



Sahi Prep Hai Toh Life Set Hai

# Special Series





## Some Basic formulas which will be used in Special Series:

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$$

**Eg1. Find the value of**  
 **$1 + 2 + 3 + 4 + \dots + 50$**

**Eg2. Find the value of**  
 **$1^2 + 2^2 + 3^2 + \dots + 60^2$**

**Eg3. Find the value of**  
 **$1^3 + 2^3 + 3^3 + \dots + 20^3$**

**Eg4. Find the value of**  
 **$21 + 22 + 23 + 24 + \dots + 60$**



**Eg5. Find the value of**  
 **$2^2 + 4^2 + 6^2 + \dots + 40^2$**

**Eg6. Find the value of**

$$1^3 + 3^3 + 5^3 + 7^3 + \dots + 29^3$$

**Eg7. Find the value of**

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ..... + 49^2 - 50^2 + 51^2$$

**If you know the  $n^{\text{th}}$  term of a sequence then you can calculate its sum:**

Let  $T_n = an^3 + bn^2 + cn + d$

So,  $S_n \rightarrow$  sum of  $n$  terms

$$S_n = \sum T_n$$

$$= a \sum n^3 + b \sum n^2 + c \sum n + d \sum 1$$

$$= a \left[ \frac{n(n+1)}{2} \right]^2 + b \left[ \frac{n(n+1)(2n+1)}{6} \right] + \frac{c(n \cdot n + 1)}{2} + d \cdot n$$

**Eg1.  $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots$**   
**Find the sum of first 10 terms.**

**Eg2.  $1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + 4 \cdot 7 \dots\dots\dots n$  terms.  
Find the sum of first  $n$  terms.**

**Eg3. Find the sum of first 10 terms.**

$$2 \cdot 5 + 5 \cdot 7 + 8 \cdot 9 + 11 \cdot 11 + 14 \cdot 13 + \dots\dots\dots$$

**Eg. 4 Find the sum of n terms of series:**

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + (1^2 + 2^2 + 3^2 + 4^2) + \dots$$



**Remember,**

In these kind of questions,

First calculate  $T_n \rightarrow n^{\text{th}}$  terms

and then to calculate

$$S_n = \sum T_n$$

# SPECIAL SERIES

## I. TELESCOPIC SERIES:

Telescopic series is a series whose partial sums eventually only have a fixed number of terms after cancellation.

This will be illustrated with some examples.

**Eg1.** 
$$S = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{19 \cdot 20}$$

**Eg2.**  $S = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + n \text{ terms.}$

**Eg3.**  $A = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \infty \text{ terms.}$

**Eg4.** 
$$P = \frac{1}{14 \cdot 15} + \frac{1}{15 \cdot 16} + \frac{1}{16 \cdot 17} + \dots + \frac{1}{48 \cdot 49}$$

**Eg5.**  $S = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{240}$

**Eg6.** 
$$A = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{19 \cdot 21}$$



Now, the same examples can be tested in exams like:

$$A = \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{19 \cdot 21}$$

Or

$$B = \frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots + \frac{1}{399}$$

Or

$$C = \frac{1}{2^2 - 1} + \frac{1}{4^2 - 1} + \frac{1}{6^2 - 1} + \dots + \frac{1}{20^2 - 1}$$

**Eg7.** 
$$B = \frac{1}{2 \cdot 4} + \frac{1}{4 \cdot 6} + \frac{1}{6 \cdot 8} + \dots + \frac{1}{18 \cdot 20}$$

**Eg8.**  $C = \frac{1}{11 \cdot 13} + \frac{1}{13 \cdot 15} + \frac{1}{15 \cdot 17} + \dots + \frac{1}{97 \cdot 99}$

**Eg9.**  $D = \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \frac{1}{11 \cdot 14} + \dots + \frac{1}{47 \cdot 50}$

**Now, we can Generalize:**

$$S = \frac{1}{a(a+d)} + \frac{1}{(a+d)(a+2d)} + \frac{1}{(a+2d)(a+3d)} + \dots + \frac{1}{[a+(n-1)d](a+nd)}$$

$$S = \frac{1}{d} \left[ \frac{1}{a} - \frac{1}{a+nd} \right]$$

$$S = \frac{1}{d} \left[ \frac{nd}{a(a+nd)} \right]$$

$$S = \frac{n}{a(a+nd)}$$

**Eg10.**  $Q = \frac{1}{1 \cdot 3 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 7} + \frac{1}{5 \cdot 7 \cdot 9} + \dots + \frac{1}{17 \cdot 19 \cdot 21} + \frac{1}{19 \cdot 21 \cdot 23}$

**Eg11.** 
$$R = \frac{1}{1 \cdot 4 \cdot 7} + \frac{1}{4 \cdot 7 \cdot 10} + \dots + \frac{1}{10 \cdot 13 \cdot 16}$$

**Eg12.** 
$$S = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \dots + \frac{1}{18 \cdot 19 \cdot 20}$$



# Practice Questions

**Q1.**  $A = \frac{3}{4} + \frac{5}{36} + \frac{7}{144} + \frac{9}{400} + \dots\dots\dots \frac{19}{8100}$

**Find the value of A.**

**Ans.**  $\frac{99}{100}$

**Q2.**  $B = \frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots\dots\dots 10 \text{ terms}$

**Find the value of B.**

**Ans.**  $1\frac{9}{11}$

**Q3.**  $C = \frac{1}{3 \times 7} + \frac{1}{7 \times 11} + \frac{1}{11 \times 15} + \dots \dots \dots \frac{1}{899 \times 903}$

**Find the value of C.**

**Ans.**  $\frac{25}{301}$

**Q4.**  $S = \frac{1}{1 \times 2} + \frac{1}{1 \times 4} + \frac{1}{2 \times 3} + \frac{1}{4 \times 7} + \frac{1}{3 \times 4} + \frac{1}{7 \times 10} + \dots \dots \dots 20 \text{ terms}$

**Find the value of S.**



**Ans.**  $\frac{420}{341}$

**Q5. Find the value of M.**

$$M = \frac{1}{1 \times 3 \times 5} + \frac{1}{1 \times 4} + \frac{1}{3 \times 5 \times 7} + \frac{1}{4 \times 7} + \frac{1}{5 \times 7 \times 9} + \frac{1}{7 \times 10} + \dots \dots \dots 20 \text{ terms}$$

**Ans.**  $\frac{6070}{14973}$



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Practise  
topic-wise quizzes

Keep attending  
live classes

