Mensuration 2-D

STUDY NOTES ON 2D MENSURATION

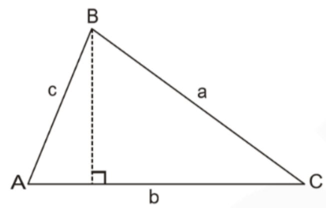
Mensuration is the section of mathematics which deals with the calculation of length, area and volume.

In these notes, we will discuss about the areas and perimeters of two-dimensional figures.

Area: The area of any plane figure is the amount of surface enclosed within its boundary lines. It is expressed in square units e.g., square meter, square centimetre, square inch, etc.

Perimeter: The perimeter of a plane figure is the total length of the sides enclosing the figure.

Area of Δ: The area of a triangle is represented by the symbol Δ. For a triangle ABC the three sides are represented by a, b and c and the angles opposite to these sides are represented by A, B and C respectively, as shown in the figure:



The formulas for areas and perimeters of various plane figures.

(i) For any triangle in general,

(a) When base (b) and height (h) perpendicular to that base are given.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} b \cdot h$$

(b) When lengths of all three sides are given a, b and c

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)} \text{ where, } S = \frac{a+b+c}{2} = \text{semi-perimeter of the triangle}$$

This is called Heron's Formula.

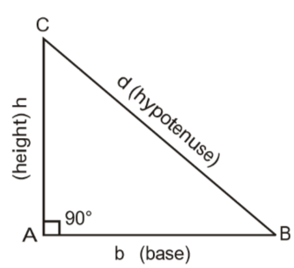
(c) If the lengths of three medians of a triangle ABC are p, q, r units, then:

$$\text{Area} = \frac{4}{3}\sqrt{S_m(S_m-p)(S_m-q)(S_m-r)}$$

$$\text{Where, } S_m = \frac{p+q+r}{2}$$

(d) Perimeter = a + b + c = 2S

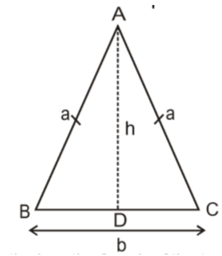
(ii) Right Angled triangle:



(a) $\text{Area} = \frac{1}{2} \times \text{Product of the sides containing the right angle}$

(b) $d^2 = b^2 + h^2$

(iii) Isosceles triangle: A triangle in which two sides are equal is said to be an isosceles triangle.



(a) Area = $\frac{b}{4}\sqrt{4a^2 - b^2}$ where 'a' is the length of each of the two equal sides and b is the third side.

(b) Perimeter = 2s = 2a + b

(c) $\text{Height}(h) = \frac{\sqrt{4a^2 - b^2}}{2}$

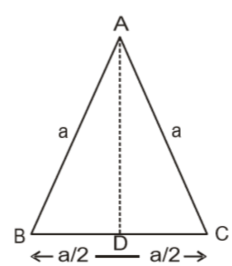
(iv) Equilateral triangle: A triangle in which all three sides are equal is said to be an equilateral triangle.

(a) $\text{Area} = \frac{\sqrt{3}}{4}a^2$, where 'a' is the side of the triangle.

(b) Height of an equilateral triangle (h) = $\frac{\sqrt{3}}{2}a$

(c) Perimeter (P) = 3a

(d) $\angle A = \angle B = \angle C = 60°$



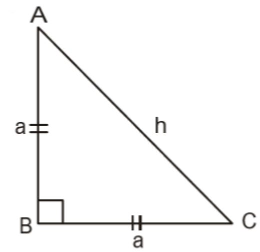
(v) Isosceles right-angled triangle

(i) AB = BC = a and $\angle B = 90°$

(ii) $h = a\sqrt{2}$

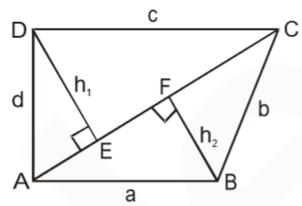
(iii) $\text{Area} = \frac{1}{2}a^2$

(iv) Perimeter = 2a + h = 2S



QUADRILATERALS

A closed figure bounded by four sides is called as quadrilateral.



(i) For any quadrilateral in general,

(a) $\angle A + \angle B + \angle C + \angle D = 360°$

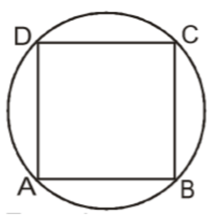
(b) Area of the Quadrilateral = $\frac{1}{2}$ × (one diagonal) × (sum of perpendicular to it from opposite vertex)

$$= \frac{1}{2} \times (AC) \times (h_1 + h_2)$$

(c) Perimeter = a + b + c + d.

(ii) For a cyclic Quadrilateral where the four sides measure, a, b, c and d respectively.

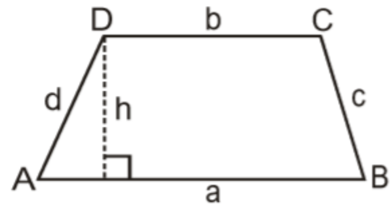
$$\text{Area} = \sqrt{(S-a)(S-b)(S-c)(S-d)}$$



Where S is the semi-perimeter, i.e, $S = \frac{a+b+c+d}{2}$

$\angle A + \angle C = \angle B + \angle D = 180°$

(iii) Trapezium: A quadrilateral where any two opposite sides are parallel is a Trapezium.

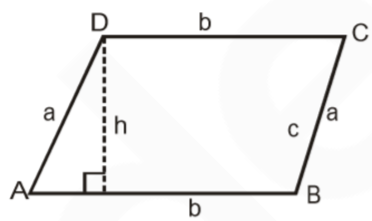


(a) Area of a trapezium = $\frac{1}{2}$ × (sum of parallel sides) × (perpendicular distance between them)

$=\frac{1}{2}(a+b)\times h$

(b) Perimeter (P) = a + b + c + d

(iv) **Parallelogram:** A parallelogram is a quadrilateral whose opposite sides are equal and parallel.



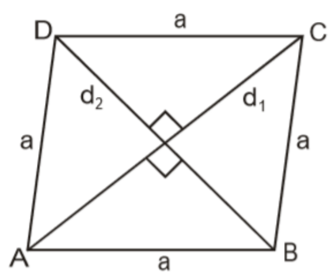
(a) Area = base × height

(b) Area = Product of two sides × sine of the angle between two adjacent sides = ab sinθ

(c) Perimeter 2(a + b)

(d) $d_1^2+d_2^2=2\left(a^2+b^2\right)$ $\quad\quad\quad (d_1, d_2 = \text{length of diagonals})$

(v) Rhombus: It is a parallelogram whose all four sides are equal

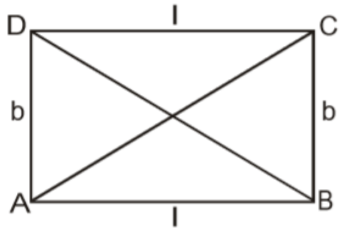


(a) Diagonals of a Rhombus bisect each other at 90°

(b) Area = $\frac{1}{2}$ × product of the diagonals

(c) Perimeter = 4 × side of the rhombus

(vi) Rectangle:

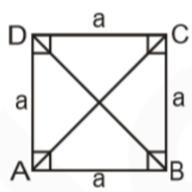


(a) Area = Length × Breadth

(b) Perimeter = 2(l + b), where l and b are the length and the breadth of the rectangle respectively.

(c) Diagonals area equal and bisect each other.

(vii) Square



All four sides of a square are equal and all four vertex angles are equal to 90°,

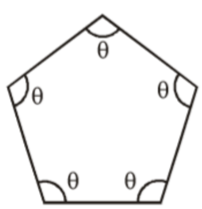
(a) Area = (side)2

(b) $\text{Area} = \frac{1}{2} \times (\text{Diagonal})^2$ (where diagonal = $\sqrt{2}$ × side)

(c) Perimeter = 4 × side

POLYGONS: A polygon is a plane figure enclosed by four or more straight line.

Regular polygon: A polygon whose all side are equal is called a Regular polygon.



All the interior angles of a regular polygon area equal.

For a regular polygon:

(i) Sum of exterior angles = 360°

(ii) Sum of interior angles (n – 2) × 180°

(iii) Interior angle + exterior angle = 180°

(iv) Each interior angle

(vii) Area of a rgular polygon = $\frac{1}{2}$ × (Perimeter) × (Perpendicular distance from the centre of the polygon to any side)

(Centre of a regular polygon is equidistant from all its sides)

Area of regular Hexagon = 6 × (Area of equilateral triangle of side a)

Since regular Hexagon is made of 6 equilateral triangle when its vertex are joined to centre.

$$= 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$$

Regular octagon: Area = $2\left(\sqrt{2}+1\right)\left(\text{side}\right)^2$

CIRCLE

(i) Area of circle = πr^2, where is the radius of the circle

(ii) Circumference of a circle = $2\pi r$

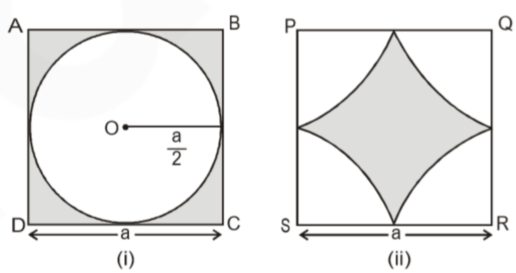
Room: If we have a room of length l, breadth b and height h, then,

Area of four walls of the room = 2h(l + b)

Area of four walls and floor = 2h(l + b) + lb

Area of floor, roof and four walls = 2h(l + b) + 2lb

Some Important results



1. If the diagonal of a square becomes x times, then the area of the square becomes x^2 times.

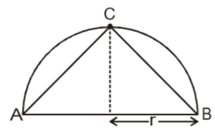
2. If each of the defining dimensions are sides of any 2-D figure are increased (or decreased) by x%, is perimeter also increases (or decreases) by x%.

3. If all the sides of a quadrilateral are increased (or decreased) by x%, its diagonals also increase (or decrease) by x%.

4. If the area of a square is a cm^2, then the area of the circle having circumference equal to the perimeter of the square is = $\frac{4a}{\pi} cm^2$.

5. In figure (i) ABCD is a square of side ‘a’ and O is the centre of the inscribed circle with radius a/2 in figure (ii) PQRS is a square of side ‘a’ and P, Q, R and S are the centres of four quadrants of circle with radius $\frac{a}{2}$. Area of shaded region in both case = $\frac{3}{14}a^2$.

6. The area of largest triangle inscribed in a semi-circle of radius r is r^2.



Area of $\Delta ACB = r^2$

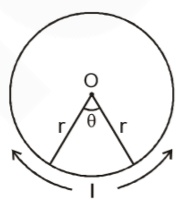
7. If each of the defining dimensions or sides of any 2-D figure is changed by x%, its area changes by $x\left(2+\frac{x}{100}\right)\%$

If the dimension is increased, take x as positive.

If the dimension is decreased, take x as negative.

8. If the length of a rectangle increased by a%, then its breadth will have to be decreased by $\left(\frac{100a}{100+a}\%\right)$ in order to maintain the same area of the rectangle.

Sector of a circle:

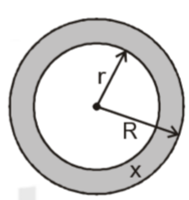


Length of arc = $\frac{\theta}{360^\circ}\times 2\pi r$

$\text{Area}=\frac{\theta}{360^\circ}\times \pi r^2$

Where θ is the angle of the sector in degrees and r is the radius of the circle.

Ring or circular path: Ring is the space enclosed between two concentric circles.



Area = $\pi(R^2 - r^2)$

Perimeter = $2\pi(R + r)$

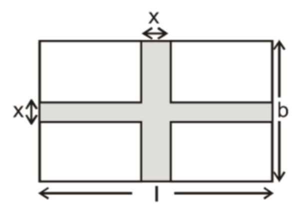
Pathway running across the middle of a rectangle.

x = width of the path.

Pathway Across the rectangle

Area of path = $(l + b - x)x$

Perimeter of path = $2(l + b) - 4x = 2(l + b - 2x)$

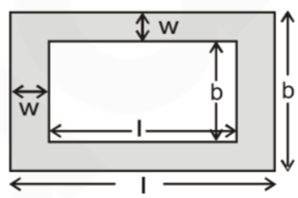


Pathways around a rectangular space:

Outer pathways:

(i) Area = $(l + b + 2w)2w$

(ii) Perimeter = outer perimeter + inner perimeter = $2(l + b) + 2(l + b + 4w) = 4(l + b + 2w)$



Inner Pathways:

Area of the path = $(l + b + 2w)2w$

Perimeter = $2(l + b) + 2(l + b - 4w) = 4(l + b - 2w)$