gradeup

Sahi Prep Hai Toh Life Set Hai

ALGEBRA-5

* Algebra - 5 (Extra session)

→ Discussing some imp Question

** Tomm we have a Doubt Session

Please send all your doubts before 7pm

Q10. If $x^{x\sqrt{x}} = \left(x\sqrt{x}\right)^{x}$, then x equals

(a) $\frac{4}{9}$ (b) $\frac{2}{3}$

(c) $\frac{9}{4}$ (d) $\frac{3}{2}$

$$x^{x^{3/2}} = \left(x^{3/2}\right)^{x}$$

$$x^{x^{3/2}} = x^{3/2 \cdot x}$$

$$x^{3/2} = \frac{3}{2}x$$

$$x \cdot x^{3/2} = \frac{9}{4}x^{2}$$

$$x = 9/4$$

Ans. (c)

Q11. If $x = \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $xy = 1$ then $\dfrac{x^2+xy+y^2}{x^2-xy+y^2}$ is

(a) $\dfrac{63}{61}$ (b) $\dfrac{67}{65}$

(c) $\dfrac{65}{63}$ (d) $\dfrac{69}{67}$

$$\rightarrow \frac{(x+y)^2-xy}{(x+y)^2-3xy} = \frac{63}{61}$$

$$x = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{8-2\sqrt{15}}{2}$$

$$= 4-\sqrt{15}$$

$$y = \frac{1}{x} = \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}} = 4+\sqrt{15}$$

$$\boxed{x+y = 8}$$

Ans. (a)

Q12. If $x=1+\sqrt{2}+\sqrt{3}$ then $(2x^4-8x^3-5x^2+26x-28)$ is

(a) $2\sqrt{2}$ (b) $3\sqrt{3}$

(c) $5\sqrt{5}$ (d) $6\sqrt{6}$

Time → 2min

$x-1=\sqrt{2}+\sqrt{3}$

$x^2-2x+1=5+2\sqrt{6}$

$x^2-2x-4=2\sqrt{6}$

$x^4-4x^3-4x^2+16x+16=24$

$x^4-4x^3-4x^2+16x-8=0$

$2x^4-8x^3-8x^2+32x-16=0$

$2x^4-8x^3-5x^2+26x-28=3x^2-6x+12$

$=3(x^2-2x-4)$

$=3\cdot 2\sqrt{6}$

$=6\sqrt{6}$

Ans. (d)

Q13. If $x = 16$, then $x^4 - 17x^3 + 17x^2 - 17x + 17$ is

(a) 0 (b) 1

(c) 4 (d) 3

$x^4 - 16x^3 - x^3 + 16x^2 + x^2 - 16x - x + 17$

$16^4 - 16 \cdot 16^3 - 16^3 + 16 \cdot 16^2 + 16^2 - 16 \cdot 16 - 16 + 17$

$= 1$

Ans. (b)

Q14. If $x = 11$ then $x^4 - 13x^3 + 12x^2 - 14x + 10 = ?$

(a) -1350 (b) -1354

(c) 0 (d) 1

$x^4 - 11x^3 - 2x^3 + 11x^2 + x^2 - 11x - 3x + 10$

$11^4 - 11 \cdot 11^3 - 2 \cdot 11^3 + 11^3 + 11^2 - 11^2 - 3 \cdot 11 + 10$

$-11^3 - 33 + 10$

$-1331 - 33 + 10$

$= -1354$

Ans. (b)

Q15. If $x = 3 + 2\sqrt{2}$, then $\frac{x^6 + x^4 + x^2 + 1}{x^3}$ is equal to–

(a) 216 (b) 192
(c) 198 (d) 204

$x = 3 + 2\sqrt{2}$

$\frac{1}{x} = 3 - 2\sqrt{2}$

$x + \frac{1}{x} = 6$

$x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$\left(x^3 + \frac{1}{x^3}\right) + \left(x + \frac{1}{x}\right)$

$6^3 - 3 \cdot 6 + 6$

$= 204$

Ans. (d)

Q16. If $x^2 + x = 5$ then $(x+3)^3 + \frac{1}{(x+3)^3}$ is

(a) 140 (b) 110

(c) 130 (d) 120

Let $(x+3) = m$

$x = m-3$

$(m-3)^2 + (m-3) = 5$

$m^2 - 5m + 1 = 0$

$m + \frac{1}{m} = 5$

$m^3 + \frac{1}{m^3}$

$= 5^3 - 3 \cdot 5$

$= 110$

Ans. (b)

Q17. If $x = 2 + 2^{\frac{2}{3}} - 2^{\frac{1}{3}}$ then find $x^3 - 6x^2 + 18x + 3$

(a) 22 (b) 24

(c) 25 (d) 0

$x - 2 = 2^{2/3} - 2^{1/3}$

$x^3 - 8 - 6x(x-2) = 4 - 2 - 3 \cdot 2^{2/3} 2^{1/3} (x-2)$

$x^3 - 8 - 6x^2 + 12x = 2 - 6x + 12$

$x^3 - 6x^2 + 18x = 22$

Ans. (c)

Q18. If $\frac{a}{1-2a}+\frac{b}{1-2b}+\frac{c}{1-2c}=1$ find $\frac{1}{1-2a}+\frac{1}{1-2b}+\frac{1}{1-2c}=?$

(a) 5 (b) 56

(c) 30 (d) 60

Sol^n Symmetricity $\rightarrow$ $\frac{1}{1-\frac{2}{5}}$

$3\left(\frac{a}{1-2a}\right)=1$

$3a=1-2a$

$a=\frac{1}{5}$

$=\frac{5}{3}\times 3$

$=5$

Ans. (a)

$$\frac{a}{1-2a} + \frac{b}{1-2b} + \frac{c}{1-2c} = 1 \quad \text{--- (1) } \times -2$$

$$\frac{1}{1-2a} + \frac{1}{1-2b} + \frac{1}{1-2c} = k \quad \text{--- (2) } \times 1$$

$$3 = -2 + k$$

$$k = 5$$

Q19. If $\frac{a}{2-3a}+\frac{b}{2-3b}+\frac{c}{2-3c}=2$ find $\frac{1}{2-3a}+\frac{1}{2-3b}+\frac{1}{2-3c}=?$

(a) 3.5 (b) 4.5

(c) 2.4 (d) None

$\frac{a}{2-3a}+\frac{b}{2-3b}+\frac{c}{2-3c}=2$ — (1) $\times -3$

$\frac{1}{2-3a}+\frac{1}{2-3b}+\frac{1}{2-3c}=K$ — (2) $\times 2$

$3=-6+2K$

$K=4.5$

Ans. (b)

Q20. If $\frac{a}{3+2a}+\frac{b}{3+2b}+\frac{c}{3+2c}=6$ find $\frac{1}{3+2a}+\frac{1}{3+2b}+\frac{2}{3+2c}=?$

(a) –2
(b) 5
(c) –3
(d) None

$$\frac{a}{3+2a}+\frac{b}{3+2b}+\frac{c}{3+2c}=6 \quad \text{—(1)} \times 2$$

$$\frac{1}{3+2a}+\frac{1}{3+2b}+\frac{1}{3+2c}=K \quad \text{—(2)} \times 3$$

$$3 = 12+3K$$

$$K=-3$$

Ans. (c)

Q21. If $\frac{a^2-bc}{a^2+bc}+\frac{b^2-ca}{b^2+ca}+\frac{c^2-ab}{c^2+ab}=1$ then the value of $\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ca}+\frac{c^2}{c^2+ab}$ is

(a) 0 (b) 1

(c) −1 (d) 2

$$\frac{a^2-bc}{a^2+bc}+\frac{b^2-ca}{b^2+ca}+\frac{c^2-ab}{c^2+ab}=1 \quad \times(-1)$$

$$\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ca}+\frac{c^2}{c^2+ab}=K \quad \times 2$$

$$3=-1+2K$$

$$K=2$$

Ans. (d)

Q22. If $a + b + c = 4\sqrt{3}$ and $a^2 + b^2 + c^2 = 16$ then $a : b : c = ?$

(a) $1 : 1 : 1$ (b) $1 : \sqrt{2} : \sqrt{3}$

(c) $1 : 2 : 3$ (d) None

II $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$48 = 16 + 2(ab+bc+ca)$

$ab+bc+ca = 16$

$a^2+b^2+c^2 = ab+bc+ca$

$a = b = c$

I symmetricity

$a = b = c$

Check

$a+a+a = 4\sqrt{3}$

$a = \frac{4}{\sqrt{3}}$

$\frac{16}{3} + \frac{16}{3} + \frac{16}{3} = 16$

Ans. (a)

Q23. If $a + b + c = 6$, $a^2 + b^2 + c^2 = 14$. Find the value of $a : b : c$ = ??

Sol^n Symmetricity

Check $3a = 6$

$a = 2$

$2^2 + 2^2 + 2^2 \neq 14$

Not satisfying

Can't be determined

Ans.

Q24. If $\sqrt{3x^2-12x+19}+\sqrt{3x^2-12x-11}=6$ then

$\sqrt{3x^2-12x+19}-\sqrt{3x^2-12x-11}=?$

(a) 4 (b) 3

(c) 0 (d) 5

— (1)

— (2)

K

Multiply (1) & (2)

$(3x^2-12x+19)-(3x^2-12x+11)=6K$

$30=6K$

$K=5$

Ans. (d)

Q25. If $x(x+y+z)=28$, $y(x+y+z)=70$, $z(x+y+z)=98$

then the value of $3(x+y+z)$ is:

(a) ± 28 (b) ± 24

(c) ± 42 (d) ± 36

$x(x+y+z) = 28$ — (1)

$y(x+y+z) = 70$ — (2)

$z(x+y+z) = 98$ — (3)

Add (1), (2) & (3)

$(x+y+z)^2 = 196$

$x+y+z = \pm 14$

Ans. (c)

Q26. If $2^x = 8^y = 32^z$ and $x + y + z = 20$. Find the value of y.

Time 1 min

$2^x = 2^{3y} = 2^{5z}$

$x = 3y = 5z$

$x + y + z = 20$

$3y + y + \frac{3y}{5} = 20$

$\frac{23y}{5} = 20$

$y = \frac{100}{23}$

Ans.

Q27. If $2^x = 3^y = 6^{-z}$ then $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ is equal to

Time 75sec

(a) 0

(b) 1

(c) $\frac{3}{2}$

(d) $\frac{-1}{2}$

$2^x = 3^y = 6^{-z} = K$

$2^x = K^1$

$2 = K^{\frac{1}{x}}$ $\quad 3 = K^{\frac{1}{y}}$ $\quad 6 = K^{\frac{-1}{z}}$

$2 \cdot 3 = 6$

$K^{\frac{1}{x}} \cdot K^{\frac{1}{y}} = K^{\frac{-1}{z}}$

$\frac{1}{x}+\frac{1}{y} = \frac{-1}{z}$

$\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$

Ans. (a)

Q28. The value of $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$ given that $pqr = 1$ is–

(a) 1 (b) 0

(c) $p+q+r$ (d) None of these

(I) $p = q = r = 1$

$\frac{1}{1+1+1} + \frac{1}{1+1+1} + \frac{1}{1+1+1}$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= 1$

Ans. (a) $pqr = 1$

$$\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$$

$$= \frac{1}{1+p+\frac{1}{q}} + \frac{r}{r+rq+1} + \frac{p}{p+pr+1}$$

$$= \frac{q}{q+qp+1} + \frac{pqr}{pqr+pqr\cdot q+pq} + \frac{pq}{pq+pqr+q}$$

$$\frac{q}{q+pq+1} + \frac{1}{1+q+pq} + \frac{pq}{pq+1+q} = 1$$

Q29. If $x^2 + 2 = 2x$ then $x^4 - x^3 + x^2 + 5 = ?$

(a) 1 (b) 2

(c) 3 (d) 4

Time 90sec

I st

$x^2 + 2 = 2x$

$x^2 = 2x - 2$

$(x^2)^2 - (x^2) \cdot x + x^2 + 5$

$(2x-2)^2 - (2x-2) \cdot x + 2x - 2 + 5$

$4x^2 - 8x + 4 - 2x^2 + 2x + 2x + 3$

$2x^2 - 4x + 7$

$2(x^2 - 2x) + 7 \quad = 2(-2) + 7$

$= 3$

Ans. (c)

$x^4 - x^3 + x^2 + 5$

$x^2 + 2 = 2x$ — (1)

$x^3 + 2x = 2x^2$ — (2)

$x^4 + 2x^2 = 2x^3$ — (3)

$x^4 + x^3 + x^2 + 2 = 2x^2$

$x^4 - x^3 + x^2 + 2 = 0$

$x^4 - x^3 + x^2 + 5 = 3$

Q30. If $x^3 + y^3 + z^3 = 3(1 + xyz)$, $P = y + z - x$, $Q = Z + x - y$, $R = x + y - z$ then what is the value of $P^3 + Q^3 + R^3 - 3PQR = ?$

(a) 9 (b) 8

(c) 12 (d) 6

I $y = z = 0$

$x^3 = 3$

$P = -x \quad Q = x \quad R = x$

$-x^3 + x^3 + x^3 + 3x^3$

$4x^3$

12

Ans. (c) Given $x^3+y^3+z^3 = 3(1+xyz)$

$P = y+z-x$

$Q = z+x-y$

$R = x+y-z$

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Detailed

$$P^3+Q^3+R^3-3PQR$$

$$\Rightarrow \frac{1}{2}(P+Q+R)\left[(P-Q)^2+(Q-R)^2+(R-P)^2\right]$$

$$\Rightarrow \frac{1}{2}(x+y+z)\left[4(y-x)^2+4(z-y)^2+4(x-z)^2\right]$$

$$\frac{1}{2}(x+y+z)(4)\left[(y-x)^2+(z-y)^2+(x-z)^2\right]$$

$$4\left(x^3+y^3+z^3-3xyz\right) = 4\cdot 3$$

$$= 12$$

Q31. If $u_n = \frac{1}{n} - \frac{1}{n+1}$ then the value of $u_1 + u_2 + u_3 + u_4 + u_5$ is–

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{5}{6}$

Ans. (d)

Q32. If $x = \sqrt[3]{a + \sqrt{a^2 + b^3}} + \sqrt[3]{a - \sqrt{a^2 + b^3}}$ then $x^3 + 3bx$ is equal to –

(a) 0 (b) a

(c) $2a$ (d) 1

Ans. (c)

Q33. If $x_1x_2x_3 = 4\ (4 + x_1 + x_2 + x_3)$ then what is the value of $\frac{1}{(2+x_1)} + \frac{1}{(2+x_2)} + \frac{1}{(2+x_3)}$

(a) 1

(b) $\frac{1}{2}$

(c) 2

(d) $\frac{1}{3}$

Ans. (b)

Q34. Let $x = \frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$ and $y = \frac{1}{x}$, then the value of $3x^2 - 5xy + 3y^2$ is–

(a) 1717 (b) 1177

(c) 1771 (d) 1171

Ans. (a)

Q35. If $a^2 + b^2 + 9 - 6b + (a + b - 4)^2 = 2ab - 6a$, then find ab = ??

(a) 7/4 (b) -7/4

(c) 7/2 (d) 12

Ans. (a)

Q36. If $a + b = 48 - ab$, $b + c = 99 - bc$ and $c + a = 3 - ca$ where a, b and c are positive number then find $7c - 15a + b = ?$

(a) 38

(b) 41

(c) 43

(d) 35

Ans. (b)

Q37. If $2.9^{x} = 841^{y} = 1000$, find $\frac{xy}{x-2y} = ?$

(a) $1/2$

(b) 2

(c) $1/3$

(d) $-1/4$

Ans. (b)

Q38. If $a^4 + b^4 - 18\,a^2b^2 = 24$ and $a^2 - b^2 - 4ab = 6$, find $ab = ?$

(a) -½

(b) -¼

(c) ½

(d) ¼

Ans. (b)

Q39. If $x^2 - 4x + 1 = 0$, find $x^9 + x^7 - 194x^5 - 194x^3 = ?$

(a) -2

(b) -4

(c) 0

(d) 1

Ans. (b)

Q40. If $\frac{8(x+y)^3-27(x-y)^3}{5(y-x)} = Ax^2+Bxy+Cy^2$

then find the value of (A + B + C) = ?

(a) 26 (b) 19

(c) 16 (d) 13