a^2-b^2}{a^2}$

Ans. (d)

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PPT → SIQ

Fri → Height & Distance I

Sat → Height & Distance II

30 min left

Sun → Off

Identities

Mon → extra session

Rem Identities + 30 min of Trig Max/Min