Lines, Angles & Polygon

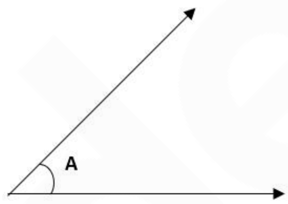
Basic Information:

Line segment: A line segment has two end points with a definite length.

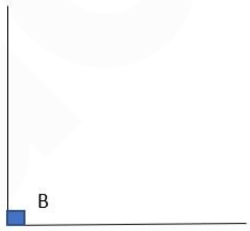
Ray: A ray has one end point and infinitely extends in one direction.

Straight line: A straight line has neither starting point nor ending point and is of infinite length.

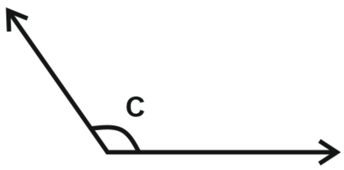
Acute angle: An angle that is between 0° and 90° is an acute angle, As ∠A in the figure below:



Right angle: An angle that is 90° is a Right angle, As ∠B as shown below:

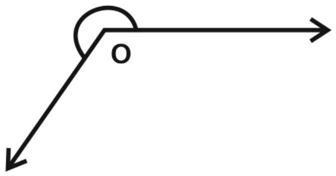


Obtuse angle: An angle that is between 90° and 180° is an obtuse angle, As ∠C as shown below:



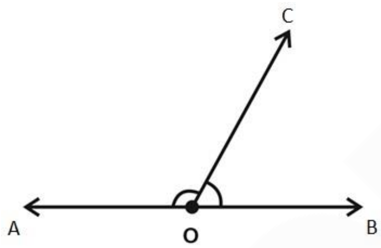
Straight angle: An angle that is 180° is a straight angle.

Reflex Angle: An angle that is between 180° and 360° is a reflex angle.

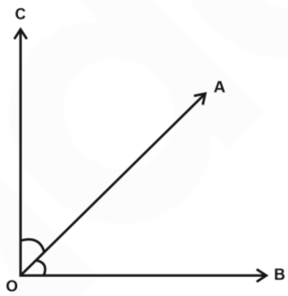


Supplementary angles:

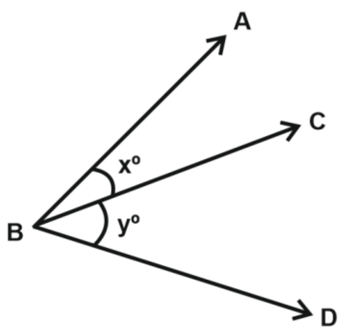
In the figure above, ∠AOC + ∠COB = ∠AOB = 180° If the sum of two angles is 180° then the angles are called supplementary angles. Two right angles always supplement each other. The pair of adjacent angles whose sum is a straight angle is called a linear pair.



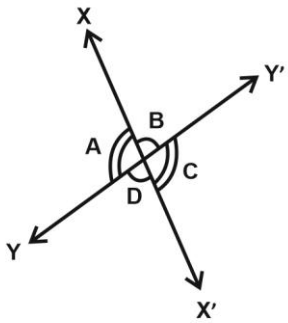
Complementary angles: ∠COA + ∠AOB = 90° If the sum of two angles is 90° then the two angles are called complementary angles.



Adjacent angles: The angles that have a common arm and a common vertex are called adjacent angles. In the figure above, ∠ABC and ∠CBD are adjacent angles. Their common arm is BC and common vertex is 'B'.

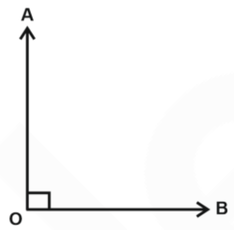


Vertically opposite angles: When two lines intersect, the angles formed opposite to each other at the point of intersection (vertex) are called vertically opposite angles.



In the figure above, x and y are two intersecting lines. ∠A and ∠C make one pair of vertically opposite angles and ∠B and ∠D make another pair of vertically opposite angles.

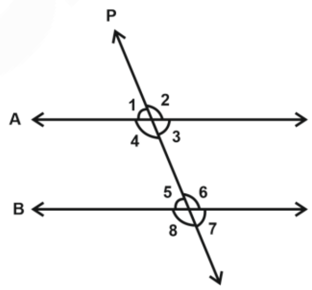
Perpendicular lines: When there is a right angle between two lines, the lines are said to be perpendicular to each other.



Here, the lines OA and OB are said to be perpendicular to each other.

Parallel lines:

Here, A and B are two parallel lines, intersected by a line p. The line p is called a transversal, that which intersects two or more lines (not necessarily parallel lines) at distinct points. As seen in the figure above, when a transversal intersects two lines, 8 angles are formed. Let us consider the details in a tabular form for easy reference.



Types of Angles	Angles
Interior Angles	∠3, ∠4, ∠5, ∠6
Exterior Angles	∠1, ∠2, ∠7, ∠8
Vertically Opposite Angles	(∠1, ∠3), (∠2, ∠4), (∠5, ∠7), (∠6, ∠8)
Corresponding Angles	(∠1, ∠5), (∠2, ∠6), (∠3, ∠7), (∠4, ∠8)
Interior Alternate Angles	(∠3, ∠5), (∠4, ∠6)
Exterior Alternate Angles	(∠1, ∠7), (∠2, ∠8)
Interior Angles on the same side of transversal	(∠1, ∠7), (∠2, ∠8)

When a transversal intersects two parallel lines,

1. The corresponding angles are equal.

2. The vertically opposite angles are equal.

3. The alternate interior angles are equal.

4. The alternate exterior angles are equal.

5. The pair of interior angles on the same side of the transversal is supplementary.

We can say that the lines are parallel if we can verify at least one of the aforementioned conditions.

Polygon:

Any closed figure, having 3 or more than sides is called a polygon.

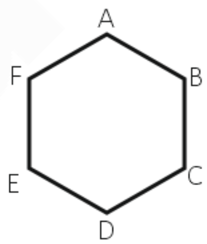
Ex.: Triangle, quadrilaterals, hexagon etc.

Types of Polygons:

It should be known that polygons are categorized as different types depending on the number of sides together with the extent of the angles. Some of the prime categories of polygons include regular polygons, irregular polygons, concave polygons, convex polygons, quadrilateral polygons, pentagon polygons and so on.

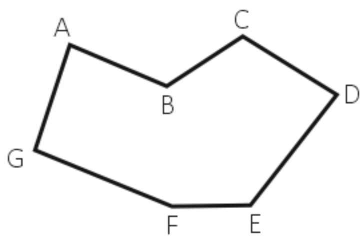
Some of the most well-known polygons are triangles, squares, rectangles, parallelograms, pentagons, rhombuses, hexagons etc.

Regular polygon: Considering a regular polygon, it is noted that all sides of the polygon tend to be equal.

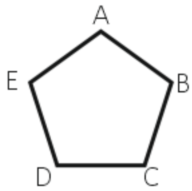


Furthermore, all the interior angles remain equivalent.

Irregular polygon: These are those polygons that aren't regular. Be it the sides or the angles, nothing is equal as compared to a regular polygon.



Concave polygon: A concave polygon is that under which at least one angle is recorded more than 180 degrees. Also, the vertices of a concave polygon are both inwards and outwards.



Convex polygon: The measure of interior angle stays less than 180 degrees for a convex polygon. Such a polygon is known to be the exact opposite of a concave polygon. Moreover, the vertices associated to a convex polygon are always outwards.

Quadrilateral polygon: Four-sided polygon or quadrilateral polygon is quite common. There are different versions of a quadrilateral polygon such as square, parallelogram and rectangle.

Pentagon polygon: Pentagon polygons are six-sided polygons. It is important to note that, the five sides of the polygon stay equal in length. A regular pentagon is a prime type of pentagon polygon.

Formulae Related to Polygon:

(N = Number of sides and S = Distance from centre to a corner)

The number of diagonals = $\frac{(N(N-3))}{2}$

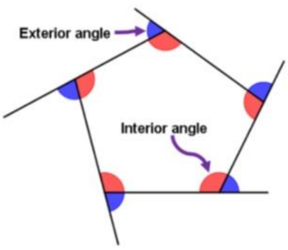
Summation of the interior angles of the polygon = (N – 2) x 180°

The count of triangles (while drawing all the diagonals through a single vertex) in a polygon = (N – 2)

Angle: The study of angles is very important whenever we are trying to understand polygons and their properties. To be precise, when two rays hold a common endpoint, in this case, the two rays together form an angle. Therefore, an angle is formed by two rays initiating from a shared endpoint. These two rays creating it are termed as the sides or arms of the angle. For representing an angle, the symbol "∠" is used in geometry.

Angles of Polygon:

One must keep in mind that all polygons possess internal angles and external angles. In addition, a polygon's external angle can be termed as that which is extended on one side. Here are certain rules which are followed regarding angles of a polygon.



• **Exterior Angle of a Polygon:** All the Exterior Angles associated to a polygon add to form a sum 360°.

• **Interior Angle of a Polygon:** The Interior and exterior angle are evaluated through the same line; therefore, they add up to 180°.

Interior Angle = 180° − Exterior Angle

Properties of Polygons:

If "n" is the total number of sides in a polygon. Then,

1. The sum of all interior angles of a polygon = (n - 2)×180°

2. Each interior angle of a regular polygon = $\frac{(n-2)\times 180°}{n}$

3. The sum of all exterior angles of a polygon = 360°

4. Each exterior angle of a regular polygon = $\frac{360°}{\text{n}}$

5. The total number of diagonals in a regular polygon = $\frac{n(n-3)}{2}$

6. The ratio of the sides of a polygon to the diagonals of a polygon = $2:(n-3)$

7. The ratio of the interior angle of a regular polygon to its exterior angle

$$= (n-2):2$$

In a regular polygon with "n" sides and the length of each side is "a" unit, "r" is the inradius of the polygon and "R" is the circumradius of the polygon then,

1. Perimeter of the polygon = na

2. Area of a polygon $= \frac{1}{2} \times (Perimeter\ of\ the\ polygon) \times inradius = \frac{1}{2} \times na\ \times r$

3. Area of a polygon = $\frac{na}{2} \times \sqrt{R^2 - \left(\frac{a}{2}\right)^2}$

4. Area of a polygon = $\frac{na^2}{4} \times \cot\left(\frac{\pi}{n}\right)$

5. Area of a polygon = $nr^2 \times \tan\left(\frac{\pi}{n}\right)$

6. Inradius (r) = $\frac{a}{2} \times \cot\left(\frac{\pi}{n}\right)$

Regular Hexagon:

Let ABCDEF is a regular hexagon with each side of length "a" unit and "O" is the centre of the given hexagon.

1. The sum of the interior angles of the Hexagon = $(6-2) \times 180° = 720°$

2. Each exterior angle = $\frac{360°}{6} = 60°$

3. Each interior angle = $\frac{(6-2)\times 180°}{6} = 4\ \times 30° = 120°$

4. Area of the Hexagon = $6 \times \left(\frac{\sqrt{3}}{4} \times a^2\right) = 6 \times Area\ of\ an\ equilateral\ triangle$

Regular Octagon:

Let ABCDEFGH is a regular octagon with each side of length "a" unit.

1. The sum of the interior angles of the Octagon = $(8 - 2) \times 180° = 1080°$

2. Each exterior angle = $\frac{360°}{8} = 45°$

3. Each interior angle = $\frac{(8-2)\times 180°}{8} = \left(\frac{3}{4}\right) \times 180° = 135°$

4. Area of the Octagon = $2(1 + \sqrt{2}) \times a^2$