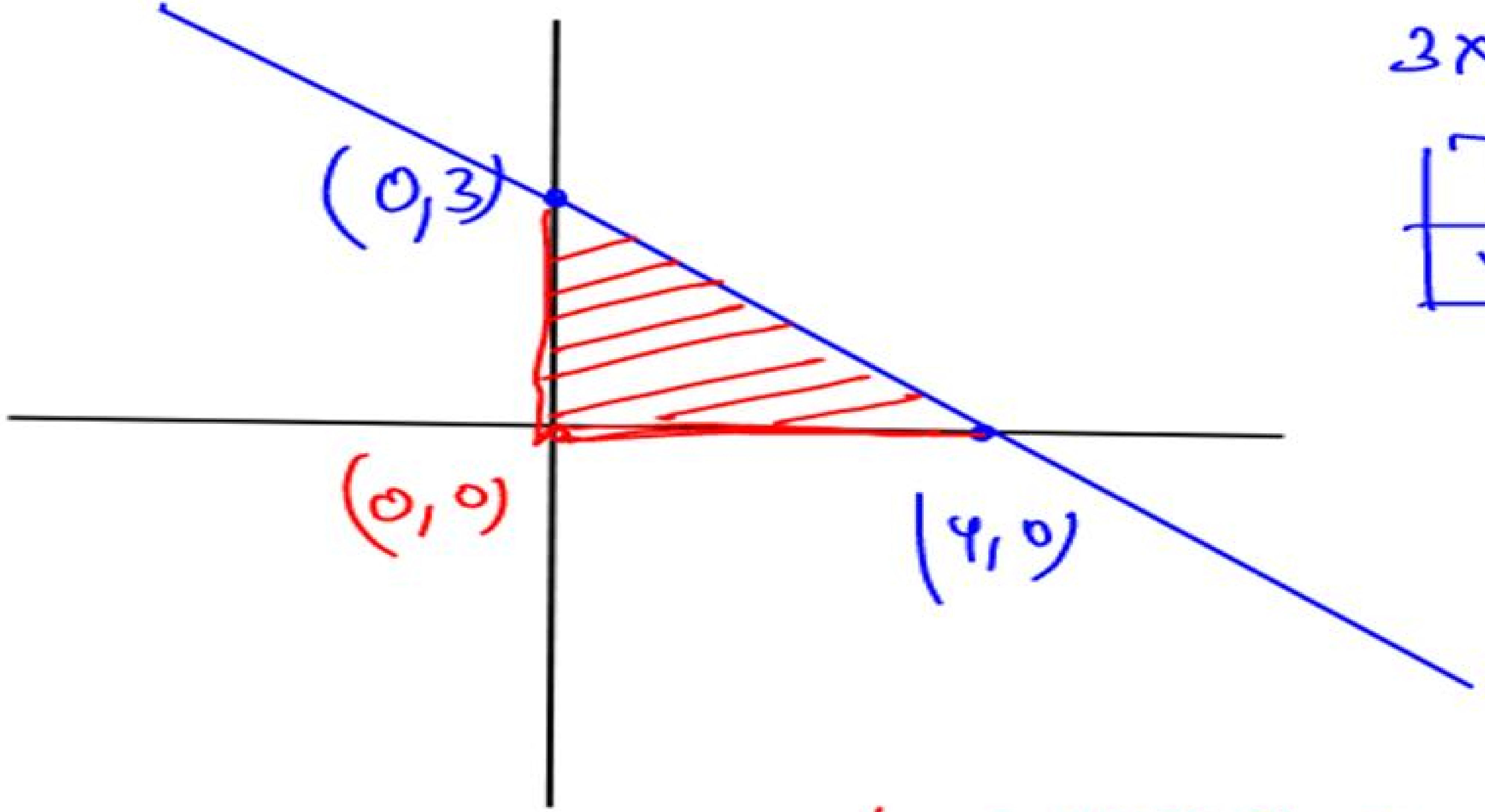
gradeup

gradeup

Sahi Prep Hai Toh Life Set Hai

COORDINATE GEOMETRY

Eg. Find the area of triangle bounded by x-axis, y-axis and $3x+4y-12 = 0$



$3x+4y-12=0$

x	0	4
y	3	0

$$\frac{1}{2} \times 4 \times 3 = \underline{\underline{6}}$$

Ans. 6

Shortcut for previous question:

X axis

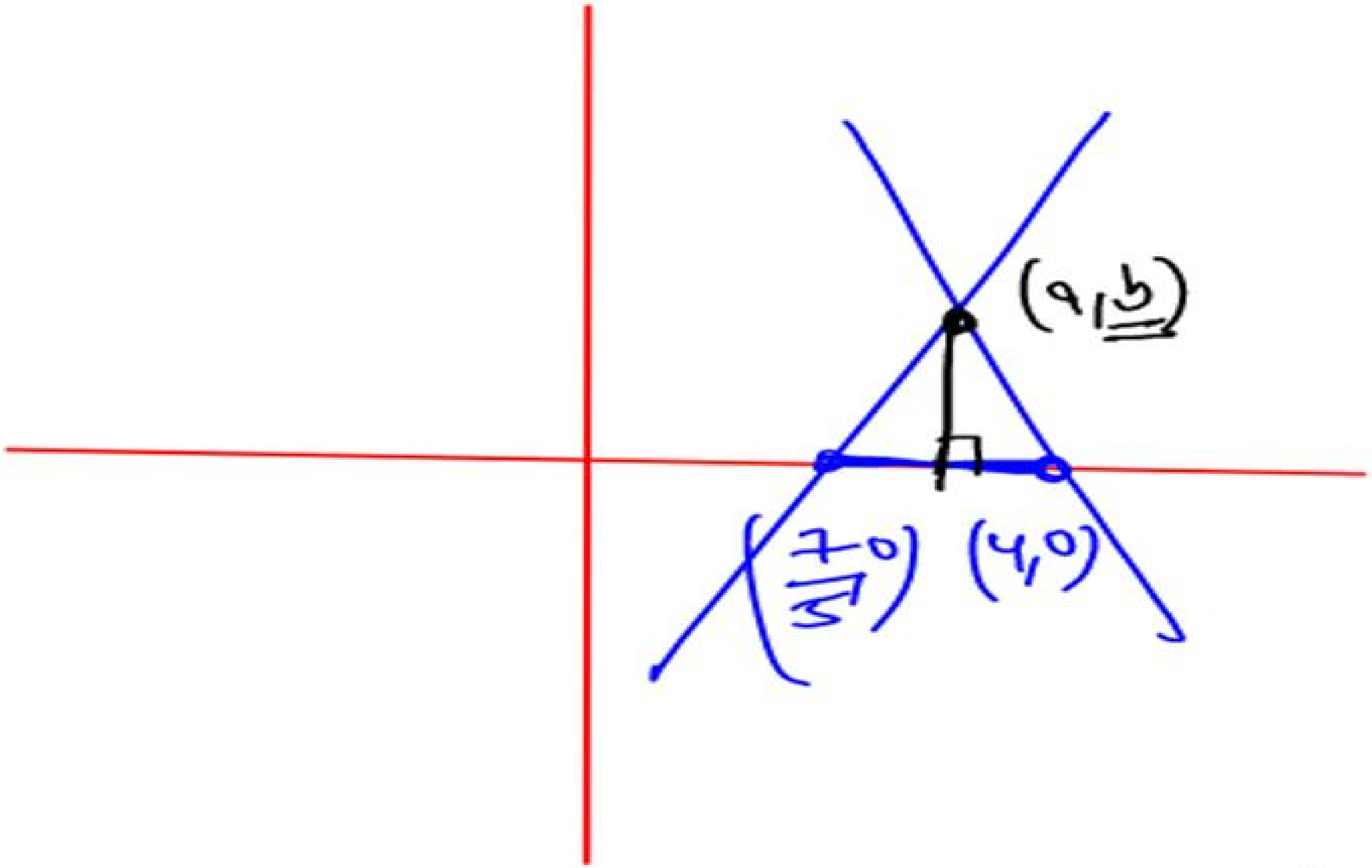
Y axis

$ax + by + c = 0$

Area of Δ bounded by these 3 lines

$$\frac{1}{2}\left|\frac{c^2}{ab}\right|$$

Eg. Find the area of the triangle formed by $3x + 4y = 12$, $5x - 2y = 7$ and x-axis:



Step 1 $y = 0$ $x = 4$

$y = 0$ $x = 7/5$

Base → $4 - \frac{7}{5} = \frac{13}{5}$

Step 2 Intersection pt

$3x + 4y = 12$

$5x - 2y = 7$

$\boxed{x = 2 \quad y = 3/2}$

Area → $\frac{1}{2} \times \frac{13}{5} \times \frac{3}{2} \rightarrow \frac{39}{20}$

Ans. 39/20

Eg find the area of Δ bounded by Y axis

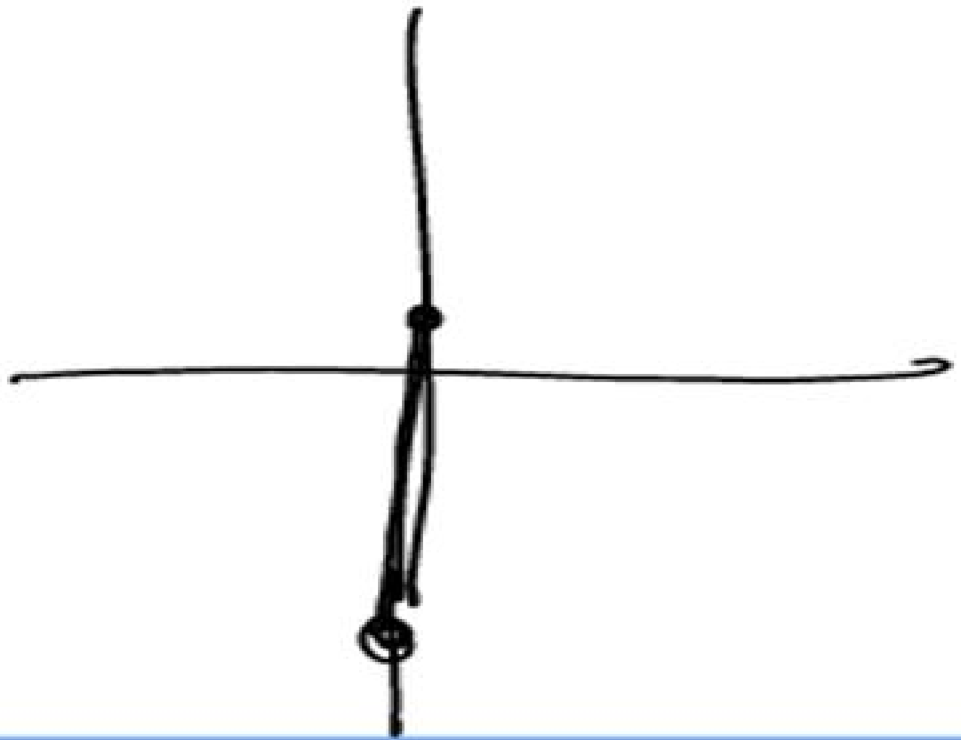
$3x+4y=1$

$5x-2y=19$

Solⁿ Step 1 $x=0$ $y=1/4$

$x=0$ $y=-19/2$

Base $\frac{1}{4}+\frac{19}{2}=\frac{39}{4}$



Step 2 $3x+4y=1$

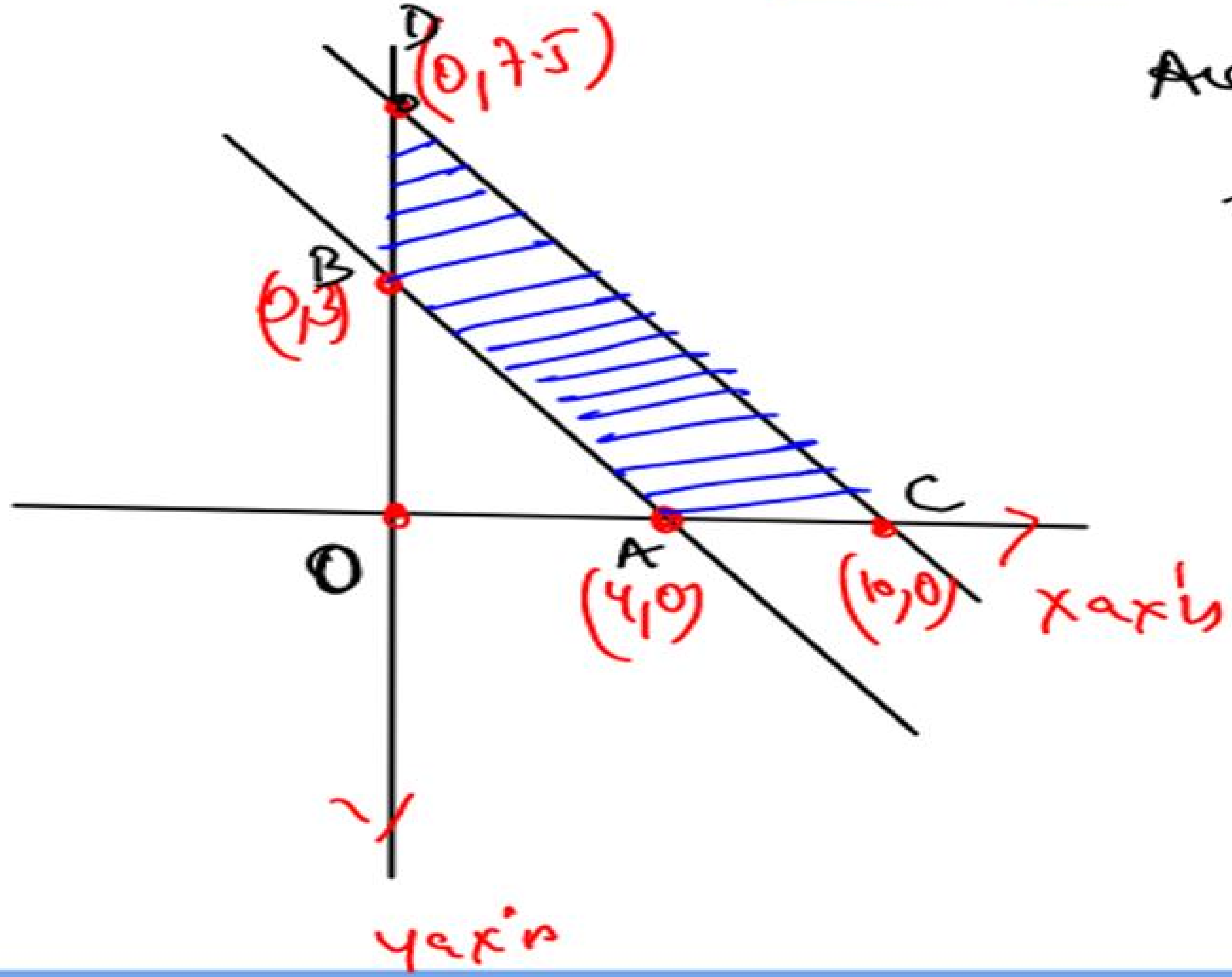
$5x-2y=19$

$x=3$ $y=-2$

$\frac{1}{2}\times\frac{39}{4}\times 3 = \frac{117}{8}$

Eg. Find the area of trapezium formed by :

x-axis, y-axis, $3x + 4y = 12$ and $6x + 8y = 60$



Area of Trapezium ABCD

$$= \text{Area of } \Delta OCD - \text{Area of } \Delta OAB$$

$$= \frac{1}{2}\left|\frac{(-60)^2}{6\cdot 8}\right| - \frac{1}{2}\left|\frac{(-12)^2}{3\cdot 4}\right|$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\overset{5\;10}{\cancel{60}}\cdot\overset{15}{\cancel{60}}}{\cancel{6}\cdot\cancel{8}\,2} - \frac{1}{2}\cdot\frac{\cancel{12}\cdot 12}{\cancel{12}}$$

$$37.5 - 6$$

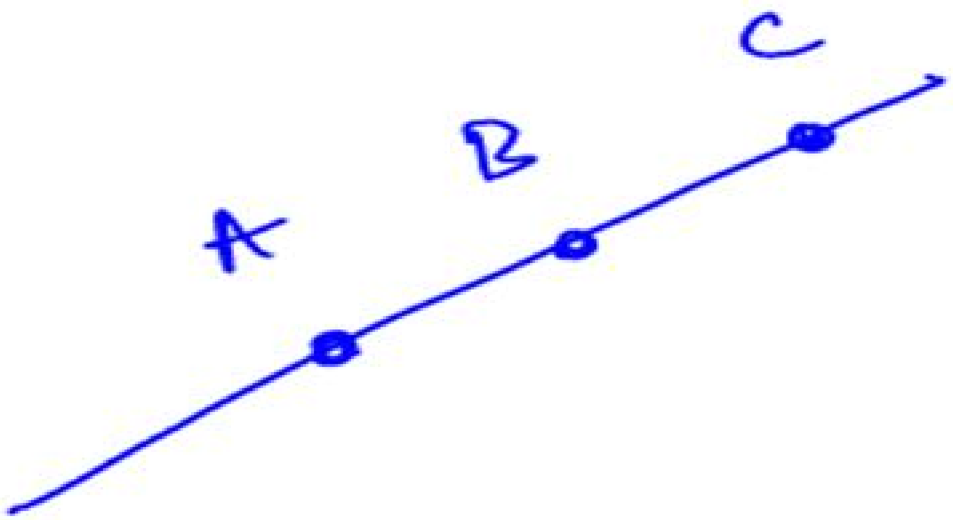
$$= \underline{31.5}$$

Ans. 31.5

HOW TO CHECK WHETHER THREE POINTS ARE COLLINEAR OR NOT?

Collinear points

If 3 or more than 3 points lie on a single line.



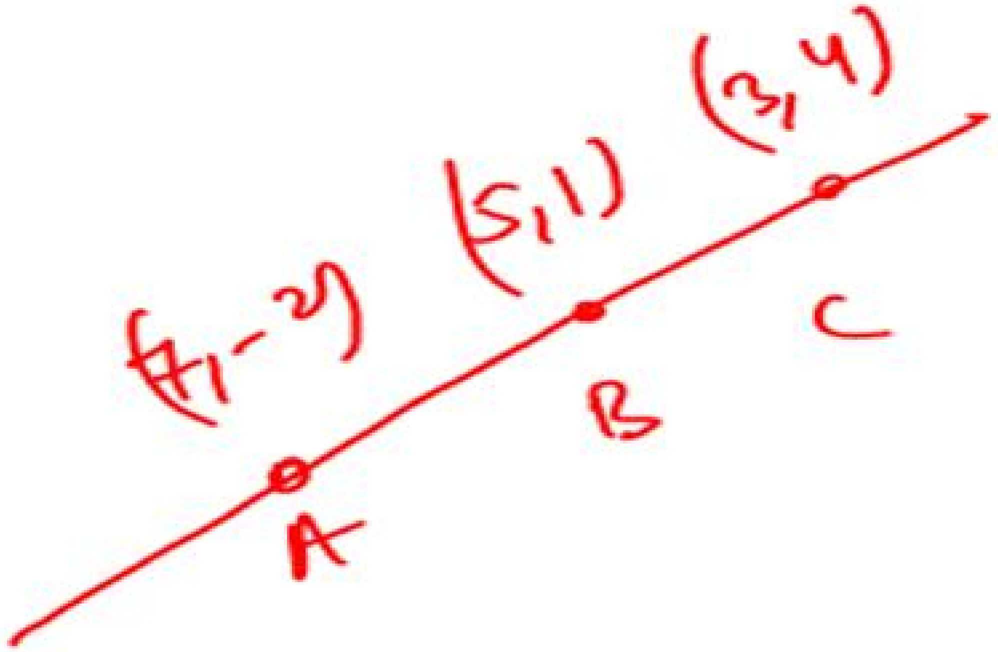
→ Area of Δ

→ Distance

→ Eq of line

→ Slope Method

Eg. Let, the three points are (7, –2), (5,1), (3, 4).
How to check whether they are collinear or not?



I If Area of $\Delta = 0$

$$\begin{array}{cccc} 7 & 5 & 3 & 7 \\ -2 & 1 & 4 & -2 \end{array}$$

$$\frac{1}{2}\left|(7+20-6)-(-10+3+28)\right|$$

$$= \frac{1}{2}|0| = 0$$

Points are collinear

II

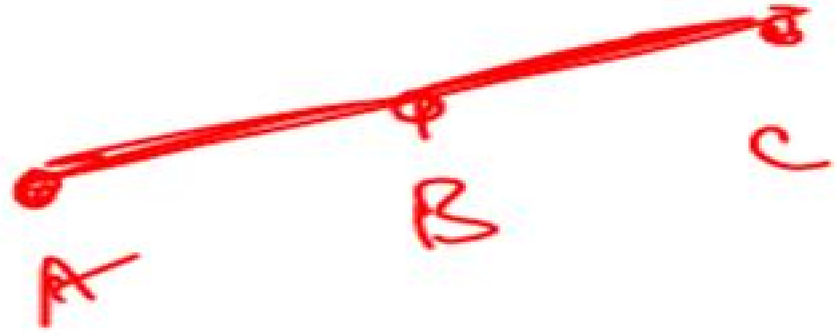
$$\underset{(7,-2)}{A} \qquad \underset{(5,1)}{B} \qquad \underset{(3,4)}{C}$$

$$AB = \sqrt{3^2+(-2)^2} = \sqrt{13}$$

$$BC = \sqrt{9+4} = \sqrt{13}$$

$$AC = \sqrt{36+16} = 2\sqrt{13}$$

$$AB + BC = AC$$



III

$$\underset{(7,-2)}{A} \qquad \underset{(5,1)}{B} \qquad \underset{(3,4)}{C}$$

Eq of AB $\Rightarrow$ $y+2 = \dfrac{3}{-2}(x-7)$

$$-2y-4 = 3x-21$$

$$\boxed{3x+2y = 17}$$

check whether C lies on that

$$3(3)+2(4) = 17$$

IV^{th} Slope Method

A B C

$(7, -2)$ $(5, 1)$ $(3, 4)$

slope of AB = slope of BC

$\frac{3}{-2}$ $\frac{3}{-2}$

Points are Collinear

Eg. Find the value of k for which three distinct points whose coordinates are (k, 2 – 2k), (– k+1, 2k) and (– 4 – k , 6 – 2k) collinear.

A B C

slope of AB = slope of BC

$$\frac{2k-(2-2k)}{-k+1-k} = \frac{6-2k-2k}{-4-k-(-k+1)}$$

$$\frac{-2(\cancel{4k-2})}{\cancel{-2k+1}} = \frac{6-4k}{-5}$$

$$10 = 6-4k$$

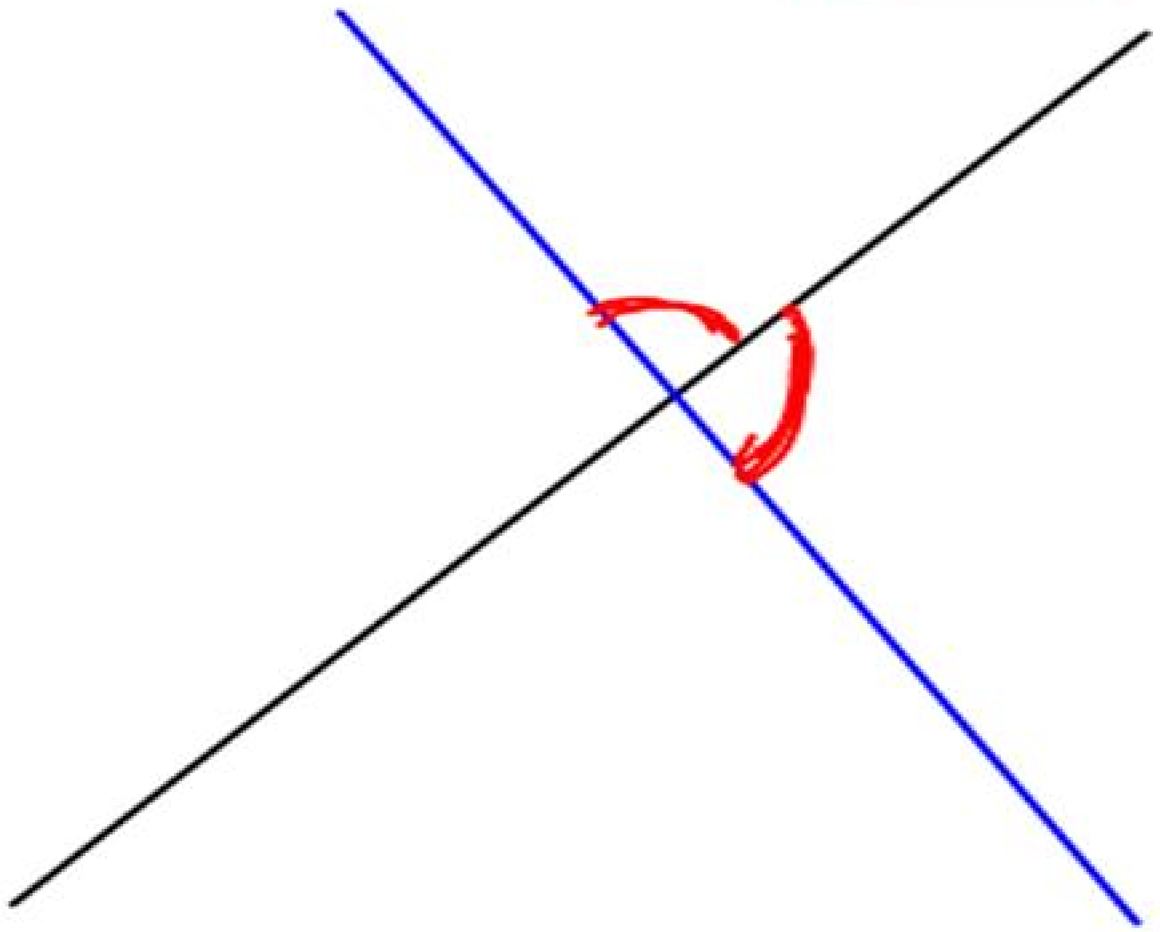
$$\boxed{k=-1}$$

Ans. -1

ANGLE BETWEEN TWO LINES

If m_1, m_2 are the slopes of two lines and θ is the acute angle between them, then:

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$



$0 < \theta < 90$

Eg. Find the angle between the lines $2x - y + 3$ and $x + y - 2 = 0$.

$2x - y + 3$ $\qquad m_1 = \frac{-(2)}{-1} = 2$

$x + y - 2$ $\qquad m_2 = \frac{-1}{1} = -1$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3}{-1}\right|$$

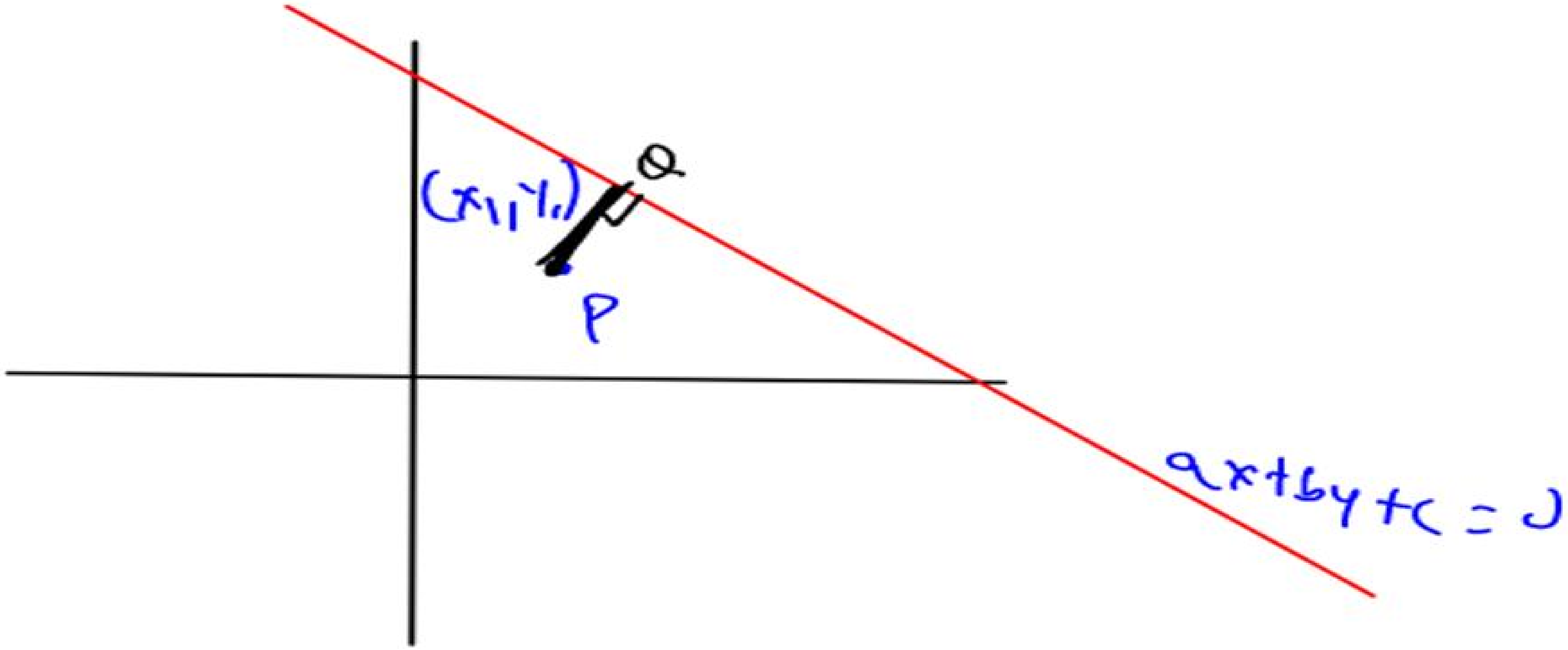
$$\tan\theta = 3$$

$$\boxed{\theta = \tan^{-1} 3}$$

DISTANCE OF A POINT FROM A LINE

Distance of a point (x_1, y_1) from a line $ax + by + c = 0$.

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$



Eg. Find the distance of point (–5, 8) from line 3x + 4y – 12 = 0.

$$\left|\frac{3(-5)+4(8)-12}{\sqrt{3^2+4^2}}\right| = \frac{5}{5} = 1$$

Ans. 1

$$ax + by + c = 0$$

Distance of a line from origin (0, 0) is $\left|\frac{c}{\sqrt{a^2 + b^2}}\right|$

Distance between 2 parallel lines :

V imp

$a_1x + b_1y + c_1 = 0$ & $a_2x + b_2y + c_2 = 0$

$$\left|\frac{c_1 - c_2}{\sqrt{a_1^2 + b_1^2}}\right|$$

Step 1 :- make coeff of x & y same

Step 2

$$\left|\frac{c_1 - c_2}{\sqrt{a_1^2 + b_1^2}}\right|$$

Eg. Find the distance between 2 parallel lines :

$$5x - 12y - 2 = 0$$
$$5x - 12y + 3 = 0$$

$$\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}$$

$$\left| \frac{-2-3}{\sqrt{5^2 + 12^2}} \right| = \frac{5}{13}$$

Ans. 5/13

gradeup

Eg. Find the distance between 2 parallel lines :

$5x - 12y + 8 = 0$

$25x - 60y + 120 = 0$

$5x - 12y + 24 = 0$

$5x - 12y + 8 = 0$

$5x - 12y + 24 = 0$

$$\left| \frac{8 - 24}{\sqrt{5^2 + 12^2}} \right| = \frac{16}{13}$$

Ans. 16/13

gradeup

Imp Eg. Find the equation of a line parallel to 5x – 12y + 26 = 0 & at a distance of 4 units from it.

$5x - 12y + 26 = 0$

$\boxed{5x - 12y + k = 0}$

$5x - 12y + 26 = 0$

$\left|\frac{k-26}{13}\right| = 4$

$\frac{k-26}{13} = \pm 4$

$\frac{k-26}{13} = 4$

$\boxed{k = 78}$

$\frac{k-26}{13} = -4$

$\boxed{k = -26}$

$5x - 12y + 78 = 0$

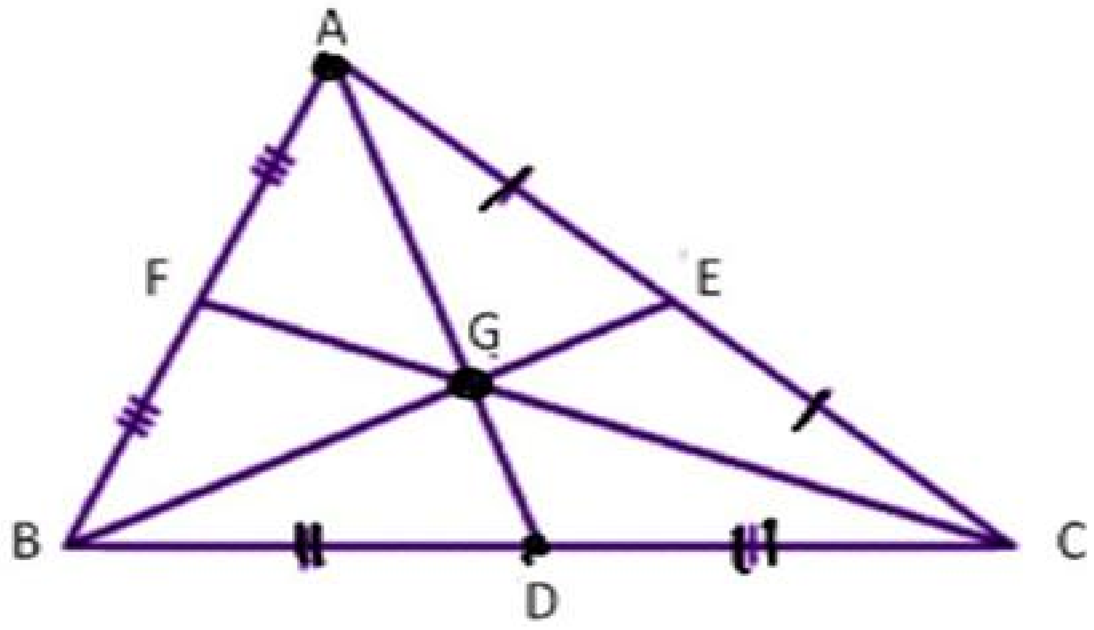
$5x - 12y - 26 = 0$

Ans. $5x - 12y - 26 = 0$

$5x - 12y + 78 = 0$

CENTROID

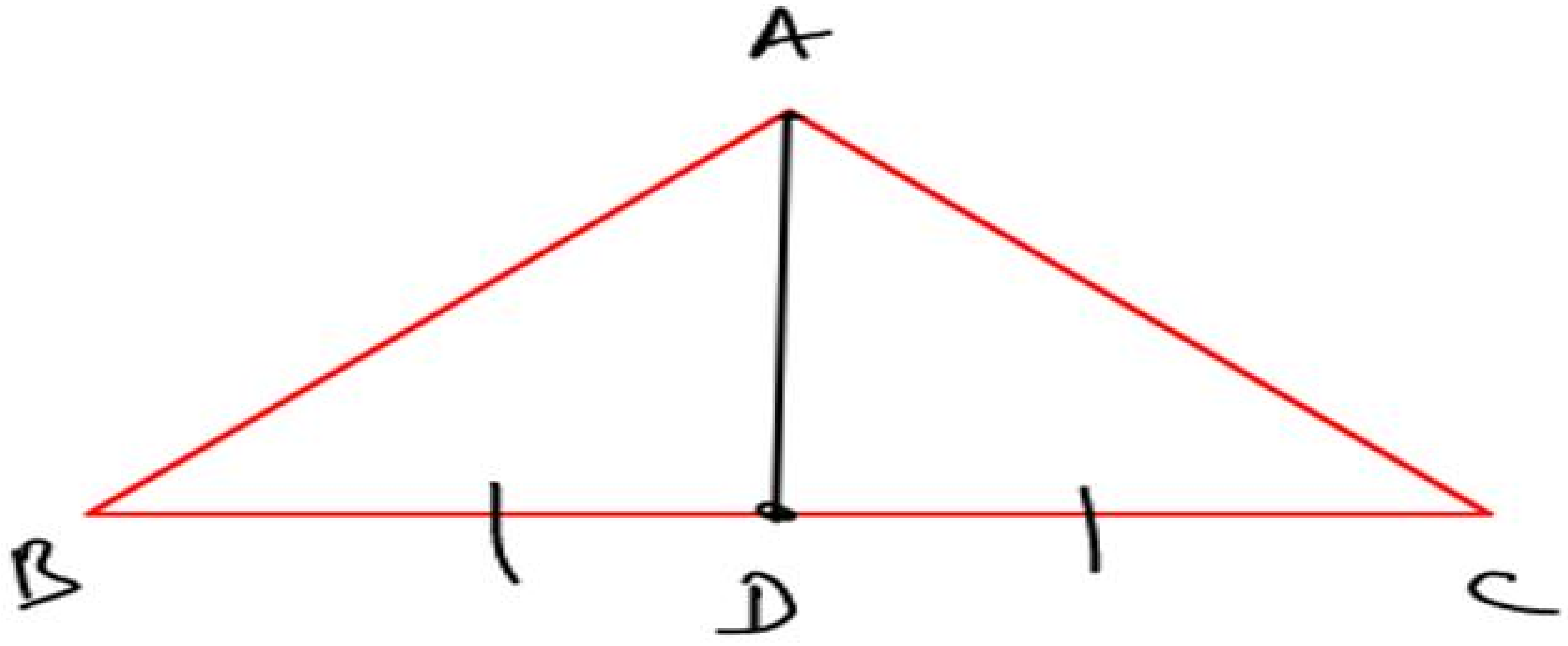
Def: Meeting point of all medians.



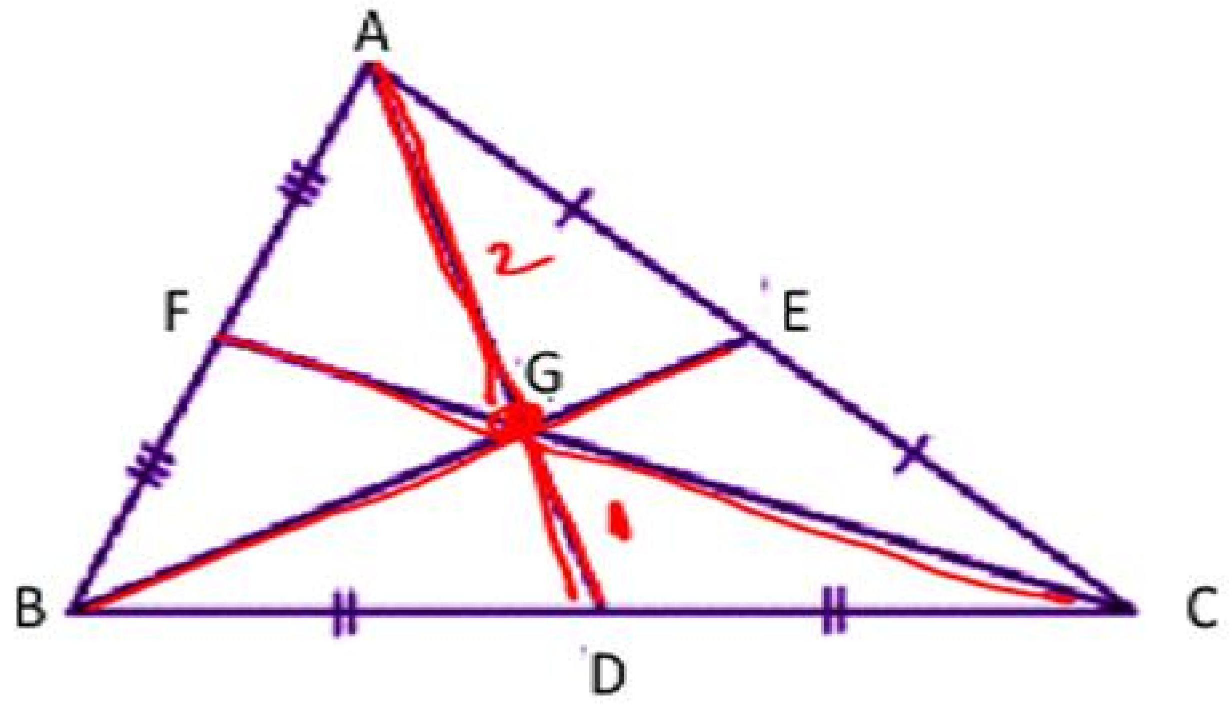
Here, G is the centroid of ΔABC.

Median

The line segment which joins one vertex to the mid point of the opposite side.



1. Centroid divides the median in 2 : 1.



AG : GD = 2 : 1

BG : GE = 2 : 1

CG : GF = 2 : 1

A triangle ABC whose vertices have coordinates :

A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3)

Coordinates of centroid $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Eg. Two vertices of a triangle are (3, -5) and (-7, 4). If its centroid is (2, -1), find the third vertex.

Centroid

$(3, -5), \quad (-7, 4), \quad (a, b) \rightarrow (2, -1)$

$$\frac{3-7+a}{3} = 2 \qquad\qquad \frac{-5+4+b}{3} = -1$$

$$-4+a = 6 \qquad\qquad -1+b = -3$$

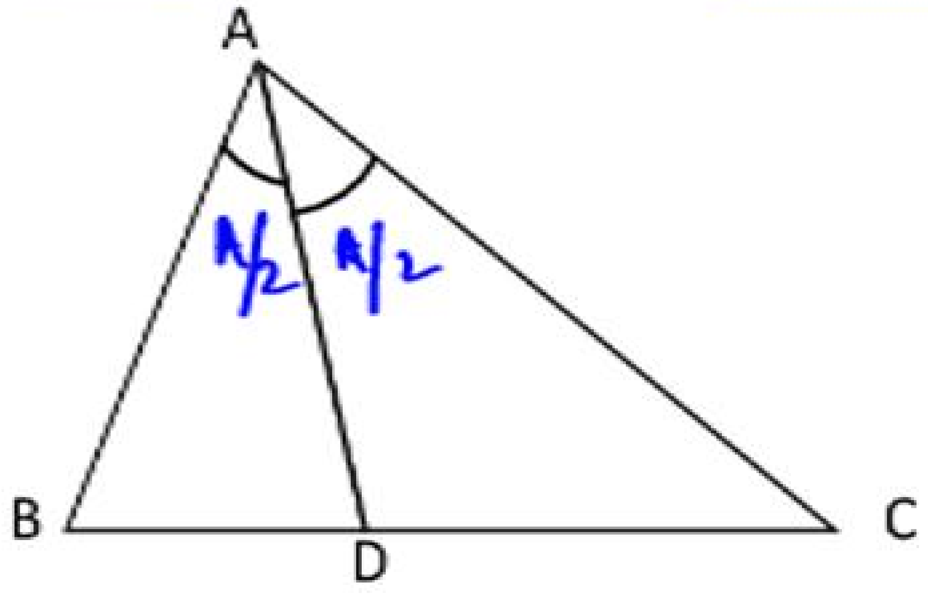
$$\underline{a = 10} \qquad\qquad b = -2$$

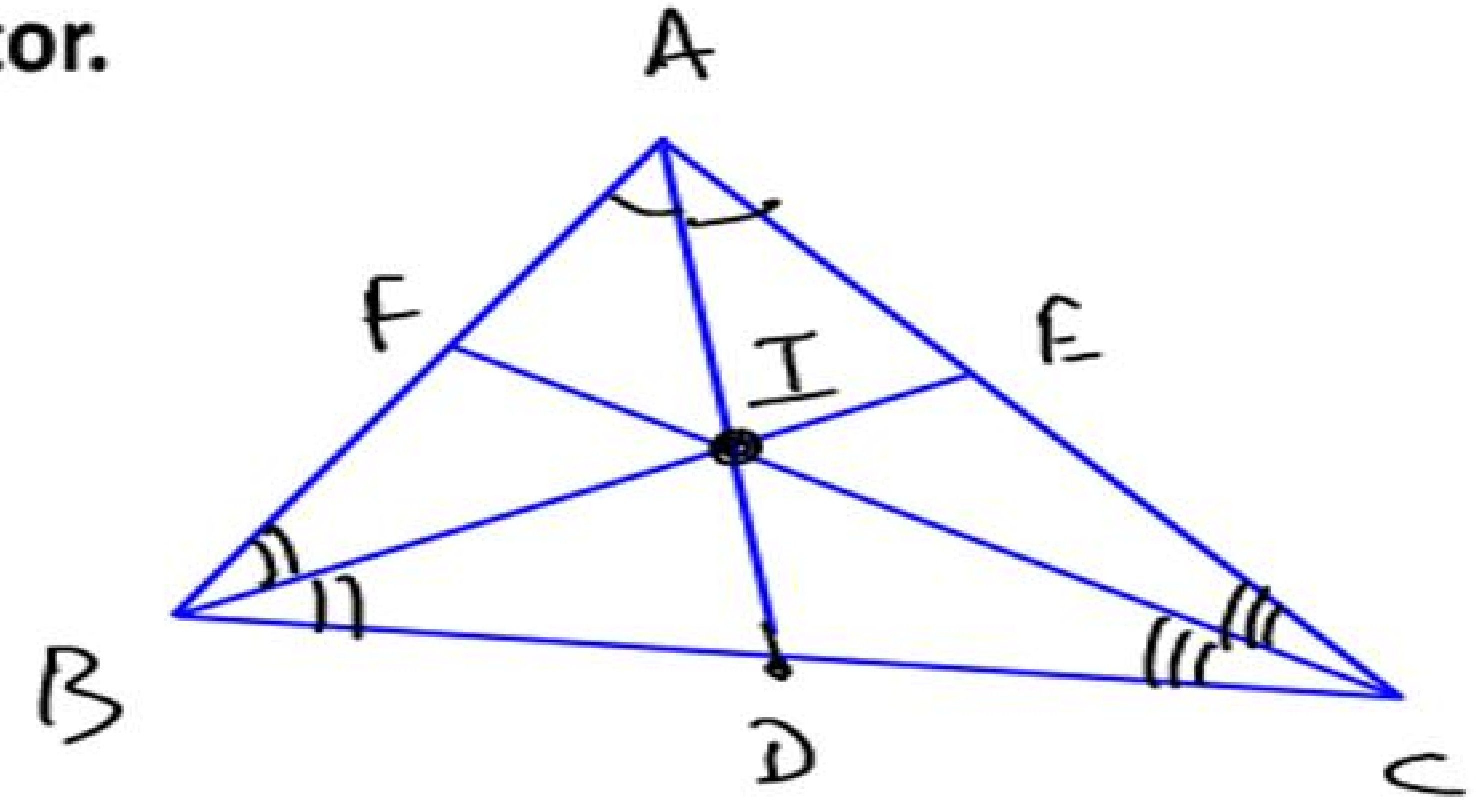
Third coordinate $(10, -2)$

Ans. (10, –2)

INCENTRE

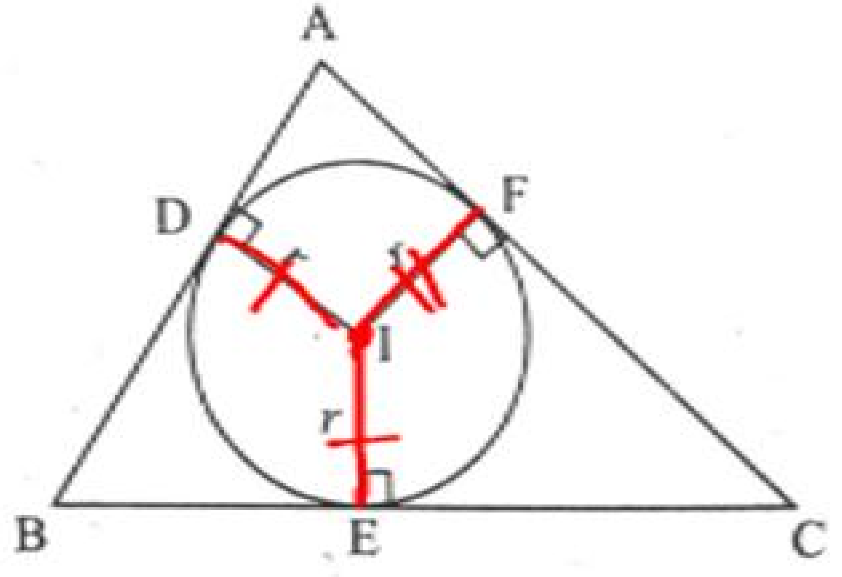
Def: Meeting point of Angle Bisector.





I → Incentre of Δ

Incentre is the centre of the circle inscribe in a triangle and it is equidistant from the sides of the triangle.

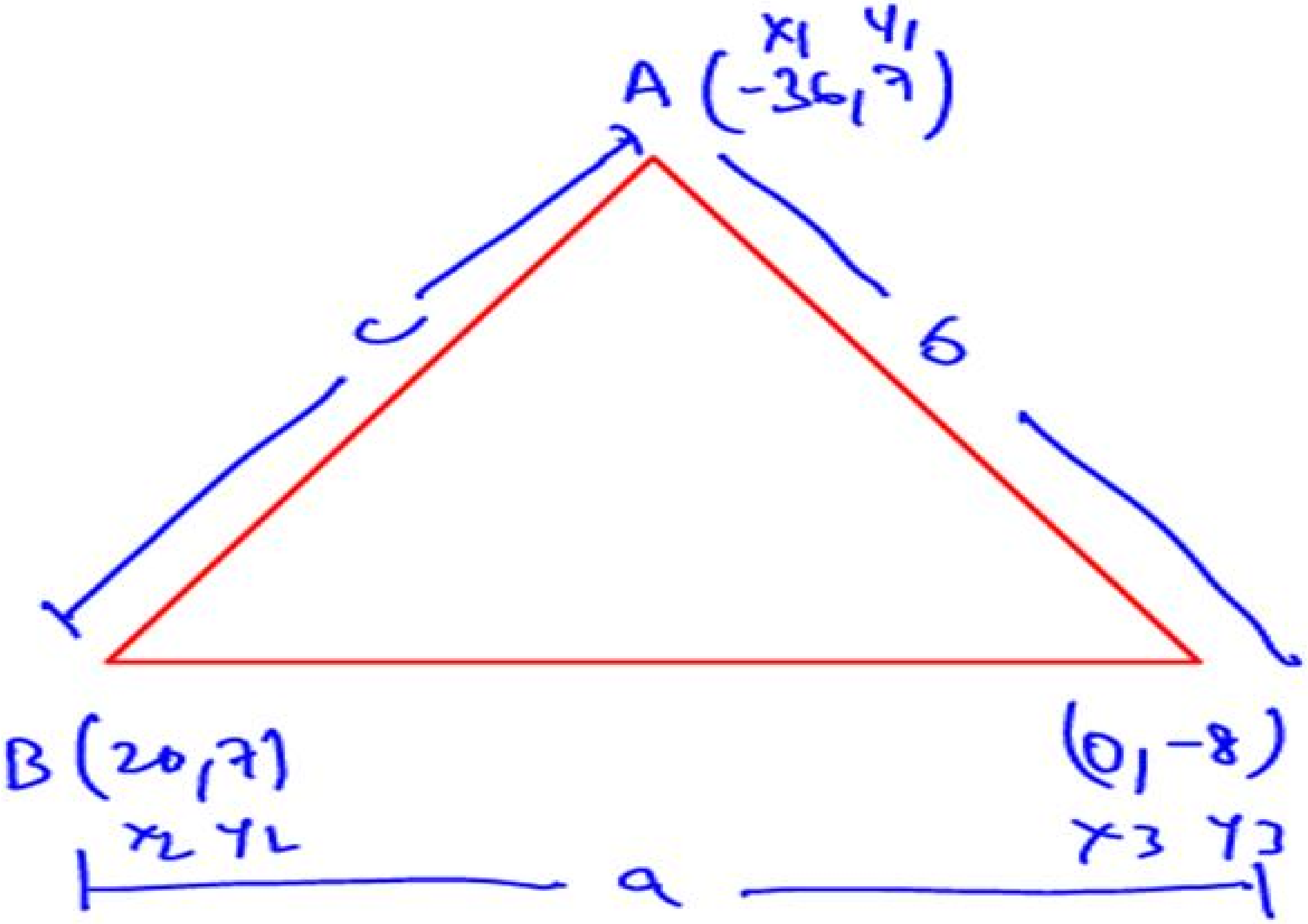


A triangle ABC whose sides are a, b & c and the coordinates of the vertices are : A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3)

$$\boxed{\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)}$$

Eg. Find the coordinates of the incentre of a triangle whose vertices are : A (-36,7) , B (20,7) and C (0,-8)

2 min



$a = \sqrt{15^2 + 20^2} = 25$

$b = \sqrt{15^2 + 36^2} = 39$

$c = \sqrt{56^2} = 56$

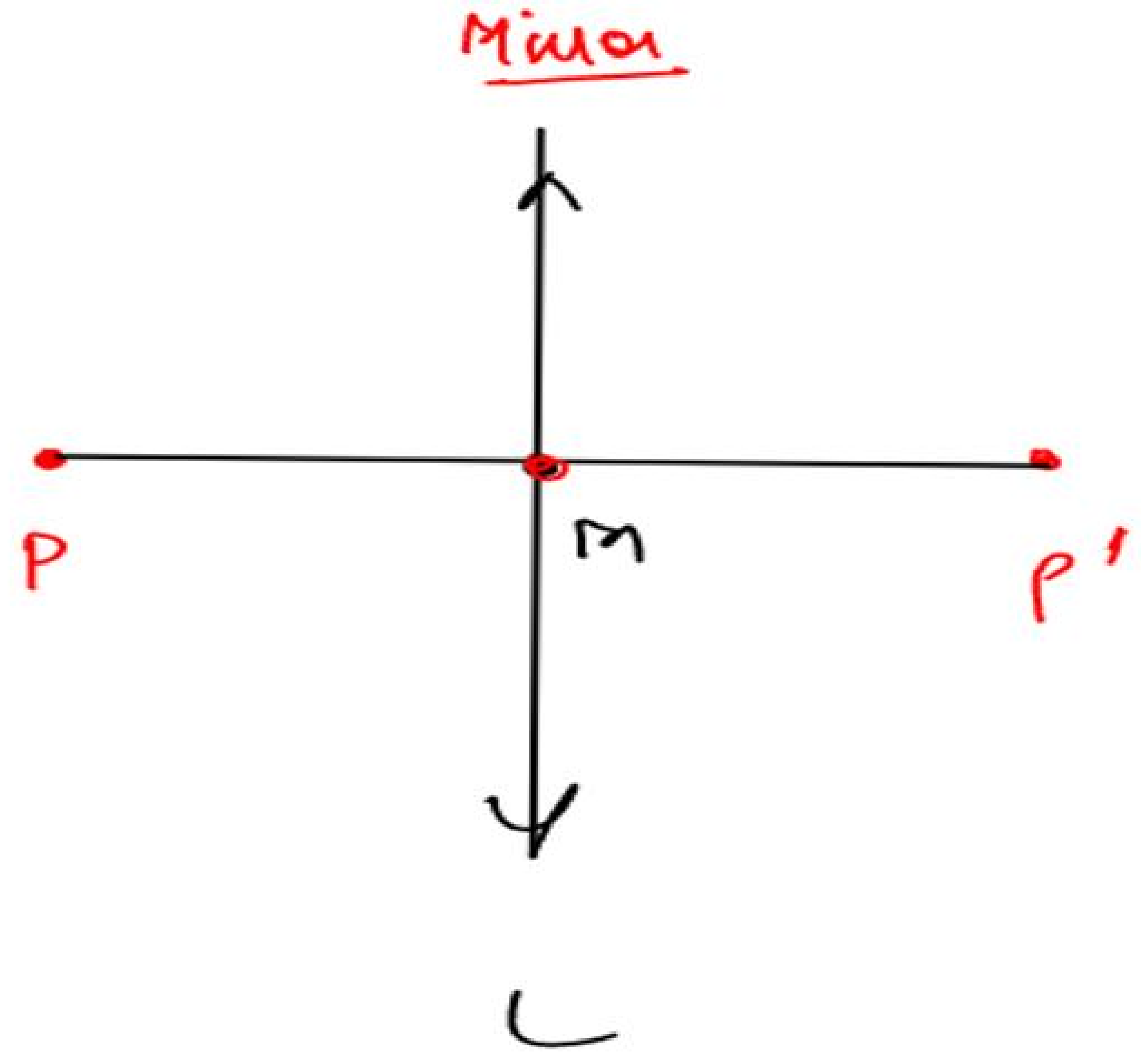
$$\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}$$

$$\frac{-900 + 780 + 0}{120}, \frac{175 + 273 - 448}{120}$$

$(-1, 0)$

Ans. $(-1, 0)$

REFLECTION OF A POINT



Basics of Reflection

① $PM = P'M$: Mid pt.

② $PP' \perp L$

$m_1 \cdot m_2 = -1$

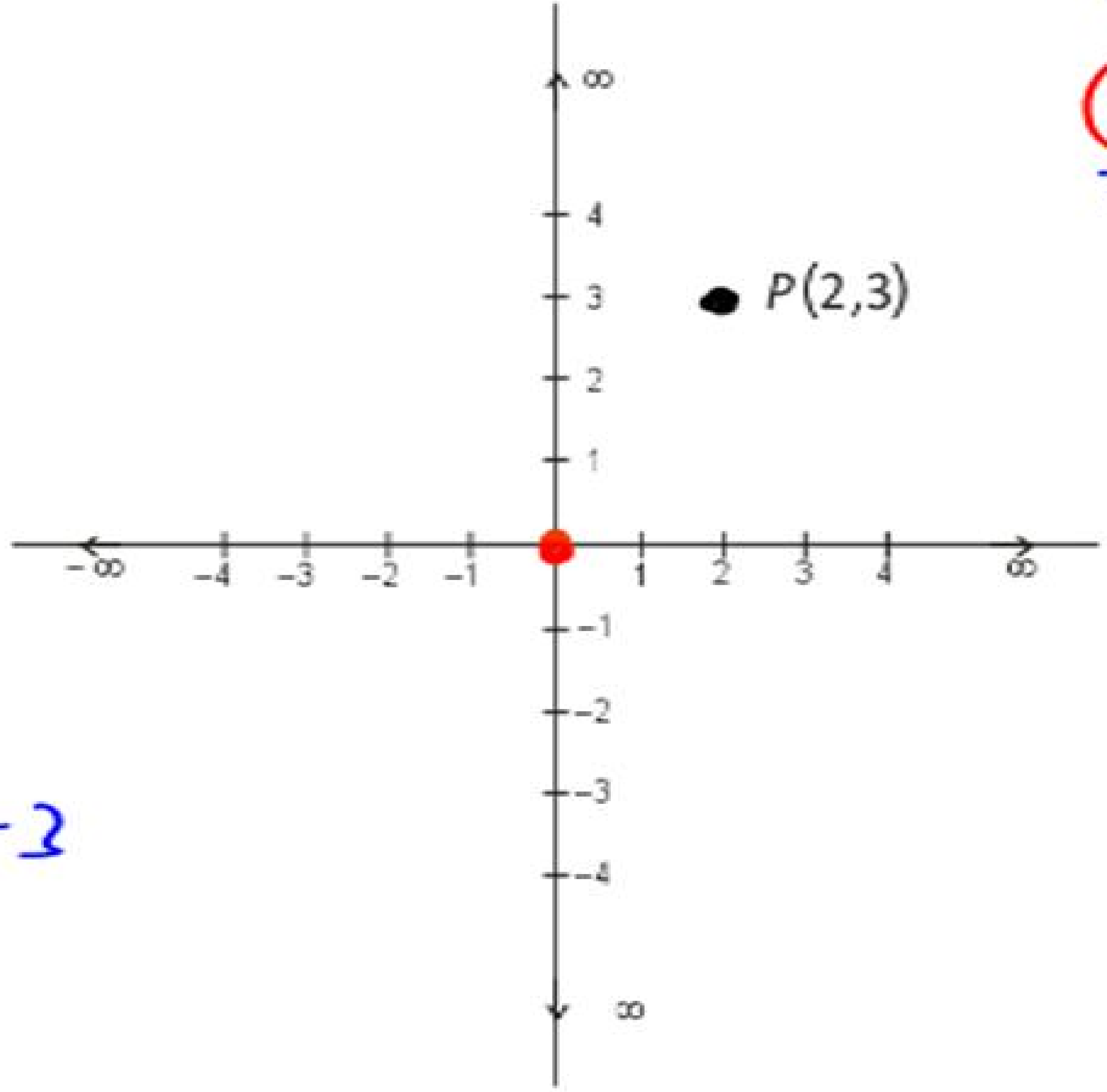
REFLECTION OF A POINT ABOUT ORIGIN

Method

$$\frac{2+a}{2} = 0$$

$$\frac{3+b}{2} = 0$$

$a = -2 \quad b = -3$

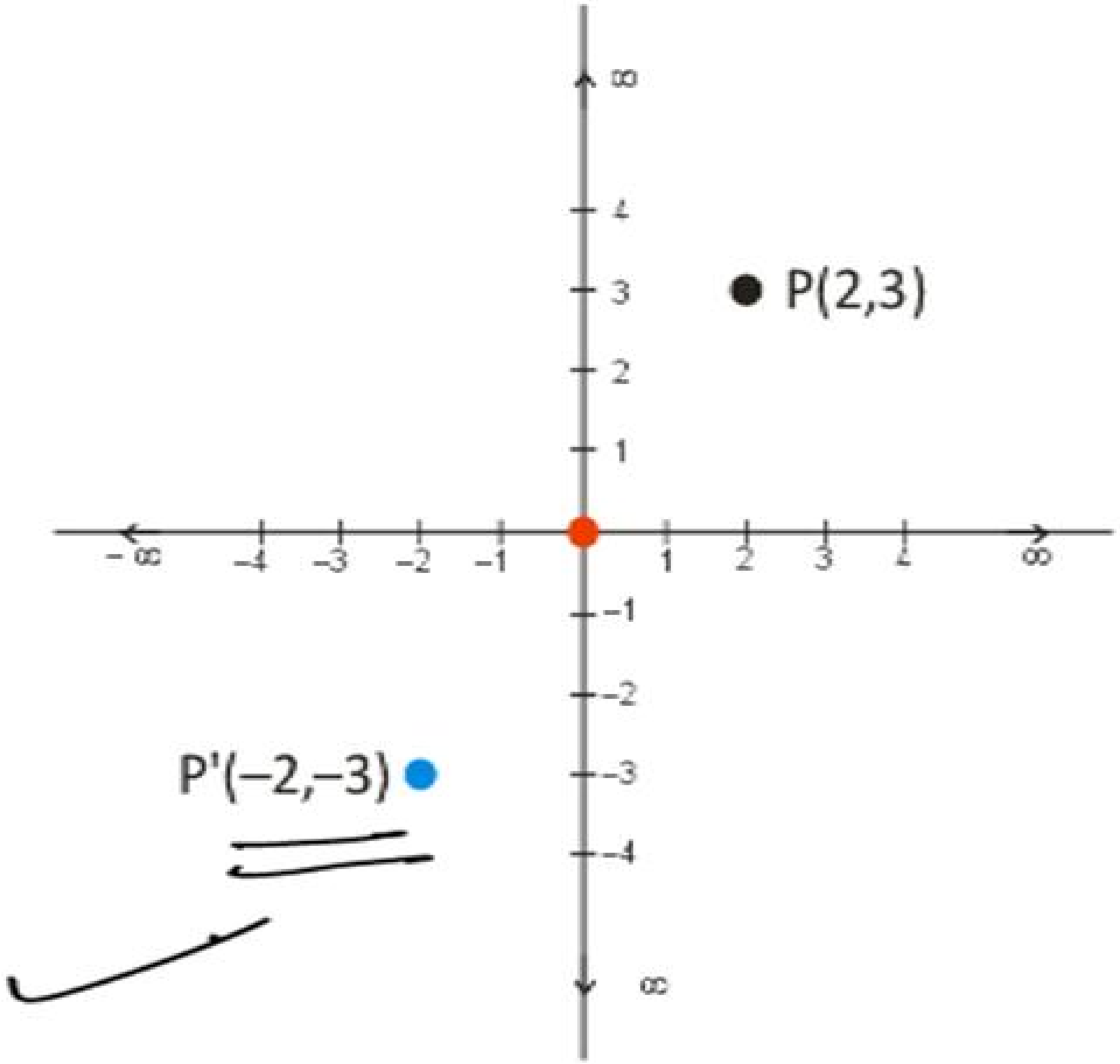


P   P'

$(2,3) \xrightarrow[\text{Origin}]{(0,0)} (a,b)$

$P'(-2,-3)$

REFLECTION OF A POINT P(2,3) ABOUT ORIGIN



$$P(x, y) \xrightarrow[(0,0)]{Origin} P'(-x, -y)$$

Eg.1 $P(5, -8) \xrightarrow{Origin}$ $(-5, 8)$ ✓

Eg.2 $P(-5, -7) \xrightarrow{Origin}$ $(5, 7)$ ✓

REFLECTION OF A POINT ABOUT ANOTHER POINT

$P \quad P'$

$(2,3) \xrightarrow{(-1,2)} (a,b)$

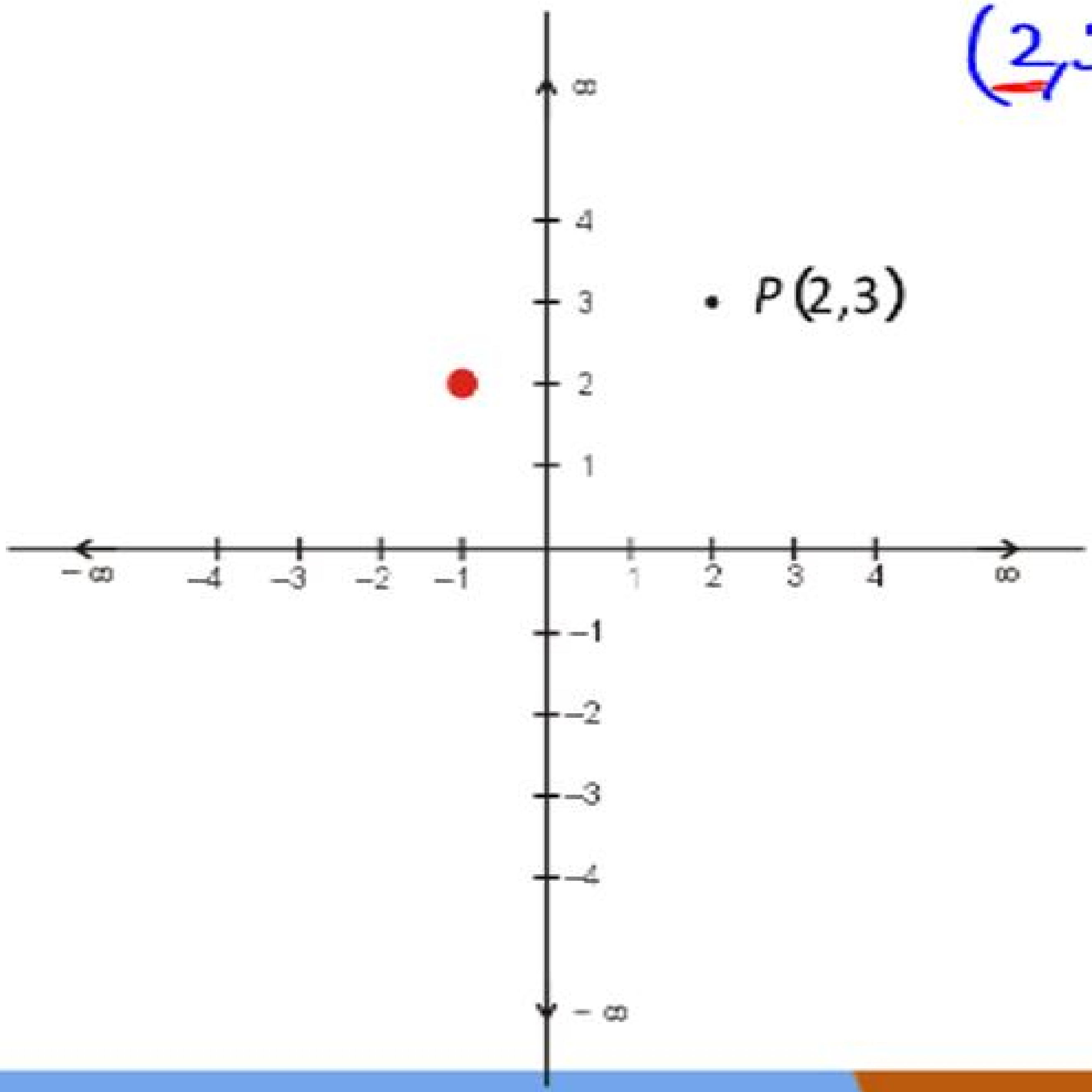
Method

$$\frac{2+a}{2} = -1$$

$$\boxed{a = -4}$$

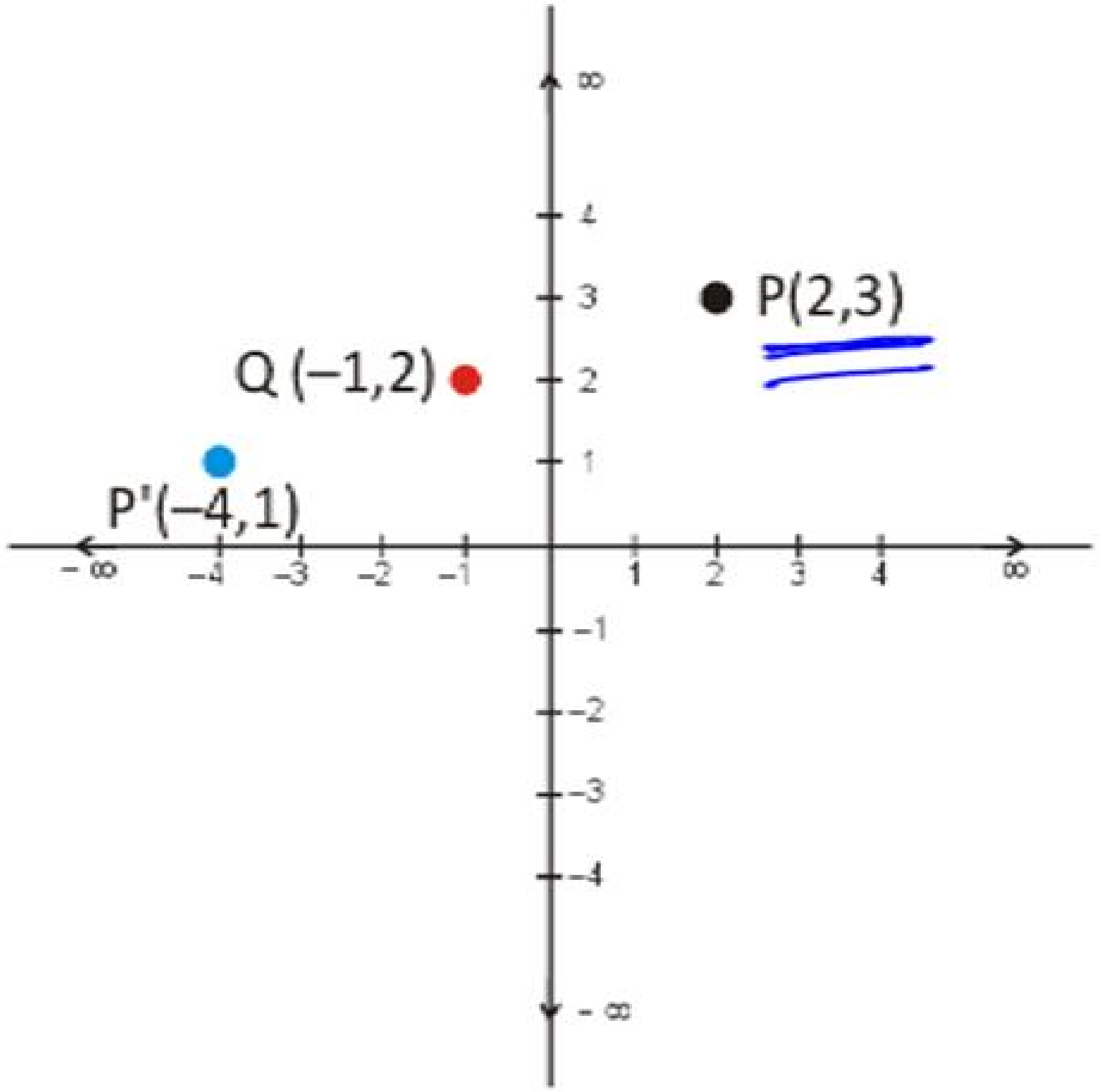
$$\frac{3+b}{2} = 2$$

$$\boxed{b = 1}$$



$(-4, 1)$

REFLECTION OF A POINT P(2,3) ABOUT POINT Q (–1,2)

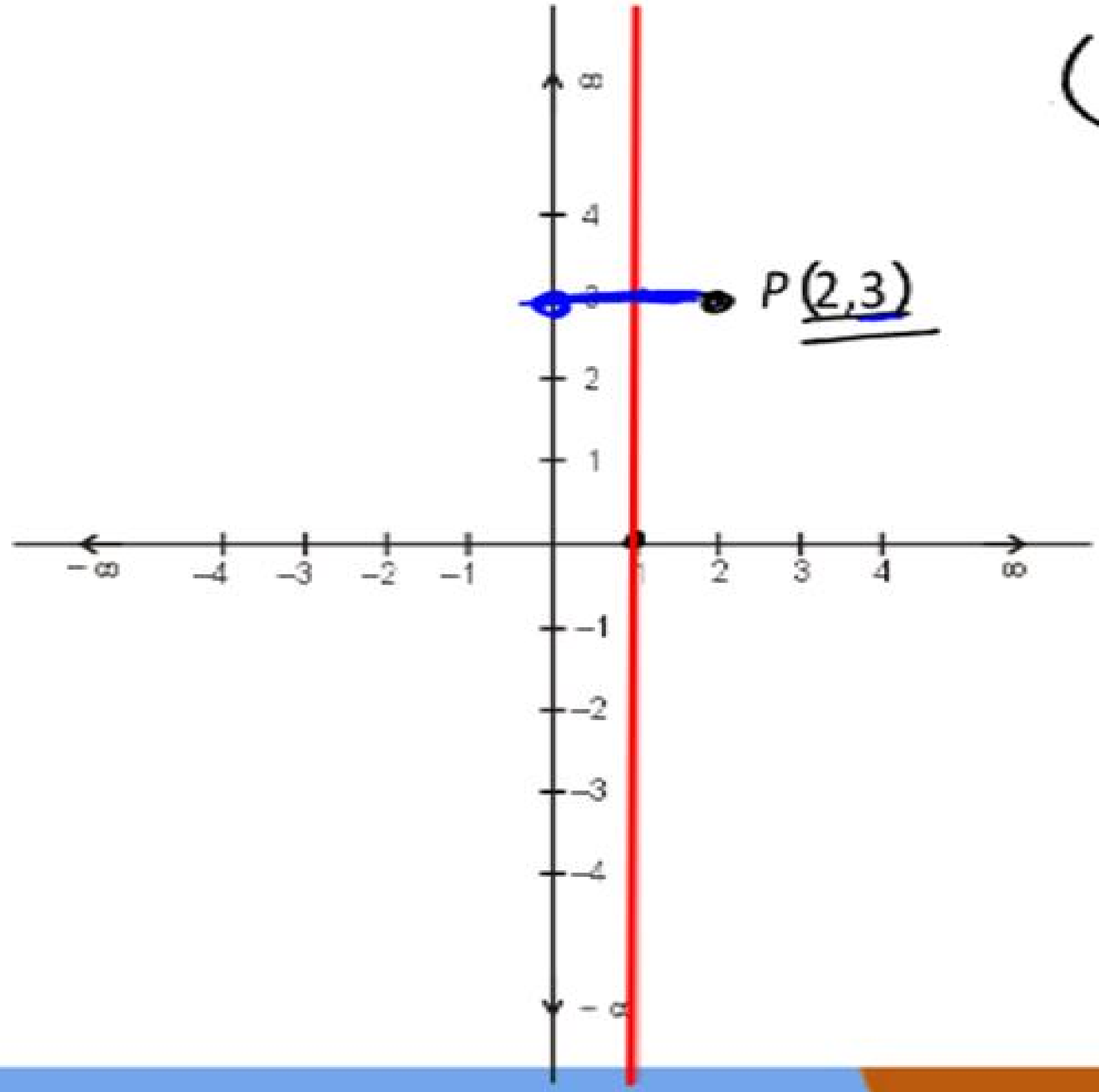


$$P(x,y) \quad \underrightarrow{Q(a,b)} \quad P'(2a-x, 2b-y)$$

Eg.1 $P\ 5,-8$ $\quad$ $\underrightarrow{Q\ 3,-4}$ $\quad$ $(1,0)$ ✓

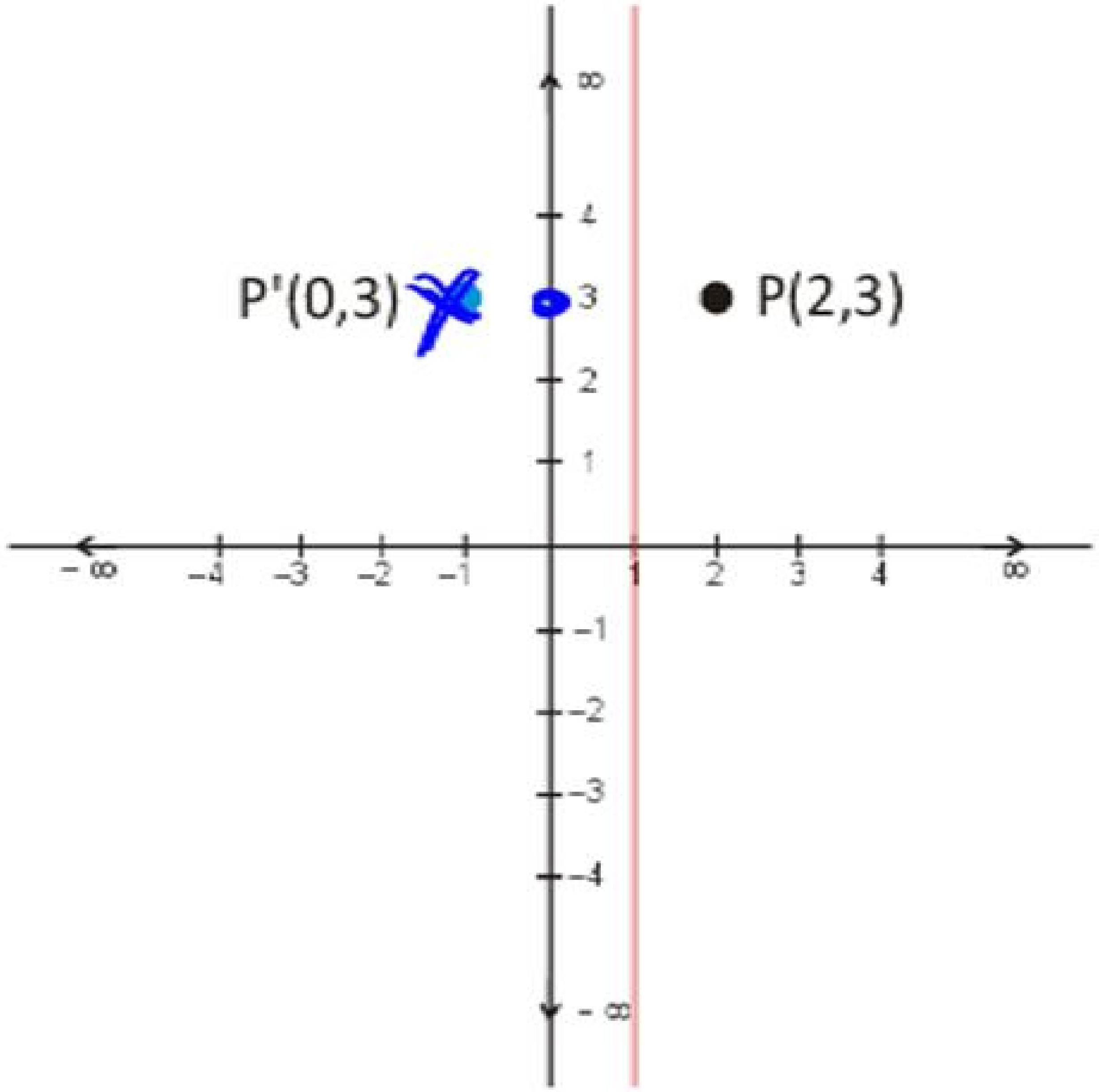
Eg.2 $P\ 5,-8$ $\quad$ $\underrightarrow{Q\ -4,1}$ $\quad$ $(-13,10)$ ✓

REFLECTION OF A POINT ABOUT X = 1



$(2,3) \xrightarrow[(1,3)]{x=1} (0,3)$

REFLECTION OF A POINT P(2,3) ABOUT X = 1

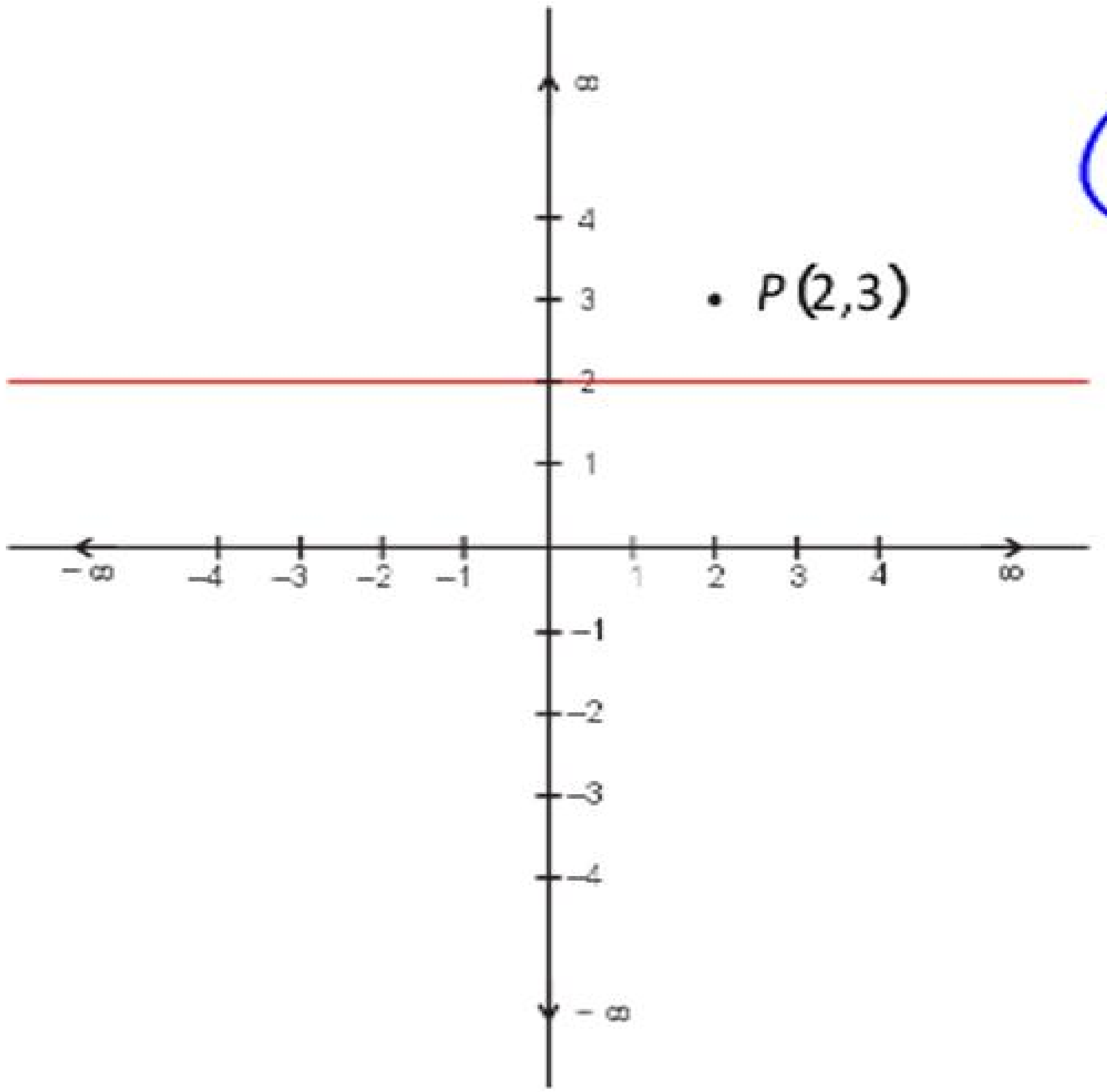


$$P(x,y) \quad (\underline{X=K}) \quad P'(2k-x,y)$$

Eg.1 $P(5,-8)$ $\underline{X=3}$ $(1,-8)$

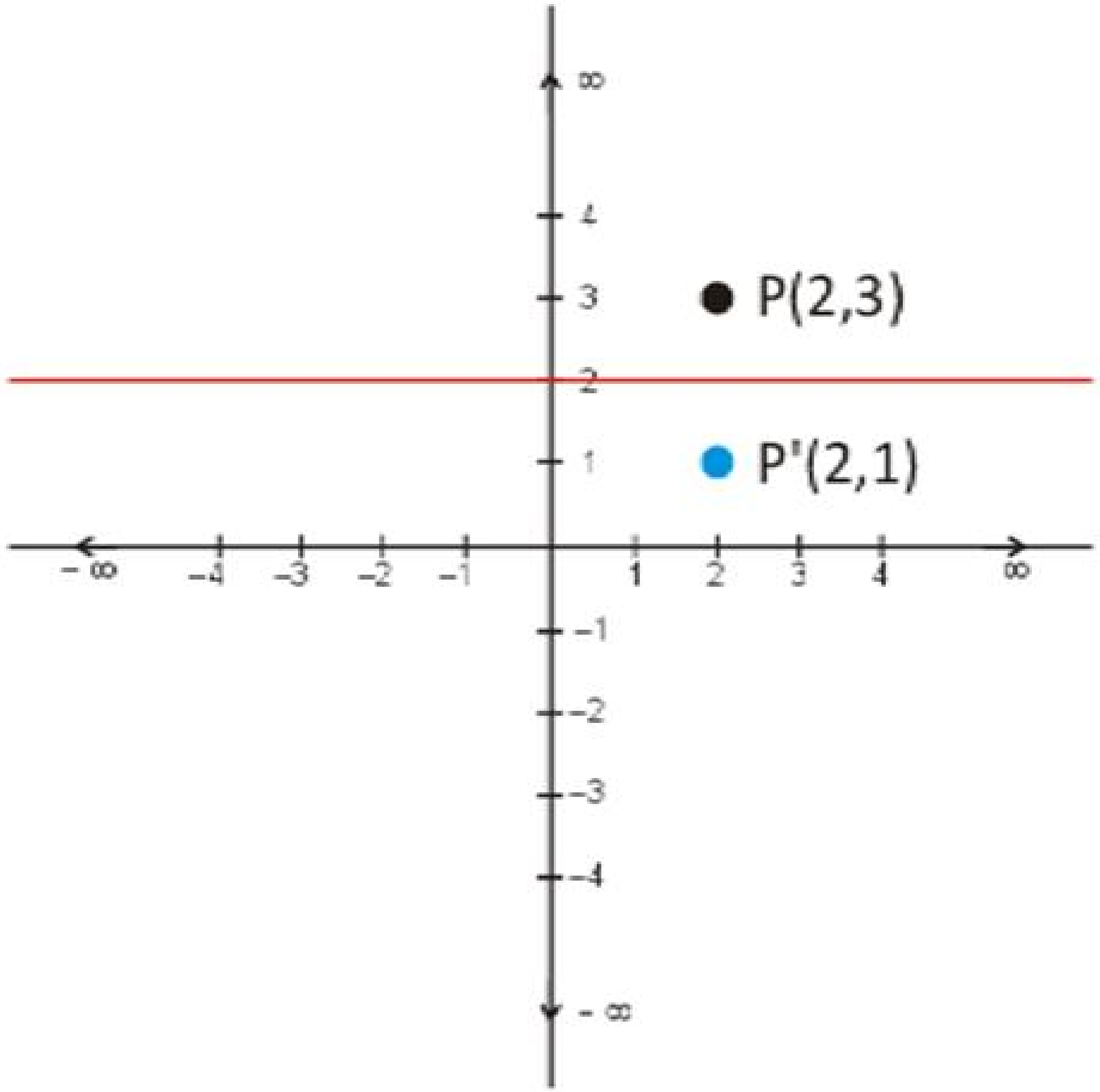
Eg.2 $P(5,-8)$ $\underline{X=-2}$ $(-9,-8)$

REFLECTION OF A POINT ABOUT Y = 2



$(2,3) \xrightarrow{y=2} (2,1)$

REFLECTION OF A POINT P(2,3) ABOUT Y = 2

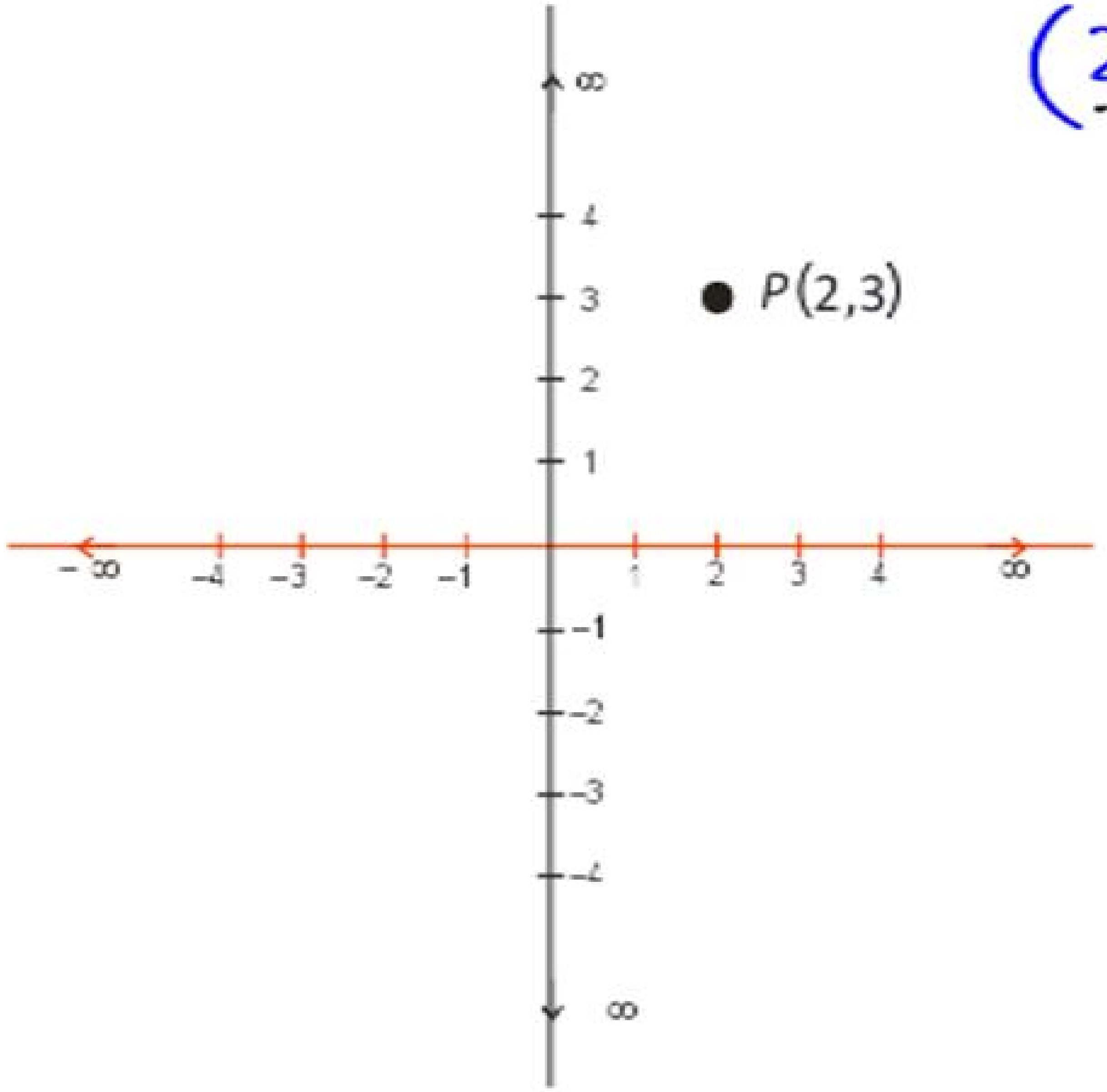


$$P(x,y) \quad (\underline{Y=K}) \quad P'(x,2k-y)$$

Eg.1 $P(5,-8)$ $\quad \underline{Y=3}$ $\quad (5, 14)$

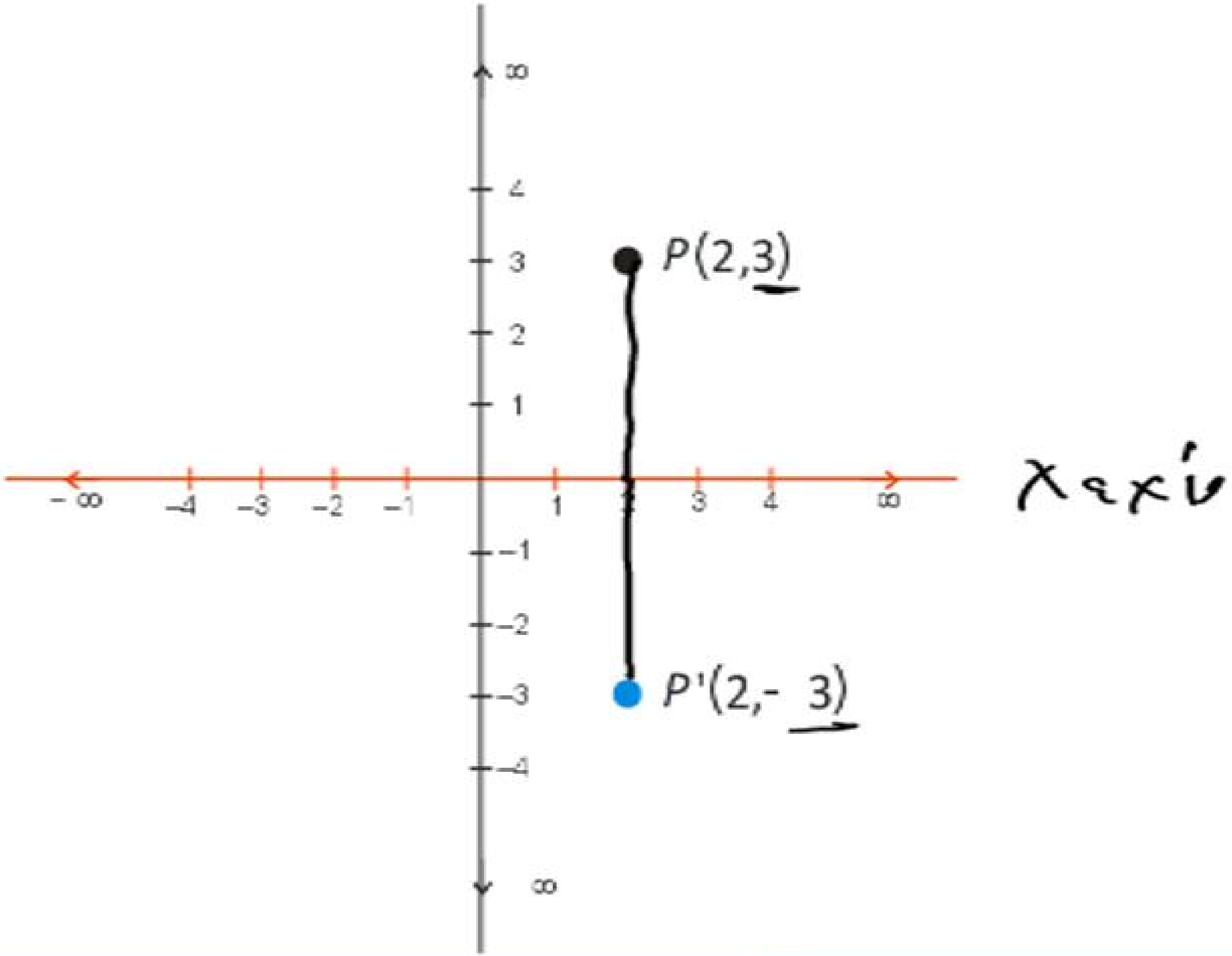
Eg.2 $P(5,-8)$ $\quad \underline{Y=-2}$ $\quad (5, 4)$

REFLECTION OF A POINT ABOUT X – AXIS



$(2,3) \xrightarrow[\text{x axis}]{(2,0)} (2,-3)$

REFLECTION OF A POINT P(2,3) ABOUT X – AXIS

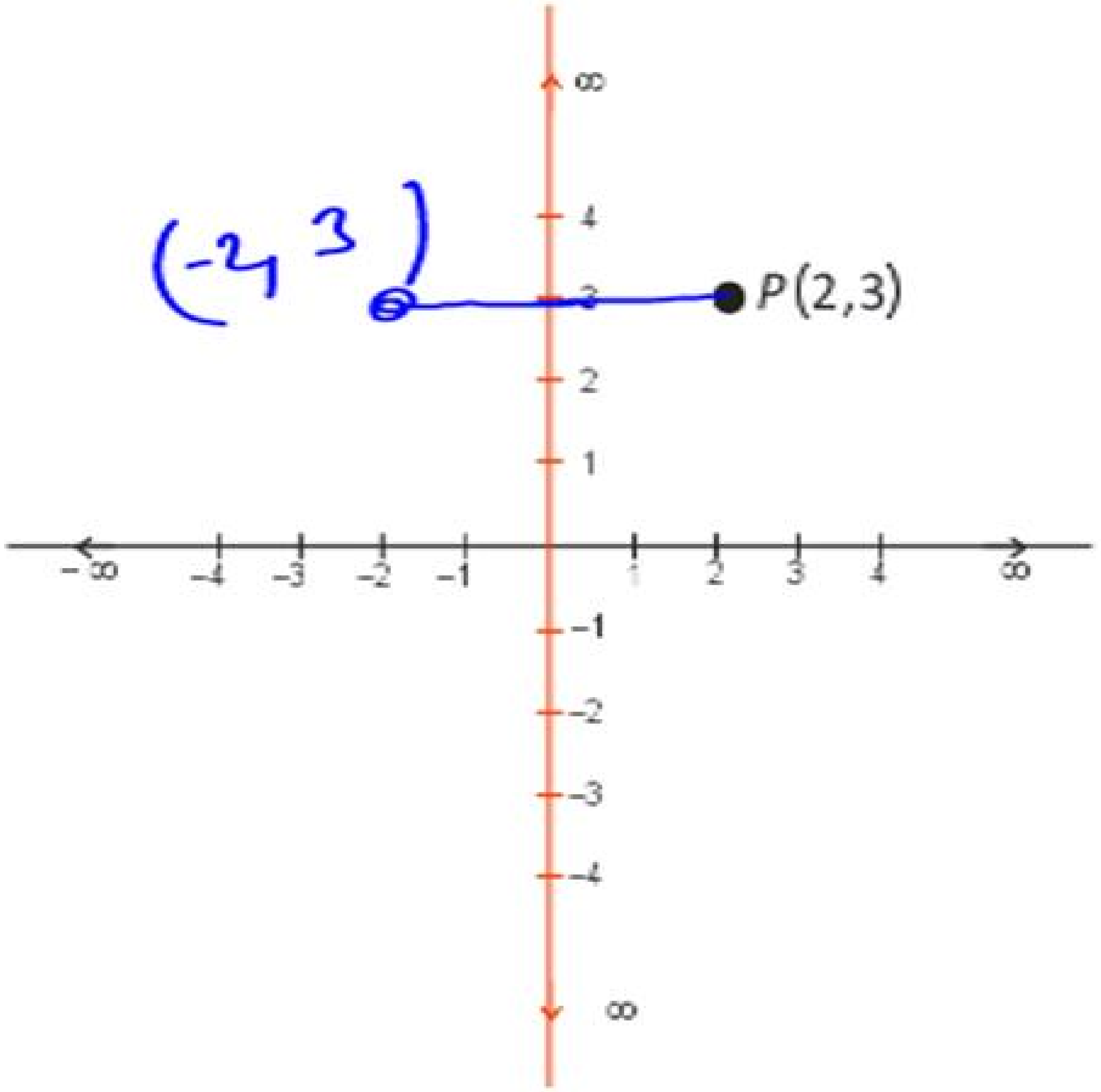


$$P(x,y) \xrightarrow{X-Axis} P'(x,-y)$$

Eg.1 $P\ -5,7$ $\xrightarrow{X-Axis}$ $(-5,-7)$

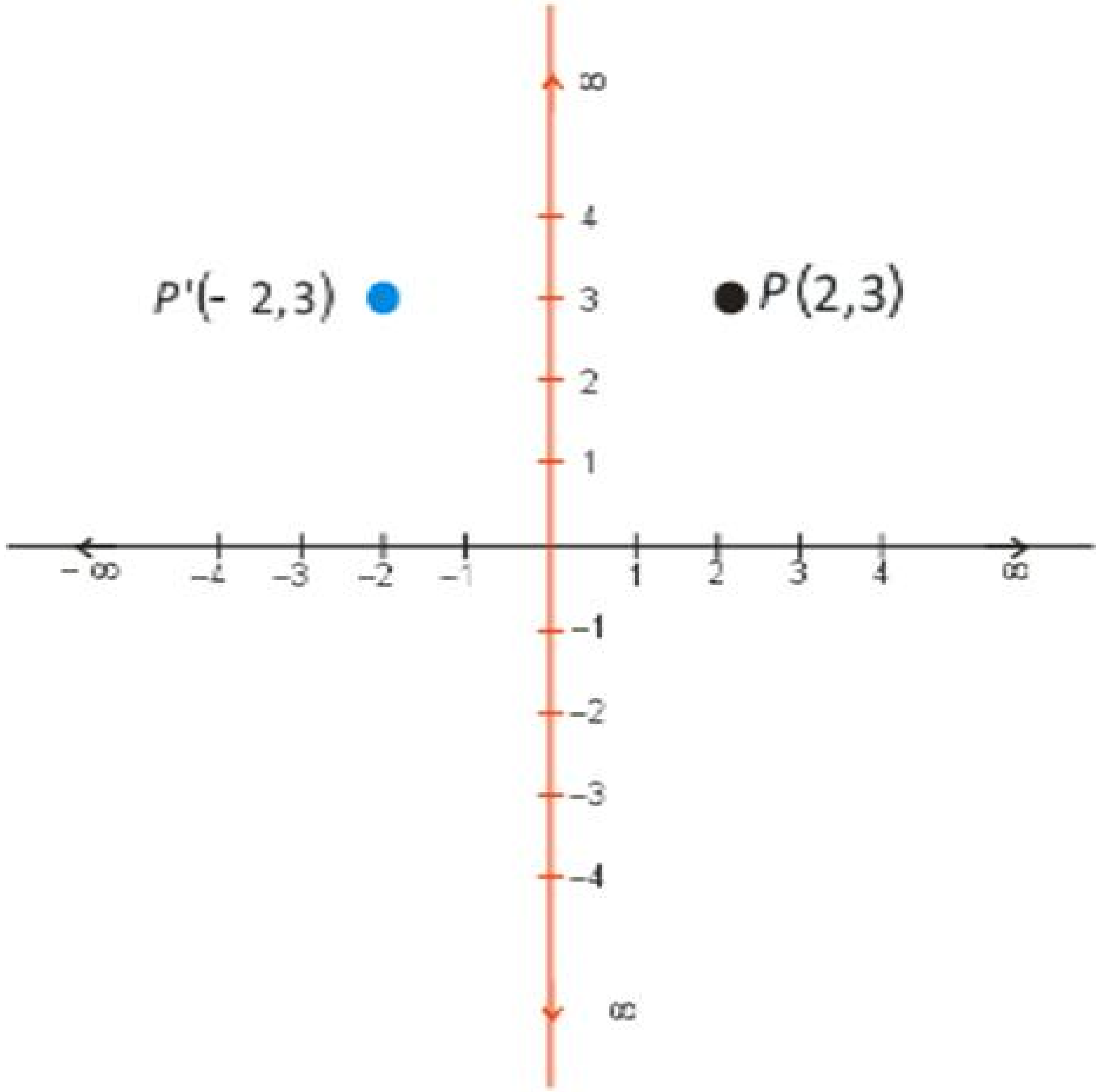
Eg.2 $P\ -5,-7$ $\xrightarrow{X-Axis}$ $(-5,7)$

REFLECTION OF A POINT ABOUT Y – AXIS



$(2,3) \xrightarrow{\text{Y axis}} -2,3$

REFLECTION OF A POINT ABOUT Y – AXIS

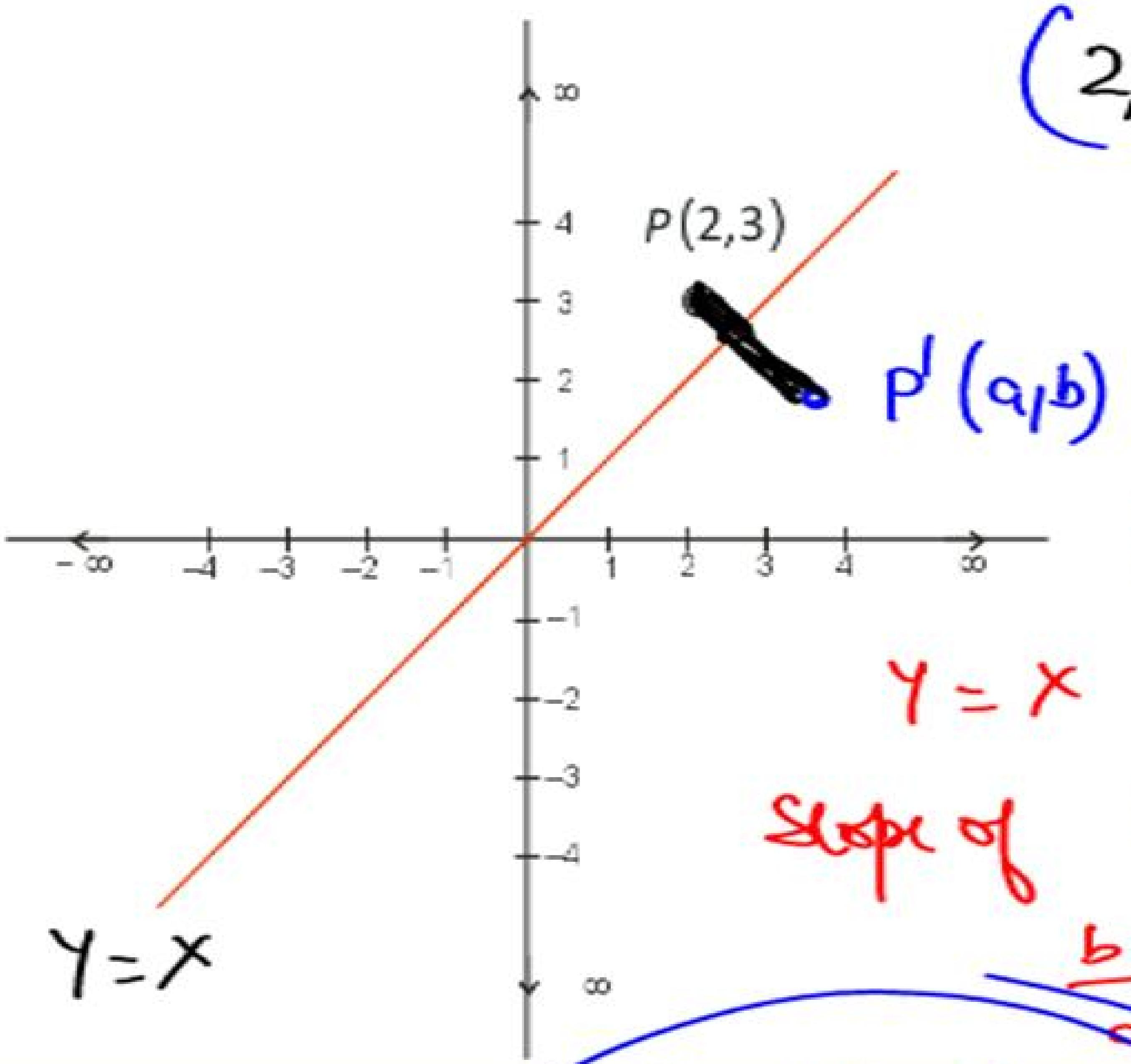


$$P\ x, y \xrightarrow{(Y-Axis)} P'(-x, y)$$

Eg.1 $P(5,-8) \xrightarrow{Y-Axis} (-5,-8)$

Eg.2 $P(5, 8) \xrightarrow{Y-Axis} (-5, 8)$

REFLECTION OF A POINT ABOUT Y = X



Method

(i) Mid Point

$$\left(\frac{2+a}{2}, \frac{3+b}{2}\right)$$

$$\frac{2+a}{\cancel{2}} = \frac{3+b}{\cancel{2}}$$

$$\boxed{a-b=1}$$

$$(2,3) \xrightarrow[Y=X]{} (a,b)$$

(ii) Slope

$Y = X \qquad X - Y = 0 \qquad m_1 = 1$

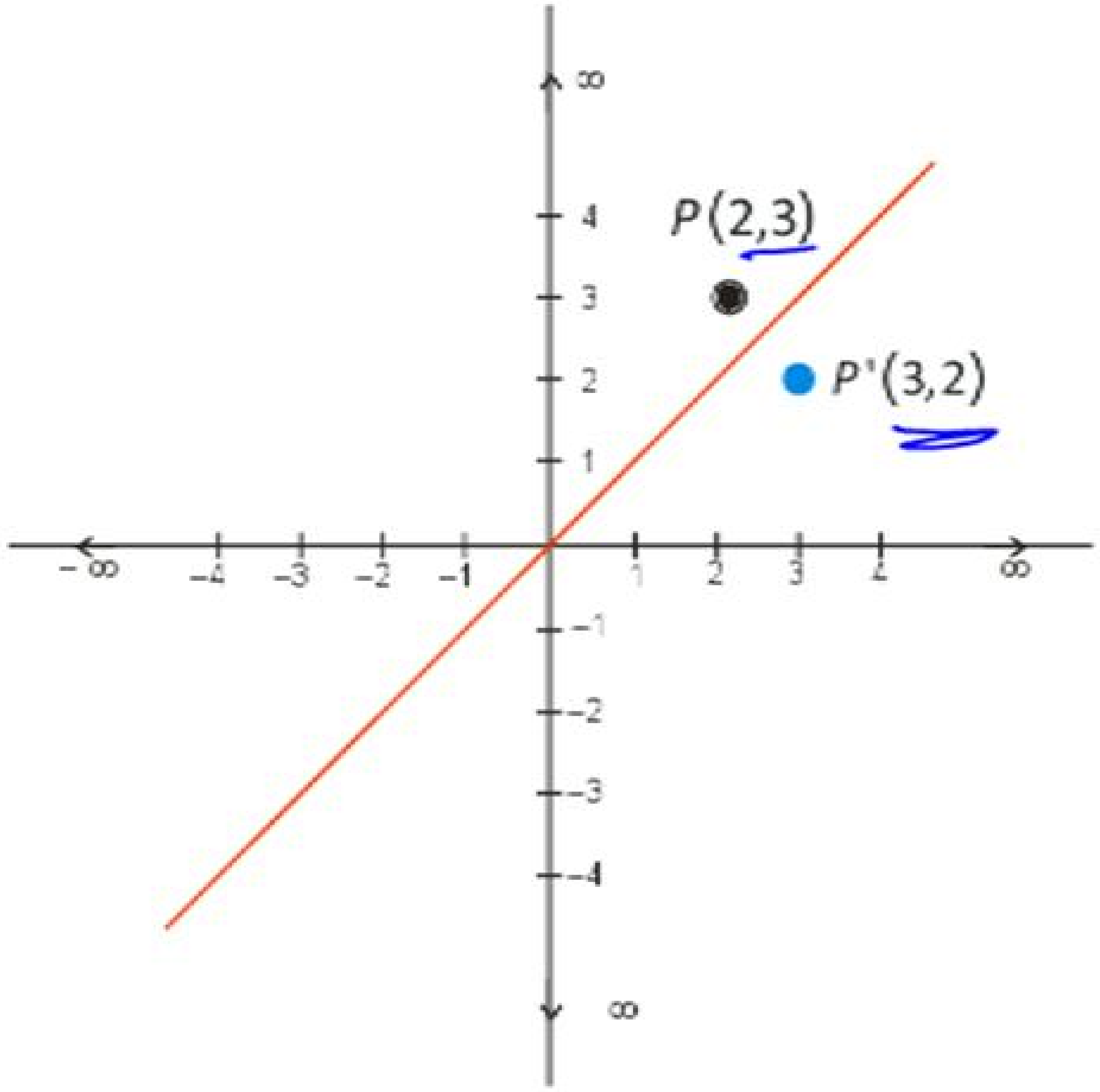
Slope of $PP' = -1$

$$\frac{b-3}{a-2} = -1 \qquad b-3 = -a+2$$

$$a+b=5$$

$$a, b = 3, 2$$

REFLECTION OF A POINT P (2,3) ABOUT Y = X

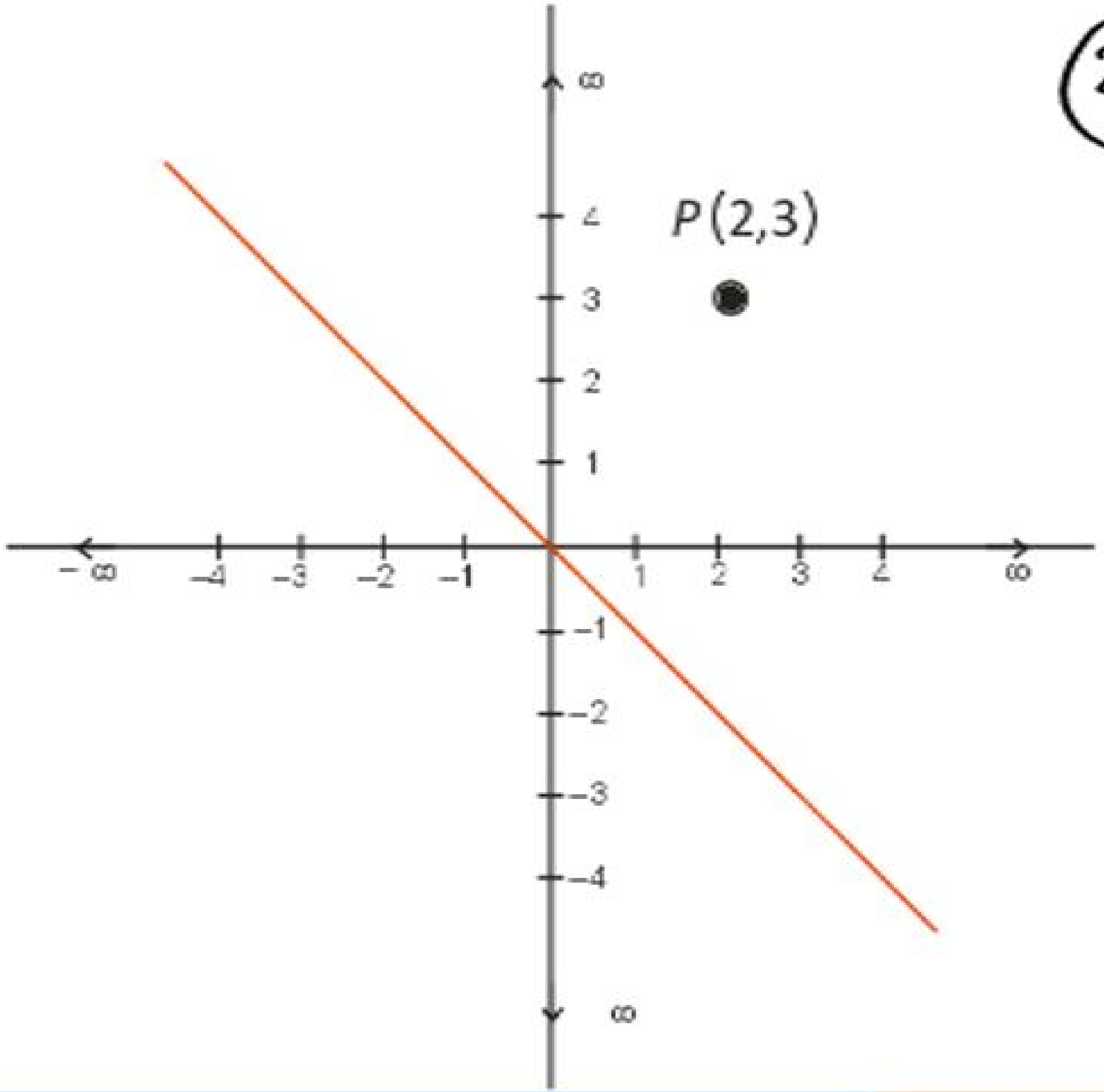


$$P(x,y) \xrightarrow{(Y=X)} P'(y,x)$$

Eg.1 $P\ 5,-8$ $\quad \xrightarrow{Y=X}$ $(-8,5)$

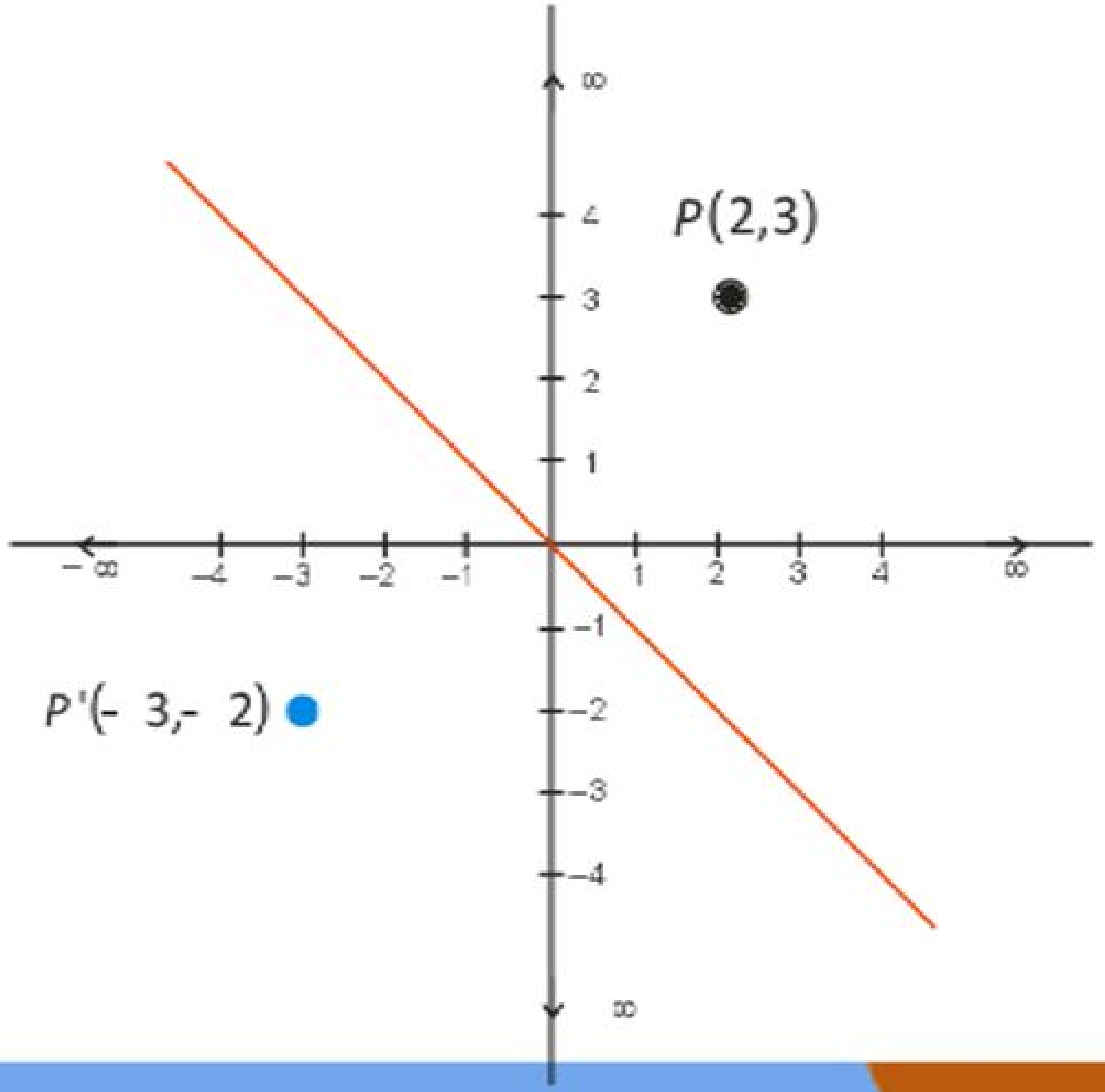
Eg.2 $P\ -5,7$ $\quad \xrightarrow{Y=X}$ $(7,-5)$

REFLECTION OF A POINT ABOUT Y = –X



$$(2,3) \xrightarrow[Y = -X]{} (-3,-2)$$

REFLECTION OF A POINT P (2,3) ABOUT Y = –X



$$P\ x,y \xrightarrow{\underline{Y=-X}} P'(-y,-x)$$

Eg.1 $P\ 5,-8 \xrightarrow{\underline{Y=-X}}$ $(+8\ -5) \longrightarrow 8,-5$

Eg.2 $P\ 5,\ 8 \xrightarrow{\underline{Y=-X}}$ $(-8\ -5) \longrightarrow -8,-5$

Eg.3 $P\ -5,-7 \xrightarrow{\underline{Y=-X}}$ $(+7\ +5) \longrightarrow 7,5$

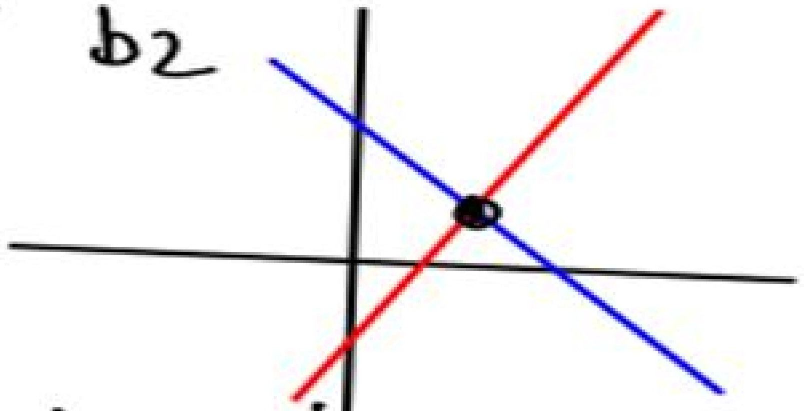
gradeup

$$\left\{\begin{aligned} a_1x + b_1y + c_1 &= 0 \\ a_2x + b_2y + c_2 &= 0 \end{aligned}\right\}$$

CONSISTENT (have atleast 1 sol^n)

Unique sol^n (1 sol^n)

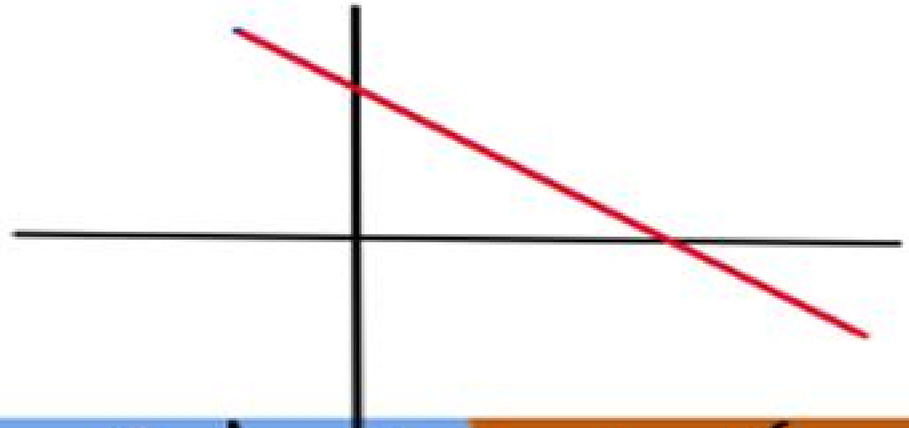
If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$



Intersecting lines

Infinite sol^n

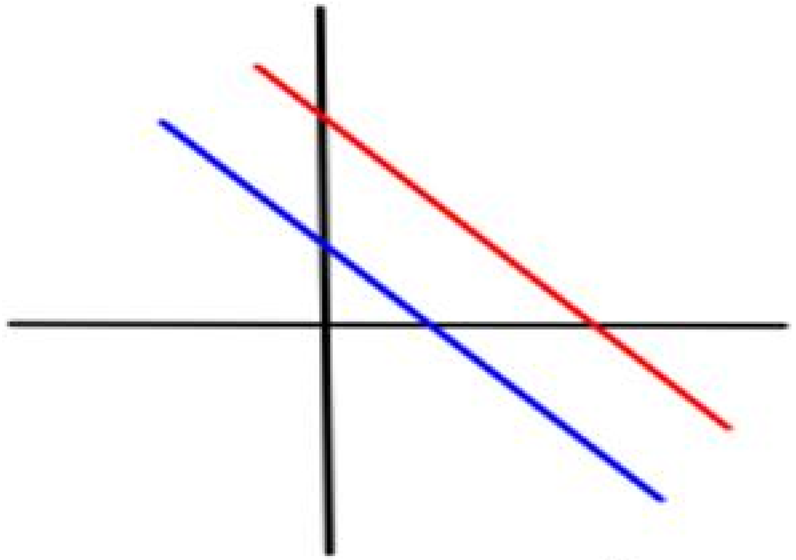
If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



Coincident lines

INCONSISTENT (No sol^n)

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



Parallel lines

Step 1

$$\frac{a_1}{a_2} \qquad \frac{b_1}{b_2}$$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Unique solⁿ)

OR

$\frac{a_1}{a_2} = \frac{b_1}{b_2}$ Calculate $\frac{c_1}{c_2}$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ & If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Infinite solⁿ NO solⁿ

gradeup

Eg1 For what value of K the following system of eq^n have infinite sol^{n}s

$$3x + 4y = 12$$
$$6x + Ky = 24$$

$$\frac{3}{6} = \frac{4}{K} = \frac{12}{24}$$

$K = 8$

Eg2 For what value of K the following system of eq^n have no sol^n

$$5x + 8y = 20$$
$$10x + 16y = K$$

$$\frac{5}{10} = \frac{8}{16} \neq \frac{20}{K}$$

$K \neq 40$

All Real no except 40

gradeup

Eq 3

For what value of K the following system of eqn is CONSISTENT

$$5x + 8y = 100$$

$$10x + ky = 200$$

Soln

Unique

$$\frac{5}{10} \neq \frac{8}{k}$$

$$k \neq 16$$

Infinite

$$\frac{5}{10} = \frac{8}{k} = \frac{100}{200}$$

$$k = 16$$

For all Real values of K