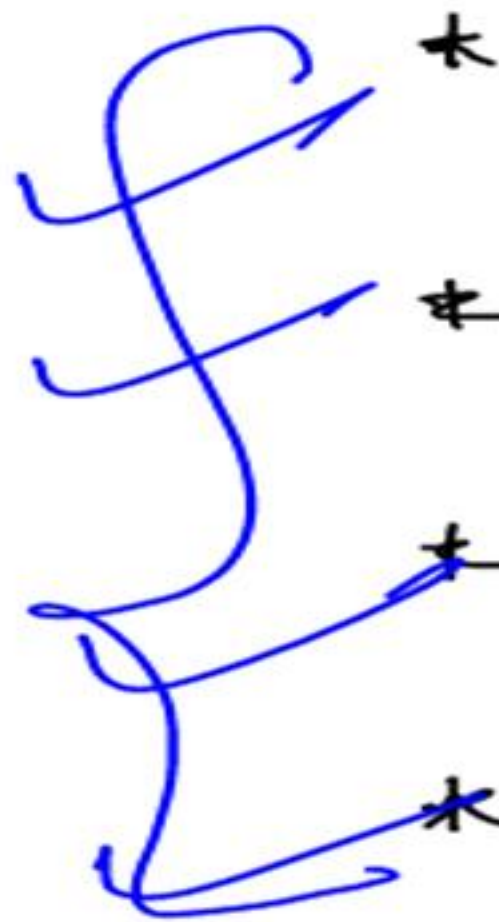




Sahi Prep Hai Toh Life Set Hai

QUADRATIC EQUATION

Quadratic Equation



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Meaning of Quadratic equation

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How to solve Quadratic eqⁿ

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How many Roots of QE

*

Condⁿ for Real Roots, —, —

*

Sum of Roots & Product of Roots

*

Maxima & Minima

Agenda

* Quadratic eq^M
 \rightarrow 70 min

* Linear eq^M left
 over Part \rightarrow 20 min

Linear Equation Remaining Part

Eg6. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

REVERSE APPROACH

Eg7. A shop stores x kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x .

- (a) $2 \leq x \leq 6$**
- (b) $5 \leq x \leq 8$**
- (c) $9 \leq x \leq 12$**
- (d) $11 \leq x \leq 14$**

Eg8. A has three sisters B, C and D. A first goes to B's house and B knows that the financial condition of his brother A is not good, so she checks the wallet of his brother and whatever amount was there she adds the same amount to his wallet. When A leaves house of B, he gives her Rs.2100 and the same process goes with other two sisters. When A left D house he has Rs.5000. Find the initial amount A has.

Quadratic Equation

Quadratic = equation
Degree 2
↓
highest power of
any variable

MEANING OF QUADRATIC EQUATION

General form $\longrightarrow ax^2 + bx + c = 0$
 $[a \neq 0]$

$a \rightarrow$ coeff of x^2

$b \rightarrow$ coeff of x

$c \rightarrow$ constant

General form of Quadratic Equation

$$ax^2 + bx + c = 0$$

$a \rightarrow$ coefficient of x^2

$b \rightarrow$ coefficient of x

$c \rightarrow$ constant term

eg $5x^2 + 3x + 8$

$$a = 5$$

$$b = 3$$

$$c = 8$$

eg $2x - 4x^2 - 5 = 0$

$$a = -4$$

$$b = 2$$

$$c = -5$$

Degree of equation tells us about the maximum number of real solutions, it can have.

E.g. $x + 4 = 7$ (1 real solution)

E.g. $x^2 + 16 = 25$ (2 real solutions)

E.g. $x^2 + 4x + 4 = 0$ (1 real solution)

E.g. $3x^2 + 4x + 6 = 0$ (0 real solution)

$$ax^2 + bx + c = 0$$

Degree = 2

How many ^{Real} solⁿs

are there

→ solⁿ can be

0, 1, 2, 2

If,

$$x^2 = 16$$

$$x = \pm 4$$

But if,

$$\overset{!}{x} = \sqrt{16}$$

$$x = 4$$

eg

$$y = \sqrt{36}$$

$$y = 6 \checkmark$$

$$y^2 = 36$$

$$y = \pm 6$$

$$y = \sqrt{x}$$

$$\begin{aligned} x &\geq 0 \\ \Delta y &\geq 0 \end{aligned}$$

$$\sqrt{(a-b)^2}$$

~~a-b~~

b-a

eg

- | | |
|--------------|-------|
| A | $x-3$ |
| B | $3-x$ |
| C | 2 |
| D | 1 |

$$y = \sqrt{(3-x)^2} \quad [x < 3]$$

\downarrow
 $3-x$
 \checkmark

\downarrow
 ~~$x-3$~~

Eg : Find the value of $\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$
if $1 < x < 3$

(a) $2x - 4$

(b) $4 - 2x$

(c) 2

(d) -2

$\sqrt{x-1}$

~~$x-3$~~

~~$1-x$~~

$3-x$

~~$x-1$~~ + ~~$3-x$~~
 $= 2$

eg

$$\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$$

if

$$x > 3$$

$$(x-1)$$

$$\cancel{1-x}$$

$$(x-3)$$

$$\cancel{3-x}$$

$$\checkmark A \quad 2x-4$$

$$B \quad 4-2x$$

$$C \quad 2$$

$$D \quad -2$$

$$\sqrt{(x-1)^2} + \sqrt{(x-3)^2}$$

$$\text{if } \underline{x < 1}$$

$$\frac{x-1}{1-x}$$

$$\frac{x-3}{3-x}$$

$$B$$

HOW TO SOLVE A QUADRATIC EQUATION

I. Method of Completion of Perfect Square.

eg

$$x^2 = 36$$

$$x = \pm 6$$

Eg. $(x - 4)^2 = 25$

$$x - 4 = \pm 5$$

$$x - 4 = 5, \quad x - 4 = -5$$

$$\underline{\underline{x = 9}}$$

$$\Delta \quad \underline{\underline{x = -1}}$$

HOW TO SOLVE A QUADRATIC EQUATION BY COMPLETION OF PERFECT SQUARE

Eg. $x^2 + 2x - 24 = 0$

$$x^2 + 2x + 1 - 1 - 24 = 0$$

$$(x+1)^2 = 25$$

$$x+1 = \pm 5$$

$$x+1 = 5$$

$$\underline{\underline{x = 4}}$$

$$x+1 = -5$$

$$\underline{\underline{x = -6}}$$

Eg. $x^2 + 7x + 5 = 0$

$$x^2 + 7x + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 5 = 0$$

$$\left(x + \frac{7}{2}\right)^2 = -5 + \frac{49}{4}$$

$$\left(x + \frac{7}{2}\right)^2 = \frac{29}{4}$$

$$x + \frac{7}{2} = \frac{\sqrt{29}}{2}$$

$$x = \frac{\sqrt{29} - 7}{2}$$

$$x + \frac{7}{2} = -\frac{\sqrt{29}}{2}$$

$$x = \frac{-\sqrt{29} - 7}{2}$$

Eg1. : $x^2 - 7x + 16 = 6$

$$x^2 - 7x + 10 = 0$$

$$x^2 - 7x + \frac{49}{4} - \frac{49}{4} + 10 = 0$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{9}{4}$$

$$x - \frac{7}{2} = \frac{3}{2}$$

$$\underline{\underline{x = 5}}$$

$$x - \frac{7}{2} = -\frac{3}{2}$$

$$\underline{\underline{x = 2}}$$

Eg2 : $4x^2 - 10x - 28 = 22$

$$4x^2 - 10x - 50 = 0$$

$$x^2 - \frac{5}{2}x - \frac{25}{2} = 0$$

$$x - \frac{5}{4} = \frac{15}{4}$$

$$x = 5$$

$$x - \frac{5}{4} = -\frac{15}{4}$$

$$x = -\frac{5}{2}$$

$$x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16} - \frac{25}{2} = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{25}{16} + \frac{25}{2}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{225}{16}$$

II. Quadratic Formula Method

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

~~v. imp~~

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

eg

$$3x^2 + 11x + 10 = 0$$

$$x = \frac{-11 \pm \sqrt{121 - 4 \cdot 3 \cdot 10}}{2 \cdot 3}$$

$$x = \frac{-11 \pm 1}{6} = \left(\frac{-5}{3}, -2 \right)$$

$$D = b^2 - 4ac$$

If $D > 0$	$D = 0$	$D < 0$
Two Real solutions (Real and Distinct)	One Real solution (Real and equal solution)	No Real solution (Imaginary solution)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

Eg. : $3x^2 + 8x + 7 = 0$

$$D = (8)^2 - 4 \cdot 3 \cdot 7$$

$$= -ve$$

No Real solⁿ

Solve the following quadratic equations by using Quadratic formula.

Eg : $9x^2 + 7x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{49 + 72}}{2 \cdot 9}$$

$$x = \frac{-7 \pm 11}{18}$$

$$\frac{2}{9}, -1$$

Eg. Find the values of k for which the given equation has real and equal roots

$$12x^2 + 4kx + 3 = 0$$

For real & equal roots $D = 0$

$$(4k)^2 - 4 \cdot 12 \cdot 3 = 0$$

$$k^2 - 9 = 0$$

$$k^2 = 9$$

$$k = \pm 3$$

Eg. Find the values of k for which the given equation has real and equal roots

$$x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$$

for Real & equal roots $D = 0$

$$[-2(1+3k)]^2 - 4 \cdot 1 \cdot 7(3+2k) = 0$$

$$(3k+1)^2 - 7(3+2k) = 0$$

$$9k^2 + 6k + 1 - 21 - 14k = 0$$

$$9k^2 - 8k - 20 = 0$$

$$k = \frac{8 \pm \sqrt{784}}{18}$$

$$\rightarrow 2, -\frac{10}{9} \quad \checkmark \checkmark$$

Eg. Find the values of k for which the given expression is a perfect square?

Note If a Quadratic expression is a Perfect Square it means

$$D = 0$$

(i) $x^2 + k^2x + 400$

Make $D = 0$

$$(k^2)^2 - 4 \cdot 1 \cdot 400 = 0$$

$$k^4 = 1600$$

$$k^2 = 40$$

$$k = \pm \sqrt{40} \\ = \pm 2\sqrt{10}$$

$$(ii) x^2 + 8x + k^2 = 0$$

$$(8)^2 - 4 \cdot 1 \cdot k^2 = 0$$

$$64 = 4k^2$$

$$k^2 = 16$$

$$k = \pm 4$$

$$(iii) \frac{x^2}{y^2} + kx + \frac{y^2}{4}$$

$$a = \frac{1}{y^2}$$

$$b = k$$

$$c = \frac{y^2}{4}$$

$$k^2 - 4 \cdot \frac{1}{y^2} \cdot \frac{y^2}{4} = 0$$

$$k^2 = 1$$

$$k = \pm 1$$

$$(iv) \ p + \frac{1}{4}\sqrt{p} + k^2$$

$$\text{let } p = x^2$$

$$x^2 + \frac{1}{4}x + k^2$$

$$\left(\frac{1}{4}\right)^2 - 4 \cdot 1 \cdot k^2 = 0$$

$$k^2 = \frac{1}{64}$$

$$k = \pm \frac{1}{8}$$

(v) $2022 \times 2023 + k$ is a perfect square, (where, k is natural number).
Find the minimum value of k .

2022 \times 2023 + k is a perfect

$$2022 \times 2022 < \underline{2022 \times 2023} < 2023 \times 2023$$

$$\begin{aligned} x(x+1) + k \\ x^2 + x + x+1 \end{aligned}$$

$k = 2023$

$$2022 \times 2023 + k = (2023)^2$$

$$\begin{aligned} k &= (2023)^2 - 2022 \times 2023 \\ &= 2023(1) \end{aligned}$$

$k = 2023$

Eg. $57365 \times 57367 + K$ is a perfect square.

Find the smallest value of K where K is a natural number.

$$57365 \times 57367 + K$$

Ist

$$(57366 - 1)(57366 + 1) + K$$

$$(57366)^2 - 1 + K$$

$$\underline{\underline{K = 1}} \quad \checkmark$$

$$57365 \times 57367 + K$$

eg

$$(57365)^2 < \underline{57365 \times 57367} < (57367)^2$$

$$57365 \times 57367 + K = (57366)^2$$

$$\begin{aligned} K &= (57366)^2 - (57365)(57367) \\ &= (57366)^2 - [(57366-1)(57366+1)] \end{aligned}$$

$$K = 1$$

INEQUALITIES BASIC CONCEPT

V. Imp

If $A > B > 0$

$$\underline{\underline{\frac{1}{A} < \frac{1}{B}}}$$

$$(2) \underline{\underline{-A < -B}}$$

When we multiply with a negative number
sign of Inequality changes.

eg

$$2 < 3$$

$$\frac{1}{2} > \frac{1}{3}$$

|

$$2 < 3$$

$$\underline{\underline{-2 > -3}}$$

$$(x - 5)(x - 8) > 0$$

What are the solution of x?

I

$$(x - 5) > 0$$

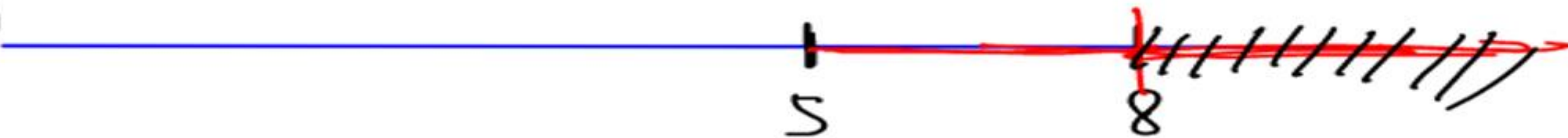
$$x > 5$$

∧

$$(x - 8) > 0$$

$$x > 8$$

$x > 8$



II

$$(x - 5) < 0$$

$$x < 5$$

∧

$$(x - 8) < 0$$

$$x < 8$$

$x < 5$



$$(x-5)(x-8) > 0$$

$$x > 8 \text{ or } x < 5$$

eg1

$$(x+4)(x-2) > 0$$

$$x > 2 \text{ or } x < -4$$

eg2

$$(x-3)(x-4) > 0$$

$$x > 4 \text{ or } x < 3$$

$$(x-5)(x-8) < 0$$

shortcut

$$5 < x < 8$$

eg

$$(x+4)(x-3) < 0$$

$$-4 < x < 3$$

Eg. $(x + 5)(x - 3) > 0$

What are the solution of x ?

$$x > 3 \text{ or } x < -5$$

Eg. $(x - 3)(x + 8) < 0$

What are the solution of x ?

$$-8 < x < 3$$

eg 1

$$(x-3)(x-7) < 0$$

$$3 < x < 7$$

eg 2

$$(x+4)(x+5) > 0$$

$$x > -4 \quad \text{or} \quad x < -5$$

eg 3

$$(x-3)(5-x) > 0$$

$$(x-3)(x-5) < 0$$

$$3 < x < 5$$

Eg1. Find the value of k for which the given equation has real roots.

$$9x^2 + 3kx + 4 = 0$$

For Real Roots

$$D \geq 0$$

$$(3k)^2 - 4 \cdot 9 \cdot 4 \geq 0$$

$$k^2 - 16 \geq 0$$

$$(k-4)(k+4) \geq 0$$

$$\underline{k \geq 4} \text{ or } \underline{k \leq -4}$$

Eg2. Find the values of k for which the equation $x^2 + 5kx + 16 = 0$ has no real roots.

No Real Roots



$$\underline{\underline{D < 0}}$$

$$(5k)^2 - 4 \cdot 1 \cdot 16 < 0$$

$$(5k)^2 - (8)^2 < 0$$

$$(5k - 8)(5k + 8) < 0$$

$$-\frac{8}{5} < k < \frac{8}{5}$$

SOLUTION OF A QUADRATIC EQUATION BY FACTORIZATION METHOD

Step I : Factorize the constant term of the given quadratic equation.

Step II : Express the coefficient of middle term as the sum or difference of the factors obtained in step I. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step III : Split the middle term in two parts obtained in step II.

Step IV : Factorize the quadratic equation obtained in step III by grouping method.

eg

which is greater

A

$$-\frac{1}{28}$$

B

$$-\frac{1}{47}$$



$$28 < 47$$

$$\frac{1}{28} > \frac{1}{47}$$

$$-\frac{1}{28} < -\frac{1}{47}$$