BYJU'S EXAM PREP

The Most Comprehensive Preparation App For All Exams

MENSURATION-3D

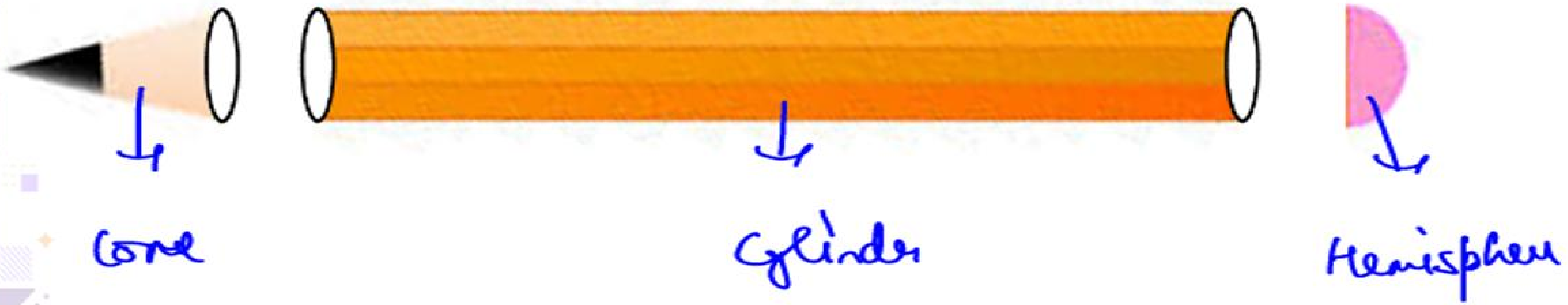
Part-5

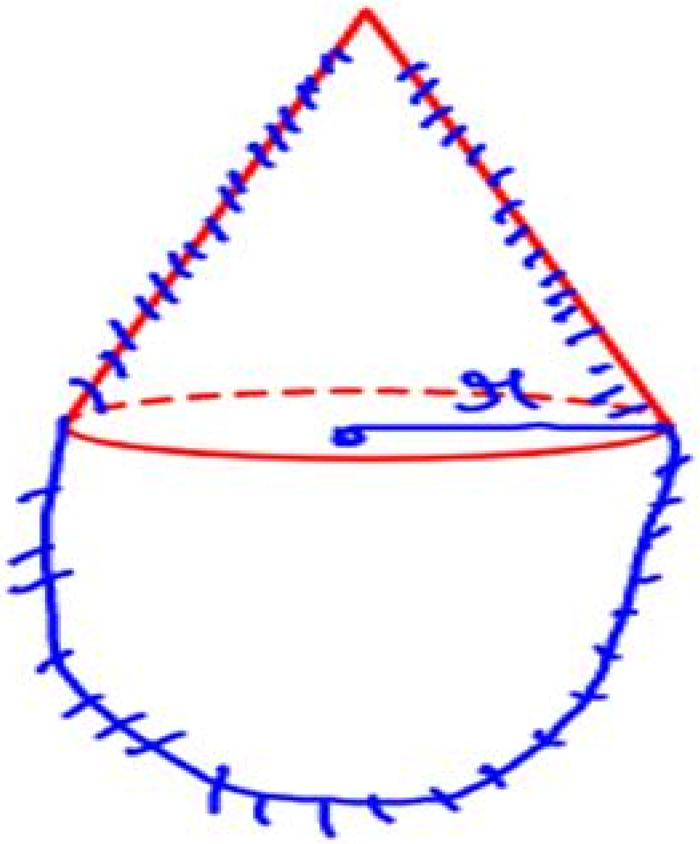
Agenda

1. Combination of solids $\longrightarrow$ (26-28) min

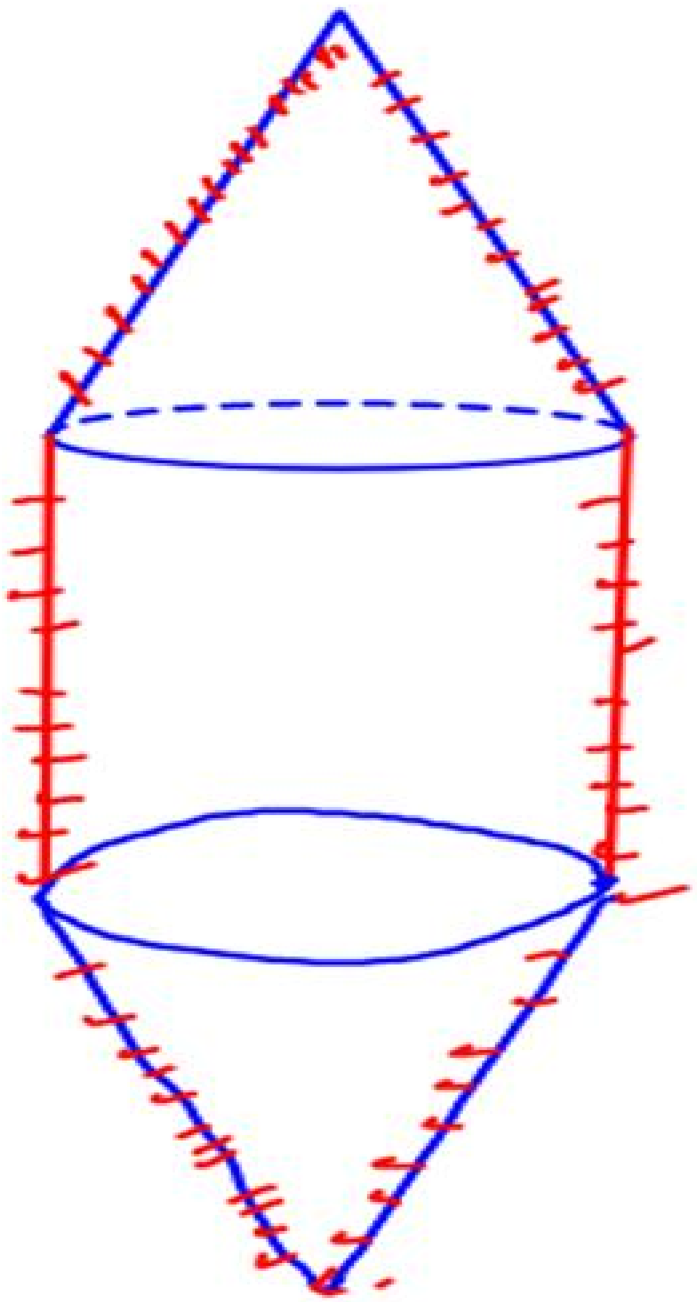
2. Optimization $\longrightarrow$ (46-48) min

COMBINATION OF SOLIDS





$$TSA = \pi r l + 2\pi r^2$$



$TSA = \pi r l_1 + 2\pi r h + \pi r l_2$

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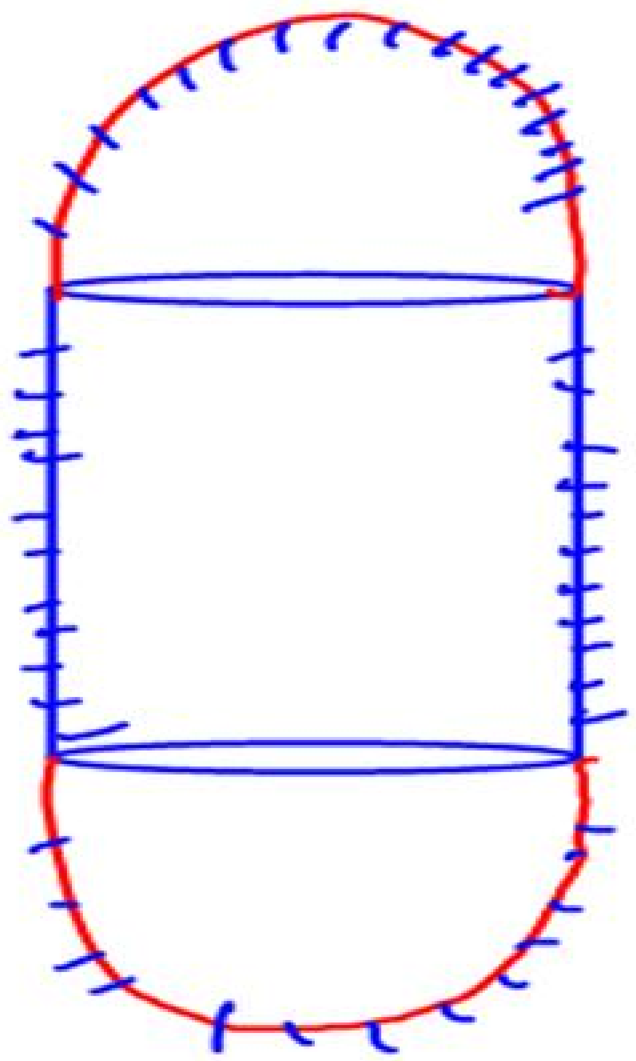
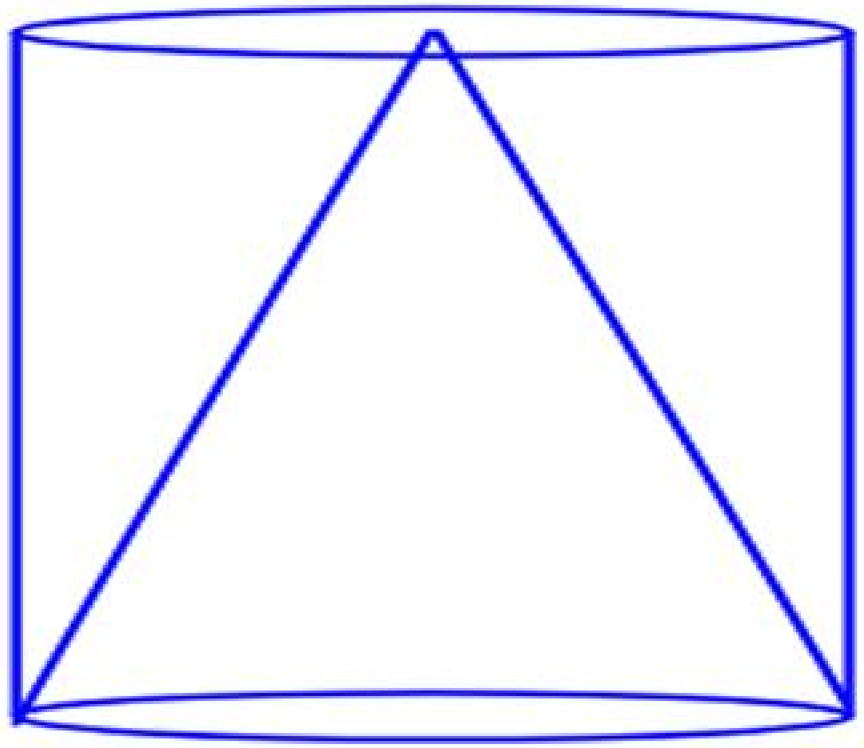
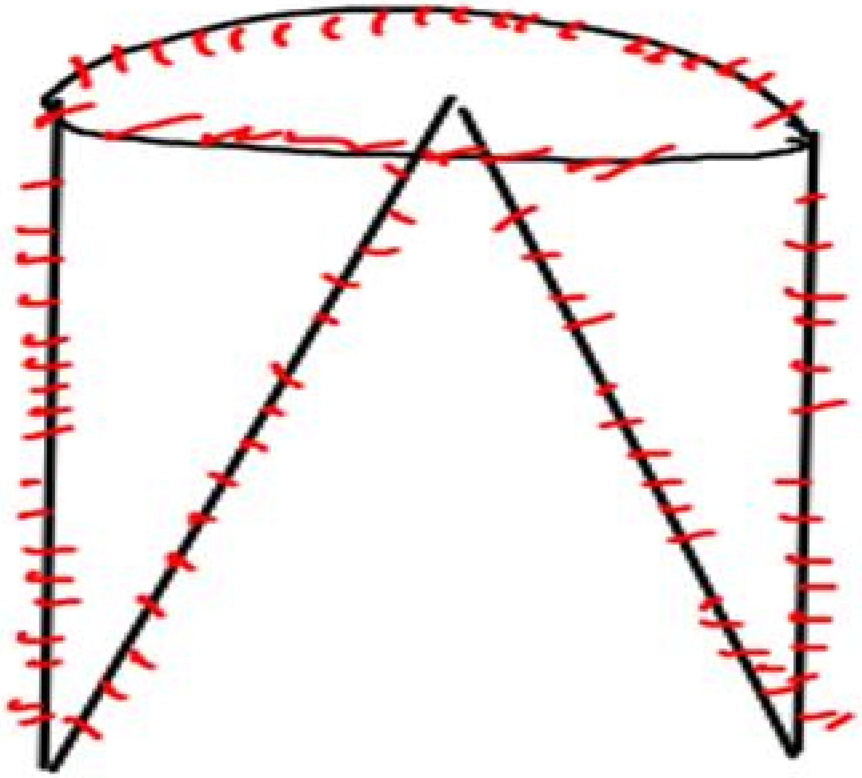


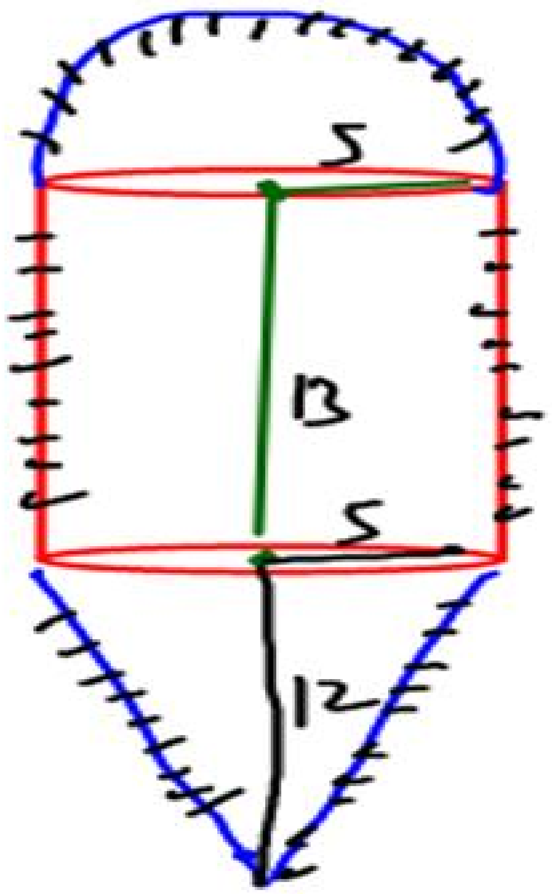
$TSA = 4\pi r^2 + 2\pi r h$





TSA $\Rightarrow \pi r^2 + 2\pi r h + \pi r l$

Eg1. : A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Calculate the surface area of the toy if height of the conical part is 12 cm.



$$TSA = 2\pi(5)^2 + 2\pi(5)(13) + \pi(5)(13)$$

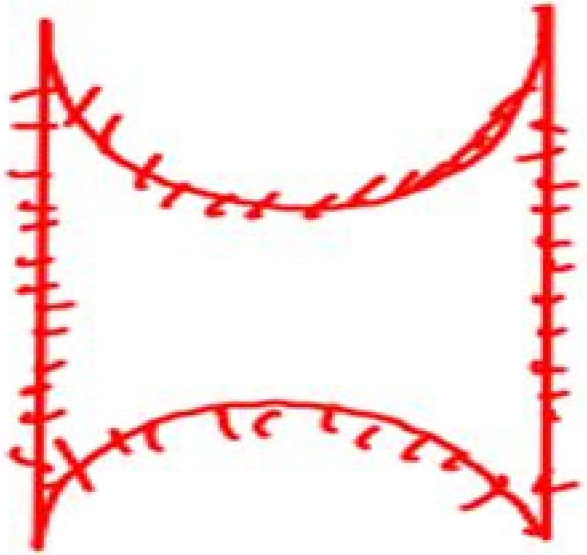
$$TSA = \pi \cdot 5\,[10 + 26 + 13]$$

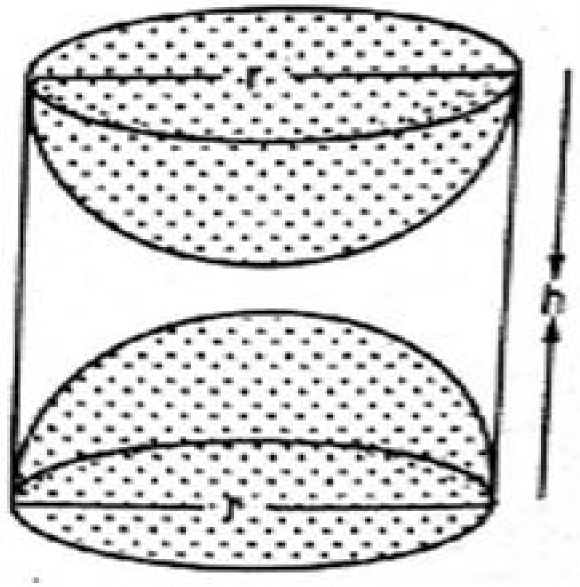
$$= \underline{\underline{245\pi \ cm^2}}$$

$$\overset{35}{\cancel{245}} \times \frac{22}{\cancel{7}} = 770\, cm^2$$

Ans. 770 cm^2

Eg. 2: A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.





$2\pi \times \frac{7}{2} \times 10 + 4\pi \cdot \frac{7}{2} \cdot \frac{7}{2}$

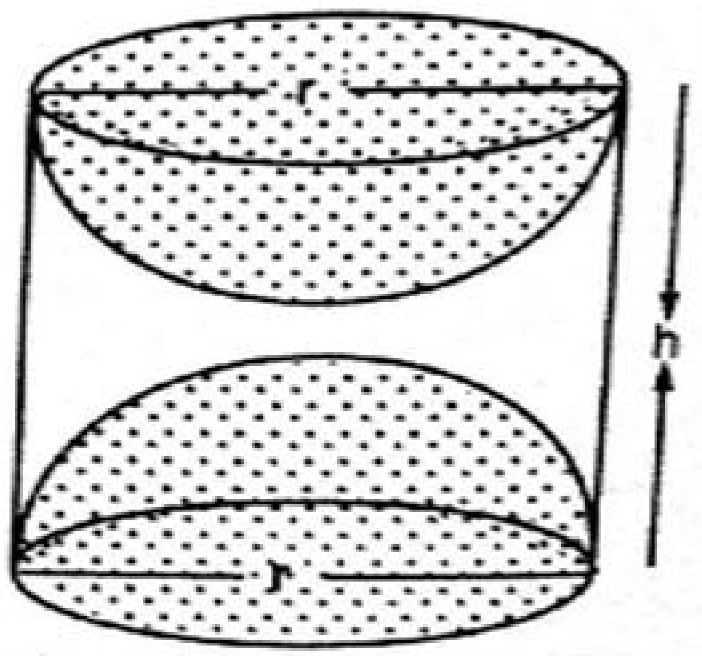
$\cancel{2} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times 10 + \cancel{4} \times \frac{22}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}}$

TSA →

$220 + 154$

$= \underline{374 \text{ cm}^2}$

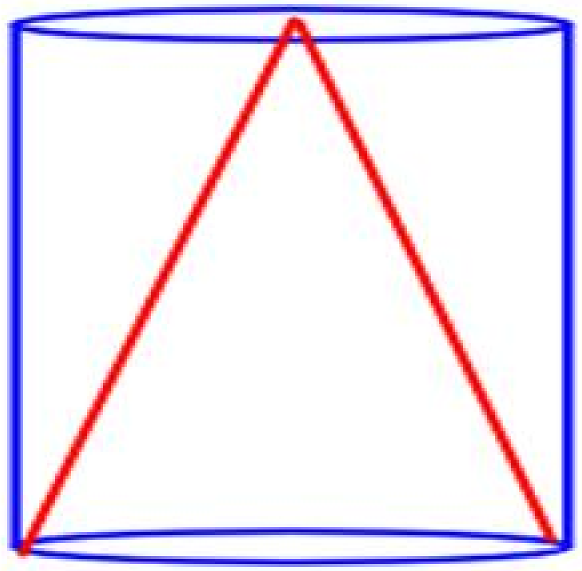
Ans. 374 cm^2

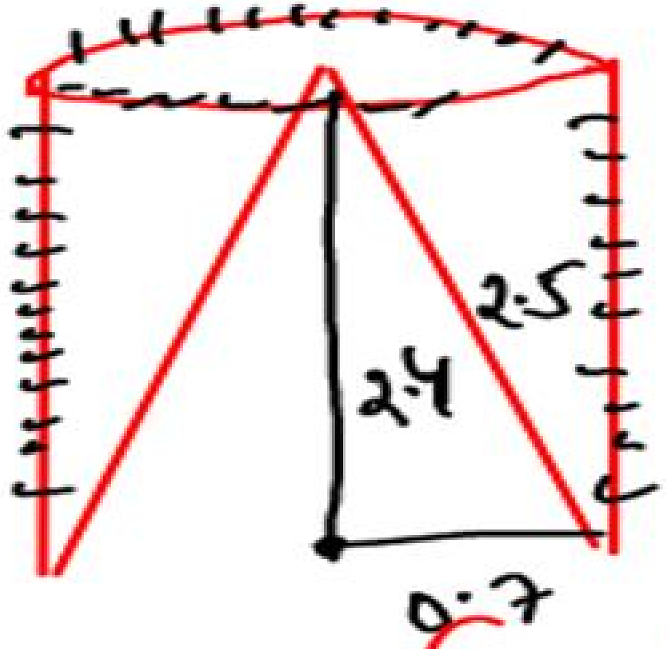


Eg. 3: From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².

Take $\left[\pi = \frac{22}{7}\right]$

90sec





$$\rightarrow \frac{22}{7}\times\frac{0.7}{10}\times\frac{0.7}{10} + 2\times\frac{22}{7}\times\frac{0.7}{10}\times\frac{2.4}{10}$$

$$+ \frac{22}{7}\times\frac{0.7}{10}\times\frac{2.5}{10}$$

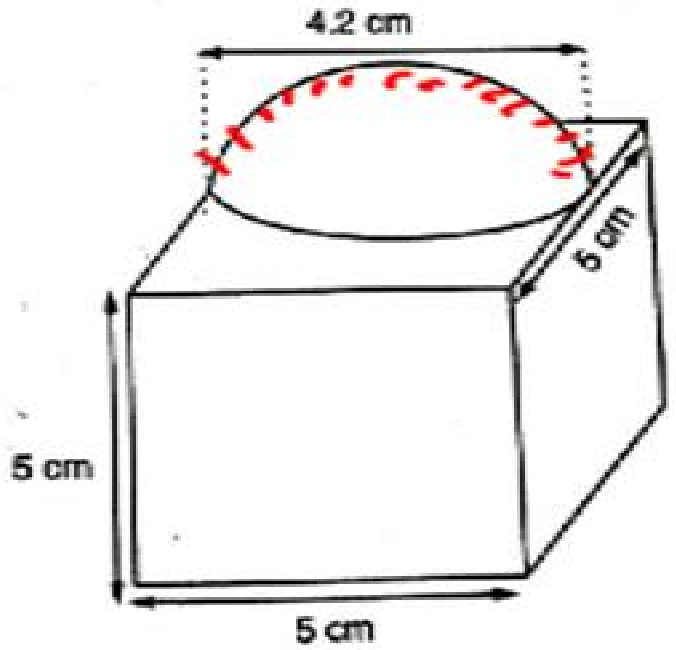
$\rightarrow 1.54 + 10.56 + 5.5$

$\rightarrow 17.6\,cm^2 \rightarrow 18$

Ans. ~~17.6 cm^2~~

$18 cm^2$

Eg. 4: A decorative block shown in Fig. is made of two solids—a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter 4.2 cm. Find the total surface area of the block.

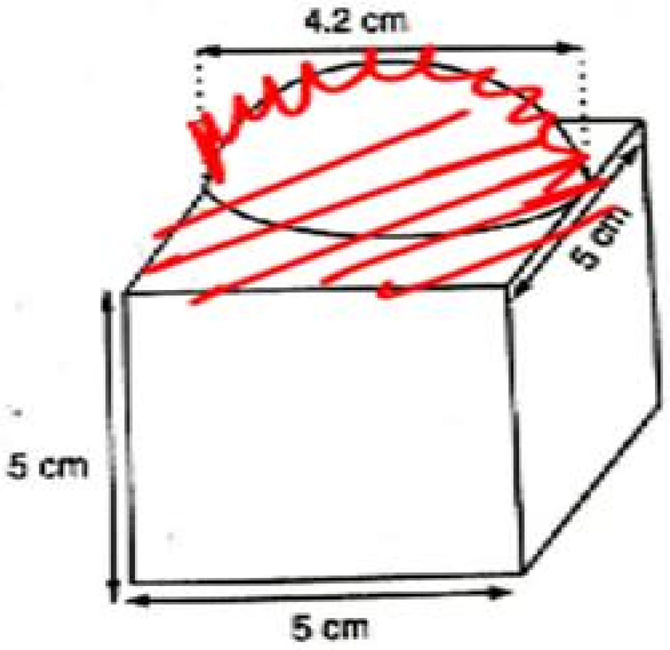


$$6 \times 5^2 + \frac{22}{\cancel{7}} \times \frac{\cancel{21}^{3}}{10} \times \frac{21}{10}$$

$$150 + \frac{1386}{100}$$

$$\underline{163.86 \text{ cm}^2}$$

Ans. 163.86 cm^2



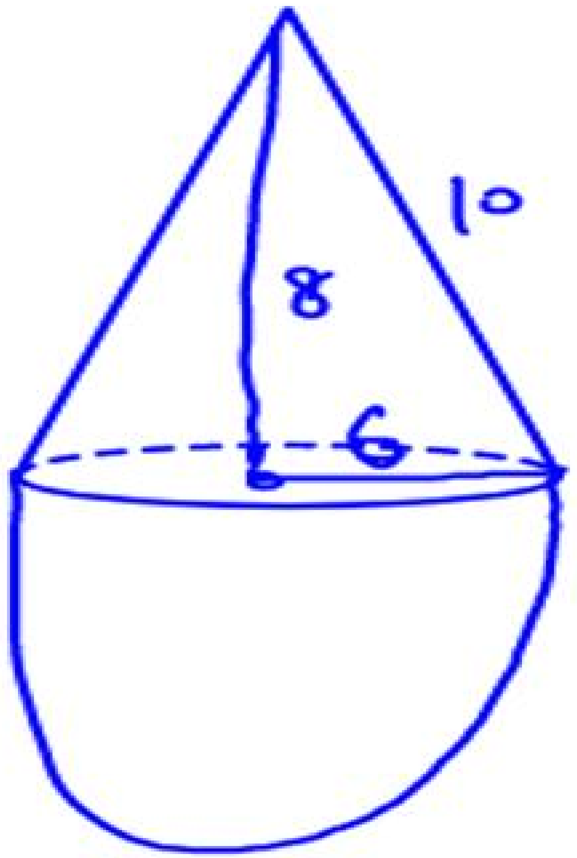
$$5s^2 + \left(s^2 - \pi r^2\right) + 2\pi r^2$$

$$6s^2 + \pi r^2$$

Eg. 5: A toy is in the form of a cone mounted on the hemisphere with the same radius. The diameter of the base of the conical portion is 12 cm and its height is 8 cm. What is the total surface area of the toy?

(a) $132\pi\ cm^2$ (b) $112\pi\ cm^2$

(c) $96\pi\ cm^2$ (d) $66\pi\ cm^2$



TSA $\Rightarrow \pi \cdot 6 \cdot 10 + 2\pi \cdot 6^2$

$\rightarrow 60\pi + 72\pi$

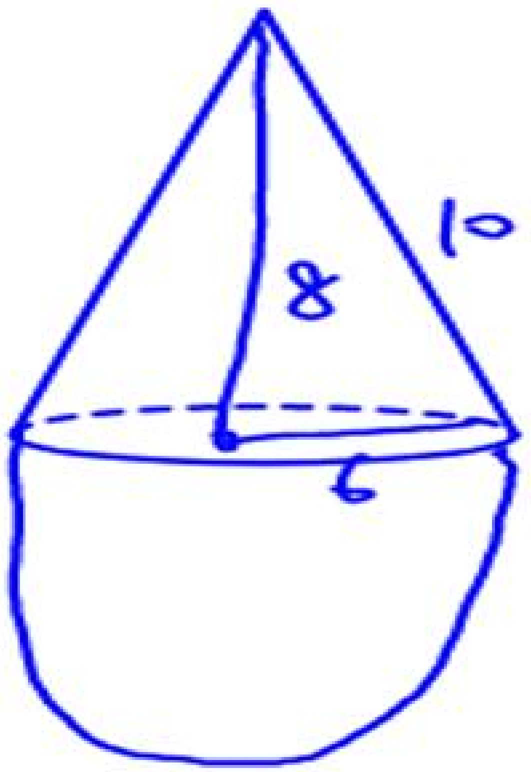
$= 132\pi\ cm^2$

Ans. (a)

Eg. 6: A toy is in the form of a cone mounted on the hemisphere with the same radius. The diameter of the base of the conical portion is 12 cm and its height is 8 cm. What is the volume of the toy?

(a) $180\pi\ cm^3$ (b) $240\pi\ cm^3$

(c) $300\pi\ cm^3$ (d) $320\pi\ cm^3$



Volume

$$\Rightarrow \frac{1}{\cancel{3}}\pi \cdot \cancel{6}^{2} \cdot 8 + \frac{2}{\cancel{3}}\pi \cdot (\cancel{6})^{3}$$

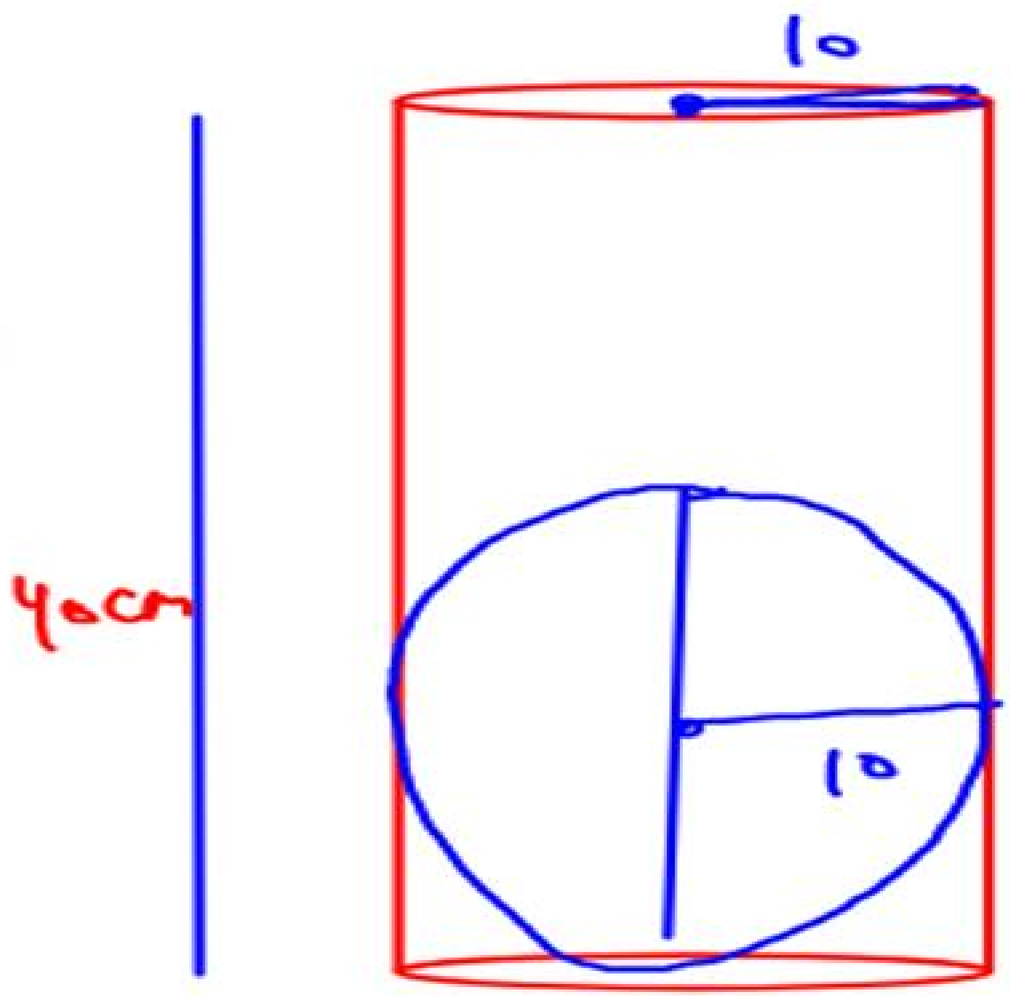
$$96\pi + 144\pi$$

$$= \underline{\underline{240\pi\ cm^3}}$$

Ans. (b)

OPTIMIZATION

Eg7. A right circular cylinder has a radius of 10 cm and height of 40 cm. What can be the max volume of a spherical ball that can be kept inside it?

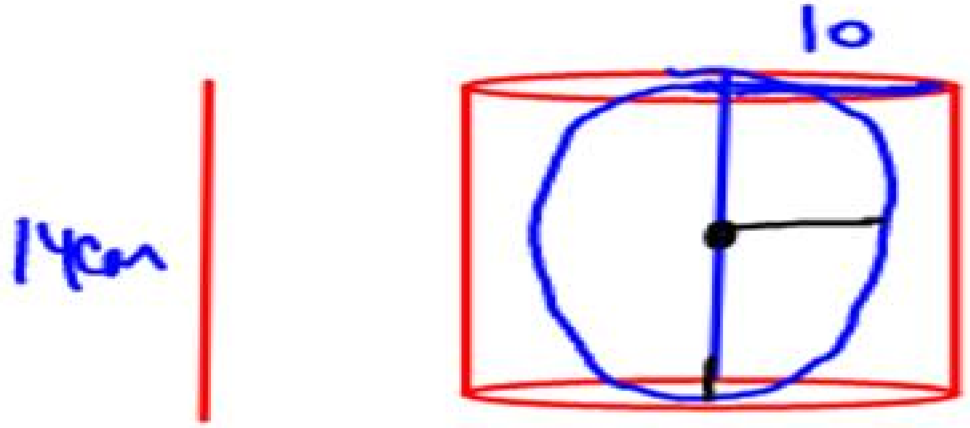


Volume of sphere $\propto \underline{\underline{R^3}}$

$\rightarrow \frac{4}{3}\pi(10)^3$

$\rightarrow \frac{4000}{3}\pi \text{ cm}^3$

Eg8. A right circular cylinder has a radius of 10 cm and height of 14 cm. What can be the max volume of a spherical ball that can be kept inside it?



$$r = 7$$

$$\frac{4}{3}\pi r^3$$

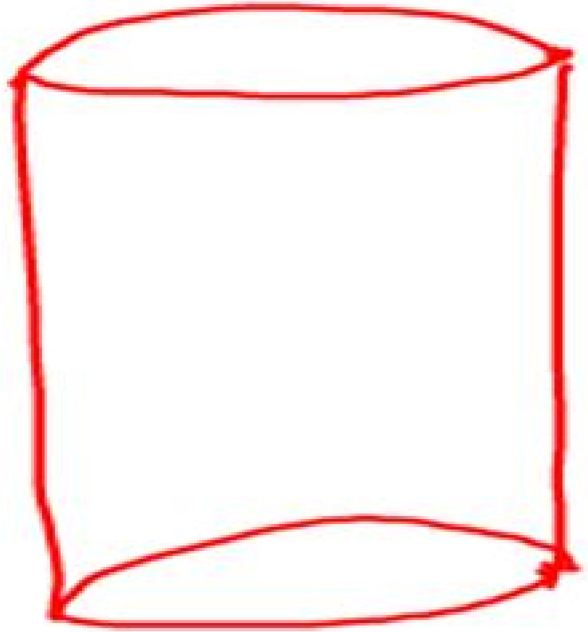
$$\frac{4}{3}\pi \cdot (7)^3$$

$$= \frac{1372}{3}\pi \ cm^3$$

Note :

If the radius of the cylinder is 'R' and its height is 'H'.

The maximum radius of a spherical ball that can be kept inside it = $\boxed{\min\left(R, \frac{H}{2}\right)}$



(i) R = 10 H = 40

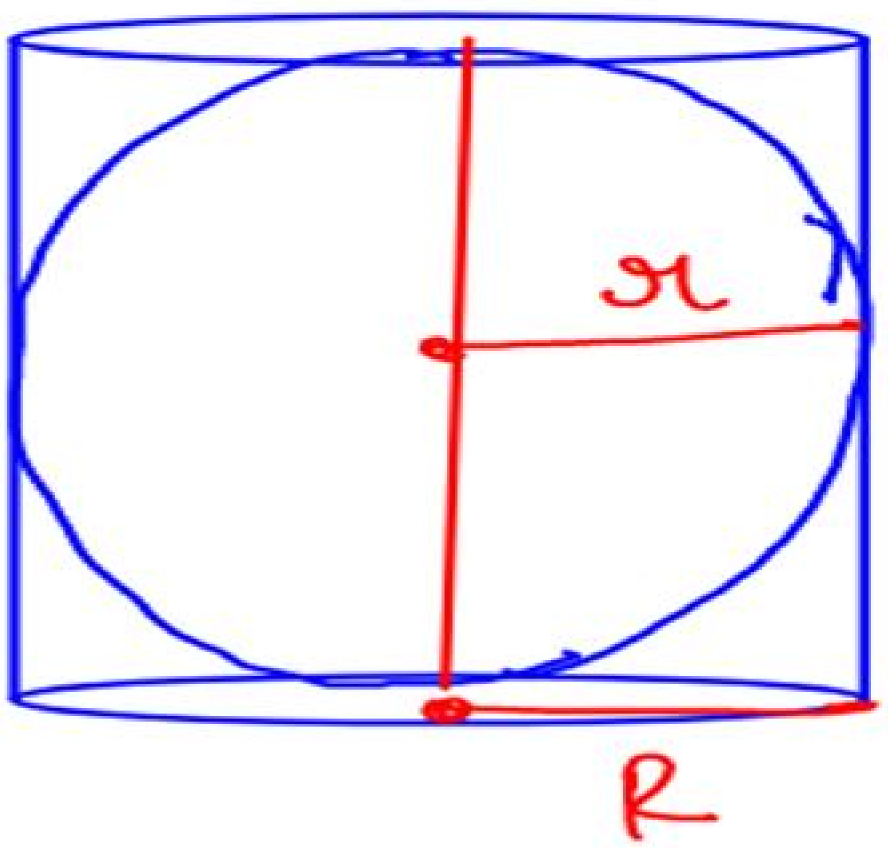
Radius of Ball = 10

(ii) R = 10 H = 14

Radius = 7

What kind of questions comes in SSC on 'OPTIMIZATION'

eg

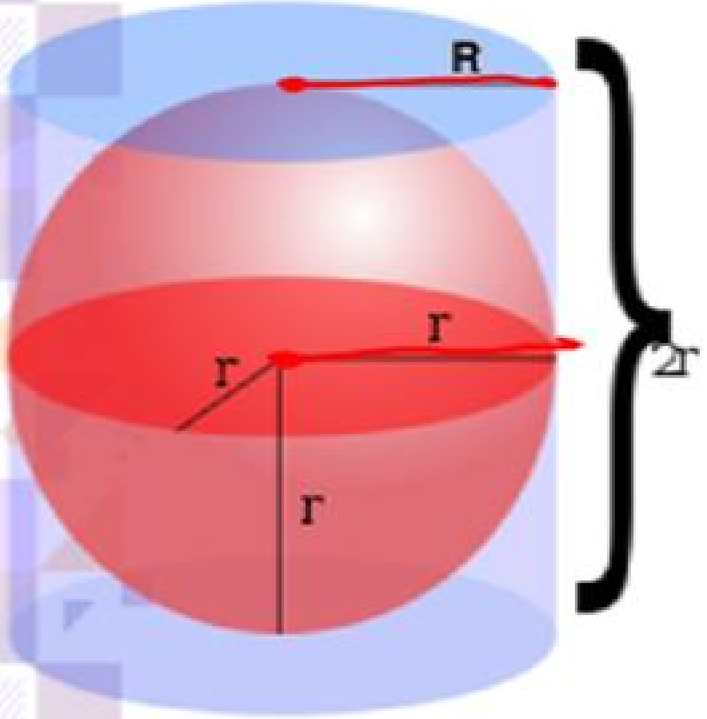


$$\frac{\text{Volume of largest sphere}}{\text{Volume of cylinder}}$$

→ Maximum value of ??

(1) Largest sphere inside a cylinder

(1) Largest sphere inside a cylinder



Let r = Radius of sphere
R = Radius of cylinder
H = Height of Cylinder

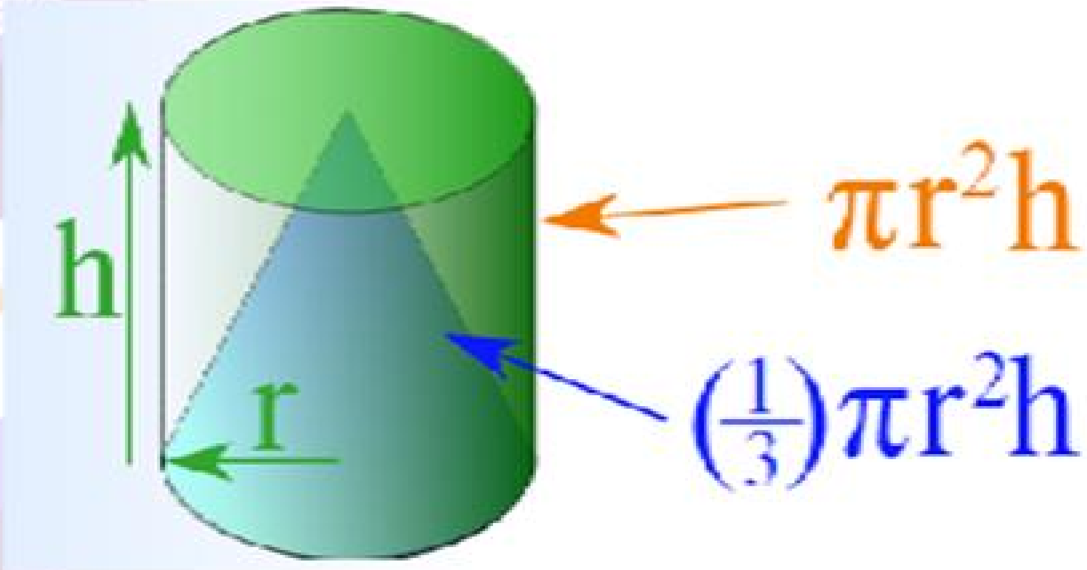
$$r = R = \frac{H}{2}$$

$$\frac{\text{Volume of sphere}}{\text{Volume of cylinder}} = \frac{\frac{4}{3}\cancel{\pi}\cancel{r^3}\,r}{\cancel{\pi}\cancel{r^2}H} = \frac{4}{3}\cdot\frac{\frac{\cancel{H}}{2}}{\cancel{H}}$$

$$= \frac{2}{3}$$

(2) Largest cone inside cylinder

(2) Largest cone inside cylinder



r = radius of cone

h = height of cone

R = Radius of cylinder

H = Height of cylinder

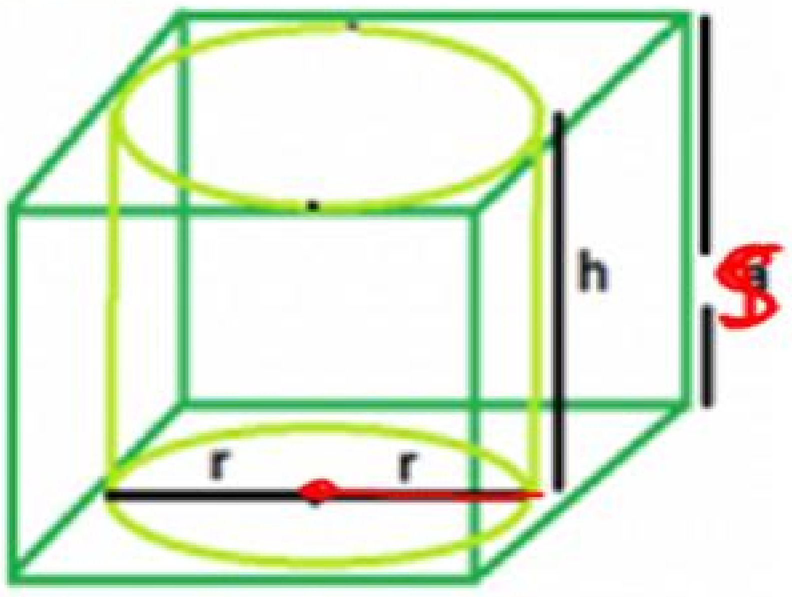
r = R ; h = H

$$\frac{\textit{Volume of cone}}{\textit{Volume of cylinder}} = \frac{\frac{1}{3}\cancel{\pi}\cancel{r^2}\cancel{h}}{\cancel{\pi}\cancel{R^2}\cancel{H}}$$

$$= \frac{1}{3}$$

(3) Largest cylinder inside a cube

(3) Largest cylinder inside a cube



$$r = \frac{S}{2}$$

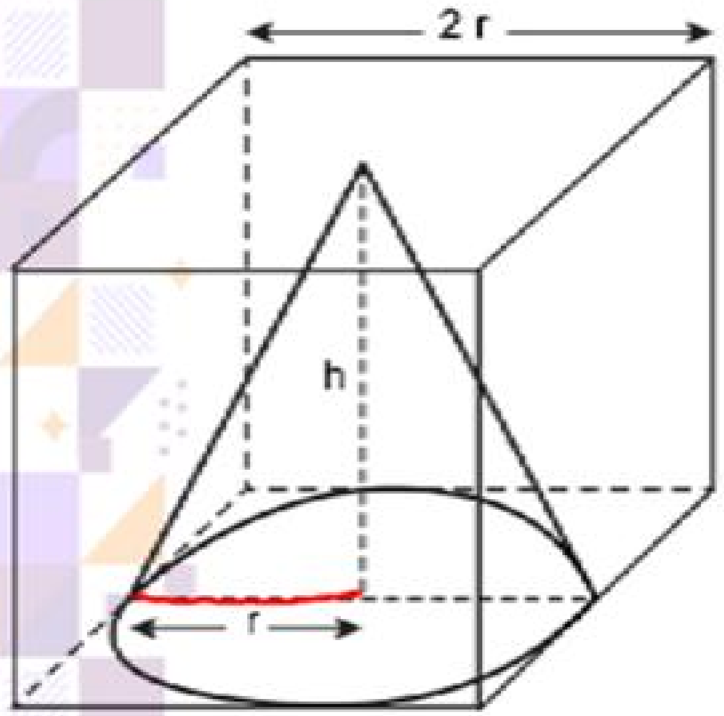
$$h = S$$

$$\frac{\textit{Volume of cylinder}}{\textit{Volume of cube}} = \frac{\pi r^2 h}{s^3} = \frac{\cancel{22}^{11}}{7}\cdot\frac{\cancel{s^2}}{\cancel{4}_{2}}\cdot\frac{\cancel{s}}{\cancel{s^3}}$$

$$\frac{11}{14}$$

(4) Largest cone inside cube

(4) Largest cone inside cube



$$r = \frac{S}{2}$$

$$h = S$$

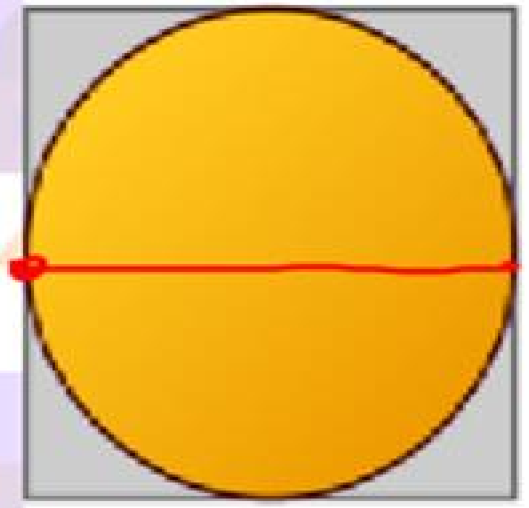
$$\frac{\textit{Volume of cone}}{\textit{Volume of cube}} = \frac{\frac{1}{3}\cdot\frac{\cancel{22}^{11}}{7}\cdot\frac{S^{\cancel{2}}}{\cancel{4}\,2}\cdot S}{\cancel{S^3}}$$

$$= \frac{11}{42}$$

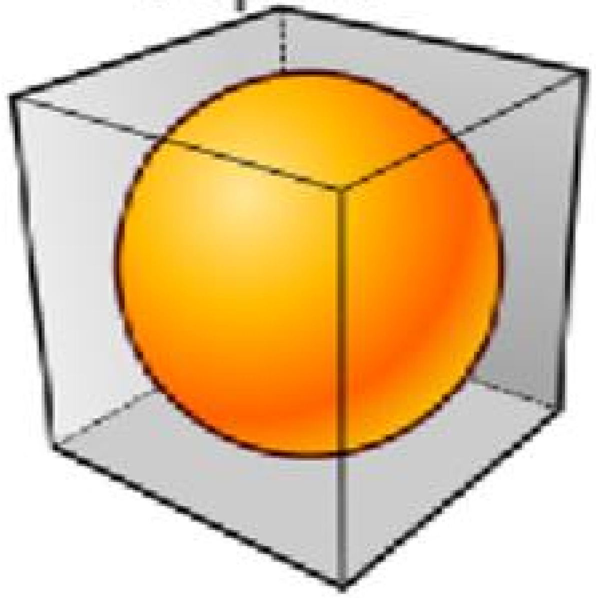
(5) Largest sphere inside cube

(5) Largest sphere inside cube

2-Sphere



3-Sphere



Diameter of sphere = side of cube

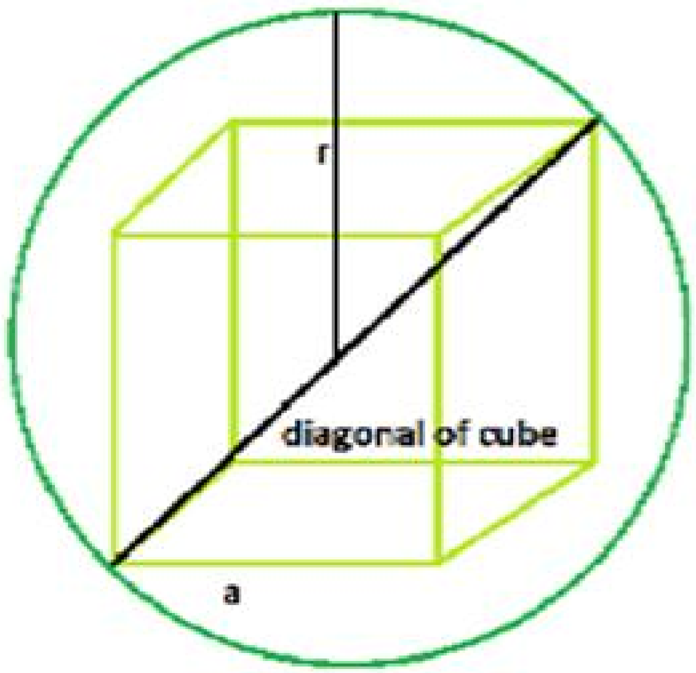
$2r = S$

$$\frac{\textit{Volume of sphere}}{\textit{Volume of cube}} = \frac{\frac{4}{3}\cdot\frac{\cancel{22}^{\,11}}{7}\cdot\frac{\cancel{8^3}}{\cancel{8}}}{\cancel{8^3}}$$

$$\frac{11}{21}$$

(6) Largest cube inside sphere

(6) Largest cube inside sphere



$\boxed{\text{Diagonal of cube = Diameter of sphere}}$

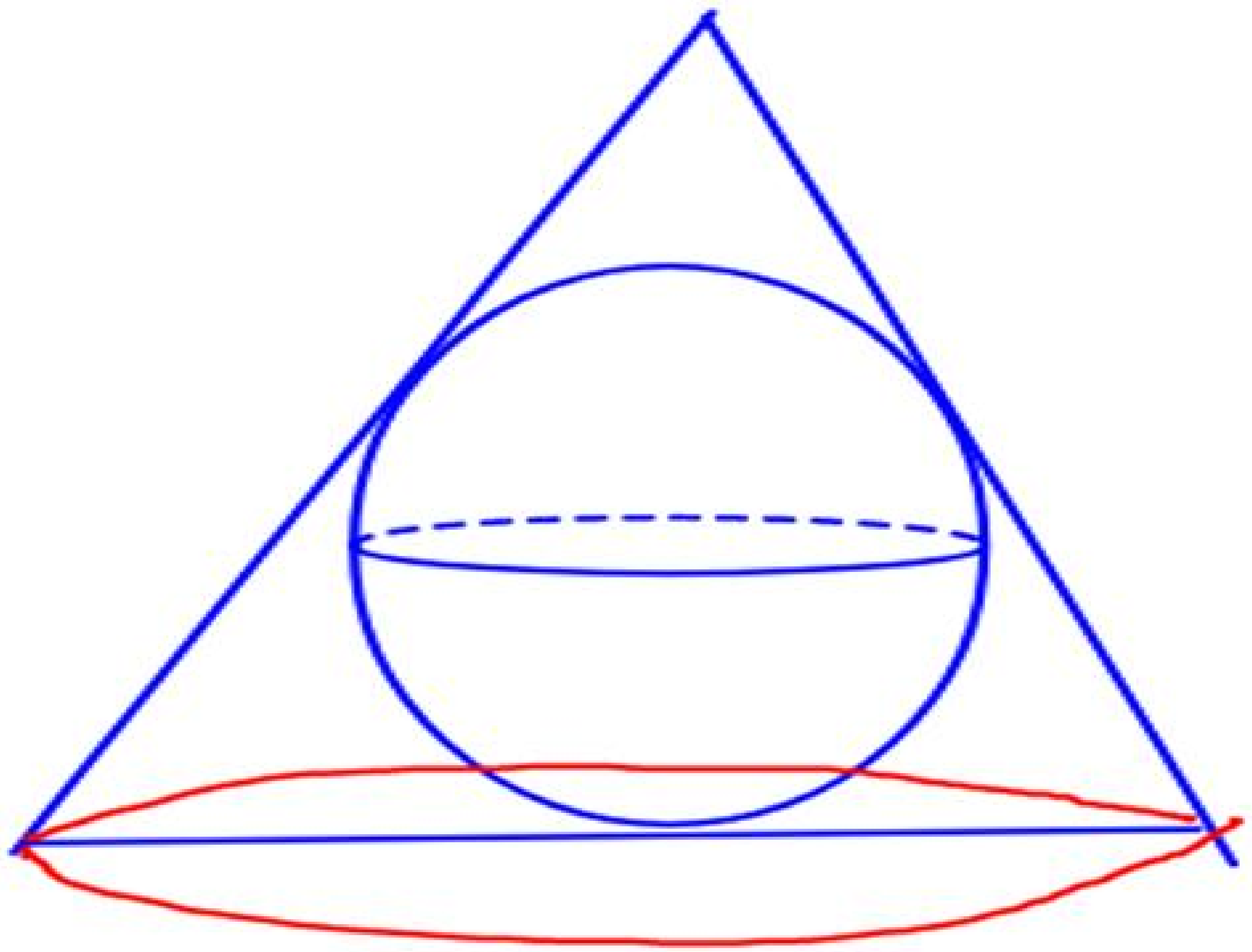
$$\sqrt{3}S = 2R$$

$$S = \frac{2R}{\sqrt{3}}$$

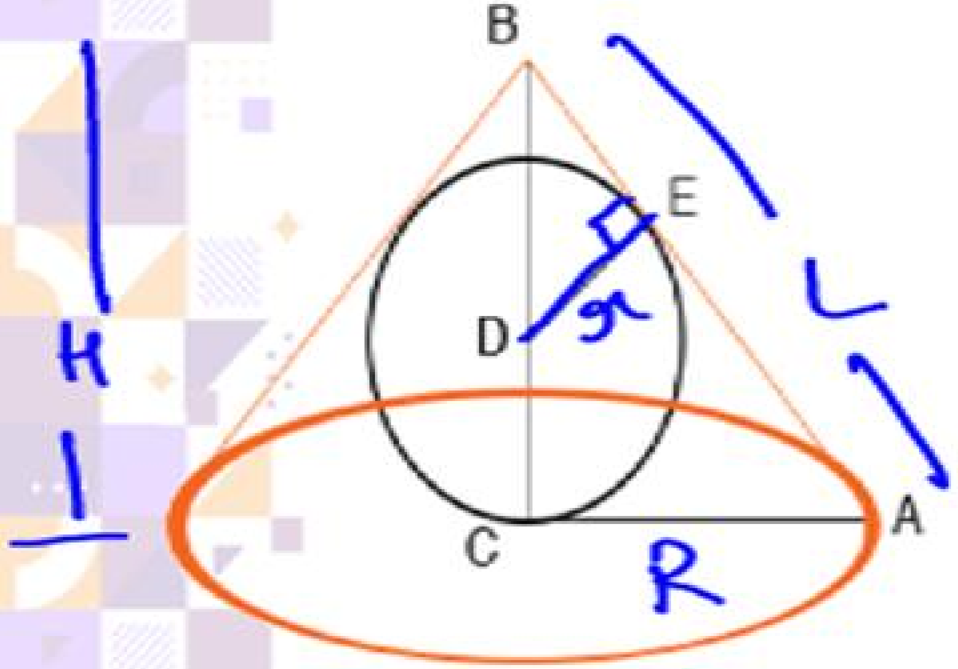
$$\frac{Volume\ of\ cube}{Volume\ of\ sphere} = \frac{S^3}{\frac{4}{3}\cdot\frac{22}{7}\cdot R^3} \Rightarrow \frac{8\cancel{R^3}}{\cancel{3}\sqrt{3}\cdot\frac{4}{\cancel{3}}\cdot\frac{22}{7}\cdot\cancel{R^3}}$$

$$\frac{\cancel{8}\cdot 7}{\cancel{4}\sqrt{3}\cdot \underset{11}{\cancel{22}}} = \frac{7}{11\sqrt{3}}$$

(7) Largest sphere inside a cone



(7) Largest sphere inside a cone



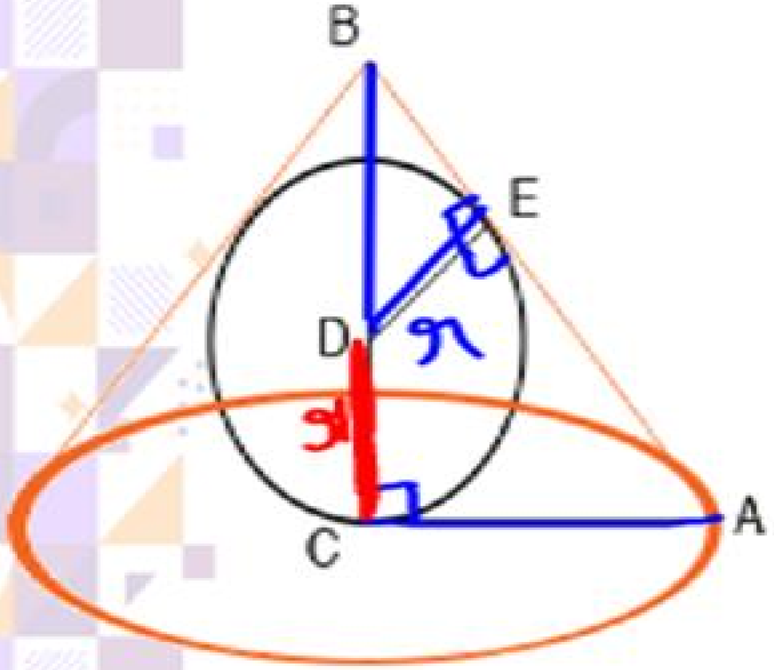
Radius of sphere = r

Radius of cone = R

Height of cone = H

Slant height of cone = L

First Approach : Similarity



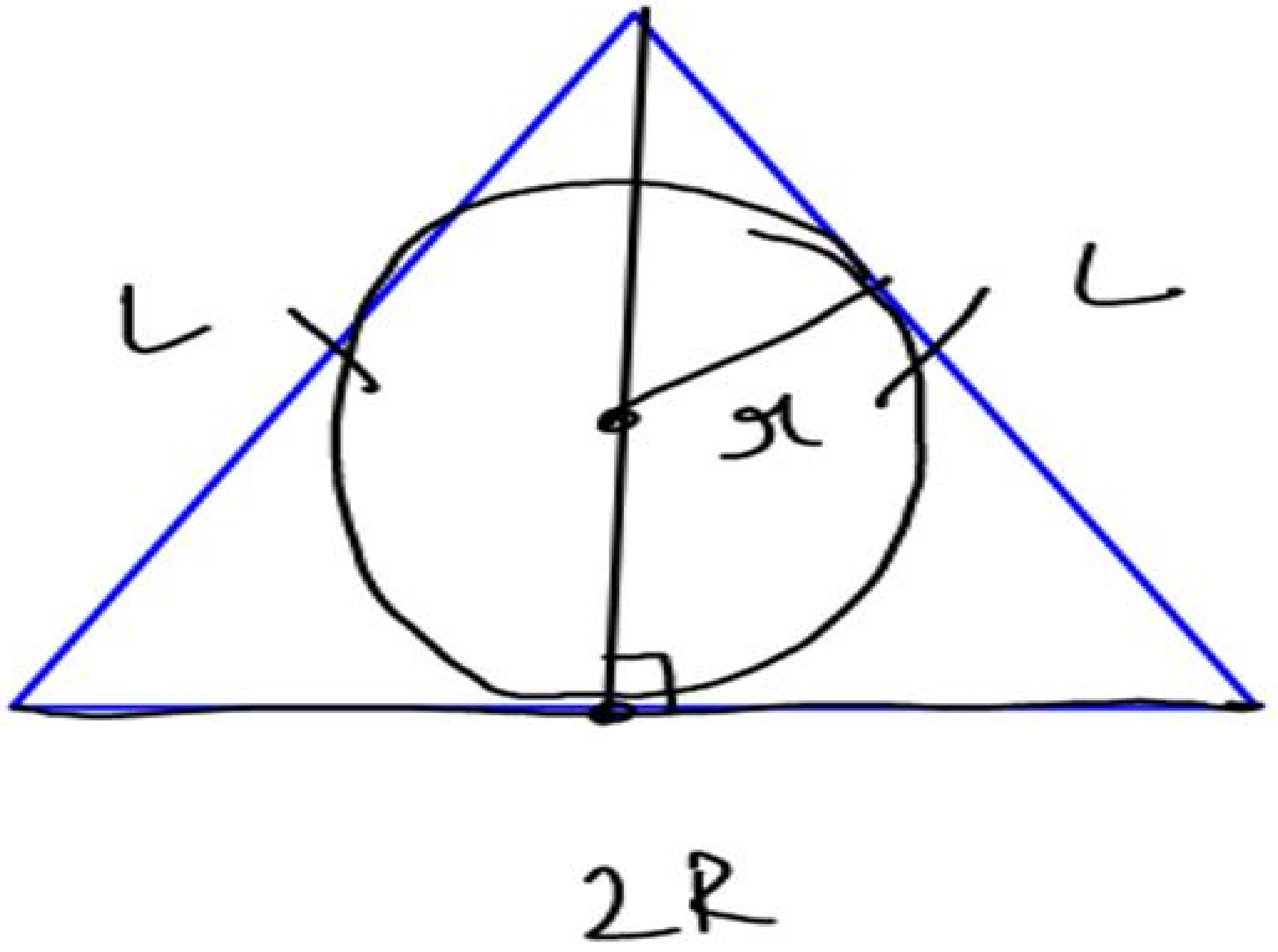
$$\Delta BCA \sim \Delta BED$$ (By AA)

$$\frac{CA}{ED} = \frac{BA}{BD}$$

$$\frac{R}{r} = \frac{L}{H-r}$$

RH – Rr = Lr

$$\boxed{r = \frac{RH}{L+R}}$$

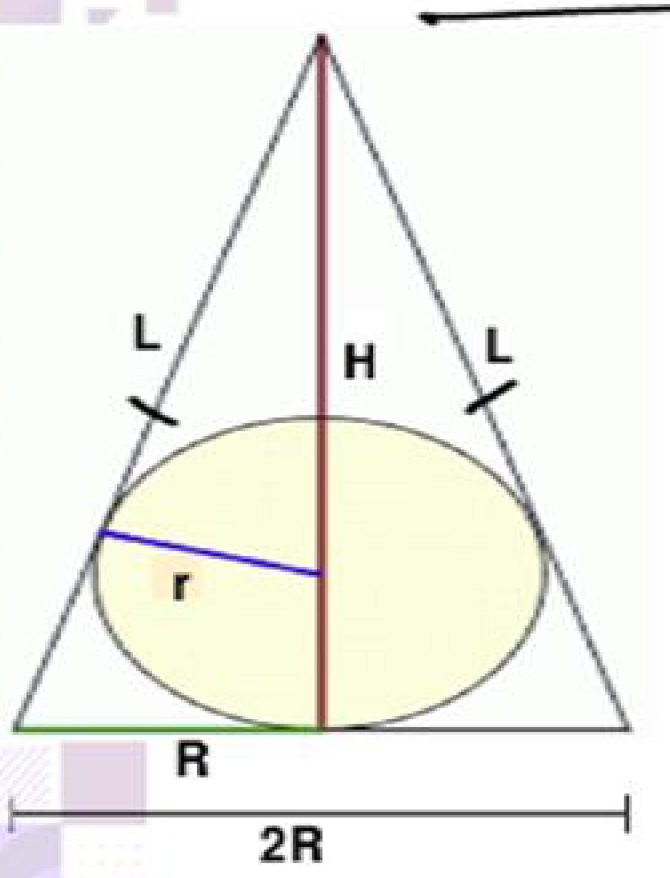


$$r = \frac{\text{Area}}{s}$$

$$\frac{\frac{1}{\cancel{2}} \cdot \cancel{2}R \cdot H}{R + L}$$

$$s = \frac{L + L + 2R}{2}$$

Second Approach : 2D-View



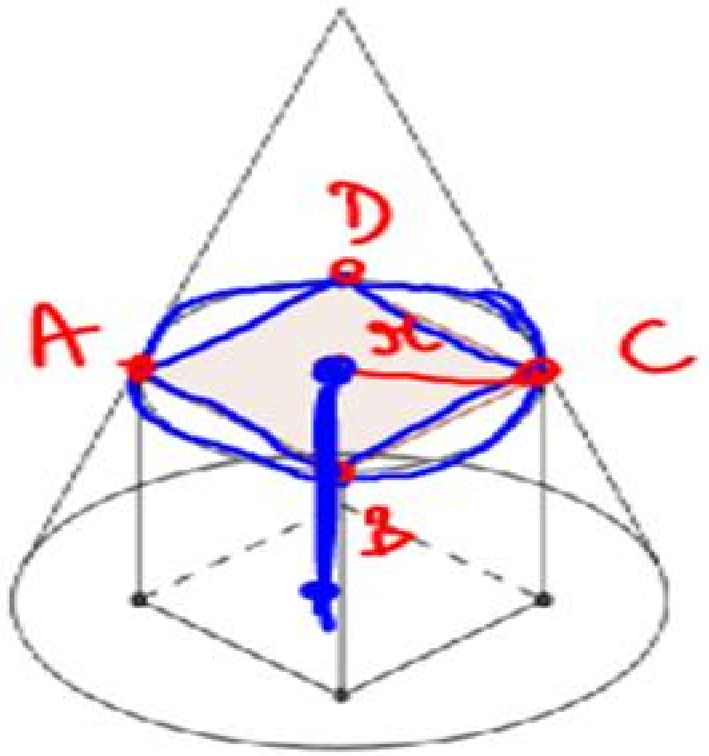
$$r = \frac{Area}{s}$$

$$= \frac{\frac{1}{2} 2R \cdot H}{L + R}$$

$$\boxed{r = \frac{RH}{L + R}}$$

(8) Largest cube inside a cone

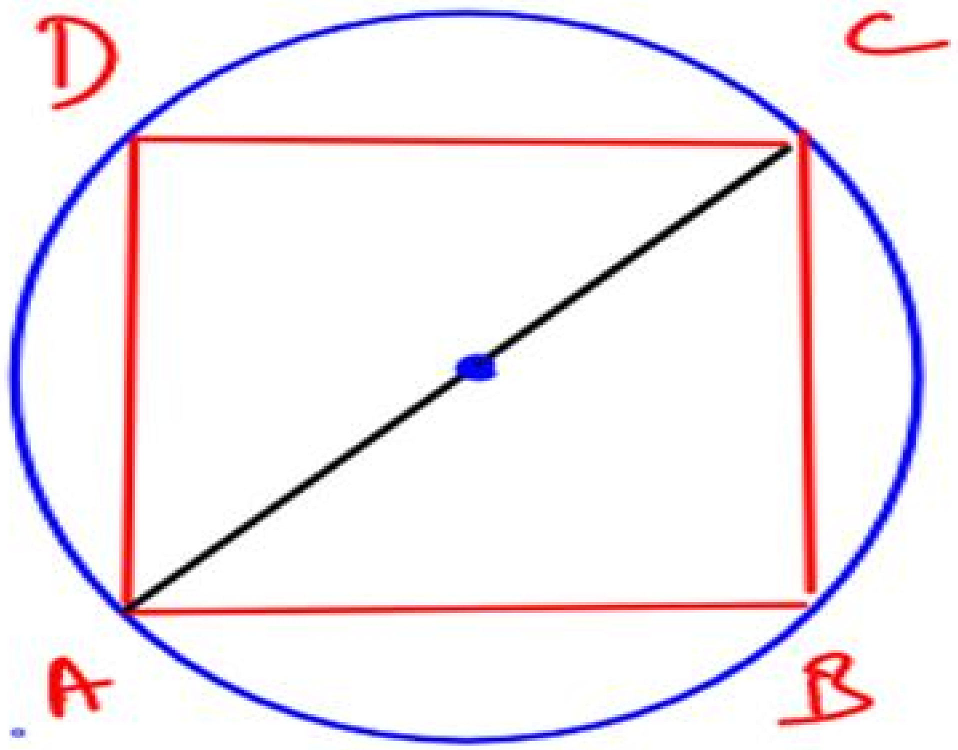
(8) Largest cube inside a cone



Side of cube = S

Radius of cone = R

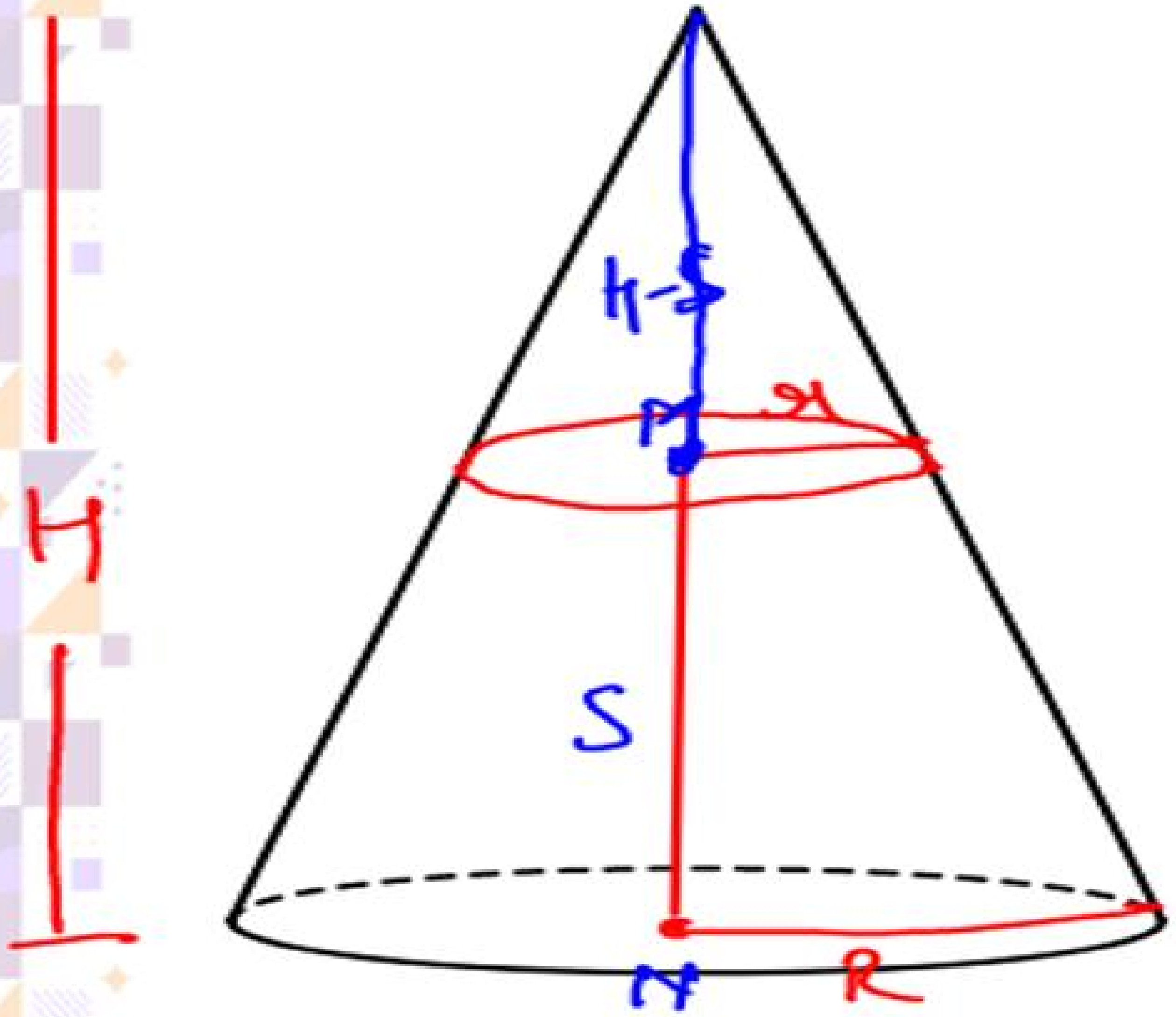
Height of cone = H



(i)

$$\sqrt{2}S = 2r$$

$$\boxed{r = \frac{S}{\sqrt{2}}}$$



$$\frac{r}{R} = \frac{H-S}{H}$$

$$\frac{S}{\sqrt{2}R} = \frac{H-S}{H}$$

$$HS = \sqrt{2}RH - \sqrt{2}RS$$

$$S(H+\sqrt{2}R) = \sqrt{2}RH$$

Imp

$$S = \frac{\sqrt{2}RH}{\sqrt{2}R + H}$$

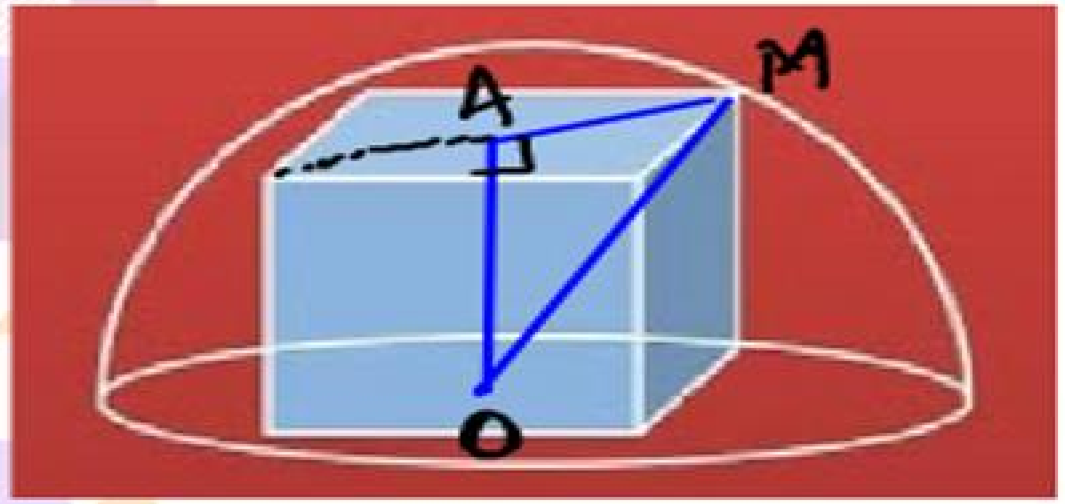
Largest cube inside a cone

$$S = \frac{\sqrt{2}R \cdot H}{\sqrt{2}R + H}$$

$$\frac{\sqrt{2}R \cdot H}{\sqrt{2}R + H}$$

(9) Largest cube inside a hemisphere

(9) Largest cube inside a hemisphere



Let side of cube = S

radius of hemisphere = r

ΔOAM

$OA = S$

$OM = r$

$AM = S/\sqrt{2}$

$$r^2 = S^2 + \frac{S^2}{2}$$

$$r^2 = \frac{3S^2}{2}$$

$$\boxed{S = \frac{\sqrt{2}}{\sqrt{3}} r}$$

BASICS OF DIFFERENTIATION

$$\frac{d}{dx}\left(x^{3}\right)=3x^{2}$$

$$\frac{d}{dx}\left(x^{4}\right)=4x^{2}$$

$$\frac{d}{dx}\left(x^{n}\right)=nx^{n-1}$$

For MAX/MIN :

p

I. First derivative = 0

Point of Min/Max

II. Second derivative

Positive $\rightarrow$ Min

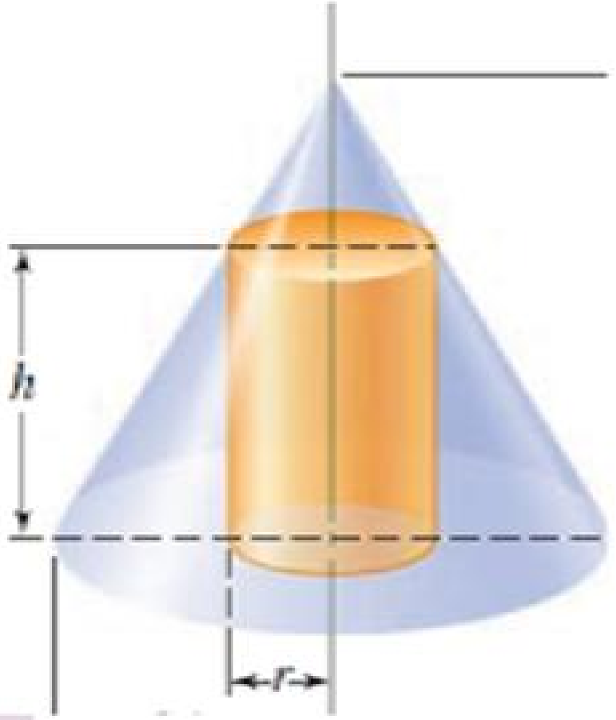
Negative $\rightarrow$ Max

Eg. $\min\left(x^2 + 6x + 8\right)$

(10) Largest cylinder inside cone

r

(10) Largest cylinder inside cone



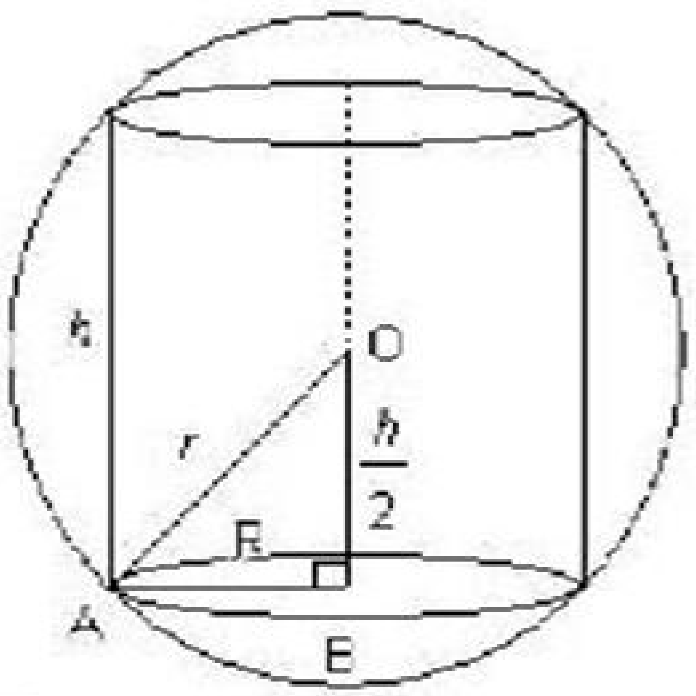
x

Eg. Find the volume of the largest cylinder that can be placed inside a cone of $R = 6$ cm and $H = 10$ cm.

x

(11) Largest Cylinder inside sphere

(11) Largest Cylinder inside sphere



r

Eg. What is the volume of the largest cylinder that can be placed inside a sphere of radius 4 cm?

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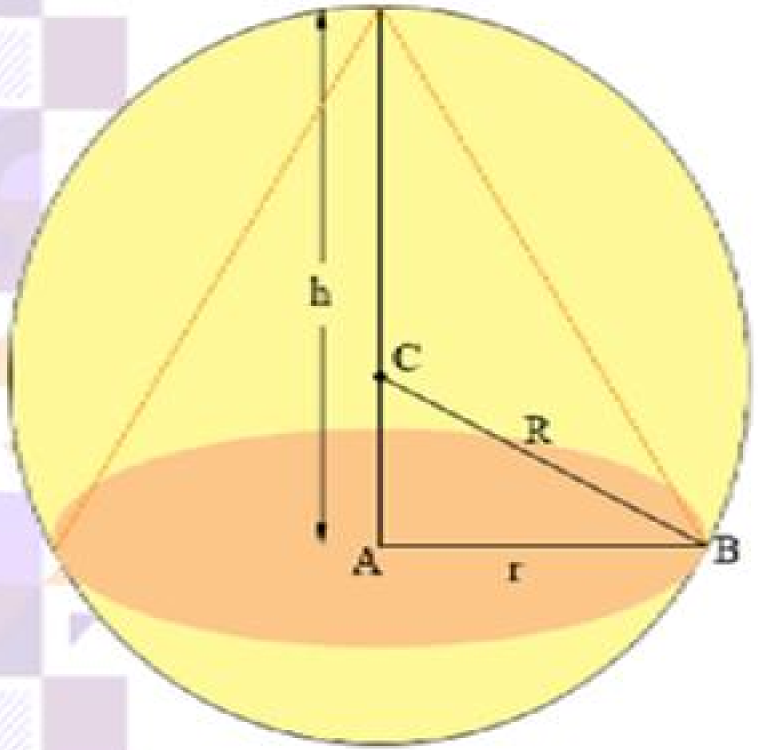
φ

(12) Largest cone inside sphere

(12) Largest cone inside sphere



x

Eg. Find the volume of the largest cone inside a sphere of radius 10 cm.

r

Eg. Consider a right circular cone of base radius 4 cm and height 10 cm. A cylinder is to be placed inside the cone with one of the flat surface resting on the base of the cone. Find the largest possible total surface area (in sq. cm) of the cylinder.

(a) $\frac{100\pi}{3}$ (b) $\frac{80\pi}{3}$ (c) $\frac{120\pi}{7}$ (d) $\frac{130\pi}{9}$

Ans. (a)

Eg. The base radius and slant height of a conical vessel is 3 cm and 6 cm respectively. Find the volume of sufficient water in the vessel such that when a sphere of radius 1 cm is placed into it, water just immersed it.

Imp

(a) $\frac{5}{2}\pi$ (b) $\frac{5}{3}\pi$ (c) $\frac{5}{4}\pi$ (d) $\frac{5}{6}\pi$

Homework

Ans. (b)

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4.5 Google Play Review

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