Arithmetic Progression → $(52-54)$ min
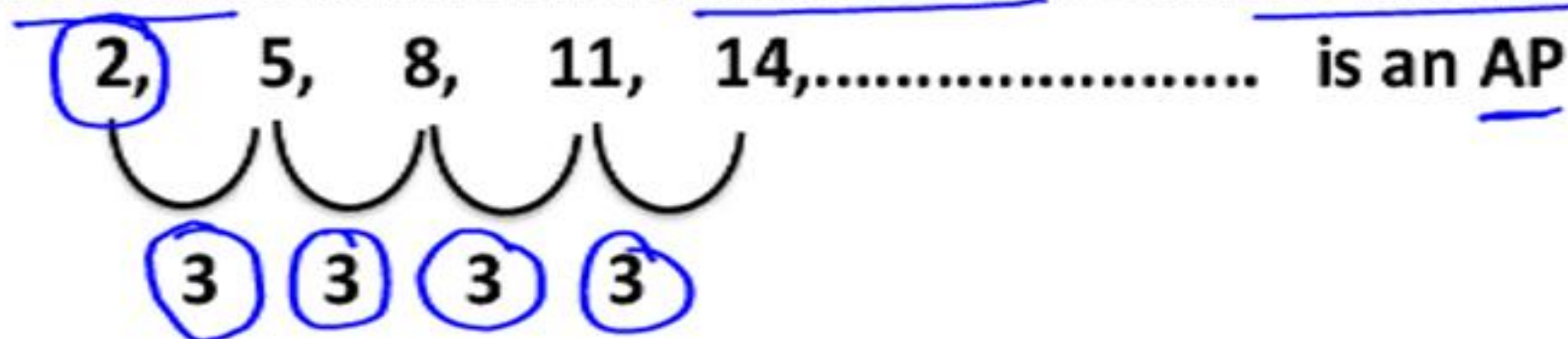
Geometric Progression → $(52-54)$ min

ARITHMETIC PROGRESSION

When the difference between two consecutive terms is constant :

e.g.   2,   5,   8,   11,   14,..................... is an AP

③ ③ ③ ③

First term $\rightarrow$ a

Common difference $\rightarrow$ d

$a = 2$ $d = T_2 - T_1$

$d = 3$ $= T_3 - T_2$

                         $= T_n - T_{n-1}$

In an AP

a, a+d, a+ 2d, a+3d, a+4d, ...

1st term	$\rightarrow$	a
2nd term	$\rightarrow$	a+d
3rd term	$\rightarrow$	a+2d
10th term	$\rightarrow$	a+9d

$$n^{th} \text{ term} = a + (n-1)d$$

Eg1. 2, 5, 8, 11, 14, ...

Find the 20th term.

$$a = 2 \qquad d = 3$$

$$n^{th} \text{ term} = a + (n-1)d$$

$$20^{th} \text{ term} = a + 19d$$

$$= 2 + 19 \cdot 3$$

$$\Rightarrow \underline{59}$$

Ans. 59

Eg2. 2, 5, 8, 11, 14, 17, ... is an AP.

Which term of the sequence is 323.

Let the nth term is 323

$$a + (n-1)d = 323$$

$$2 + (n-1)3 = 323$$

$$3n = 324$$

$$n = 108$$

108th term

Ans.   108

Eg3. 5, 11, 17, 23, ... 359 (AP)

Find the 20th term from end.

Time → 60 sec

Sol^n

5, 11, 17, 23, ── ── ── ── ── 359

$\boxed{359,}$ 353, 347, ── ── ── 5

$a = 359$

$d = -6$

20th term

$a + 19d$

$359 + 19(-6)$

$= \underline{245}$

Ans.   **245**

$\underline{a}$ $\longrightarrow$ **First Term**

$\underline{T_n}$ $\longrightarrow$ **n^{th} Term**

$\underline{d}$ $\longrightarrow$ **Common Difference**

$\underline{S_n}$ $\longrightarrow$ **Sum of n Terms**

$T_5 \longrightarrow 5^{th}$ term

$T_8 \longrightarrow 8^{th}$ term

$S_5 \longrightarrow$ Sum of first 5 term

Eg4. If $T_{12} = 79$

$\quad\quad$ $T_{20} = 111$

$\quad\quad$ Find T_{16}
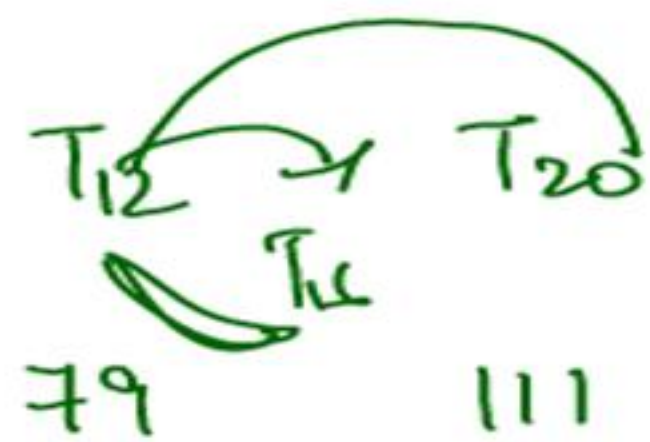
$\underline{\underline{I}}^{st}$ &

$$a + 11d = 79 - \text{①}$$

$$a + 19d = 111 \quad \text{②}$$

$$\text{②} - \text{①}$$

$$8d = 32 \quad \boxed{d = 4}$$

$$a = 35$$

$$a + 15d$$

$$35 + 15 \cdot 4 = 95$$

$\underline{\underline{II}}^{nd}$

$T_{12} \longrightarrow T_{20}$

$\quad\quad T_{16}$

$79 \quad\quad\quad 111$

$8d = 32$

$d = 4$

$79 + 16$

$= \boxed{95}$

Ans. 95

T_{11}^{nd}

T_{12} T_{16} T_{20}

79   111

$$\frac{79+111}{2} = 95$$
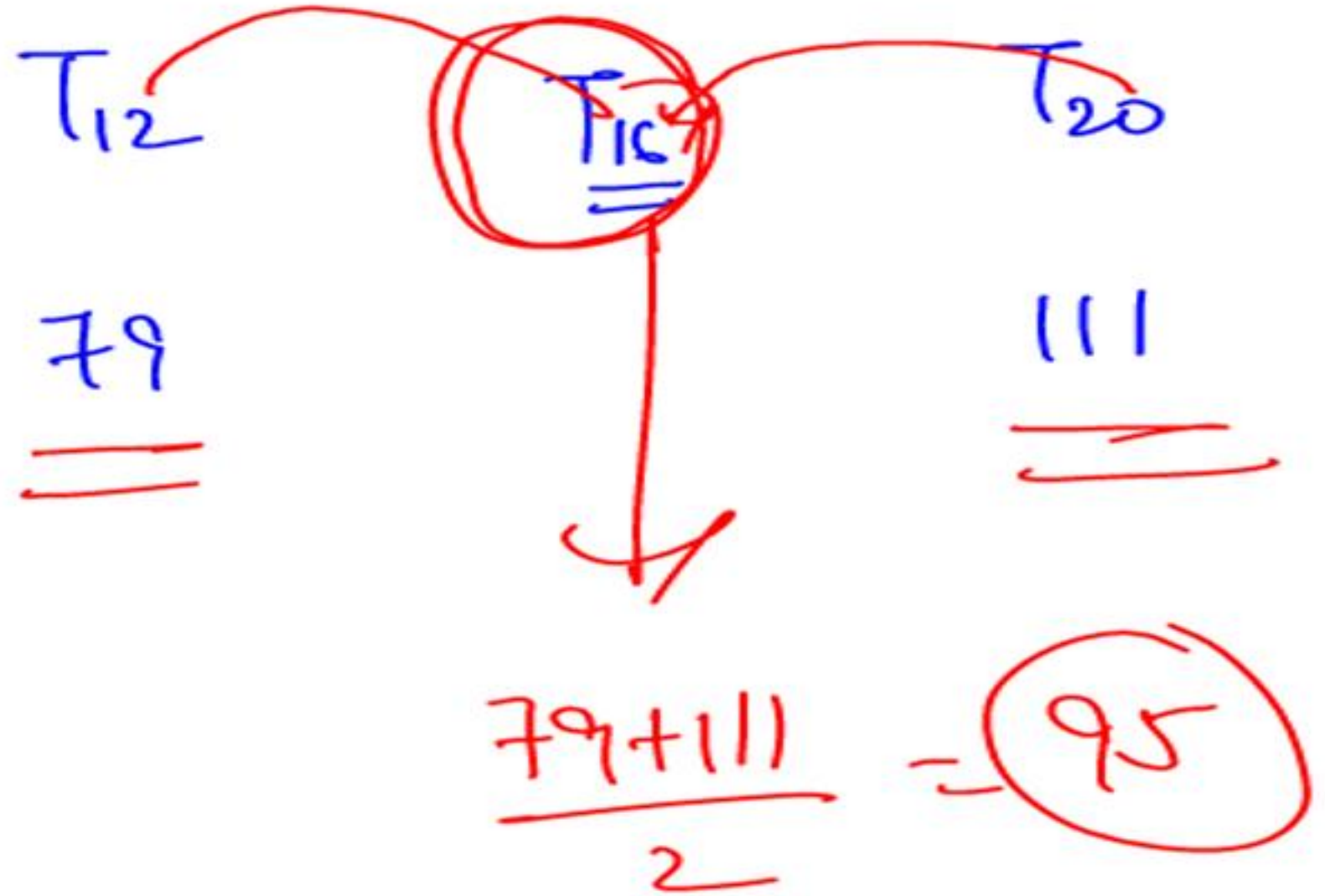
Eg5. Which term of the sequence

$$20, 19\frac{1}{2}, 19, 18\frac{1}{2}, 18, \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

is the first negative term?

60 sec

Sol^n

$a = 20$

$d = 19\frac{1}{2} - 20 = \frac{-1}{2}$

$$20 + (n-1)\left(\frac{-1}{2}\right) < 0$$

$$40 + (1-n) < 0$$

$$41 - n < 0$$

$$41 < n$$

42nd term

Ans. 42

Eg6. If $T_m = n$

$\quad\quad T_n = m$

Find $T_{m+n} = ??$

$a + (m+n-1)d$

$(n+m-1) + (m+n-1)(-1)$

$= \underline{\underline{0}}$

$a + (m-1)d = n \quad — ①$

$a + (n-1)d = m \quad — ②$

$(m-n)d = n-m$

$\boxed{d = -1}$

$a + (m-1)(-1) = n$

$\boxed{a = n+m-1}$

Shortcut:

If $T_m = n$
  $T_n = m$
 then $\boxed{T_{m+n} = 0}$

eg

$T_{(5)} = 10$

$T_{(10)} = 5$

$T_{15} \longrightarrow \underline{0}$

eg

$T_{100} = 200$

$T_{200} = 100$

$T_{300} \longrightarrow \underline{0}$

Selection of terms in an AP

3 terms $\rightarrow$ $a - d, a, a + d$

5 terms $\rightarrow$ $a - 2d, a - d, a, a + d, a + 2d$

4 terms $\rightarrow$ $a - 3d, a - d, a + d, a + 3d$

Eg7. If the sum of first 3 consecutive terms of an increasing AP (d > 0) is 51 & the product of first and third term is 273, then the third term is:

Sol^{N}

$$(a-d), \ (a) \ \& \ (a+d)$$

$$(a-d) + (a) + (a+d) = 51$$

$$3a = 51 \qquad \boxed{a = 17}$$

$$17+d$$

$$= \boxed{21}$$

$$(17-d)(17+d) = 273$$

$$289 - d^2 = 273$$

$$\boxed{d = 4}$$

Sum of n terms of an Arithmetic Progression

Sum = $\dfrac{n}{2}\Big[First + Last\Big]$

or

$$\dfrac{n}{2}\Big[2a + (n-1)d\Big]$$

Eg8. Find the sum of first 20 terms of 2, 5, 8, 11, ...

Sol^n

$d = 5 - 2$

$\quad = 3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{20} = \frac{20}{2}\left[4 + 19 \cdot 3\right]$$

$$= \frac{\overset{10}{\cancel{20}}}{\cancel{2}} \cdot 61$$

$$\boxed{S_{20} = 610}$$

Ans. 610

Eg9. Find the sum of first 20 terms of 5, 12, 19, 26, ...698

$$S_{20} = \frac{20}{2}\left[10 + 19 \cdot 7\right]$$

$$10 \cdot 143$$

$$S_{20} = 1430$$

Ans. 1430

Eg

$$5, 13, 21, 29, \text{----} 997$$

find sum of all terms

$$997 = 5 + (n-1)8$$

$$\frac{992}{8} + 1 = n$$

$$n = 125$$

$$S_{125} = \frac{125}{2}\left[5 + 997\right]$$

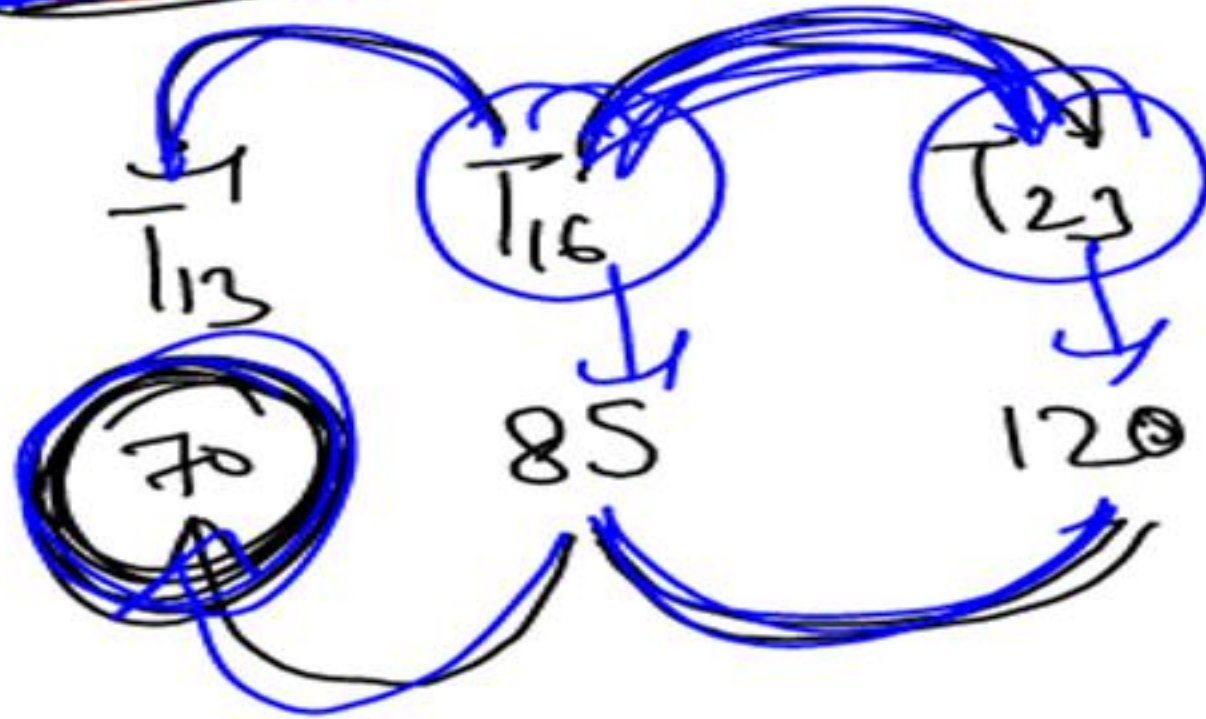$$= \frac{125}{2} \times 1002$$

Eg10. If $T_{16} = 85$

$T_{23} = 120$

$$S_{25} = ??$$

Sol^n

$$T_{13}$$

$$70$$

$$85$$

$$120$$

$$70 \times 25$$
$$= 1750$$

Ist

$$a + 15d = 85$$

$$a + 22d = 120$$

$$a = 10 \quad d = 5$$

$$S_{25} = \frac{25}{2}\left[20 + 24 \cdot 5\right]$$

$$= \frac{25}{2} \cdot \overset{70}{140}$$

$$1750$$

Ans. 1750

Note

In an Arithmetic Progression

$$\text{Average} = \text{Middle Term}$$

eg If 25 terms are there

$$\frac{25+1}{2} \longrightarrow 13^{th} \text{ term}$$

Eg11. If $T_{15} = 100$

Find : (i) $S_{29} =$

(ii) S_{50}

Middle $= \left(\dfrac{29+1}{2} \right)$

15th term

100×29

$= 2900$

Middle $= \left(\dfrac{50+1}{2} \right)$

$25\cdot 5$ term

$\rightarrow$ Can't be det

$$T_{15} = 100 \qquad \boxed{a + 14d = 100}$$

(i) $\underline{S_{29}}$

$$\frac{29}{2}[2a + 28d]$$

$$\frac{29}{2} \cdot 2[a + 14d]$$

$$29 \cdot 100$$

$$= 2900$$

(ii) $\underline{S_{50}}$

$$\frac{50}{2}[2a + 49d]$$

$$25[2a + 49d]$$

Can't be det

Ans. (i) 2900 (ii) Can't be determined

ARITHMETIC MEAN

gradeup

If a & b are 2 numbers :

$$A.M = \frac{a+b}{2}$$

If a, b, c, d, ... (n numbers are there)

$$AM = \frac{a+b+c+d..........................}{n}$$

Eg12. Find AM of 9 & 19.

$$A.M = \frac{9+19}{2} = 14$$

Q If $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the A.M of a & b find $n = ?? \; a \neq b$

Sol^n

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a^1 + b}{2}$$

$$\boxed{n = 0}$$

Detailed

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{a+b}{2}$$

$$2a^{n+1} + 2b^{n+1} = a^{n+1} + a^n \cdot b + b^n \cdot a + b^{n+1}$$

$$a^{n+1} - a^n b + b^{n+1} - b^n \cdot a = 0$$

$$a^n(a-b) + b^n(b-a) = 0$$

$$(a-b)(a^n - b^n) = 0 \qquad\qquad a^n = b^n$$

$$\boxed{a = b} \quad \text{or} \quad \underline{a^n = b^n} \qquad \left(\frac{a}{b}\right)^n \left(\frac{a}{b}\right)^0$$

$$n = 0$$

gradeup

GP $\longrightarrow$ **It is a sequence of numbers where consecutive terms are in a fixed ratio.**

Eg. 2, 6, 18, 54, 162, 486,..........................

$a = 2$

$r = \dfrac{6}{2} = 3$

a = 1ˢᵗ term

r = Common ratio

$$r = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{T_n}{T_{n-1}}$$

Eg. 3, 6, 12, 24, 48, 96,..........................

a = 3 ,

$$r = \frac{6}{3} = 2$$

In GP, the terms are

a, ar, ar², ar³, ar⁴,.....................

$$n^{th} \text{ term} = ar^{n-1}$$

Eg1. 3, 6 , 12, 24, 48, ...

Find the 10th term.

$$a = 3 \qquad x = 2$$

$$10^{th} \text{ term} \longrightarrow a \cdot x^9$$

$$3 \cdot 2^9$$

$$3 \cdot 512$$

$$= 1536$$

Ans. 1536

Eg2. Which term of the GP

243, 81, 27, ... is $\dfrac{1}{729}$

Soln   Let n^{th} term is $\dfrac{1}{729}$

$$a \cdot r^{n-1} = \dfrac{1}{729}$$

$$243 \cdot \left(\dfrac{1}{3}\right)^{n-1} = \dfrac{1}{729}$$
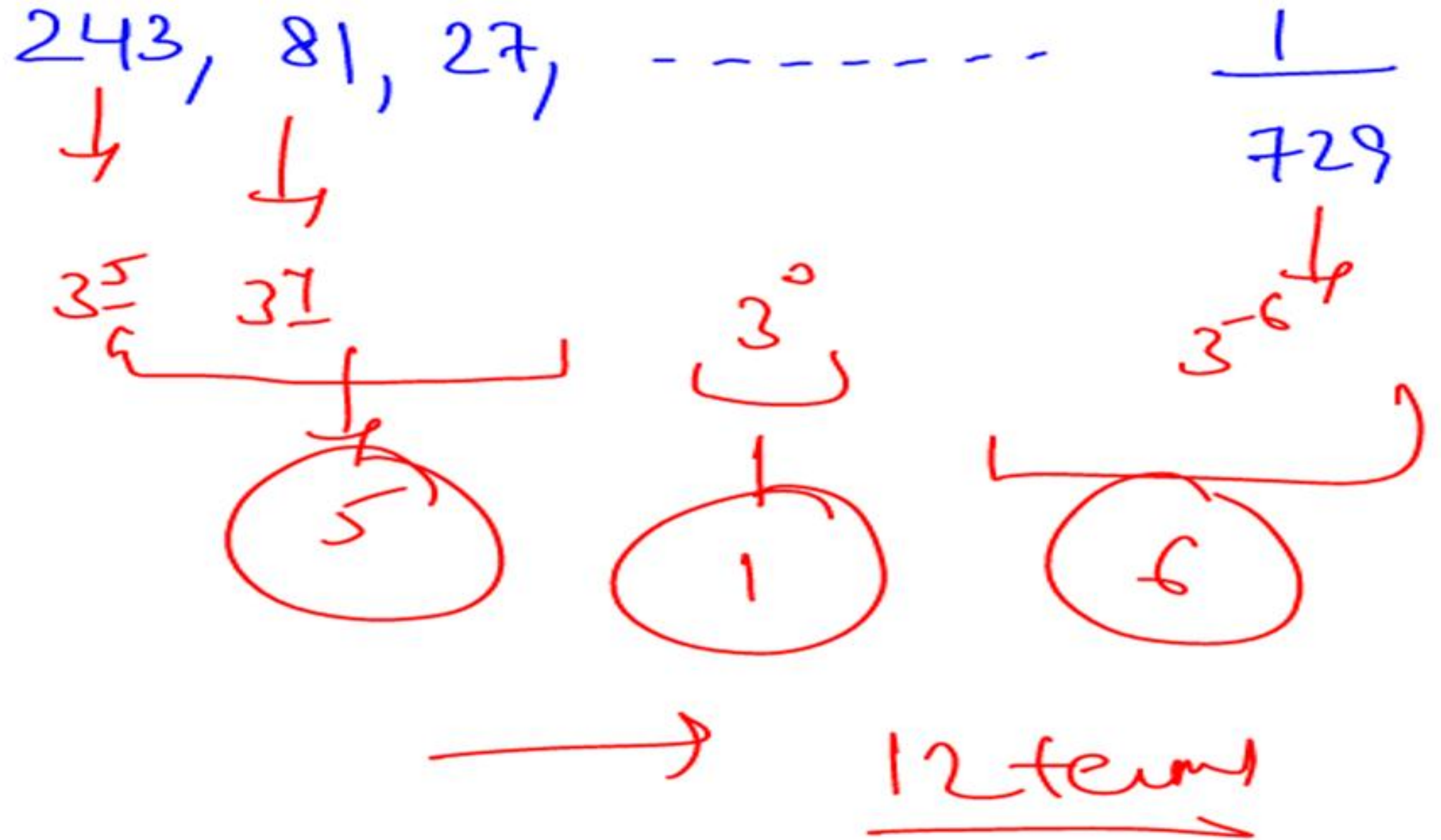
$\longrightarrow$

$$3^5 \cdot 3^{1-n} = 3^{-6}$$
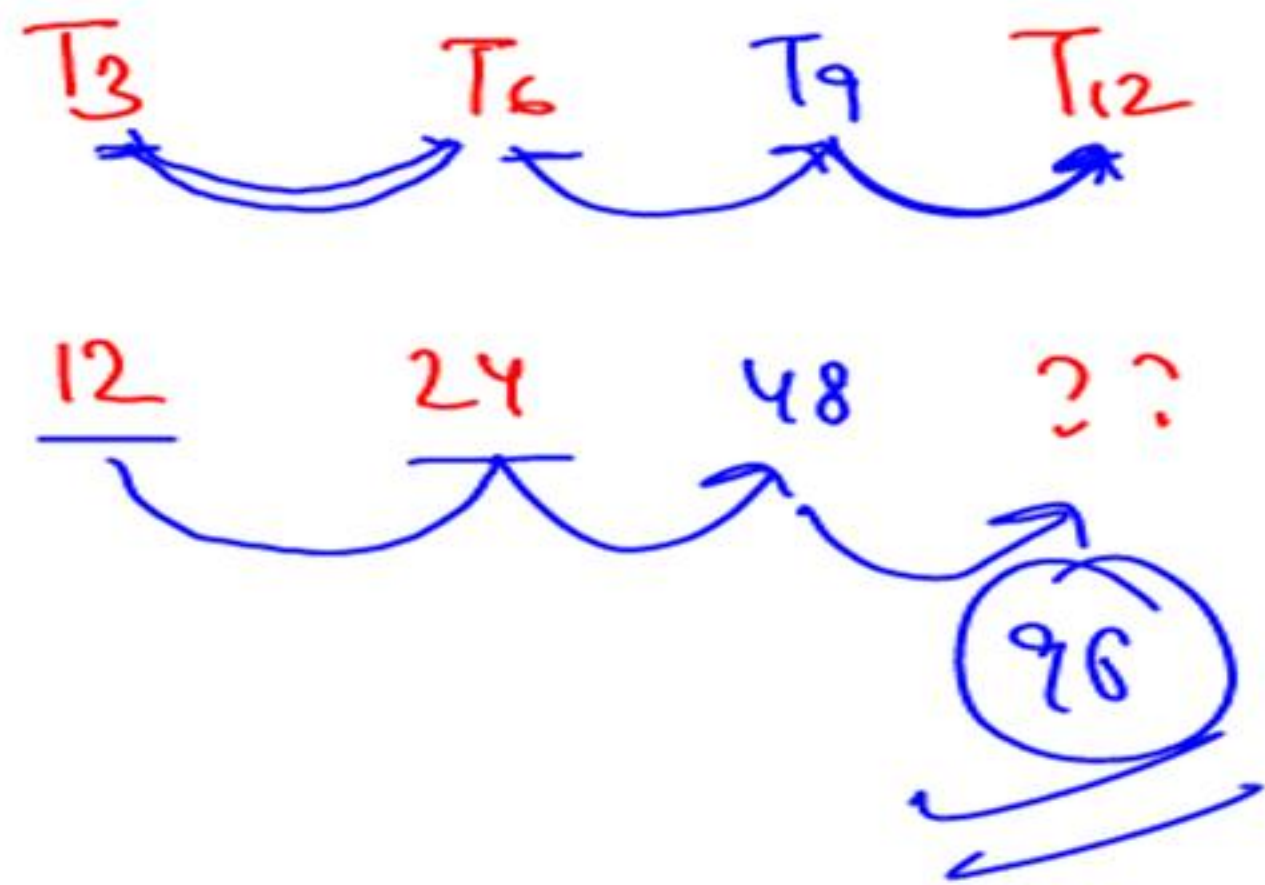
$$3^{6-n} = 3^{-6}$$

$$6-n = -6$$

$$\boxed{n = 12}$$

IInd

$$243, \quad 81, \quad 27, \quad ------- \quad \frac{1}{729}$$

$3^5 \qquad 3^1 \qquad\qquad 3^0 \qquad\qquad 3^{-6}$

⑤   ①   ⑥

→   12 term

Eg3. If $T_3 = 12$
$T_6 = 24$
$T_{12} = ??$

I^{st}

$a \cdot r^2 = 12$ — ①

$a r^5 = 24$ — ②

$\dfrac{a r^{11}}{} = ??$

② $\div$ ①

$$r^3 = 2$$

$(a r^2) \cdot r^9$

$12 \cdot (2)^3 = \boxed{96}$

IInd

$\underset{12}{T_3} \qquad \underset{24}{T_6} \qquad \underset{48}{T_9} \qquad \underset{??}{T_{12}}$

96

Ans. 96

Eg4. If $T_4 = 10$

$T_7 = 80$

$T_n = 2560$

$n = ??$

2min

I^{st}

$ar^3 = 10$ — ①

$ar^6 = 80$ — ②

$n = ??$

$ar^{n-1} = 2560$

② ÷ ①

$\dfrac{5}{4} \cdot (2)^{n-1} \to \dfrac{512}{2560}$

$r^3 = 8$

$r = 2$

$(2)^{n-1} = 2048$

$a \cdot 8 = 10$

$(2)^{n-1} = 2^{11}$

$a = 5/4$

$n = 12$

Ans. $n = 12$

T_4 T_7 T_n

10 80 2560

$r = 2$ $\dfrac{2560}{80}$ 32
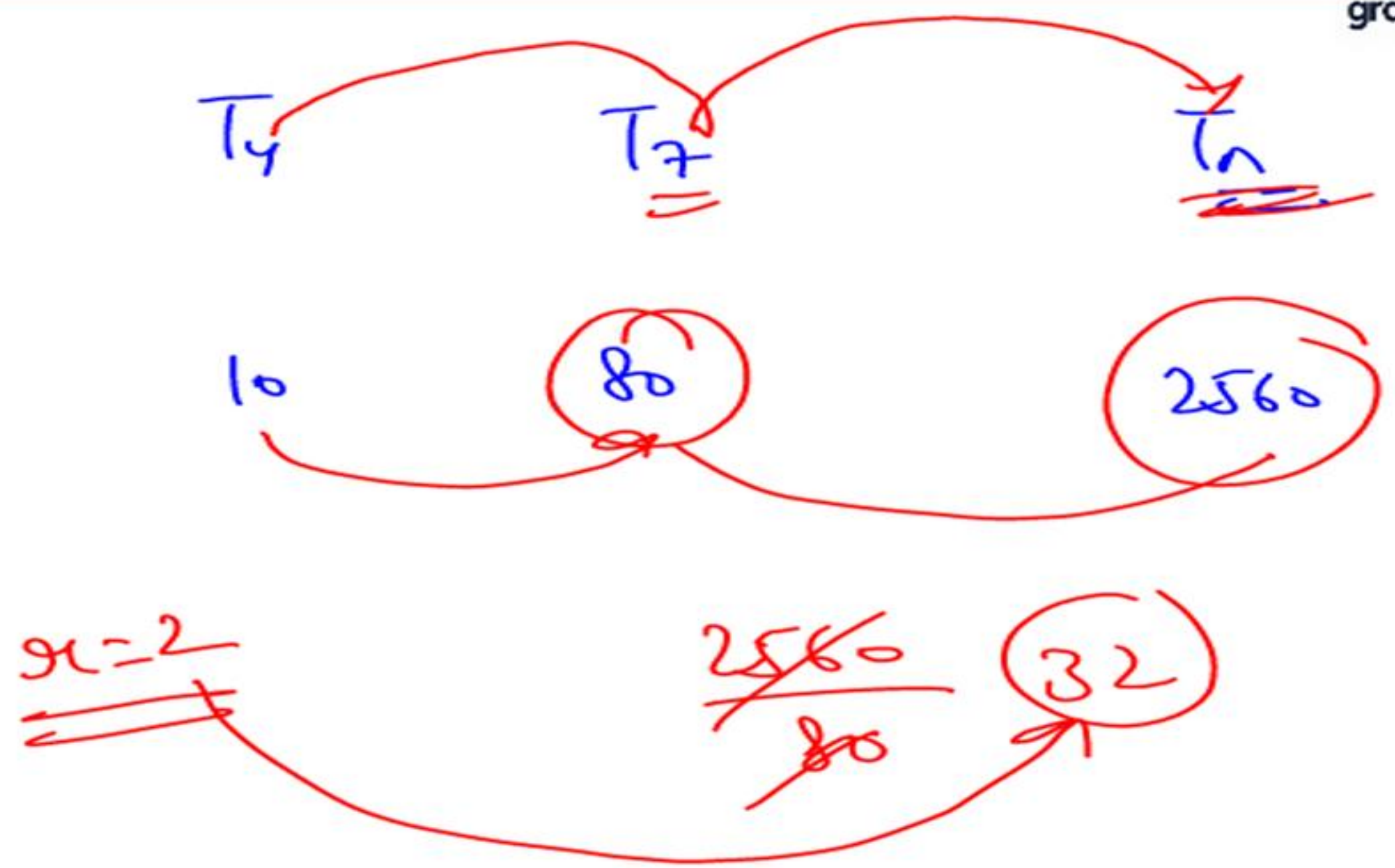
$n = 12$

Eg5. If $T_7 + T_9 = 40$

$T_8 + T_{10} = 80$

Find $T_{13} = ??$

I^{st}

$$ax^6 + ax^8 = 40$$

$$ax^6(1+x^2) = 40 \quad \text{——} \quad \textcircled{1}$$

$$ax^7 + ax^9 = 80$$

$$ax^7(1+x^2) = 80 \quad \text{——} \quad \textcircled{2}$$

$$\textcircled{2} \div \textcircled{1}$$

$$x = 2$$

$$ax^6(5) = 40$$

$$\boxed{ax^6 = 8}$$

$$T_{13} = ax^{12}$$

$$ax^6 \cdot x^6 \rightarrow 8 \cdot 2^6 = 512$$

$$T_7 + T_9 = 40$$

$$T_8 + T_{10} = 80$$

$$T_{13} = ??$$

5 unit $\longrightarrow$ 40

1 unit $\longrightarrow$ 8

$$d = 2$$

T_7 T_8 T_9 .... T_{13}

1 unit   2   4 unit

6 units

512
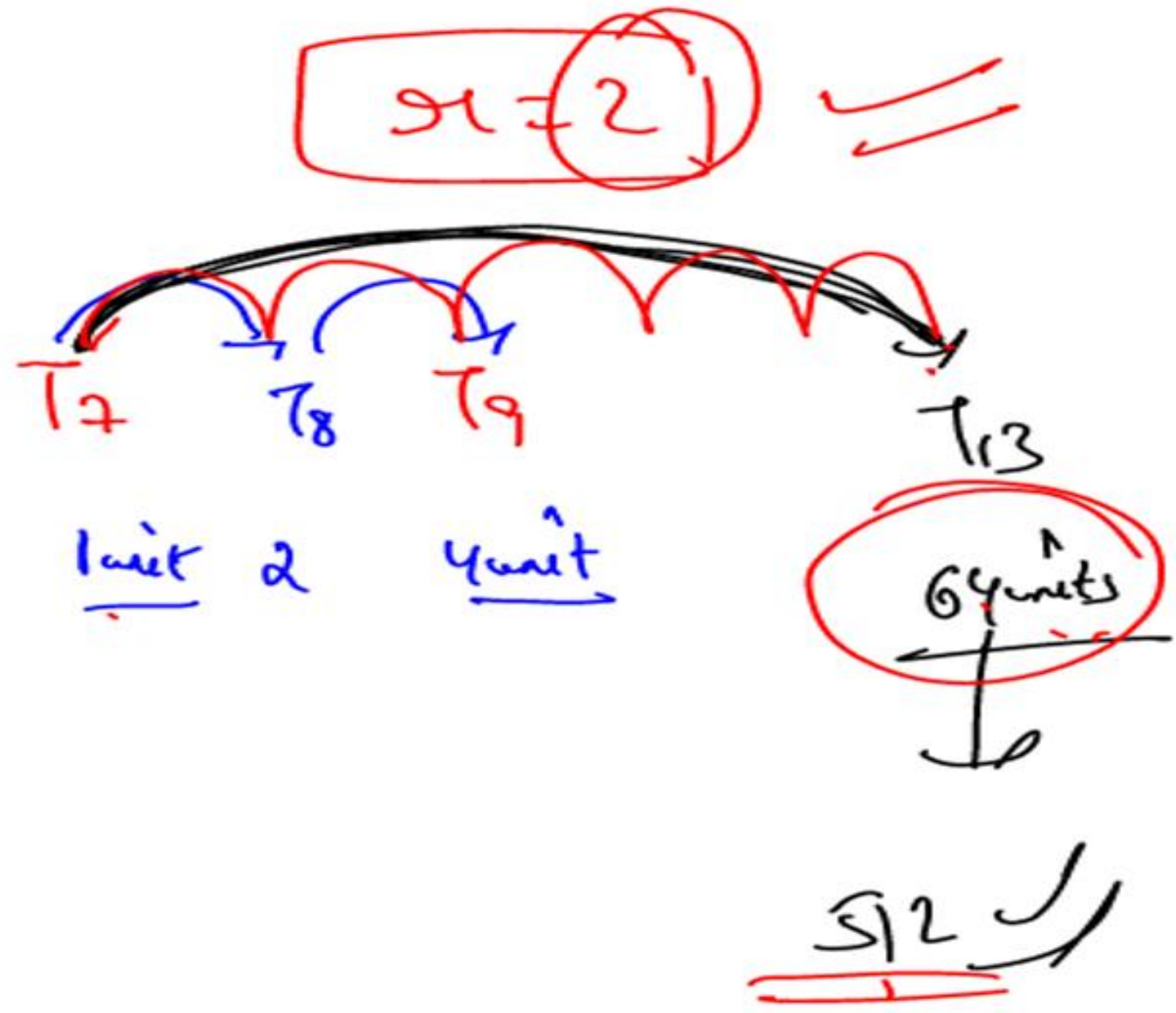
Ans.   512

Selection of terms in GP

$$3 \text{ terms} \rightarrow \frac{a}{r}, a, ar$$

$$5 \text{ terms} \rightarrow \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

$$4 \text{ terms} \rightarrow \frac{a}{r^3}, \frac{a}{r}, ar, ar^3$$

Eg6. If sum of 3 consecutive terms in GP is 38 & their product is 1728. Find r [if r > 1].

$$\frac{a}{r} + a + ar = 38$$

$$\frac{a}{r} \cdot a \cdot ar = 1728$$

$$a^3 = 1728 \qquad a = 12$$

$$r = \frac{2}{3}, \frac{3}{2}$$

$$\frac{12}{r} + 12r = 26$$

$$12r^2 - 26r + 12 = 0$$

$$6r^2 - 13r + 6 = 0$$

Ans. $\dfrac{3}{2}$

Sum of 'n' terms of GP

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

where, $r \neq 1$

Eg7. Find the sum of the series :

2+6+18+54+.......................... till (7 terms)

$$S_n = \frac{a\left(x^n - 1\right)}{x - 1}$$

$$= \frac{2\left(3^7 - 1\right)}{3 - 1} = \frac{2186}{}$$

Ans. 2186

Eg8. If $T_1 = 14$

$\qquad T_n = 896$

$\qquad S_n = 1778$

Find r = ??

$$a = 14$$

$$14 \cdot x^{n-1} = 896$$

$$\boxed{x^{n-1} = 64}$$

$$a(x^n - 1) = 1778$$

$$14 \cdot (x^{n-1} \cdot x - 1) = 1778 \quad \overset{127}{}$$

$$64x - 1 = 127$$

$$\boxed{x = 2}$$

Ans.   r = 2

Eg9. Find the value :

2+22+222+2222+ .... upto n terms.

$$= 2\left[1 + 11 + 111 + \text{-----} \quad n \text{ terms} \right]$$

$$= \frac{2}{9}\left[9 + 99 + 999 + \text{-----} \quad n \text{ terms} \right]$$

$$= \frac{2}{9}\left[(10-1) + (100-1) + (1000-1) + \text{----} \quad n \text{ terms} \right]$$

$$= \frac{2}{9}\left[(10 + 100 + 1000 + \text{---}) - n \right]$$

$$= \frac{2}{9}\left[\frac{10(10^n - 1)}{9} - n \right]$$

Sum of infinite terms of GP

$\rightarrow$ a, ar, ar^2, ar^3, ... (∞ terms)

$$S_\infty = \frac{a}{1-r}$$

$$if \ \ 0 < r < 1$$

eg

$$100, \ 50, \ 25, \ - - - - - - - - -$$

$$S_\infty = \frac{100}{1-\frac{1}{2}} = 200$$

Eg10. Find the sum of infinite terms of this GP

$$80, 40, 20, 10, 5, \frac{5}{2}, \dots\dots\dots\dots\dots\dots\dots$$

$$S_\infty = \frac{80}{1 - \frac{1}{2}} \Rightarrow \underline{160} \checkmark$$

Eg11. $5^{8+4+2+1+\ldots\ldots\ldots\ldots\ldots\infty} = (125)^K$

Find the value of K.

$$5^{\frac{8}{1-\frac{1}{2}}} = (5^3)^K$$

$$5^{16} = 5^{3K}$$

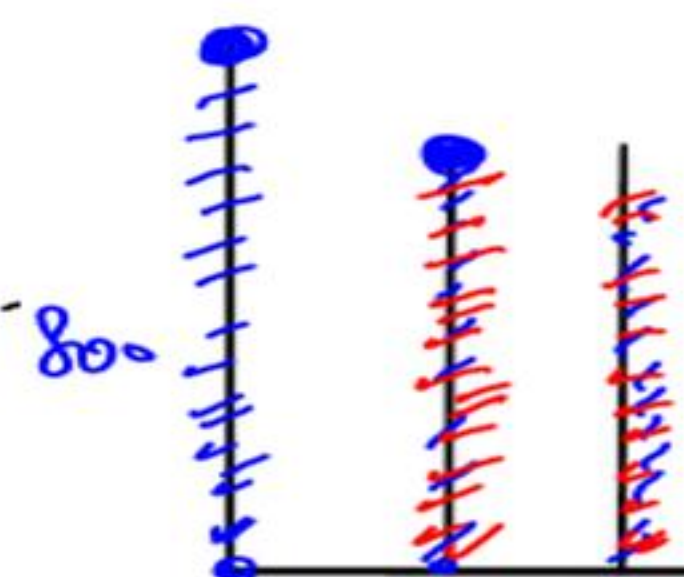$$\boxed{K = \frac{16}{3}}$$

Ans

Q1. After striking a floor, a certain ball rebounds to $\left(\frac{3}{4}\right)^{th}$ of the height from which it has fallen.

Find the distance it travels before coming to rest, if it is dropped from a height of 800 m.

$$800 + 600 + 600 + 450 + 450 + \cdots$$

$$\left(800 + 800 + 600 + 600 + \cdots\right) - 800$$

$$2\left[800 + 600 + \cdots\right] - 800$$

$$2\left[\frac{800}{1 - \frac{3}{4}}\right] - 800$$

$$= 5600$$

Shortcut for ball rebound:

H → **Height from which the ball is dropped.**

$\dfrac{a}{b}$ → **Fraction with which it rebounds (here a < b)**

D → **Total distance travelled by the ball**

$$D = H\left(\frac{b+a}{b-a}\right)$$

$$D = 800\left(\frac{4+3}{4-3}\right)$$

$$= 5600$$

GEOMETRIC MEAN

If a and b are two numbers and G is their GM (Geometric Mean)
a, G & b are in GP.

$$\frac{G}{a} = \frac{b}{G}$$

$$G^2 = ab$$

$$G = \sqrt{ab}$$