gradeup

Time and Work

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Work: Work is defined as something which has an effect or outcome; often the one desired or expected. **Efficiency**: Efficiency is rate of work done with respect to time.

$$Efficiency = \frac{Work}{Time}$$

Or $Time = \frac{Work}{Efficiency}$

Efficiency is inversely proportional to the Time taken when the amount of work done is constant.

$$Efficiency \propto \frac{1}{Time}$$

This above the relation forms the crux of Time and Work problems.

This can be used to compare efficiencies and Time taken across different groups.

If A does a work in n days

Then per day Efficiency of A = $\frac{1}{n}$

i.e. A does 1/n amount of Work in 1 day

The basic concept of Time and Work is similar to that across all Arithmetic topics, i.e. the concept of Proportionality.

In Time Speed and Distance, efficiency is replaced by Speed; i.e. Speed(Efficiency) is inversely proportional to Time when the Distance(Work) is constant. Pipes and Cisterns are just an application of Time and Work. Concept wise, it is one and the same. In the above proportionality, Efficiency is replaced by Rate of filling.

$$Rate\ of\ filling \propto \frac{1}{Time}$$

Example 1: The ratio of efficiencies of A and B is 2:5. If B does a work in 25 days, find the time taken by A to complete the same work.

As

$Efficiency \propto \frac{1}{Time}$

So $E_A/E_B = T_B/T_A$

$2/5 = 25/T_A$

T_A = 10 days

Example 2: If A does a work in x days and B does the same work in y days. Find the time required to complete the work if they work together.

One day work of A = 1/x

One day work of B = 1/y

Combined 1 day work of A and B = $\frac{1}{x}+\frac{1}{y} = \frac{x+y}{xy}$

This is also the per day efficiency of A and B combined

So Time required = $\frac{xy}{x+y}$

The concept of percentage of work done can be used to solve most of the related questions. A few basic points one needs to know to use the percentage concept are: When it is said that someone has done a work, it means he has done 100 % of the work. Hence, if A finishes a work in 4 days, it means- in 4 days he will do 100% of the work. Hence, in one day he finishes 25% (100/4) of the work. Similarly, in 3 days he finishes 75 % of the work.

Time and Work Shortcut Tricks:

Table of commonly used numbers:

Number of days taken to complete a work	**Percentage values of amount of work done**
1	100%
2	50%

3	33.33%
4	25%
5	20%
6	16.67%
7	14.28%
8	12.50%
9	11.11%

The complete work can also be considered as 1 unit or 100 unit or anything as far as it is constant. Then if A takes 4 days to finish a work, it means he can finish 1/4th of the work in 1 day.

Illustrations: Here are some basic questions to illustrate time and work shortcut tricks.

Example 3: Rahim can finish a work in 10 days and Ram can finish the same work in 40 days. If Ram and Rahim both work together then what is the total number of days taken?

Solution: The problem can be solved in three different approaches.

Approach 1: Using Fractions Ram can finish the work in 10 days i.e. in one day he will do 1/10th of the work. Rahim can finish the work in 40 days i.e. in one day he will do 1/40th of the work. So, in one day, both working together can finish = (1/10) + (1/40) = 5/40 = 1/8th of the work. So, to complete the work they will take 8 days.

Approach 2: Using Percentage (Shortcut- Recommended)

Rahim can finish 100 % of work in 10 days

i.e. in one day he finishes 10% of the work.

Ram can finish 100% of the work in 40 days

i.e. in one day he finishes 2.5 % of the work.

So, working together, in a single day they can finish 12.5% of the work.

So, to complete 100% of the work, they will take 100/12.5 = 8 days.

Approach 3: LCM approach

AS the work here is constant, thus it can be taken to be = LCM (Time taken by each entity) = LCM (10,40) = 40 units

We do this for the ease of calculation

As Efficiency = Work/Time

So the efficiency value will always be an integer in this case.

Efficiency of Rahim = 40/10 = 4 units per day

Efficiency of Ram = 40/40 = 1 unit per day

Total efficiency of both working together = 4+1 = 5 units/day

Time taken (When both are working together) = Total Work / Total efficiency = 40/5 = 8 days.

Example 4: Ravi can do a job in 10 days. Raman can do the same job in 20 days. They together start doing the job but after 4 days Raman leaves. How many more days will be required by Ravi to complete this job alone?

Solution: Ravi can finish a job in 10 days i.e in one day he can finish 10 % of the job.

Raman can finish the same job in 20 days i.e. in one day he can finish 5 % of the job.

So, working together, in a day they can do 10 + 5 = 15 % of the job.

In the 4 days, if they worked together, they would have finished 4 × 15% = 60% of the job.

So, job left = 100 – 60 = 40%.

This work has to be done by Ravi who does 10 % of the job in a day.

So, to finish the remaining 40%, he will take 40/ 10 = 4 more days.

Negative Work: Negative work increases the time in which a work is to be completed. This application can be extended to cases involving Pipes and cisterns. Suppose there are two pipes in a Cistern. Pipe A is used to fill the Cistern and Pipe B is used to empty the Cistern. Here we say that Pipe B and Pipe A are working against each other. When a leak is developed in the Cistern, the leak forms the component of negative work, which slows down the completion of the task (in this case, the filling of the Cistern)

Example 5: Two pipes A and B can fill a cistern in 20 and 30 minutes respectively, and a third pipe C can empty it in 40 minutes. How much time will it take to fill the cistern if all the three are opened simultaneously?

Solution: LCM of 20, 30, and 40 = 120.
Let us assume that the capacity of the cistern is 120 liters.
Therefore,
Rate of pipe A = 120/20 = 6 liters/min
Similarly, Rate of pipe B = 120/30 = 4 liters/min
And rate of pipe C = -120/40 = -3 liters/min (Negative work)
As pipes are opened simultaneously,
So total rate = 6+4-3 = 7 liters/min
Time required = Total work or Capacity / total rate or efficiency
$= 120 / 7 = 17\frac{1}{7}$ minutes

Man –Work –Hour Formula:

If M1 men having efficiency E1 each can do W1 work in D1 days working H1 hours per day and M2 men having efficiency E2 each can do W2 work in D2 days working H2 hours per day, the the following relation exists:

$$\frac{M1\ D1\ H1\ E1}{W1} = \frac{M2\ D2\ H2\ E2}{W2}$$

Example 6: 10 men can complete a job in 10 days. 12 women can complete the same job in 10 days. If 15 men and 6 women are employed to complete the job, then in how many days will they complete it?

Solution: Work done by 10 men = Work done by 12 women
10 men = 12 women
Men / Women = 6 / 5
These can be taken as their respective efficiencies.
Now,

$$\frac{M1\ D1\ H1\ E1}{W1} = \frac{M2\ D2\ H2\ E2}{W2}$$

$$\frac{10 \times 6 \times 10 \times h}{\mathrm{W}} = \frac{(15 \times 6 + 6 \times 5)\mathrm{D2} \times \mathrm{h}}{\mathrm{W}}$$

D2 = 5 days …. Answer

Important formulas:

1. Time = Work/Efficiency
2. Work = time x efficiency
3. Men x Days = Work
4. $\frac{M1D1H1E1}{W1} = \frac{M2D2H2E2}{W2}$