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Sahi Prep Hai Toh Life Set Hai

TRIANGLE-3

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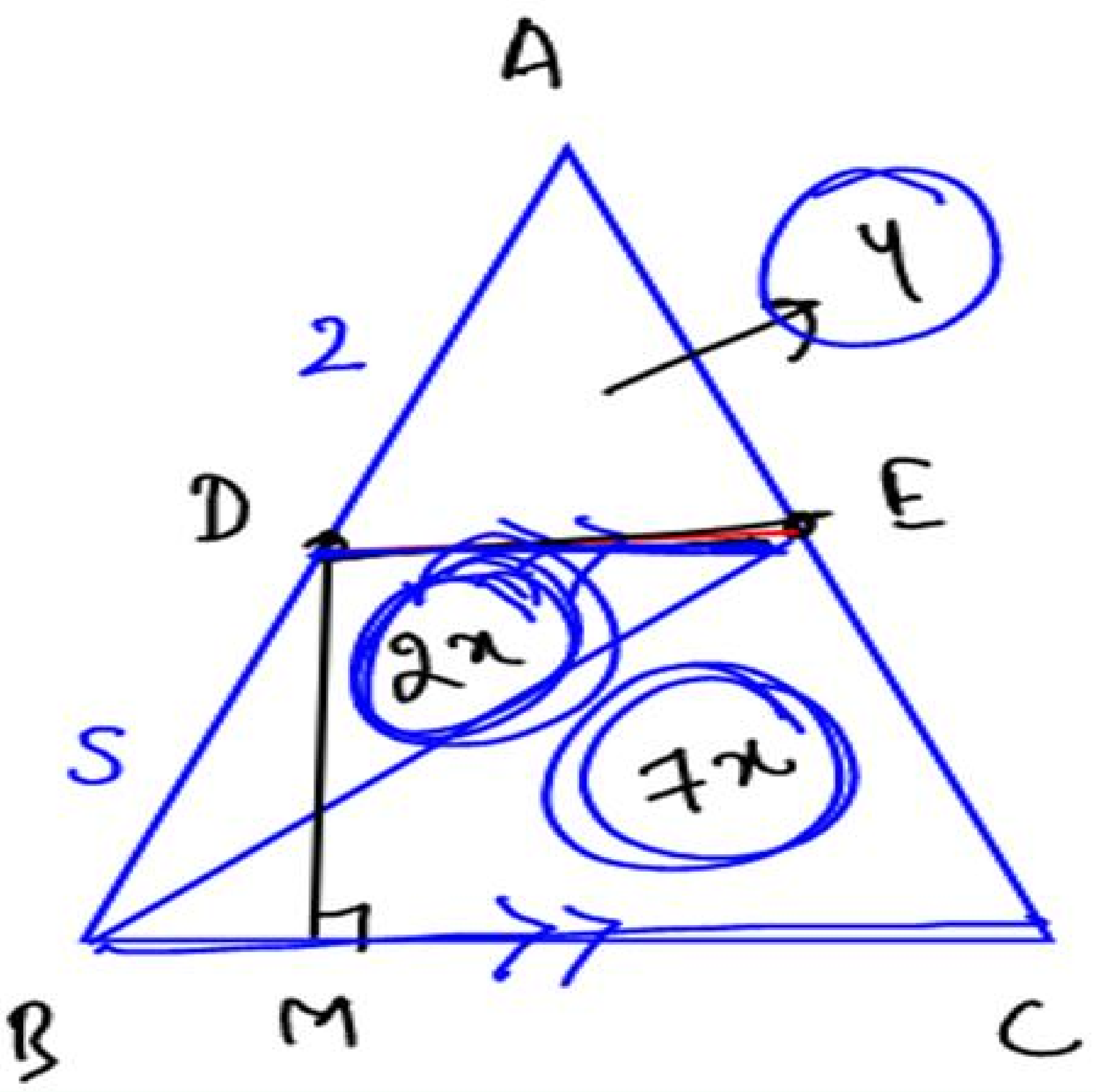
Eg. In a ΔABC, points D and E are taken on AB & AC in such that DE || BC.

4 10 35

If $\frac{AD}{DB} = \frac{2}{5}$, find (Area of ΔADE : Area of ΔDEB : Area of ΔBEC)



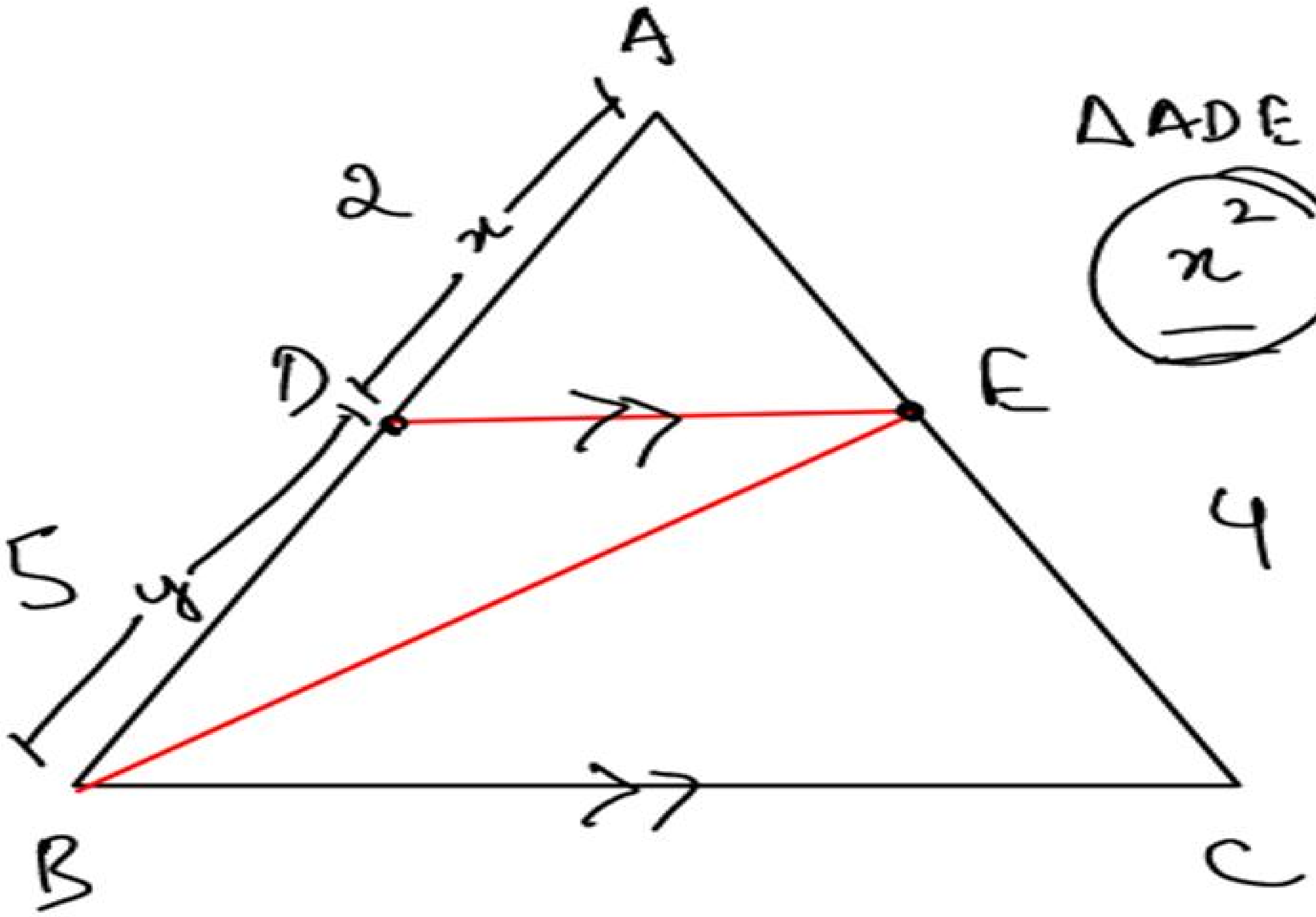
$$\frac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \left(\frac{2}{7}\right)^2 \Rightarrow \frac{4}{49}$$

ΔDEB & ΔBEC

$$\frac{\text{area of } \Delta DEB}{\text{area of } \Delta BEC} = \frac{\cancel{\frac{1}{2}} \cdot DE \cdot \cancel{DM}}{\cancel{\frac{1}{2}} \cdot BC \cdot \cancel{DM}} = \frac{2}{7}$$

$4 + 9x = 45$

$x = 5$

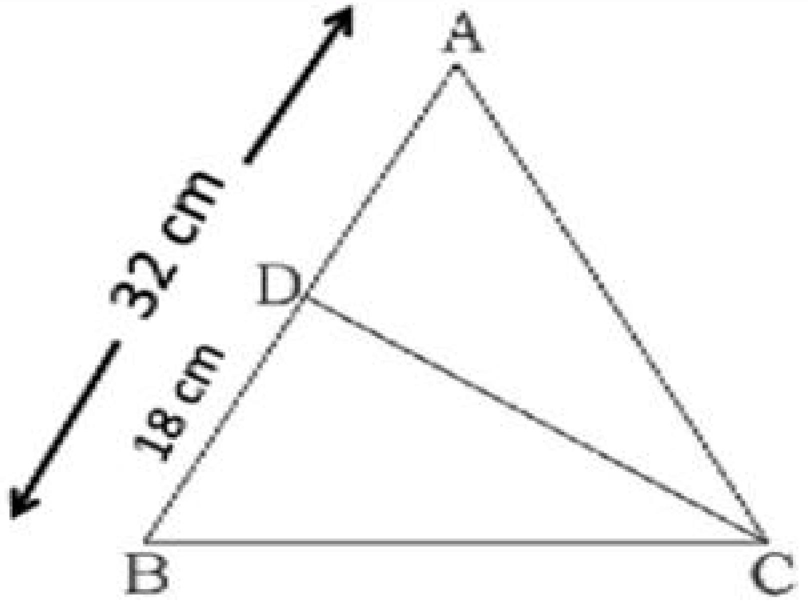


$\Delta ADE : \Delta DEB : \Delta BEC$

$x^2 \quad xy \quad (x+y)y$

$4 : 10 : 35$

Eg. In the given figure, $\angle BAC = \angle BCD$, AB = 32 cm and BD = 18 cm, then the ratio of perimeter of ΔBDC and ΔABC is:



(a) 4 : 3 (b) 8 : 5

(c) 5 : 8 (d) 3 : 4

Done in last class

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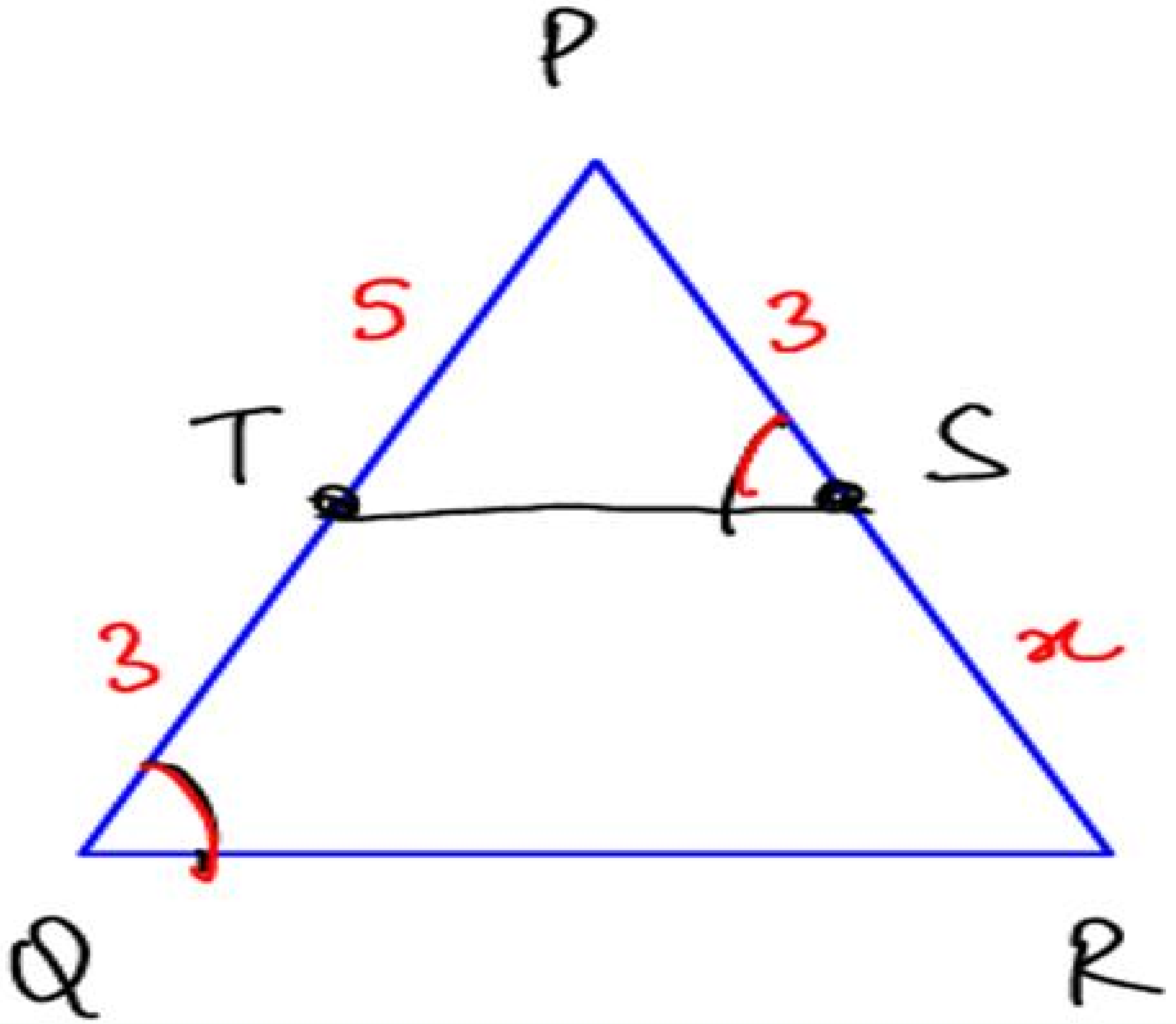
Imp

Eg. In ΔPQR, S and T are points on side PR and PQ respectively such that, $\angle PQR = \angle PST$. If PT = 5 cm, PS = 3 cm and TQ = 3 cm, then length of SR is

(a) 5 cm (b) 6 cm

(c) $\frac{31}{3}$ cm (d) $\frac{41}{3}$ cm



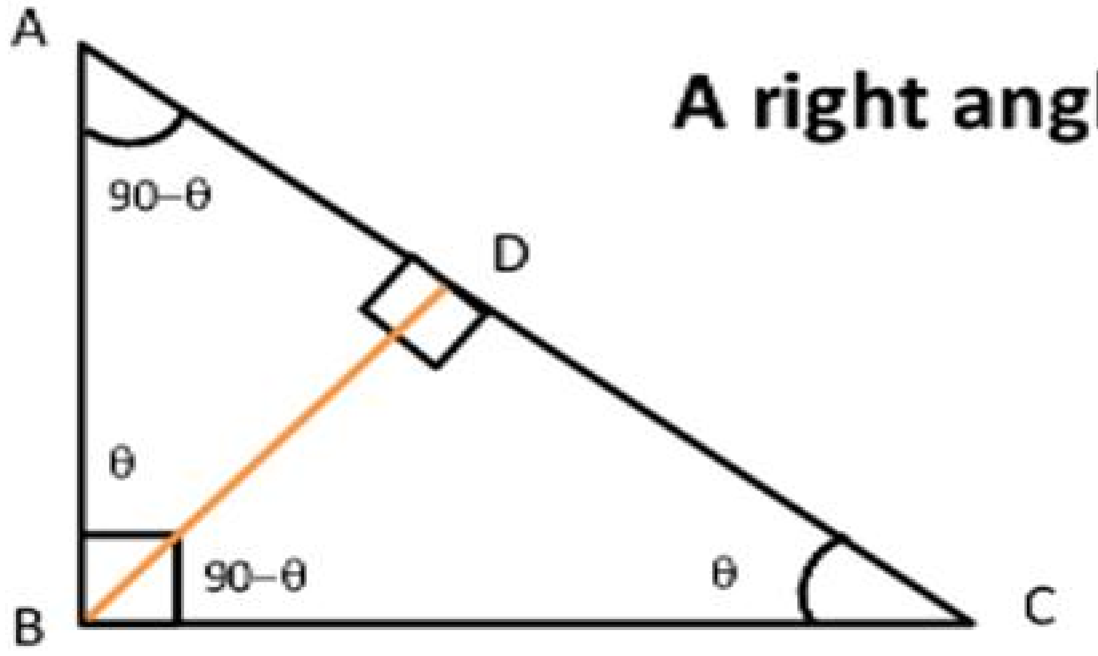
$$\Delta PST \sim \Delta PQR$$

$$\frac{3}{8} = \frac{5}{3+x}$$

$$9 + 3x = 40$$

$$x = \frac{31}{3}$$

SIMILARITY IN RIGHT ANGLE TRIANGLE

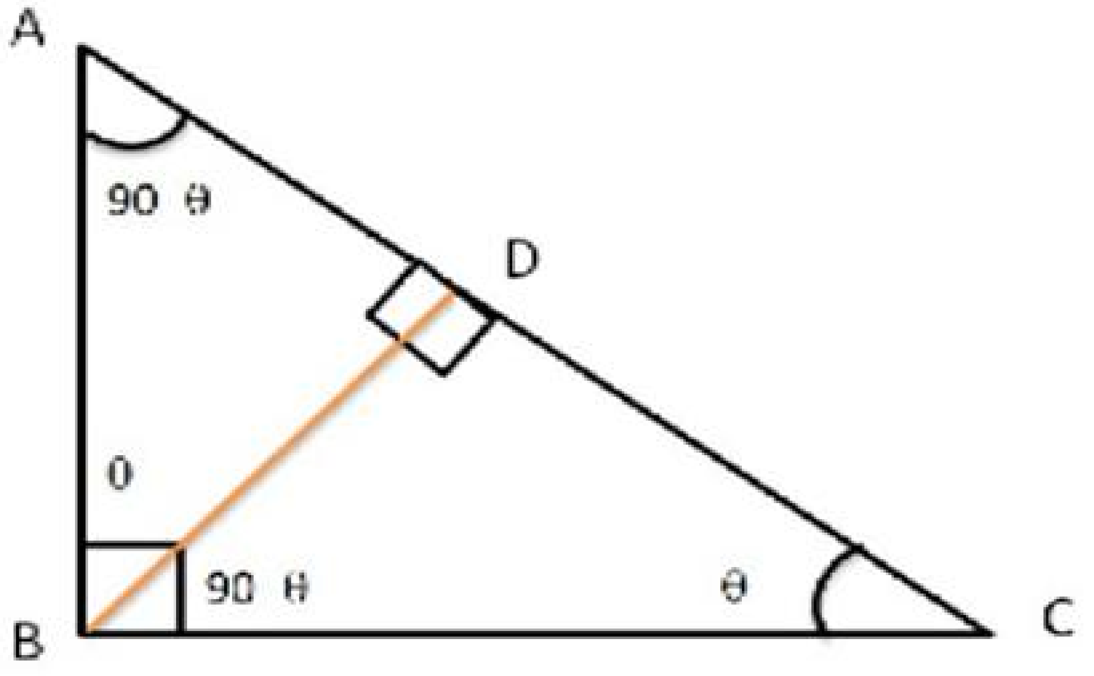


A right angle Δ, right angle at B and BD is perpendicular to AC.

90−θ 90 θ   90−θ 90 θ   90−θ 90 θ

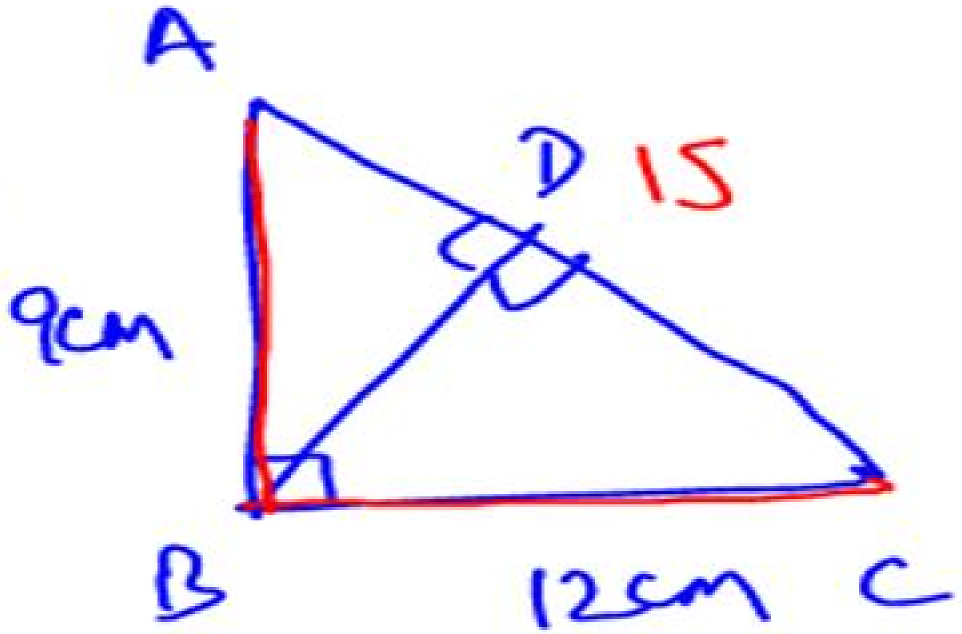
$\Delta ABC \sim \Delta ADB \sim \Delta BDC$

(1) A right angle Δ, right angle at B and BD is perpendicular to AC.



$$\boxed{BC \cdot AB = AC \cdot BD}$$

eg

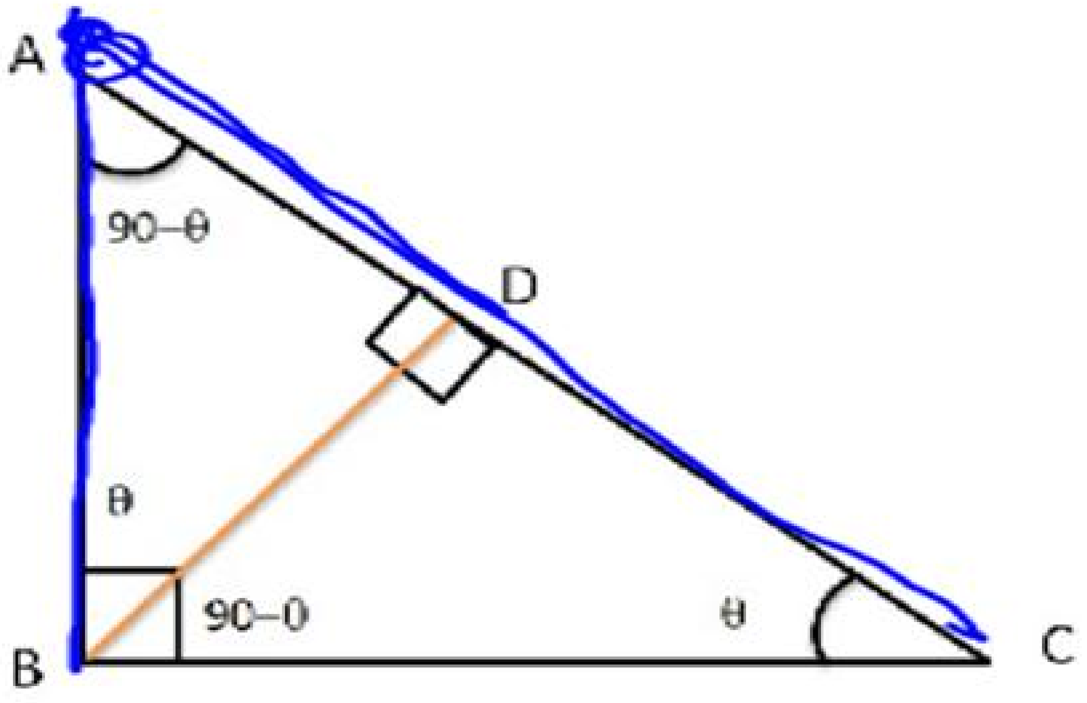


Find BD ??

$$12 \cdot \overset{3}{\cancel{9}} = \overset{5}{\cancel{15}} \cdot BD$$

$$\boxed{BD = 7.2\,cm}$$

(2) A right angle Δ, right angle at B and BD is perpendicular to AC.

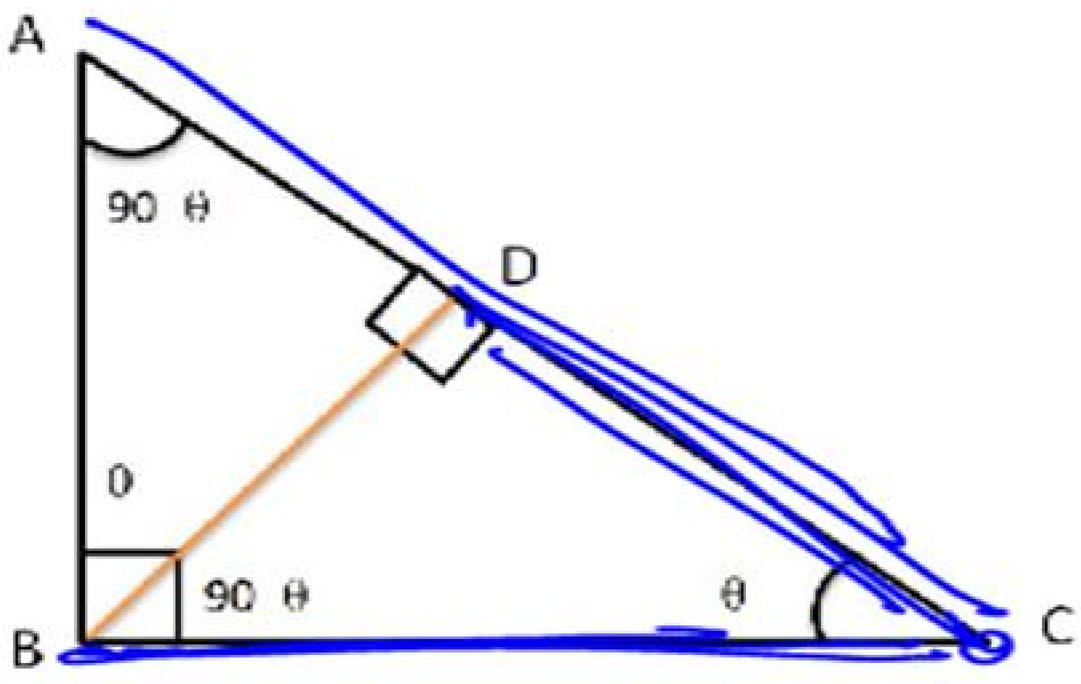


$\Delta ABC \sim \Delta ADB$

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$$AB^2 = (AD)(AC)$$

(3) A right angle Δ, right angle at B and BD is perpendicular to AC.



90−θ 90 θ   90−θ 90 θ

$\Delta ABC \sim \Delta BDC$

$$\frac{AC}{BC} = \frac{BC}{DC}$$

$$\boxed{BC^2 = AC \times DC}$$

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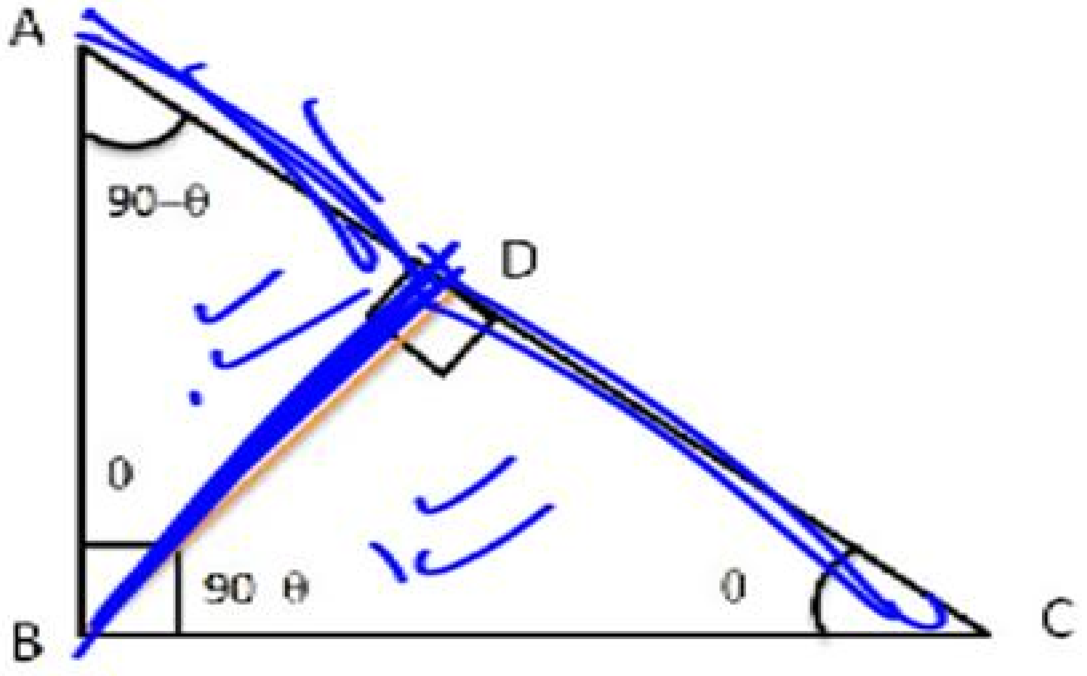
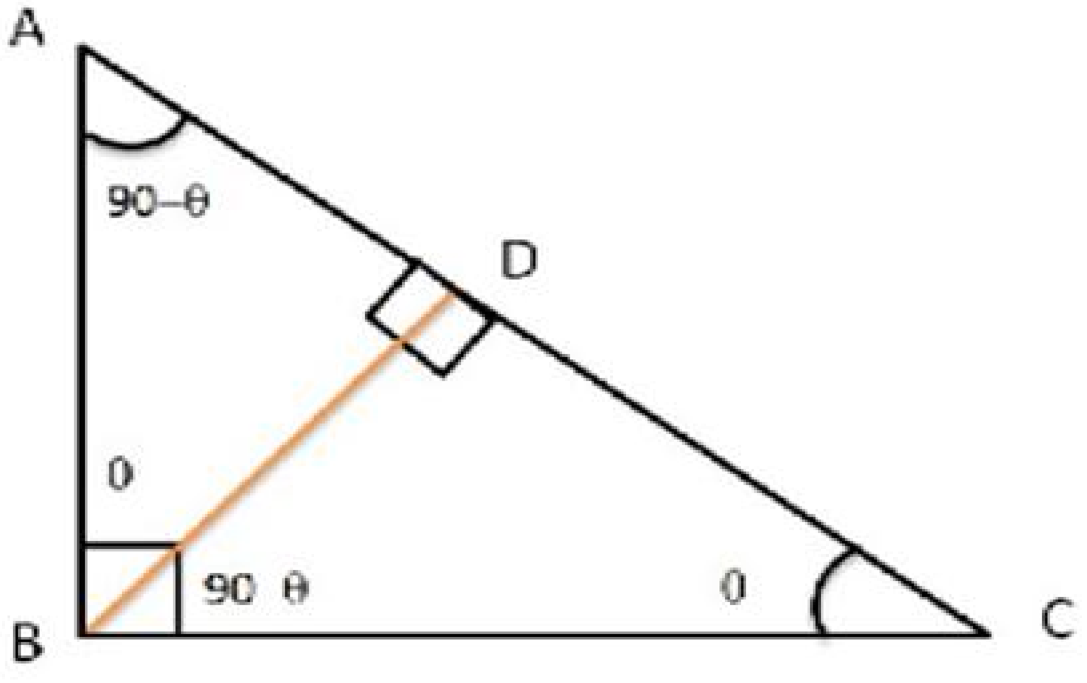
(4) A right angle Δ, right angle at B and BD is perpendicular to AC.



$$BD^2 = (DA)(DC)$$

(5) A right angle Δ, right angle at B and BD is perpendicular to AC.

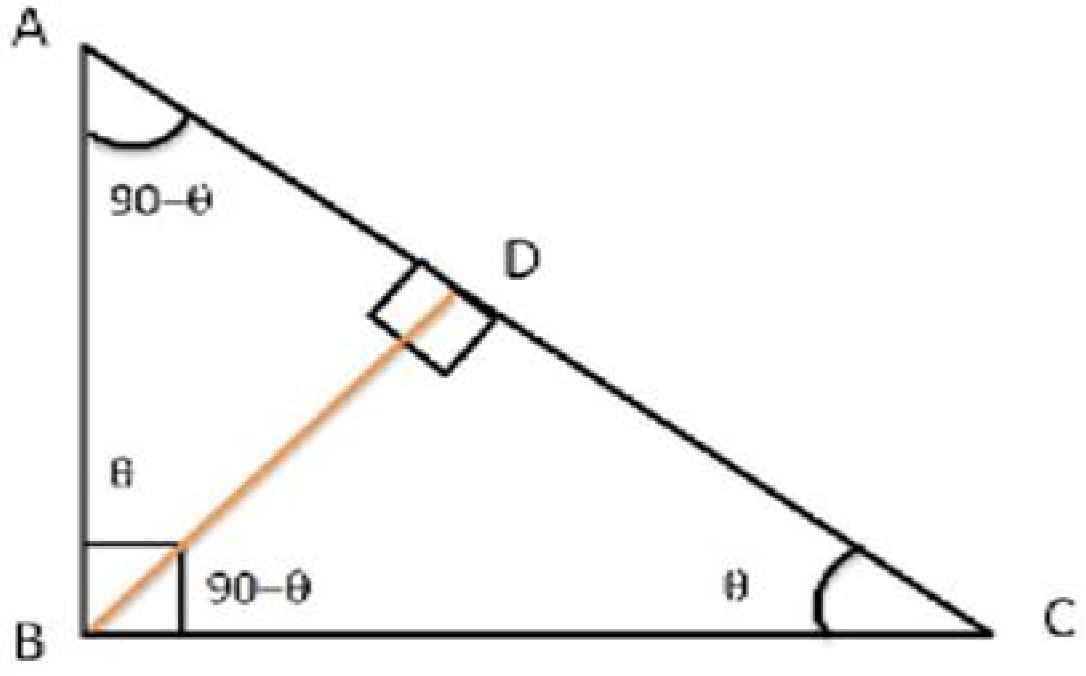


$$(BC)^2 \cdot (AB)^2 = (AC)^2 \cdot (BD)^2$$

$$(BC)^2 (AB)^2 = (AB^2 + BC^2)(BD)^2$$

$$\frac{1}{(BD)^2} = \frac{AB^2 + BC^2}{(AB)^2 \cdot (BC)^2}$$

$$\frac{1}{(BD)^2} = \frac{1}{(BC)^2} + \frac{1}{(AB)^2}$$



(1) $AB \times BC = AC \cdot BD$

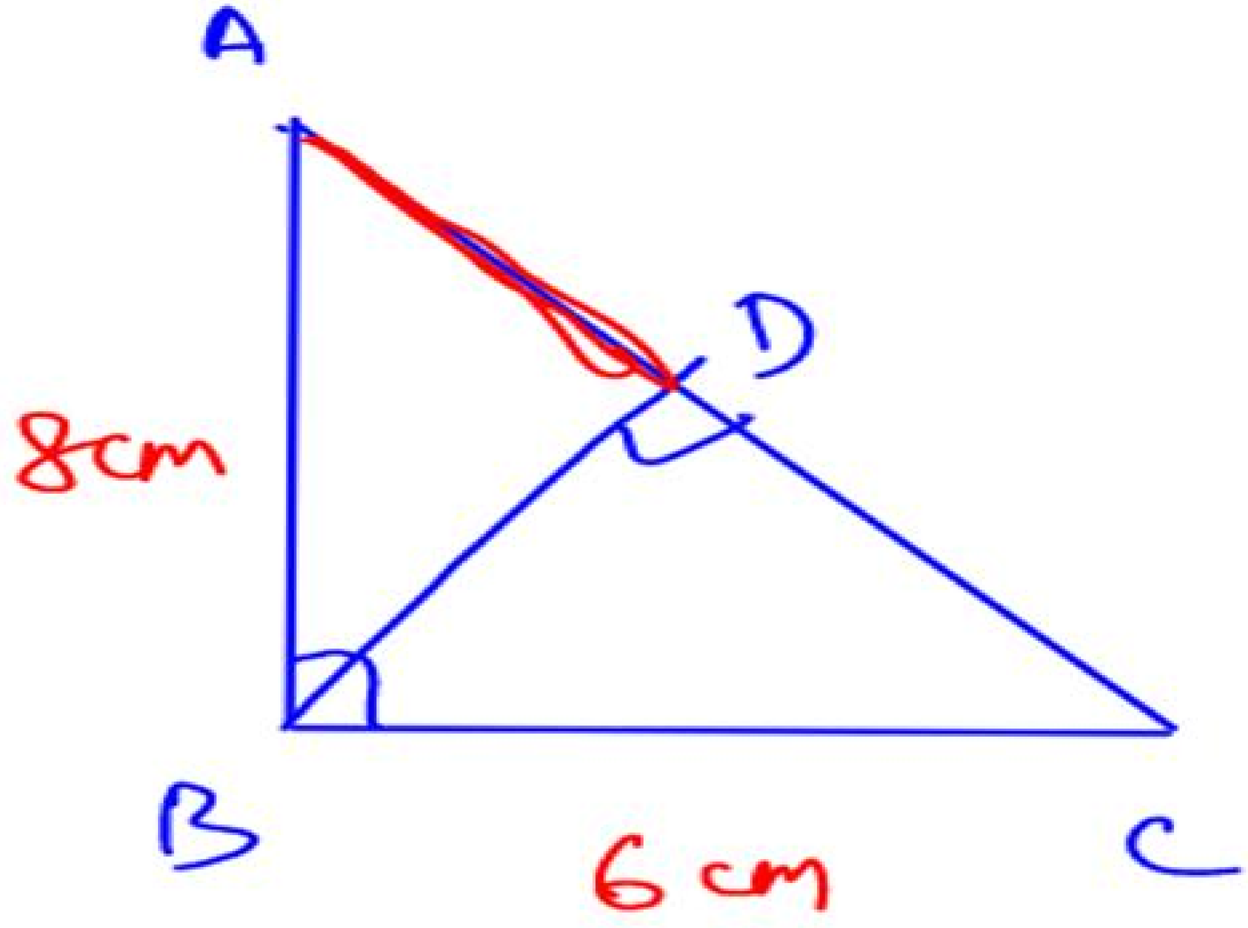
(2) $BA^2 = AD \cdot AC$

(3) $BC^2 = CD \cdot CA$

(4) $BD^2 = DA \cdot DC$

(5) $\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$

Eg1



If $BC = 6\,cm$

$AB = 8\,cm$

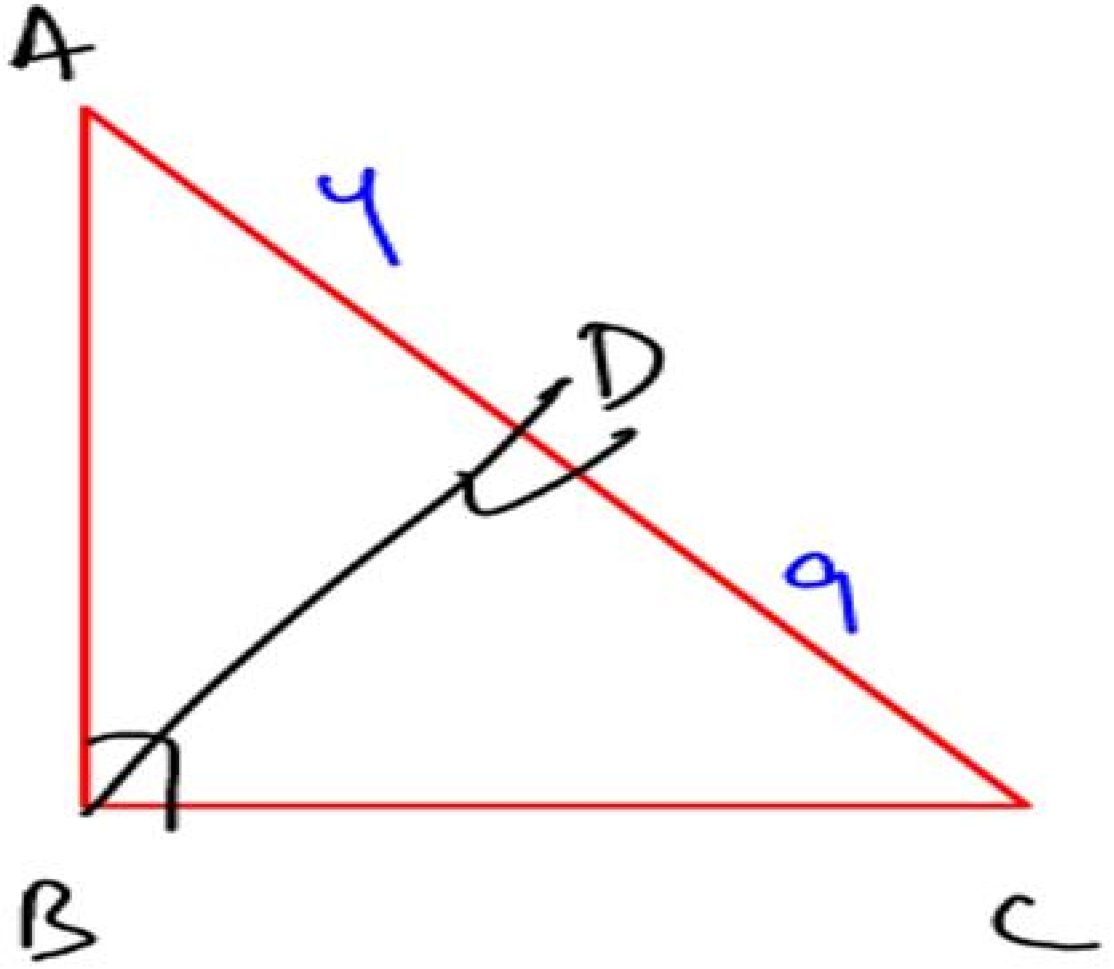
$AD = ??$

$$(BA)^2 = (AD)(AC)$$

$$64 = AD\ 10$$

$$AD = 6.4$$

Eg3



AD = 4 cm

AC = 13 cm

BD = ??

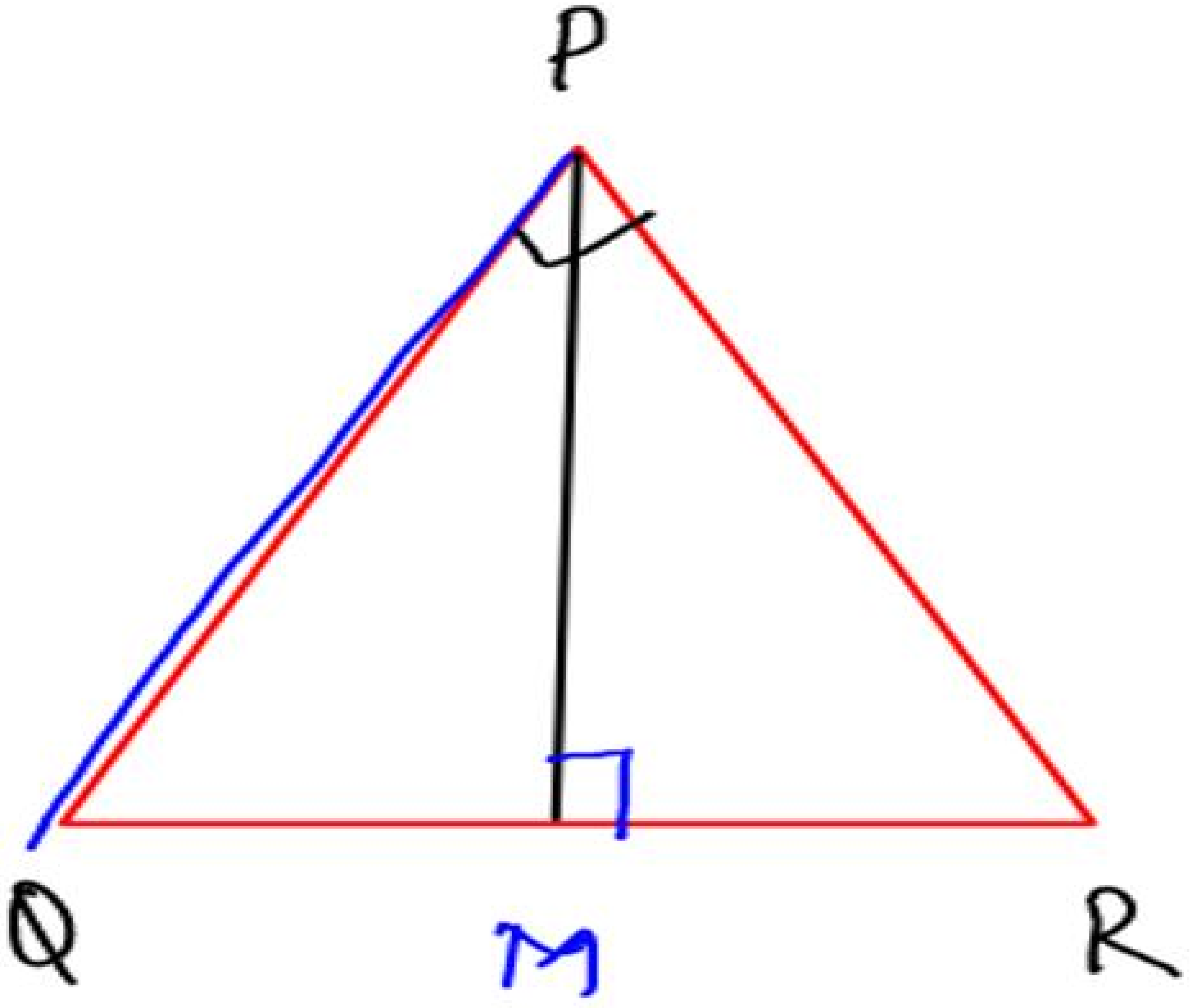
6 cm

$$(BD)^2 = (AD)(CD)$$

$$= 4 \cdot 9$$

$$(BD)^2 = 36$$

EXAMPLES ON SIMILARITY IN RIGHT ANGLE Δ



1. $(QP)(PR) = (QR)(PM)$
2. $(PQ)^2 = (QM)(QR)$
3. $(PR)^2 = (RM)(RQ)$
4. $(PM)^2 = (QM)(MR)$
5. $\frac{1}{(PM)^2} = \frac{1}{(PQ)^2} + \frac{1}{(PR)^2}$

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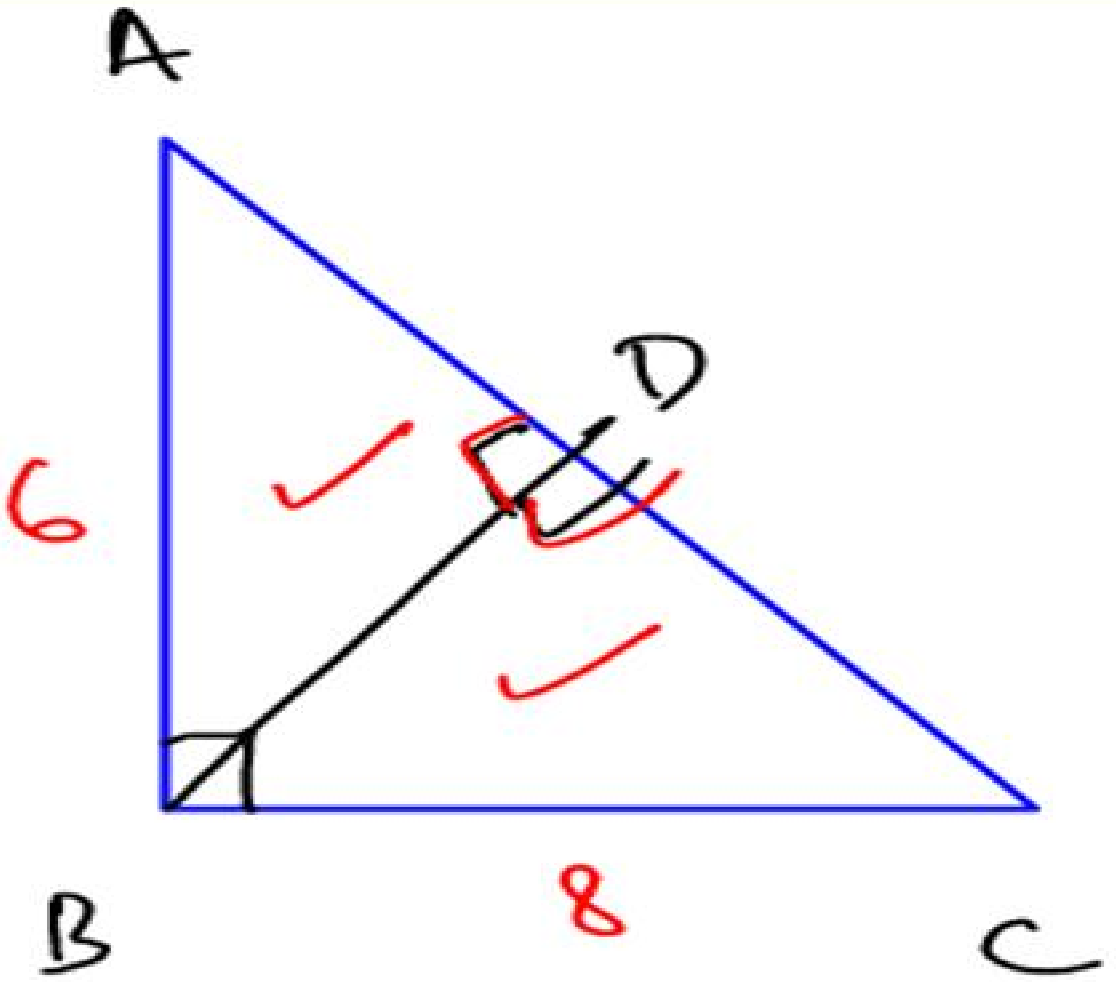
Eg



If AB = 6cm

BC = 8 cm

$$\frac{\text{area of } \triangle ABD}{\text{area of } \triangle BDC} = ??$$

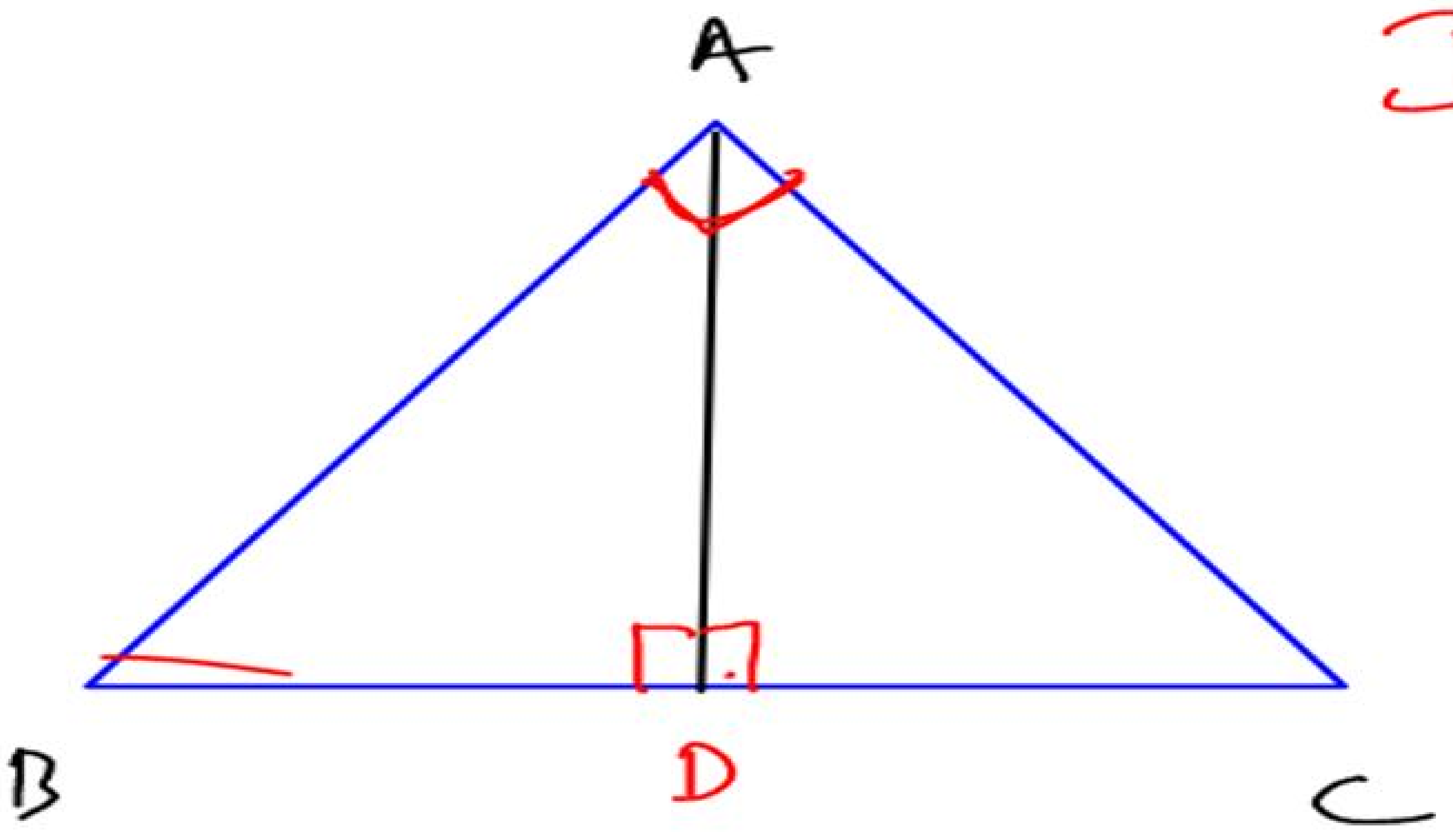
$$\left(\frac{6}{8}\right)^2$$

$$= \frac{9}{16}$$

Eg. In a ΔABC,

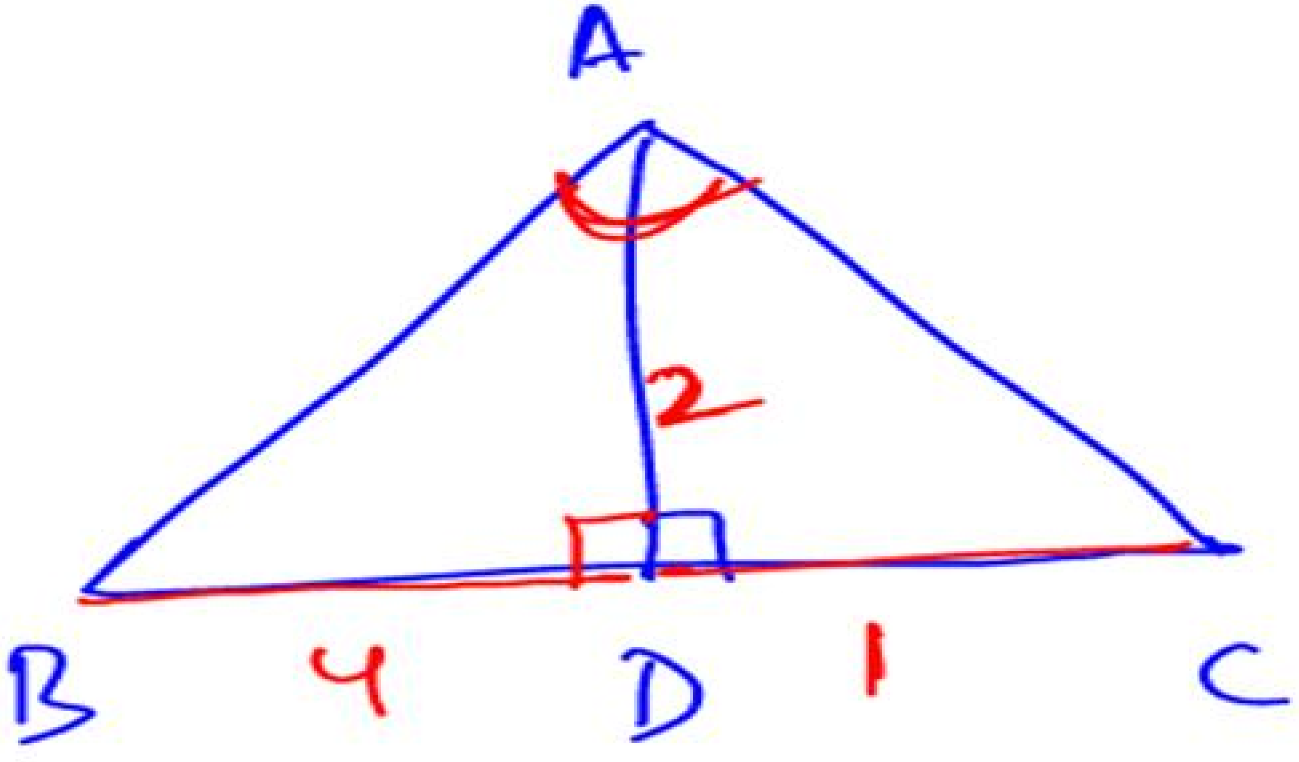
$AD \perp BC$ & $AD^2 = BD \cdot DC$

Find $\angle BAC = ??$



I If you know the result

$\angle BAC = \underline{\underline{90^\circ}}$



$AD^2 = BD \cdot DC$

find $\angle BAC = ??$

$\underline{\underline{II^{nd}}}$

let $AD = 2$

$BD = 4$

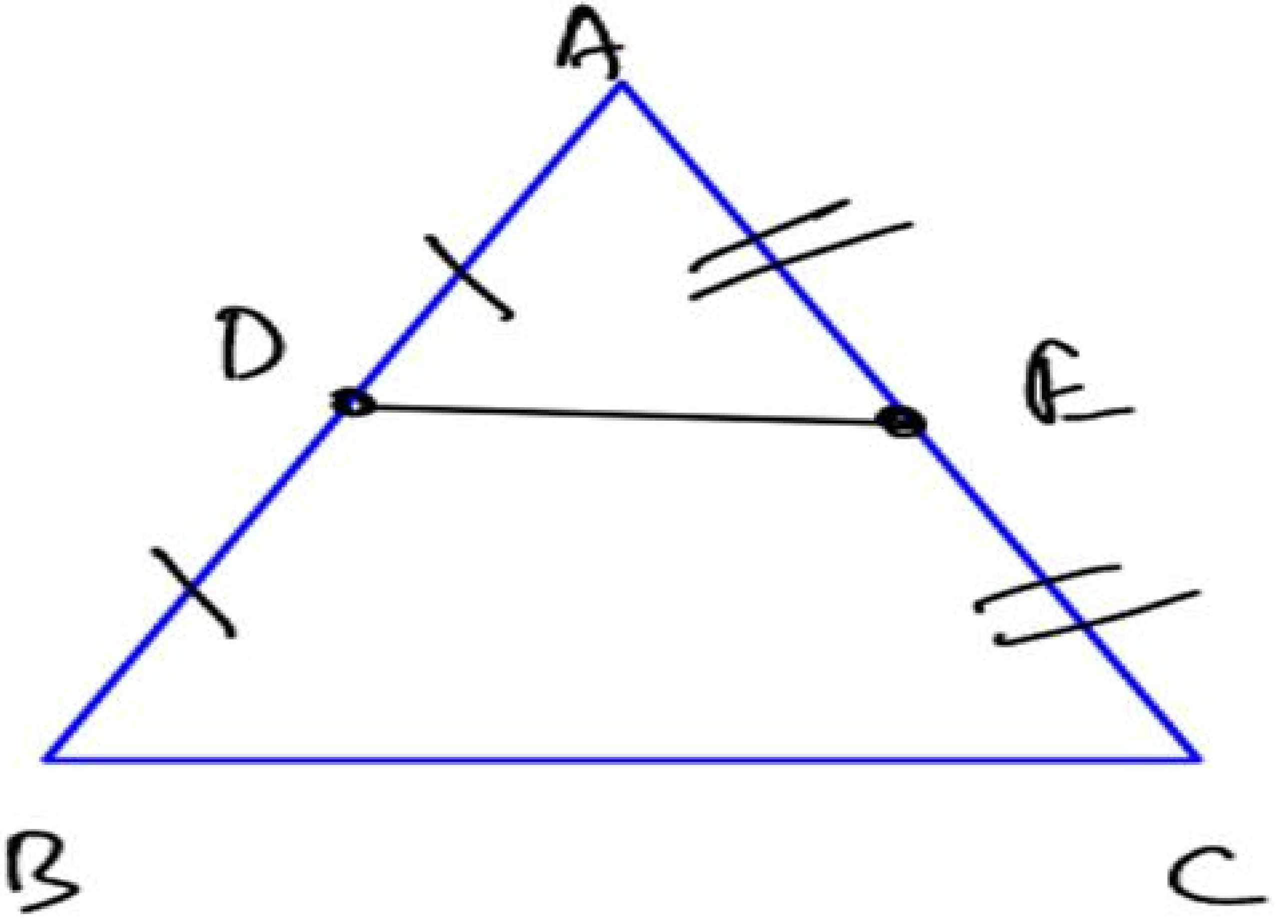
$CD = 1$

$AB = \sqrt{20}$ $AC = \sqrt{5}$

$AB^2 + AC^2 = BC^2$

MID-POINT THEOREM

If we join mid-points of any 2 sides of a Δ by a line segment then that line segment will be parallel to the third side and half of it.



Given D, E are mid points of AB & AC

$$DE \parallel BC$$

$$\& \quad DE = \frac{1}{2} BC$$

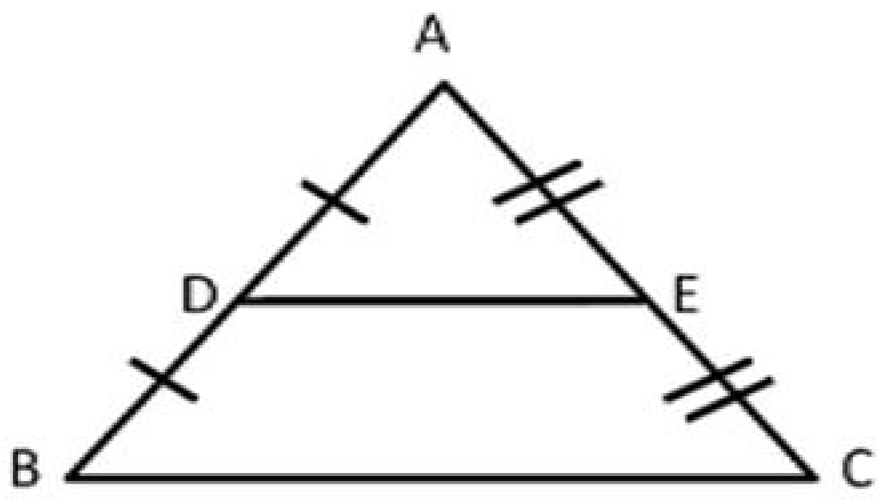
Eg. Given,

D is mid-point of AB.

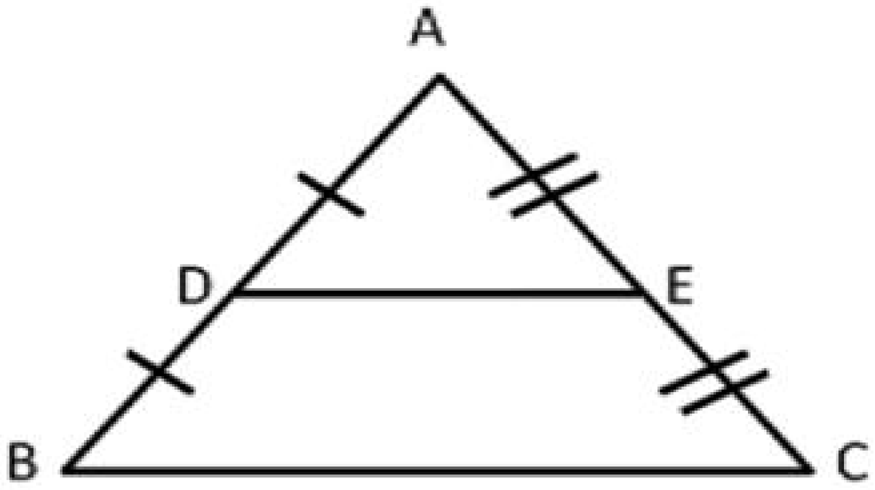
E is mid-point of AC.

$$DE \,||\, BC$$

$$DE = \frac{1}{2} BC$$



Proof of Mid-point theorem:



Given, D, E are mid-point of AB & AC.

To prove: (i) $DE \,||\, BC$

(ii) $DE = \frac{1}{2} BC$

Proof: AD : AB = 1 : 2

AE : AC = 1 : 2

ΔADE ~ ΔABC (SAS similarity)

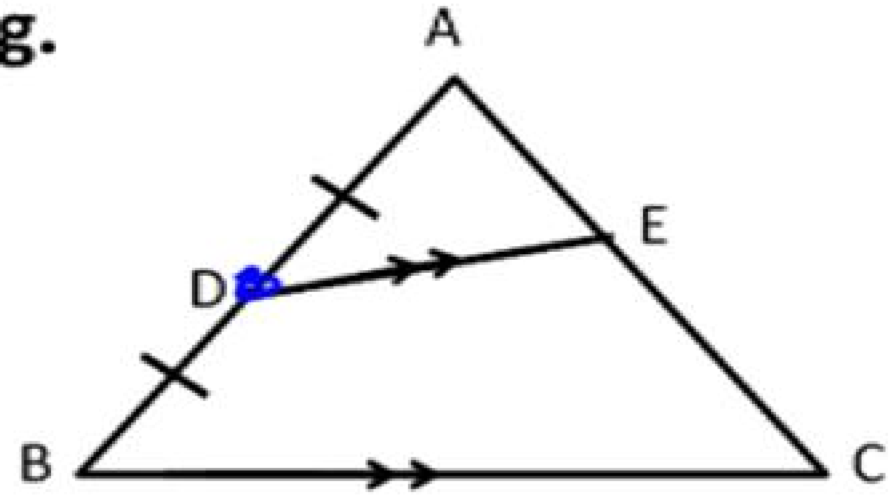
$\angle$ADE = $\angle$ABC (Corresponding angles)

$DE \,||\, BC$

$DE = \frac{1}{2} BC$

CONVERSE OF MID-POINT THEOREM

Eg.



Given,
D is mid-point of AB.
DE | |BC

E is mid-point of AC.

CONGRUENCY

Two figures are said to be congruent, if they are exactly same in every aspect.

2 line segments are congruent ? when their length are equal

2 circles are congruent ? when their radius are equal

2 squares are congruent ? when sides are equal

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$\Delta ABC \cong \Delta DEF$

Then

symbol of congruent

* $\angle A = \angle D, \quad \angle B = \angle E, \quad \angle C = \angle F$

* $AB = DE, \quad BC = EF, \quad AC = DF$

Triangles

3 Angles Δ 3 sides

CONDITIONS OF CONGRUENCY

(1) SSS

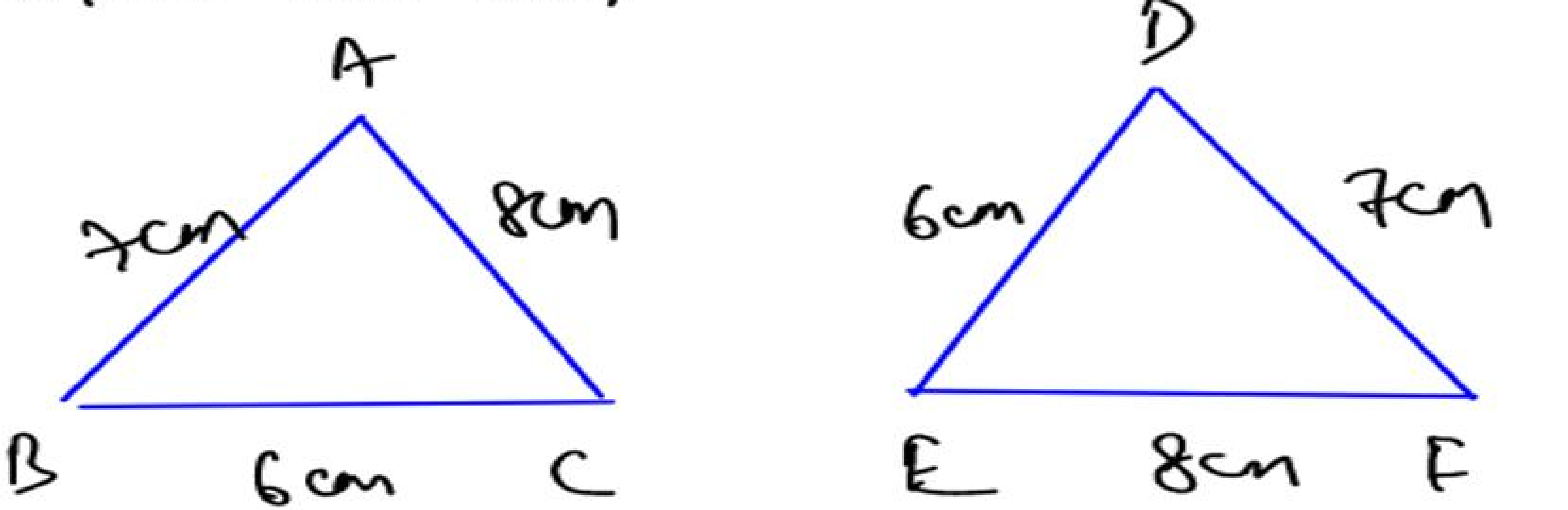
(2) SAS

(3) ASA

(4) AAS

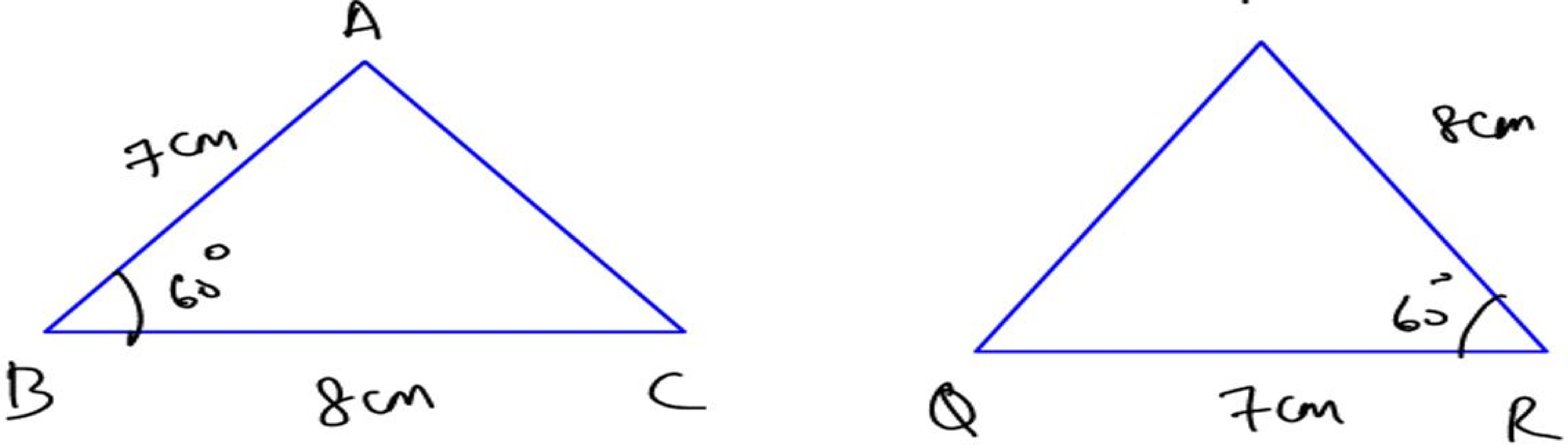
(5) RHS

SSS (Side – Side – Side)



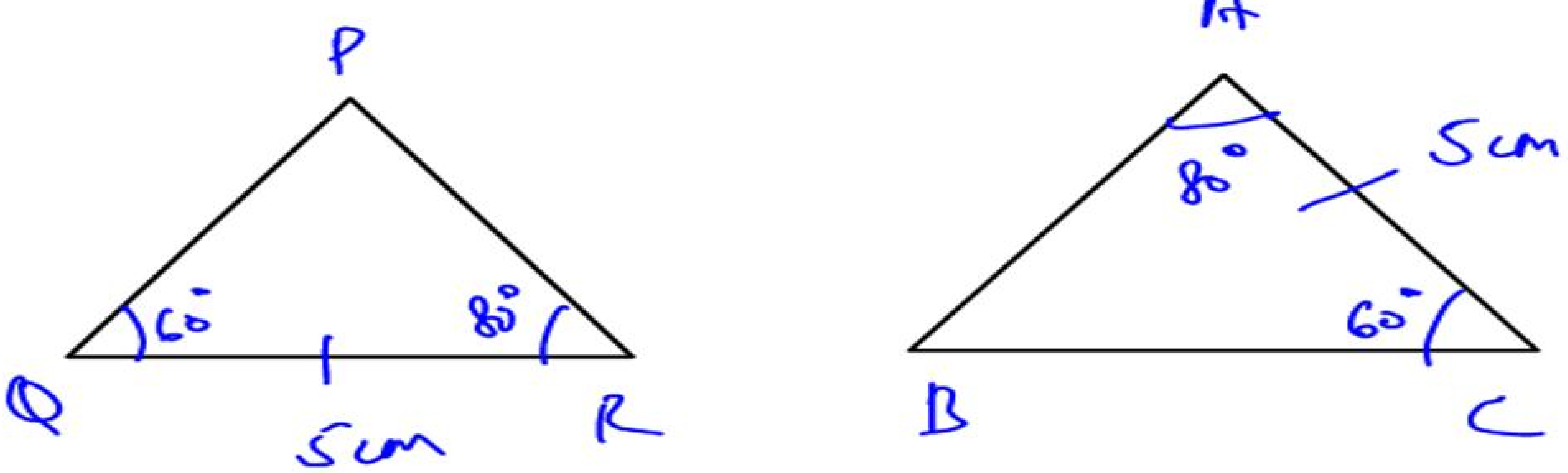
$\triangle ABC \cong \triangle FDE$

SAS (Side – Angle – Side)



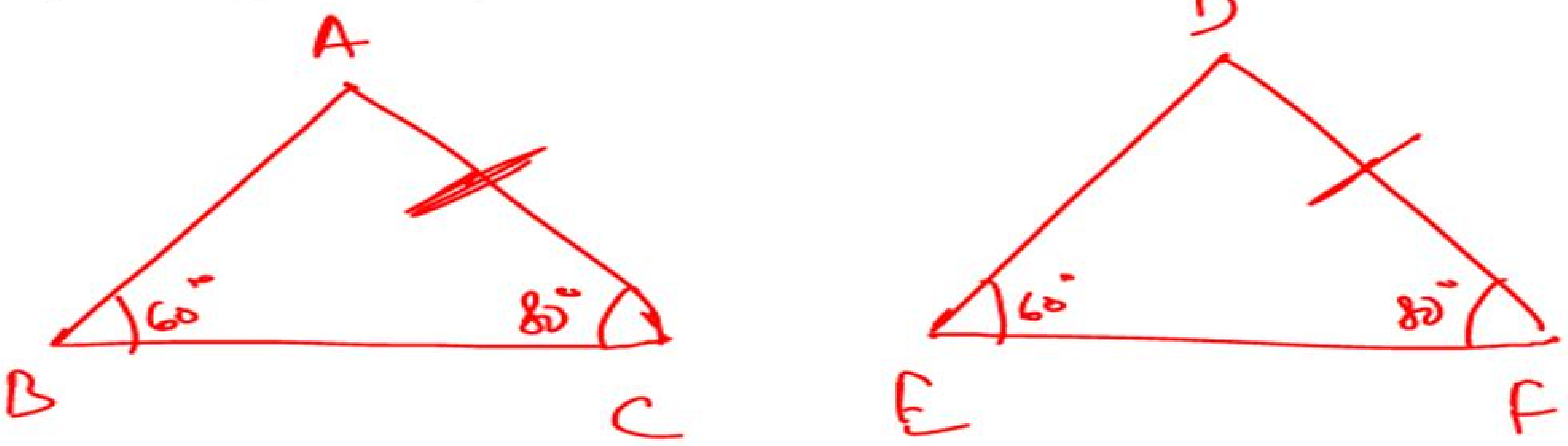
$\Delta ABC \cong \Delta QRP$

ASA (Angle – Side – Angle)



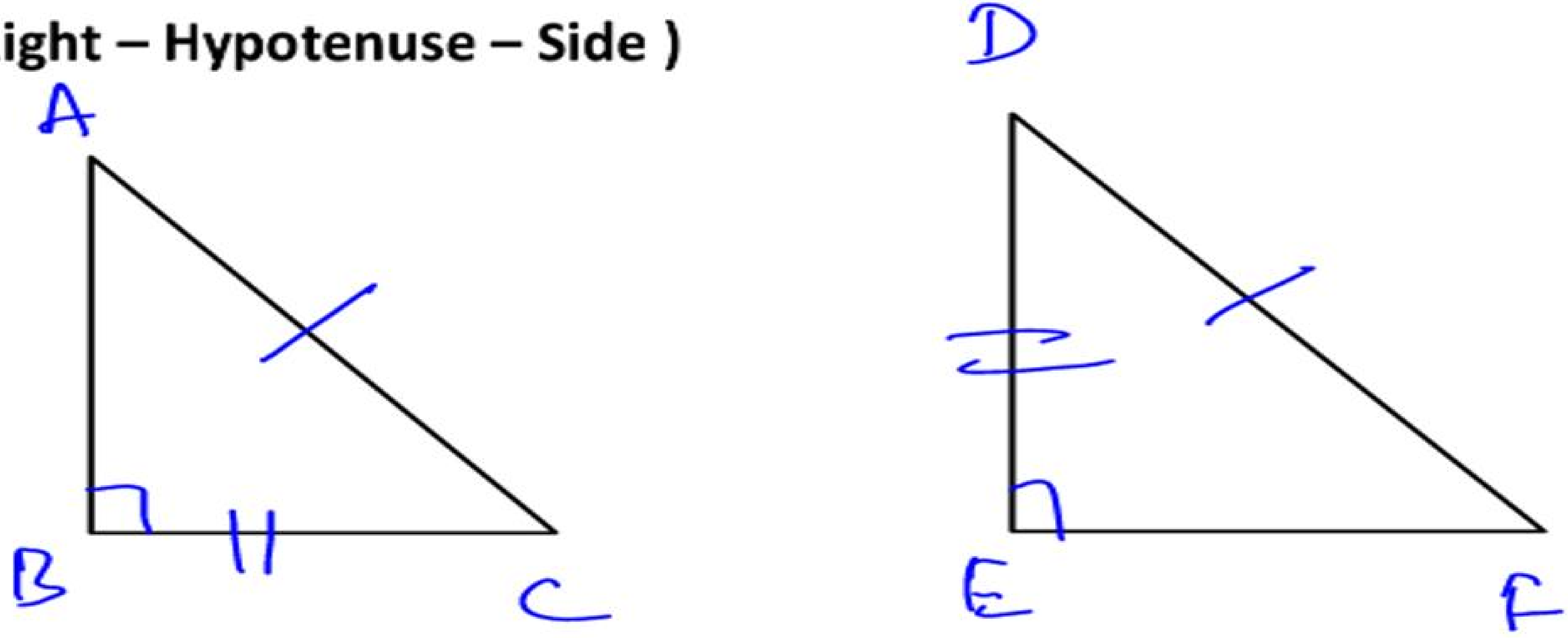
$\Delta PQR \cong \Delta BCA$

AAS (Angle – Angle – Side)



$\Delta ABC \cong \Delta DEF$

RHS (Right – Hypotenuse – Side)



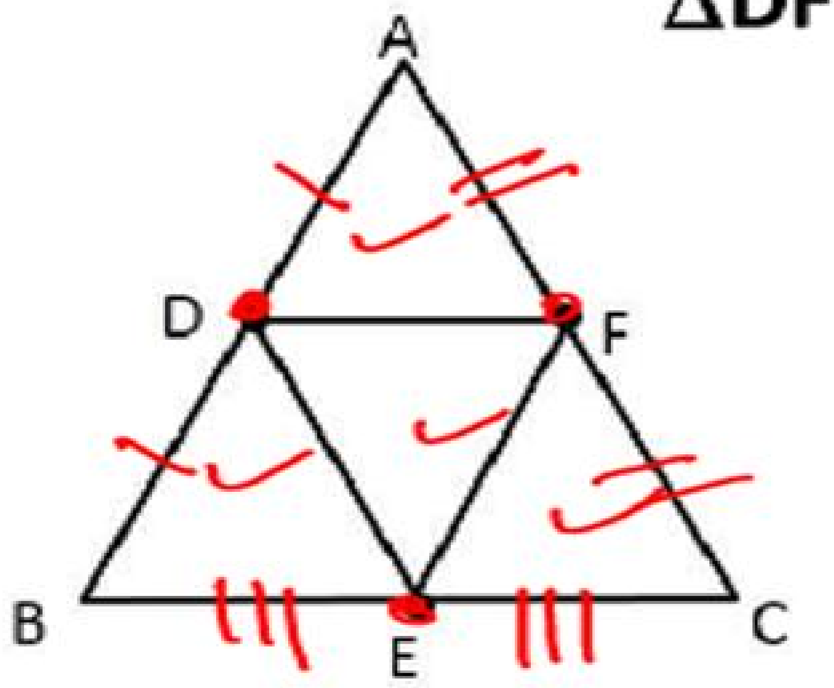
$\Delta ABC \cong FED$

AAA & SSA does not guarantee congruency.

If D, E & F are midpoints of the sides AB, BC, CA

Then,

$$\Delta DFE \cong \Delta FDA \cong \Delta EBD \cong \Delta CEF$$



Area of $\Delta DFE = \frac{1}{4}$ (Area of ΔABC)

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If Congruent → Similar

If Similar → Congruent ??

If Congruent → Area same

If Area same → Congruent ??

Similar + Area same → Congruent

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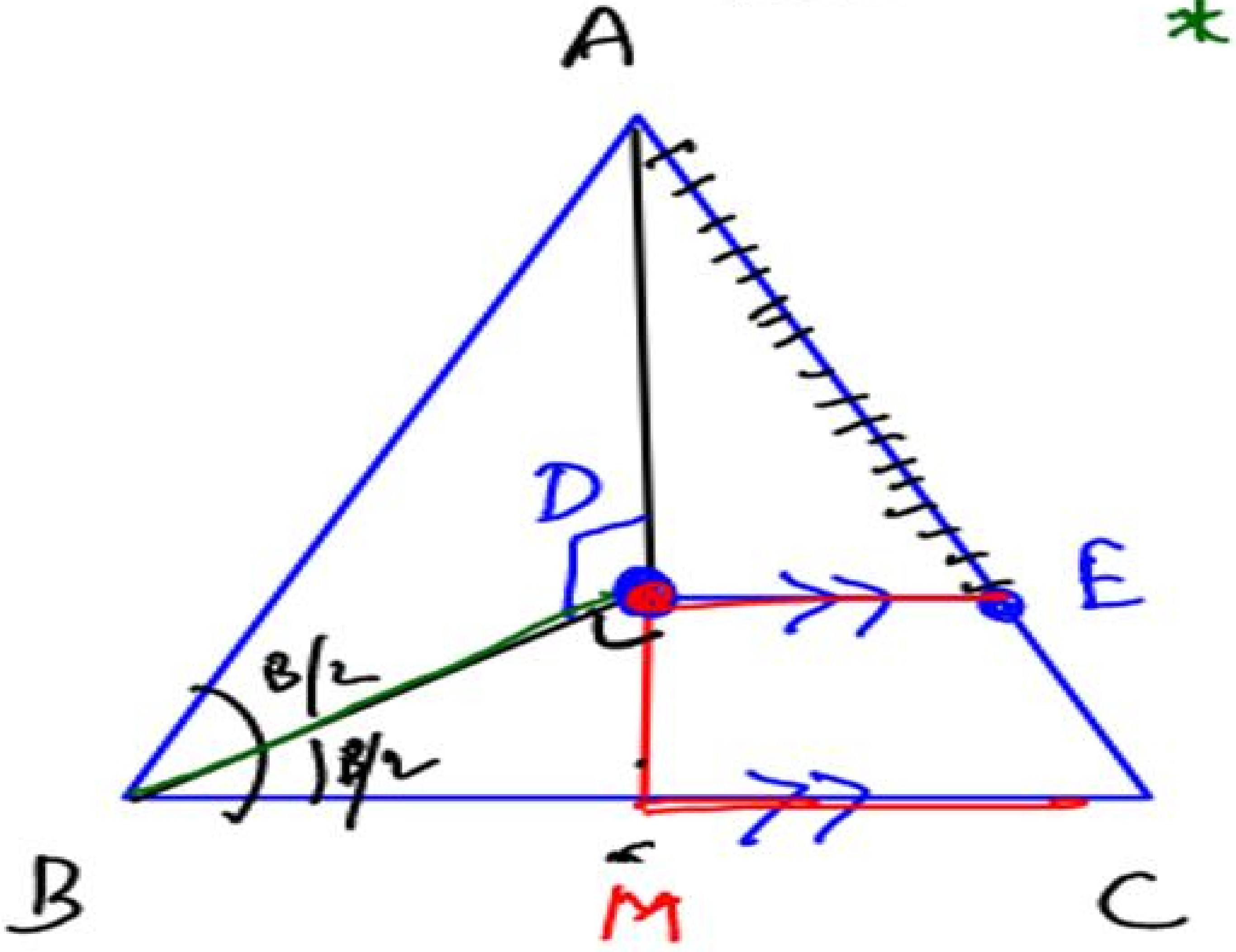
v. imp

Q. AD is perpendicular to the internal bisector of $\angle ABC$ of ΔABC. DE is drawn through D parallel to BC to meet AC at E. If the length of AC is 12 cm, then the length of AE (in cm.) is :

(a) 8 (b) 6 (c) 3 (d) 4



* $\Delta A\bar{D}\bar{B}$ & $\Delta M\bar{D}\bar{B}$

$\Delta \bar{A}DB \cong \Delta \bar{M}DB$ [ASA]

$AD = MD$

D is mid point of AM

ΔAMC DE ∥ MC

By converse of mid pt Theorem

$AE = EC$

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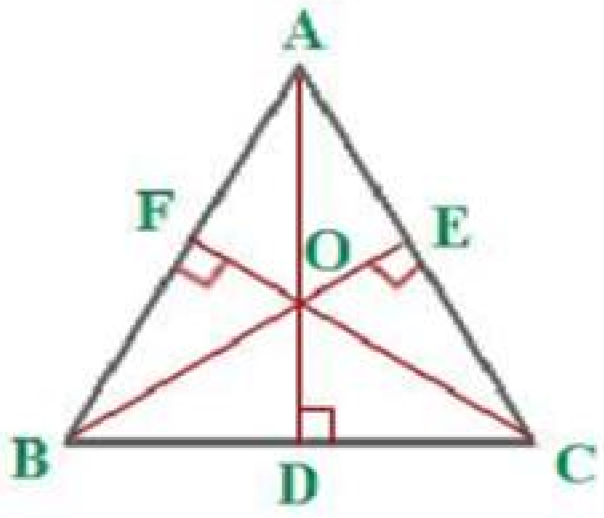
Ans. (b)

Centres of Triangles

* Orthocentre → (15-17) min
* Circumcentre → (10-12) min
* Incentre → (38-42) min
* Centroid → (52-56) min

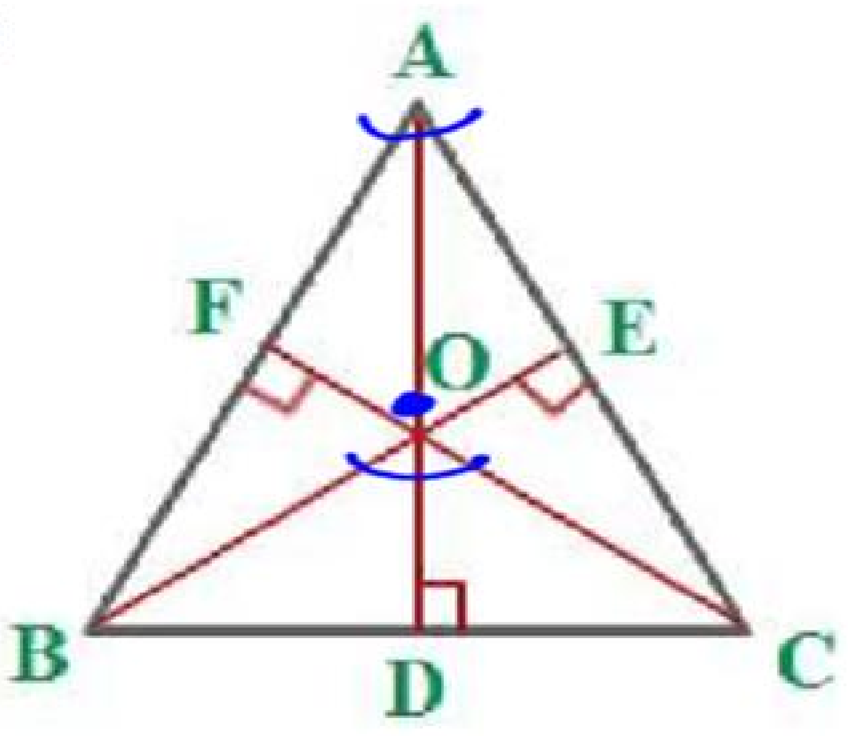
ORTHOCENTRE

Def: Meeting point of all altitudes



AD, BE and CF are altitudes of triangle.

O $\rightarrow$ Orthocentre



imp

$\angle A + \angle BOC = 180°$

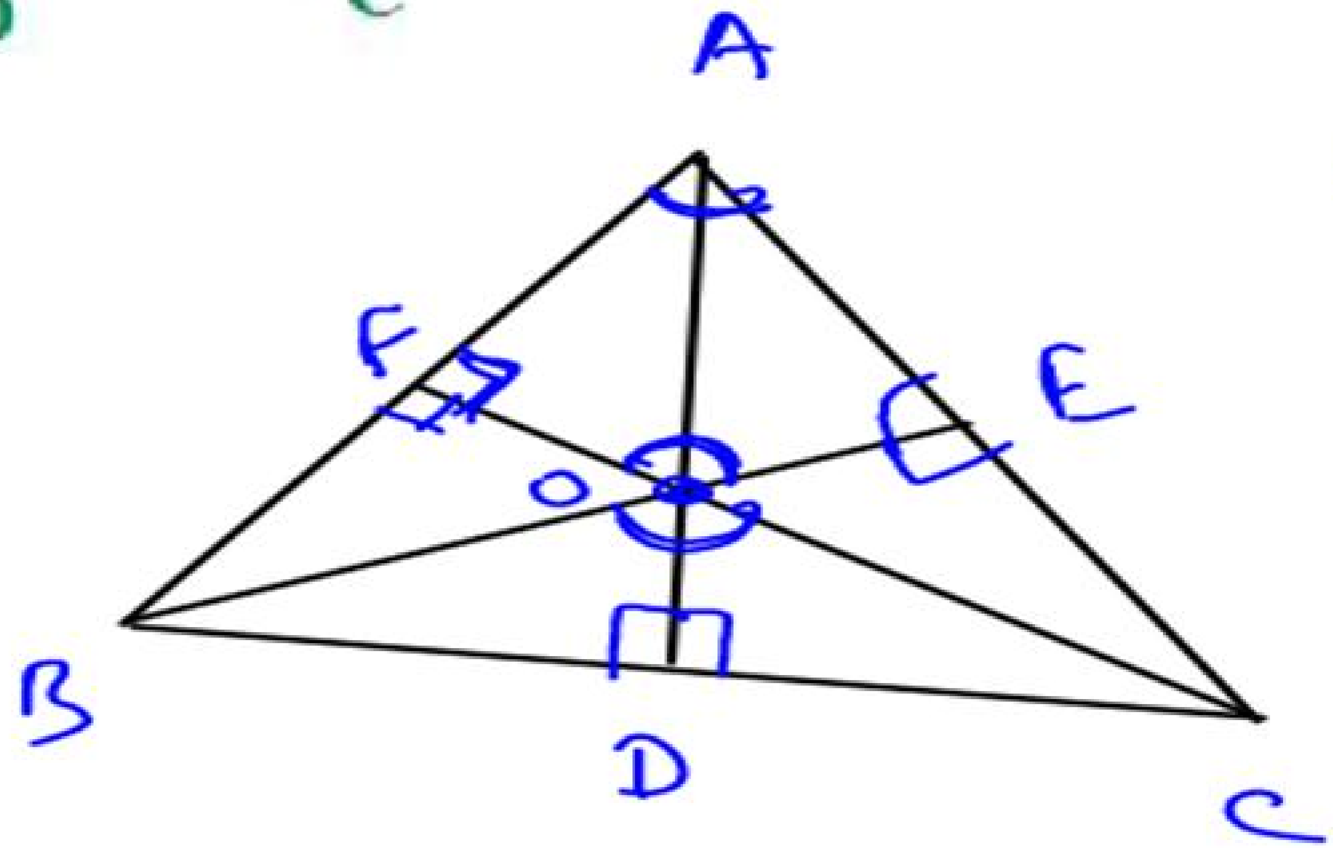
Reason ??

Quad AFOE

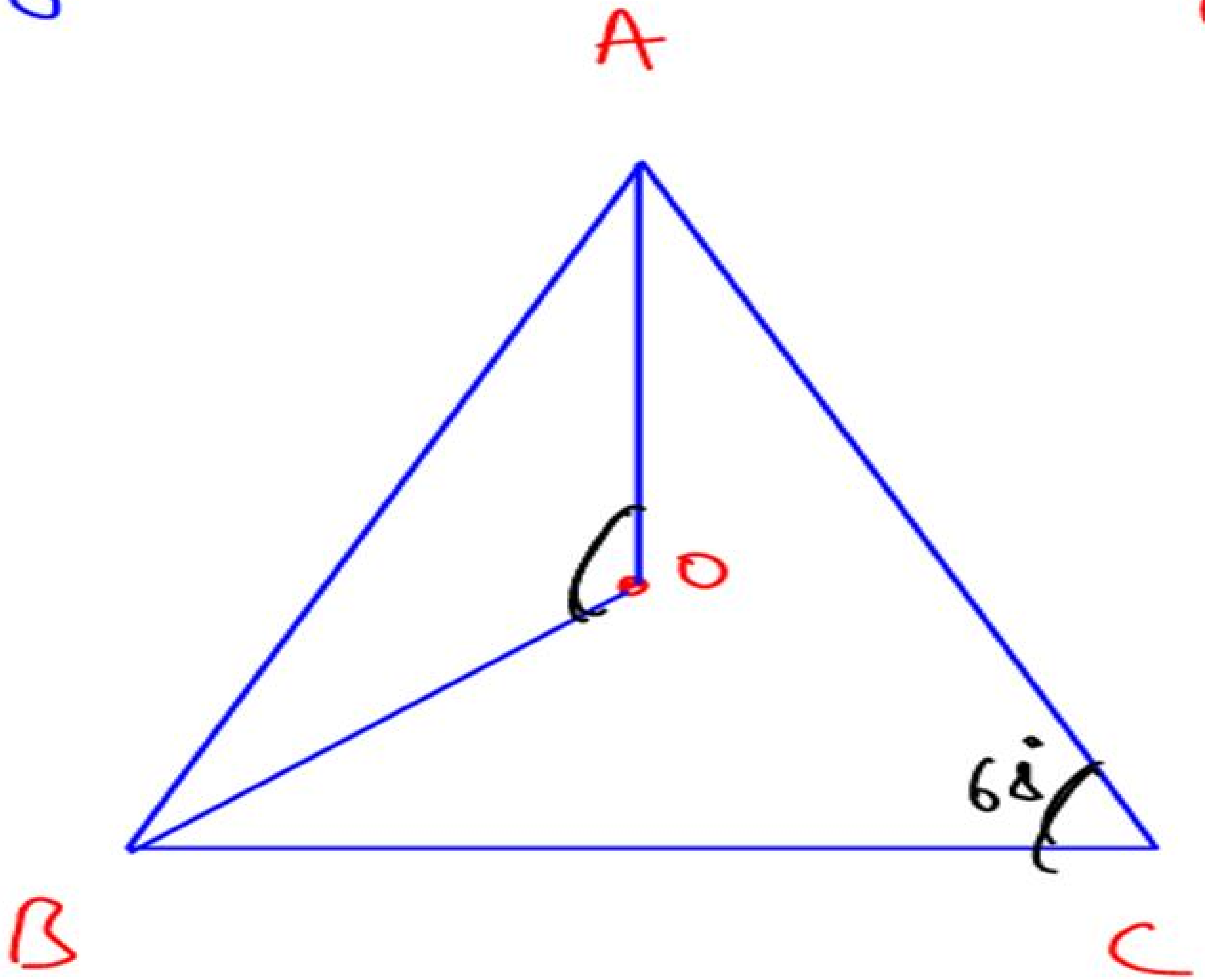
$\angle A + 90 + \angle FOE + 90 = 360$

$\angle A + \angle FOE = 180$

$\boxed{\angle A + \angle BOC = 180°}$



eg



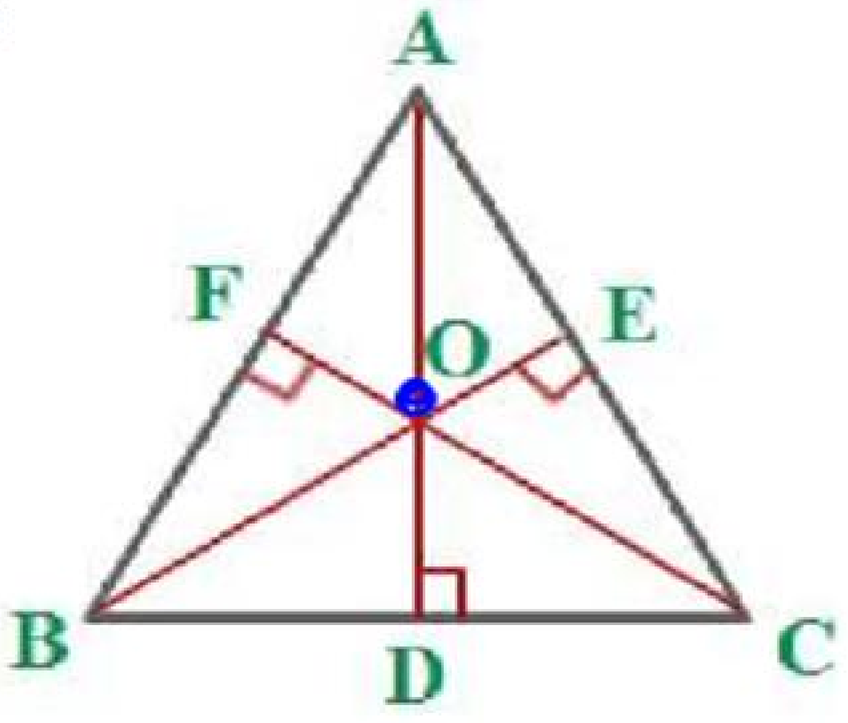
$O \rightarrow$ orthocentre

If $\angle C = 68^\circ$

find $\angle AOB = ??$

$68 + \angle AOB = 180^\circ$

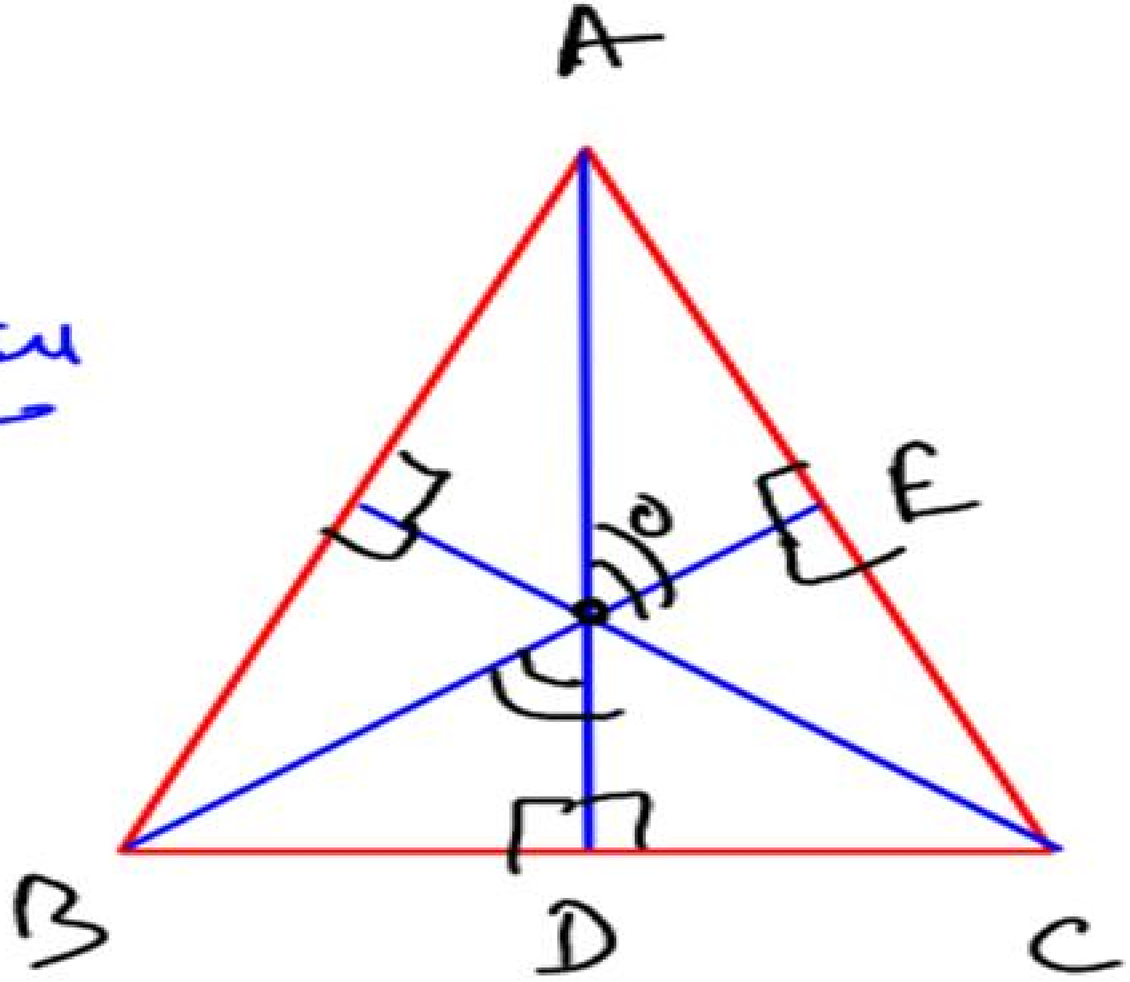
$\angle AOB = \underline{\underline{112^\circ}}$



$$\boxed{AO \cdot OD = BO \cdot OE = CO \cdot OF}$$

Reason ??

O → orthocentre

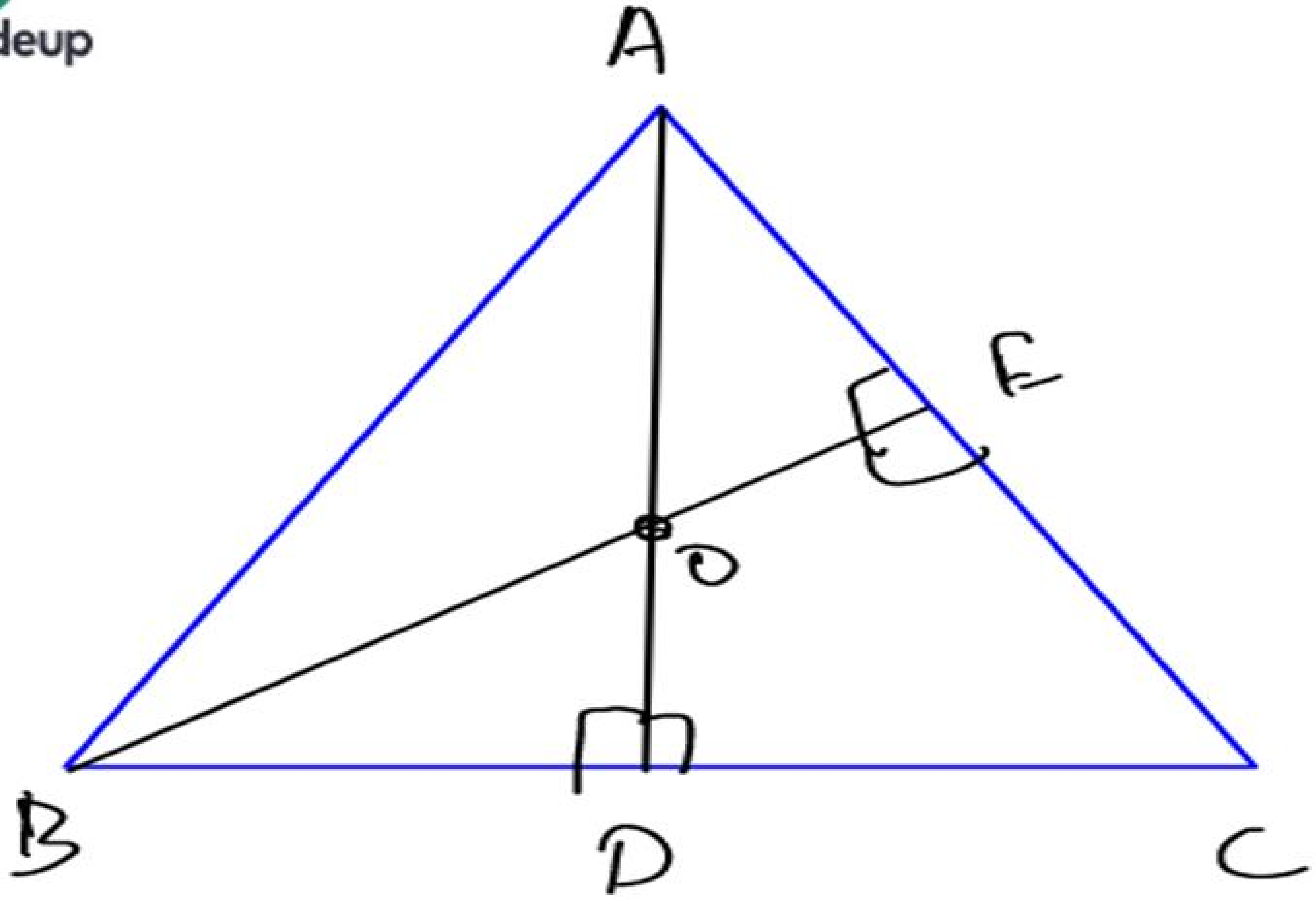


$\Delta AOE \sim \Delta BOD$

$\Delta AOE \simeq \Delta BOD$

$$\frac{AO}{BO} = \frac{EO}{DO}$$

$(AO)(DO) = (BO)(EO)$



$AO = 10$ cm

$BO = 12$ cm

$EO = 5$ cm

find $DO = ??$

$10 \times DO = 12 \times 5$

$\underline{\underline{DO = 6 \text{ cm}}}$

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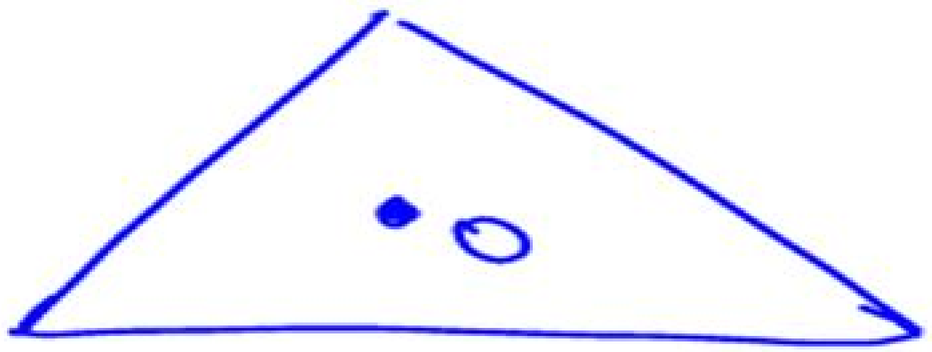
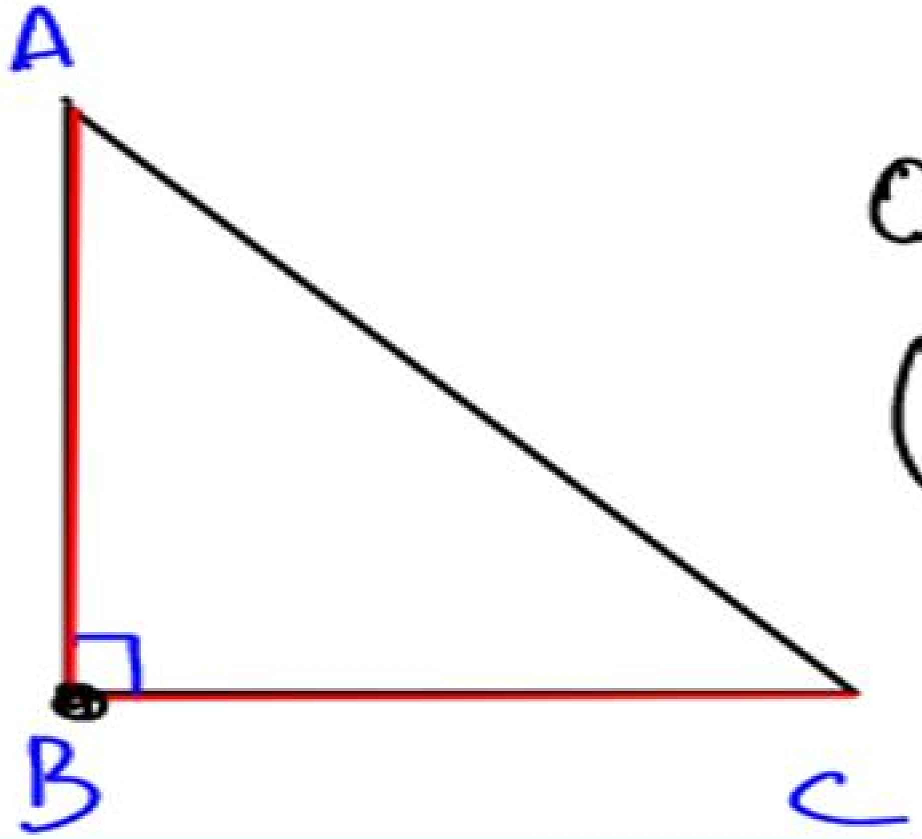
POSITION OF ORTHOCENTRE

1. **Acute Angle Triangle**



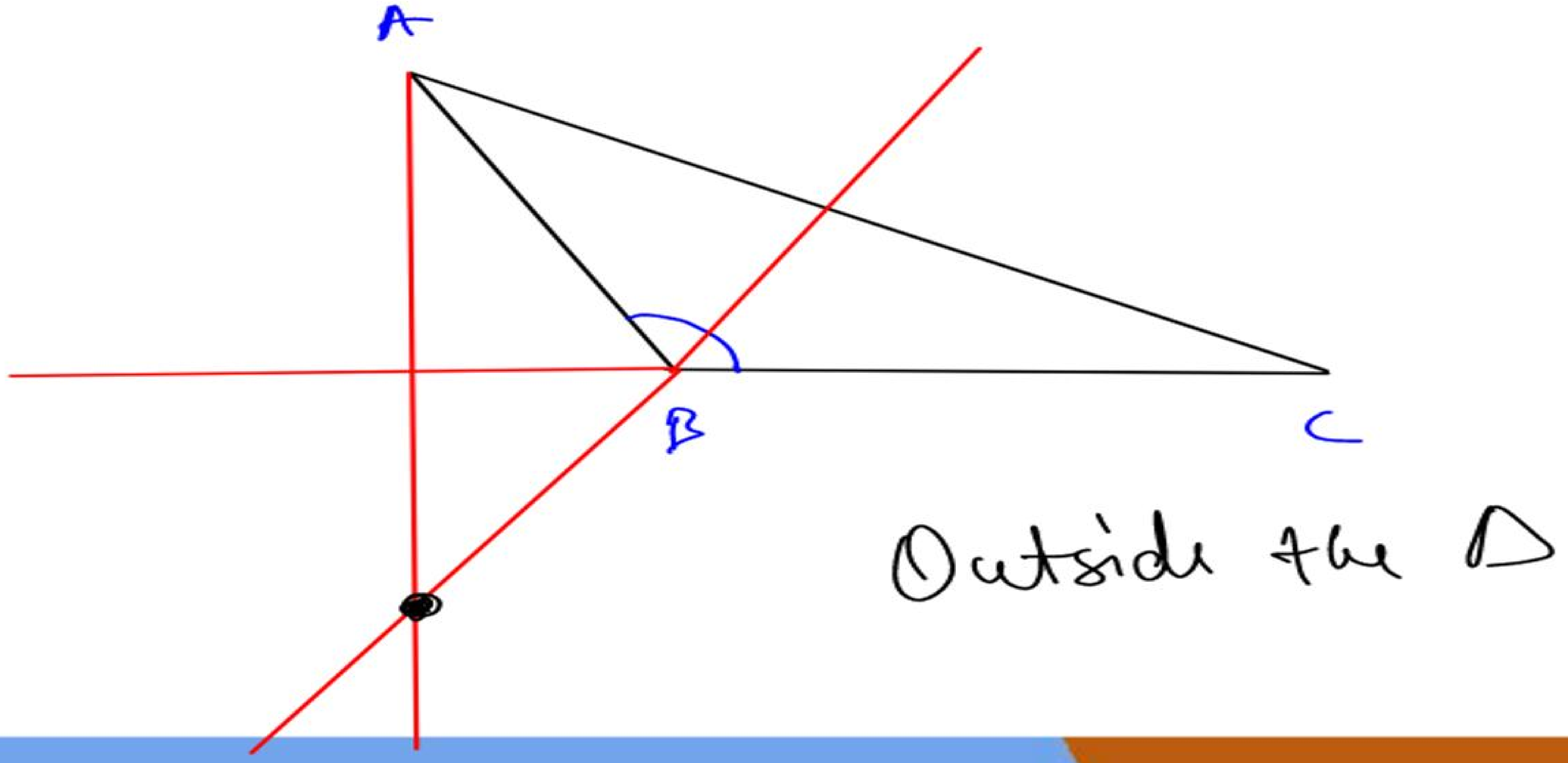
lies inside the Δ

2. **Right Angle Triangle**



Orthocentre is at B (The vertex where 90° is formed)

3. Obtuse Angle Triangle

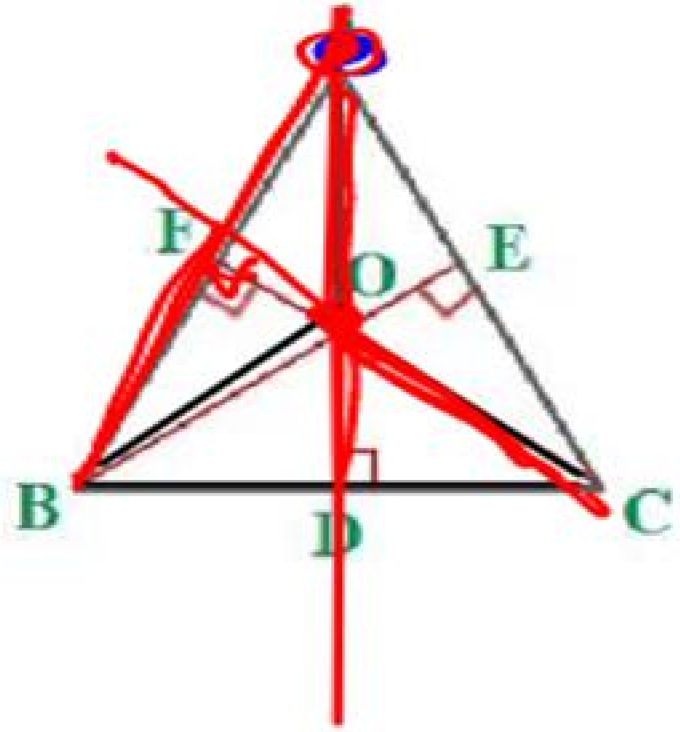


In a ΔABC, O is the orthocentre. Which point is the orthocentre of ΔBOC?

V. Imp

Δ BOC

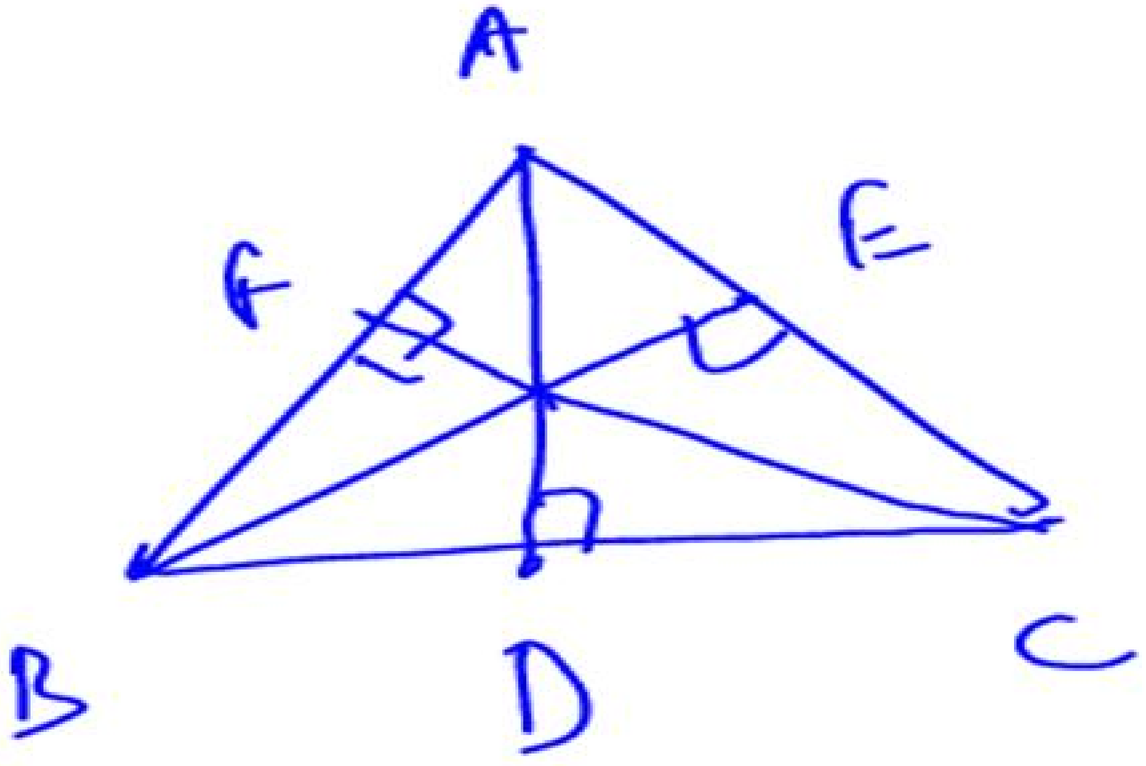
Def → Meeting pt of all altitudes



→ A

Sum of all altitudes of a triangle is less than perimeter of triangle.

किसी त्रिभुज के सभी शीर्षलंबों का योग त्रिभुज के परिमाप से कम होता है।



$$(AD + BE + CF) < (AB + BC + CA)$$

CIRCUMCENTRE

Def: Meeting point of all perpendicular bisector.