The Most Comprehensive
Preparation App For All Exams

MENSURATION-3D

Part-2

Agenda

Time 11:00 - 12:30

I Right Circular Cone → (60 - 65) min

II Leftover Part of cuboid → 20 min

IInd session

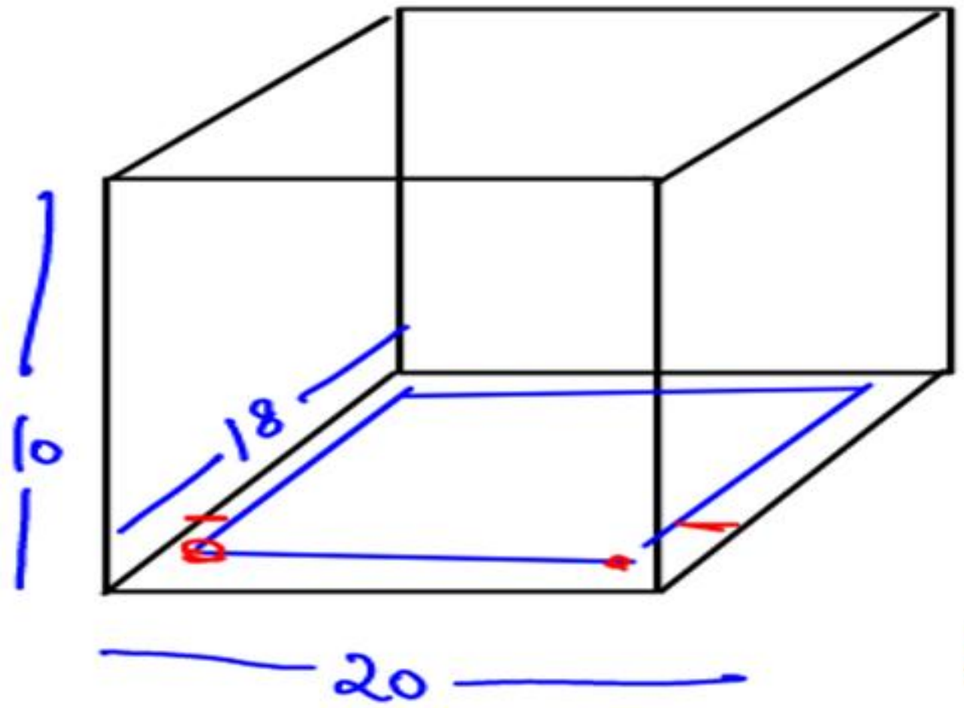
Sphere

1pm - 2:30pm

Eg. A wooden box measures 20 cm by 12 cm by 10 cm. Thickness of wood is 1 cm. Volume of wood to make the box (in cubic cm) is

(a) 960 (b) 519

(c) 2400 (d) 1120



$20 \times 12 \times 10$

$18 \times 10 \times 8$

Volume of wood

$20 \times 12 \times 10 - 18 \times 10 \times 8$

$10 (240 - 144)$

$960\ cm^2$

Ans. (a)

Eg. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is :

(a) 1 m (b) 1.1 m

(c) 1 dm (d) 90 cm

Let thickness of bottom x

$$(3.3 - 0.1)(2.6 - 0.1)(1.1 - x) = 8$$

$$(3.2)(2.5)(1.1 - x) = 8$$

$$1.1 - x = 1$$

$$x = 0.1 \text{ m} \quad 1 \text{ dm}$$

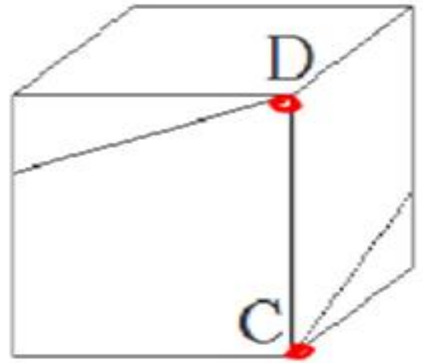
Ans. (c)

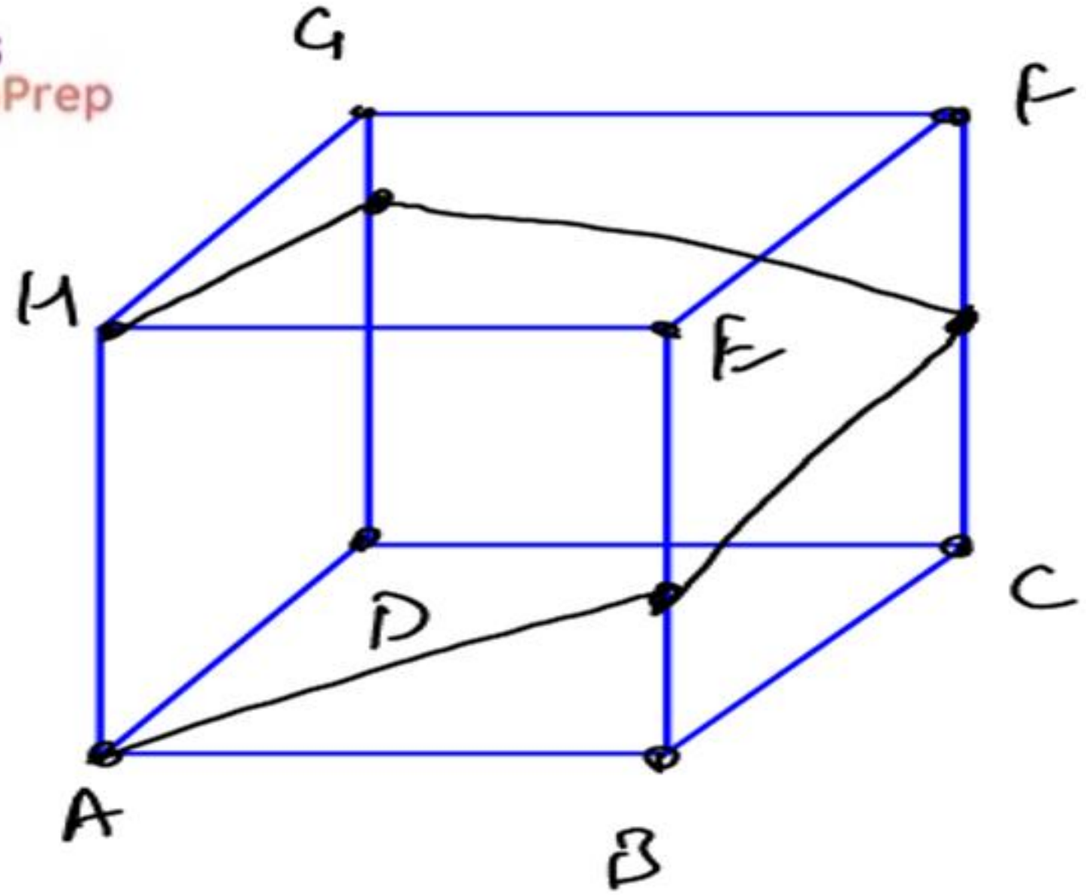
Eg. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale).The length of the string, in cm, is

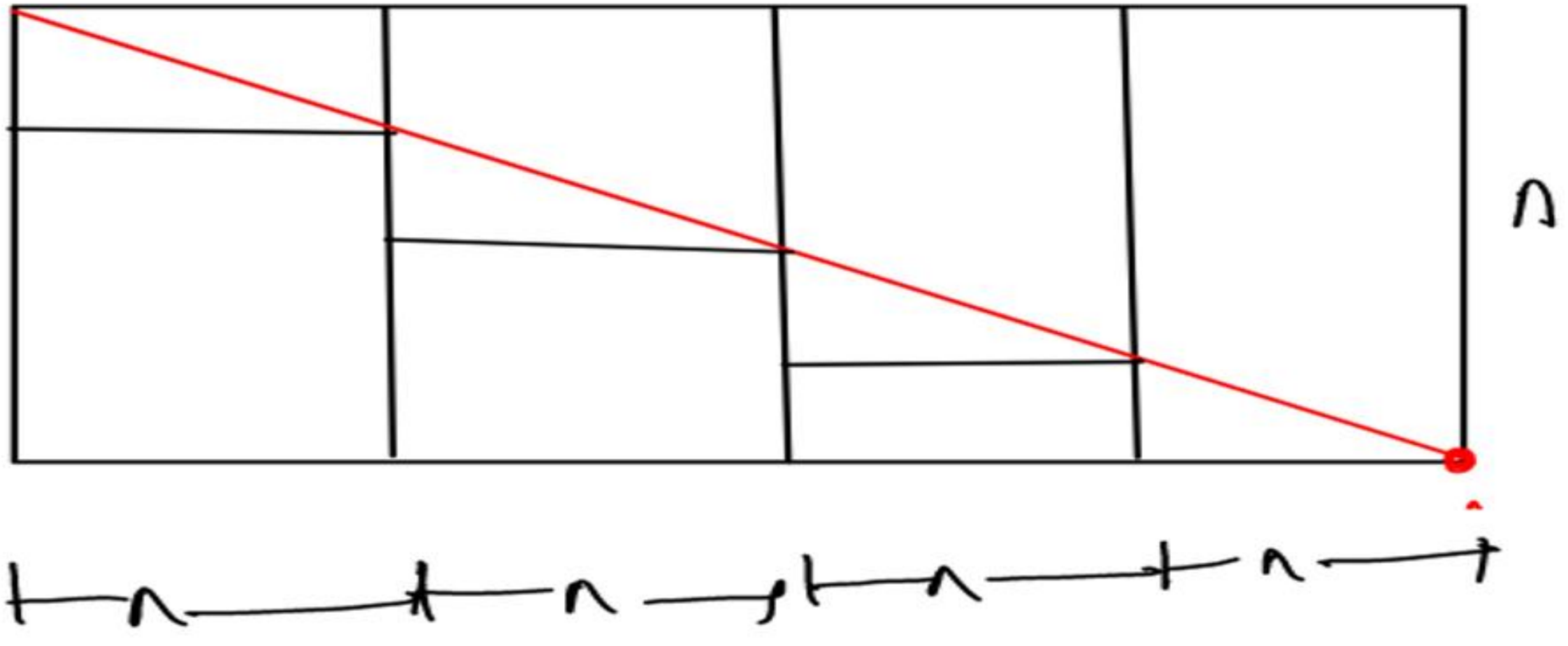
CAT

BYJU'S EXAM PREP

(a) $\sqrt{2}n$ (b) $\sqrt{17}n$ (c) n (d) $\sqrt{13}n$




G
F
H
E
C
D
A
B



$$\sqrt{(4n)^2+(n)^2} = \sqrt{17}\,n$$

Ans. (b)

Eg. A solid cube has side 8 cm. It is cut along diagonals of top face to get 4 equal parts. What is the total surface area (in cm^2) of each part.

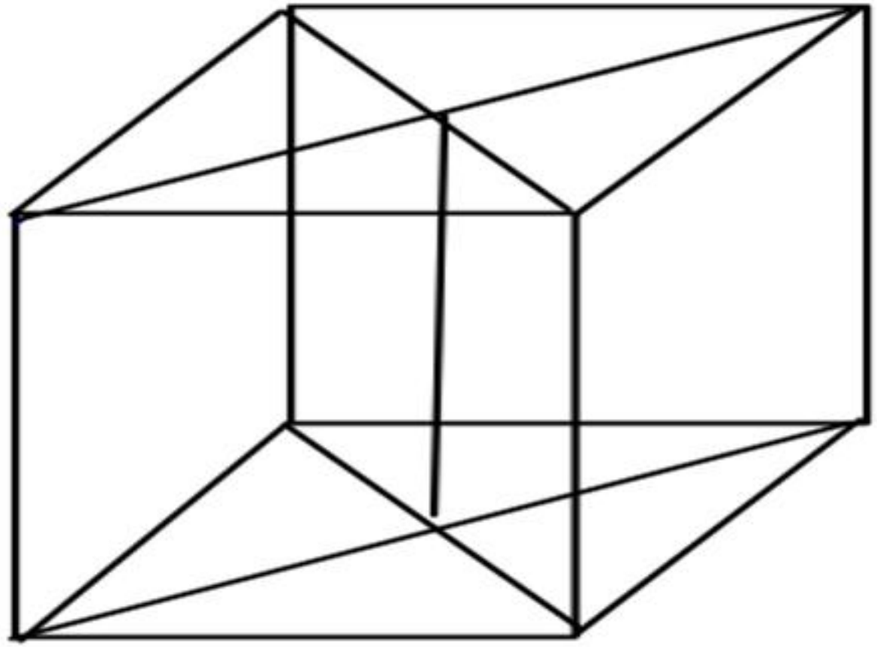
PYQ of SSC

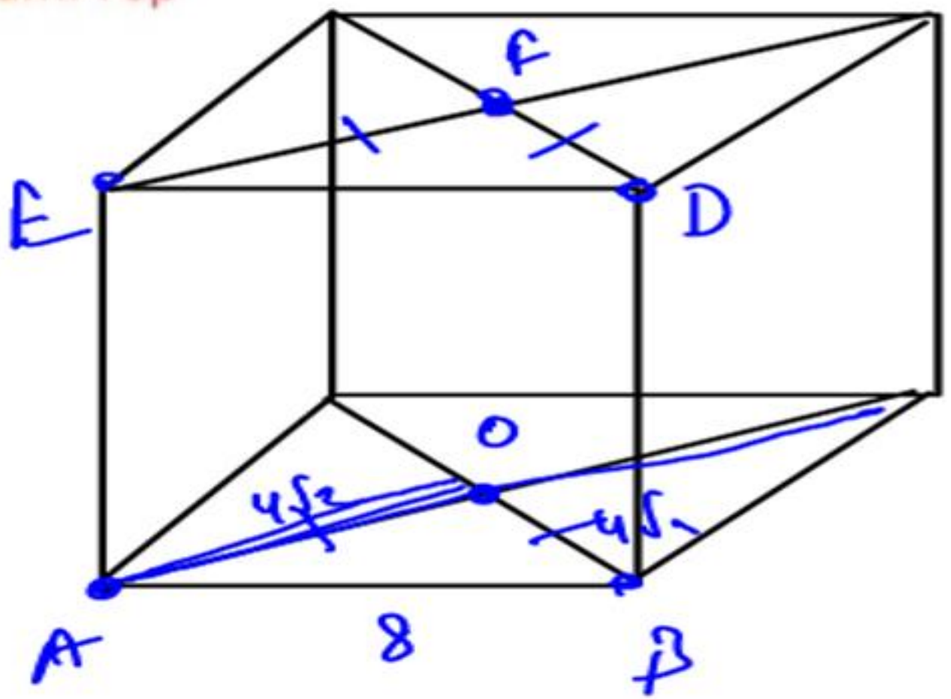
$(a)96+64\sqrt{2}$

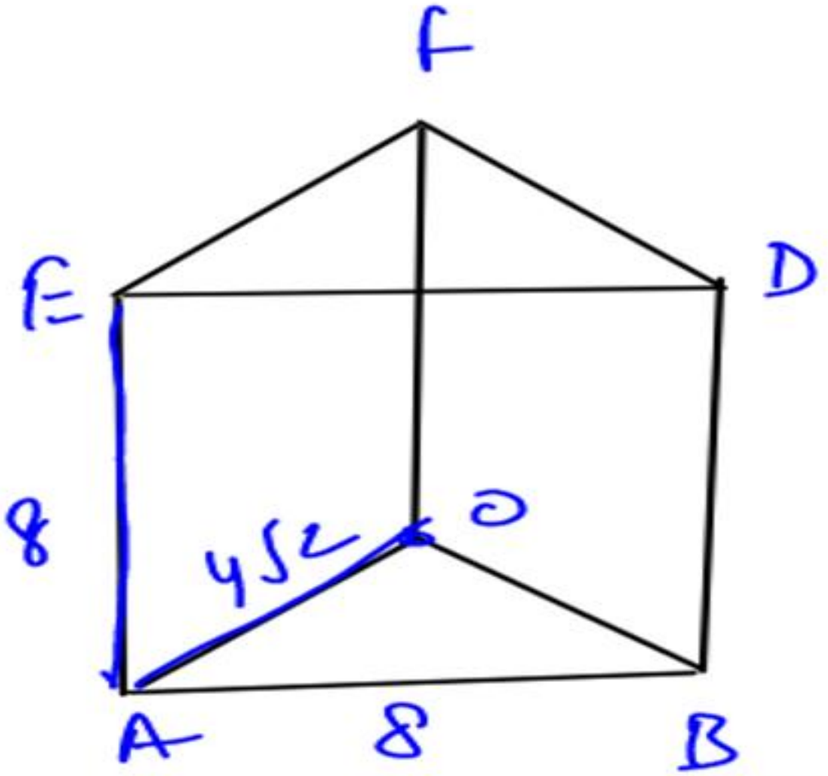
$(b)80+64\sqrt{2}$

$(c)96+48\sqrt{2}$

$(a)80+48\sqrt{2}$







ΔAOB & ΔEFD $\rightarrow$ $2\left(\frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2}\right)$

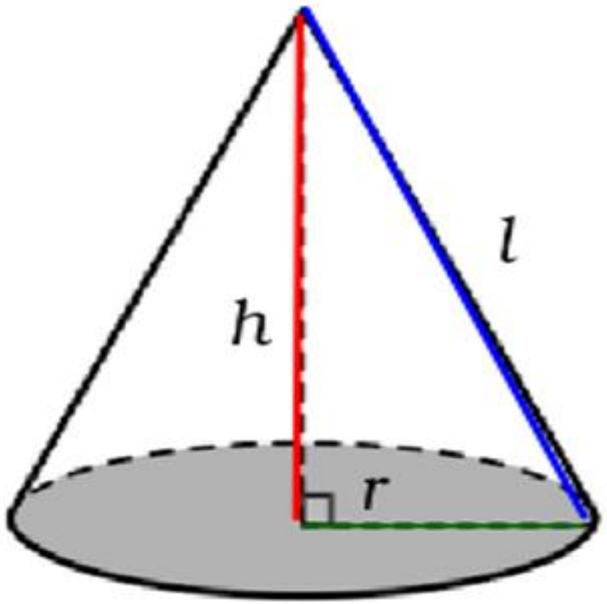
$\rightarrow$ $32\ cm^2$

$AOEF$ & $DOBF$ $\rightarrow$ $2 \times 8 \times 4\sqrt{2} = 64\sqrt{2}$

$DEAB$ $\rightarrow$ 8×8 $\rightarrow$ 64

Ans. (a)

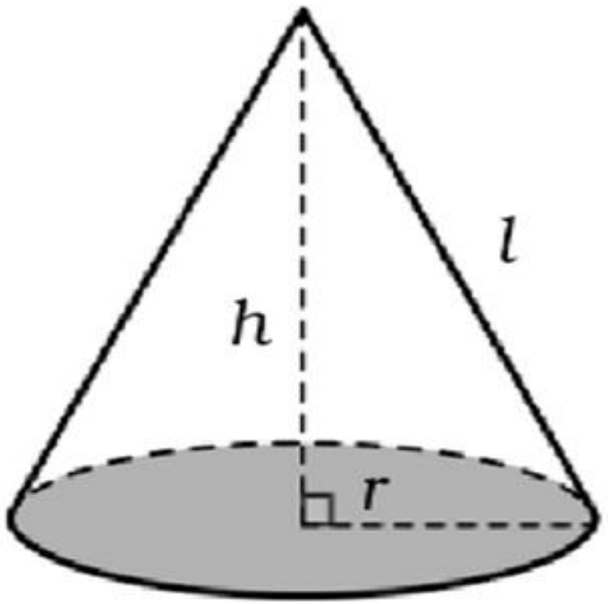
CONE



h = height
l = slant height
r = radius

$$l^2 = h^2 + r^2$$

Right circular cone

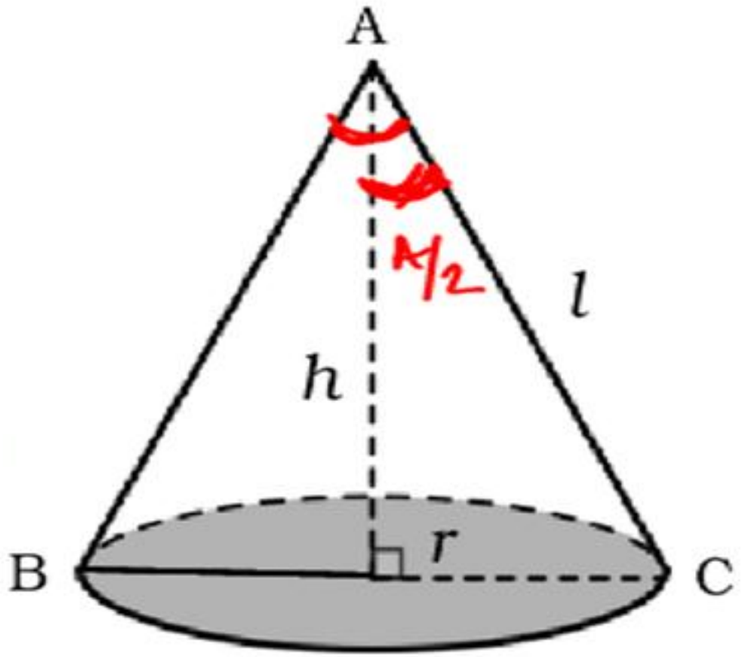


CSA of Cone	$= \pi r l$
TSA of Cone	$= \pi r l + \pi r^2$ $= \pi r (l + r)$
Volume of Cone	$= \frac{1}{3} \pi r^2 h$

Amount of air / water inside the container → Volume

Canvass (cloth) required → CSA

SEMI-VERTICAL ANGLE



$\angle A \rightarrow$ vertical

$\frac{\angle A}{2} \rightarrow$ Semi-vertical

Eg. The volume of a right circular cone is 1232 cm^3 and its vertical height is 24 cm. Its curved surface area is

(a) 154 cm^2 (b) 550 cm^2

(c) 604 cm^2 (d) 704 cm^2

$$\frac{1}{3}\pi r^2 h = 1232$$

$$h = 24$$

$$l = \sqrt{24^2 + 7^2}$$

$$CSA = ??$$

$$\frac{1}{3} \cdot \frac{22}{7} \cdot r^2 \cdot 24 = 1232$$

$$r^2 = 49$$

$$r = 7$$

$$CSA = \frac{22}{7} \times 7 \times 25$$

$$= 550 cm^2$$

Ans. (b)

Eg. The volume of a conical tent is 1232 m^3 and the area of its base is 154 sq.m. Find the length of the canvas required to build the tent, if the width of canvas is 2 m.

(a) 270 m (b) 272 m

(c) 276 m (d) 275 m

60sec

8

$\frac{1}{3}\pi r^2 h = 1232$

$\pi r^2 = 154$

$\frac{1}{3} \cdot 154 \cdot h = 1232$

$h = 24m$

$\frac{22}{7} \cdot r^2 = 154$

$r = 7m$

$CSA = \frac{22}{7} \cdot 7 \cdot 25$

$l = 25$

$22 \cdot 25 = l \cdot 2$

$l = 275m$

Ans. (d)

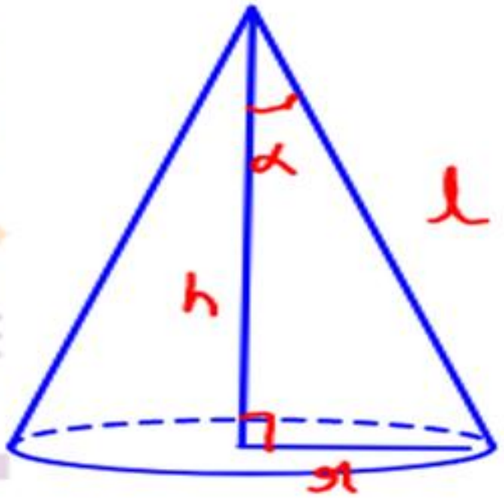
Eg. If S denotes the area of the curved surface of a right circular cone of height h and semi vertical angle α then S equals.

(a) $\pi h^2 \tan^2\alpha$

(b) $\frac{1}{3}\pi h^2 \tan^2\alpha$

(c) $\pi h^2 \sec\alpha.\tan\alpha$

(d) $\frac{1}{3}\pi h^2 \sec\alpha.\tan\alpha$



$S = \pi r l$

$\sec\alpha = \frac{l}{h}$ $\tan\alpha = \frac{r}{h}$

$l = h\sec\alpha$ $r = h\tan\alpha$

$\pi \cdot (h\tan\alpha)(h\sec\alpha)$

Ans. (c)

Eg. The radius and the height of a cone are in the ratio 4 : 3. The ratio of the curved surface area and total surface area of the cone is:

(a) 5 : 9 (b) 3 : 7

(c) 5 : 4 (d) 16 : 9

6 sec c

$r : h = 4 : 3$

$= \quad =$

$$\frac{CSA}{TSA} = \frac{\pi r l}{\pi r (l + r)} \Rightarrow \frac{5}{5+4}$$

$$= \frac{5}{9}$$

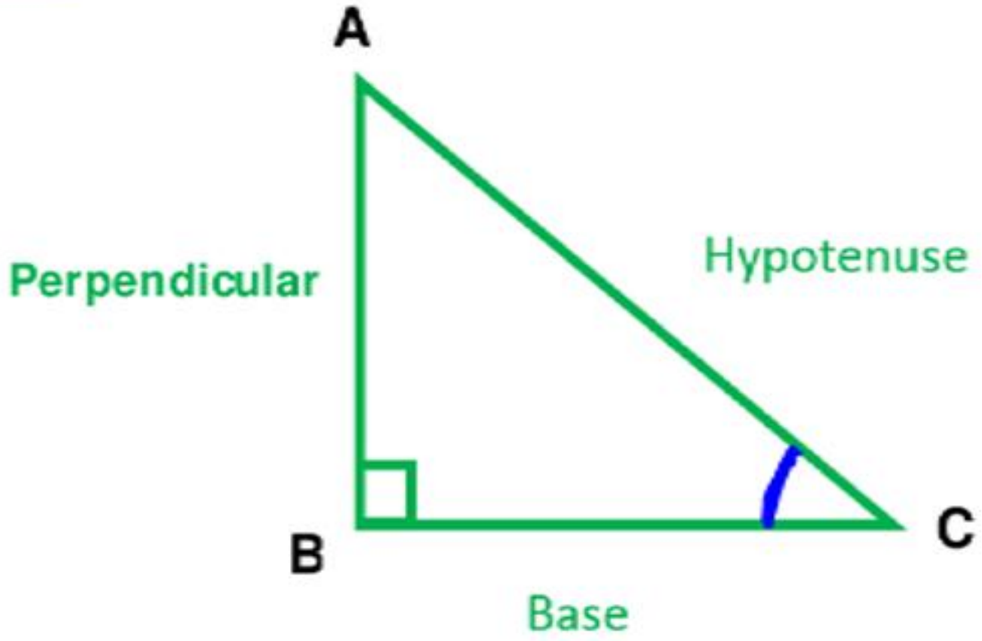
Ans. (a)

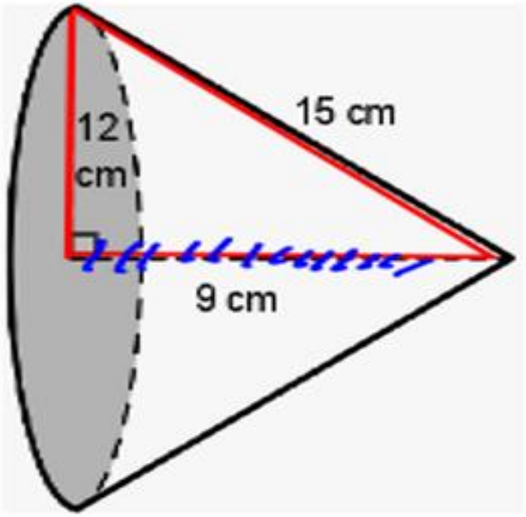
Rotation of a Right Angle Triangle

(i) Along Base

(ii) Along Perpendicular

(iii) Along Hypotenuse



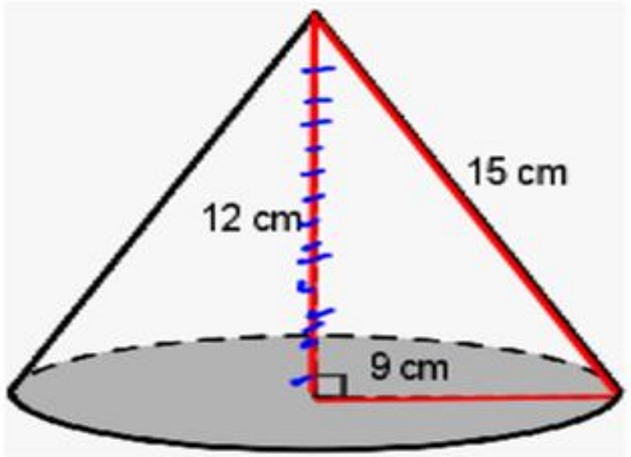


(i) Rotate along = 9 cm

h = 9 cm

r = 12 cm

l = 15 cm



(ii) Rotate along side

h = 12 cm

r = 9 cm

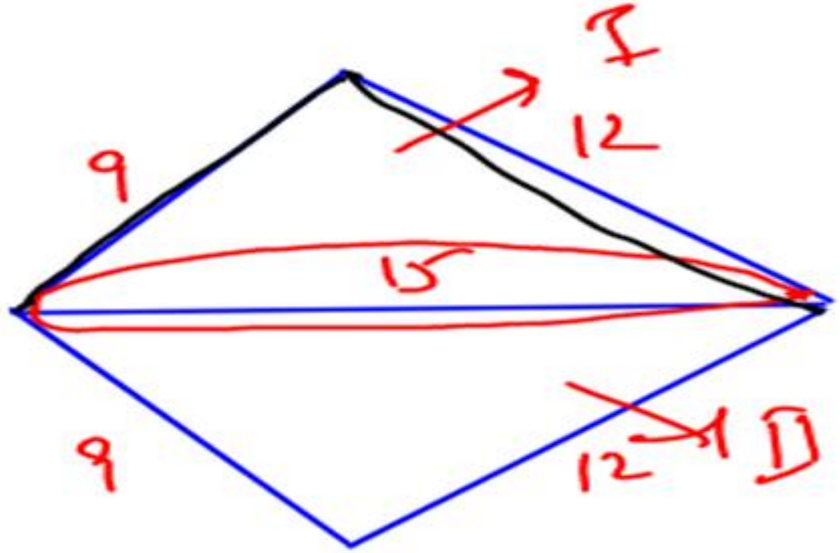
l = 15 cm

Ans

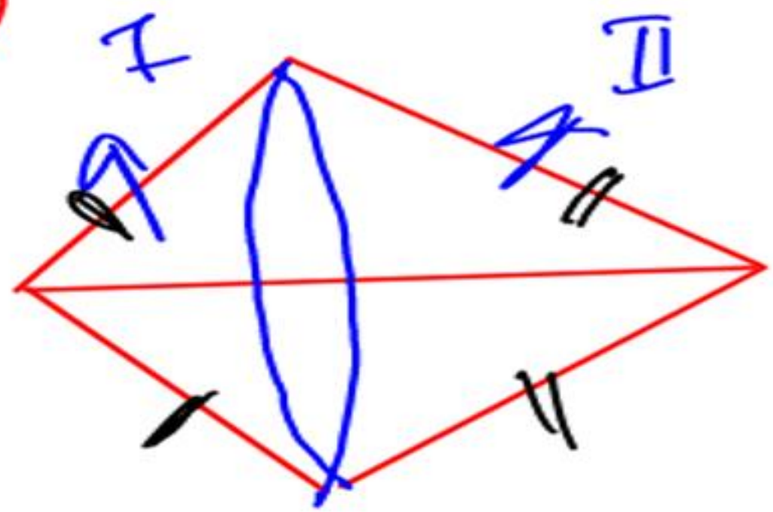
(iii) If a right angled Δ whose sides are 9, 12 and 15 cm is rotated along its hypotenuse then find the volume of the double cone formed.

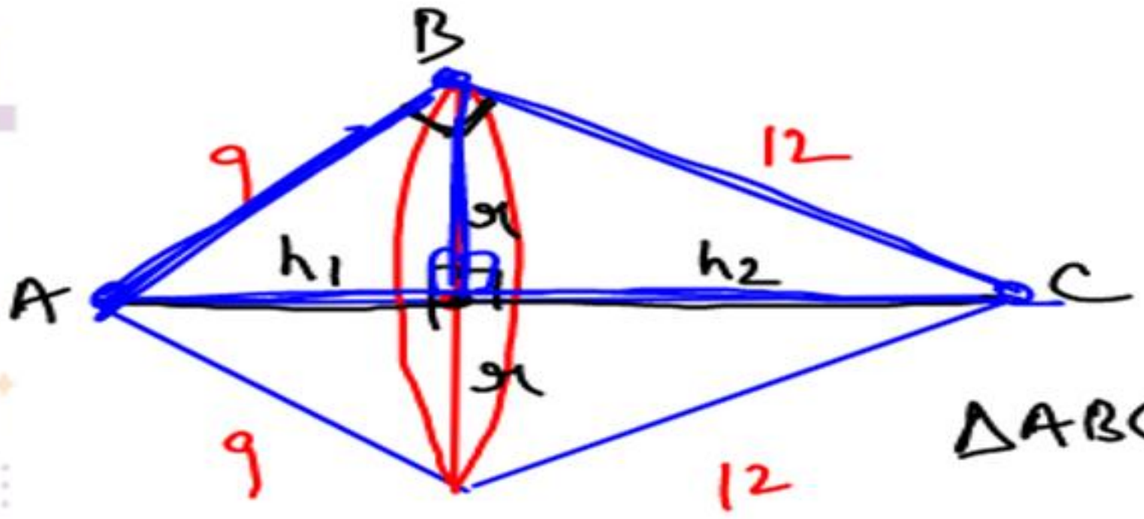
BYJU'S EXAM PREP

A



B





$h_1 + h_2 = 15$

ΔABC

$9 \cdot 12 = 15 \cdot x$

$x = \frac{36}{5}$

$\frac{1}{3}\pi x^2 h_1 + \frac{1}{3}\pi x^2 h_2$

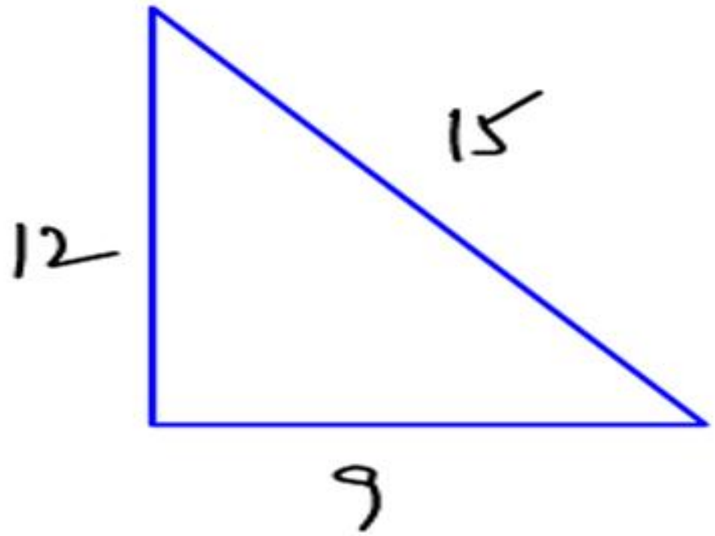
$\frac{1}{3}\pi \cdot \frac{36}{5} \cdot \frac{36}{5} \cdot 18 = \frac{1296}{5}\pi \, cm^3$

Shortcut: Right angle Δ is rotated along its hypotenuse

BYJU'S EXAM PREP

Volume of double cone $\frac{1}{3}\pi\frac{b^2p^2}{h}$

Here, b = base ; p = perpendicular & h = hypotenuse

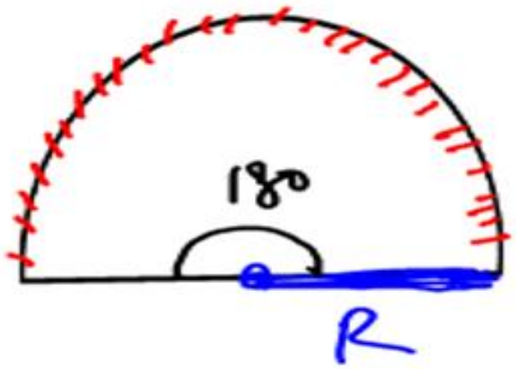


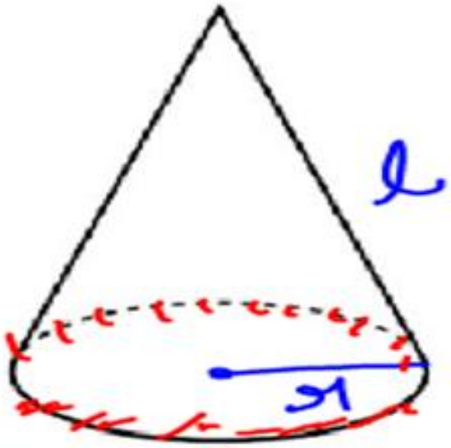
$$\frac{1}{3}\pi\cdot\frac{9^2\cdot 12^2}{15}$$

$$\frac{1}{3}\pi\cdot\frac{9\cdot 9\cdot 12\cdot 12}{5}$$

$$\frac{\pi\ 1296}{5}\ cm^3$$

Semi-circular sheet is folded to form an open conical cup



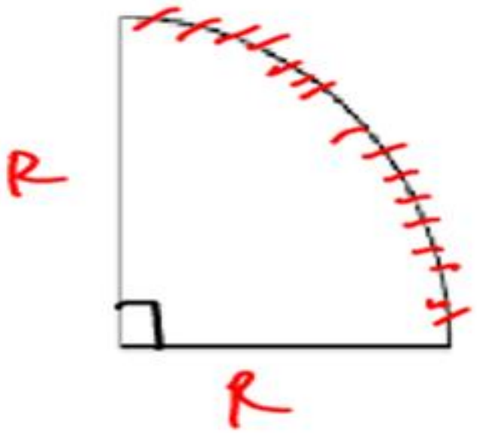


$$2\pi r = 2\pi r$$

$$r = \frac{R}{2}$$

$$l = R$$

Quadrant of a circle is folded to form an open conical cup



$$\frac{\pi R}{2} = 2\pi r$$

$$r = \frac{R}{4}$$

$$l = R$$

When a sector of a circle is folded to form an open conical cup

Sector of central angle θ & Radius R $\rightarrow$ Open conical cup

(i) $r = \frac{\theta}{360} R$

(ii) $l = R$

Where r, l are radius and slant height of cone.

logic

$\frac{1}{2} \pi R^2 \Rightarrow \pi r l$

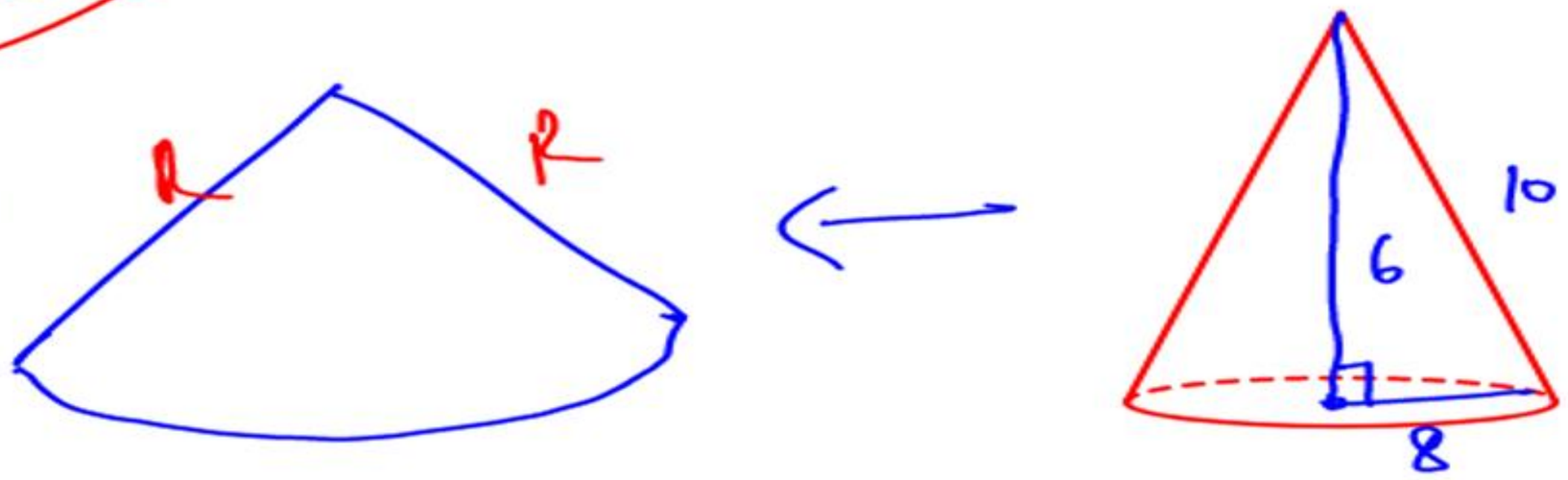
$l = R$

Area of sector of a Circle = CSA of cone

Eg. A sector is formed by opening out a cone of base radius 8 cm and height 6 cm. The radius of the sector is (in cm.)

(a) 4 (b) 8

(c) 10 (d) 6



R = 10

Ans. (c)

Eg. A sector of circle of radius 3 cm has an angle of 120°. If it is moulded into a cone, find the volume of the cone.

(a) $\frac{\pi}{\sqrt{3}}\text{cm}^3$

(b) $\frac{2\sqrt{2\pi}}{3}\text{cm}^3$

(c) $\frac{2\sqrt{3}}{\pi}\text{cm}^3$

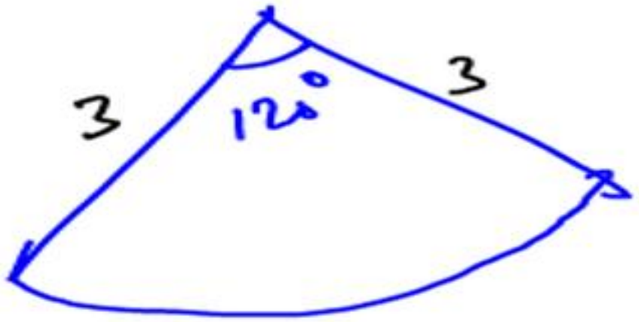
(d) $\frac{\sqrt{3}}{\pi}\text{cm}^3$

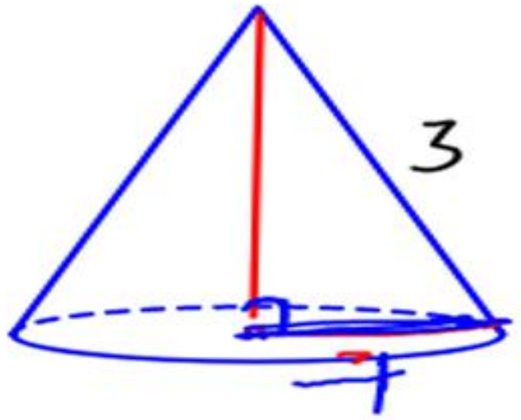
Volume of Cone

$\Rightarrow \frac{1}{3}\pi \cdot 1 \cdot 2\sqrt{2}$

π is outside root

$\frac{2\sqrt{2}\,\pi}{3}$





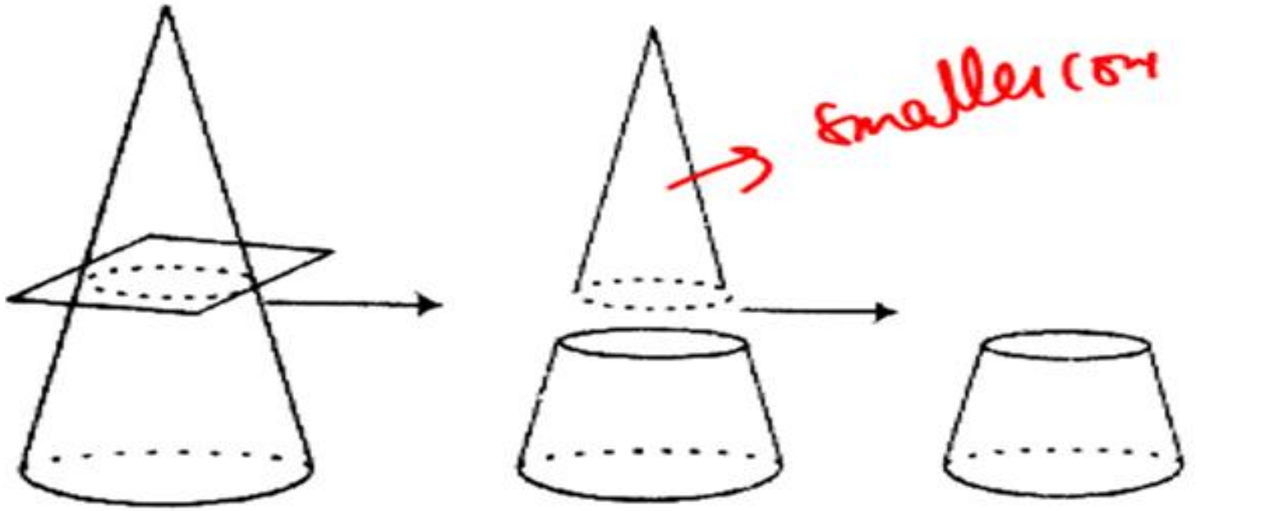
$r = \frac{120 \cdot 3}{360} = 1$

$h = \sqrt{3^2 - 1^2}$

$= 2\sqrt{2}$

Ans. (b)

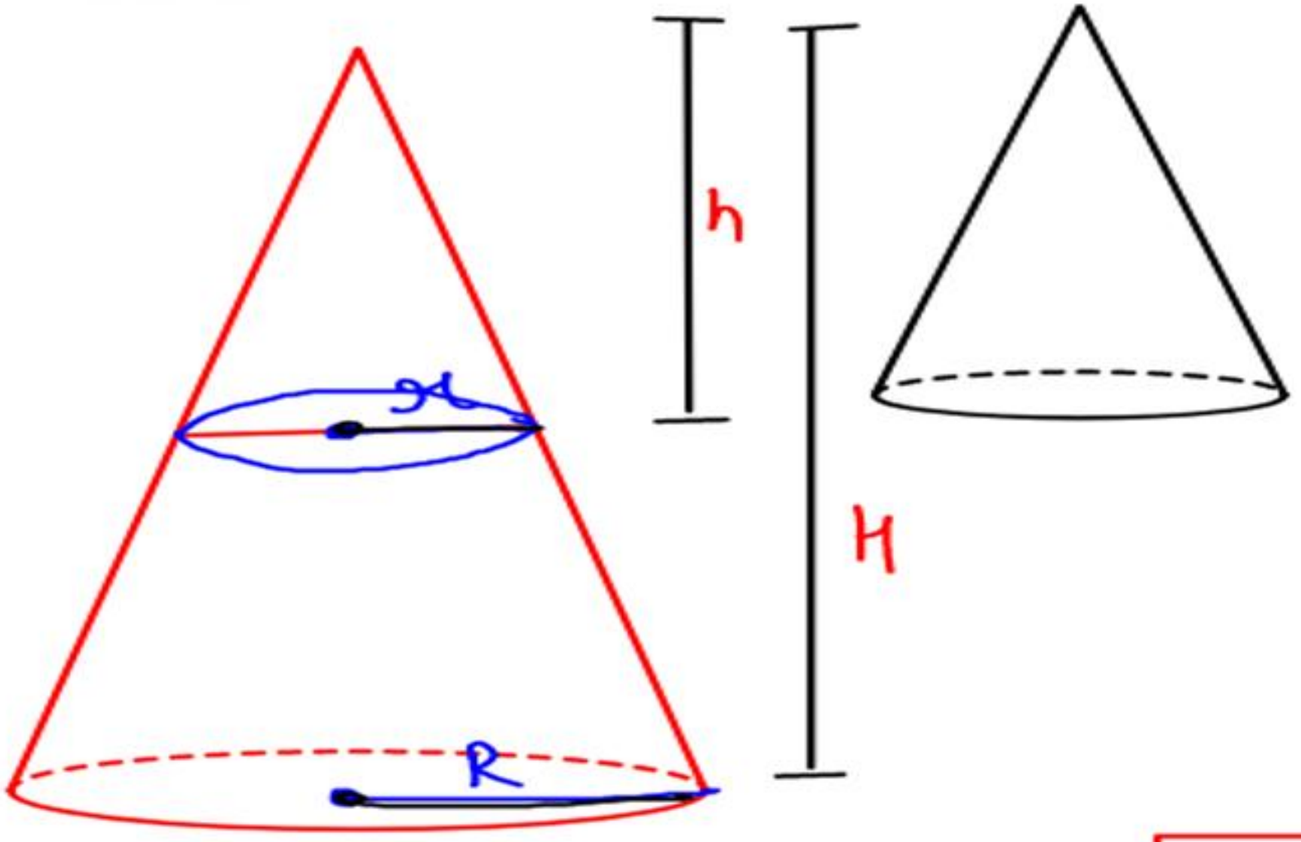
CUTTING OF A CONE PARALLEL TO ITS BASE

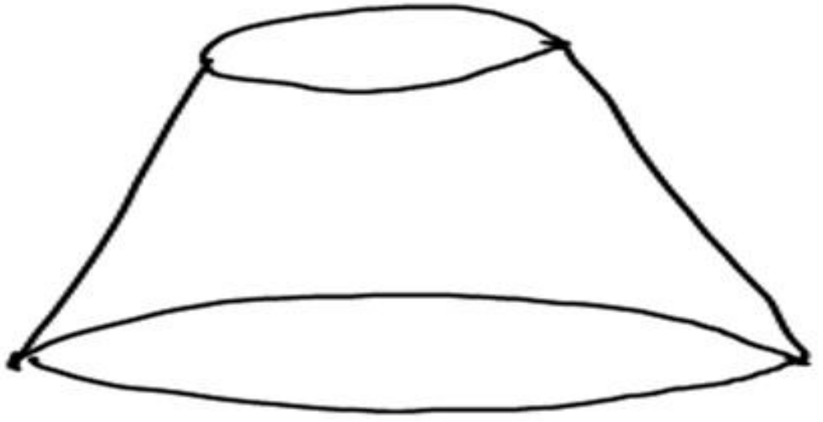


A cone sliced by a plane parallel to base

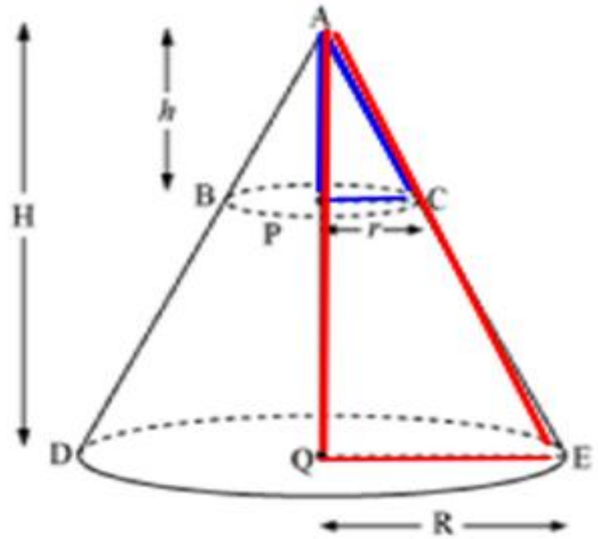
The two parts separated

Frustum of a cone





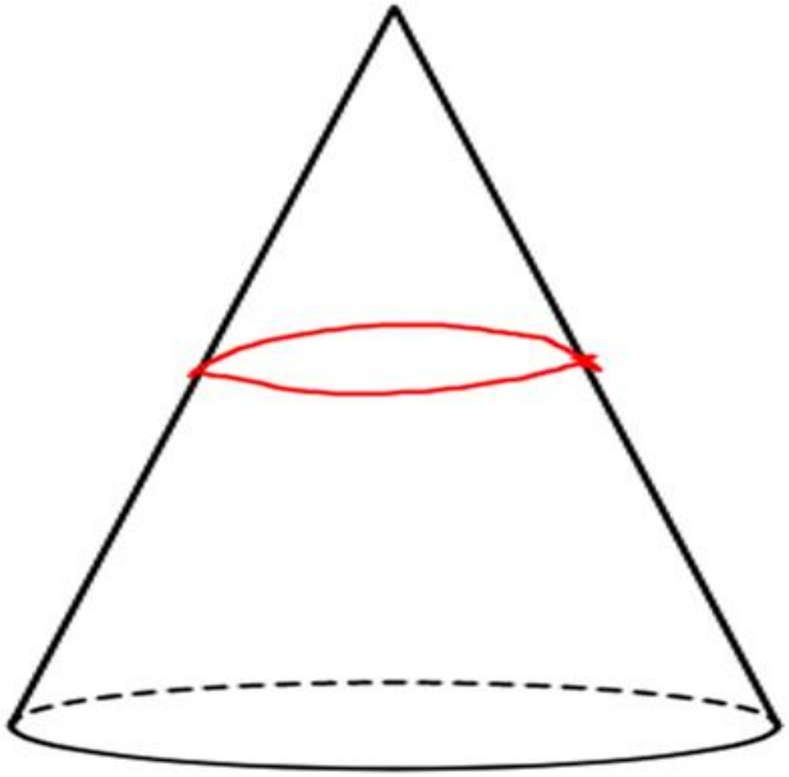
$$\frac{r}{R} = \frac{h}{H}$$



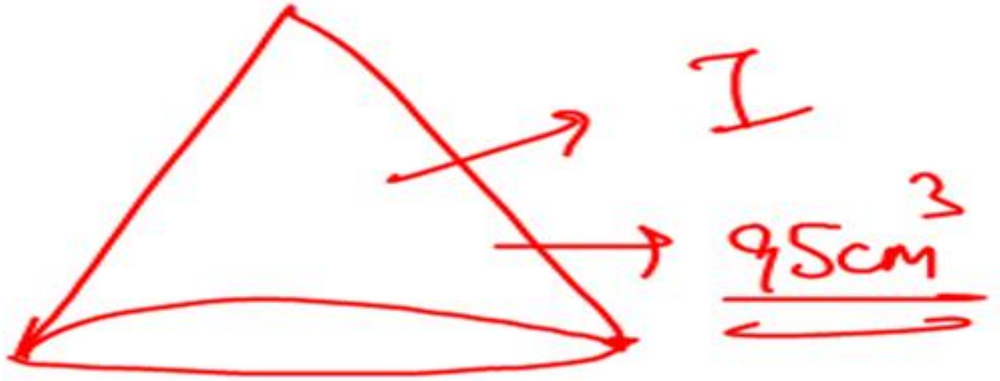
$\Delta APC \sim \Delta AQE$

$$\frac{h}{H} = \frac{r}{R}$$

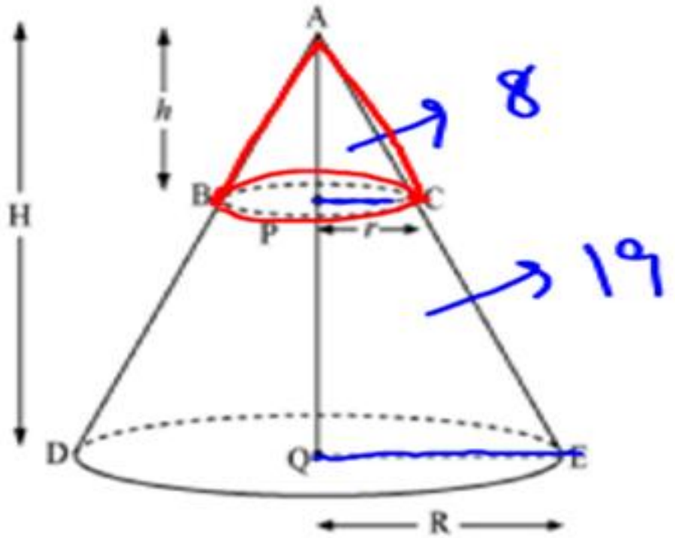
$$\frac{\text{Volume of smaller cone}}{\text{Volume of larger cone}} = \frac{r^2h}{R^2H} = \frac{r^3}{R^3} = \frac{h^3}{H^3}$$



$100 cm^3$



Eg. If $\frac{r}{R} = \frac{2}{3}$



8

19

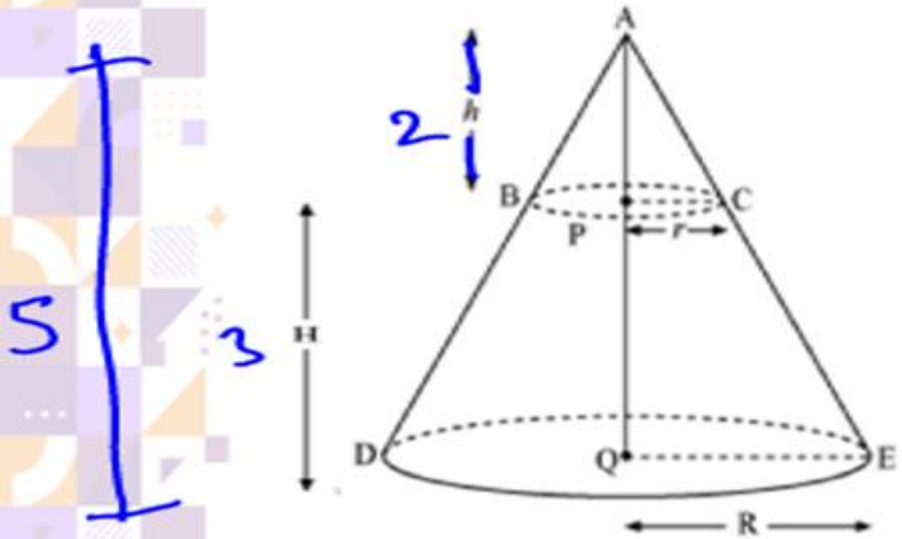
then $\frac{\text{Volume of I part}}{\text{Volume of II part}} = ??$ $\frac{8}{19}$

Volume of smaller cone → 8 unit³

Volume of bigger cone → 27 unit³

Eg. If $\frac{h}{H} = \frac{2}{3}$

then $\frac{\text{Volume of I part}}{\text{Volume of II part}} = ??$



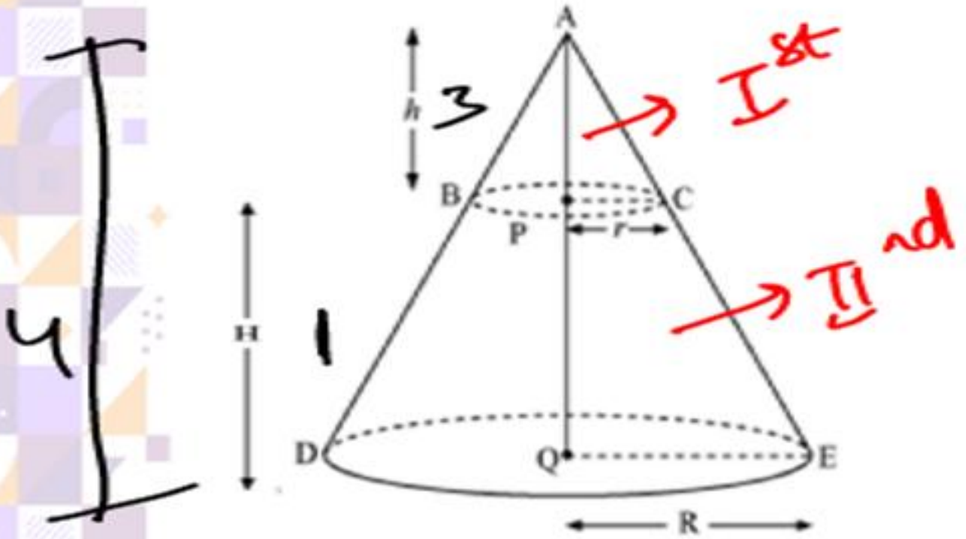
Volume of smaller cone → 8 units

Volume of bigger cone → 125 units

$\frac{8}{117}$

Eg. If $\frac{\textbf{Volume of I part}}{\textbf{Volume of II part}} = \frac{27}{37}$

Find: $\frac{h}{H} = \frac{3}{1}$



$$\frac{\text{Volume of smaller cone}}{\text{Volume of bigger cone}} = \frac{27}{64}$$

$$\frac{\text{Height of smaller cone}}{\text{Height of bigger cone}} = \frac{3}{4}$$

V. Imp

Eg. A plane divides a cone into two parts of equal volume. If the plane is parallel to the base, then the ratio in which the height of the cone is divided, is-

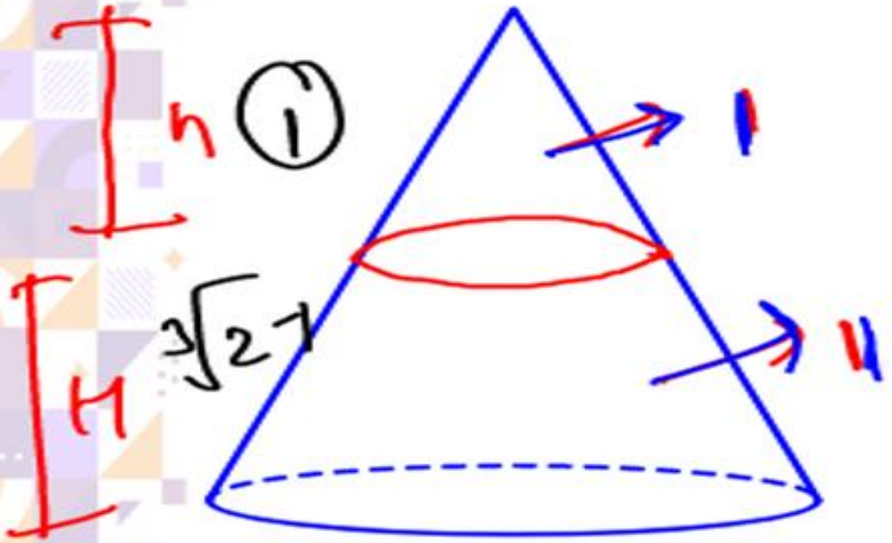
PYQ of SSC

(a) $1 : \sqrt{2}$

(b) $1 : \sqrt[3]{2} - 1$

(c) $1 : \sqrt[3]{2}$

(d) $1 : \sqrt[3]{2} + 1$



$$\frac{\text{Volume of smaller cone}}{\text{Volume of bigger cone}} = \frac{1}{2}$$

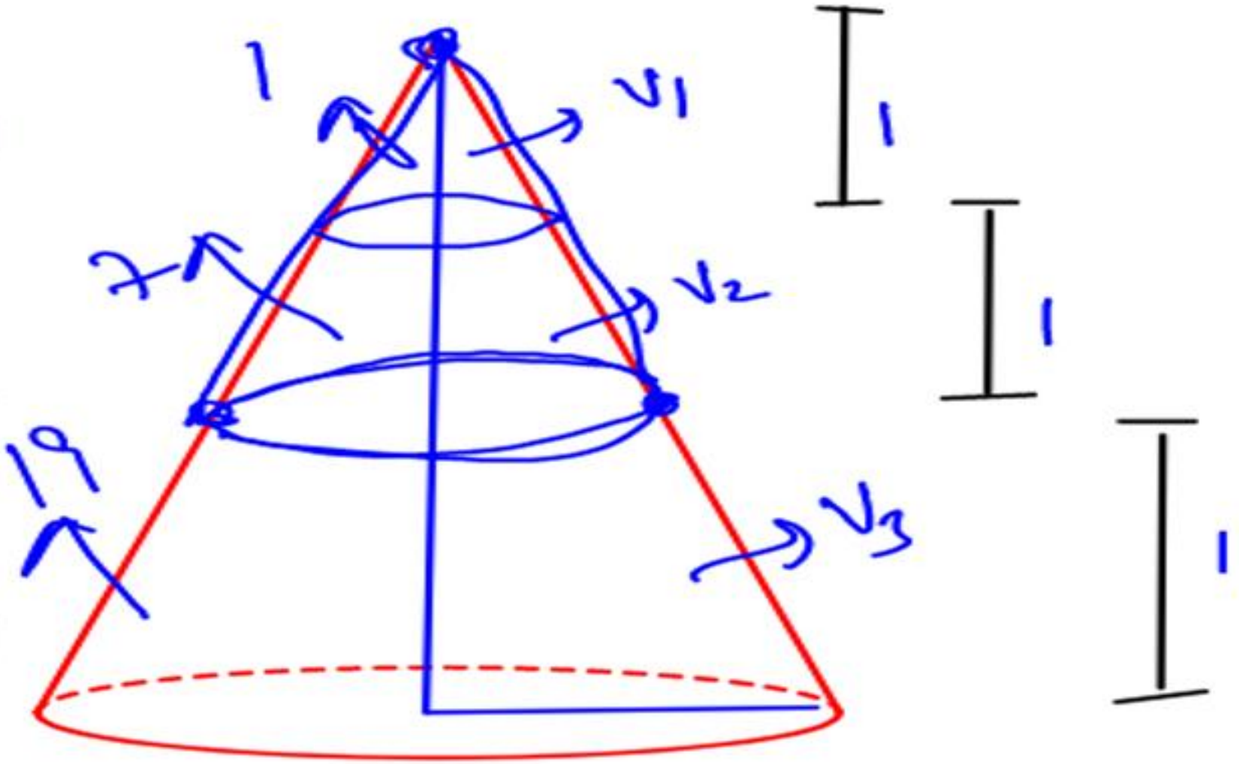
$$\frac{\text{Height of smaller cone}}{\text{Height of bigger cone}} = \frac{1}{\sqrt[3]{2}}$$

Ans. (b)

Eg. If a right circular cone is separated into solids of volumes V_1, V_2, V_3 by two planes parallel to the base which also trisect the altitude, then $V_1 : V_2 : V_3$ is-

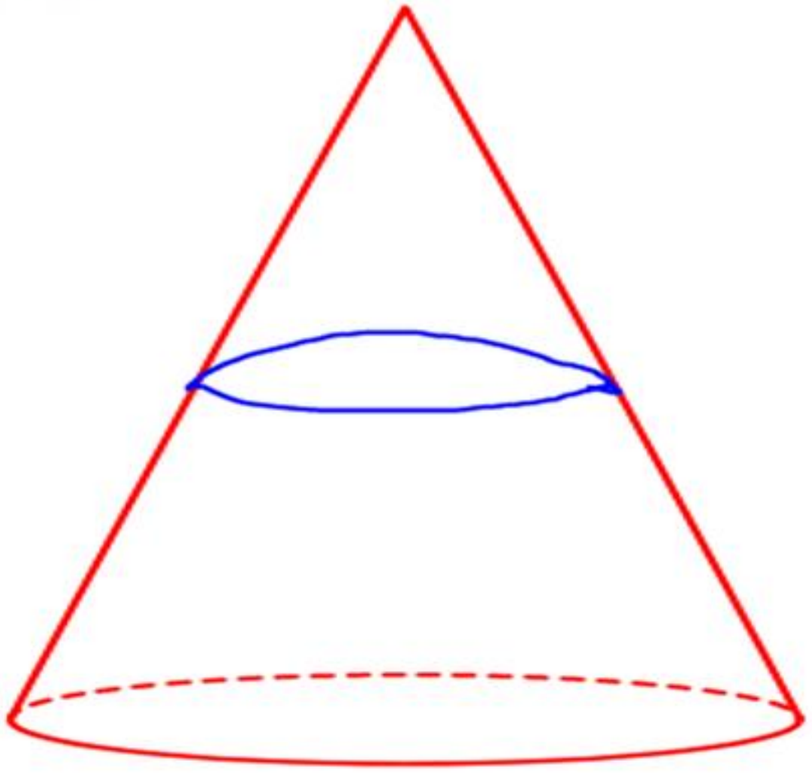
(a) 1 : 2 : 3 (b) 1 : 4 : 6

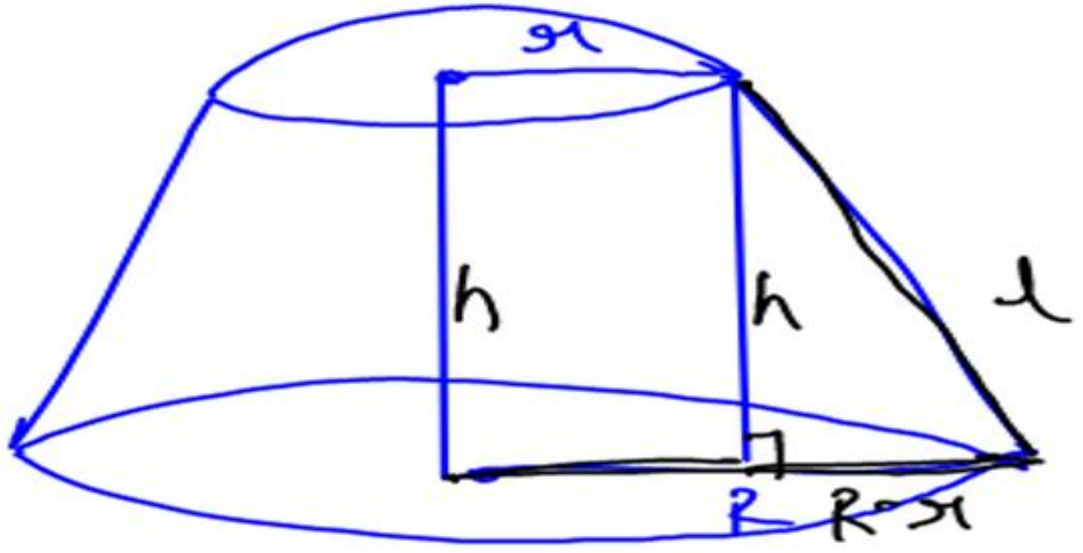
(c) 1 : 6 : 9 (d) 1 : 7 : 19



1 : 7 : 19

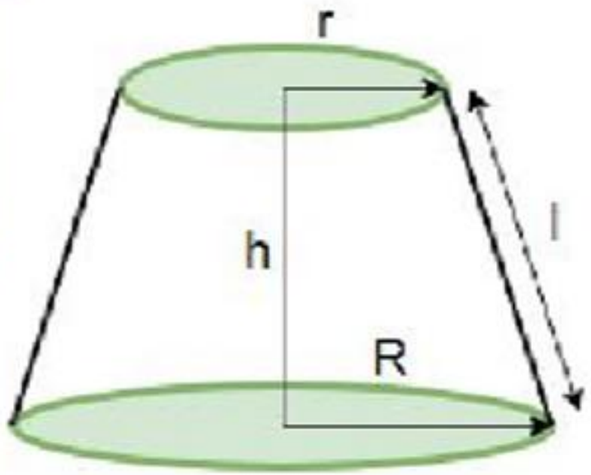
Ans. (d)





$$l^2 = h^2 + (R-r)^2$$

FRUSTUM OF CONE



$$CSA = \pi (R + r)\, l$$

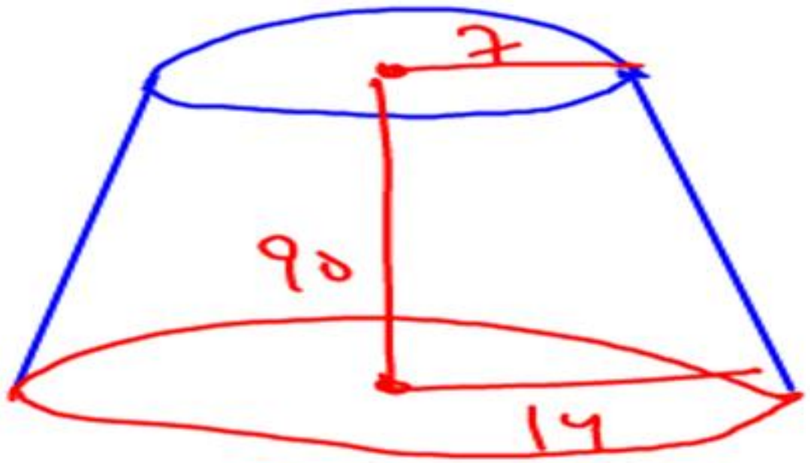
$$TSA = \pi (R + r)\, l + \pi (R^2 + r^2)$$

$$\text{Volume} = \frac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$$

$$l = \sqrt{h^2 + (R - r)^2}$$

Eg. If the radii of the circular ends of a truncated conical bucket which is 90 cm high be 14 cm and 7 cm, then the capacity of the bucket in cubic centimetre is (use $\pi = 22/7$)

(a) 9485
(b) 32340
(c) 4815
(d) 48050



$$\frac{1}{3}\pi h\left(R^2 + R\cdot r + r^2\right)$$

$$\frac{1}{3}\cdot\frac{22}{7}\cdot 90\left[14^2 + 14\cdot 7 + 7^2\right]$$

$$\frac{1}{3}\cdot\frac{22}{7}\cdot 90\cdot 7\cdot 7\left[4 + 2 + 1\right]$$

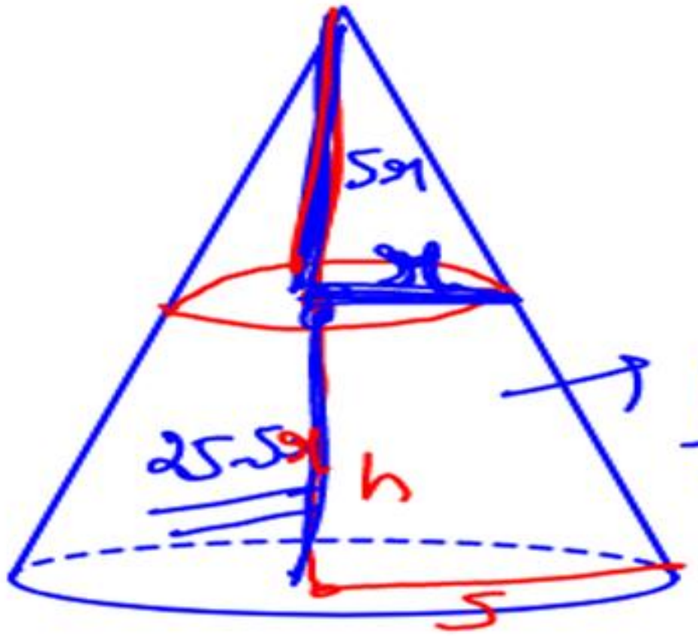
$$660\cdot 49 \rightarrow 40$$

Ans. (b)

Eg. The base radius and height of a cone is 5 cm and 25 cm respectively. If the cone is cut parallel to its base at a height of h from the base. If the volume of this frustum is 110 cm^3. Find the radius of smaller cone ?

(a) $(104)^{1/3}$ cm (b) $(104)^{1/2}$ cm

(c) 5 cm (d) None of these



110 cm^3

$\frac{1}{3} \cdot \frac{22}{7} (25-5x) \left[25 + 5x + x^2 \right] = 110$

$125 - x^3 = 21$

$x^3 = 104$

$x = (104)^{1/3}$

Ans. (a)

PRACTICE QUESTIONS

Q1. A conical tent is to accommodate 11 persons such that each person occupies 4 m^2 space on the ground and has 220 m^3 of air to breathe. The height of the cone is :

(a) 145 m (b) 155 m

(c) 165 m (d) 205 m

Ans. (c)

Q2. If h, c, v are respectively the height, curved surface area and volume of a right circular cone, then the value of $3\pi vh^3 - c^2h^2 + 9v^2$ is :

(a) 2 (b) –1

(c) 1 (d) 0

Ans. (d)

Q3. A solid cone of height 8 cm and base radius 6 cm is melted and recast into identical cones, each of height 2 cm and radius 1 cm. What is the number of cones formed ?

(a) 36 (b) 72

(c) 144 (d) 180

Ans. (c)

Q4. Find the volume of a right circular cone formed by joining the edges of a sector of a circle of radius 4 cm where the angle of the sector is 90°.

(a) $\frac{2\sqrt{3}}{\pi}\text{cm}^3$

(b) $\frac{2\sqrt{2}\pi}{3}\text{cm}^3$

(c) $\frac{\pi\sqrt{5}}{\sqrt{3}}\text{cm}^3$

(d) $\frac{\sqrt{3}}{\pi}\text{cm}^3$

Ans. (c)

Q5. The radii of the circular end of a conical bucket are 14 cm and 6 cm, whose height is 6 cm, find the total surface area of bucket.

(a) 1357.71 cm^2
(b) 1257.71 cm^2
(c) 1468.67 cm^2
(d) None of these

Ans. (a)

Q6. Numerical values of curved surface area and volume of a right circular cone are equal. If h & r be the height and radius of the cone then $\frac{\mathbf{1}}{\boldsymbol{h^2}}+\frac{1}{\boldsymbol{r^2}}$ =

(a) 3
(b) 1/3
(c) 1/9
(d) 9

Ans. (c)

Q7. The radius of the base of a conical tent is 16 metre. If $427\frac{3}{7}$ sq. metre canvas is required to construct the tent, the slant height of the tent is :

(a) 17 metre
(b) 15 metre
(c) 19 metre
(d) 8.5 metre

Ans. (d)

Q8. A right circular cone is 3.6 cm high and radius of its base is 1.6 cm. It is melted and recast into a right circular cone with radius of its base as 1.2 cm. Then the height of the cone (in cm.) is

(a) 3.6 (b) 4.8

(c) 6.4 (d) 7.2

Ans. (c)

Q9. A cardboard sheet in the form of a circular sector of radius 30 cm and central angle 144° is folded to make a cone. What is the radius of the cone?

(a) 12 cm
(b) 18 cm
(c) 21 cm
(d) None of these

Ans. (a)

Q10. The height of a solid cone is 20 cm. A small cone is cut off from the top of it such that base of the cone cut off and the base of a given cone are parallel to each other. If the volume of the cone cut and the volume of the original cone are in the ratio of 1 : 8, find the height of the frustum.

(a) 6 cm
(b) 8 cm
(c) 10 cm
(d) 12 cm

Ans. (a)