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Sahi Prep Hai Toh Life Set Hai

TRIGONOMETRIC IDENTITIES

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Agenda

Trigonometric Identities

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Trigonometric Identities

Basics of School

Prove

L.H.S = R.H.S

We will do Question with Option

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Trigonometric equation is true for some values of θ.

Eg. $\sin\theta = \frac{1}{2}$ $\quad 0 < \theta < 180$

$\theta = 30^\circ$ *or* 150°

Trigonometric eq^n $\longrightarrow$ It is true for some values of θ

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Trigonometric Identity is true for all values of θ.

Eg. $\sin^2\theta + \cos^2\theta = 1$

Put $\theta = 30$

$\frac{1}{4} + \frac{3}{4} = 1$

$\theta = 45$

$\frac{1}{2} + \frac{1}{2} = 1$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$[\cos\theta \neq 0]$

$\sin\theta = ½$ (Trigonometric Equation)

$\sin^2\theta + \cos^2\theta = 1$ (Trigonometric Identity)

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TRIGONOMETRIC IDENTITIES

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \text{cosec}^2\theta$

Note :

If you are not able to understand which identity should be used then convert everything into **sin θ and cos θ.**

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Prove the following identities:

Eg1. $$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

L.H.S

$$\frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$\frac{2}{\cos^2\theta} \longrightarrow 2\sec^2\theta \quad \text{R.H.S}$$

Proved

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Eg2. $\operatorname{cosec}^2\theta + \sec^2\theta = \operatorname{cosec}^2\theta \sec^2\theta$

L.H.S

$$\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta \cdot \cos^2\theta} \Rightarrow \frac{1}{\sin^2\theta} \cdot \frac{1}{\cos^2\theta}$$

$$\operatorname{cosec}^2\theta \cdot \sec^2\theta$$

R.H.S

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Eg3. $(\sin\theta + \mathrm{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

L.H.S

$\sin^2\theta + \mathrm{cosec}^2\theta + 2\sin\theta\,\mathrm{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$

$\sin^2\theta + \cos^2\theta + \mathrm{cosec}^2\theta + \sec^2\theta + 4$

$1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 4$

$7 + \cot^2\theta + \tan^2\theta$

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Eg4. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

L.H.S

$$\sec^2\theta\ (\sec^2\theta - 1)$$

$$(\tan^2\theta + 1)\ (\tan^2\theta)$$

$$\tan^4\theta + \tan^2\theta \quad \text{R.H.S}$$

Hence Proved

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Eg5. $$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$$

$\sin\theta = S \quad \cos\theta = C$

V.imp

I^{st}

L.H.S

$$\frac{\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}-\frac{1}{\cos\theta}+1} = \frac{\sin\theta+1-\cos\theta}{\sin\theta-1+\cos\theta}$$

$$= \frac{S+(1-C)}{S-(1-C)} \times \frac{S+(1-C)}{S+(1-C)}$$

$$= \frac{S^2+(1-C)^2+2S(1-C)}{S^2-(1-C)^2}$$

$$= \frac{S^2+C^2-2C+1+2S-2SC}{S^2-C^2-1+2C}$$

$$\frac{2 - 2c + 2s - 2sc}{\cancel{s^2} - c^2 - \cancel{s^2} - c^2 + 2c}$$

$$\Rightarrow \frac{2(1-c) + 2s(1-c)}{2c(1-c)}$$

Very lengthy $= \frac{\cancel{(1-c)}\left[2+2s\right]}{2c\cancel{(1-c)}} = \frac{s+1}{c}$

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$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$$

L.H.S

N^R

$$\sec\theta + \tan\theta - (\sec^2\theta - \tan^2\theta)$$

$$(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

$$\frac{(\sec\theta + \tan\theta)\left[\cancel{1 - \sec\theta + \tan\theta}\right]}{\cancel{\tan\theta - \sec\theta + 1}}$$

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

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Eg6. $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

L.H.S

$$\frac{t}{1-\frac{1}{t}}+\frac{\frac{1}{t}}{1-t}$$

$$\frac{t^2}{t-1}+\frac{1}{t(1-t)}$$

$$\Rightarrow \frac{t^2}{t-1}-\frac{1}{t(t-1)}$$

$$\Rightarrow \frac{t^3-1}{t(t-1)}$$

$$\Rightarrow \frac{\cancel{(t-1)}(t^2+t+1)}{(t)\cancel{(t-1)}}$$

$$t+1+\frac{1}{t}$$

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Eg7. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

L.H.S

$\sin^4\theta + \cos^4\theta \Rightarrow (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$1 - 2\sin^2\theta\cos^2\theta$

$(a+b)^2 = a^2 + b^2 + 2ab$

$a^2 + b^2 = (a+b)^2 - 2ab$

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Eg8. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

→ L.H.S

$(\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$

$1 - 3\sin^2\theta\cos^2\theta$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

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V. imp

$$\sin^4\theta + \cos^4\theta = \left(1 - 2\sin^2\theta\cos^2\theta\right)$$

$$\sin^6\theta + \cos^6\theta = \left(1 - 3\sin^2\theta\cos^2\theta\right)$$

Find Max & Min value of $\sin^6\theta + \cos^6\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

Max $\rightarrow$ $1 - 0 = 1$

Min $\rightarrow$ $1 - 3\left(\frac{1}{4}\right) = \frac{1}{4}$

Eg9. $\sin^8\theta - \cos^8\theta = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

$$\sin^8\theta - \cos^8\theta = (\sin^4\theta)^2 - (\cos^4\theta)^2$$

$$= \underline{(\sin^4\theta - \cos^4\theta)}(\sin^4\theta + \cos^4\theta)$$

$$(\sin^2\theta - \cos^2\theta)\underline{(\sin^2\theta + \cos^2\theta)}(1 - 2\sin^2\theta\cos^2\theta)$$

$$(\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$$

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Eg10a.

If (secA + tanA) (secB + tanB) (secC + tanC) = (secA – tanA) (secB – tanB) (secC – tanC)
Prove that each side is equal to ±1.

L = R

Multiply Both sides by R

$$L \cdot R = R^2$$

$$(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C) = R^2$$

$$R^2 = 1$$

$$\boxed{R = \pm 1}$$

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Eg10b.

If (cosecA+cotA)(cosecB+cotB)(cosecC+cotC) = (cosecA–cotA)(cosecB–cotB)(cosecC–cotC)

Prove that each side is equal to ±1.

$$L = R$$

$$L \cdot R = R^2$$

$$1 \cdot 1 \cdot 1 = R^2$$

$$\boxed{R = \pm 1}$$

Eg11. If $(1+\cos x)(1+\cos y)(1+\cos z) = (1-\cos x)(1-\cos y)(1-\cos z)$
Prove that each side is equal to $\pm \sin x \sin y \sin z$.

$$L = R$$

Multiply both sides by R

$$L \cdot R = R^2$$

$$(1-\cos^2 x)(1-\cos^2 y)(1-\cos^2 z) = R^2$$

$$\sin^2 x \sin^2 y \sin^2 z = R^2$$

$$\pm \sin x \sin y \sin z = R$$

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Eg12. If $\sin\theta + \cos\theta = p$ & $\sec\theta + \text{cosec}\,\theta = q$

Prove $q(p^2 - 1) = 2p$

L.H.S

$$(\sec\theta + \text{cosec}\,\theta)\left[\cancel{\sin^2\theta + \cos^2\theta} + 2\sin\theta\cos\theta \cancel{-1}\right]$$

$$\left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)\left(2\sin\theta\cos\theta\right)$$

$$\frac{\sin\theta + \cos\theta}{\cancel{\sin\theta}\,\cancel{\cos\theta}} \cdot 2\cancel{\sin\theta}\,\cancel{\cos\theta}$$

$$= 2p$$

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Eg13. If $X = a \sin\theta$ $Y = b \tan\theta$ prove $\frac{a^2}{X^2} - \frac{b^2}{Y^2} = 1$

$$\frac{a}{x} = \frac{1}{\sin\theta} \qquad \frac{b}{y} = \frac{1}{\tan\theta}$$

$$\frac{a^2}{x^2} - \frac{b^2}{y^2}$$

$$\underline{\operatorname{cosec}^2\theta - \cot^2\theta = 1}$$

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Eg14. If X = r sin A cos C, Y = r sin A sin C & Z = r cos A

Prove that $r^2 = X^2 + Y^2 + Z^2$

R.H.S

$$X^2 + Y^2 + Z^2$$

$$r^2 \sin^2 A \cos^2 C + r^2 \sin^2 A \sin^2 C + r^2 \cos^2 A$$

$$r^2 \sin^2 A \,[1] + r^2 \cos^2 A$$

$$r^2 [\sin^2 A + \cos^2 A] = r^2$$

Eg15(a). If $\sec\theta + \tan\theta = 3$, find value of $\sin\theta$.

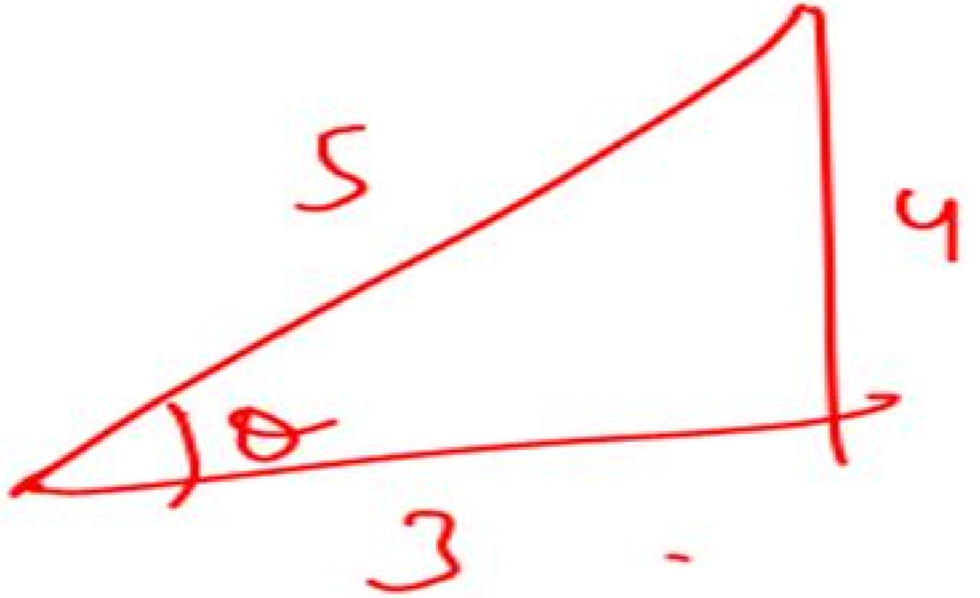
Imp

$$\sec\theta + \tan\theta = 3$$

$$\sec\theta - \tan\theta = \frac{1}{3}$$

$$2\sec\theta = \frac{10}{3}$$

$$\sec\theta = \frac{5}{3}$$



$$\boxed{\sin\theta = \frac{4}{5}}$$

Eg15(b). If $\sec\theta + \tan\theta = 2+\sqrt{5}$, find value of $\sin\theta$.

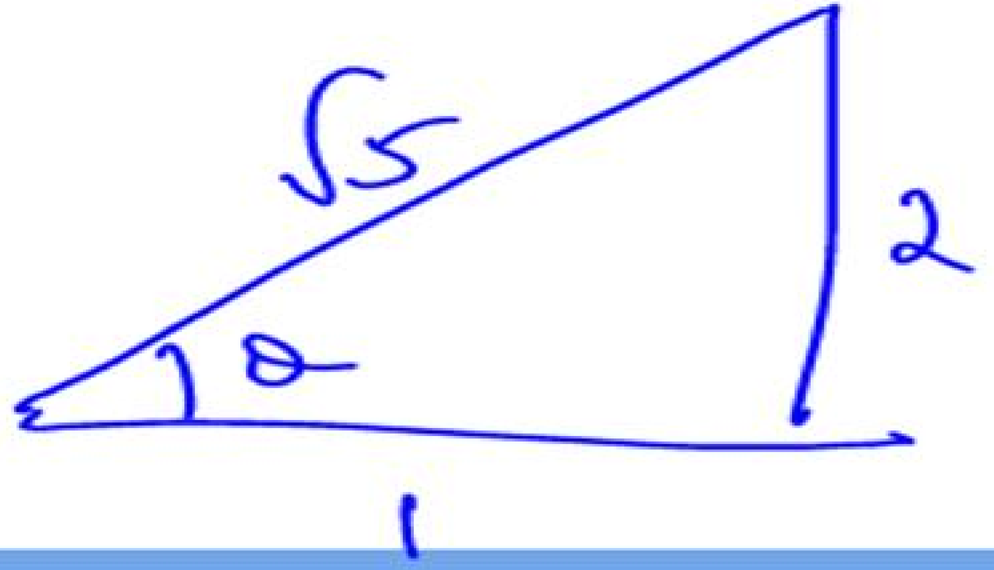
$$\frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$\frac{2-\sqrt{5}}{-1} = \sqrt{5}-2$$

$$\sec\theta + \cancel{\tan\theta} = 2+\sqrt{5}$$

$$\sec\theta - \cancel{\tan\theta} = \sqrt{5}-2$$

$$\cancel{2}\sec\theta = \cancel{2}\sqrt{5}$$



$$\boxed{\sin\theta = \frac{2}{\sqrt{5}}}$$

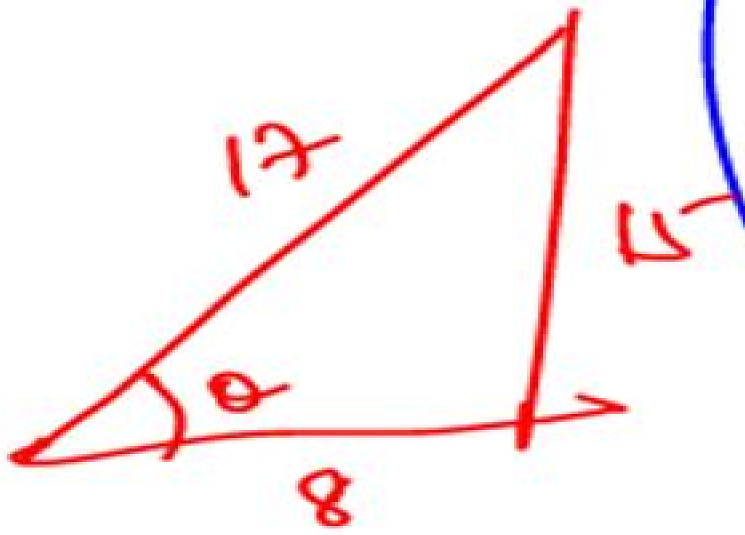
Eg15(c). If $\sec\theta + \tan\theta = \frac{1}{4}$, find value of $\sin\theta$.

$$\sec\theta + \tan\theta = \frac{1}{4}$$

$$\sec\theta - \tan\theta = 4$$

$$2\sec\theta = \frac{17}{4}$$

$$\boxed{\sec\theta = \frac{17}{8}}$$



$\sec\theta = +ve$

I, IV

$\tan\theta = -ve$

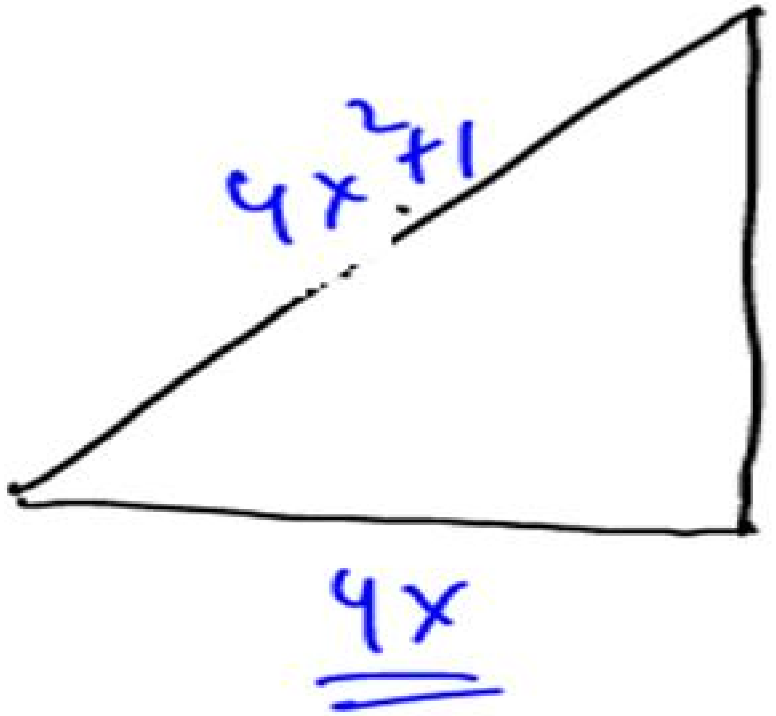
II, IV

$$\sin\theta = \frac{-15}{17}$$

Eg15(d). If ~~$\sec\theta = X + \frac{1}{4}X$, find value of $\sec\theta + \tan\theta$ in terms of X~~.

If $\sec\theta = x + \frac{1}{4x}$ find $(\sec\theta + \tan\theta)$ in terms of x

$$\sec\theta = \frac{4x^2+1}{4x}$$



$$P\left(4x^2-1 \,\middle|\, 1-4x^2\right)$$

$$P^2 + (4x)^2 = (4x^2+1)^2$$

$$P^2 = (4x^2+1)^2 - (4x)^2$$

$$P^2 = 16x^4 - 8x^2 + 1$$

$$P^2 = (4x^2-1)^2$$

$$P = 4x^2-1 \text{ or } 1-4x^2$$

$\sec\theta + \tan\theta$

$$\frac{4x^2+1}{4x} + \frac{4x^2-1}{4x}$$

$$\frac{8x^2}{4x}$$

$$\underline{2x}$$

$$\frac{4x^2+1}{4x} + \frac{1-4x^2}{4x}$$

$$\frac{2}{4x}$$

$$\frac{1}{2x}$$

$$x^2 = 36$$

$$x^2 = (6)^2$$

$$\boxed{x = \pm 6}$$

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Eg16. If $X\sin^3\theta + Y\cos^3\theta = \sin\theta\cos\theta$ & $X\sin\theta = Y\cos\theta$. Find value of $X^2 + Y^2$.

90sec

$$X\sin^3\theta + Y\cos^3\theta = \sin\theta\cdot\cos\theta$$

$$X\sin\theta\cdot\sin^2\theta + Y\cos^3\theta = \sin\theta\cos\theta$$

$$Y\cos\theta\cdot\sin^2\theta + Y\cos^3\theta = \sin\theta\cos\theta$$

$$Y\cancel{\cos\theta}\,(1) = \sin\theta\cdot\cancel{\cos\theta}$$

$\cos^2\theta + \sin^2\theta$

$= 1$

$$Y = \sin\theta \qquad X = \cos\theta$$

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V.Imp **Eg17(a) .** If $a\cos\theta - b\sin\theta = c$ prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

$$a\cos\theta - b\sin\theta = c \quad — (1)$$

$$a\sin\theta + b\cos\theta = K \quad — (2)$$

$$(1)^2 + (2)^2$$

$$a^2 + b^2 = c^2 + K^2$$

$$K^2 = a^2 + b^2 - c^2$$

$$K = \pm\sqrt{a^2+b^2-c^2}$$

Eg17(b) . If $5\sin\theta + 8\cos\theta = 9$

Find the value of $5\cos\theta - 8\sin\theta = ??$

$$5\sin\theta + 8\cos\theta = 9$$

$$5\cos\theta - 8\sin\theta = k$$

$$5^2 + 8^2 = 9^2 + k^2$$

$$k^2 = 8$$

$$\boxed{k = \pm 2\sqrt{2}}$$

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Eg18. If $\sin\theta + \sin^2\theta = 2$ find the value of $\cos^2\theta + \cos^4\theta$.

$\sin^2\theta + \sin\theta - 2 = 0$

$\sin\theta = \frac{-1 \pm \sqrt{9}}{2}$

$\sin\theta \Rightarrow 1$, $\cancel{-2}$

$\theta = 90^\circ$

$ax^2 + bx + c = 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$0 + 0$

$\longrightarrow 0$

Eg19. If $\tan^2\theta = 1 - a^2$

Prove that : $\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (2 - a^2)^{3/2}$

Homework

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Eg20. If cosec $\theta - \sin\theta = l$ & $\sec\theta - \cos\theta = m$

Find the value of $l^2m^2(l^2 + m^2 + 3)$

Homework