Mensuration 3D

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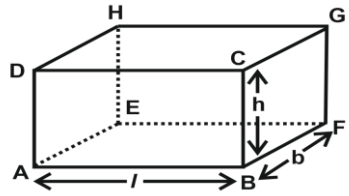
3-D Mensuration

1. Cuboid:

Definition: A **cuboid** is a three-dimensional shape with a length, width, and a height. The **cuboid** shape has six sides called faces. Each face of a **cuboid** is a rectangle, and all of a **cuboid's** corners (called vertices) are 90-degree angles. Ultimately, a **cuboid** has the shape of a rectangular box.

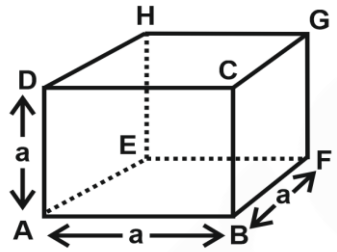


If $l, b, and\ h$ represent Length, Breadth, and Height of the cuboid respectively.

- Total surface area of cuboid = $2(lb + bh + lh)$
- Length of diagonal of cuboid= $\sqrt{l^2 + b^2 + h^2}$
- Volume of cuboid = $l \times b \times h$
- If area of faces of cuboid are x, y and z then volume of cuboid = $\sqrt{xyz}$

2. Cube:

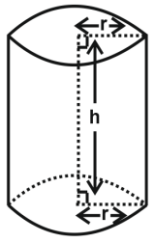
Definition: A **cube** is a three-dimensional solid object closed by six **square** faces, facets or sides, with three of them meeting at each vertex. A cube has 6 faces, 12 edges, and 8 vertices. All the sides of a cube are equal in length.



- Volume of cube = $(Side)^3 = a^3$
- Total surface area of cube = $6 \times (Side)^2 = 6a^2$
- Length of Leading Diagonal of Cube = $\sqrt{3} \times Side = a\sqrt{3}$

3. Cylinder:

Definition: A cylinder is a closed solid that is formed by connecting two parallel (usually circular) bases by a curved surface. The two parallel bases are circular in shape. The radius of the circular bases is called the radius of cylinder and the straight distance between these two bases is called the height of the Cylinder.

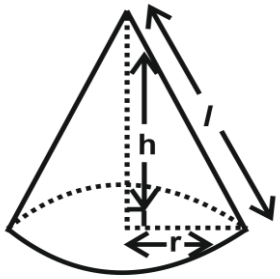


If, r *and* h represent radius of the base of the cylinder and Height of the cylinder respectively.

- Curved surface area of a cylinder = $2\pi rh$
- Total surface area of a cylinder = $2\pi r(r + h)$
- Volume of a cylinder = $\pi r^2 h$

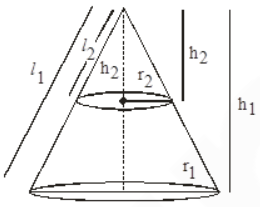
4. Cone:

Definition: A cone is a solid that has a circular base connected to a point vertex. In a cone the flat base smoothly tapers to a point called the apex or vertex as shown in the figure.



If, r, h *and* l represent radius of the base of the cone, Height of the cone, and Slant height of the cuboid respectively.

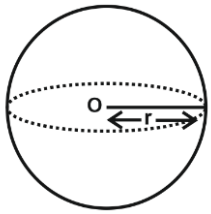
- Slant height of a cone = $l = \sqrt{h^2 + r^2}$
- Curved surface area of a cone $(C.S.A.) = \pi \times r \times l$
- Total surface area of a cone $(T.S.A.) = \pi \times r \times (r + l)$
- Volume of right circular cone $= \frac{1}{3} \times \pi r^2 h$
- Important relation between radius, height and slant height of similar cone.



$$\frac{r_1}{r_2} = \frac{h_1}{h_2} = \frac{l_1}{l_2}$$

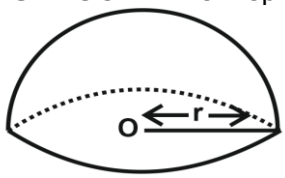
5. Sphere:

Definition: A **sphere** is a **geometrical** figure that is perfectly round, 3-dimensional and circular - like a ball. In Geometry, a **sphere** is **defined** as the set of all points equidistant from a single point in space.



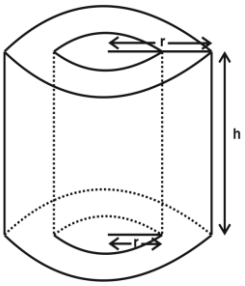
- Surface area of a sphere = $4\pi r^2 = \pi d^2$
- Volume of a sphere = $\left(\frac{4}{3}\right)\pi r^3$

6. Hemisphere:
Definition: A hemisphere is a half-cut portion of a Sphere that is cut along one of its axis**.**



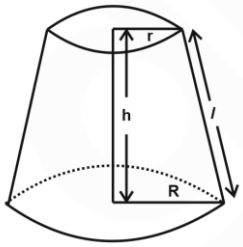
- Volume of a hemisphere = $\left(\frac{2}{3}\right)\pi r^3$
- Curved surface area of a hemisphere = $2\pi r^2$
- Total surface area of a hemisphere = $3\pi r^2$

7. Hollow Cylinder:
Definition: A hollow Cylinder is a Cylinder that is vacant from inside and has a thin layer of sheet to form the shape.



If R, r and h represent Outer radius of cylinder, Inner radius of cylinder and Height of hollow cylinder respectively.
- Volume of hollow cylinder = $\pi(R^2 - r^2)h$

8. Frustum of a cone:
Definition: Frustum of a cone is obtained by cutting a right circular cone by a plane parallel to the base of the cone.

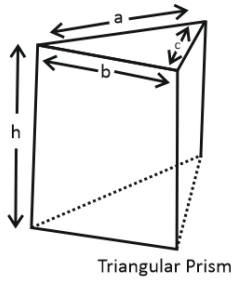


If, R, r, h *and* l represent radius of the base of the frustum, radius of the top of the frustum, height of the frustum, and slant height respectively.

- Slant height of the frustum = $l = \sqrt{h^2 + (R - r)^2}$
- Curved surface area of frustum = $\pi(R + r)l$
- Total surface area of frustum = $\pi(R + r)l + \pi(R^2 + r^2)$
- Volume of the frustum = $\left(\frac{1}{3}\right)\pi h(R^2 + r^2 + Rr)$

9. Prism:

Definition: Prism is a three-dimensional solid that is formed by two similar parallel (of any shape) bases connected by a surface.



Triangular Prism

- Volume of prism = Base area × height
- Lateral surface area of prism = perimeter of base × height
- Total surface area of prism = Lateral surface area + (2 × base area)

10. Pyramid:

Definition: A **pyramid** is a polyhedron that has a base (of any shape) whose points meet at a point called the apex.

- Total surface area of pyramid =

$$\textbf{base area} + (\textbf{number of sides} \times \frac{\mathbf{1}}{\mathbf{2}} \times \textbf{slant height} \times \textbf{base length})$$

- Volume of pyramid = $\left(\frac{1}{3}\right) \times \textbf{area of base} \times \textbf{height}$

Important facts:

- From a solid cylinder no. of maximum solid cone of same height and radius as cylinder are 3.
- From a solid sphere, no. of maximum solid cone having height and radius equal can be made are 4.
- From a solid hemisphere, no. of maximum solid cone having height and radius equal can be made are 2.