gradeup

Sahi Prep Hai Toh Life Set Hai

ALGEBRA-2

Agenda

Some Common Pattern
Based Question $\longrightarrow$ (64-66) min

Remainder Theorem
$\longrightarrow$ (28-30) min
Factor - Theorem
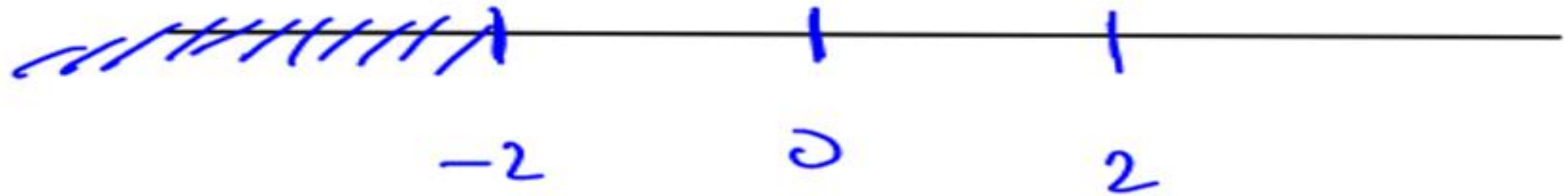
$x \rightarrow$ Real no

$$x + \frac{1}{x}$$

$x \rightarrow$ Number

$\frac{1}{x} \rightarrow$ Reciprocal of Number

If $x > 0$

$$x + \frac{1}{x} \geq 2$$

If $x < 0$

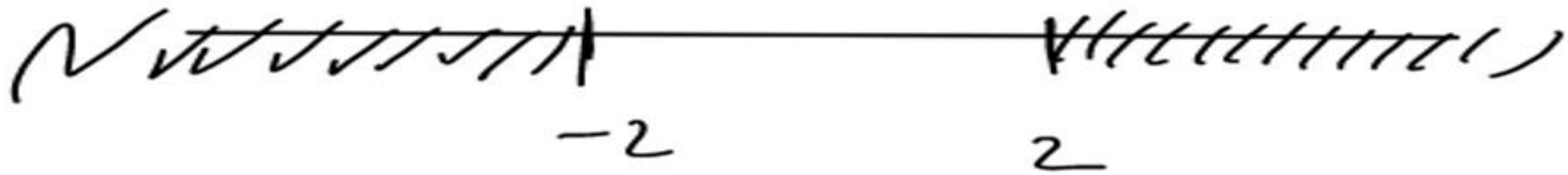$$x + \frac{1}{x} \leq -2$$

$$\boxed{x + \frac{1}{x}}$$

$x < 0$

$x > 0$

-2 2

If $x + \dfrac{1}{x} = 2$ then $x = 1$

Reason:

$$x + \frac{1}{x} = 2$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$\boxed{x = 1}$$

If $x + \dfrac{1}{x} = -2$ then $x = -1$

Reason:

$$x + \frac{1}{x} = -2$$

$$x^2 + 1 = -2x$$

$$x^2 + 2x + 1 = 0$$

$$(x+1)^2 = 0$$

$$\boxed{x = -1}$$

$$(-1)^{odd} = -1$$

$$(-1)^{even} = 1$$

Q1. If $x + \dfrac{1}{x} = -2$ $\qquad\qquad x = -1$

Find : (i) $x^{98} - x^{97} + x^{96} - x^{95} + \ldots\ldots\ldots x^2 - x^1 + 1$

$(-1)^{98} - (-1)^{97} + (-1)^{96} - (-1)^{95} + \cdots (-1)^2 - (-1) + 1$

$1 + 1 + 1 + 1 + \cdots 1 + 1$

$= \boxed{99}$

$$x + \frac{1}{x} = -2 \qquad x = -1$$

(ii) $x^{19} - \dfrac{1}{x^{42}}$

$$(-1)^{19} - \frac{1}{(-1)^{42}} \qquad = \quad -1 - 1$$

$$= \boxed{-2}$$

$x \to$ imaginary

If $x + \dfrac{1}{x} = 1$ then $x^3 = -1$

Reason:

$$x + \frac{1}{x} = 1$$

$$x^2 + 1 = x$$

$$(x+1)(x^2 - x + 1) = 0$$

$$x^3 + 1 = 0$$

$$x^3 = -1$$

Q2. If $x + \dfrac{1}{x} = 1$ $\longrightarrow$ $\underline{\underline{x^3 = -1}}$

Find : (i) $x^{12} + x^9 + x^6 + x^3 + 1$

$$\left(x^3\right)^4 + \left(x^3\right)^3 + \left(x^3\right)^2 + \left(x^3\right)^1 + 1$$

$$\cancel{1} \quad -\cancel{1} \quad +\cancel{1} \quad -\cancel{1} \quad + 1$$

$$= 1$$

$$x + \frac{1}{x} = 1 \longrightarrow x^3 = -1$$

(ii) $x^{96} + x^{90} + x^{84} + \ldots\ldots\ldots + x^{12} + x^6 + 1$

$$\underbrace{(x^3)^{32} + (x^3)^{30} + (x^3)^{28} + \cdots\cdots (x^3)^{4} + (x^3)^{2} + 1}$$

$$1 + 1 + 1 + \cdots\cdots\cdots\cdots\cdots 1 + 1$$

$$= \boxed{17}$$

$$x + \frac{1}{x} = 1 \longrightarrow x^3 = -1$$

(iii) $x^{54} - x^{51} + x^{48} - x^{45} + \ldots\ldots\ldots\ldots x^{12} - x^9 + x^6 - x^3 + 1$

$$\left(x^3\right)^{18} - \left(x^3\right)^{17} + \left(x^3\right)^{16} - \left(x^3\right)^{15} + \cdots - \left(x^3\right)^2 - \left(x^3\right)^1 + 1$$

$$1 \quad + 1 \quad + 1 \quad + 1 \quad + \cdots \cdots \quad 1 \quad + 1$$

$$= \boxed{19}$$

(iv) $x^{30} + x^{50} + x^{16} + 1$

$$x + \frac{1}{x} = 1 \qquad\qquad x^2 + 1 = x$$

$$x^2 - x + 1 = 0$$

$$\left(x^3\right)^{10} + \left(x^3\right)^{16} \cdot x^2 + \left(x^3\right)^5 \cdot x^1 + 1$$

$$1 \quad + \quad x^2 - x + 1$$

$$1 + 0 \quad = \quad 1$$

$x \to$ imaginary

If $x + \dfrac{1}{x} = -1$ then $x^3 = 1$

Reason:

$$x + \frac{1}{x} = -1$$

$$x^2 + 1 = -x$$

$$(x-1)(x^2 + x + 1) = 0$$

$$x^3 - 1 = 0$$

$$\boxed{x^3 = 1}$$

$x \rightarrow$ Imaginary

V. Imp

If $x + \dfrac{1}{x} = \sqrt{3}$ then $x^6 = -1$

Reason:

$$x + \frac{1}{x} = \sqrt{3}$$

$$x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 3\sqrt{3}$$

$$x^3 + \frac{1}{x^3} = 0$$

$$x^6 + 1 = 0 \qquad \boxed{x^6 = -1}$$

Q3. If $x + \dfrac{1}{x} = \sqrt{3}$

$$x^6 = -1$$

Find : (i) $x^{90} + x^{84} + x^{78} + \ldots\ldots\ldots x^{12} + x^6 + 1$

$$(x^6)^{15} + (x^6)^{14} + (x^6)^{13} + \cdots (x^6)^2 + (x^6)^1 + 1$$

$$-1 \quad + 1 \quad -1 \quad + \cdots\cdots \quad + 1$$

$$= \underline{0}$$

$$x + \frac{1}{x} = \sqrt{3} \longrightarrow x^6 = -1$$

(ii) $x^{54} - x^{48} + x^{42} - x^{36} + \ldots\ldots\ldots x^{18} - x^{12} + x^6 + 1$

$$(x^6)^9 - (x^6)^8 + (x^6)^7 - (x^6)^6 + \cdots (x^6)^1 + 1$$

$$-1 \quad -1 \quad -1 \quad -1 -1 \quad -1 -1 \quad +1$$

$$= \boxed{-8}$$

$$x + \frac{1}{x} = \sqrt{3} \longrightarrow x^6 = -1$$

(iii) $x^{204} + x^{198} + x^{79} + x^{73} + x^{60} + 1$

$$x^{198}\left[x^6 + 1\right] + x^{73}\left[x^6 + 1\right] + (x^6)^{10} + 1$$

$$1 + 1$$

$$= \boxed{2}$$

$$x^6 = -1$$
$$x^6 + 1 = 0$$

gup

If $x + \dfrac{1}{x} = 2$ then $x = 1$

If $x + \dfrac{1}{x} = -2$ then $x = -1$

If $x + \dfrac{1}{x} = 1$ then $x^3 = -1$

If $x + \dfrac{1}{x} = -1$ then $x^3 = 1$

If $x + \dfrac{1}{x} = \sqrt{3}$ then $x^6 = -1$

Eg1. If $x + \dfrac{1}{x} = \boxed{5}$ find : $x^2 + \dfrac{1}{x^2}$

$$x + \frac{1}{x} = 5$$

$$x^2 + \frac{1}{x^2} + 2 = 25$$

$$x^2 + \frac{1}{x^2} = \boxed{23}$$

If $x + \dfrac{1}{x} = k$ then $x^2 + \dfrac{1}{x^2} = k^2 - 2$

eg   If $x + \dfrac{1}{x} = 9$

$x^2 + \dfrac{1}{x^2} = 79$

$x + \dfrac{1}{x} = 35$

$x^2 + \dfrac{1}{x^2} = 35^2 - 2$

$1225 - 2$

$\boxed{1223}$

(i) If $x^{34} + \dfrac{1}{x^{34}} = 11$ $\quad$ find : $x^{68} + \dfrac{1}{x^{68}}$

$\Rightarrow 11^2 - 2$

$\Rightarrow 119$

(ii) If $x + \dfrac{1}{x} = 7$ find: $x^4 + \dfrac{1}{x^4}$

$$x + \frac{1}{x} = 7$$

$$x^2 + \frac{1}{x^2} = 49 - 2 = 47$$

$$x^4 + \frac{1}{x^4} = (47)^2 - 2$$

$$\Rightarrow \underline{\underline{2207}}$$

(iii) If $x + \dfrac{1}{x} = 3$ find : $x^8 + \dfrac{1}{x^8}$

$$x + \frac{1}{x} = 3$$

$$x^2 + \frac{1}{x^2} = 3^2 - 2 = \boxed{7}$$

$$x^4 + \frac{1}{x^4} = 7^2 - 2 \Rightarrow 47$$

$$x^8 + \frac{1}{x^8} = 47^2 - 2 = \underline{\underline{2207}}$$

Eg2. If $x + \dfrac{1}{x} = 5$ find : $x^3 + \dfrac{1}{x^3}$

Detailed

$$x + \frac{1}{x} = 5$$

$$x^3 + \frac{1}{x^3} + 3 \cdot \cancel{x} \frac{1}{\cancel{x}}\left(x + \frac{1}{x}\right) = 125$$

$$x^3 + \frac{1}{x^3} + 15 = 125$$

$$x^3 + \frac{1}{x^3} = 110$$

If $x + \dfrac{1}{x} = k$ $\qquad x^3 + \dfrac{1}{x^3} = k^3 - 3k$

eg $\qquad x + \dfrac{1}{x} = 10 \qquad\qquad x^3 + \dfrac{1}{x^3} = 10^3 - 3 \cdot 10$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Rightarrow 970$

eg $\qquad x + \dfrac{1}{x} = 14 \qquad\qquad x^3 + \dfrac{1}{x^3} = 14^3 - 3 \cdot 14$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = \underline{\underline{2702}}$

(i) If $x + \dfrac{1}{x} = 8$ find : $x^3 + \dfrac{1}{x^3}$

$$= 8^3 - 3 \cdot 8$$

$$512 - 24$$

$$= 488$$

(ii) If $2m + \dfrac{1}{2m} = 11$

find : $8m^3 + \dfrac{1}{8m^3}$

$\Rightarrow 11^3 - 3 \cdot 11$

$\Rightarrow 1331 - 33$

$= 1298$

(iii) If $x^5 + \dfrac{1}{x^5} = 3$ find: $x^{45} + \dfrac{1}{x^{45}}$

$$x^5 + \frac{1}{x^5} = 3$$

$$x^{15} + \frac{1}{x^{15}} = 18$$

$$x^{45} + \frac{1}{x^{45}} = 18^3 - 3 \cdot 18$$

$$= 5832 - 54$$

$$= \boxed{5778}$$

(iv) Find the value of $(5 + 2\sqrt{6})^3 + (5 - 2\sqrt{6})^3$

$$x = 5 + 2\sqrt{6}$$

$$\frac{1}{x} = 5 - 2\sqrt{6}$$

$$x + \frac{1}{x} = 10$$

$$x^3 + \frac{1}{x^3} = 10^3 - 3\cdot10$$

$$= 970$$

Eg3. If $x + \dfrac{1}{x} = 5$ find : $x^6 + \dfrac{1}{x^6}$

I

$$x + \frac{1}{x} = 5$$

$$x^2 + \frac{1}{x^2} = 23$$

$$x^6 + \frac{1}{x^6} = (23)^3 - 3 \cdot 23$$

$$= 12167 - 69$$

$$= 12068$$

Betta

II

$$x + \frac{1}{x} = 5$$

$$x^3 + \frac{1}{x^3} = 110$$

$$x^6 + \frac{1}{x^6} = 12100 - 2$$

$$= 12098$$

Eg4. If $x + \dfrac{1}{x} = 3$ find : $x^5 + \dfrac{1}{x^5}$

$$x^2 + \frac{1}{x^2} = 7 \qquad\qquad x^3 + \frac{1}{x^3} = 18$$

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) = 126$$

$$x^5 + \frac{1}{x} + x + \frac{1}{x^5} = 126$$

$$\boxed{x^5 + \frac{1}{x^5} = 123}$$

If $x + \dfrac{1}{x} = k$ $\qquad$ $x^5 + \dfrac{1}{x^5} = \left[(k^2 - 2)(k^3 - 3k) - k \right]$

$\mathscr{g}$ $\qquad$ If $\qquad$ $x + \dfrac{1}{x} = 4$ $\qquad\qquad$ $x^5 + \dfrac{1}{x^5} = ??$

$$(4^2 - 2)(4^3 - 3 \cdot 4) - 4$$

$$14 \cdot 52 - 4$$

$$\boxed{724}$$

REMAINDER THEOREM

$$\dfrac{x^2 - 6x + 8}{x - 1}$$

$$Rem = \; ? \; ?$$

$$x = 1$$

$$1^2 - 6(1) + 8$$

$$= \boxed{3}$$

$$
\begin{array}{r}
x - 5 \\
x - 1 \enclose{longdiv}{x^2 - 6x + 8} \\
\underline{x^2 - x} \quad (+) \\
-5x + 8 \\
\underline{-5x + 5} \quad (-) \\
\boxed{3}
\end{array}
$$

$$Rem$$

Eg1. Find the remainder when $x^2 - 7x + 4$ is divided by $x - 2$.

$$x^2 - 7x + 4$$

$$x - 2$$

$$Rem = 2^2 - 7 \cdot 2 + 4$$

$$= -6$$

Eg2. Find the remainder.

$$\frac{x^3 - 5x^2 + 11x + 9}{x + 2}$$

Rem

$$(-2)^3 - 5(-2)^2 + 11(-2) + 9$$

$$-8 - 20 - 22 + 9$$

$$= \boxed{-41}$$

Eg3. Find the remainder.

$$\frac{12x^3 - 13x^2 - 5x + 7}{3x + 2}$$

$$3x + 2 = 0$$

$$x = -2/3$$

$$12\left(\frac{-2}{3}\right)^3 - 13\left(\frac{-2}{3}\right)^2 - 5\left(\frac{-2}{3}\right) + 7$$

$$12^4\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right) - \frac{52}{9} + \frac{10}{3} + 7$$

$$-\frac{32}{9} - \frac{52}{9} + \frac{10}{3} + 7$$

$$\frac{-32 - 52 + 30 + 63}{9} = \frac{9}{9} = 1 \quad \text{①}$$

Eg4. Find the remainder.

$$\frac{x^{40} + 2}{x + 1}$$

$$(-1)^{40} + 2$$

$$= \boxed{3}$$

Eg5. Find the remainder.

$$\frac{x^{40} + 2}{x^4 + 1}$$

$$x^4 = -1$$

$$x^{40} + 2$$

$$(x^4)^{10} + 2$$

$$= \boxed{3}$$

Eg6. Y = ax² + bx + c

Y is divisible by (x+1) & (x+2) but leaves a remainder of 12 when divided by (x-1). Find c.

$$\frac{ax^2 + bx + c}{x+1} \qquad R = 0$$

$$\frac{ax^2 + bx + c}{x+2} \qquad R = 0$$

$$\frac{ax^2 + bx + c}{x-1} \qquad R = 12$$

$$a - b + c = 0 \quad -①$$

$$4a - 2b + c = 0 \quad ②$$

$$a + b + c = 12 \quad ③$$

③ - ①

$$2b = 12$$

$$\boxed{b = 6}$$

$$a + c = 6$$

$$4a + c = 12$$

③ $a = 2$

$$\boxed{c = 4}$$

$y = ax^2 + bx + c$ It's div by $(x+1)$ & $(x+2)$ but
on div by $(x-1)$, it gives 12
as remainder. Find c ??

IInd App

$$ax^2 + bx + c = K(x+1)(x+2)$$

$$ax^2 + bx + c = K(x^2 + 3x + 2)$$

$a = K$

$b = 3K$

$$\boxed{c = 2K}$$

$a + b + c = 12$

$6K = 12$ $\boxed{c = 4}$

$K = 2$

$$\nearrow \frac{x^2 - 3x + 2}{\boxed{x - 2}} \longrightarrow \qquad Rem = 0$$

$$, \quad Rem = 0 \quad then \quad (x-2)' \text{ is a factor}$$
$$of \quad x^2 - 3x + 2$$

Eg1. Check whether (x+3) is a factor of x³ + 6x² – 11x + 6.

$$(-3)^3 + 6(-3)^2 - 11(-3) + 6$$

$$-27 + 54 + 33 + 6$$

$$\neq 0$$

So $(x+3)$ is not a factor of

$$x^3 + 6x^2 - 11x + 6$$

Eg2. If $x^3 + mx^2 + 3x + m$ is divisible by $(x+3)$. Find m?

$$(-3)^3 + m(-3)^2 + 3(-3) + m = 0$$

$$-27 + 9m - 9 + m = 0$$

$$10m = 36$$

$$m = \frac{\overset{18}{\cancel{36}}}{\underset{5}{\cancel{10}}}$$

Eg3. If (x-1) & (x-2) are factors of $x^3 + 10x^2 + ax + b$. Find a & b.

$$x = 1$$

$$x = 2$$

$$1 + 10 + a + b = 0$$

$$8 + 40 + 2a + b = 0$$

$$a + b = -11 \quad \textcircled{1}$$

$$2a + b = -48 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1}$$

$$a = -37$$

$$b = 26$$

Eg4. If $mx^3 - 4x^2 - nx + 5$ is divisible by $(x^2 - x - 2)$.
Find $m - n$.

$$(x-2)(x+1)$$

$x = 2$

$x = -1$

$-m - 4 + n + 5 = 0$

$m - n = -1$