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Sahi Prep Hai Toh Life Set Hai

QUADRATIC EQUATION

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Agenda

- Factorization
- Roots of Q.E → (60-65) min
- Maxima & Minima

+ Linear eqn → 20 min

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SOLUTION OF A QUADRATIC EQUATION BY FACTORIZATION METHOD

Step I : Factorize the constant term of the given quadratic equation.

Step II : Express the coefficient of middle term as the sum or difference of the factors obtained in step I. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step III : Split the middle term in two parts obtained in step II.

Step IV : Factorize the quadratic equation obtained in step III by grouping method.

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$ax^2 + bx + c = 0$

$- \quad x \quad - \quad = ac$

$- \quad + \quad - \quad = b$

Eg1. $5x^2 + 14x + 8 = 0$

$5x^2 + 10x + 4x + 8 = 0$

$5x(x+2) + 4(x+2) = 0$

$(5x+4)(x+2) = 0$

$x = -4/5$ $\qquad$ $x = -2$

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(i) $3x^2 + 11x + 10 = 0$

$- x - = 30$

$- + - = 11$

$3x^2 + 6x + 5x + 10 = 0$

$3x(x+2) + 5(x+2) = 0$

$(3x+5)(x+2) = 0$

$x = -5/3 \;,\; x = -2$

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(ii) $5x^2 - 18x - 8 = 0$

$5x^2 - 20x + 2x - 8 = 0$

$5x(x-4) + 2(x-4) = 0$

$(5x+2)(x-4) = 0$

$x = -2/5 \quad x = 4$

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Shortcut for Factorization Method

$$3x^2 + 11x + 10 = 0$$

$\rightarrow$ -5 and -6

$\frac{-5}{3}$ 1 $\frac{-6}{3}$

$\Rightarrow$ $\frac{-5}{3}$ 1 -2

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eg $5x^2 - 18x - 8 = 0$

$-20 \quad 1 \quad 2$

$\frac{20}{5} \; 1 \quad -\frac{2}{5}$ → $4, -\frac{2}{5}$

eg $4x^2 + 18x + 14 = 0$

$\frac{-14}{4} \quad -\frac{4}{4}$ → $-7/2, -1$

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eg $7x^2 - 12x - 4 = 0$

$-14 \qquad 2$

$\frac{14}{7}, \frac{-2}{7}$

$2, \frac{-2}{7}$

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$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Let one Root is $\alpha = \frac{-b + \sqrt{b^2-4ac}}{2a}$

other Root is $\beta = \frac{-b - \sqrt{b^2-4ac}}{2a}$

$$\alpha+\beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Sum of Roots (S) $= \frac{-b}{a}$

Product of Roots (P) $= \frac{c}{a}$

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eg $5x^2 - 8x + 11 = 0$

$S = 8/5$ ✓

$P = 11/5$ ✓

Eg2. $5x^2 - 11x + 6 = 0$

Find the sum of roots and product of roots of the equation.

Sum of Roots $(\alpha+\beta) = \frac{-b}{a}$

$= \frac{11}{5}$

Product of Roots $(\alpha\beta) = \frac{c}{a}$

$= \frac{6}{5}$

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If sum of roots = S
Product of roots = P
then, the quadratic equation will be

$$x^2 - Sx + P = 0$$

Eg. Write a quadratic equation whose sum of roots = 7 and product of roots = 12.

$S = 7 \qquad P = 12$

Quadratic eqⁿ $\Rightarrow$ $x^2 - 7x + 12 = 0$

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BASIC ALGEBRAIC FORMULAS

$(a+b)^2 - (a-b)^2 = 4ab$

$(a-b)^2 = (a+b)^2 - 4ab$

(1) $(a + b)^2 = a^2 + 2ab + b^2$

(2) $(a - b)^2 = a^2 - 2ab + b^2$

(3) $a^2 - b^2 = (a + b)(a - b)$

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$ax^2+bx+c=0$ $\quad \alpha+\beta=\frac{-b}{a}$ $\quad \alpha\beta=\frac{c}{a}$

(i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

(ii) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

(iii) $\alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

$\Rightarrow \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2\alpha^2\beta^2$

(iv) $\alpha-\beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

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$ax^2 + bx + c = 0$

$$\alpha + \beta = \frac{-b}{a} \qquad and \qquad \alpha\beta = \frac{c}{a}$$

$$\alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$$

$$\alpha^4 + \beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$$

$$= \left[(\alpha+\beta)^2 - 2\alpha\beta\right]^2 - 2\alpha^2\beta^2$$

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Eg3. Find the value of k, so that sum of roots of equation $3x^2 + (2k + 1)x - k - 5 = 0$ is equal to product of the roots?

$S = P$

$\frac{-b}{a} = \frac{c}{a}$

$b + c = 0$

$(2k+1) - k - 5 = 0$

$\boxed{k = 4}$

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Eg4. $5x^2 + 8x - 7 = 0$ [α, β are the roots of the equation]
Find $\alpha^2 + \beta^2$ = ??

$$\alpha+\beta = \frac{-8}{5} \qquad \alpha\beta = \frac{-7}{5}$$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= \frac{64}{25} + \frac{14}{5} \Rightarrow \frac{64+70}{25}$$

$$\frac{134}{25}$$

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Eg5. $ax^2 + bx + c = 0$ [α, β are the roots of the equation]

Find $\dfrac{1}{a\alpha + b} + \dfrac{1}{a\beta + b}$

$\alpha + \beta = \dfrac{-b}{a}$

$\alpha\beta = \dfrac{c}{a}$

$$\frac{a\beta + b + a\alpha + b}{a^2\alpha\beta + ab\alpha + ab\beta + b^2}$$

$$\frac{a(\alpha+\beta) + 2b}{a^2\alpha\beta + ab(\alpha+\beta) + b^2} = \frac{-b + 2b}{ac - b^2 + b^2}$$

$$\Rightarrow \frac{b}{ac}$$

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$\alpha = 2\beta$

$\alpha + \beta = \frac{-b}{a}$

$3\beta = 9$

$\beta = 3$

$\alpha = 6$

Eg6. For what value of k, the equation has 2 roots, where one root is twice of the other.

$x^2 - 9x + k = 0$

$18 = k$

$K = 18$

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Eg7. If one root of a quadratic equation is $5+\sqrt{2}$, then find the equation.

Note : If one Root of a Quadratic eq^n is $a+\sqrt{b}$, then the other root is $a-\sqrt{b}$

$\alpha = 5+\sqrt{2}$ $\quad \beta = 5-\sqrt{2}$

$S = 10$ $\quad P = 23$

$x^2 - Sx + P = 0$

$$x^2 - 10x + 23 = 0$$

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Eg8. $x^2 + kx + 12 = 0$ [α, β are the roots of the equation]

If $\alpha - \beta = 1$

Find $5 - 2k$ = ??

Sol^n

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$$

$$1 = k^2 - 4 \cdot 12$$

$$k^2 = 49$$

$$\boxed{k = \pm 7}$$

when $k = 7$ $\quad 5 - 2(7) = -9$

$k = -7$ $\quad 5 - 2(-7) = 19$

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Eg9. $3x^2 - 5x + 8 = 0$ [α, β are the roots of the equation]

Then write the equation whose roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$??

I st.

Detailed Approach

$\alpha+\beta = 5/3 \qquad \alpha\beta = 8/3$

New Roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$

$$S = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5/3}{8/3} = \frac{5}{8}$$

$$P = \frac{1}{\alpha}\cdot\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{3}{8}$$

$x^2 - Sx + P = 0$

$x^2 - \frac{5}{8}x + \frac{3}{8} = 0$

$8x^2 - 5x + 3 = 0$

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$$ax^2 + bx + c = 0 \quad [\alpha, \beta]$$

$$cx^2 + bx + a = 0 \quad \left[\frac{1}{\alpha}, \frac{1}{\beta}\right]$$

eg $3x^2 - 5x + 8 = 0 \quad [\alpha, \beta]$

$8x^2 - 5x + 3 = 0 \quad \left[\frac{1}{\alpha}, \frac{1}{\beta}\right]$

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Eg10. If α, β are roots of quadratic equation $ax^2 + bx + c = 0$

Equation whose roots are $\frac{1}{\alpha}$ & $\frac{1}{\beta}$

$cx^2 + bx + a = 0$

eg $-3x^2 + 11x + 8 = 0$ [α & β]

$8x^2 + 11x - 3 = 0$ [$\frac{1}{\alpha}$ & $\frac{1}{\beta}$]

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Eg11. If α, β are roots of $5x^2 - 8x + 2 = 0$ Then write a quadratic equation whose roots are $(\alpha + 2)$ & $(\beta + 2)$.

$\alpha+\beta = \frac{8}{5}$

$\alpha\beta = \frac{2}{5}$

New Roots are $(\alpha+2)$ & $(\beta+2)$

$S = (\alpha+2)+(\beta+2) \Rightarrow \alpha+\beta+4 \Rightarrow \frac{8}{5}+4 = \frac{28}{5}$

$P = (\alpha+2)(\beta+2) \Rightarrow \alpha\beta+2\alpha+2\beta+4$

$\Rightarrow \frac{2}{5}+2\left(\frac{8}{5}\right)+4$

$\Rightarrow \frac{38}{5}$

$x^2 - \frac{28}{5}x + \frac{38}{5} = 0$

$5x^2 - 28x + 38 = 0$

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V-qp

$ax^2 + bx + c = 0$ $[\alpha \& \beta]$

If you have to write a quadratic eq^n whose roots are $(\alpha+k)$ & $(\beta+k)$ then just substitute x by $x-k$

$5x^2 - 8x + 2 = 0$ $[\alpha, \beta]$

$5(x-2)^2 - 8(x-2) + 2 = 0$ $[\alpha+2 \& \beta+2]$

$5x^2 - 28x + 38 = 0$

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DIFFERENCE BETWEEN ROOTS AND FACTORS OF A QUADRATIC EQUATION

$$ax^2 + bx + c = 0$$

Roots are α & β

factors are $(x-\alpha)$ & $(x-\beta)$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$$x = 2, 3$$

Roots are 2 & 3

factors are $(x-2)$ & $(x-3)$

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eg If the Roots are 5 & -3 of the Q.E then the factors of Q.E are

$(x-5)$ & $(x+3)$

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Eg12. If one factor of the equation $x^2 - x(\alpha + \beta) + (\alpha - 1)(\beta + 1) = 0$ is $(x - \alpha + 1)$, then the other factor is:

(a) $x - \alpha$ **(b) $x - \alpha - \beta$**

(c) $x - \beta + 1$ **(d) $x - \beta - 1$**

One factor = $x - \alpha + 1$

One Root = $\alpha - 1$

Product of Roots → $(\alpha - 1)(\beta + 1)$

other Root → $\beta + 1$

Other factor → $x - (\beta + 1)$

$x - \beta - 1$

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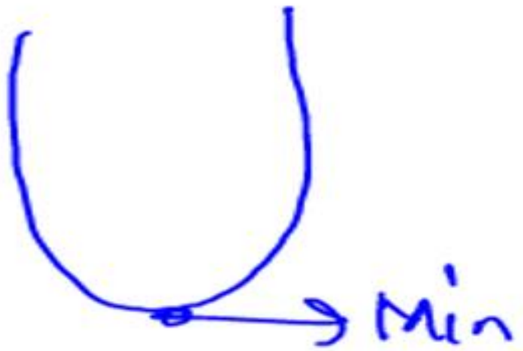
V-imp

Minimum and Maximum value of a Quadratic Expression

$$ax^2 + bx + c \qquad [a \neq 0]$$

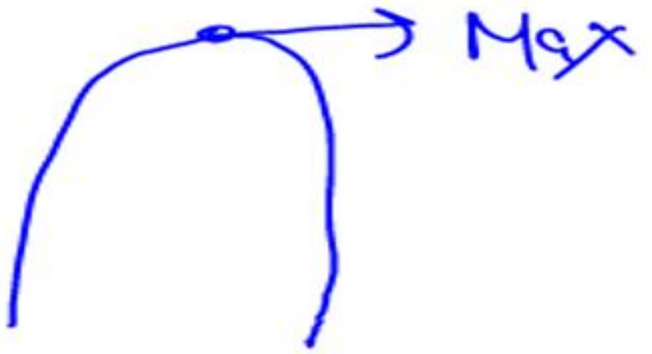
If a > 0



Min ⟶ $-\infty$

Max ⟶ ∞

If a < 0



Min ⟶ $-\infty$

Max ⟶

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minimum value of $x^2 = 0$ when $x = 0$

min $(x+2)^2 = 0$ when $x = -2$

min $(x-4)^2 = 0$ when $x = 4$

min $(x+3)^2 + 5 = 5$ when $x = -3$

min $(x-4)^2 - 8 = -8$ when $= 4$

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min $(x^2 + 6x + 8) = ??$

$(x^2 + 6x + 9) - 9 + 8$

$y = (x+3)^2 - 1$

$y_{min} = -1$ when $x = -3$

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$$\min \left(x^2 - 5x + 11 \right)$$

$$\left(x^2 - 5x + \frac{25}{4} \right) - \frac{25}{4} + 11$$

$$y = \left(x - \frac{5}{2} \right)^2 + \frac{19}{4}$$

$$y_{min} = \frac{19}{4} \quad \text{when } x = \frac{5}{2}$$

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Eg13. Find minimum value of $x^2 + 6x + 10$

$a = 1 \qquad b = 6 \qquad c = 10$

$$\frac{4 \cdot 1 \cdot 10 - 6^2}{4 \cdot 1} = 1$$

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$$Y = ax^2 + bx + c$$

$$Y = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$$

$$= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{-b^2 + 4ac}{4a^2}\right]$$

$$Y\min = \frac{4ac - b^2}{4a} \quad when \quad x = \frac{-b}{2a}$$

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Eg14. Find the minimum value of $x^2 - 6x + 8$.

$a = 1 \qquad b = -6 \qquad c = 8$

$$\frac{4 \cdot 1 \cdot 8 - 36}{4 \cdot 1} = -1$$

When $x = \frac{6}{2 \cdot 1} = 3$

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Eg15. Find the maximum value of $-3x^2 + 11x + 4$.

$$y = -3x^2 + 11x + 4$$

$$a = -3 \qquad b = 11 \qquad c = 4$$

$$y_{max} = \frac{4ac - b^2}{4a}$$

$$\frac{4 \cdot (-3)(4) - 121}{4(-3)} = \frac{169}{12}$$

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Linear Equation
Remaining Part

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Eg6. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

Let Fraction $= \frac{x}{y}$

Fraction $\frac{5}{9}$

$x + y = 2x + 4$

$-x + y = 4$ — (1)

$\frac{x+3}{y+3} = \frac{2}{3}$

$3x + 9 = 2y + 6$

$3x - 2y = -3$ — (2)

$x = 5 \quad y = 9$

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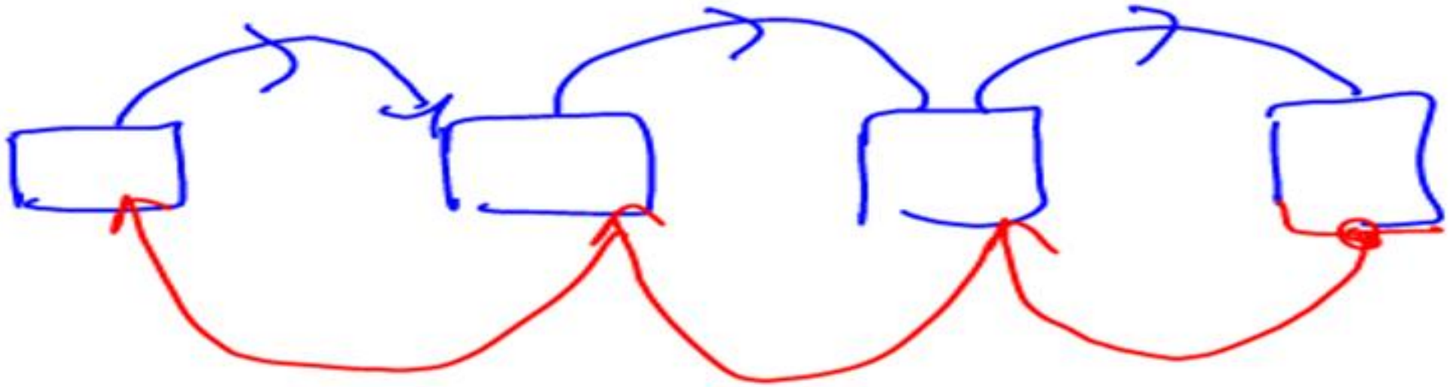
REVERSE APPROACH



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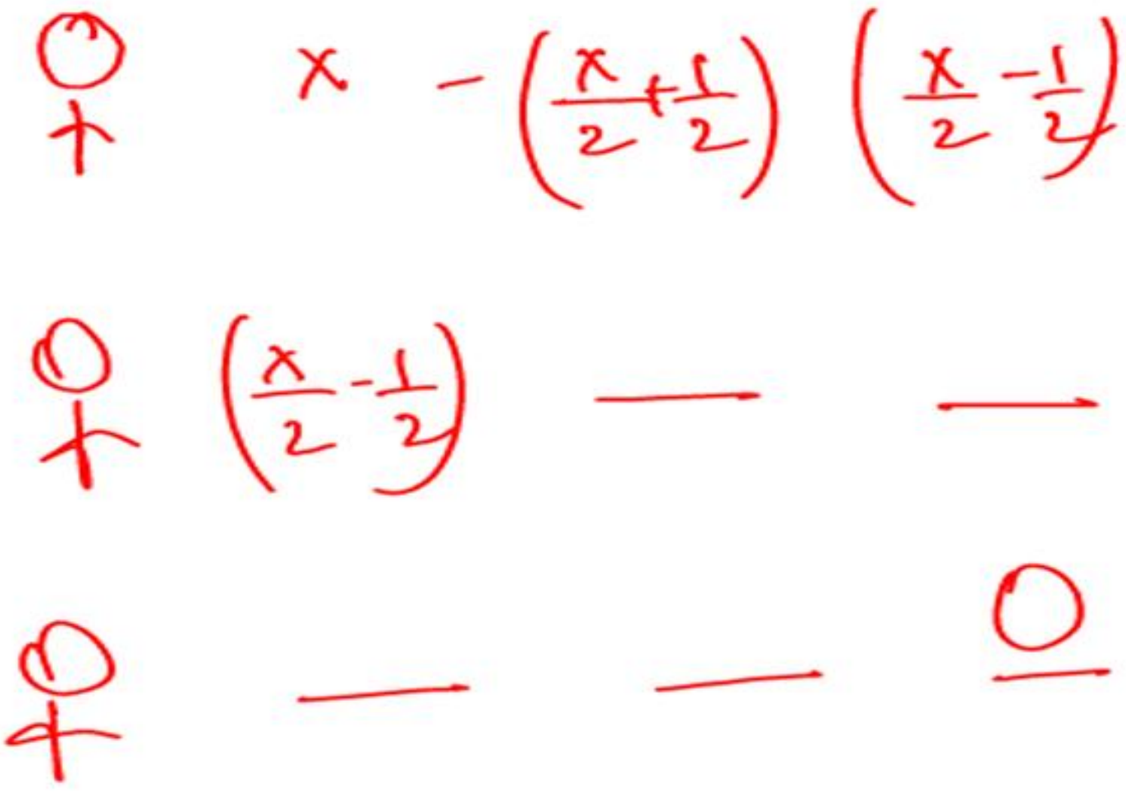
Initial Bought Left



Eg7. A shop stores x kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x.

(a) $2 \leq x \leq 6$
(b) $5 \leq x \leq 8$
(c) $9 \leq x \leq 12$
(d) $11 \leq x \leq 14$

Time → 2min

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		Initial	Bought	Left
Kushagra		7kg	4kg	3kg
Durga		3kg	2kg	1kg
Vishabha		1		0kg

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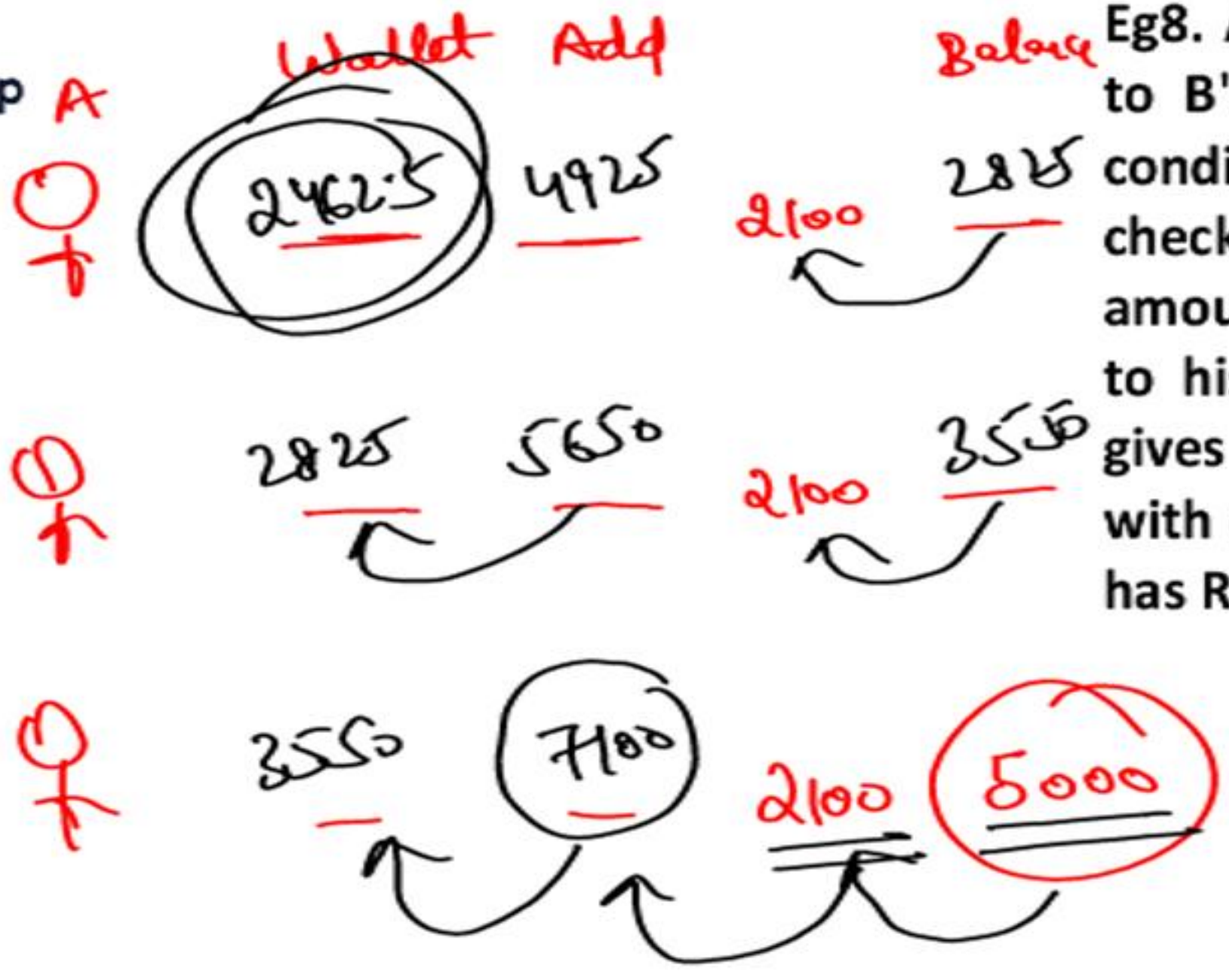

Eg8. A has three sisters B, C and D. A first goes to B's house and B knows that the financial condition of his brother A is not good, so she checks the wallet of his brother and whatever amount was there she adds the same amount to his wallet. When A leaves house of B, he gives her Rs.2100 and the same process goes with other two sisters. When A left D house he has Rs.5000. Find the initial amount A has.

Initial Amt → 24625

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Practice Questions

Q. If α and β are the roots of the quadratic equation $x^2+kx-15=0$ such that $\alpha-\beta=8$, then what is the positive value of k ?

A. 2 B. 3 C. 4 D. 5

Ans. A

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Q. If p and q are the roots of the equation $x^2 - 15x + r = 0$ and $p - q = 1$, then what is the value of r ?

A. 55 B. 56 C. 60 D. 64

Ans. B

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Q. If α and β are the roots of the equation $x^2 - 6x + 6 = 0$, what is $\alpha^3 + \beta^3 + \alpha^2 + \beta^2 + \alpha + \beta$ equal to?

A. 150 B. 138 C. 128 D. 124

Ans. B

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Q .If one root of $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice the other, then what is the value of 'a'?

A. $\frac{2}{3}$ B. $-\frac{2}{3}$ C. $\frac{1}{3}$ D. $-\frac{1}{3}$

Ans. A

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Q. If α and β are the roots of the equation $x^2 + px + q = 0$, then what is $\alpha^2 + \beta^2$ equal to?

A. $p^2 - 2q$ **B.** $q^2 - 2p$ **C.** $p^2 + 2q$ **D.** $q^2 - q$

Ans. A

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Q. Aman and Alok attempted to solve a quadratic equation. Aman made a mistake in writing down the constant term and ended up in roots (4, 3). Alok made a mistake in writing down the coefficient of X to get roots (3, 2). The correct roots of the equation are

A. -4, -3 B. 6, 1 C. 4, 3 D. -6, -1

Ans. B

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Q. If the roots of the equation $px^2 + x + r = 0$ are reciprocal to each other, then which one of the following is correct?

A. P = 2r B. P = r C. 2p = r D. P = 4r

Ans. B

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Q. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then what is the value of the expression $(\alpha + 1)(\beta + 1)$?

A. $\frac{a + b + c}{a}$

B. $\frac{b + c - a}{a}$

C. $\frac{a - b + c}{a}$

D. $\frac{a + b - c}{a}$

Ans. C

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Q. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{a\alpha+b} + \frac{1}{a\beta+b}$ is

A. $\frac{a}{bc}$ B. $\frac{b}{ac}$ C. $\frac{c}{ab}$ D. $-\frac{1}{abc}$

Ans. B

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Q. If one root of the equation $ax^2 + x - 3 = 0$, is -1, then what is the other root ?

A. 1/4 B. 1/2 C. 3/4 D. 1

Ans. C

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Q. What is the ratio of sum of squares of roots to the product of the roots of the equation $7x^2 + 12x + 18 = 0$?

A. 6 : 1 B. 1 : 6 C. -6 : 7 D. -1 : 6

Ans. C

Q. If the roots of the equation $Ax^2 + Bx + C = 0$ are -1 and 1, then which one of the following is correct?

A. A and C are both zero

B. A and B are both positive

C. A and C are both negative

D. A and C are of opposite sign

Ans. D

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Q. If α and β are the roots of the equation $x^2 - x - 1 = 0$, then what is the value of $(\alpha^4 + \beta^4)$?

A. 7 B. 0 C. 2 D. None of the above

Ans. A

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Q. If one of the roots of the equation $px^2 + qx + r = 0$ is three times the other, then which one of the following relations is correct?

A. $3q^2 = 16\ pr$

B. $q^2 = 24\ pr$

C. $p = q + r$

D. $p + q + r = 1$

Ans. A

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Q. The sign of the quadratic polynomial $ax^2 + bx + c$ is always positive if

A. a is positive and $b^2 - 4ac \leq 0$

B. a is positive and $b^2 - 4ac \geq 0$

C. a can be any real number and $b^2 - 4ac \leq 0$

D. a can be any real number and $b^2 - 4ac \geq 0$

Ans. A

Q. For the inequality $x^2 - 7x + 12 > 0$, which one of the following is correct ?

A. $3 < x < 4$

B. $-\infty < x < 3$ only

C. $4 < x < \infty$ only

D. $-\infty < x < 3$ or $4 < x < \infty$

Ans. D

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Q. The value of $x^2 - 4x + 11$ can never be less than

A. 7 B. 8 C. 11 D. 22

Ans. A

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Q. In the quadratic equation $x^2 + ax + b = 0$, a and b can take any value from the set {1, 2, 3, 4}. How many pairs of values of a and b are possible in order that the quadratic equation has real roots?

A. 6 B. 7 C. 8 D. 16

Ans. B

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Q. For what value of $k, z + \frac{1}{4}\sqrt{z} + k^2$ is a perfect square?

A. -1/8

B. 1/8

C. A and B

D. None of these

Ans. C

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Q. If the equation $x^2+2(1+k)x+k^2=0$ has equal roots, then what is the value of k?

A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. 1 D. -1

Ans. B

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Q. If the equation $\left(a^2+b^2\right)x^2-2\left(ac+bd\right)x+\left(c^2+d^2\right)=0$ **has equal roots, then which one of the following is correct ?**

A. $ab=cd$

B. $ad=bc$

C. $a^2+c^2=b^2+d^2$

D. $ac=bd$

Ans. B

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Q. Which on is one of the factors of $x^2+\frac{1}{x^2}+8\left(x+\frac{1}{x}\right)+14$?

A. $x+\frac{1}{x}+1$

B. $x+\frac{1}{x}+3$

C. $x+\frac{1}{x}+6$

D. $x+\frac{1}{x}+7$

Ans. C

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Q. What are the roots of the equation?

$$4^x - 3.2^{x+2} + 32 = 0$$

A. 1, 2 **B. 3, 4** **C. 2, 3** **D. 1, 3**

Ans. C

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Q. The sum of the squares of two positive integers is 208. If the square of the larger number is 18 times the smaller number, then what is the difference of the larger and smaller numbers?

A. 2 B. 3 C. 4 D. 6

Ans. C

eg $3x^2 + 5x + 1 = 0$ $[\alpha \& \beta]$

$3(x+3)^2 + 5(x+3) + 1$ $[(\alpha-3) \& (\beta-3)]$

$3x^2 + 23x + 43 = 0$