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Sahi Prep Hai Toh Life Set Hai

QUADRILATERAL

PART-1

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Agenda

Quad

80 - 90

(i) Quadrilateral

→ (30 - 35) min

(ii) Parallelogram

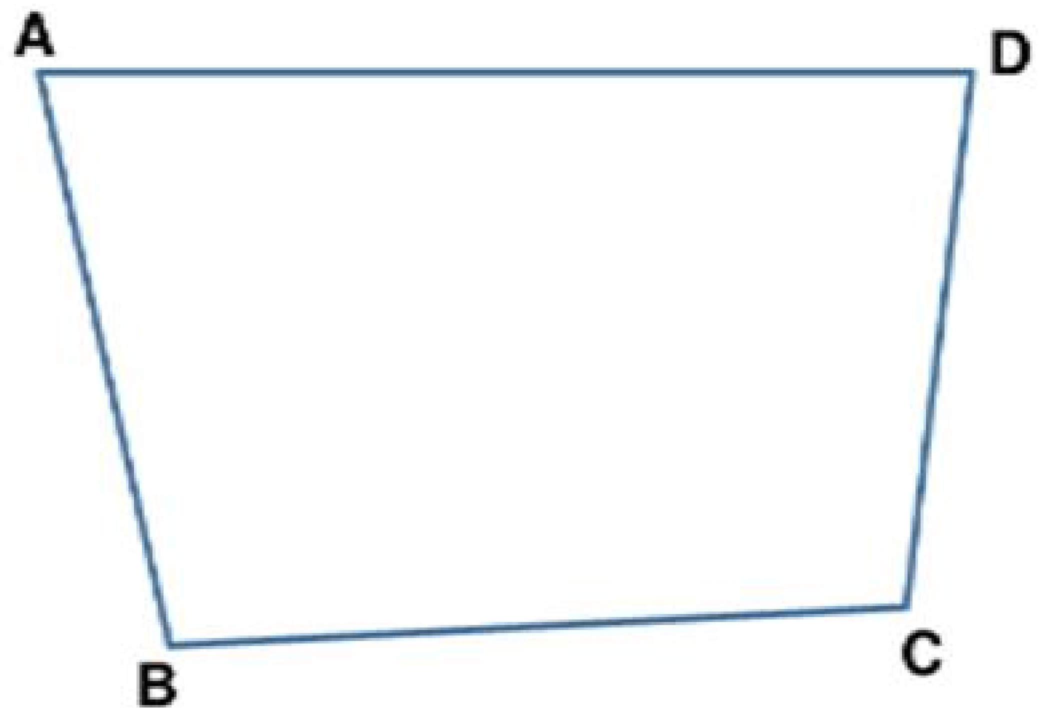
(50 - 55) min

(iii) Rhombus

(15 - 20) min

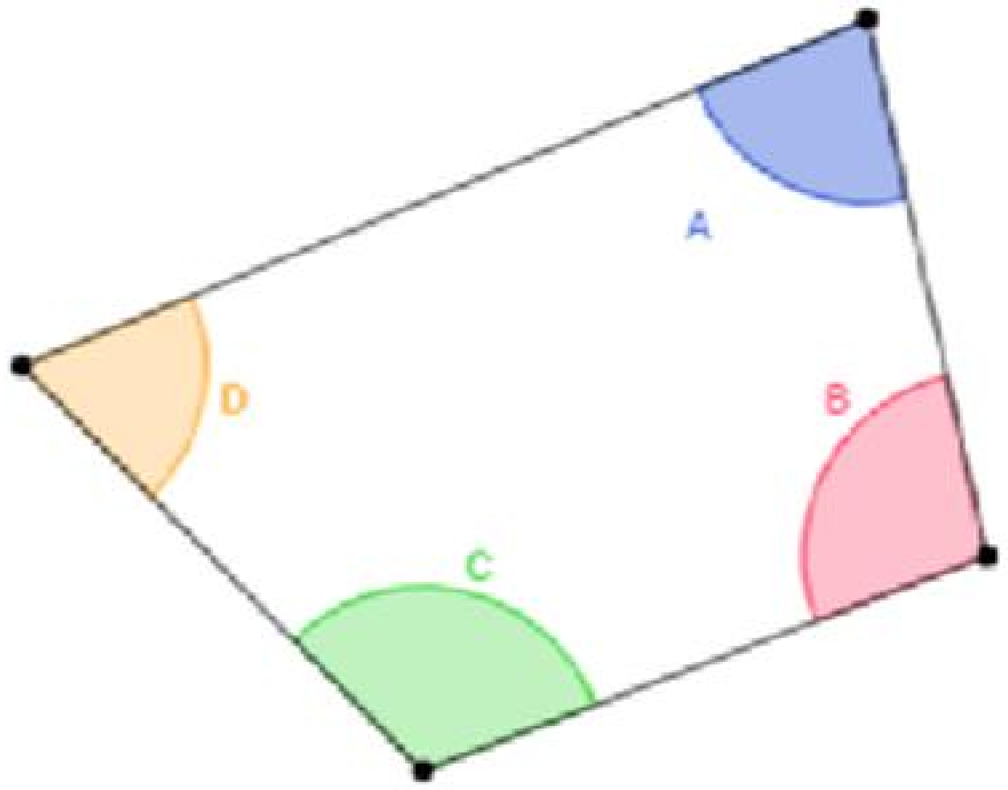
QUADRILATERAL

Def: Any four sided closed figure is called as Quadrilateral.



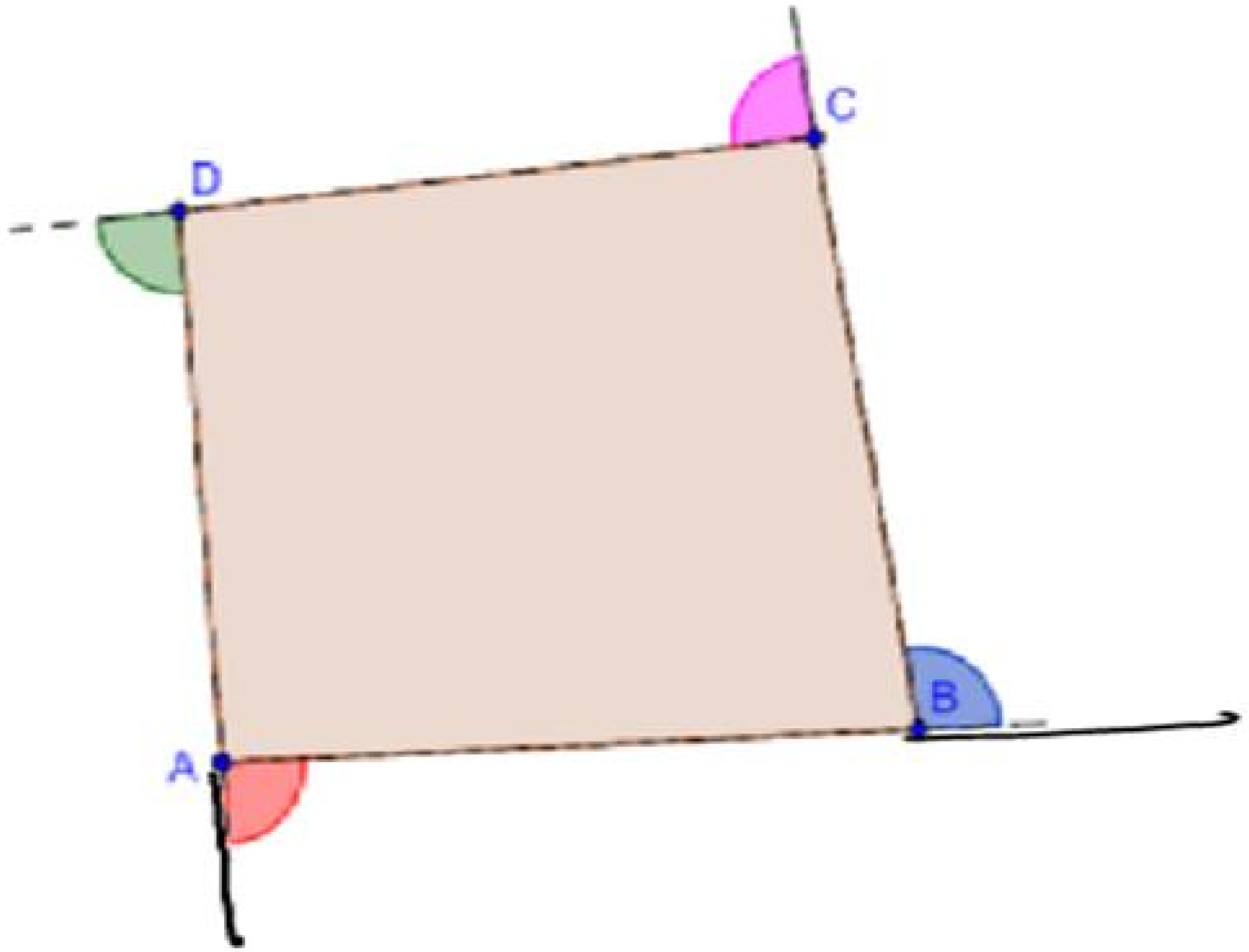
PROPERTIES

1. Sum of all interior angles of a quadrilateral = 360°

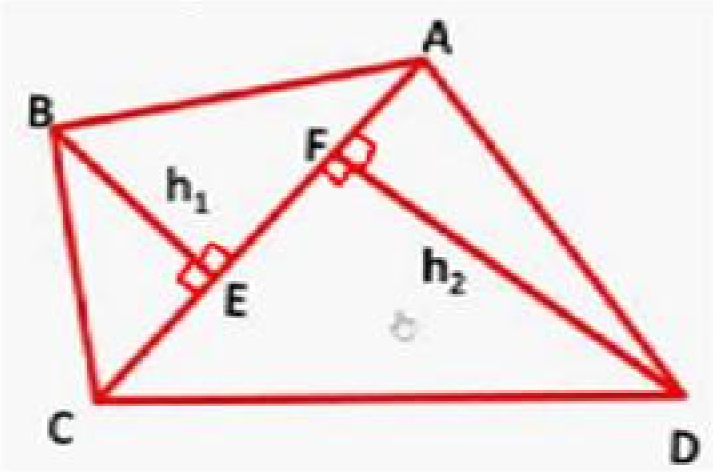


$\angle A + \angle B + \angle C + \angle D = 360^\circ$

2. Sum of all exterior angles of a quadrilateral = 360°



3. Area of quadrilateral ABCD :

(i) 

$$= \frac{1}{2} \times One\ of\ the\ diagonals \times Sum\ of \perp dropped\ on\ it$$

$$= \frac{1}{2} AC(BE + DF)$$

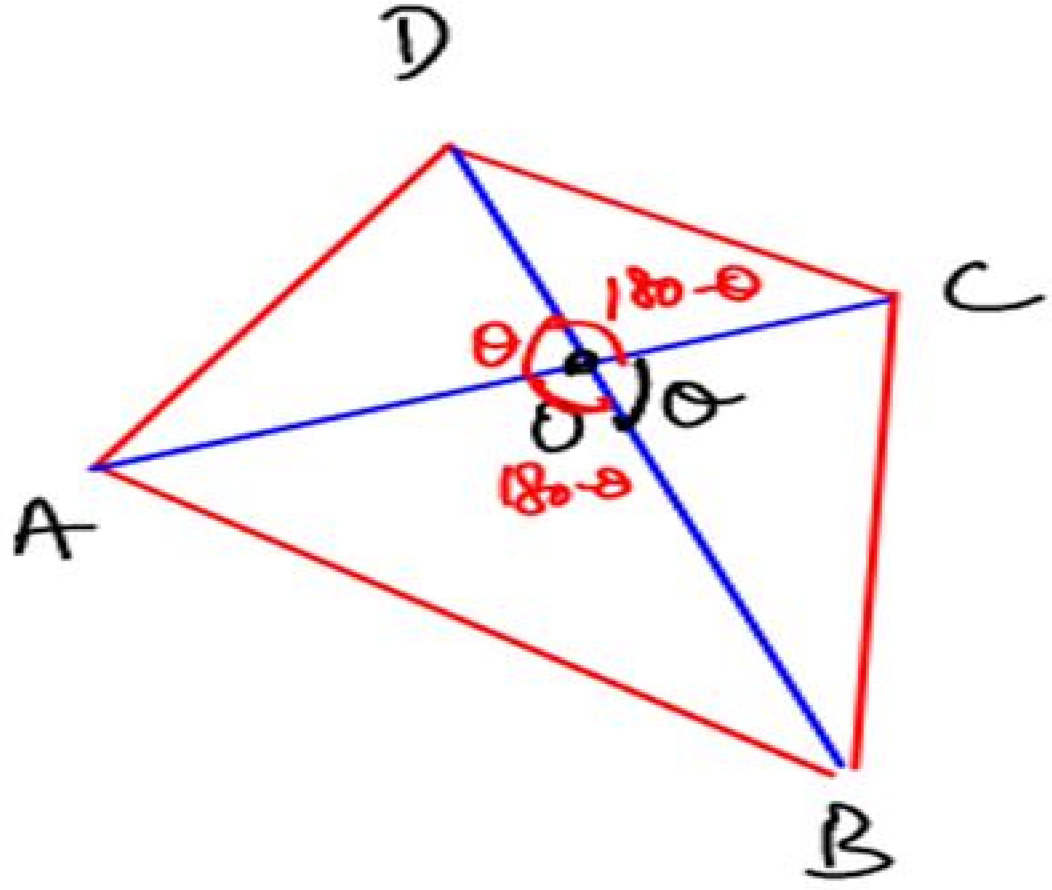
area of Quad ABCD = area of $(\Delta ABC + \Delta ACD)$

$$= \frac{1}{2} AC\ BE + \frac{1}{2} \times AC \cdot DF$$

$$= \frac{1}{2} AC\,(BE + DF)$$

(ii) **Area of quadrilateral** $= \frac{1}{2} D_1 D_2 \sin\theta$

where, D_1, D_2 are diagonals of quadrilateral and θ is the angle between the diagonal.



$\sin\theta = \sin(180-\theta)$

Area of ABCD

$= \text{Area}(\Delta AOB + \Delta BOC + \Delta COD + \Delta DOA)$

$\frac{1}{2} AO \cdot OB \sin\theta + \frac{1}{2} BO \cdot CO \sin\theta$

$\frac{1}{2} CO\, DO \sin\theta + \frac{1}{2} DO\, AO \sin\theta$

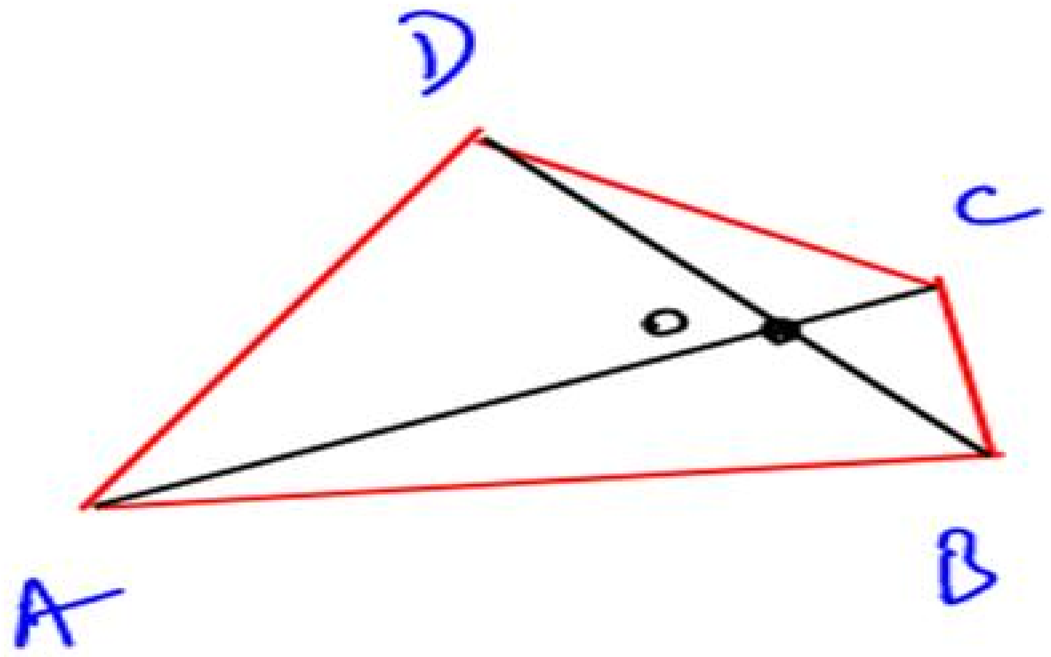
$\frac{1}{2} AO \sin\theta (BD) + \frac{1}{2} CO \sin\theta (BD)$

$\frac{1}{2} \sin\theta\, BD (AC)$

v.imp

4. In a quadrilateral, if AC and BD are the diagonals and they intersect at O, then

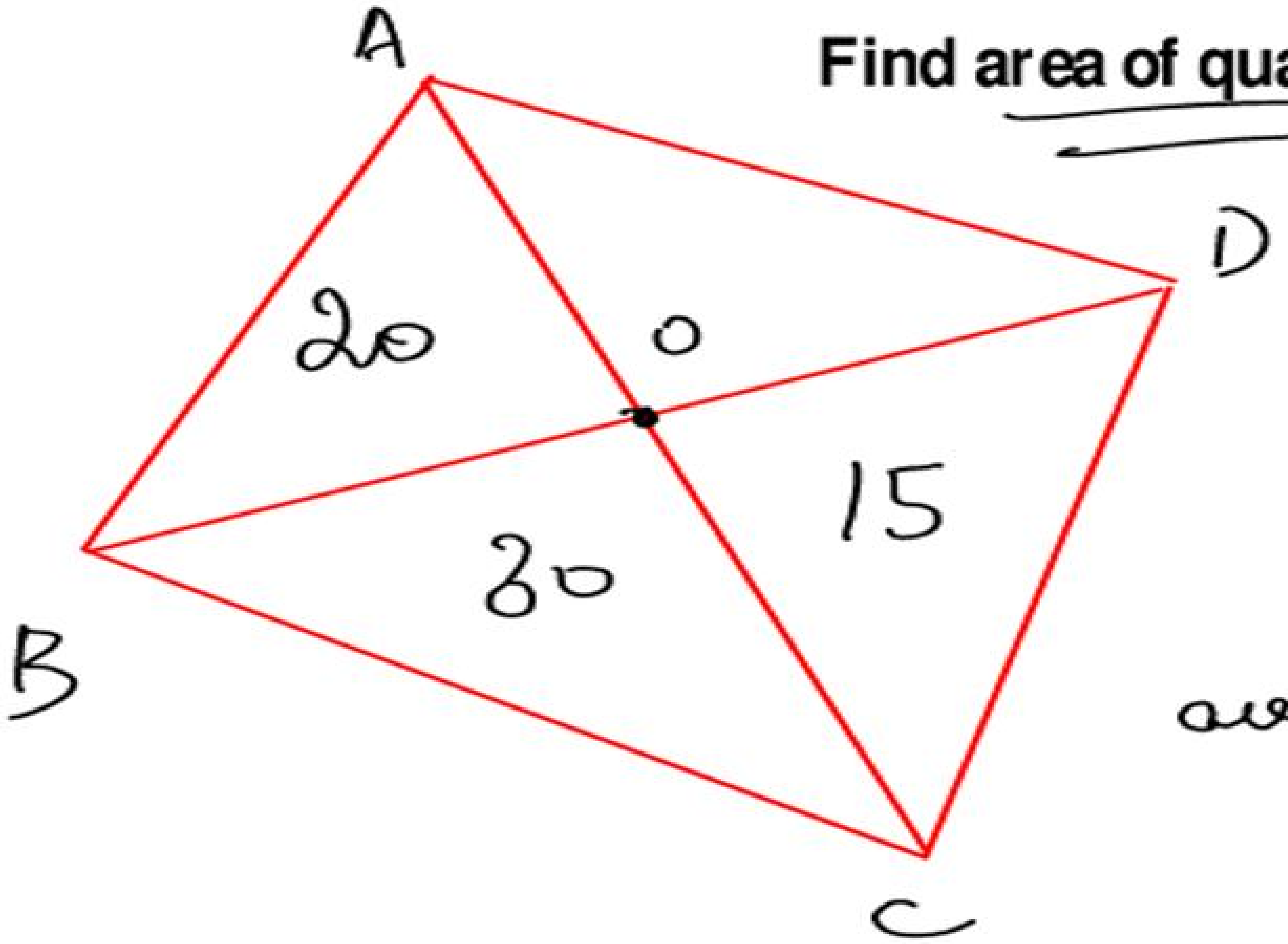
$$(\text{area of } \Delta AOB) \cdot (\text{area of } \Delta COD) = (\text{area of } \Delta BOC) \cdot (\text{area of } \Delta AOD)$$



Eg. In a quadrilateral diagonals AC and BD intersect each other at O.

If area of : $\Delta AOB = 20\ cm^2$, $\Delta BOC = 30 cm^2$ and $\Delta COD = 15\ cm^2$

Find area of quadrilateral ABCD.



area $\Delta AOD = \cancel{20} \times \cancel{15}^{5}$

area of $\Delta AOD = 10$

area of ABCD

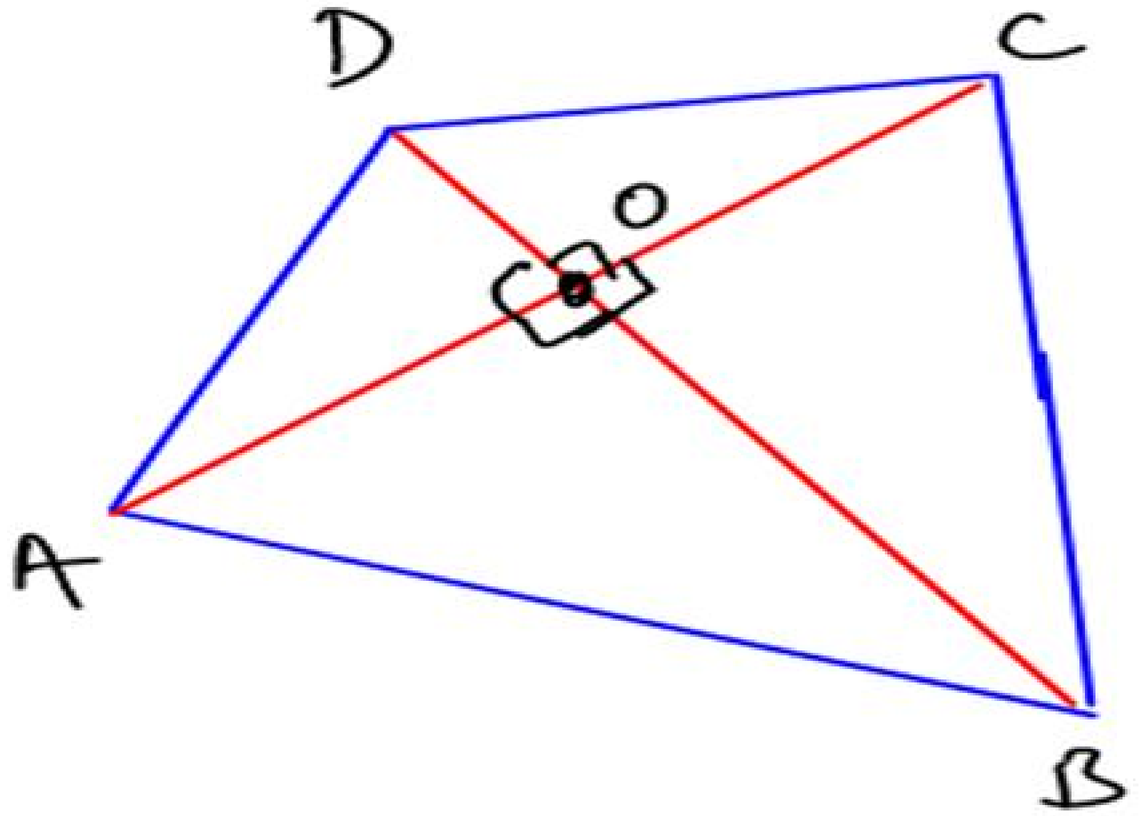
$= 20 + 30 + 15 + 10$

$= 75 cm^2$

Ans. 75 cm^2

5. If diagonals of a quadrilateral intersect each other at 90°, then :

$$AB^2 + CD^2 = BC^2 + AD^2$$



Reason

ΔAOB $\quad AO^2 + BO^2 = AB^2$ — (1)

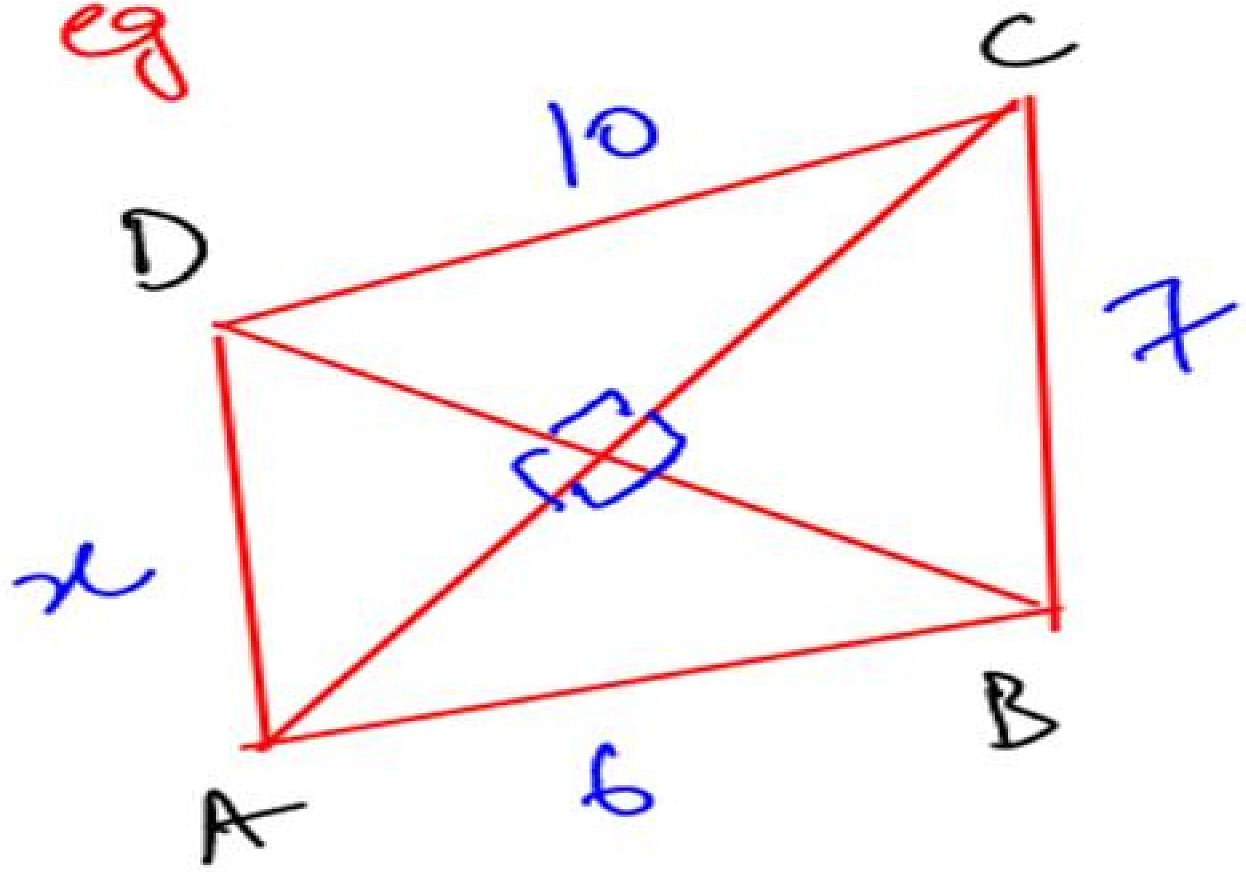
ΔBOC $\quad BO^2 + CO^2 = BC^2$ — (2)

ΔCOD $\quad CO^2 + DO^2 = CD^2$ — (3)

ΔDOA $\quad DO^2 + AO^2 = AD^2$ — (4)

ans $\boxed{AB^2 + CD^2 = BC^2 + AD^2}$

eg



Find x ??

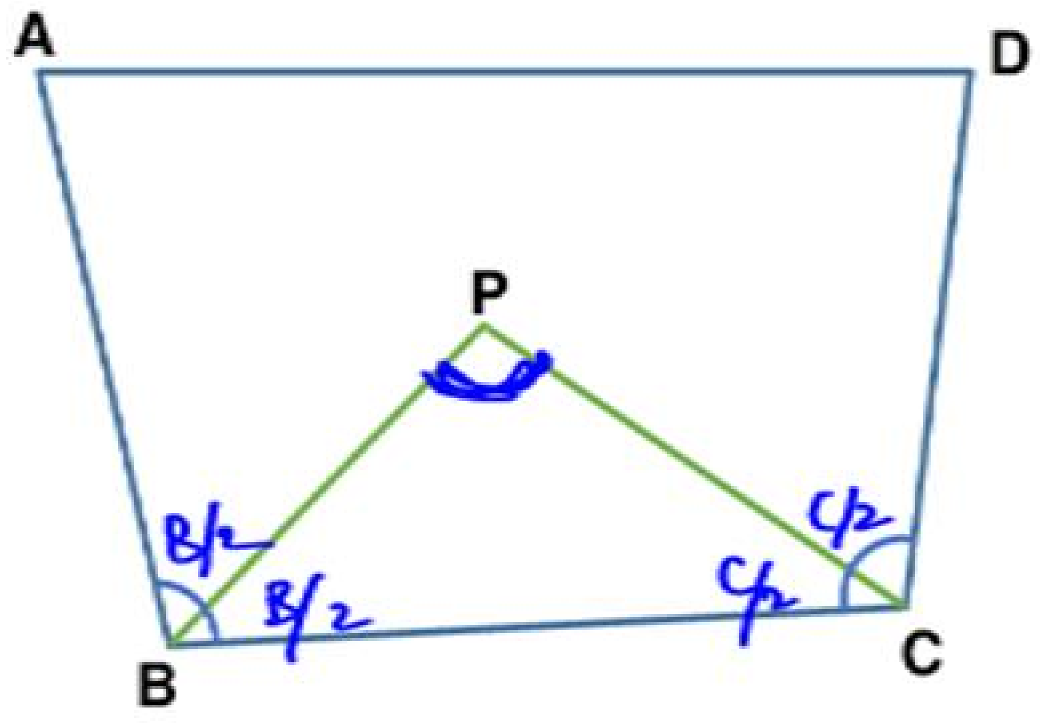
$$6^2 + 10^2 = 7^2 + x^2$$

$$136 - 49 = x^2$$

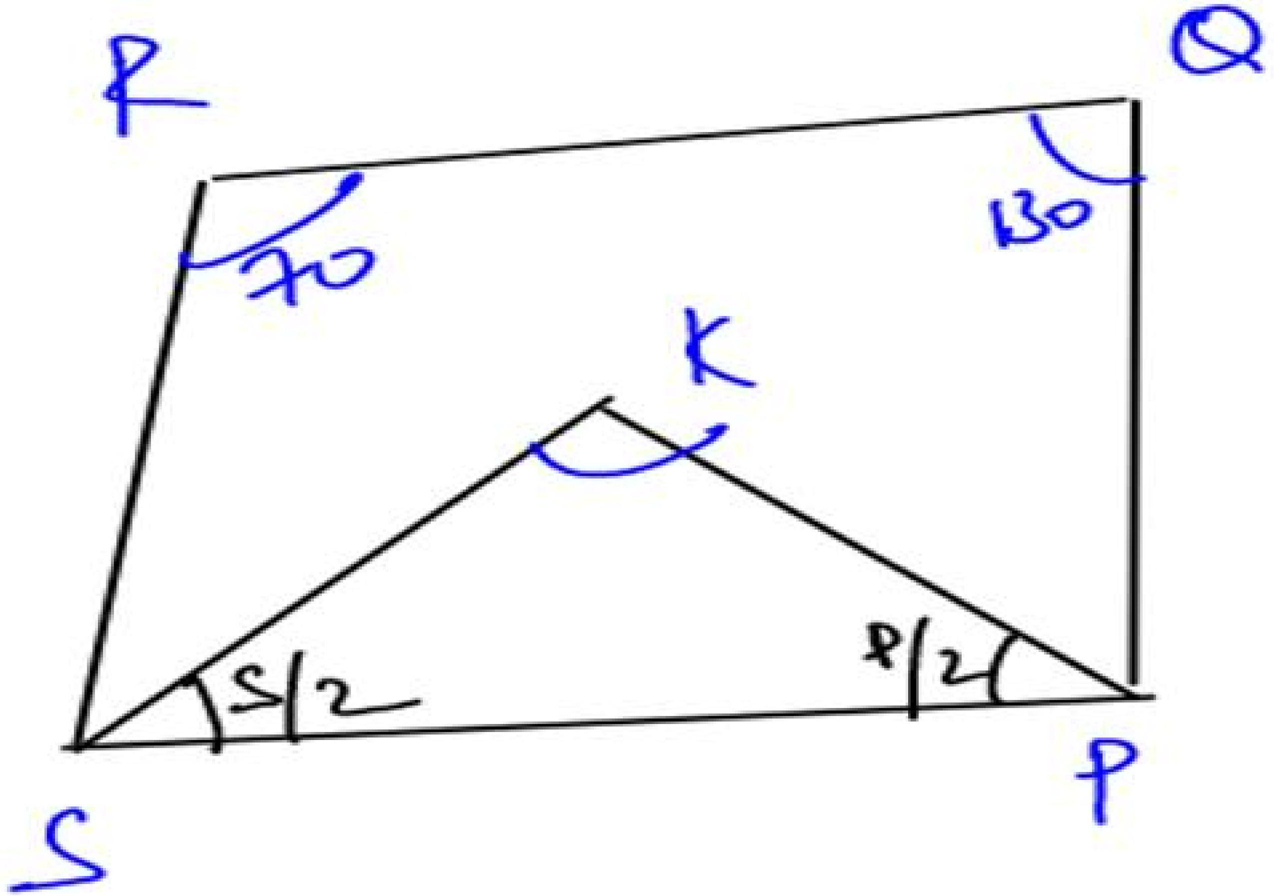
$$x^2 = 87$$

$$x = \sqrt{87}$$

6. If bisectors of $\angle B$ and $\angle C$ of a quadrilateral intersect each other at P, then $\angle BPC = \frac{1}{2}(\angle A + \angle D)$



Eg. In a quadrilateral PQRS, bisectors of $\angle S$ and $\angle P$ meet at K.
If $\angle R = 70^\circ$ & $\angle Q = 130^\circ$. Find $\angle SKP$.



Shortcut

$\rightarrow \angle SKP = \frac{1}{2}(70 + 130)$

$= \underline{\underline{100^\circ}}$

Detailed App

$\angle P + \angle Q + \angle R + \angle S = 360$

$\angle P + \angle S = \underline{\underline{160}}$

$\frac{\angle S}{2} + \frac{\angle P}{2} + \angle SKP = 180$

$80 + \angle SKP = 180$

$[\angle SKP = 100^\circ]$

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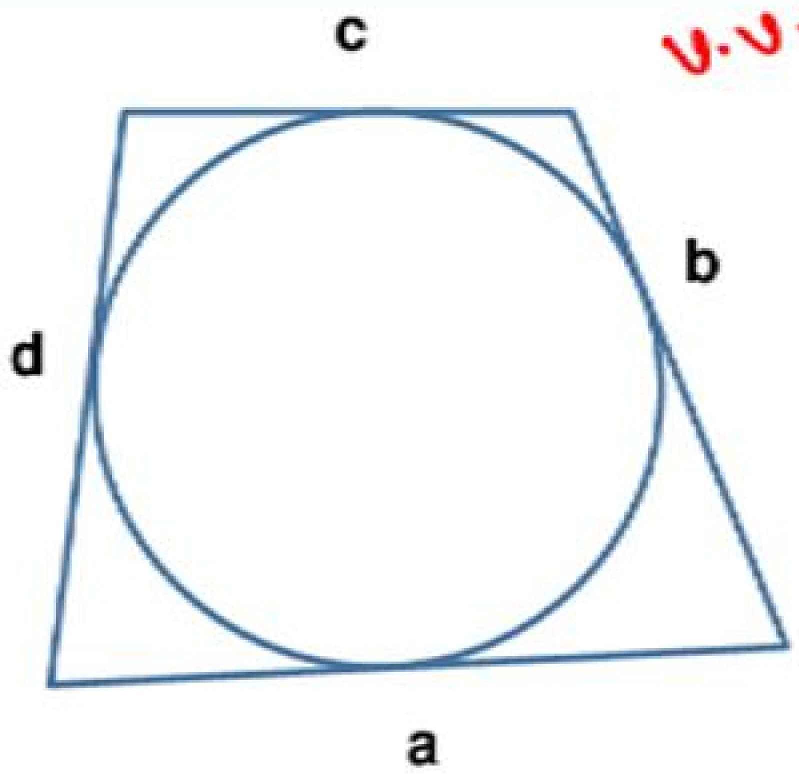
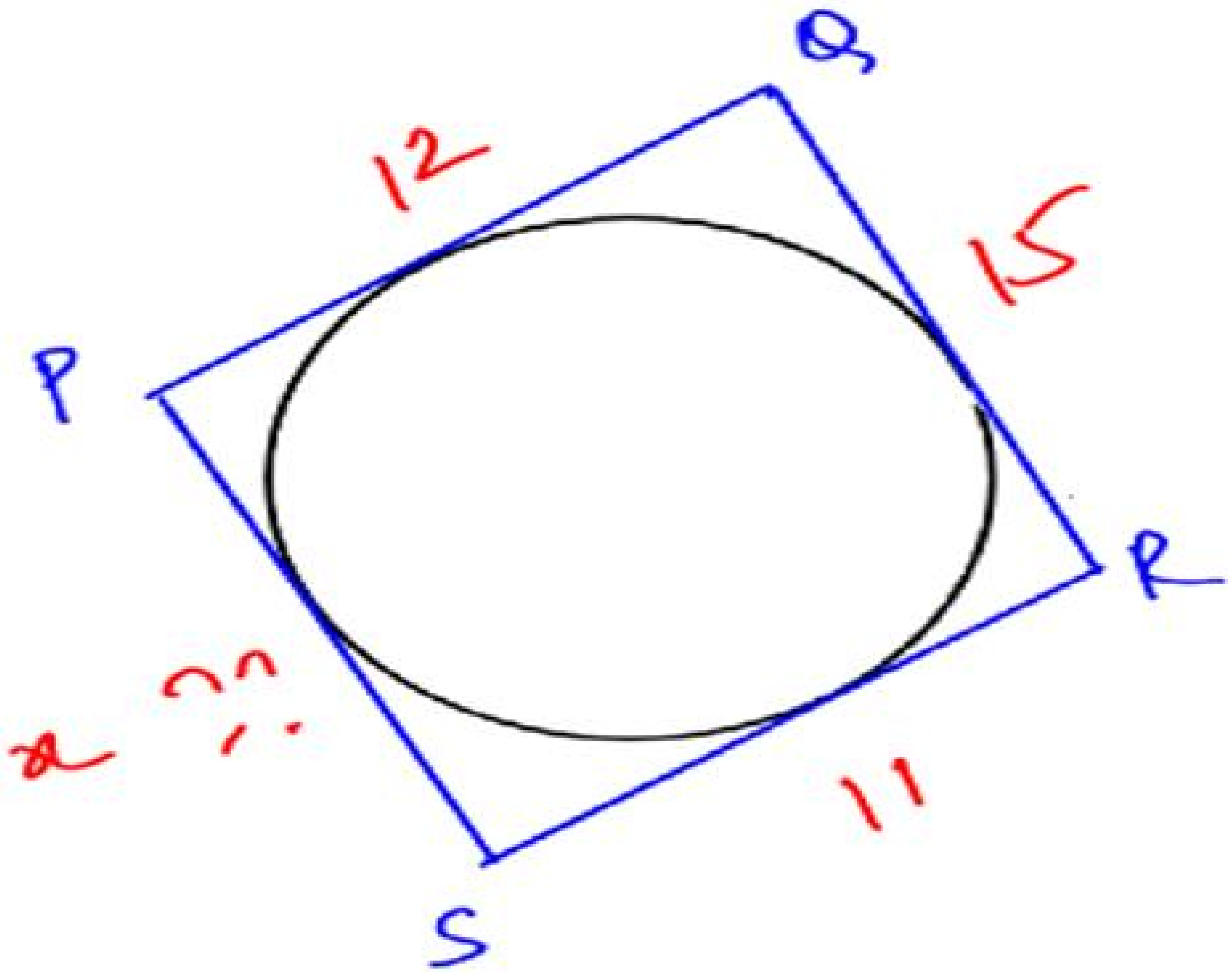
7. If a circle is inscribed in quadrilateral :



$$a + c = b + d$$

Reason : Concept of Tangents



Eg. If PQ = 12 cm

QR = 15 cm

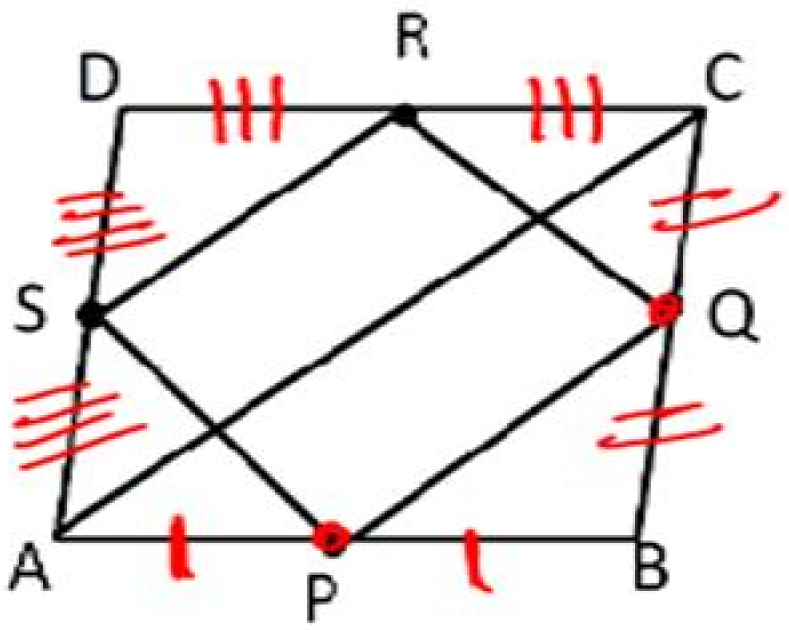
RS = 11 cm

Find PS = ??

$12 + 11 = 15 + x$

$x = 8$

8. Figure formed by joining the mid-points of all sides of a quadrilateral is a parallelogram and its area is half of the quadrilateral.

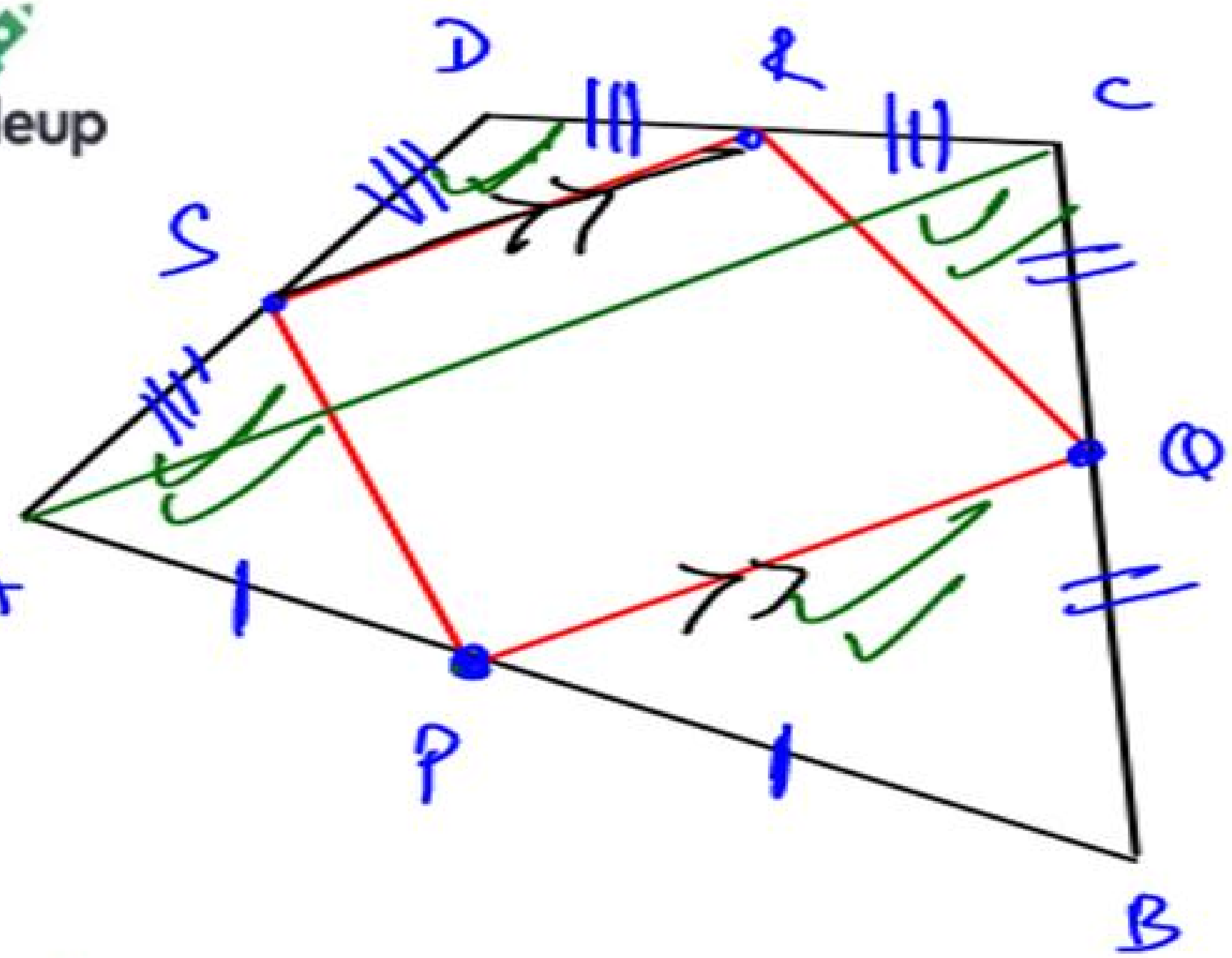


ABCD → Quad

& P, Q, R & S are mid pts of AB, BC, CD & DA

PQRS → ||gm

area of PQRS = $\frac{1}{2}$ area of ABCD



Given: ABCD is a Quad

P, Q, R & S are mid pts of AB, BC, CD & DA

To prove (i) PQRS is a ||gm

(ii) area of PQRS = $\frac{1}{2}$ area ABCD

Proof:- ΔABC

PQ || AC & PQ = $\frac{1}{2}$ AC

ΔACD

RS || AC & RS = $\frac{1}{2}$ AC

PQ || RS & PQ = RS

PQRS is a ||gm

(ii)

$\Delta PQB = \frac{1}{4} \Delta ABC$

$\Delta RDS = \frac{1}{4} ACD$

$\Delta PQB + \Delta RDS = \frac{1}{4} ABCD$

$PQRS \rightarrow \frac{1}{2} ABCD$

SUFFICIENT CONDITIONS FOR A QUADRILATERAL TO BE A PARALLELOGRAM

1. If opposite sides of a quadrilateral are equal, then that is a parallelogram.

2. If opposite angles of a quadrilateral are equal, then that is a parallelogram.

3. If diagonals of a quadrilateral bisect each other, then that is a parallelogram.

4. If one pair of sides of a quadrilateral is equal and parallel, then that is a parallelogram.

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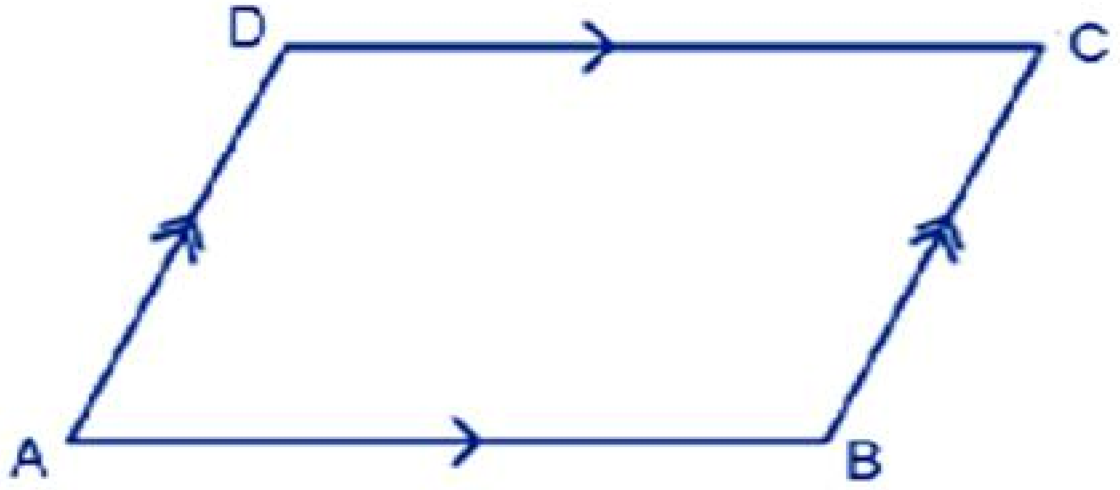
PARALLELOGRAM

Def: A quadrilateral in which opposite sides are parallel.

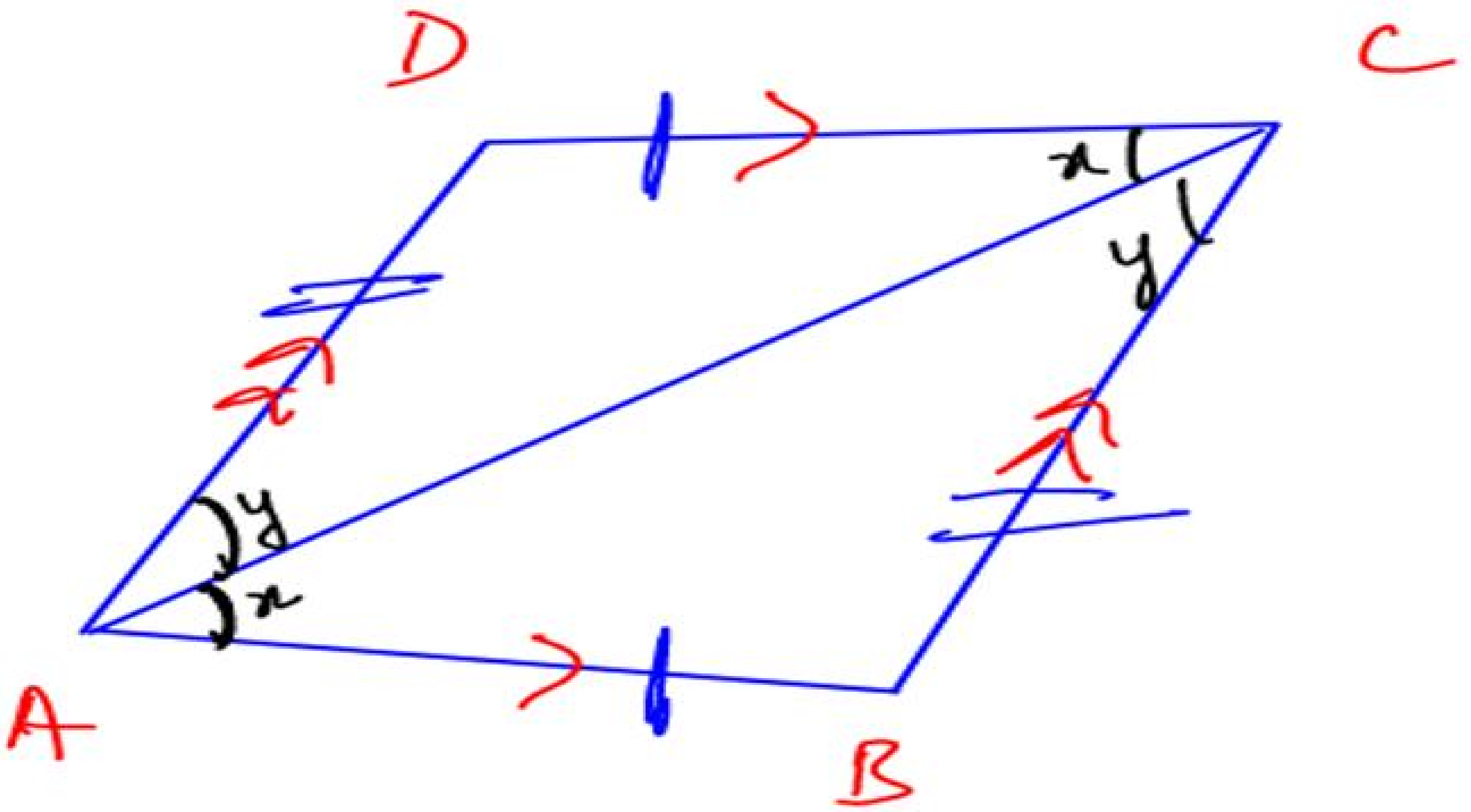


Def :- ABCD is a Quad

AB || CD

& BC || AD

ABCD is a ||gm



$\Delta ABC \qquad \Delta CAD$

$\angle BAC = \angle ACD$

$\angle ACB = \angle CAD$

$AC = AC$

$\Delta ABC \cong \Delta CDA$

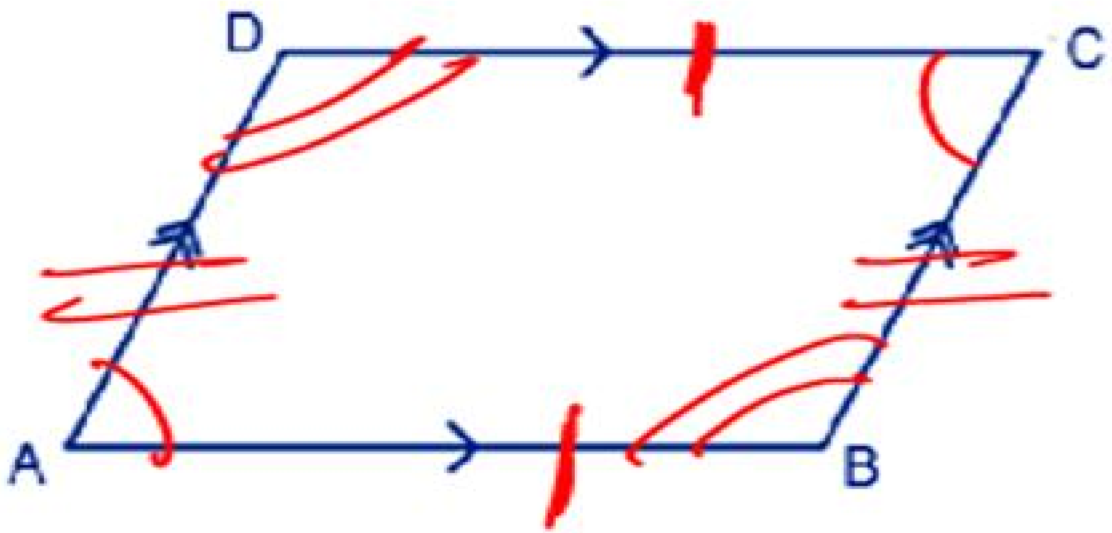
[ASA]

$AB = CD$

$BC = AD$

PROPERTIES OF PARALLELOGRAM

1. Opposite sides and opposite angles of parallelogram are equal.



(i) $AB = CD$

$BC = AD$

(ii) $\angle A = \angle C$

$\angle B = \angle D$

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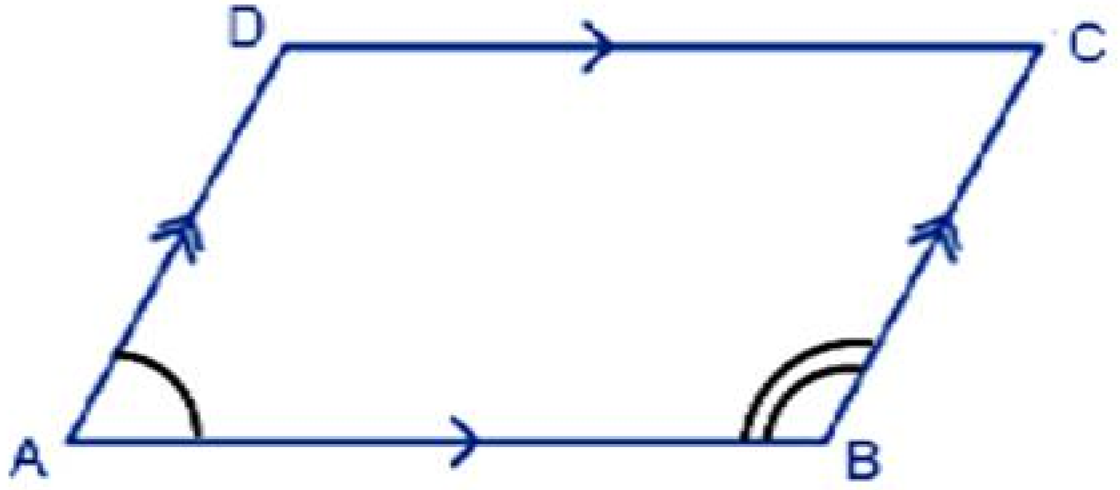
2. Sum of adjacent angles of a parallelogram is 180°.



$\angle A + \angle B = 180°$

Similarly

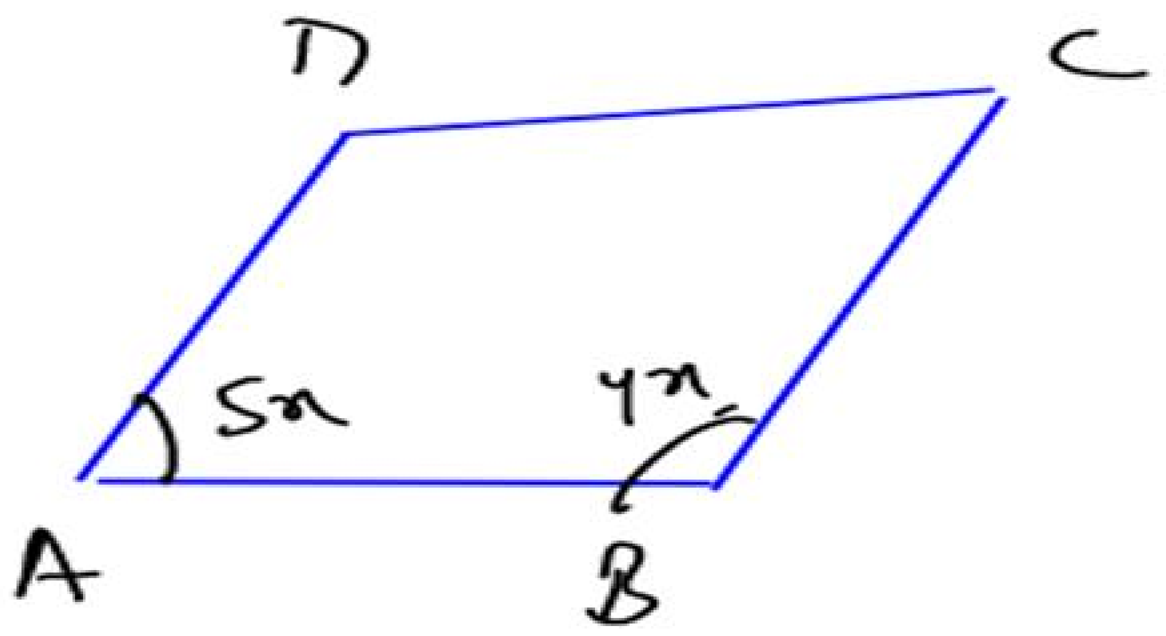
$\angle B + \angle C = 180$

$\angle C + \angle D = 180$

$\angle D + \angle A = 180°$

Eg. In a parallelogram ABCD, $\angle A : \angle B = 5 : 4$

Find the value of $\angle D$.



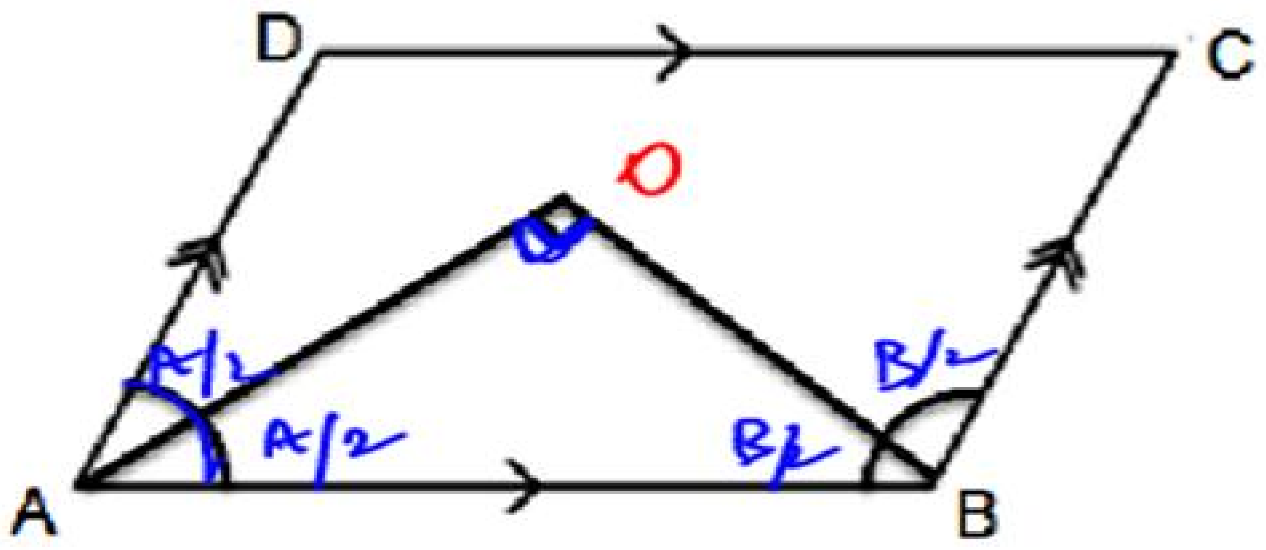
$9x = 180$

$x = 20$

$\angle B = 80$

$\angle D = 80^\circ$

3. (i) Angle bisectors of adjacent angles of a parallelogram intersect each other at 90°.



$\angle AOB = 90°$

Reason

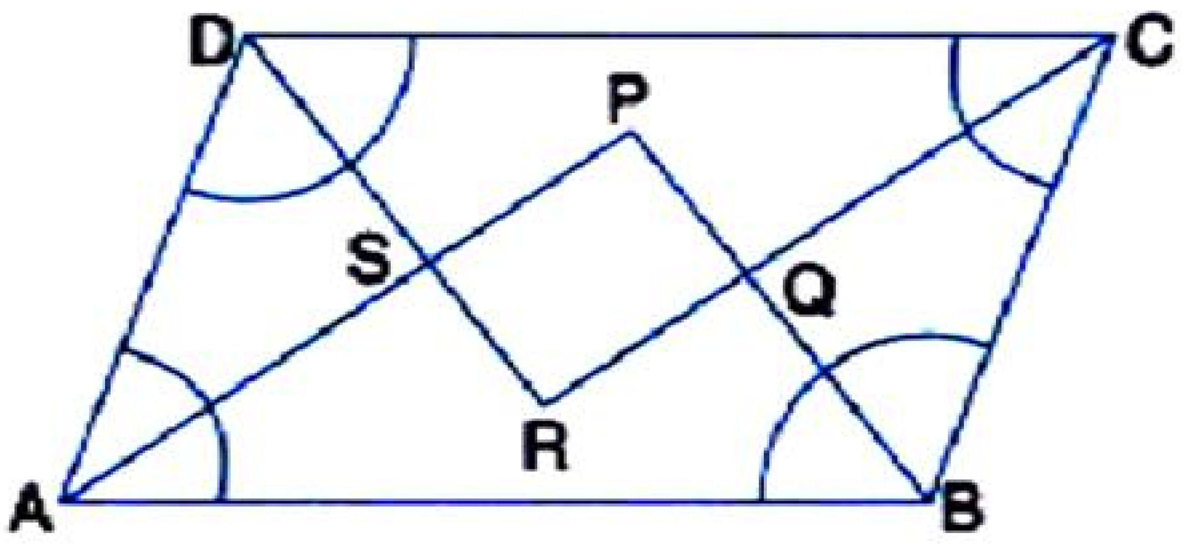
In ΔAOB

$\frac{A}{2} + \angle AOB + \frac{B}{2} = 180$

$90 + \angle AOB = 180$

$\angle AOB = 90°$

3. (ii) Angle bisector of a parallelogram forms a rectangle.

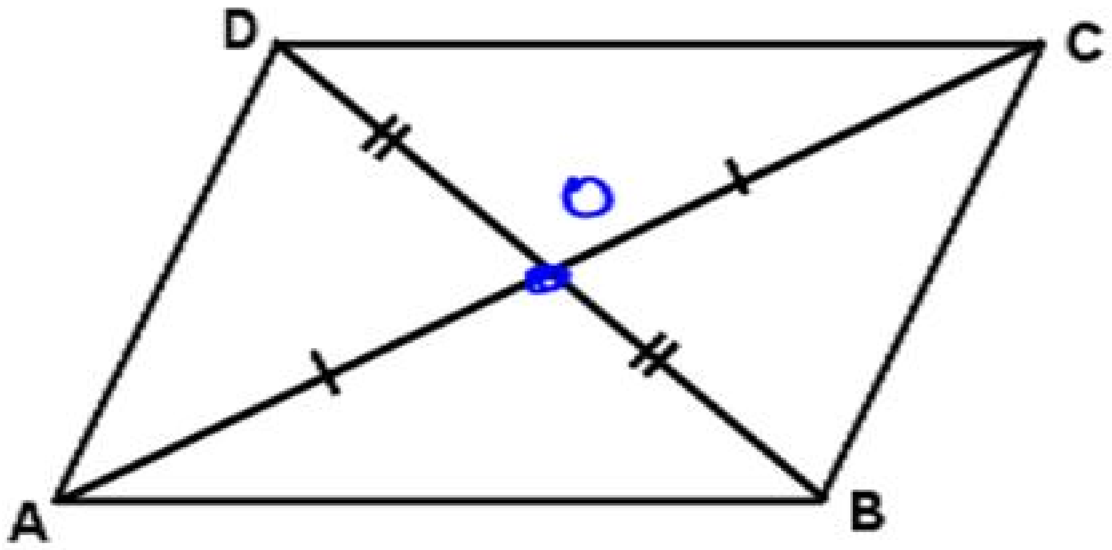


ABCD is a parallelogram.

AP, BP, CR and DR are bisectors of $\angle A$, $\angle B$, $\angle C$ & $\angle D$.

Then, PQRS is a rectangle.

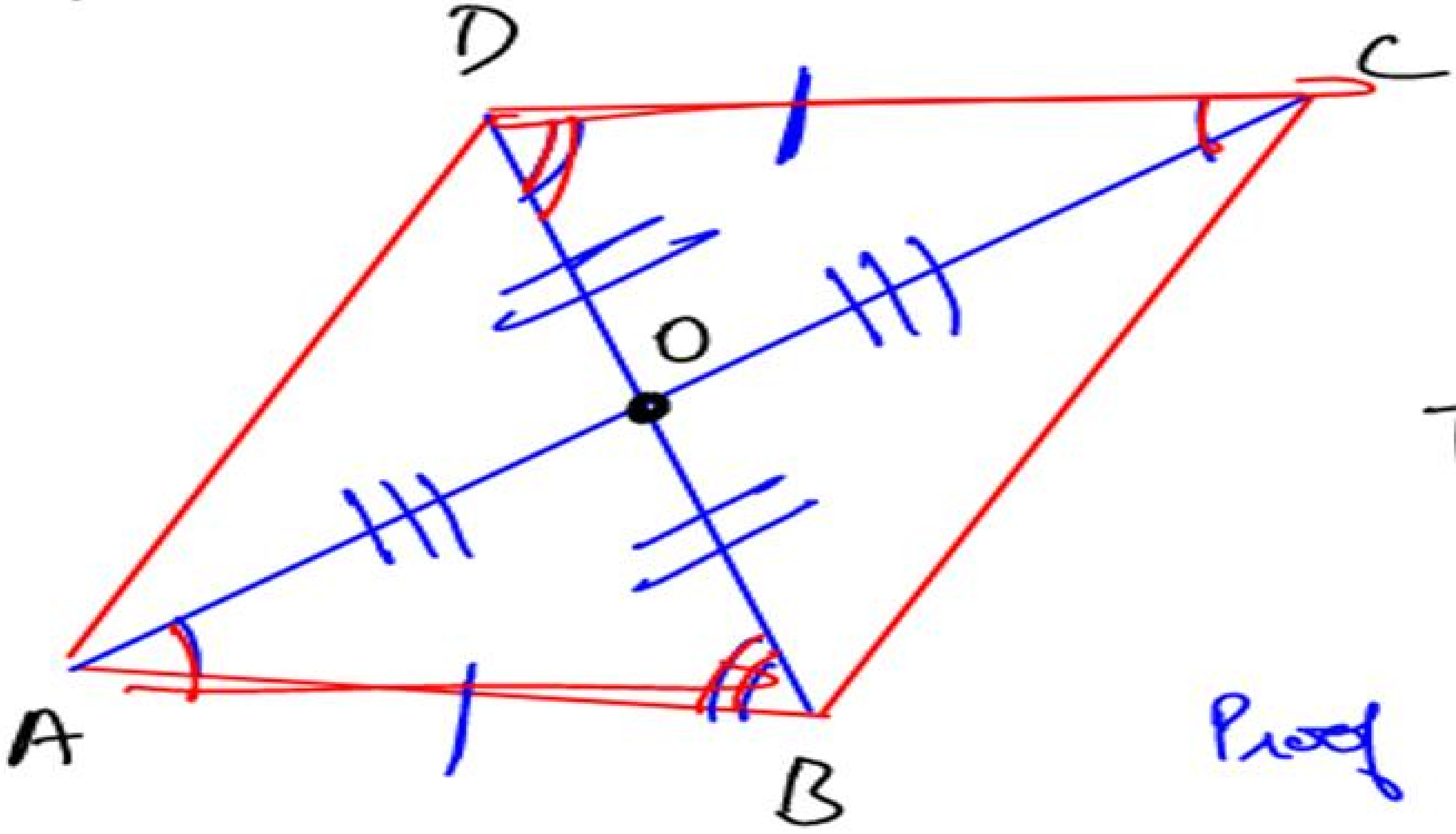
4. (i) Diagonals of a parallelogram bisect each other, but not necessarily at 90°.



(i) $AO = OC$

& $BO = OD$

(ii) $\angle BOC \neq 90^\circ$



ABCD is a ||gm

AC & BD intersect at

O

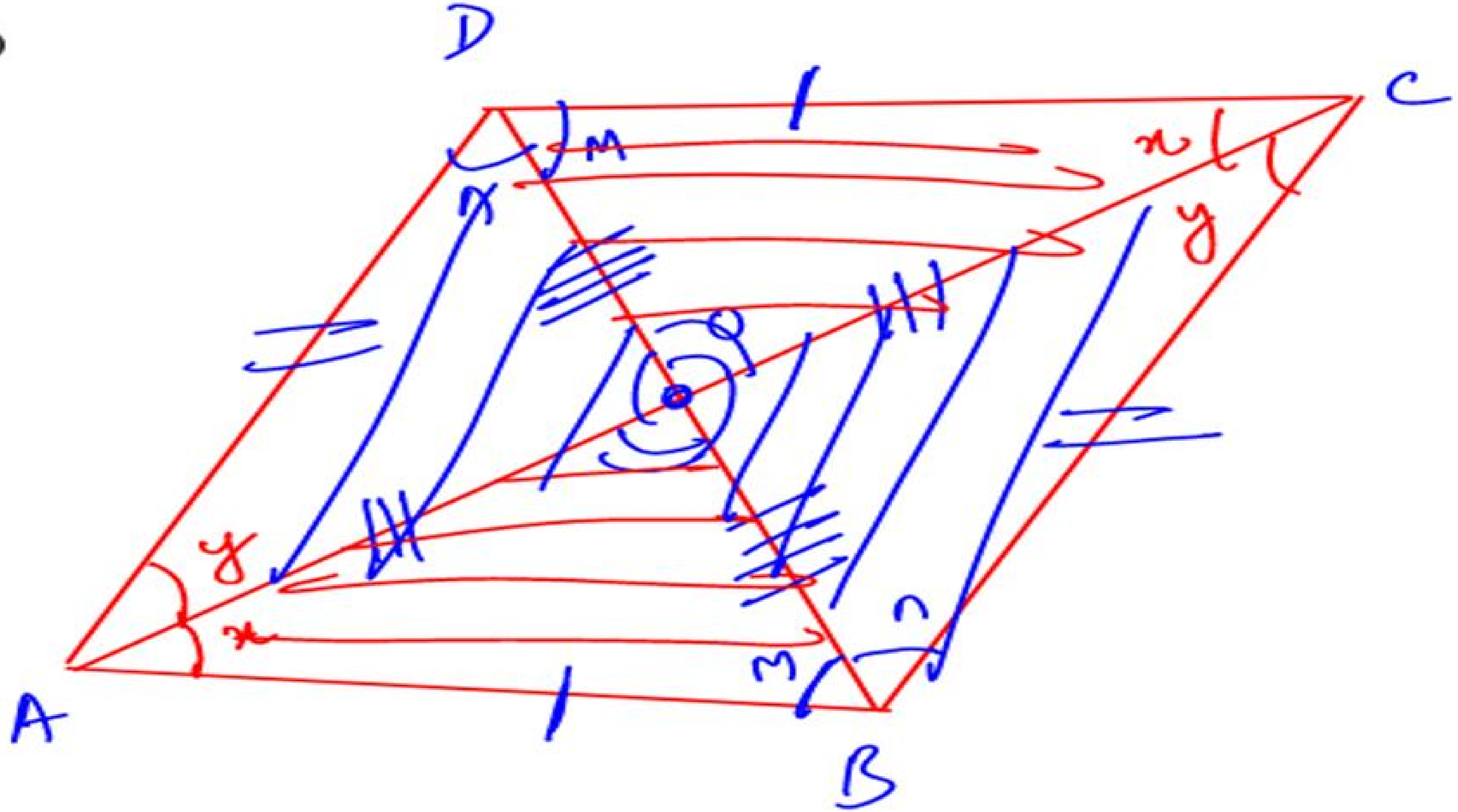
To prove AO = OC &

BO = OD

Proof ΔAOB & ΔCOD

$\Delta AOB \cong \Delta COD$ (ASA)

AO = CO


D
C
A
B
M
M
x
y
x
y
O

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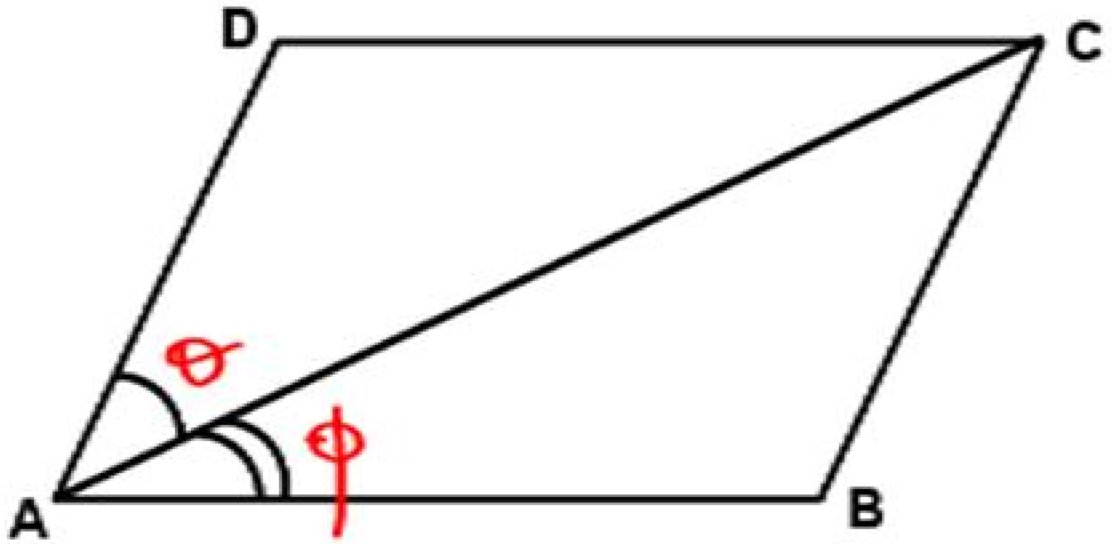
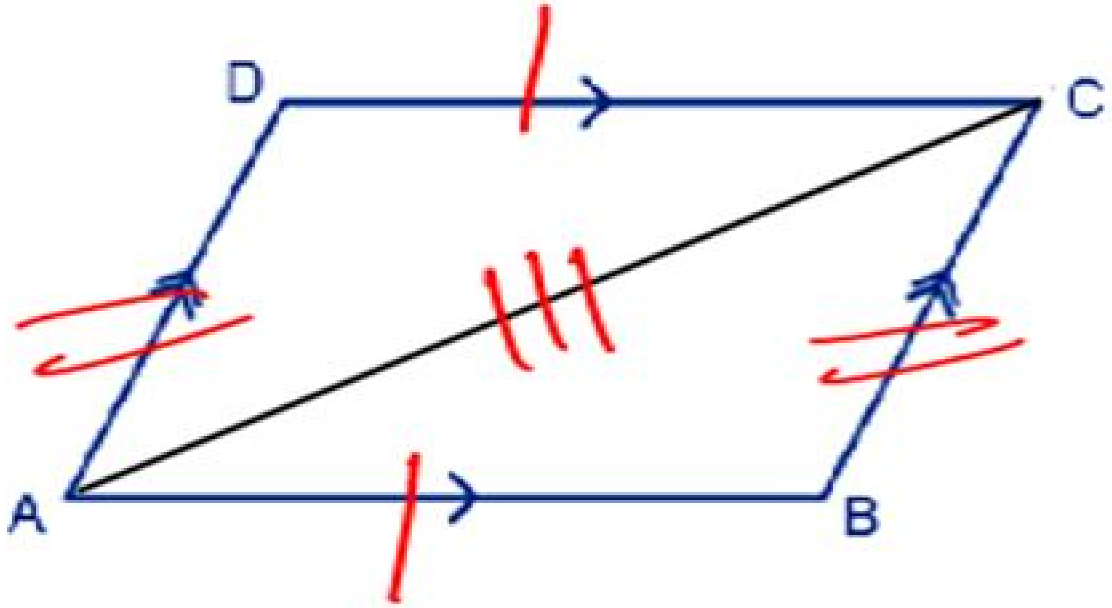
4. (ii) Diagonals of a parallelogram need not be angle bisector.



5. Diagonal of a parallelogram divides it into 2 congruent triangles.



$\Delta ABC \cong \Delta CDA$

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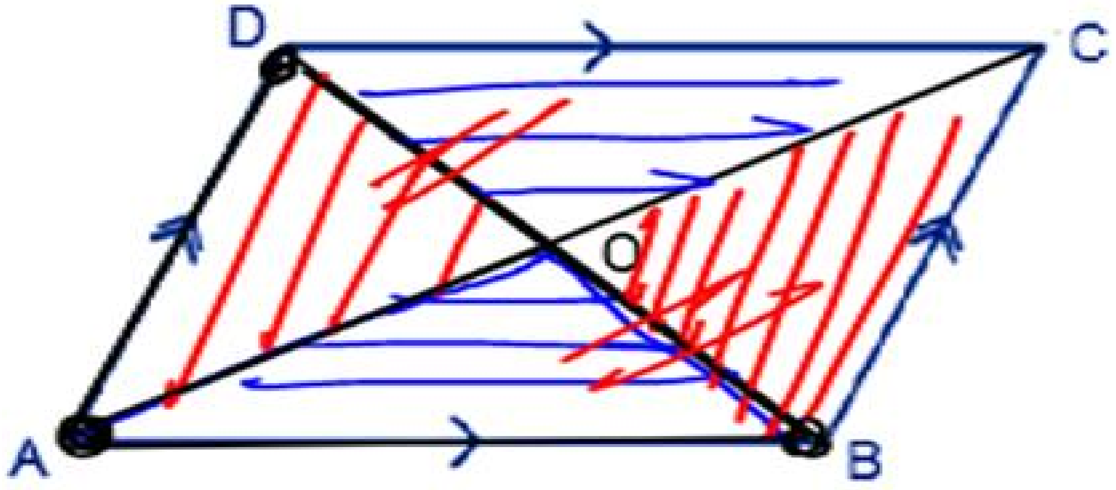
6. If diagonals AC and BD of a parallelogram intersect each other at O.



Area of ($\Delta AOB = \Delta BOC = \Delta COD = \Delta DOA$)

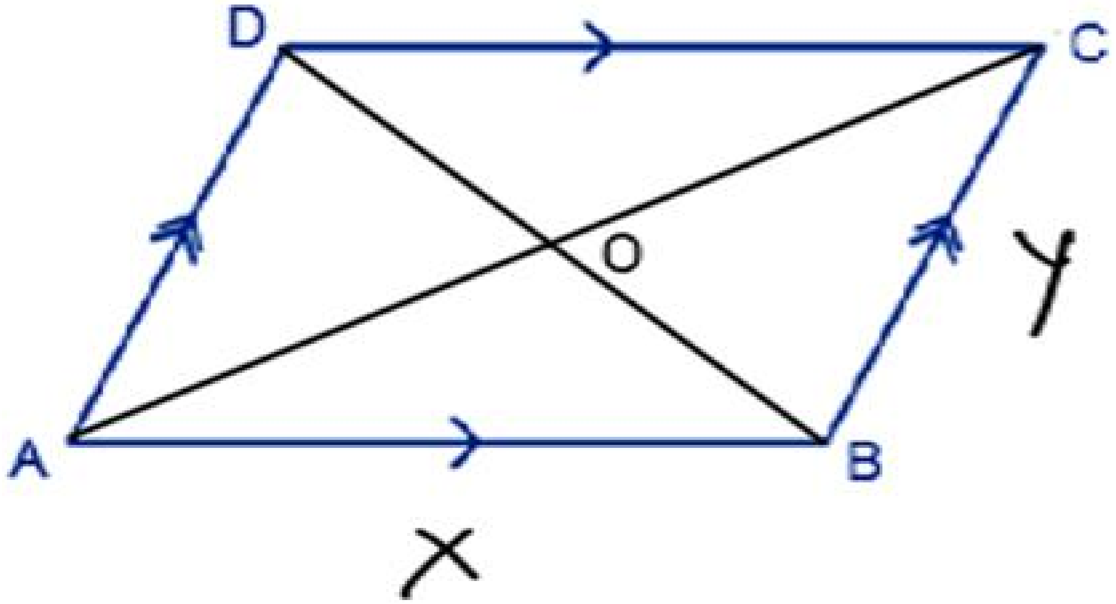
Reason

ΔABD

AO is median

area of ΔAOD = area of ΔAOB

7. $AC^2 + BD^2 = 2(AB^2 + BC^2)$



$$D_1^2 + D_2^2 = 2(x^2 + y^2)$$

Reason → Appolonius

→ cosine Rule

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Eg. If the 2 sides of a parallelogram are 12 cm and 15 cm and one of its diagonal is of length 17 cm. Find length of 2nd diagonal.

$$D_1^2 + D_2^2 = 2(x^2 + y^2)$$

$$17^2 + D_2^2 = 2(12^2 + 15^2)$$

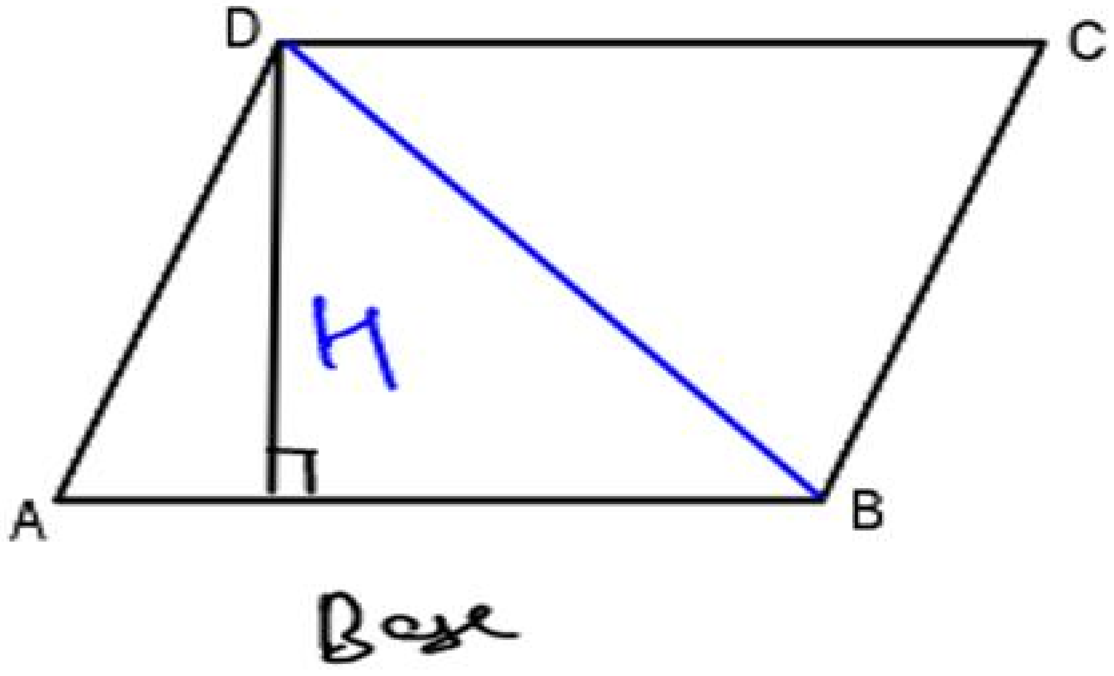
$$D_2^2 = 738 - 289$$

$$D_2 = \sqrt{449}$$

Ans. $\boldsymbol{x = \sqrt{449}}$

8. Area of parallelogram :

(i) Base × Height



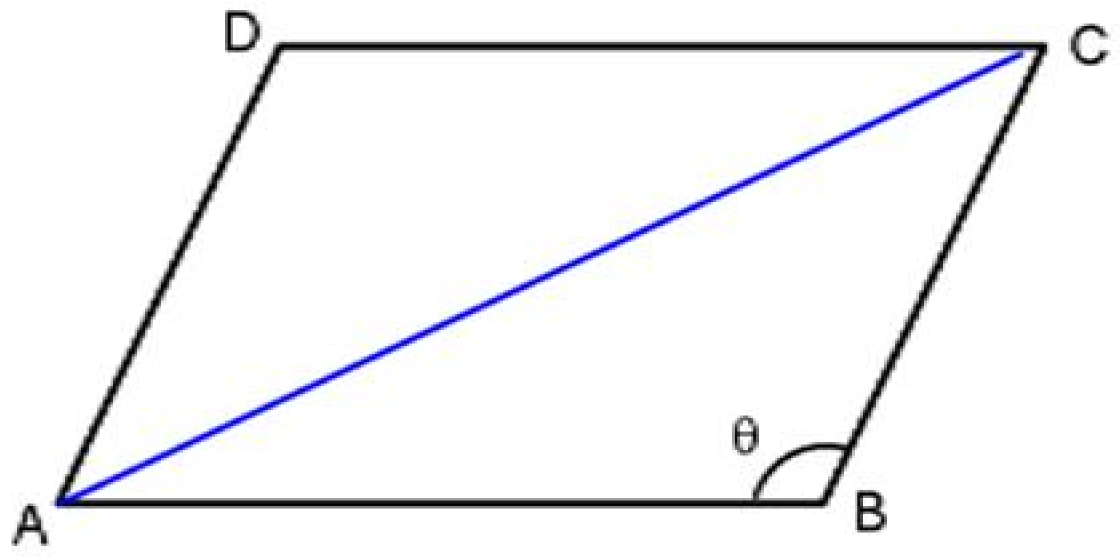
Area of ABCD

$= 2$ area of ΔABD

$= \cancel{2} \times \frac{1}{\cancel{2}} BH$

$\boxed{\text{Base} \times \text{Height}}$

(ii) Area of parallelogram = $AB \cdot BC \cdot \sin\theta$



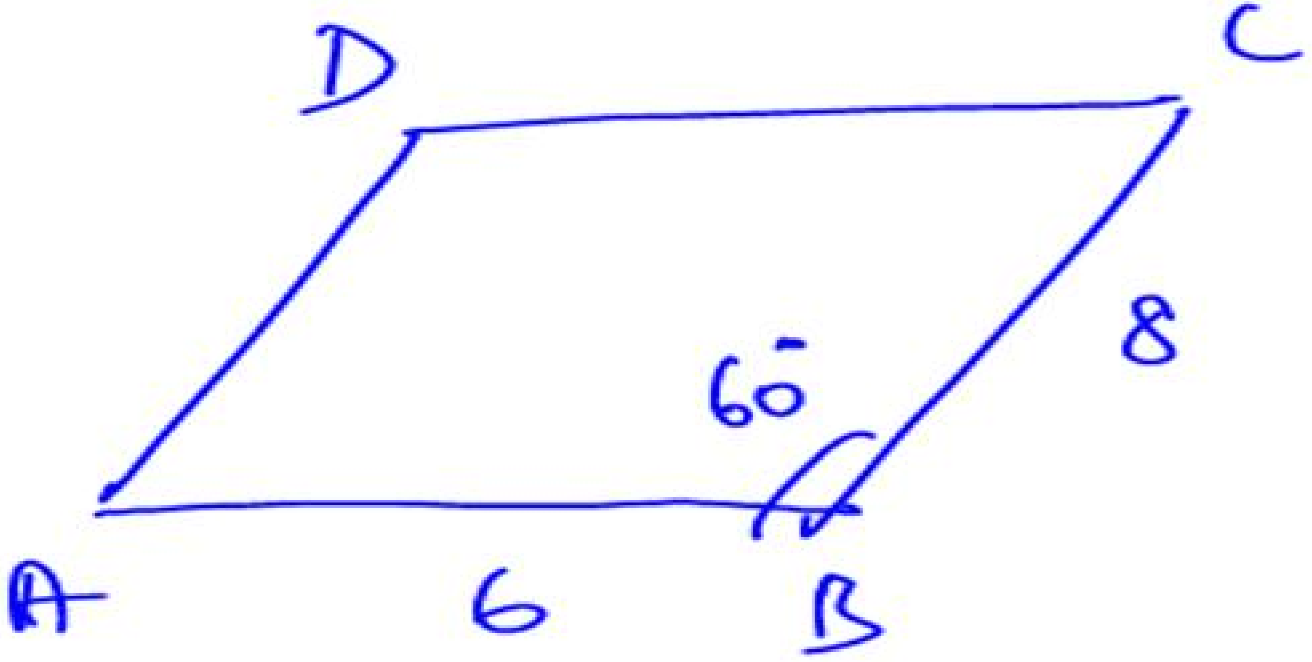
where, AB and BC are adjacent sides of a || gm and θ is the angle between them.

Area of ABCD = 2 area of ΔABC

$$= \cancel{2}\left(\frac{1}{\cancel{2}} AB \cdot BC \sin\theta\right)$$

$$AB \cdot BC \sin\theta$$

Eg. If 2 sides of a parallelogram are 6 cm and 8 cm and angle between them is 60°. Find area of parallelogram.



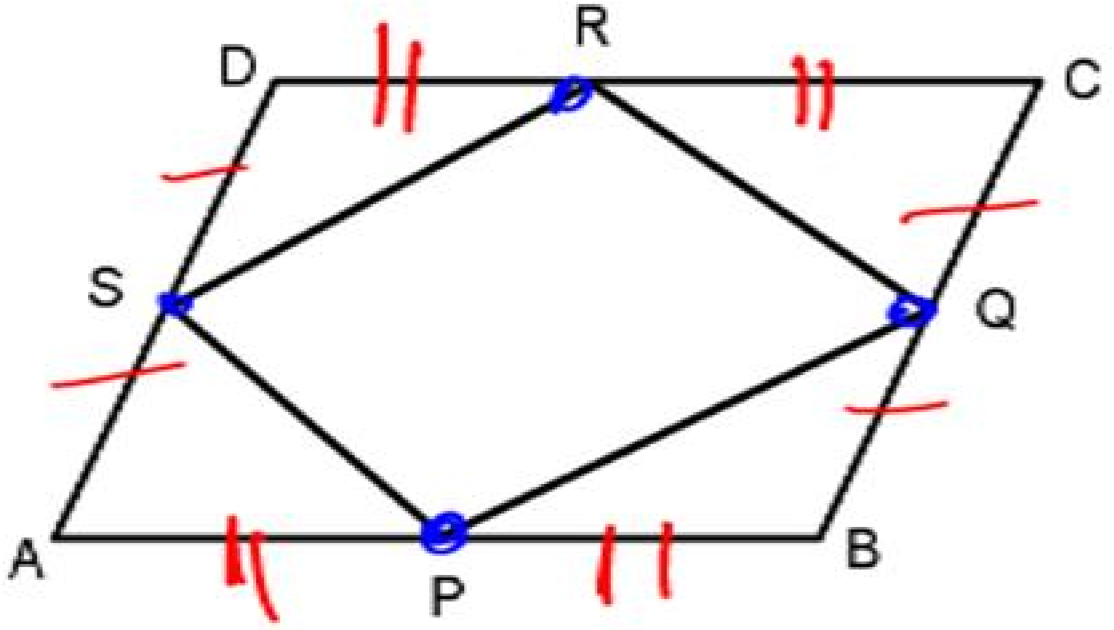
Area of Parallelogram

$$6 \cdot 8 \cdot \sin 60$$

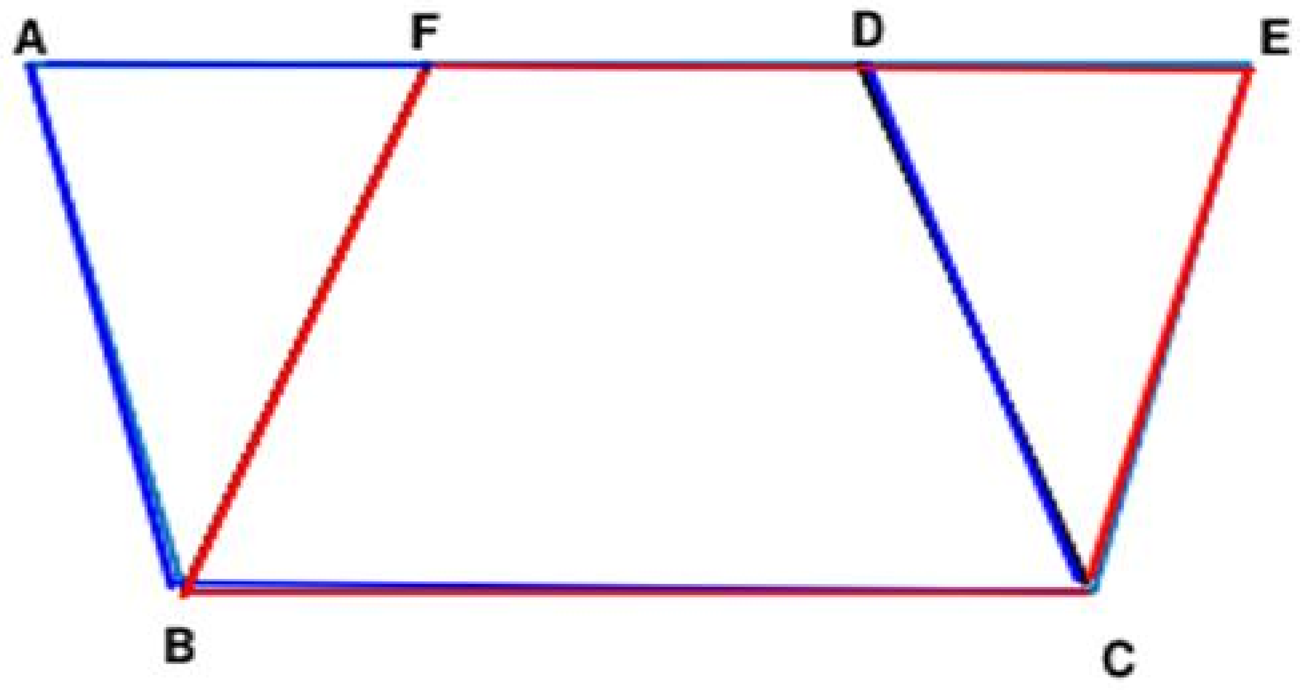
$$6 \cdot \overset{4}{\cancel{8}} \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$\underline{\underline{24\sqrt{3}\ cm^2}}$$

9. Figure formed by joining the mid-point of all sides of a parallelogram, is a PARALLELOGRAM and its area is half of the parallelogram.



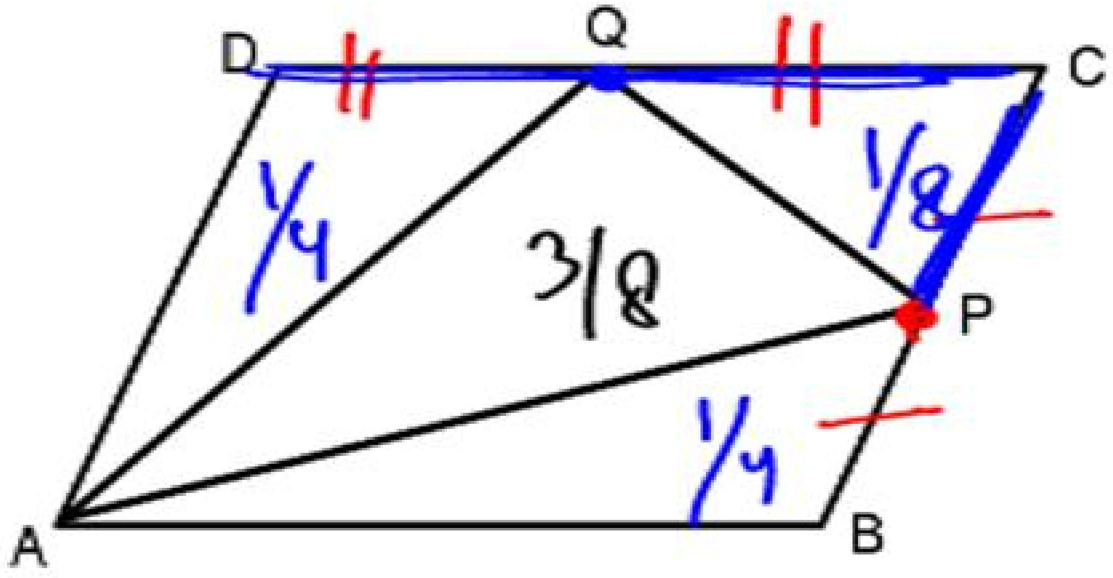
10. Parallelogram drawn on the same base and between same parallels have equal areas.



Area of | | gm ABCD = Area of | | gm BCEF

11. In a parallelogram ABCD, P, Q are mid points of BC and CD respectively.

$$\boldsymbol{Area\ of\ \Delta APQ = \frac{3}{8} Area\ of\ ABCD}$$

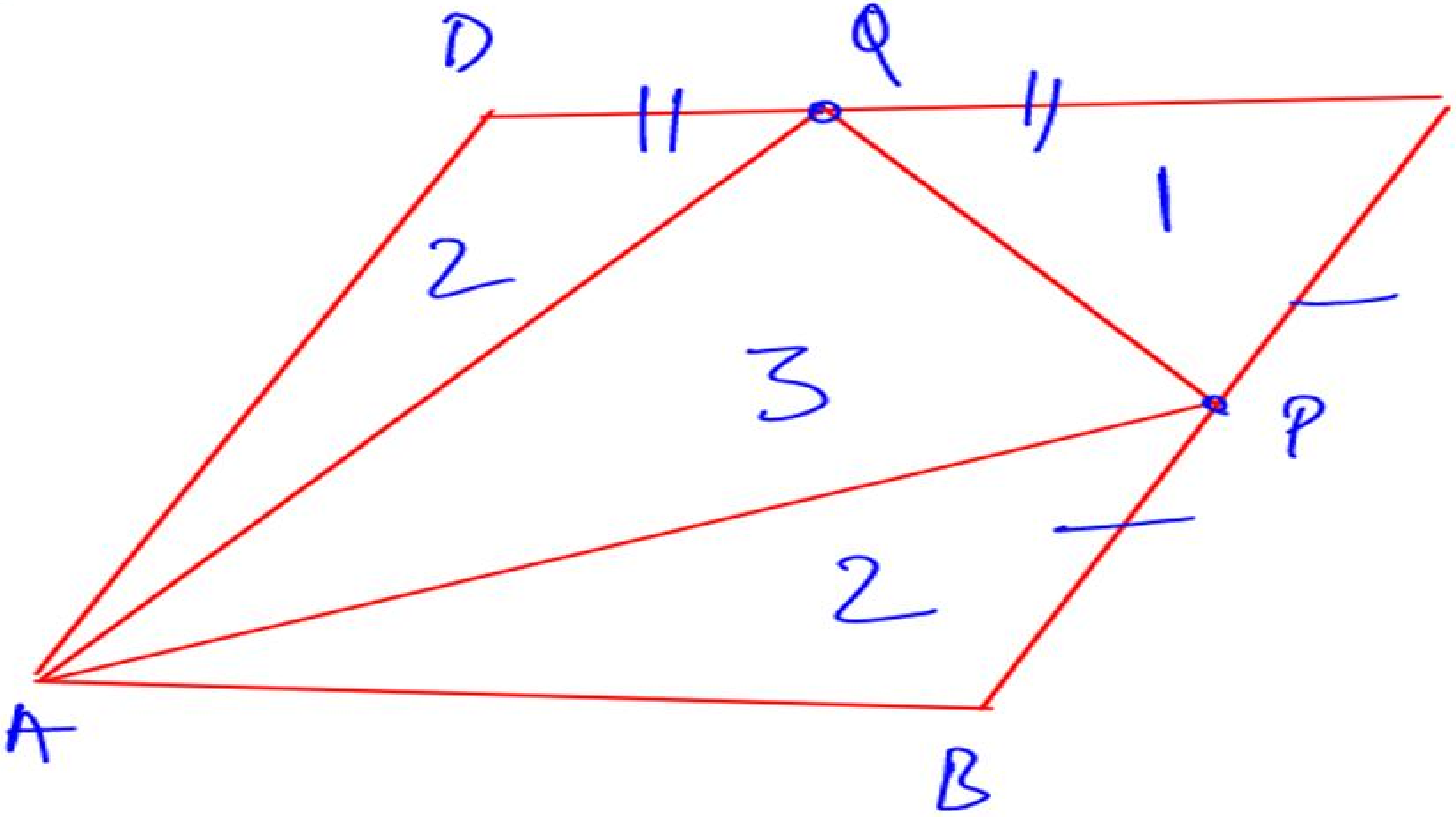


Reason

$$\frac{\text{area of } \Delta ABP}{\text{area of ||gm } ABCD} = \frac{\frac{1}{2}\cancel{B_1}\,\cancel{H_1}}{\cancel{B_2}\,\cancel{H_2}} = \frac{1}{4}$$

$$\frac{\text{area of } \Delta CPQ}{\text{area of } ABCD} = \frac{\frac{1}{2}\cancel{B_1}\,\cancel{H_1}}{\cancel{B_2}\,\cancel{H_2}} = \frac{1}{8}$$

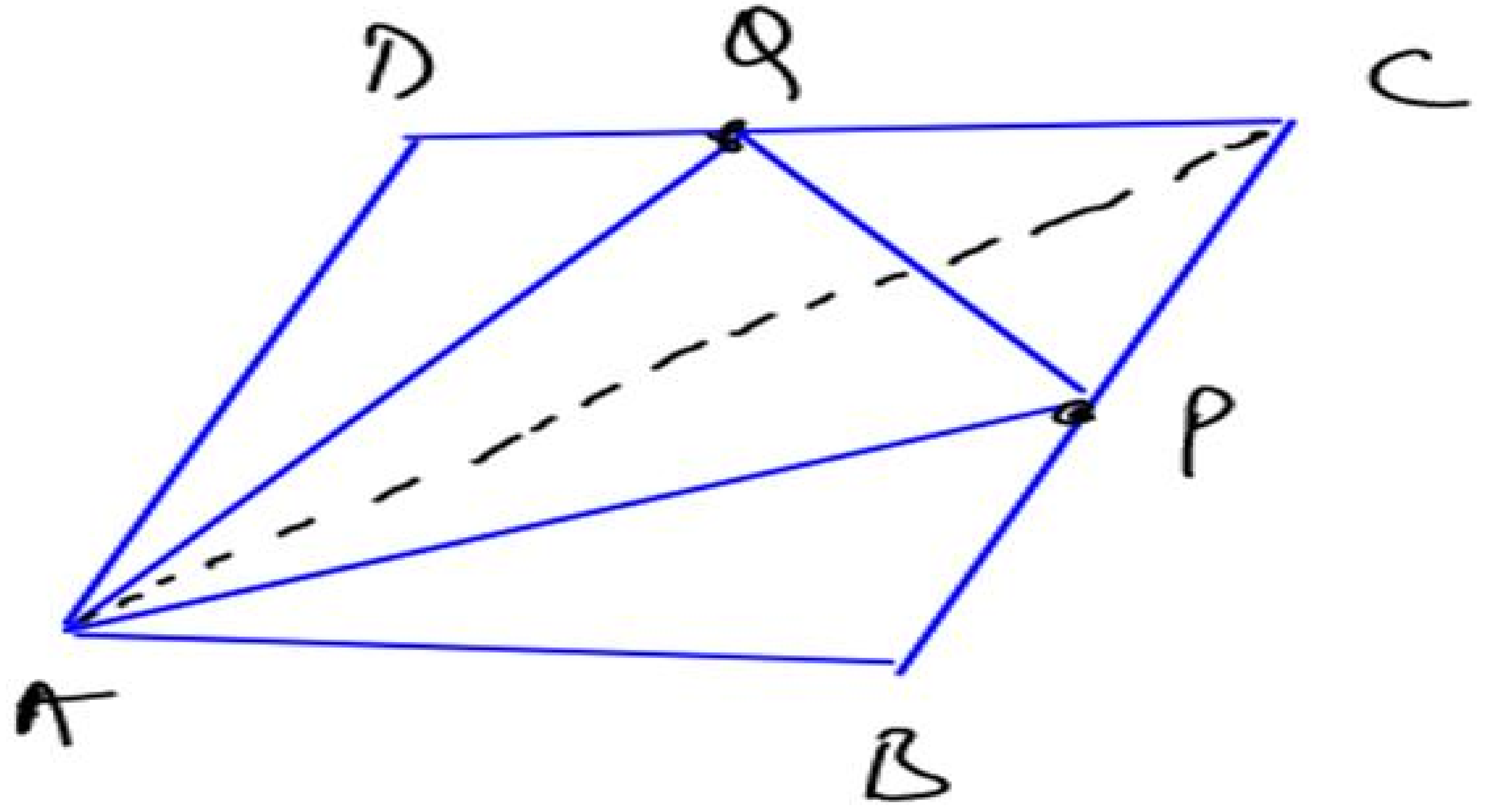
$$1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{8} \Rightarrow 3/8$$



If Area of ||gm ABCD

→ 8 units

Eg ABCD is a ||gm. P is the mid pt of BC & Q is the mid point of CD. If area of $\Delta ABC = 12cm^2$. Find Area of ΔAPQ ??



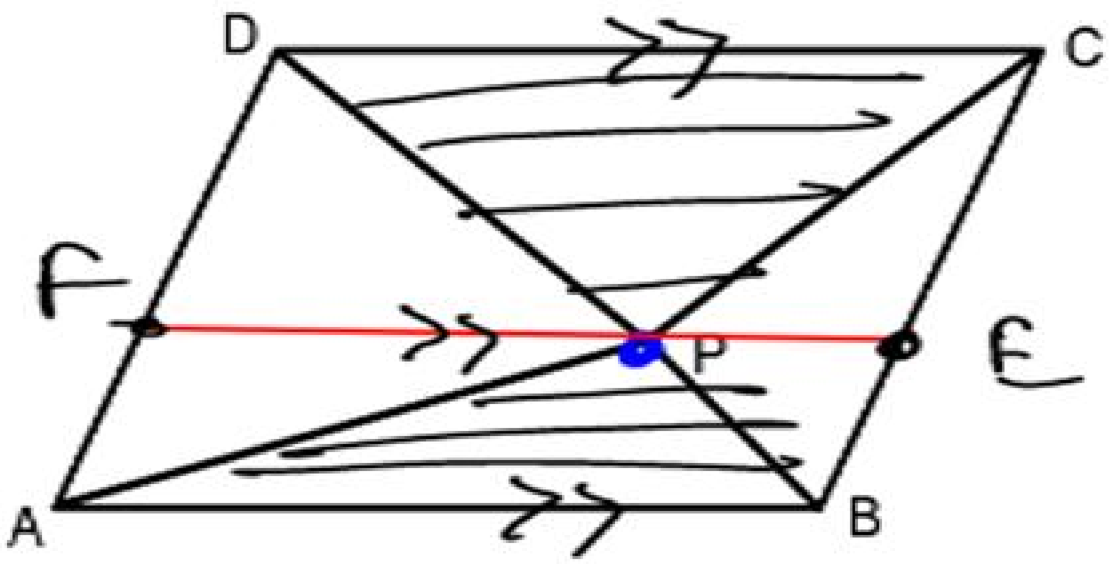
$4 \longrightarrow 12cm^2$

$1 \longrightarrow 3cm^2$

$3 \longrightarrow 9cm^2$

12. If P is any point in the interior of || gm ABCD, then

Area of ($\Delta APB + \Delta CPD$) = Area of ($\Delta BPC + \Delta APD$) = ½ || gm ABCD



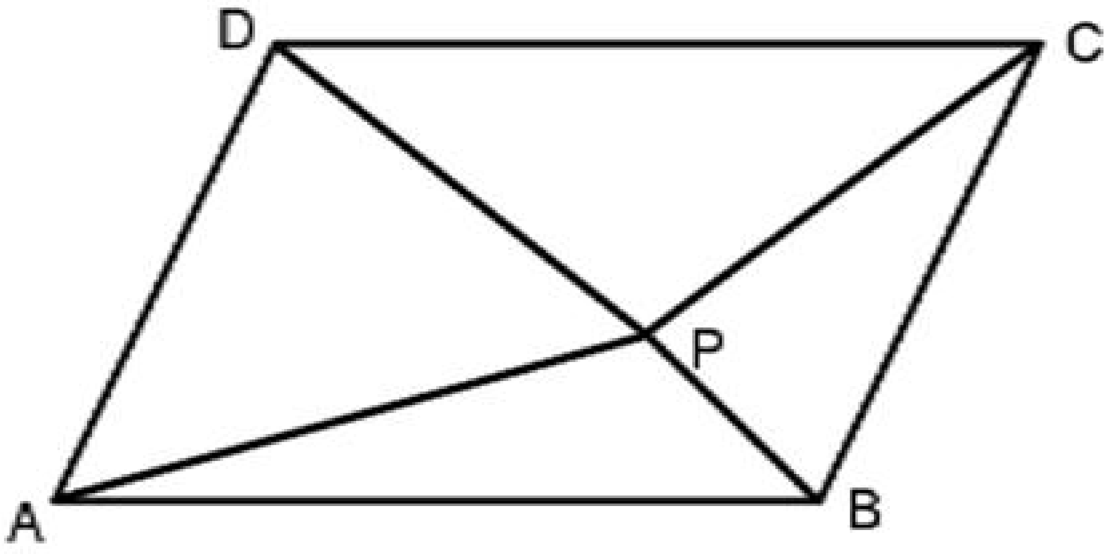
Reason

area of $\Delta ABP = \frac{1}{2}$ area of ||gm ABEF

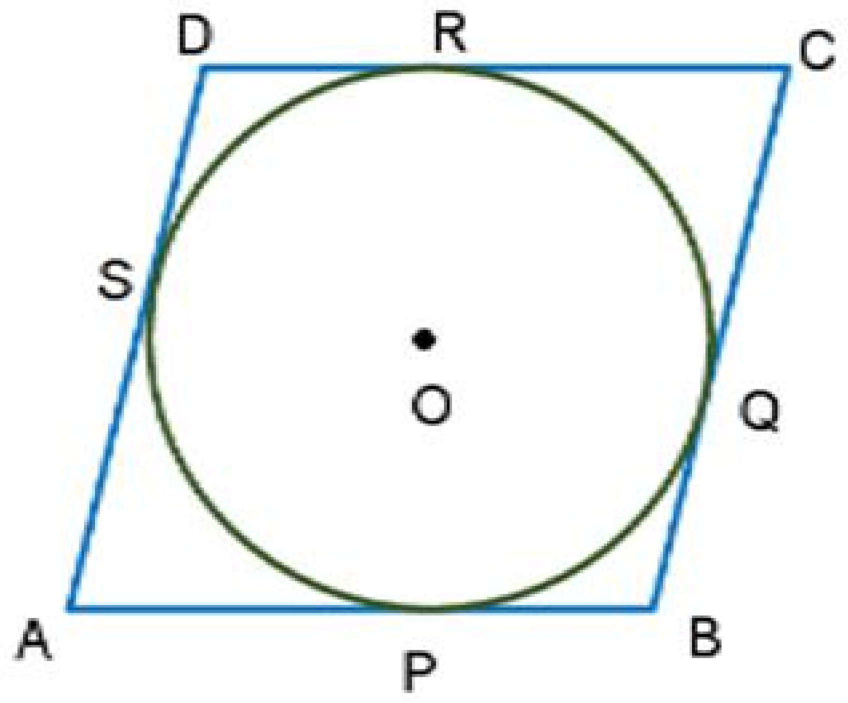
area of $\Delta CPD = \frac{1}{2}$ area of ||gm ECDF

area of $\Delta ABP + \Delta CPD = \frac{1}{2}$ area of ||gm ABCD

Reason:



PARALLELOGRAM CIRCUM SCRIBING A CIRCLE IS A RHOM BUS



AB = BC = CD = DA

In quad

$\underline{AB} + \underline{CD} = BC + AD$

$\cancel{2}AB = \cancel{2}BC$

$AB = BC$

ABCD is a Rhombus

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Given: ABCD be a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus.

We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, AP = AS, BP = BQ, CR = CQ and DR = DS.

Adding the above equations,

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$

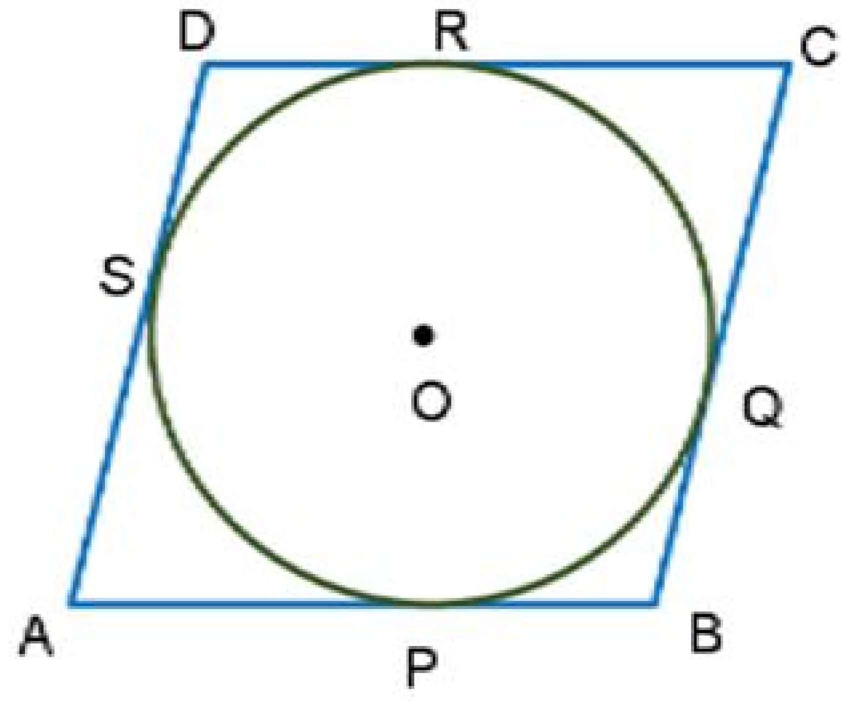
$2AB = 2BC$

(Since, ABCD is a parallelogram so AB = DC and AD = BC)

$AB = BC$

Therefore, AB = BC = DC = AD.

Hence, ABCD is a rhombus.



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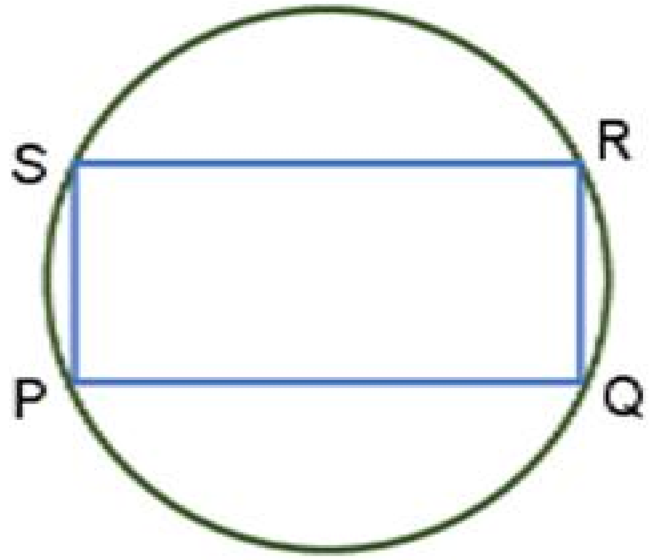
Parallelogram inscribe in a circle is rectangle.

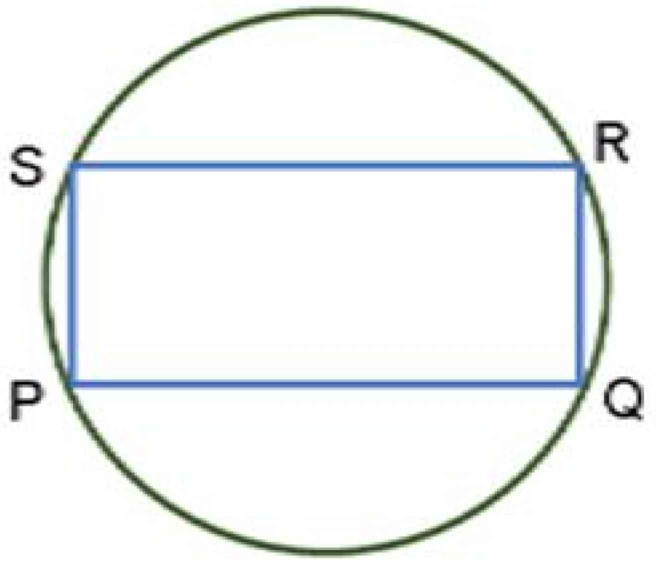


If PQRS is || gm, then PQRS is rectangle.

Reason $\longrightarrow$ Cyclic Quad

Given: PQRS is a parallelogram inscribed in a circle.

To prove: PQRS is a rectangle.



Proof: Since, PQRS is a cyclic quadrilateral.

$\therefore \angle P + \angle R = 180°$

($\because$ Sum of opposite angles in a cyclic quadrilateral is 180°) ...(i)

But $\angle P = \angle R$ ($\because$ In a || gm opposite angles are equal) ...(ii)

From Eqs. (i) and (ii), we get

$\angle P = \angle R = 90°$

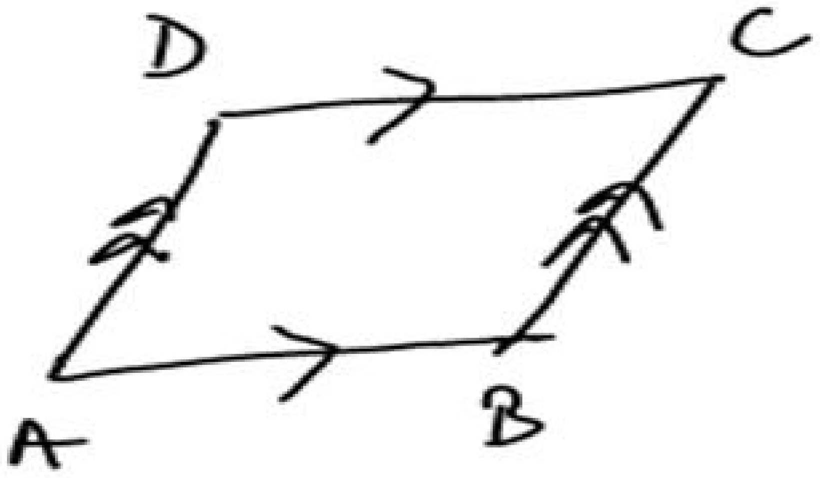
Similarly, $\angle Q = \angle S = 90$

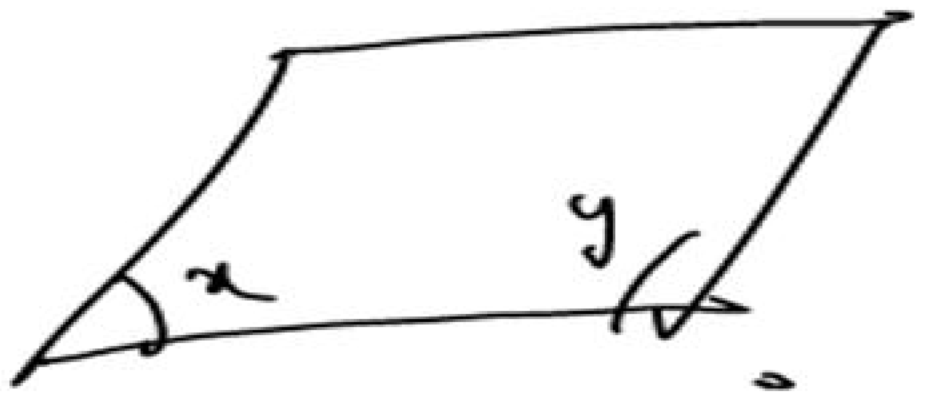
$\therefore$ Each angle of PQRS is 90°.

Hence, PQRS is a rectangle.

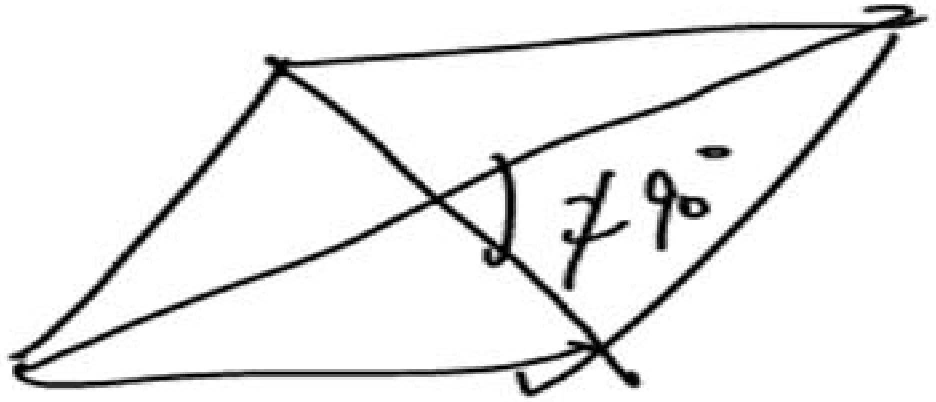
Parallelogram

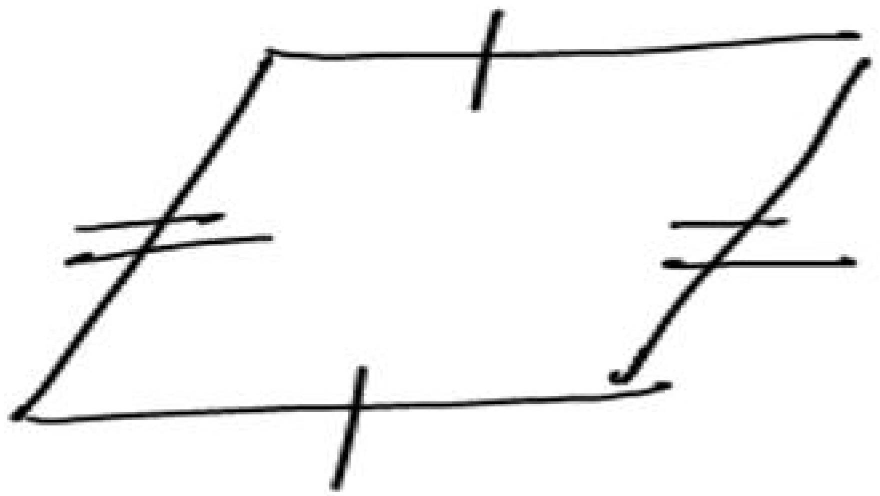
* Def

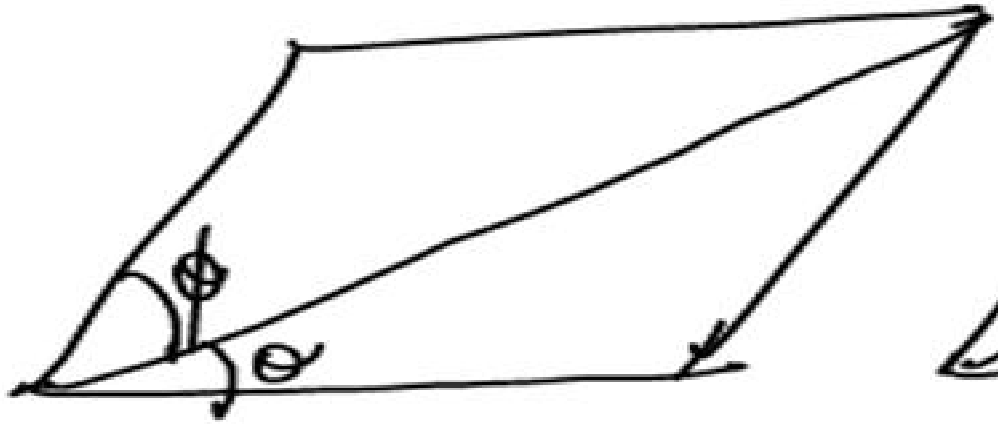


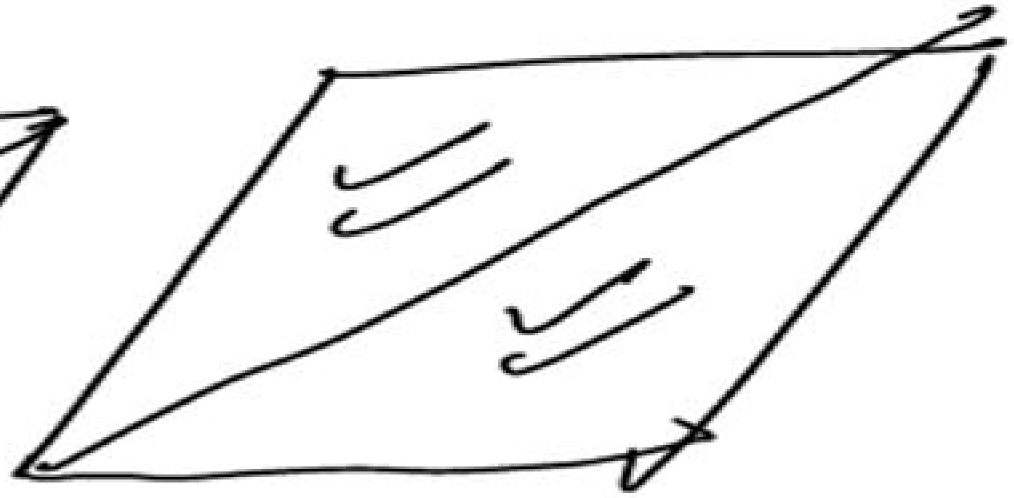


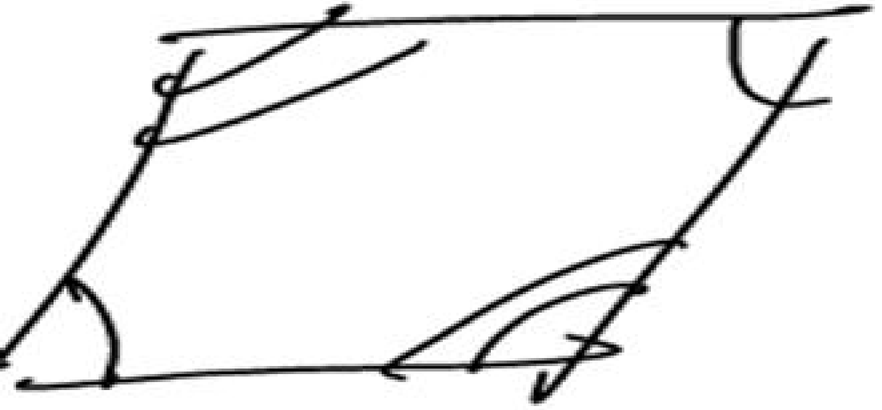
$x+y = 180°$

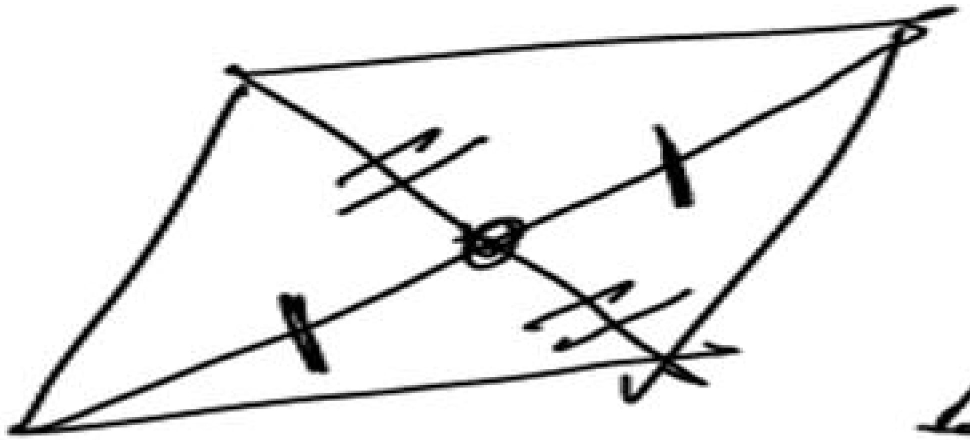


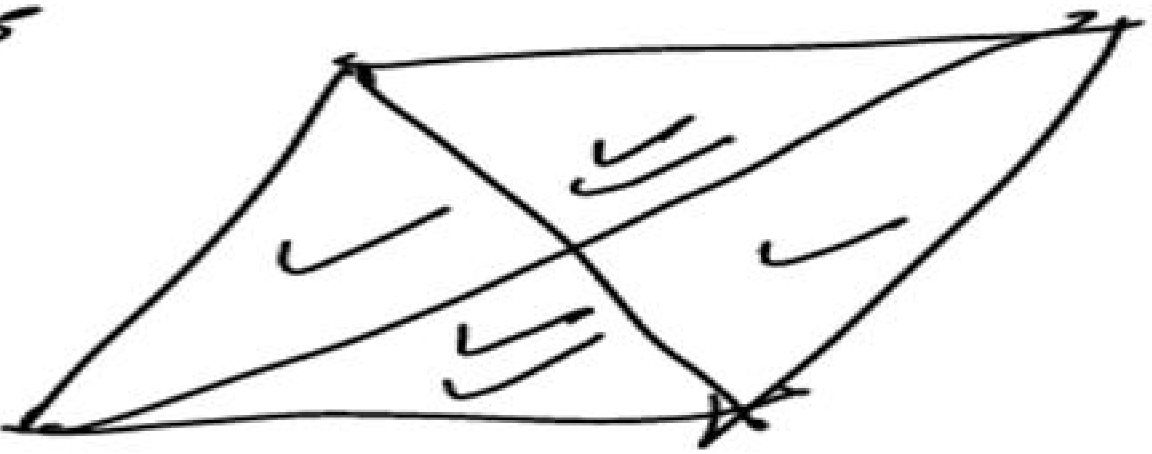


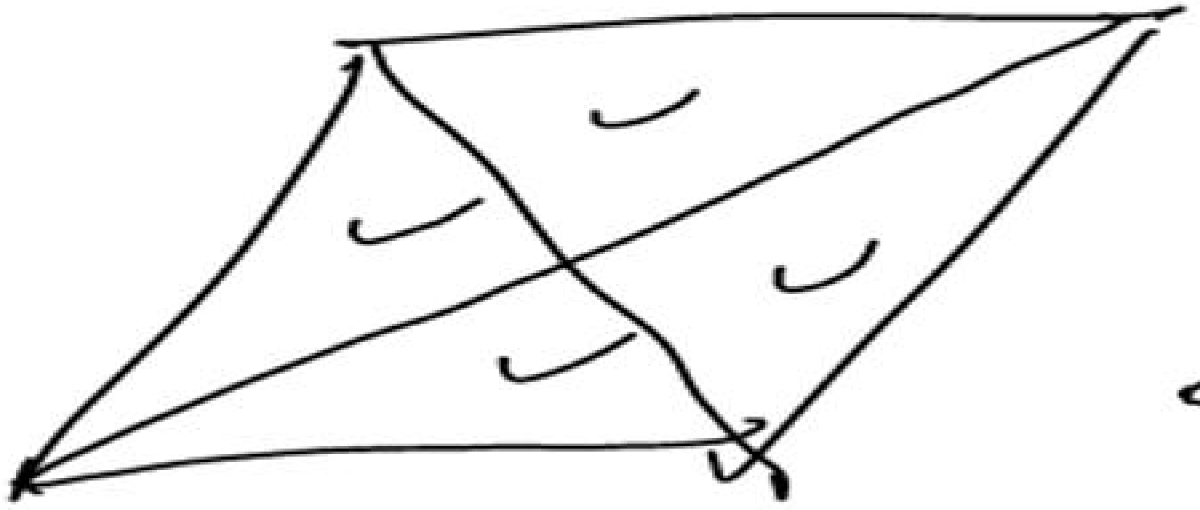


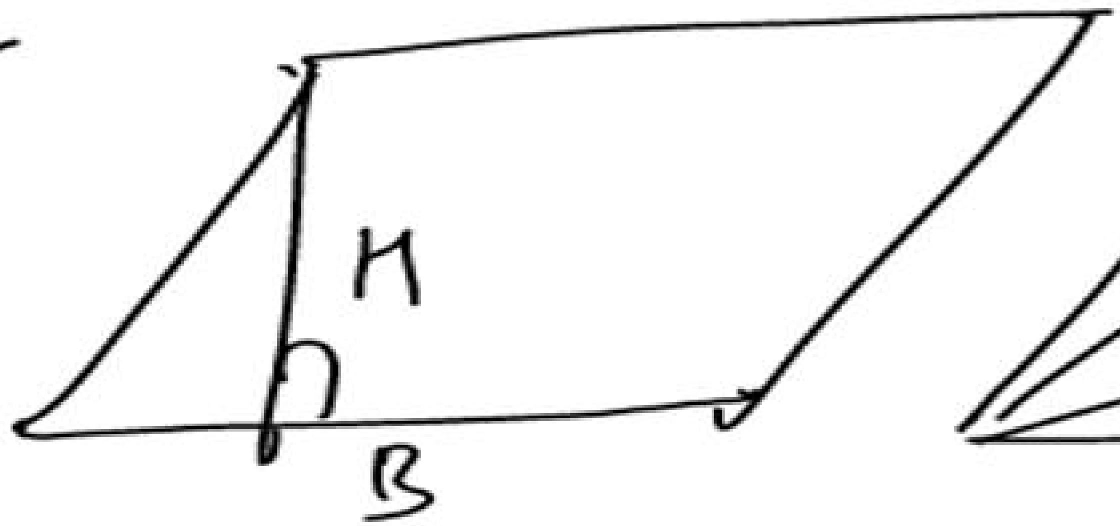




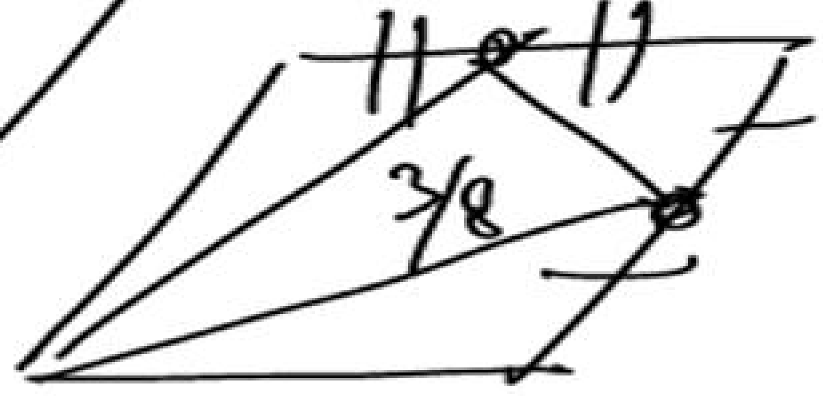


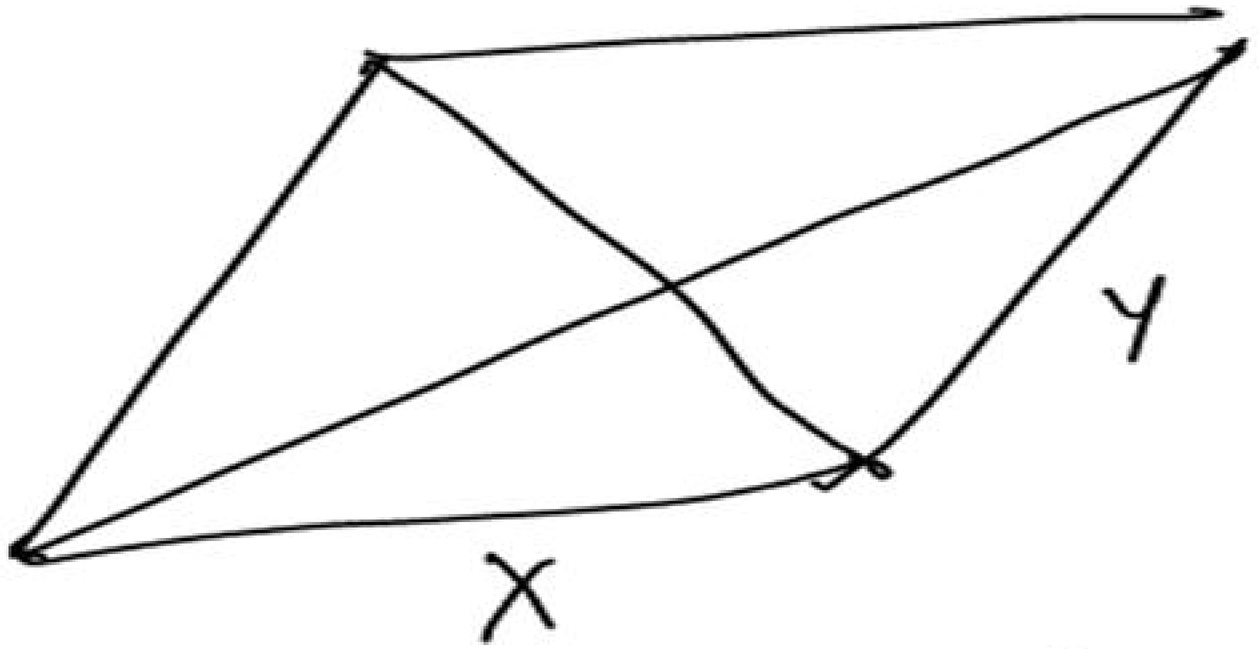


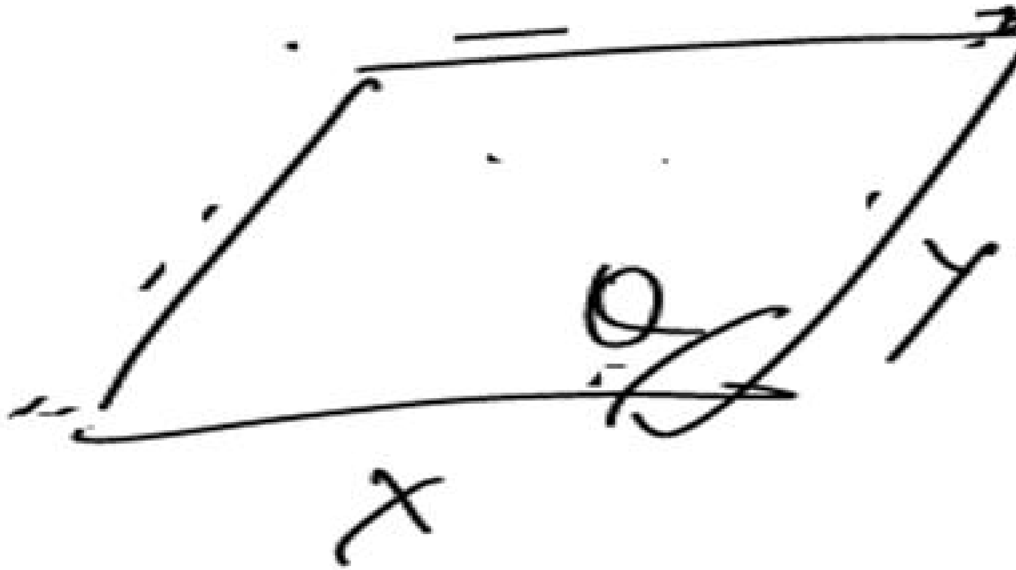


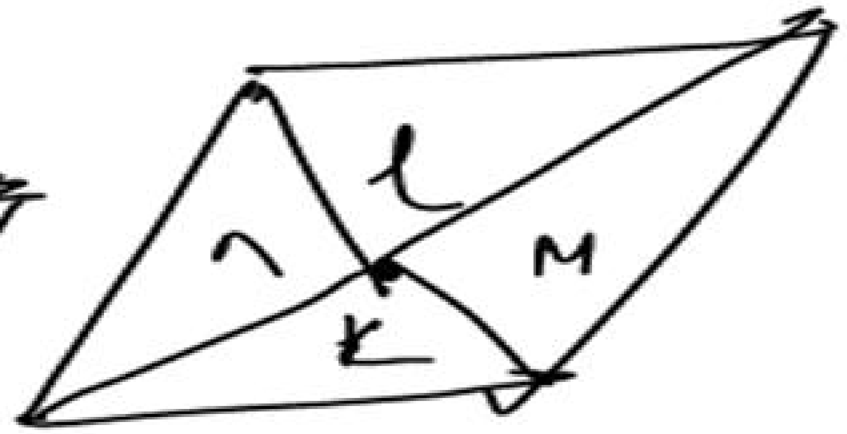


Area = $B \cdot H$





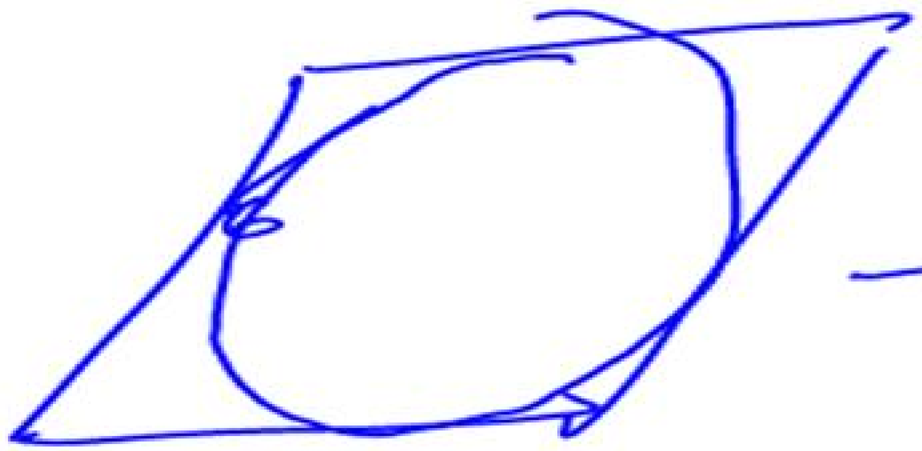




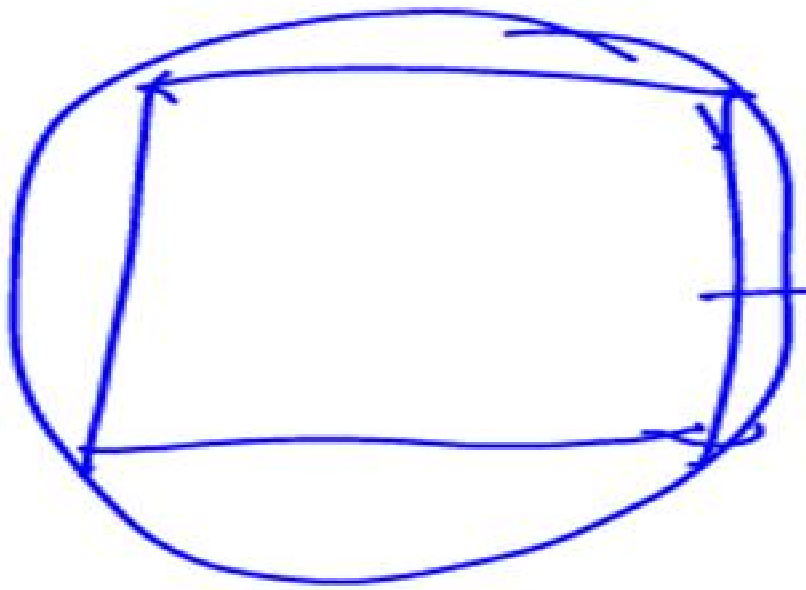
$K + l = m + n$

$D_1^2 + D_2^2 = 2(X^2 + Y^2)$

Area $= XY \sin\theta$



→ Rhombus



→ Rectangle

RHOMBUS

Def: Rhombus is a parallelogram in which adjacent sides are equal.