gradeup

Sahi Prep Hai Toh Life Set Hai

TRIGONOMETRY
(Minimum & Maximum)

* Maxima & Minima

→ Some Results

$$0° \leq \theta \leq 90°$$

$$\theta \rightarrow \text{Any Angle}$$

	MIN	MAX		MIN	MAX
$\sin\theta / \cos\theta$	0	1		-1	1
$\tan\theta / \cot\theta$	0	∞		$-\infty$	∞
$\sec\theta / \csc\theta$	1	∞		$-\infty$	∞

$$0 \leq \theta \leq 90$$

	Minimum	Maximum
sin θ / cos θ	0	1
tan θ / cot θ	0	∞
sec θ / cosec θ	1	∞

For any value of θ

	Minimum	Maximum
$\sin\theta$ / $\cos\theta$	-1	1
$\tan\theta$ / $\cot\theta$	$-\infty$	∞
$\sec\theta$ / $\mathrm{cosec}\,\theta$	$-\infty$	∞

Representation of Maximum & Minimum Value on Number Line
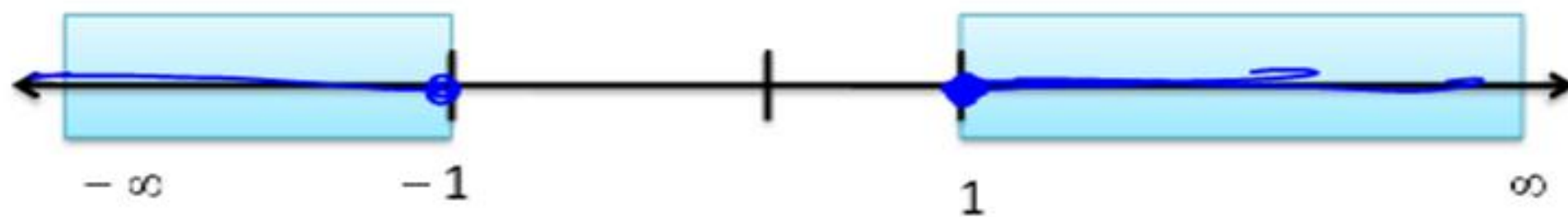
$\theta \longrightarrow$ Any Angle

sin θ / cos θ

tan θ / cot θ

$-\infty$

∞

sec θ / cosec θ

$$-\infty \qquad -1 \qquad\qquad 1 \qquad\qquad \infty$$

Any
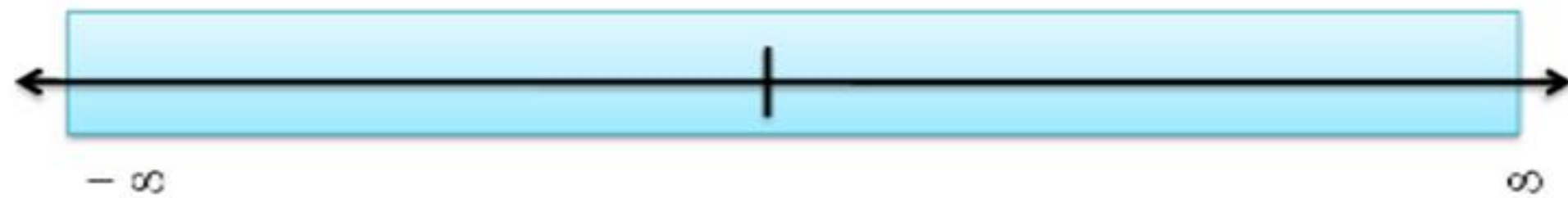
θ → Angle

Eg. If, $0 \leq \theta \leq 90$
Then find:

(i) min (sin θ)   0

(ii) min (tan θ)   0

(iii) max (sec θ)   ∞

Eg. Find:

$\theta \longrightarrow$ Any Angle

(i) min (sin θ) $\longrightarrow$ -1

(ii) min (sec θ) $\longrightarrow$ $-\infty$

Eg. Find the min & max value of given expression.

S.No.	Expression	Min	Max
1.	$\sin\theta$	-1	1
2.	$\tan\theta$	$-\infty$	∞
3.	$\sin 2\theta$	-1	1
4.	$2\sin\theta$	-2	2
5.	$3\sin 5\theta$	-3	3
6.	$\sin^2\theta$	0	1

$x^2 \to$ can never be negative

$\min(x^2) \to 0$

	MIN	MAX
$\sin\theta$	-1	1
$\sin^2\theta$	0	1

Square of a number can never be negative

S.No.	Expression	Min	Max
7.	$\sin^3 \theta$	-1	1
8.	$5 \sin^2 \theta$	0	5
9.	$5 + \sin \theta$	4	6
10.	$8 + 3 \sin^3 \theta$	5	11
11.	$3 - 2 \sin^5 \theta$	1	5
12.	$5 \sin^3 \theta - 6$	-11	-1

$$Y = 3 - 2\sin^5 \theta$$

$$Y_{min} = 3 - [2] = 1$$

$$Y_{max} = 3 - [-2] = 5$$

Y = A − B

$$Y_{min} = A_{min} - B_{max}$$

$$Y_{max} = A_{max} - B_{min}$$

Eg. Find the min & max value of 3 − 2 sin³θ

$$Y = 3 - 2\sin^3\theta$$

$$Y_{min} = 3 - [2] = 1$$

$$Y_{max} = 3 - [-2] = 5$$

Eg. **Find minimum and maximum value of :**

$$\text{MIN} \qquad \text{MAX}$$

(a) $\underline{\sec \theta} + 5$ $\qquad -\infty \qquad \infty$

(b) $\underline{\sec^2 \theta} + 5$ $\qquad 6 \qquad \infty$

(c) $\underline{\sec^3 \theta} - 5$ $\qquad -\infty \qquad \infty$

(d) $4 - \sec^2 \theta$ $\qquad -\infty \qquad 3$

Eg. Find minimum and maximum value of the following :

(a) $2 \sin \theta + 3 \cos \phi$

MIN   MAX

−5   5

(b) $3 \sin A - 4 \cos B$

MIN   MAX

−7   7

$y = 3 \sin A - 4 \cos B$

$y_{min} = (3 \sin A)_{min} - (4 \cos B)_{max} = -3 - 4 = -7$

$y_{max} = (3 \sin A)_{max} - (4 \cos B)_{min} = 3 + 4 = 7$

$$3 \cdot 1 - (4 \times -1)$$

$$= \boxed{7}$$

Eg. Find the minimum and maximum value of 3 sin θ + 4 cos θ

$$y = 3\sin\theta + 4\cos\theta$$

$$y_{min} \rightarrow -\sqrt{3^2 + 4^2}$$

$$-\sqrt{25}$$

$$\underline{\underline{-5}}$$

$$y_{max} = \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$= 5$$

Ans.

Min = −5   Max = 5

$$Y = a \sin \theta + b \cos \theta$$

Min

$$-\sqrt{a^2 + b^2}$$

Max

$$\sqrt{a^2 + b^2}$$

$$y = a\sin\theta + b\cos\theta$$

$$\text{Let } \begin{cases} a = r\cos\phi \\ b = r\sin\phi \end{cases} \qquad \boxed{a^2 + b^2 = r^2}$$

$$r = \sqrt{a^2 + b^2}$$

$$y = r\cos\phi\sin\theta + r\sin\phi\cos\theta$$

$$= r\left[\sin\theta\cos\theta + \cos\theta\sin\phi\right]$$

$$= r\left[\sin(\theta+\phi)\right]$$

$$y_{min} = -\sqrt{a^2+b^2} \qquad y_{max} = \sqrt{a^2+b^2}$$

Eg. Find the min & max value of the following expressions.

S.No.	Expression	Min	Max
1.	$3 \sin \theta + 4 \cos \theta$	-5	5
2.	$7 \sin \theta - 24 \cos \theta$	-25	25
3.	$\sin \theta + \cos \theta$	$-\sqrt{2}$	$\sqrt{2}$
4.	$5 \sin \theta + 12 \cos \theta - 3$	-16	10

$-13 \mid +3.$

Q1. Find minimum and maximum value of the following

(a) $25 \cos^2 \theta - 30 \sin \theta \cos \theta + 9\sin^2 \theta$

This we will after 2-3 classes

(b) $10 \cos^2 \theta + 6 \sin \theta \cos \theta + 2 \sin^2 \theta$

Ans.

(a) Min = 0   Max = 34

(b) Min = 1   Max = 11

Q2. Find max and min value of the following expression.

$$Y = 16^{\sin\theta}\ 4^{\sin\theta}\ 2^{\cos\theta}\ 128^{\cos\theta}$$

$$Y = \left(2^4\right)^{\sin\theta} \cdot \left(2^2\right)^{\sin\theta} \cdot 2^{\cos\theta} \cdot \left(2^7\right)^{\cos\theta}$$

$$= 2^{4\sin\theta + 2\sin\theta + \cos\theta + 7\cos\theta}$$

$$Y = 2^{6\sin\theta + 8\cos\theta}$$

$$Y_{\min} = 2^{-10}$$

$$= \frac{1}{1024}$$

$$Y_{\max} = 2^{10}$$

$$= 1024$$

Ans.

$Y_{Min} = 1/1024$ $Y_{Max} = 1024$

* $$\sin^2\theta + \cos^2\theta = 1$$

* $$\tan^2\theta + 1 = \sec^2\theta$$

* $$1 + \cot^2\theta = \csc^2\theta$$

Eg. Find minimum and maximum value of

$$Y = \textcircled{3} \sin^2 \theta + \textcircled{4} \cos^2 \theta$$

Detailed

$$Y = 3\sin^2\theta + 4\cos^2\theta$$

$$= 3\sin^2\theta + 3\cos^2\theta + \cos^2\theta$$

$$= 3(1) + \cos^2\theta$$

$$Y_{min} = 3 \checkmark$$

$$Y_{max} = 4 \checkmark$$

Ans.

$Y_{Min} = 3$ $Y_{Max} = 4$

$$Y = a \sin^2 \theta + b \cos^2 \theta \quad (a < b)$$

Min	Max
a	b

* The coeff which is smaller is the minimum value & the coeff which is larger is max value

Eg. Find minimum and maximum value of

	MIN	MAX
(a) $5 \sin^2 \theta + 4 \cos^2 \theta$	4	5
(b) $3 \sin^2 \theta - 5 \cos^2 \theta$	-5	3
(c) $\dfrac{1}{2} \sin^2 \theta + \dfrac{1}{3} \cos^2 \theta$	$\dfrac{1}{3}$	$\dfrac{1}{2}$
(d) $-5 \sin^2 \theta - 3 \cos^2 \theta$	-5	-3

$$Y = a \tan^2 \theta + b \cot^2 \theta \quad (a > 0, b > 0)$$

min → $2\sqrt{ab}$

max → ∞

Eg. $Y = 4\tan^2\theta + 9\cot^2\theta$

$Y_{min} = ??$

$2\sqrt{4 \cdot 9} = 2 \cdot 6$

$= \boxed{12}$

Eg. $Y = 8\tan^2\theta + 2\cot^2\theta$

$Y_{min} = ??$

$2\sqrt{8 \cdot 2} = 8$

$Y = a \sin^2 \theta + b \csc^2 \theta$ ($a > 0, b > 0$)

$\min \to 2\sqrt{ab}$ ($a \geq b$)

$\min \to a + b$ ($a \leq b$)

Eg. $Y = 3 \sin^2 \theta + \underline{12} \csc^2 \theta$ $\qquad Y_{min} = ??$

$$3 + 12 = 15$$

Eg. $Y = 12 \sin^2 \theta + 3 \csc^2 \theta$ $\qquad Y_{min} = ??$

$$2\sqrt{12 \cdot 3}$$

$$= 12$$

$Y = a \cos^2\theta + b \sec^2\theta \ (a > 0, b > 0)$

$\min \rightarrow 2\sqrt{ab} \quad (a \geq b)$

$\min \rightarrow a + b \quad (a \leq b)$

Eg. $Y = 8 \cos^2\theta + 18 \sec^2\theta$ $\qquad Y_{min} = ??$

$8 + 18 = 26$

Eg. $Y = 18 \cos^2\theta + 8 \sec^2\theta$ $\qquad Y_{min} = ??$

$2\sqrt{18 \cdot 8} = 24$

Eg. Find the minimum value of:

$$1 + \tan^2 \theta = \sec^2 \theta$$

(a) $32 \cos^2 \theta + 2 \tan^2 \theta$

$$32 \cos^2 \theta + 2(\sec^2 \theta - 1)$$

$$32 \cos^2 \theta + 2 \sec^2 \theta - 2$$

$$2\sqrt{32 \cdot 2} - 2$$

$$16 - 2 = 14$$

(b) $4 \sin^2 \theta + 64 \cot^2 \theta$

$$1 + \cot^2 \theta = cosec^2 \theta$$

$$4 \sin^2 \theta + 64 \left(cosec^2 \theta - 1 \right)$$

$$4 \sin^2 \theta + 64 \, cosec^2 \theta \quad - 64$$

$$4 + 64 - 64$$

$$= 4$$

(c) $4 \sec^2 \theta + 9 \csc^2 \theta$

$$Y = 4(1 + \tan^2 \theta) + 9(1 + \cot^2 \theta)$$

MIN

$$= 13 + 4\tan^2\theta + 9\cot^2\theta$$

$$Y_{min} \rightarrow 13 + 2\sqrt{4 \cdot 9}$$

$$= \underline{\underline{25}}$$

Shortcut $MINI \rightarrow \left(\sqrt{4} + \sqrt{9}\right)^2 \Rightarrow 25 \checkmark$

Ans.
(a) 14
(b) 4
(c) 25

$Y = a \sec^2 \theta + b \csc^2 \theta$ (a > 0, b > 0) min → $(\sqrt{a} + \sqrt{b})^2$

Eg. $Y = 16 \sec^2 \theta + 25 \csc^2 \theta$

Y_{min} = ??

$(4 + 5)^2$

$= 81$

$$\boxed{\sin 2\theta = 2\sin\theta \cos\theta}$$

$$-1 \le \sin 2\theta \le 1$$

$$-1 \le 2\sin\theta \cos\theta \le 1$$

$$\boxed{\frac{-1}{2} \le \sin\theta \cos\theta \le \frac{1}{2}}$$

$$\text{Min} \ (\sin^n \theta \cdot \cos^n \theta) = \left(\frac{-1}{2}\right)^n \quad , \quad (n \rightarrow \text{odd})$$

$$= 0 \quad , \quad (n \rightarrow \text{even})$$

$$\text{Max} \ (\sin^n \theta \cdot \cos^n \theta) = \left(\frac{1}{2}\right)^n$$

			MIN	MAX
(i)	$(\sin\theta - \cos\theta)$	$\Rightarrow$	$-1/1$	$1/2$
(ii)	$\sin^3\theta \cdot \cos^3\theta$	$\Rightarrow$	$-1/8$	$1/8$
(iii)	$\sin^2\theta \cdot \cos^2\theta$	$\Rightarrow$	0	$1/4$
(iv)	$\sin^4\theta \cdot \cos^4\theta$	$\Rightarrow$	0	$1/16$

Eg. Find the minimum and Maximum value of 2 sin θ cos θ

$$Y = \sin 2\theta$$

$$Y_{min} = -1 \qquad\qquad Y_{max} = 1$$

Eg. Find the minimum and Maximum value of

$$Y = \sin^3 \theta \cdot \cos^3 \theta$$

$$Y_{min} = -\frac{1}{8} \qquad\qquad Y_{max} = \frac{1}{8}$$

Eg. Find the minimum and Maximum value of

$$Y = \sin^4 \theta \cdot \cos^4 \theta$$

$$Y_{min} = 0 \qquad\qquad Y_{max} = \frac{1}{16}$$

Eg. Find the minimum and Maximum value of

$$Y = 32 \sin^5 \theta \cdot \cos^5 \theta$$

$$Y = \left(2\sin\theta \cos\theta\right)^5 = \left(\sin 2\theta\right)^5$$

$$Y_{min} = -1 \qquad\qquad Y_{max} = 1$$

$$\max (\sin^n \theta + \cos^m \theta) = 1 \qquad [m, n \geq 2]$$

$$(0 \leq \theta \leq 90)$$

Eg. $\max(\sin^6\theta + \cos^{10}\theta)$

Eg. $\max(\sin^{20}\theta + \cos^{40}\theta)$

$$\sin \theta + \cos \theta \geq 1$$

$$(0 \leq \theta \leq 90)$$

Eg. If $0 \le \theta \le 90$

min $(\sin\theta + \cos\theta)$

Eg. If $0 < \theta < 90$

$\sin\theta + \cos\theta = ?$

(a) < 1 (b) > 1

(c) ≥ 1 (d) 0

Eg. min (sin θ + cos θ)

Q3. Find the minimum and maximum value of :

$$Y = \sin^4 \theta + \cos^4 \theta$$

Q4. Find the minimum and maximum value of :

$$Y = \sin^6 \theta + \cos^6 \theta$$

Q5. $A = \sin^2\theta + \cos^4\theta$

Which of the following statement is true?

$a \quad \dfrac{1}{4} \leq A \leq \dfrac{1}{2}$

$b \quad \dfrac{3}{4} \leq A \leq 1$

$c \quad \dfrac{1}{2} \leq A \leq 1$

$d \quad \dfrac{3}{4} \leq A \leq \dfrac{3}{2}$

Q6. Find the minimum and maximum value of:

$$Y = \sin^2 \theta + 6 \sin \theta + 5$$

Q7. Find the minimum and maximum value of:

$$B = \sin^2 \theta - \sin \theta - 1$$

Q8. Find the minimum and maximum value of:

$$C = \sin^2 \theta + \cos \theta$$

Q9. Find the minimum value of:

$$\sin^2 A + \operatorname{cosec}^2 A + \sec^2 B + \cos^2 B + \tan^2 C + \cot^2 C$$

Q10. Find the minimum value of:

$$\sin^2 A + \cos^2 A + \tan^2 A + \csc^2 B + \sec^2 B + \cot^2 B$$

Q11. Find the minimum value of:

$$\sin^2 A + \cos^2 A + \tan^2 A + \cot^2 A + \sec^2 A + \csc^2 A$$

Practice Questions

Q1. Find minimum and maximum value of the following :

(a) $\sin 2\theta$

(b) $3 \sin \theta$

(c) $5 \sin 4\theta$

(d) $3 \sin^2 \theta$

(e) $5 \sin^3 \theta$

(f) $3 \sin^2 \theta - 8$

(g) $5 \sin^3 \theta - 4$

(h) $10 - \sin^2 \theta$

(i) $1 + \cos 2\theta$

(j) $4 + \sin \theta + \cos \phi$

(k) $5 \sin^2 \theta + 4 \cos^2 \phi$

(l) $\sec^2 \theta + 4$

Ans.

	Min	Max
(a)	-1	1
(b)	-3	3
(c)	-5	5
(d)	0	3
(e)	-5	5
(f)	-8	-5
(g)	-9	1
(h)	9	10
(i)	0	2
(j)	2	6
(k)	0	9
(l)	5	∞

Q2. Find minimum and maximum value of the following

    (a) $3 \sin \theta + \cos \theta$

    (b) $5 \sin \theta + 12 \cos \theta - 8$

Ans.

(a) Min = $-\sqrt{10}$ Max = $\sqrt{10}$

(b) Min = -21 Max = 5

Q3. Find minimum and maximum value of the following

(a) $-3\sin^2\theta - 4\cos^2\theta$

(b) $3\sin^2\theta - 4\cos^2\theta + 8$

(c) $\dfrac{1}{2}\sin^2\theta + \dfrac{1}{3}\cos^2\theta$

(d) $-5\,(2\sin^2\theta + 3\cos^2\theta)$

Ans.

	Min	Max
(a)	-4	-3
(b)	4	11
(c)	1/3	1/2
(d)	-15	-10

Q4. Find minimum and maximum value of the following.

(a) $4 \sin^2 \theta + 9 \csc^2 \theta$

(b) $8 \cos^2 \theta + 18 \sec^2 \theta$

(c) $12 \sin^2 \theta + 3 \csc^2 \theta$

(d) $4 \sec^2 \theta + 9 \cos^2 \theta$

Ans.

	Min	Max
(a)	13	∞
(b)	26	∞
(c)	12	∞
(d)	12	∞

Q5. Find minimum value of the following

(a) $4\tan^2\theta + 9\cot^2\theta$

(b) $18\sin^2\theta + 2\csc^2\theta$

(c) $72\cos^2\theta + 18\sec^2\theta$

(d) $18\tan^2\theta + 8\cot^2\theta$

(e) $6\sin^2\theta + 150\csc^2\theta$

(f) $14\cos^2\theta + 56\sec^2\theta$

(g) $4\sin^2\theta + 9\csc^2\theta + 5$

(h) $\cos^2\theta + \sec^2\theta$

(i) $4\csc^2\theta + 4\sin^2\theta$

(j) $\tan^{10}\theta + \cot^{10}\theta$

Ans.

(a) 12

(b) 12

(c) 72

(d) 24

(e) 156

(f) 70

(g) 18

(h) 2

(i) 8

(j) 2

Q6. Find minimum value of the following

(a) $4\sec^2\theta + 25\ \mathrm{cosec}^2\theta$

(b) $100\sec^2\theta + 9\ \mathrm{cosec}^2\theta$

Ans.
(a) 49
(b) 169