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Sahi Prep Hai Toh Life Set Hai

MENSURATION-2D

Part – 1

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** Yesterday class → Doubt session

Will be on coming Monday

→ 6th Sep

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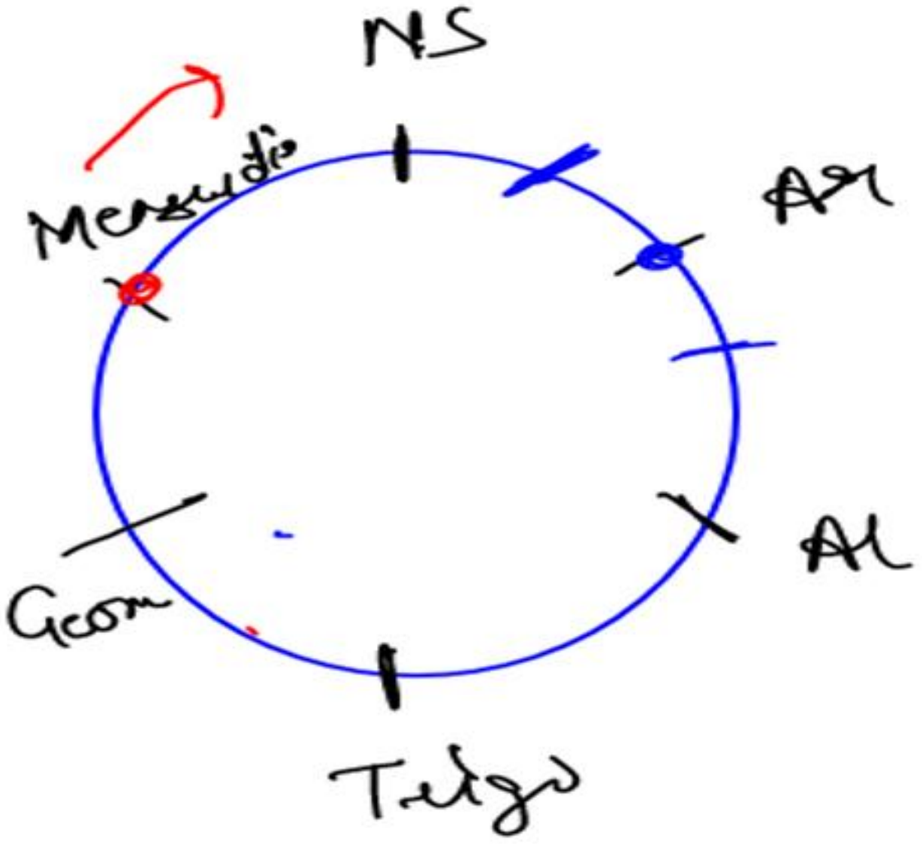
How we go through Grade up

Quant Syllabus



I Number System

II Arithmetic

III Algebra

IV Trigo

V Geometry

VI Mensuration

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Mensuration → (2-2½) weeks

2D

- Triangles
- Circles
- Comb of Triangles & Circles
- Quad | Square
- Comb of Square & Circle

3D

- Cube | Cuboid
- Cylinder | Hollow
- Cone | Frustum
- Sphere | Hollow
- Prism, Pyramid
- Tetrahedron
- Comb of Fig (MIX)

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UNITS

1 Km = 1000 M

1 m = 100 cm

1 M = 1000 MM

1 Hectar = 10000 M^2

mm	millimeter
cm	centimeter
dm	decimeter
m	meter
Dm	Decameter
Hm	Hectometer
Km	Kilometer

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Eg Area of a Field → 4 Hectares

Area → $4 \times 10000\ M^2$

$= 40000\ M^2$ ✓

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REVISION OF TRIANGLE FORMULAS

1. Area of triangle
2. Inradius & Circumradius
3. Equilateral, Right Angle and Isosceles Triangle
4. Medians of Triangle
5. Similarity and Congruency

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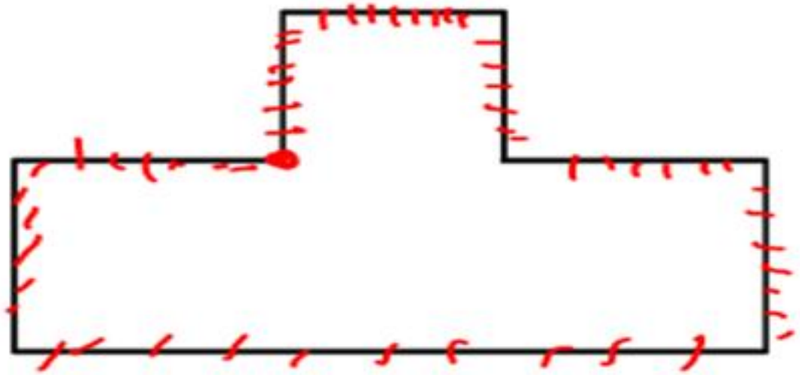
What is Area ?

The area can be defined as the space occupied by a flat shape or the surface of an object.

The area of a figure is the number of unit squares that cover the surface of a closed figure. Area is measured in square units such as square centimeters, square meter, etc.

PERIMETER

Perimeter can be defined as the path or the boundary that surrounds a figure. It can also be defined as the length of the outline of a shape.



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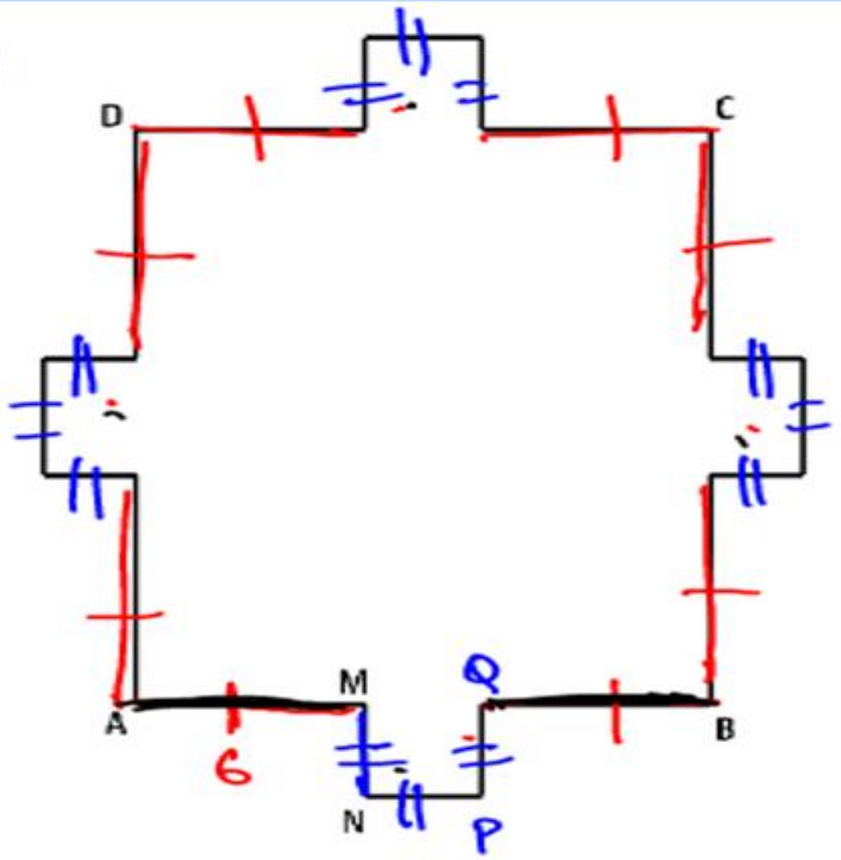

Eg. Find the area and perimeter of the following figure.

AM = 6 cm **MN = 2 cm**

ABCD → Square

MNPQ → Square

Perimeter → $6 \times 8 + 12 \times 2$

$\Rightarrow 48 + 24 = 72\text{cm}$

Area → $14^2 + 4 \cdot 2^2$

$196 + 16$

212cm^2

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Ans. 72 cm

↓

Perimeter

Area = 212 cm^2

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TYPES OF TRIANGLES

(1) Scalene Triangle : Is a Δ in which all sides are distinct.

$a \neq b \neq c$

(2) Isosceles Triangle : Is a Δ in which atleast 2 sides are equal.

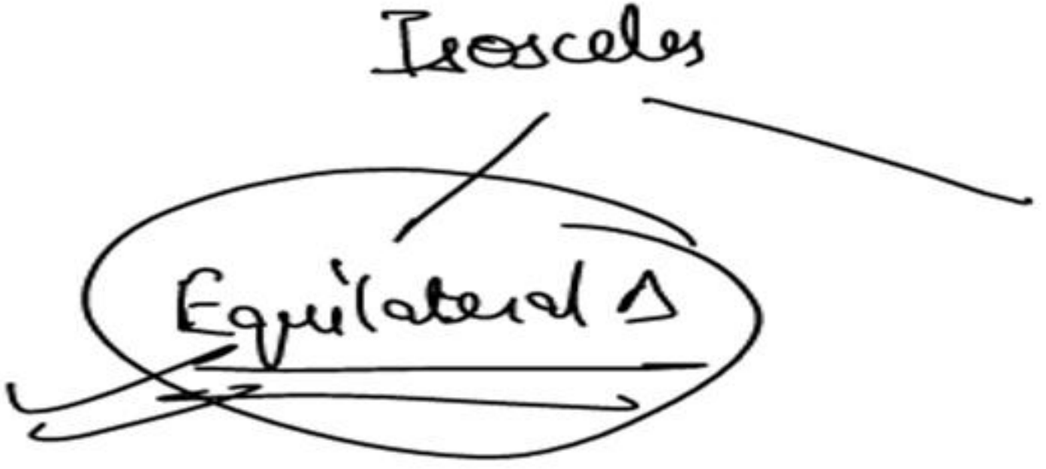
$a = b \neq c$

(3) Equilateral Triangle : Is a Δ in which all sides are equal.

$a = b = c$

All equilateral Δ's are Isosceles

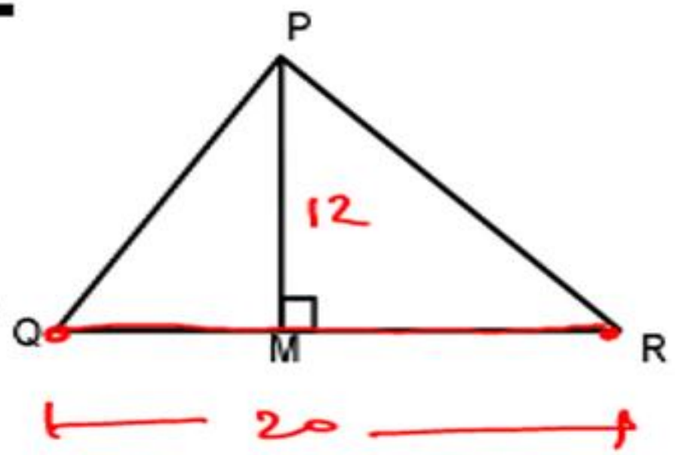
But all Isosceles Δ's are not equilateral.



AREAS OF TRIANGLE

(1) $$Area = \frac{1}{2} \times Base \times Height$$

Eg1. If QR = 20 cm
PM = 12 cm
Find area of triangle.



Area $= \frac{1}{2} \times 20 \times 12$

$= 120\ cm^2$

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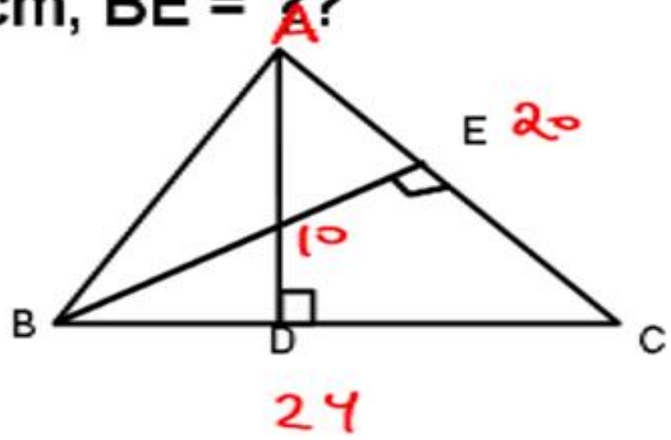
Q1. If AD = 10 cm, BC = 24 cm, AC = 20 cm, BE = ?



$\frac{1}{2} \times 24 \cdot 10 = \frac{1}{2} \times 20 \times BE$

$BE = 12\ cm$

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Q2. Area of 2 triangles are in the ratio 16 : 25 and their altitudes are in the ratio 5 : 4. Find the ratio of their corresponding base?

$$\frac{\text{Area of } \Delta_1}{\text{Area of } \Delta_2} = \frac{\frac{1}{2} B_1 H_1}{\frac{1}{2} B_2 H_2} \Rightarrow \frac{16}{25}$$

$$\frac{B_1}{B_2} \cdot \frac{5}{4} = \frac{16}{25}$$

$$\frac{B_1}{B_2} = \frac{64}{125}$$

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(2)

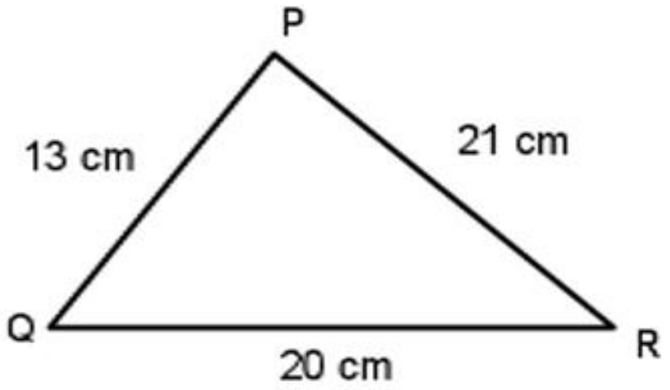
$$\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$s \rightarrow semi-perimeter$

$$s = \frac{a+b+c}{2}$$

a, b & c are sides of Δ.

Eg2. Find the area of given triangle.



$$S = \frac{13+20+21}{2} = 27$$

$$\text{Area} = \sqrt{(27)(14)(7)(6)}$$

$$= \sqrt{3 \cdot 3 \cdot 3 \cdot 7 \cdot 2 \cdot 7 \cdot 2 \cdot 3}$$

$3 \cdot 3 \cdot 7 \cdot 2 \Rightarrow 126 \text{ cm}^2$

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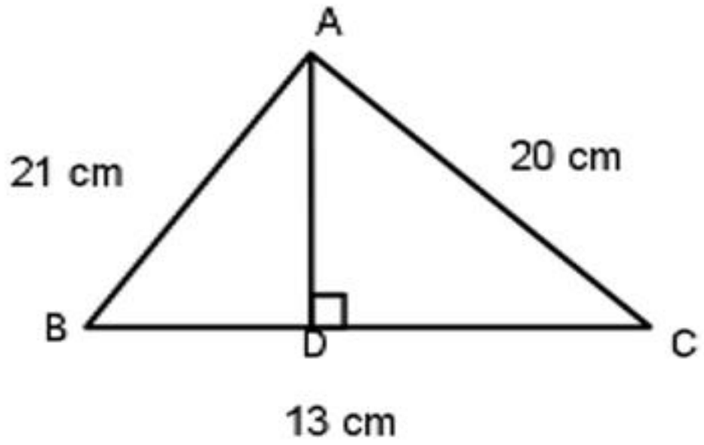
Q3. Find AD = ??



Area of $\Delta = 126\ cm^2$

$126 = \frac{1}{2} \times 13 \times AD$

$AD = \frac{252}{13}$

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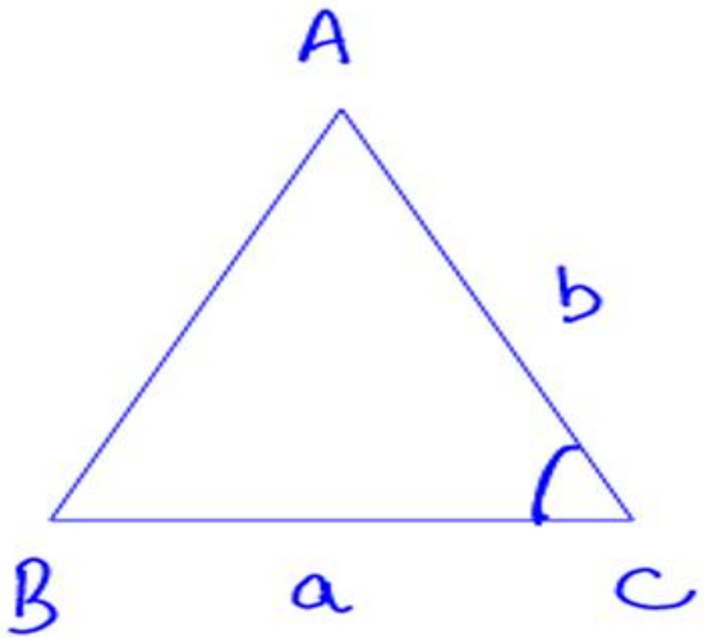
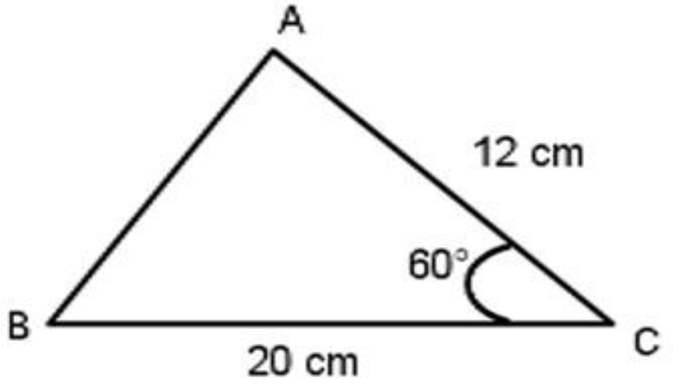
(3) $$\textbf{Area of } \Delta = \frac{1}{2}ab\sin C$$



Eg3. Find area of triangle.



$$\frac{1}{2} \cdot \overset{10}{\cancel{20}} \cdot 12 \sin 60$$

$$= 10 \cdot \overset{6}{\cancel{12}} \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$60\sqrt{3}\ cm^2$$

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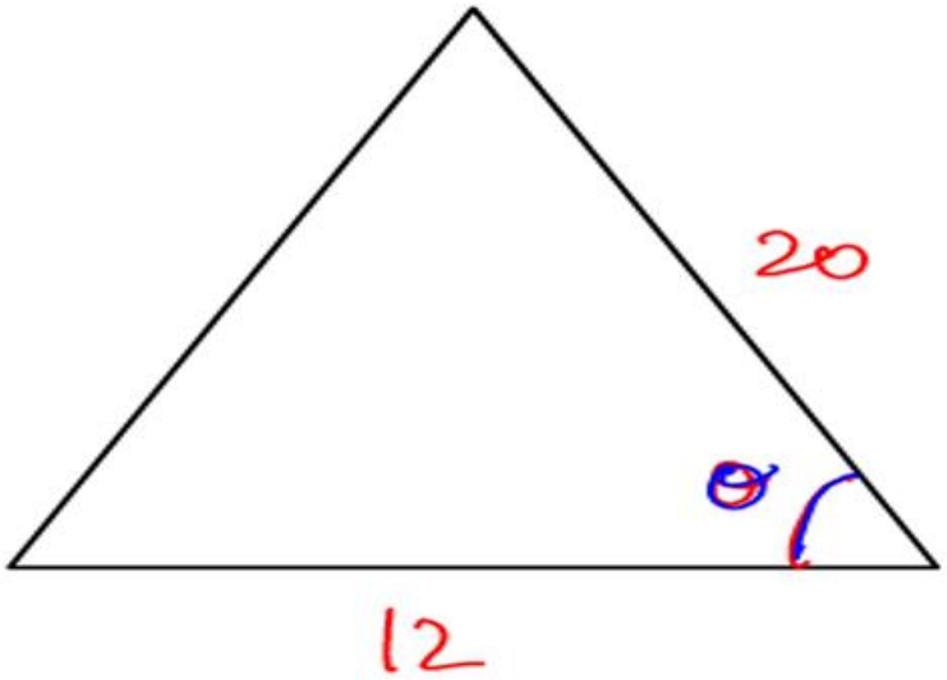
Q4. If 2 sides of a triangle are 12 cm and 20 cm, what can be the maximum area of triangle?



$$\text{Area} = \frac{1}{2} \cdot 12 \cdot 20 \sin\theta$$

$$\text{Area}_{\text{Max}} = \frac{1}{\cancel{2}} \cdot \cancel{12}^{6} \cdot 20 \cdot 1$$

$$120\ \text{cm}^2$$

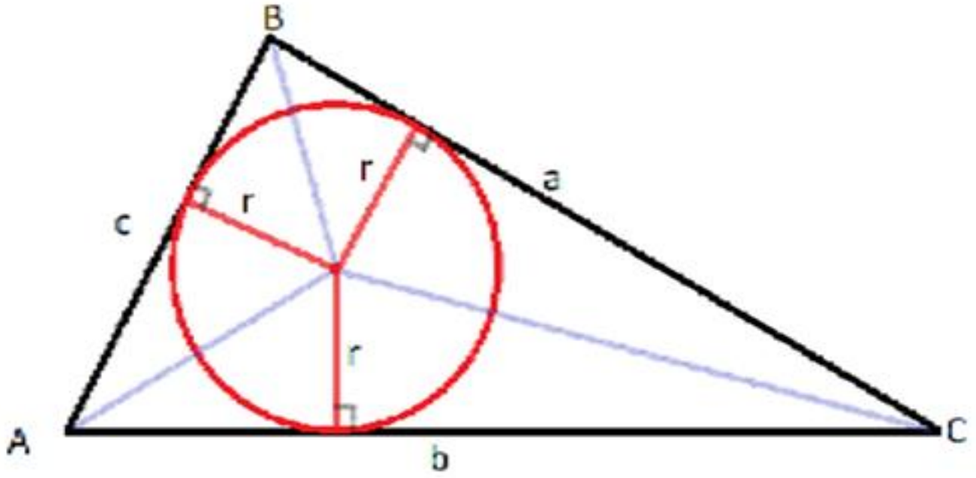
If 2 sides of Δ are given then maximum area is always of a Right Angled Triangle.

If a, b are 2 sides of a Δ :

$$\textbf{Max Area} = \frac{1}{2}ab$$

(4) $\text{Area} = r \cdot s$

where :
r - inradius
s - semi-perimeter



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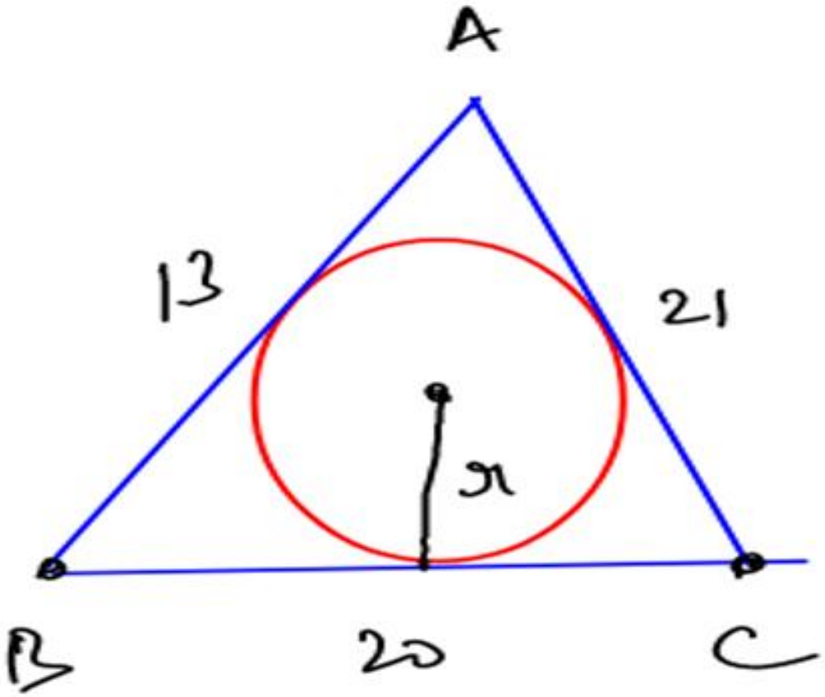
Q5. Find the in-radius of triangle whose sides are 13 cm, 21 cm and 20 cm.



$$\text{Area} = r \cdot s$$

$$s = \frac{13+20+21}{2} = 27$$

$$126 = r \cdot s$$

$$126 = r \cdot 27$$

$$r = \frac{126}{27} \; 14$$

$$3$$

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$r \cdot s = \text{Area}$

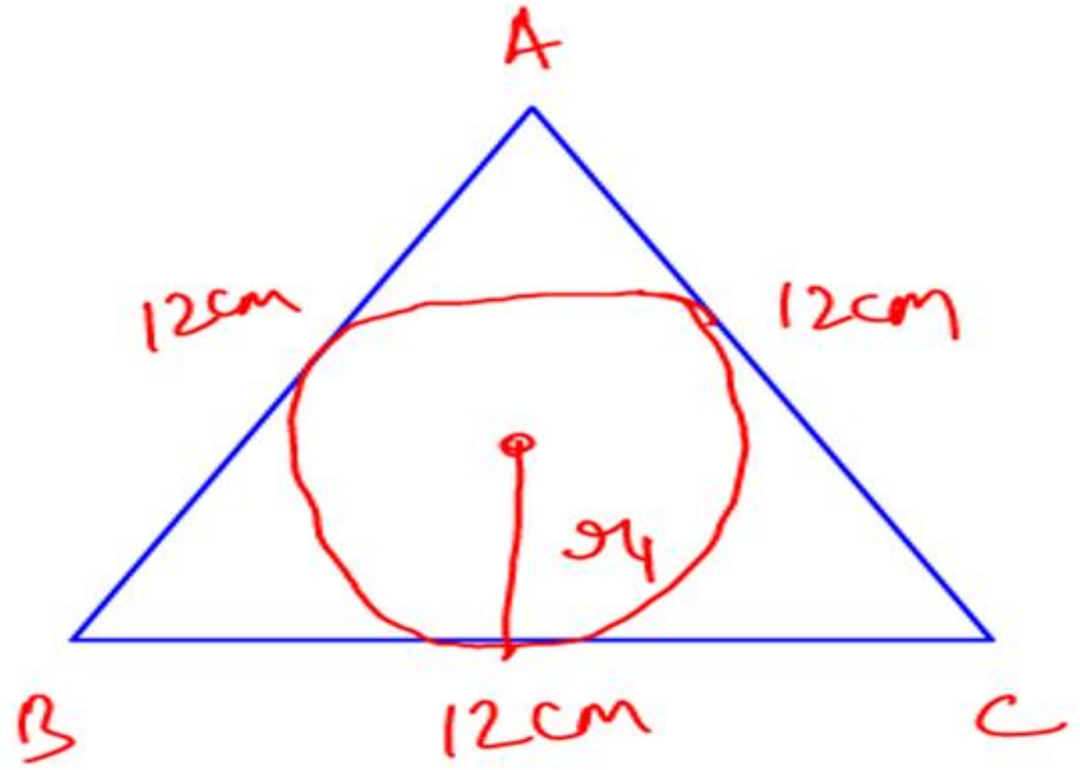
$$r = \frac{Area}{s}$$

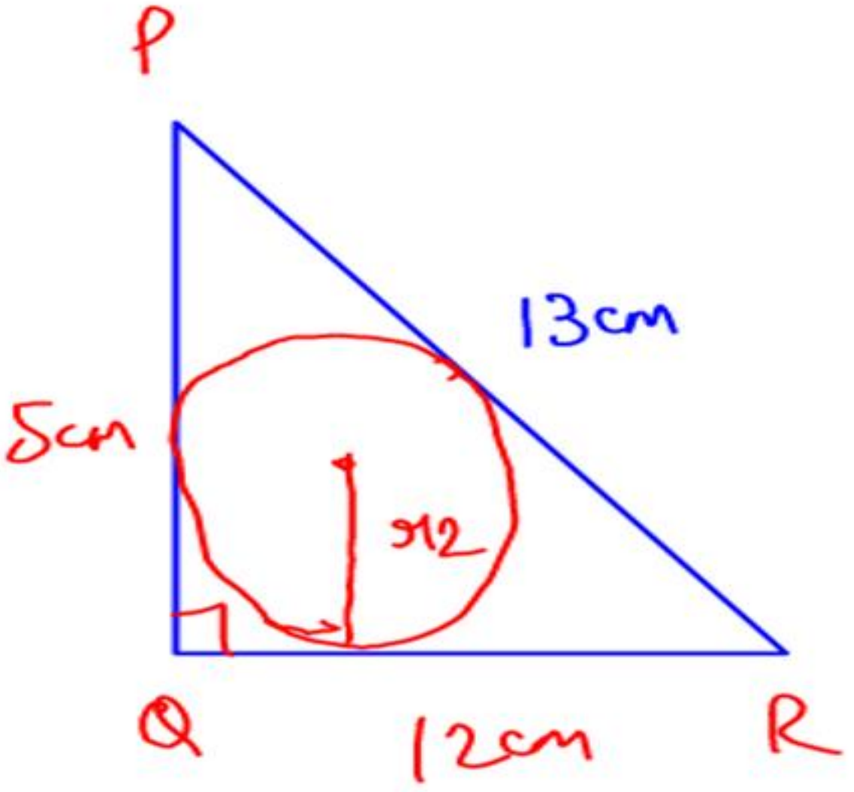
Inradius (r)

For any Δ $= \dfrac{Area}{s}$

Equilateral Δ $= \dfrac{Side}{2\sqrt{3}}$

Right angle Δ $= \dfrac{Base + Perpendicular - Hypotenuse}{2}$





Equilateral $\triangle$

$$r_1 = \frac{\text{side}}{2\sqrt{3}} = \frac{12}{2\sqrt{3}}$$

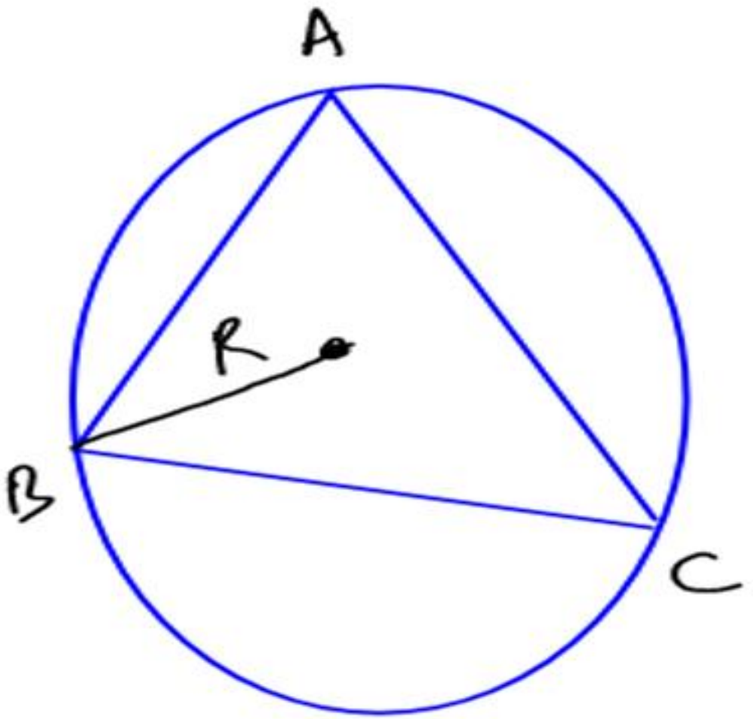
$$\frac{12^{6}}{2\cdot\sqrt{3}} \Rightarrow \frac{2\cdot\sqrt{3}\cdot\sqrt{3}}{\sqrt{3}}$$

$$= 2\sqrt{3}\ \text{cm}$$

$$r_2 = \frac{5+12-13}{2}$$

$$= 2\text{cm}$$

CIRCUM RADIUS



The circumradius is the radius of the circumscribed circle of that polygon.

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(5) $$\textbf{Area of } \Delta = \frac{a \cdot b \cdot c}{4R}$$

where, a, b, c are sides of triangle.

R → Circum-radius

Eg4. Find the circum-radius of triangle whose sides are 13 cm, 21 cm and 20 cm.

$$126 = \frac{13 \cdot 20 \cdot 21}{4 \cdot R}$$

$$R = \frac{65}{6} \text{ cm}$$

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$$\text{Area of } \Delta = \frac{a \cdot b \cdot c}{4R}$$

$$R = \frac{a \cdot b \cdot c}{4 \text{Area of } \Delta}$$

Circumradius (R)

For any Δ	=	$\frac{a \cdot b \cdot c}{4 \text{Area of } \Delta}$
Equilateral Δ	=	$\frac{\text{Side}}{\sqrt{3}}$
Right angle Δ	=	$\frac{\text{Hypotenuse}}{2}$

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AREA OF TRIANGLE (For any Δ)

(1) $Area = \frac{1}{2} \times Base \times Height$

(2) $Area\ of\ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

(3) $Area\ of\ \Delta = \frac{1}{2}ab\sin C$

(4) $Area\ of\ \Delta = r \cdot s$

(5) $Area\ of\ \Delta = \frac{a \cdot b \cdot c}{4R}$

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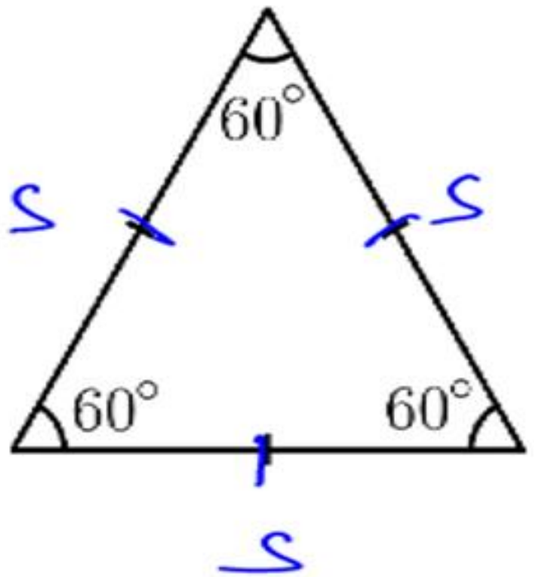
EQUILATERAL TRIANGLE

An **equilateral triangle** is a **triangle** in which all three sides are equal.



Height of equilateral $\Delta = \frac{\sqrt{3}}{2} \times S$

Area of equilateral $\Delta = \frac{\sqrt{3}}{4} \times S^2$

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Eg. If height of equilateral triangle = 12 cm.
Find area of equilateral triangle.

$$\frac{\sqrt{3}}{2} s = 12 \qquad s = 8\sqrt{3}$$

$$\text{Area} = \frac{\sqrt{3}}{4} \cdot (8\sqrt{3})^2 \quad (8\sqrt{3})$$

$$= 48\sqrt{3} \text{ cm}^2$$

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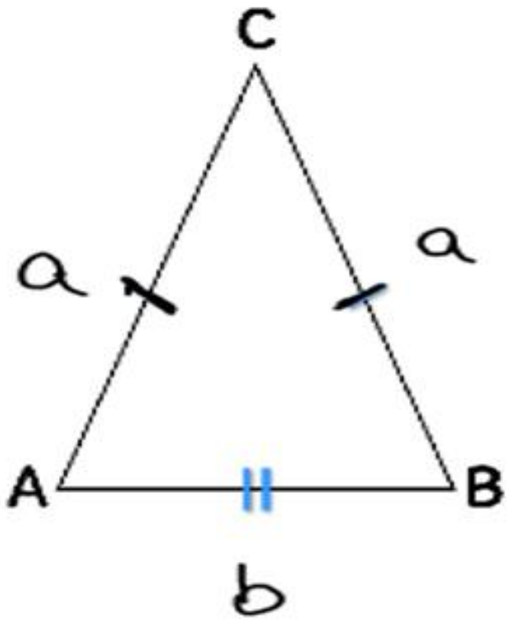
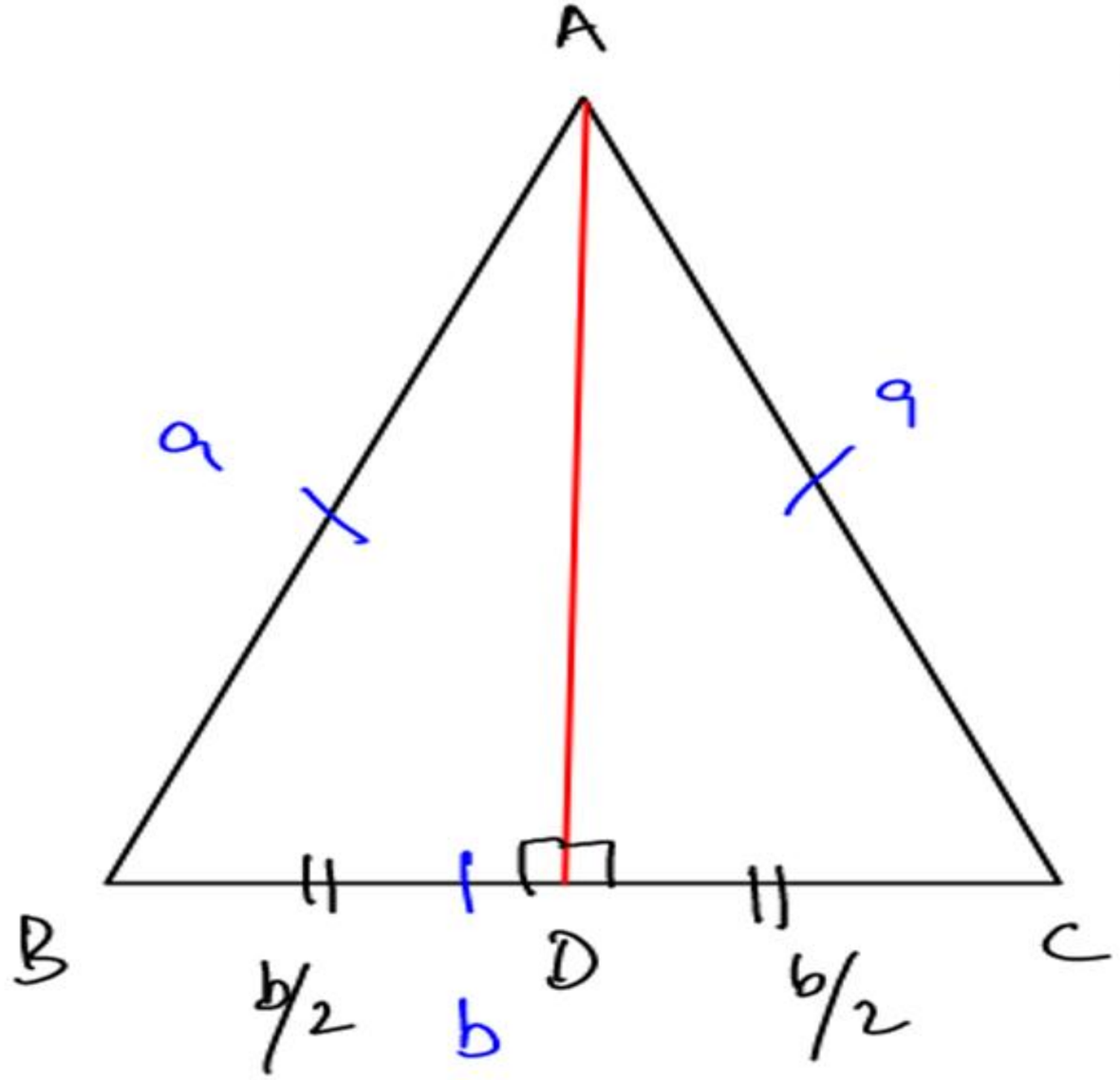
ISOSCELES TRIANGLE

An **isosceles triangle** is a **triangle** that has two sides of equal length.

$$\textbf{Area of isosceles } \Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$$

Where, b is base of isosceles Δ.
and a is length of equal sides.





$\frac{b}{4}\sqrt{4a^2-b^2}$

ΔADC

$(AC)^2 = (AD)^2 + (DC)^2$

$a^2 = AD^2 + \frac{b^2}{4}$

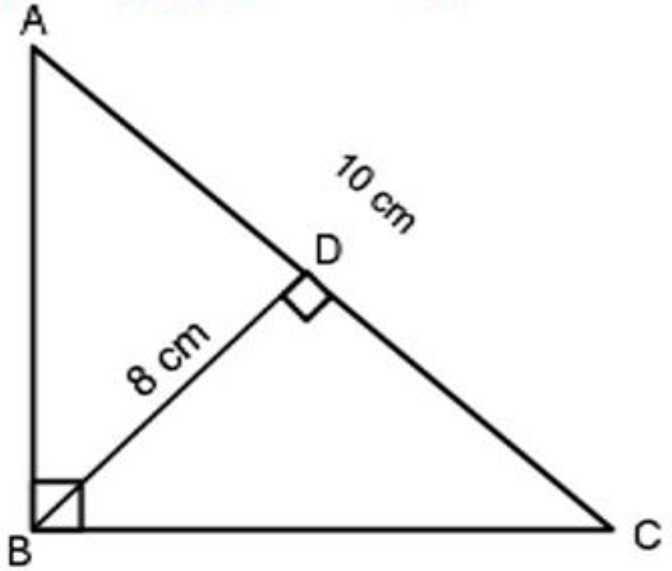
$AD = \sqrt{a^2 - \frac{b^2}{4}}$

$= \frac{\sqrt{4a^2-b^2}}{2}$

$\text{Area} = \frac{1}{2} \cdot b \cdot \frac{\sqrt{4a^2-b^2}}{2}$

Geometry

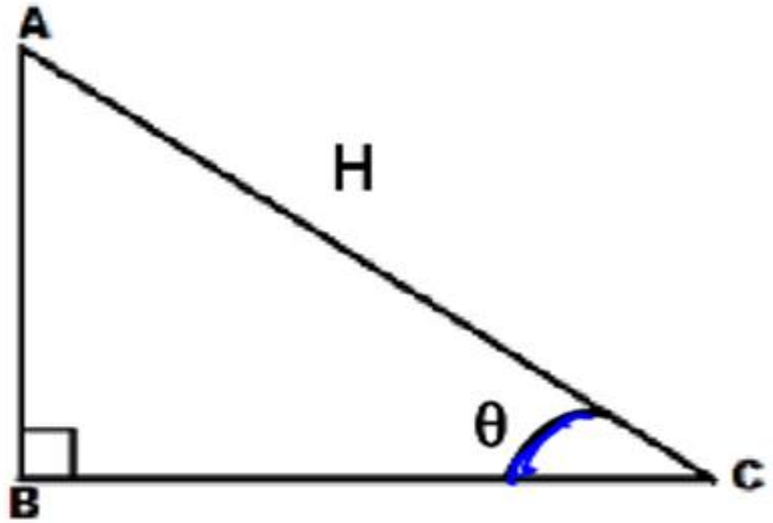
Eg. In a ΔABC, AC = 10 cm ; BD = 8 cm
Find area of ΔABC.

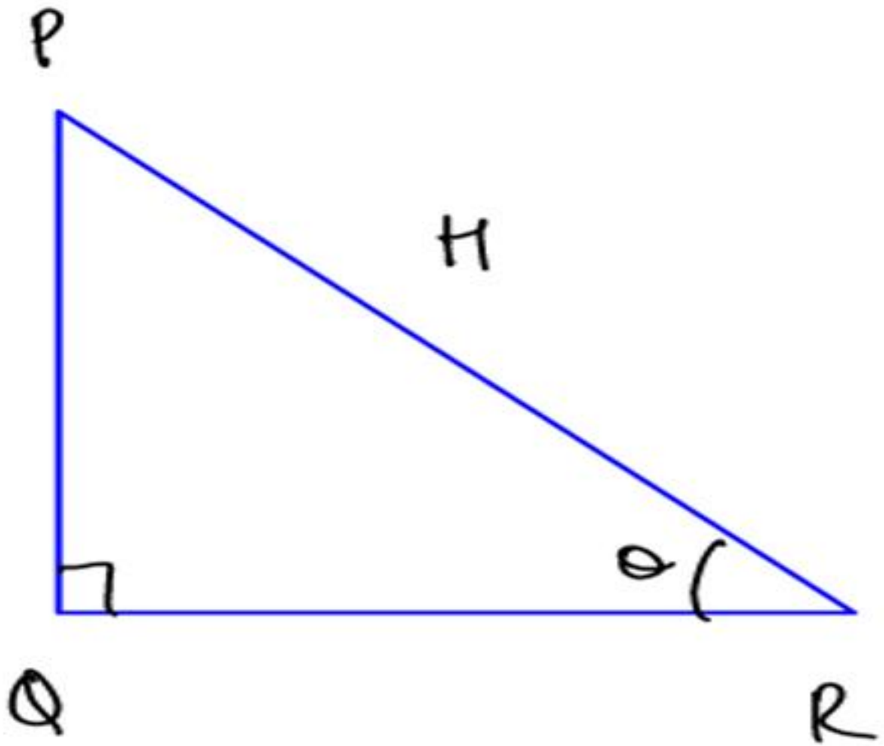


RIGHT ANGLE TRIANGLE

$$\textit{Area of right angle } \Delta = \frac{H^2}{4}\sin 2\theta$$

Where, H → Hypotenuse
and, θ → one of the acute angle of
right angle triangle.





$$\sin\theta = \frac{PQ}{H}$$

$$\cos\theta = \frac{QR}{H}$$

Area of $\triangle PQR$

$$= \frac{1}{2}(QR)PQ$$

$$= \frac{1}{2}(H\cos\theta)(H\sin\theta)$$

$$= \frac{1}{2} H^2 (2\sin\theta\cos\theta)$$

$$\frac{H^2}{4}\sin 2\theta$$

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Eg. If hypotenuse of a right angle Δ is 10 cm. What can be its maximum area?

$$\text{Area} = \frac{H^2}{4}\sin 2\theta$$

$$(\text{Area})_{max} = \frac{(10)^2}{4}$$

$$= 25\ cm^2$$

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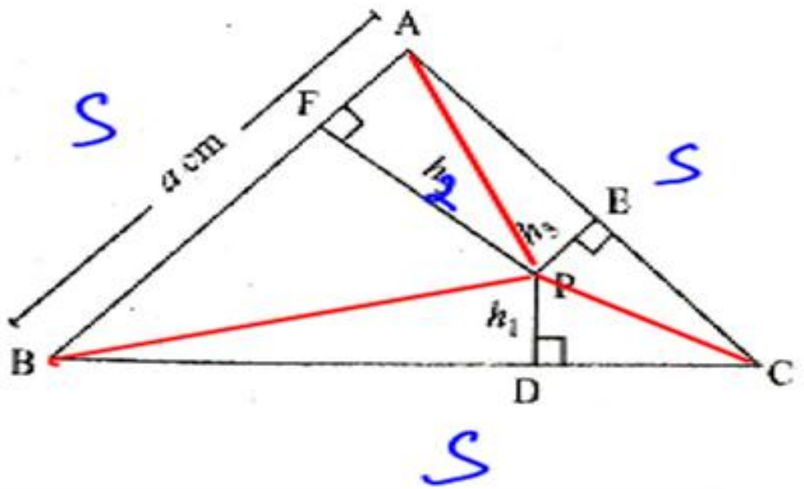
If P is a point inside an equilateral triangle ABC, the sum of altitudes from the point P to the sides AB, BC and AC is equal to the altitude of the ΔABC.



S S S

Area of eq ΔABC = Area of ($\Delta ABP + \Delta BPC + \Delta APC$

$$\frac{\sqrt{3}}{4}S^2 = \frac{1}{2}\cdot S h_2 + \frac{1}{2}S\cdot h_1 + \frac{1}{2}S\cdot h_3$$

$$\frac{\sqrt{3}}{4}S^2 = \frac{1}{2}S(h_1 + h_2 + h_3)$$

$$\frac{\sqrt{3}}{2}S = h_1 + h_2 + h_3$$

Height of eq Δ = $h_1 + h_2 + h_3$

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Eg. If the length of the three perpendiculars from a point in the interior of an equilateral triangle to the sides are 4 cm, 5 cm and 6 cm, then find the area of the triangle.

(a) $50\sqrt{3}\ cm^2$

(b) $75\sqrt{3}\ cm^2$

(c) $225\ cm^2$

(d) $100\ cm^2$

$H = 4+5+6 \rightarrow 15\ cm$

$\frac{\sqrt{3}}{2}s = 15$

$s = 10\sqrt{3}$

$Area = \frac{\sqrt{3}}{4} \cdot 10\sqrt{3} \cdot 10\sqrt{3}$

$= 75\sqrt{3}\ cm^2$

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Ans. (b)

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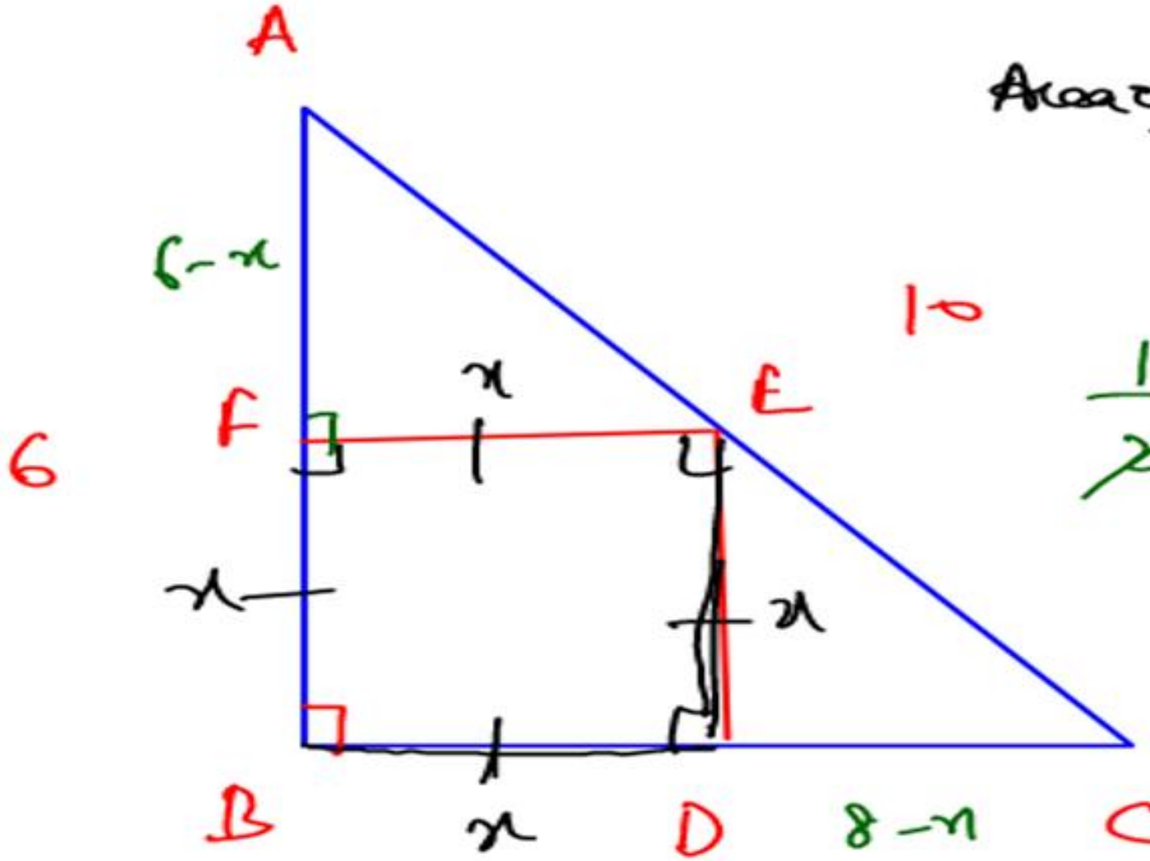
Eg. Find the area of the largest square inscribed in a right angle triangle whose sides are 6, 8 and 10 cm.



Area of ΔABC = Area (ΔAFE + Sq BDEF + ΔDEC)

$$\frac{1}{2} \cdot 6 \cdot 8 = \frac{1}{2} x(6-x) + x^2 + \frac{1}{2}(8-x)(x)$$

$$24 = 3x - \frac{x^2}{2} + x^2 + 4x - \frac{x^2}{2}$$

$$24 = 7x$$

$$\boxed{x = \frac{24}{7}}$$

$$\text{Area} = \left(\frac{24}{7}\right)^2 = \frac{576}{49}$$

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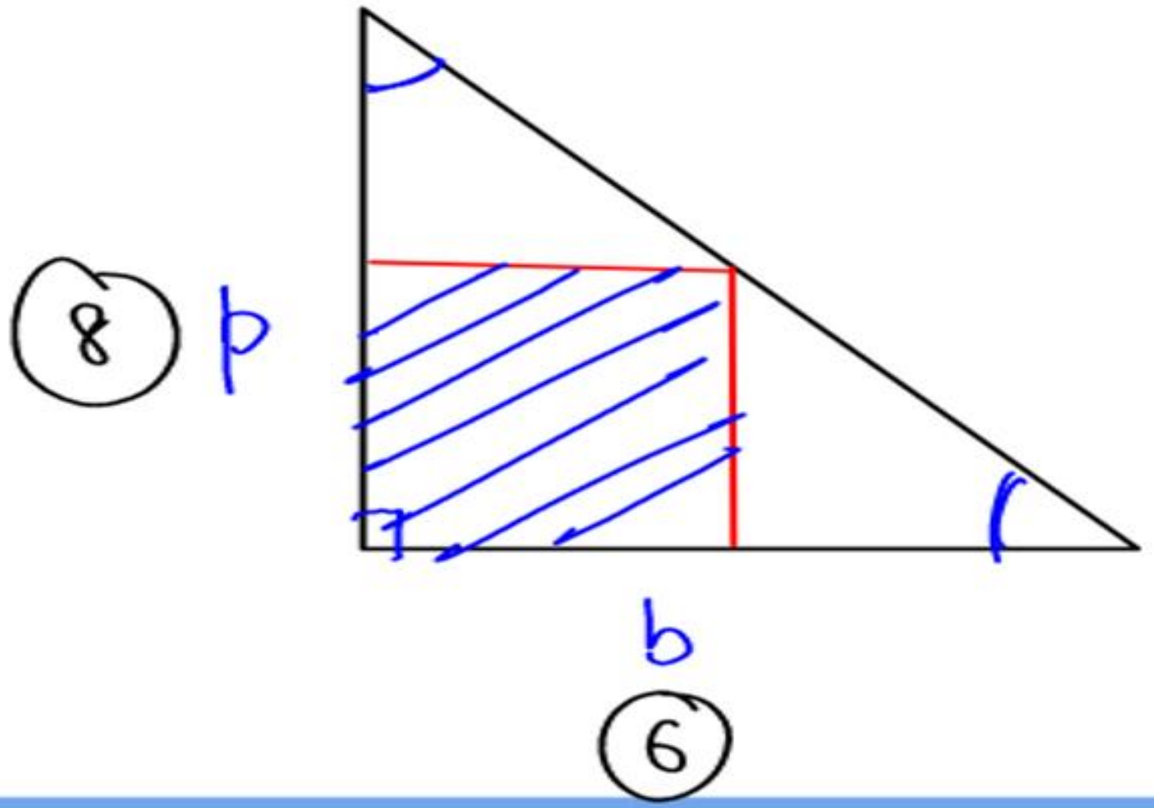
Area of largest square inside a right angle triangle whose base is b and perpendicular is p =

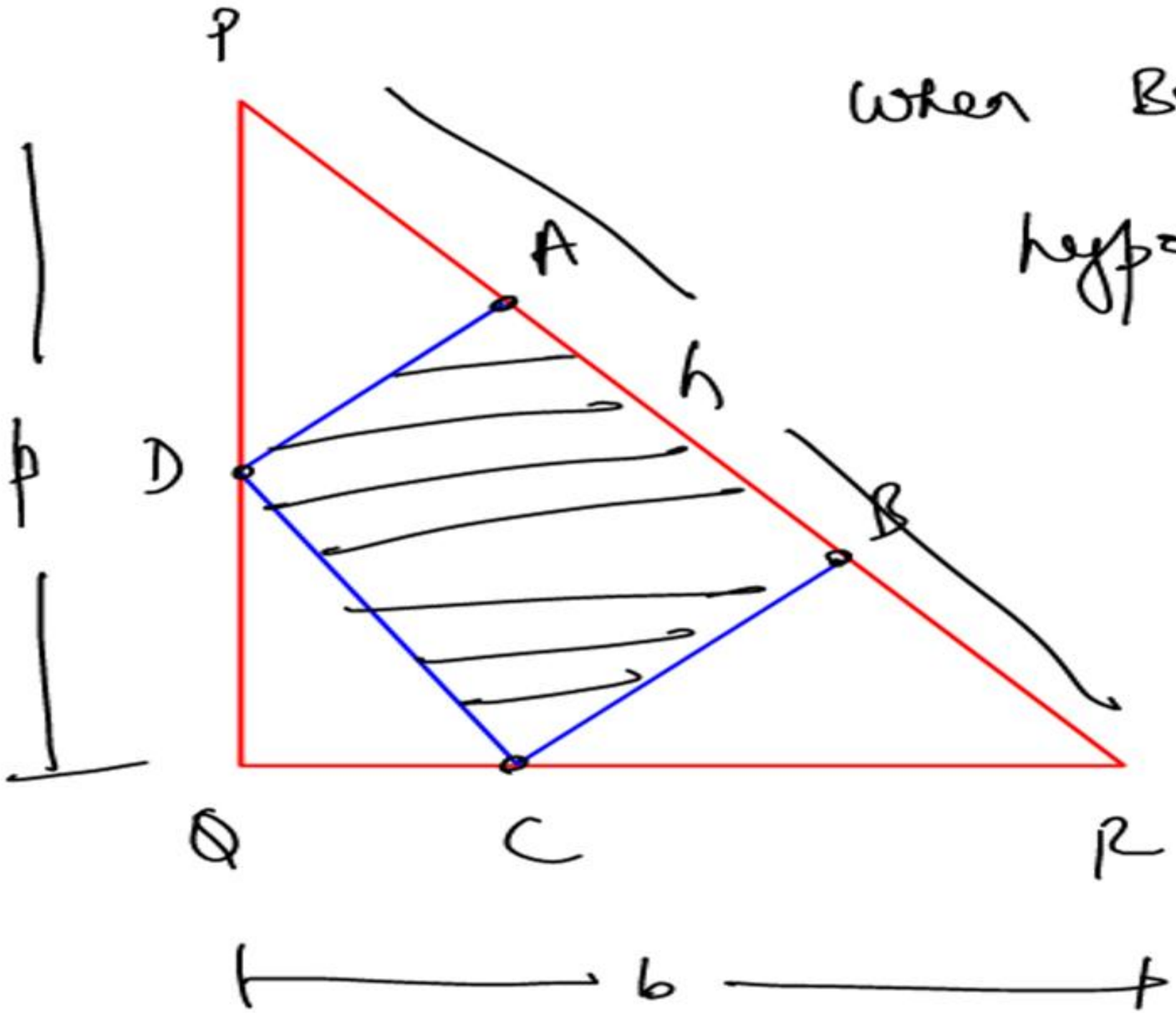
$$\left(\frac{b \cdot p}{b + p}\right)^2$$



$$\text{Area} = \left(\frac{6 \cdot 8}{6+8}\right)^2$$

$$= \left(\frac{24}{7}\right)^2 = \frac{576}{49}$$

When Base is on hypotenuse



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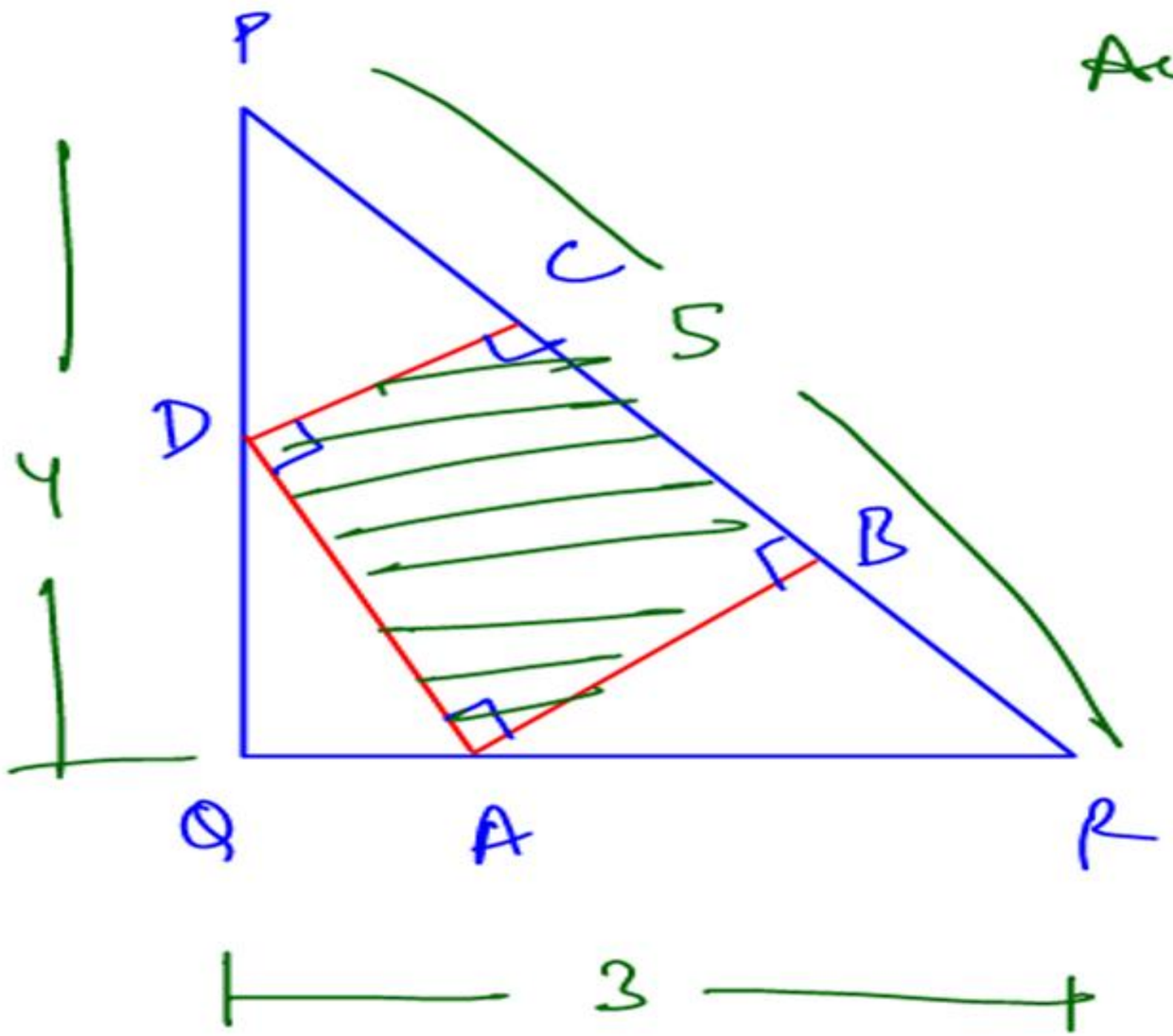
Area of largest square that can be inscribed in a right angle triangle in such a way that one of its side lies on the hypotenuse.

$$\left(\frac{b \cdot p \cdot h}{b^2 + bp + p^2}\right)^2$$



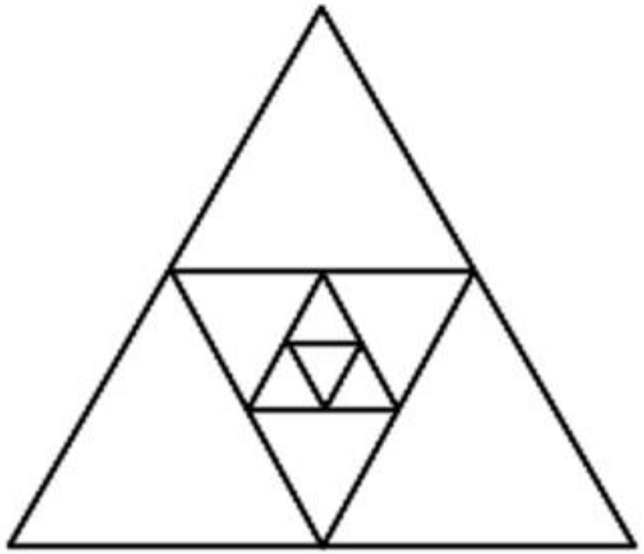
Area of square ABCD

$$\rightarrow \left(\frac{b \cdot p \cdot h}{b^2 + bp + p^2}\right)^2$$

$$\left(\frac{3 \cdot 4 \cdot 5}{9 + 12 + 16}\right)^2$$

$$= \left(\frac{60}{37}\right)^2 = \frac{3600}{1369}\ cm^2$$

USAGE OF GP or Ratio



(i) $\frac{\text{Area of } T_3}{\text{Area of } T_6}$

(ii) $\frac{\text{Perimeter of } T_{11}}{\text{Perimeter of } T_6}$

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Ans. (i) 64 : 1

(ii) 1 : 32

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Infinite GP

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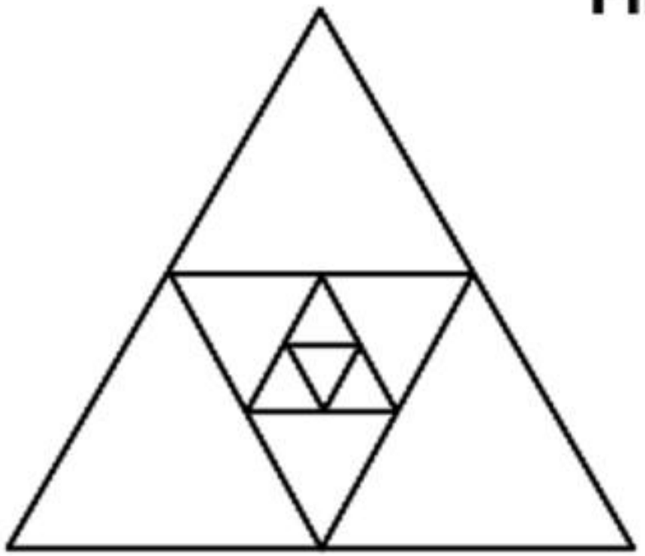
(iii) If side of T_1 = 20 cm

Find area of ($T_1 + T_2 + \ldots\ldots T_\infty$)



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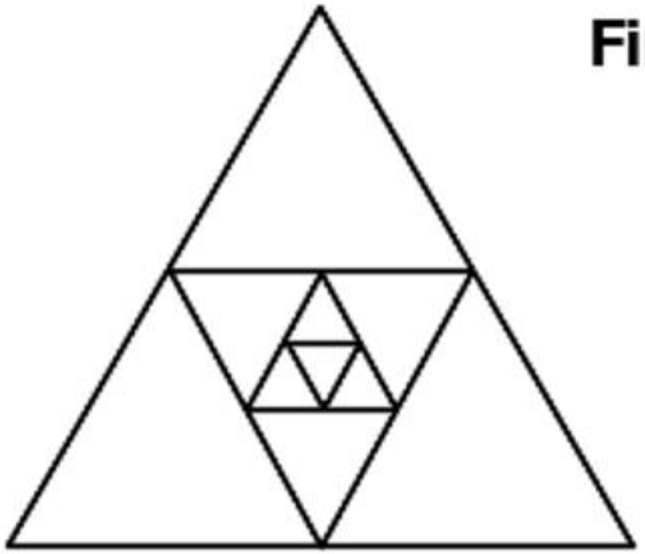
Ans. $\frac{400\sqrt{3}}{3} cm^2$

(iv) If side of T_1 = 20 cm

Find perimeter of ($T_1 + T_2 + \ldots\ldots T_\infty$)



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Ans. 120 cm

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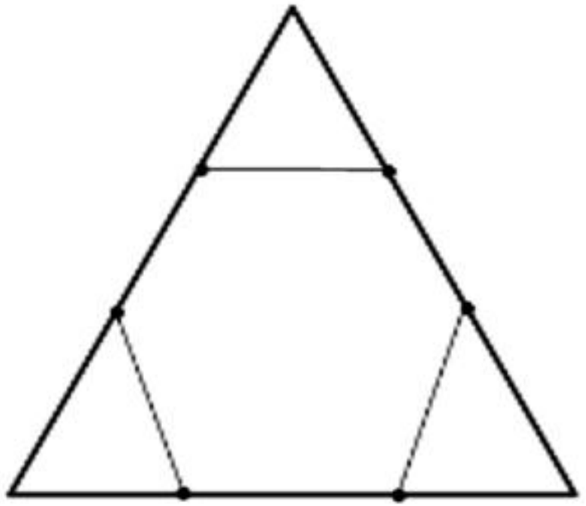
If corners of equilateral triangle are cut to form a regular hexagon.



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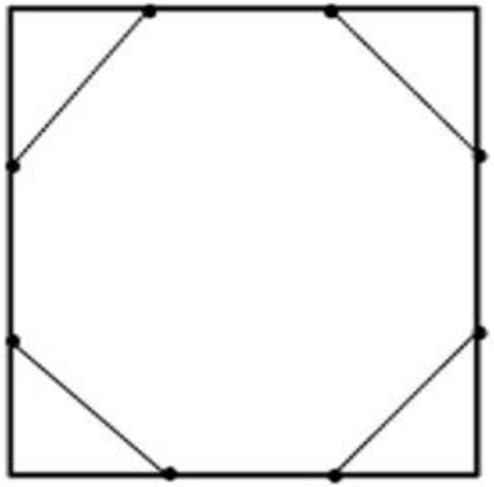
Eg. If corners of an equilateral triangle of side 12 cm are cut to form a regular hexagon. Find the area of regular hexagon.

gradeup

Ans. $24\sqrt{3}\ cm^2$

gradeup

Eg. If corners of a square of side 10 cm are cut to form a REGULAR OCTAGON. Find the side of the REGULAR OCTAGON.

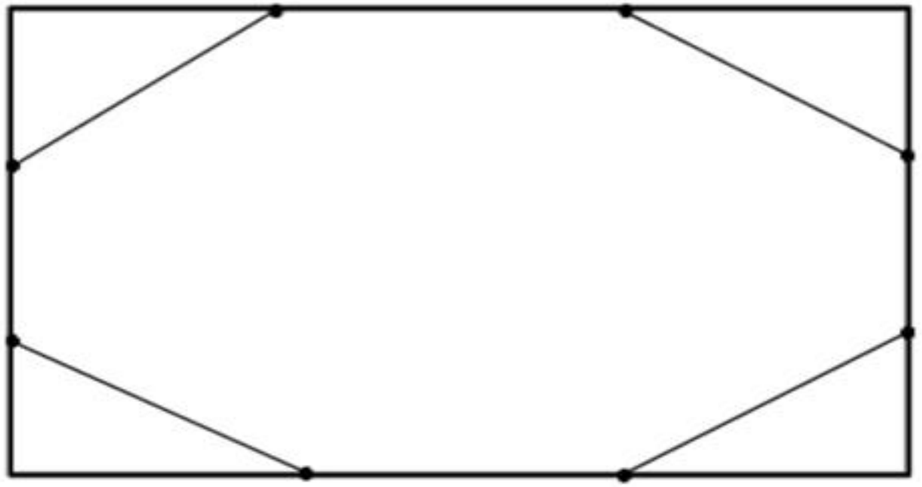


gradeup

Ans. $10(\sqrt{2}-1)$

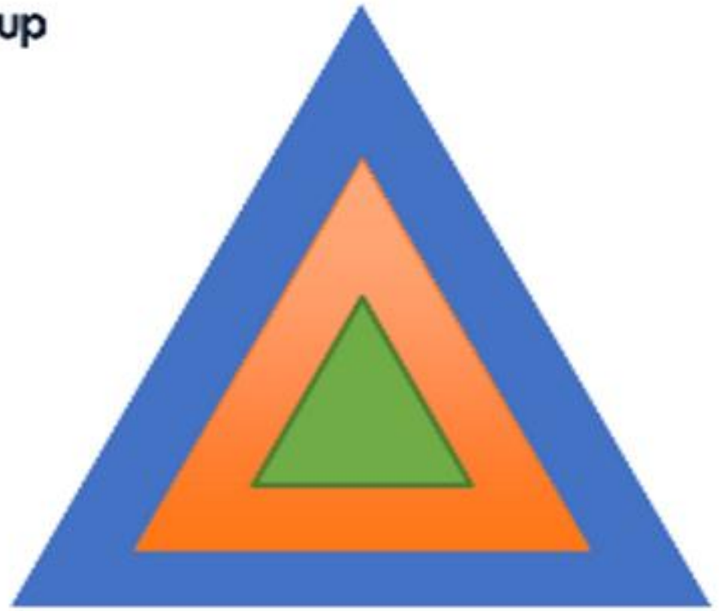
gradeup

Eg. If corners of a rectangle of sides 6 & 8 cm, are cut to form a OCTAGON (whose all sides are equal). Find the side of the OCTAGON.



gradeup

Ans. $-7+\sqrt{99}$



USAGE OF RATIO

All are equilateral Δ's

If area of Green : Orange : Blue = 1 : 3 : 5

Find the ratio of their sides.

gradeup

If two sides of a triangle are given, then the maximum area is of right angle triangle.

Eg. If two sides of a triangle are 8 and 10 cm, find the maximum area of the triangle.

gradeup

Ans. 40 cm^2

gradeup

If perimeter of a triangle is given, then the maximum area is of equilateral triangle.

Eg. If perimeter of a triangle is 60 cm. What is the maximum area of Δ?

gradeup

Ans. $100\sqrt{3}$ cm^2

gradeup

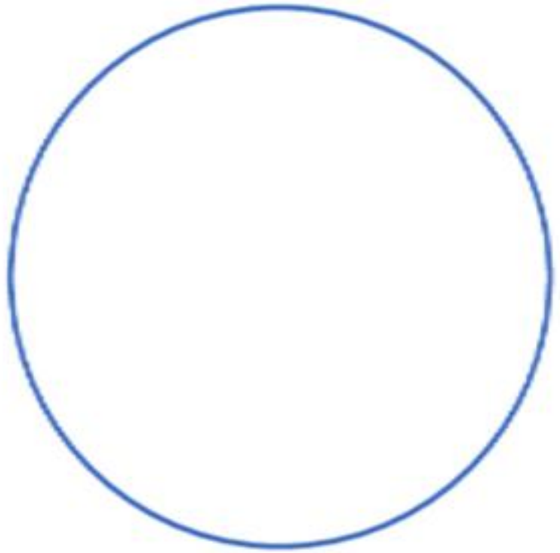
Eg. If hypotenuse of a right angle triangle is 12 cm. Find the maximum area of a triangle.

gradeup

Ans. 36 cm^2

gradeup

BASICS OF CIRCLE



Area of Circle = πr^2

Circumference of Circle = $2\pi r$

$$\text{Area of Semi-Circle} = \frac{\pi r^2}{2}$$

$$\text{Circumference of Semi-Circle} = \pi r + 2r$$

gradeup

Eg. If circumference of a semi-circle is 72 cm. Find its area.

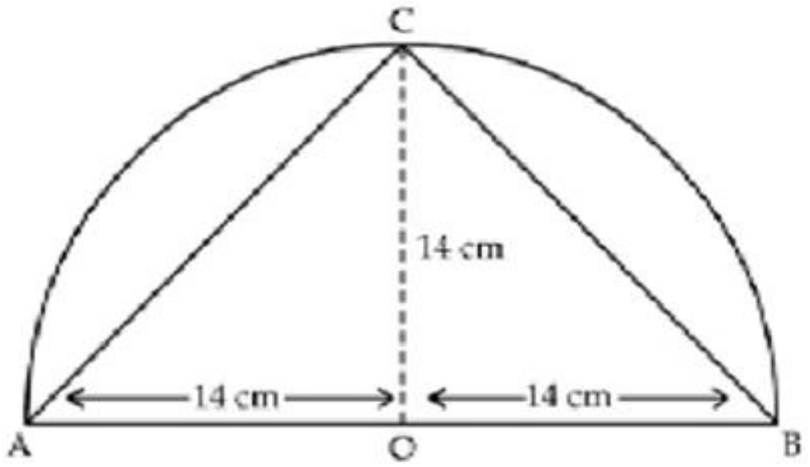
(Take $\pi = 22/7$)

gradeup

Ans. 308 cm^2

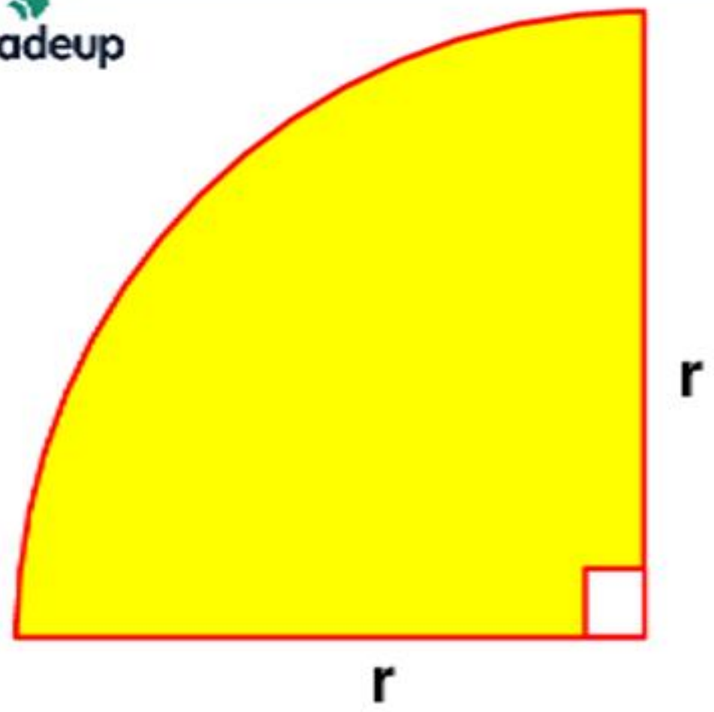
gradeup

Eg. Find the area of the largest triangle that can be drawn inside a semi-circle of radius 14 cm.


C
14 cm
14 cm
14 cm
A
O
B

gradeup

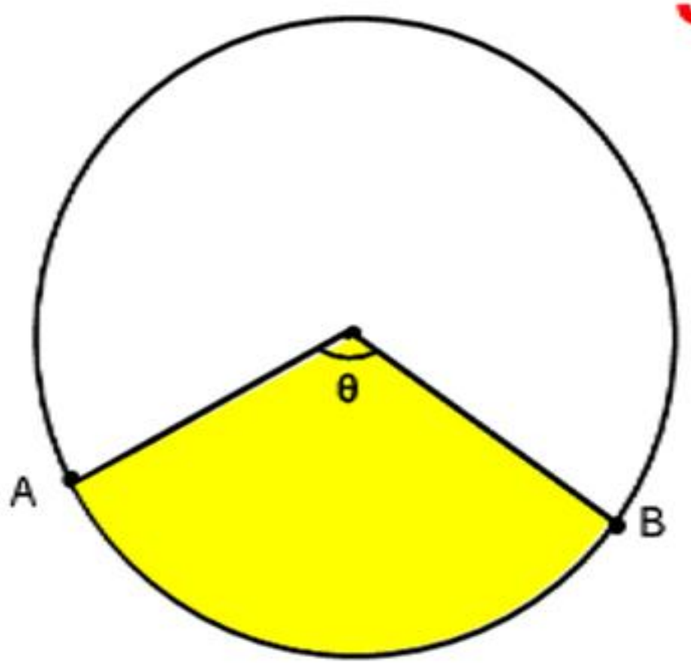
Ans. 196 cm^2



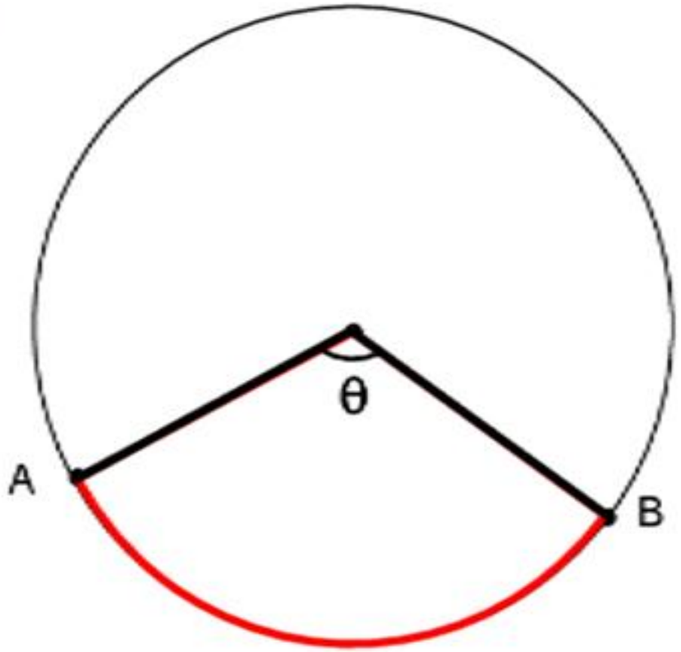
Area of Quadrant of a Circle = $\frac{1}{4}\pi r^2$

Circumference of Quadrant of a Circle = $\frac{\pi r}{2} + 2r$

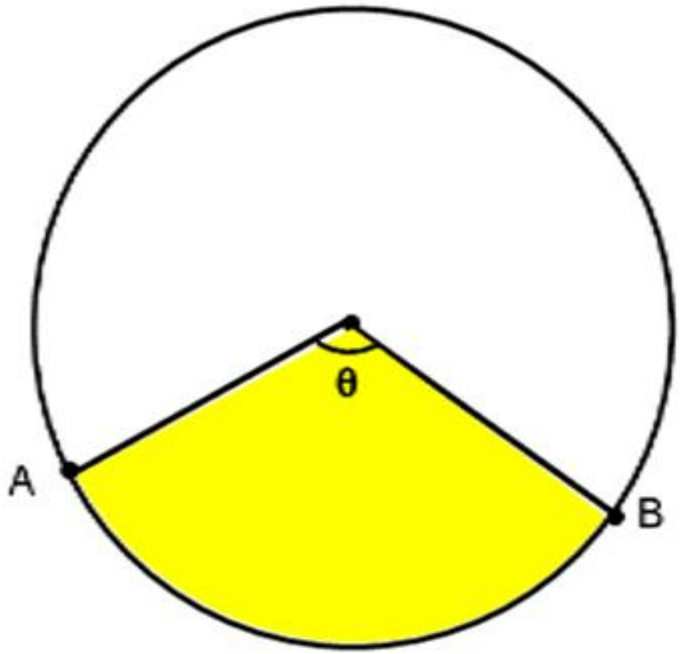
SECTOR OF A CIRCLE



AREA OF SECTOR = $\dfrac{\pi r^2 \theta}{360^\circ}$



$$\text{Length of the Arc AB } (l) = \frac{2\pi r\theta}{360^\circ}$$



Area of Sector = $\frac{1}{2}lr$

Eg. If length of the arc = 6 cm and radius of circle = 5 cm.

Find area of sector of a circle.

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Ans. 15 cm^2

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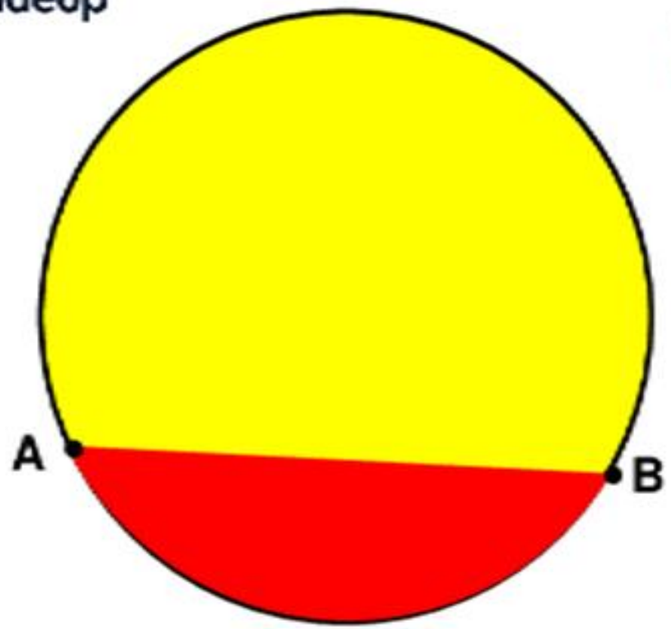
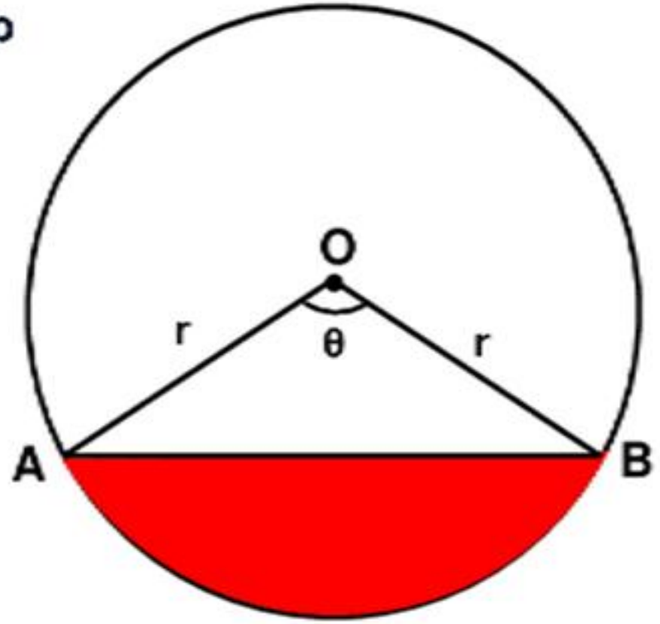
SEGMENT OF A CIRCLE



Chord of a circle divides a circle in 2 segments.

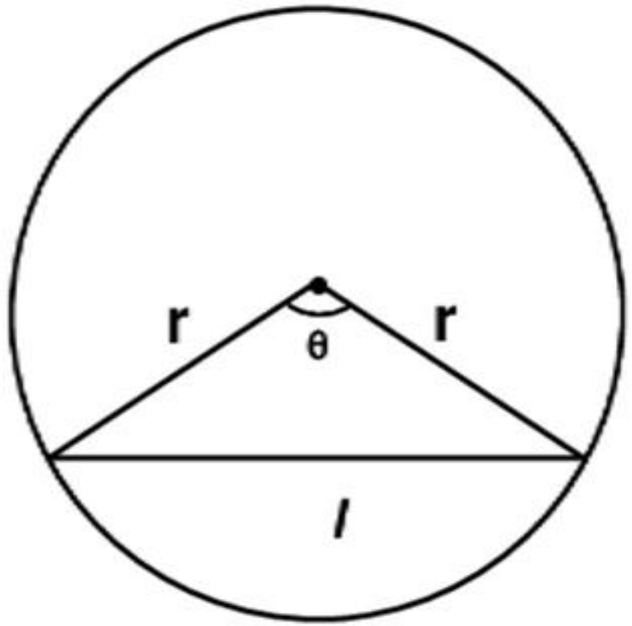


AREA OF SEGMENT

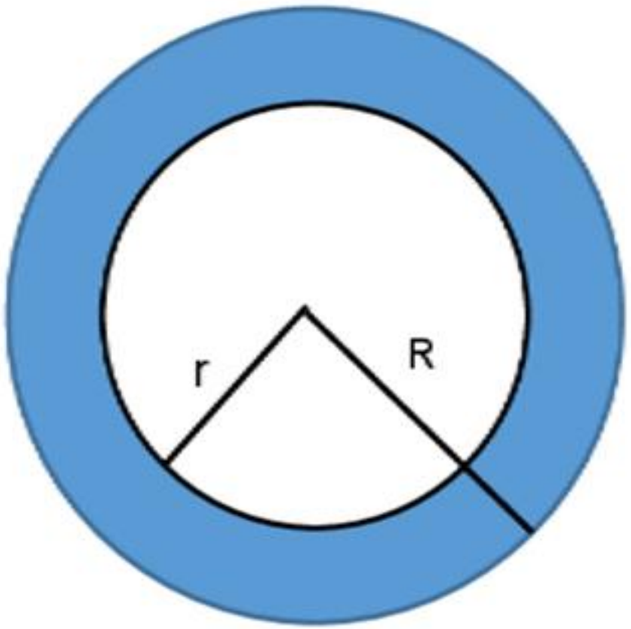
= Area of Sector − Area of ΔAOB

$$\frac{\pi r^2 \theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$$

LENGTH OF CHORD OF A CIRCLE



AREA ENCLOSED BY TWO CONCENTRIC CIRCLES



If R and r are radii of two concentric circles, then

Area enclosed by the two circles $= \pi R^2 - \pi r^2$

$= \pi\left(R^2 - r^2\right)$

$= \pi(R + r)(R - r)$

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Some useful results:

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) the number of revolutions completed by a rotating wheel in one minute = $\frac{Distance\ moved\ in\ one\ minute}{Circumference}$

gradeup

Q. Two circles touch externally. The sum of their areas is 130 π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

Ans. R = 11 cm and r = 3 cm.

Q. A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 min. when the car is travelling at a speed of 66 km per hour?

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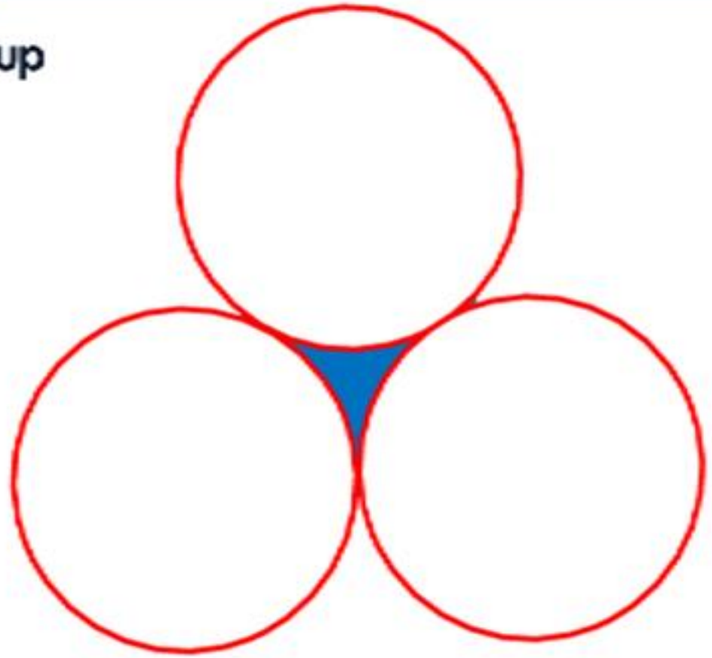
Ans. 4375



Eg. Radius of each of the circle is 10 cm. Find the area of shaded region.

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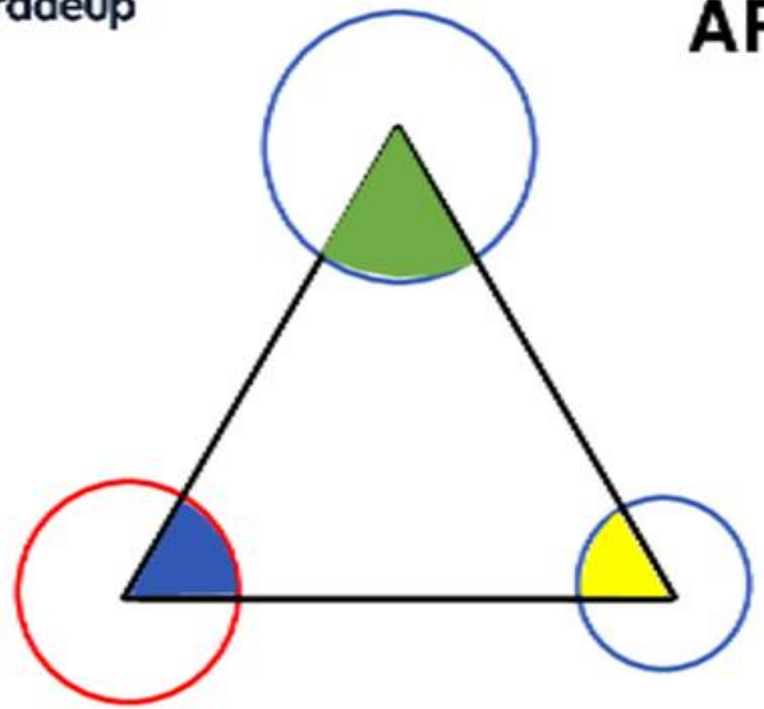
Ans. $\left(100\sqrt{3}-50\pi\right)cm^2$

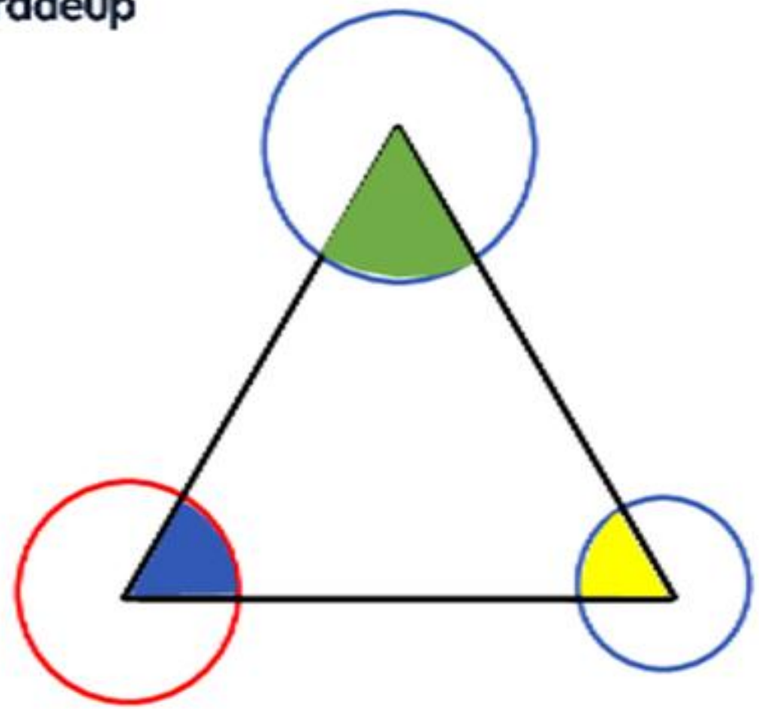
gradeup

AREA GRAZED BY COW



An equilateral triangle whose side is 20 cm. Find the area grazed by the cows if they are attached by rope of length 6 cm, 5 cm and 4 cm on the 3 vertices.

Ans. $\frac{121}{3}\,cm^2$



Eg. A triangle whose sides are 20 cm, 18 cm and 16 cm. Find the area grazed by the cows if they are attached by rope of length 6 cm on all the 3 vertices.

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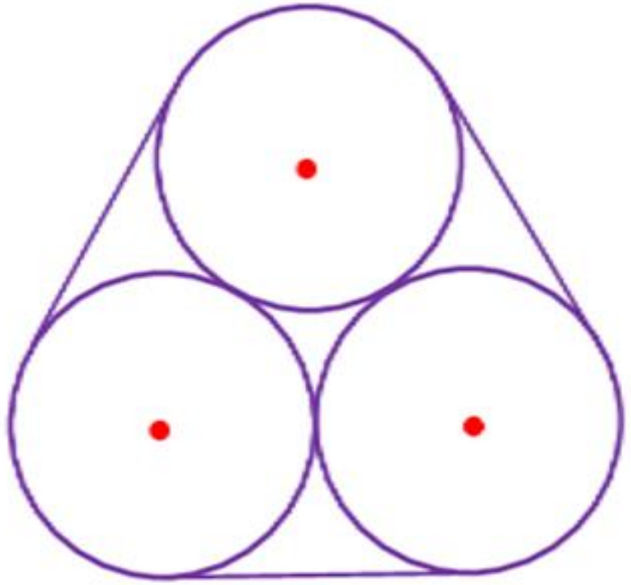
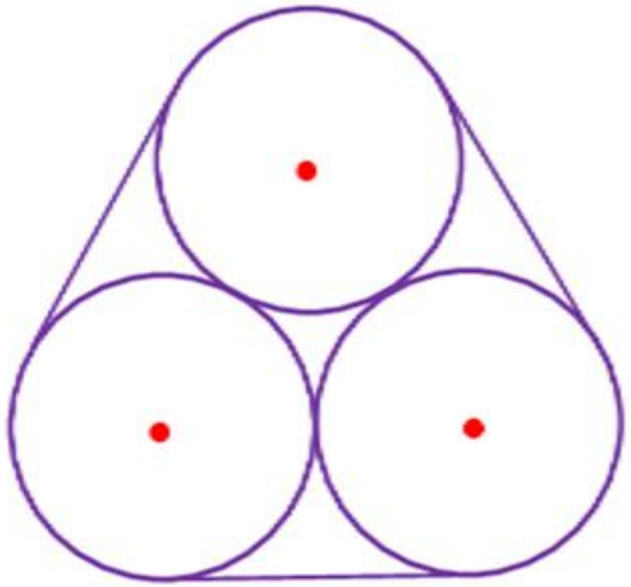
Ans. $\frac{396}{7}\ cm^2$

LENGTH OF RUBBER BAND



If the radius of each circle is 10 cm.

Find the length of rubber band.



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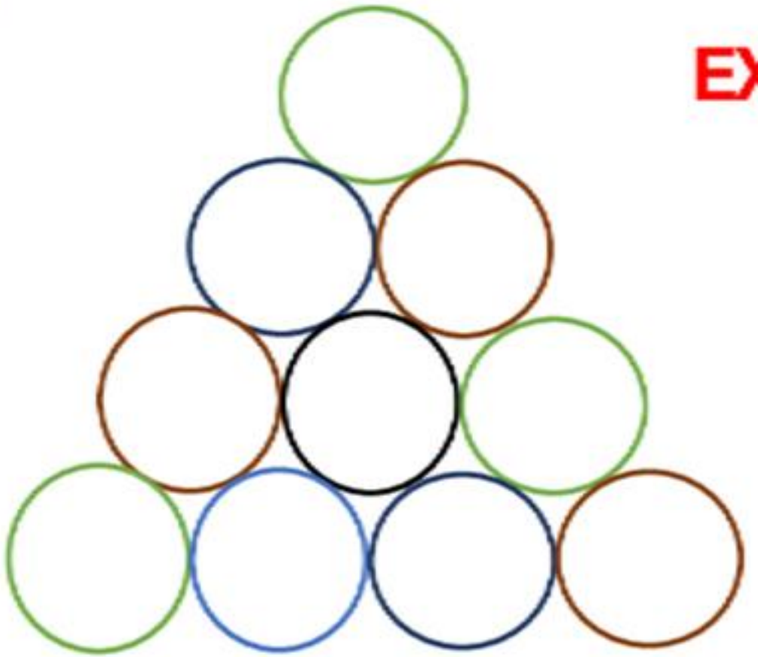

EXTENSION OF RUBBER BAND QUESTIONS

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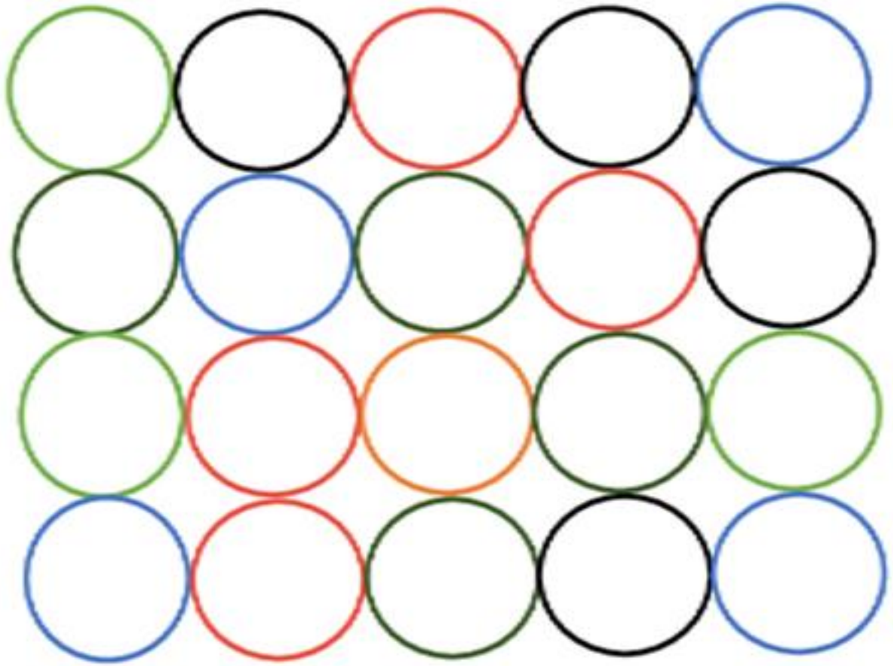
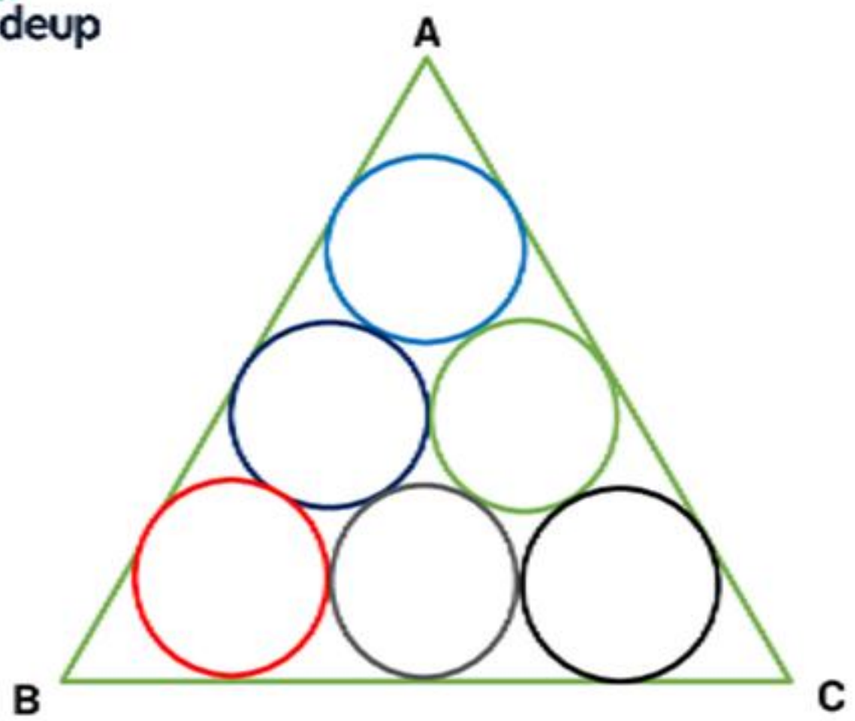
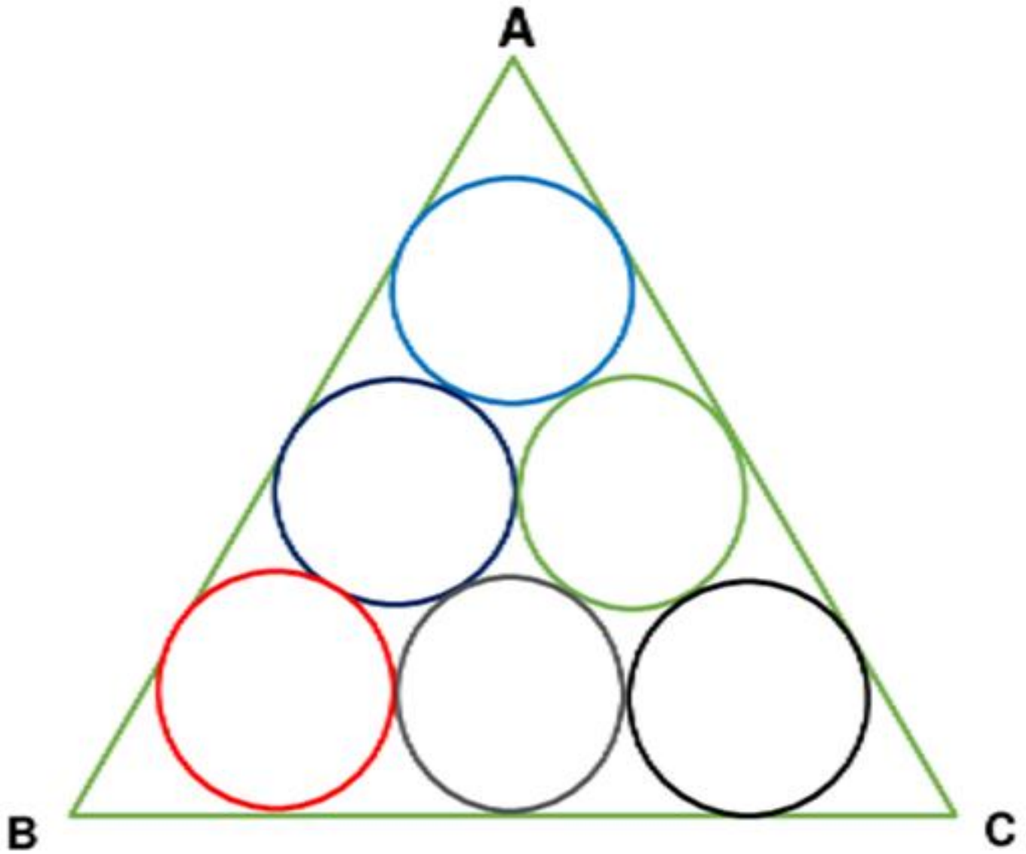


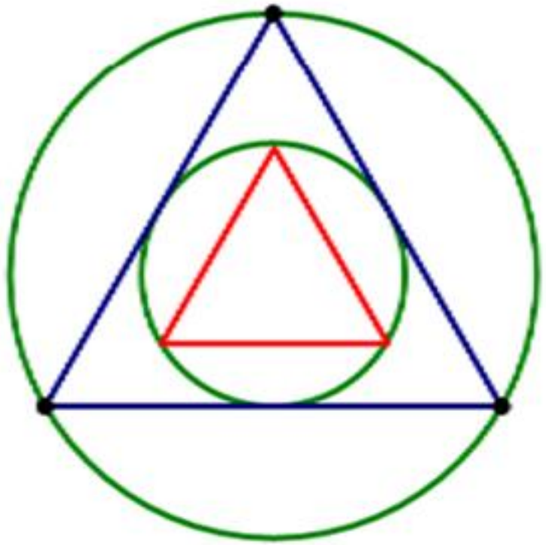


Equilateral Δ

If radius of each circle is 10 cm.

Find the side of equilateral Δ.


A
B
C



Equilateral Δ

If r = inradius

R = circumradius

S = Side of equilateral triangle

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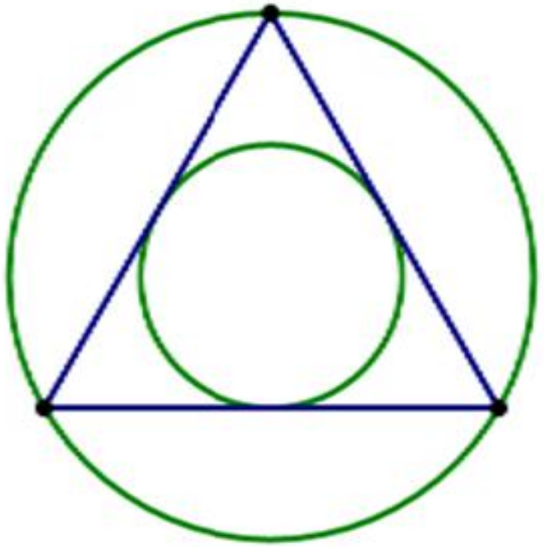

Equilateral Δ

(i) Find : $\frac{r}{R}$

(ii) $\frac{\text{Area of incircle}}{\text{Area of circumcircle}}$

(iii) Find the ratio of r : S : R

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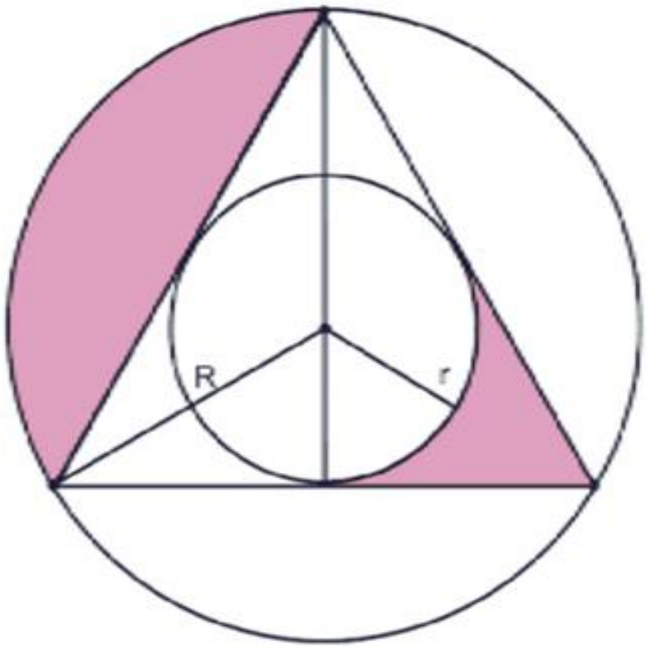
Eg. If side of an equilateral triangle is 12 cm. Find the area of shaded region.



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Ans. 12π

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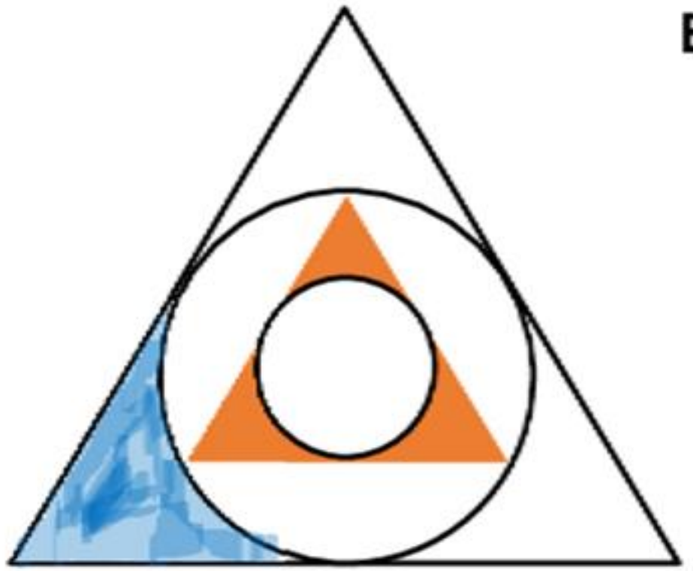
Equilateral Δ



Eg. Find : $\dfrac{\textit{Area of Orange region}}{\textit{Area of Blue region}}$

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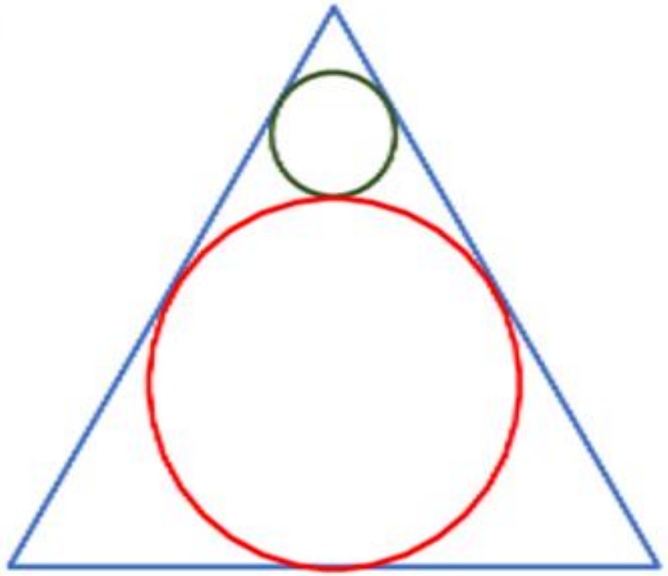
Ans. 3 : 4



Equilateral Δ

Eg. Find : $\frac{r}{R}$

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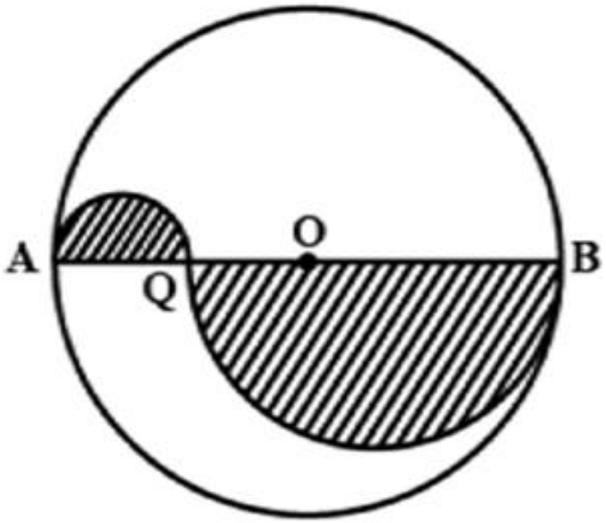
Ans. 1 : 3



Diameter of bigger circle (AB) = 12 cm

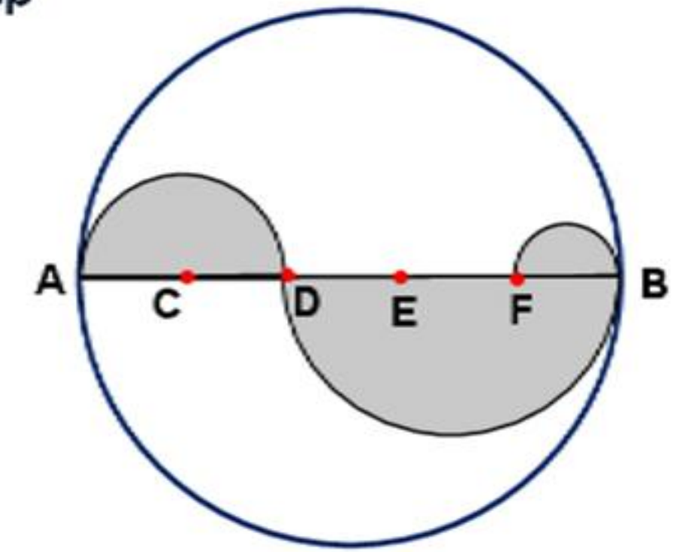
If AQ = QO and O is centre of the bigger circle

Semi-circles are drawn taking AQ and QB as diameter as shown in the figure.

Find : $\frac{\textit{Area of shaded part}}{\textit{Area of complete circle}}$

gradeup

Ans. 5 : 16



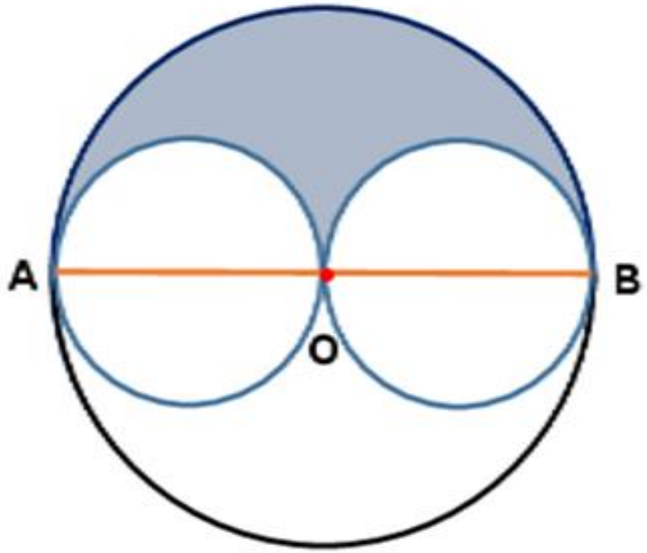
AB = Diameter of bigger circle

AC = CD = DE = EF = FB

Find : $\frac{\textit{Area of shaded part}}{\textit{Area of complete circle}}$

gradeup

Ans. 7 : 25



Eg. O is centre of larger circle.

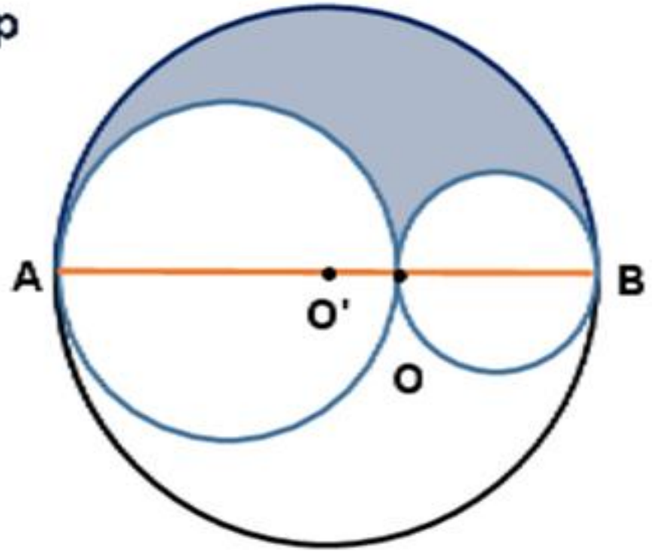
AB is diameter

AB = 20 cm

Find the area of shaded part.

gradeup

Ans. 25π



O' is the centre of the larger circle

AB = 20 cm

AO and OB are diameters of smaller circle

(i) Find the area of shaded region.

(ii) Find circumference of shaded region.

Ans. (i) Can't be determined

(ii) 20π