gradeup

gradeup

Sahi Prep Hai Toh Life Set Hai

CIRCLE

Part 1

CIRCLES

I^{st} → Basic Terminologies

Theorems of Circles

II^{nd} → Practice of Question (20Q)

→ 20 Ques Homework

III^{rd} → Common Tangent & Common Chord

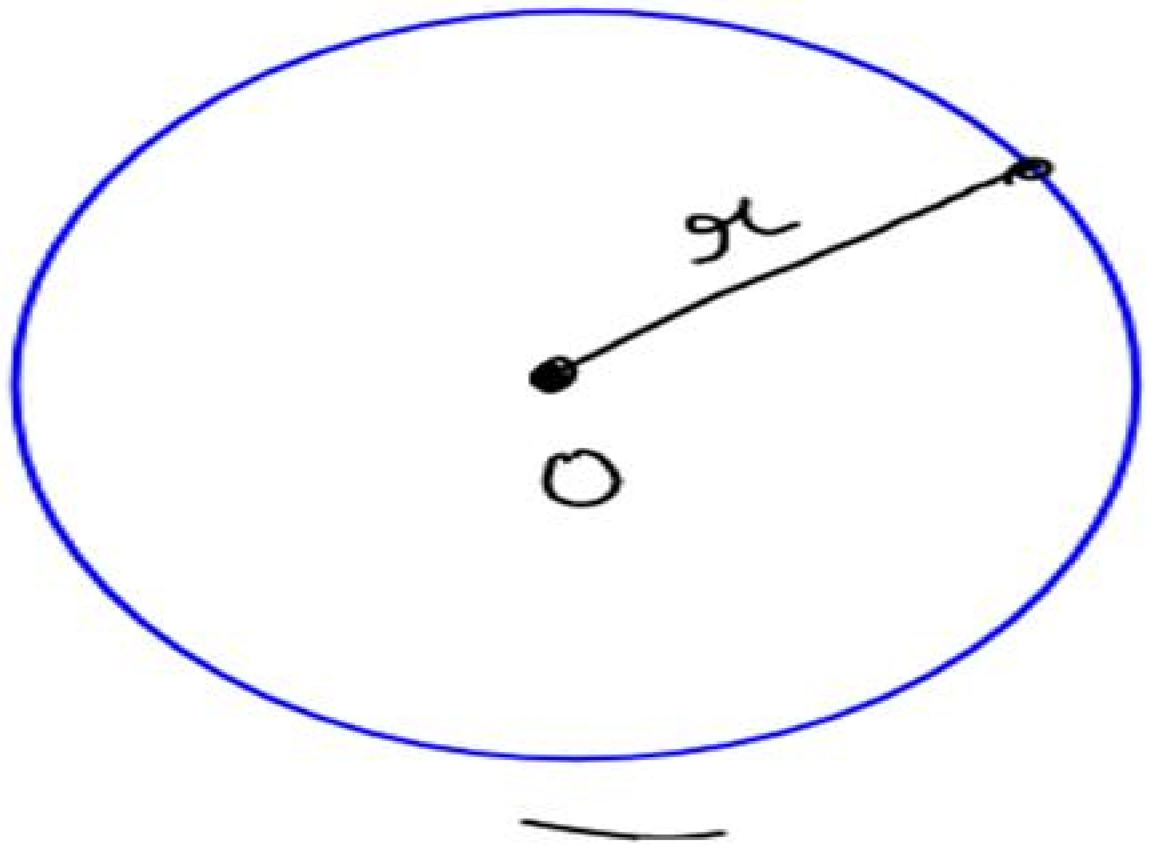
→ 20 Question

IV^{th} → Doubt Session

gradeup

BASIC TERMINOLOGIES

Circle is a collection of all those points which are at a fixed distance from a certain given point.

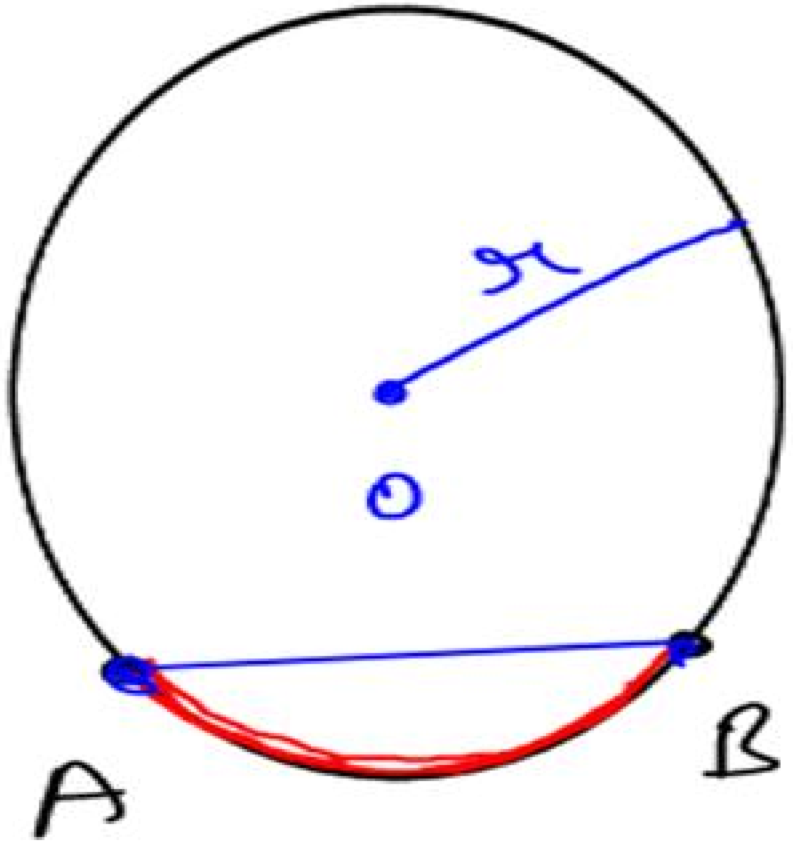


Centre

Circumference

Radius

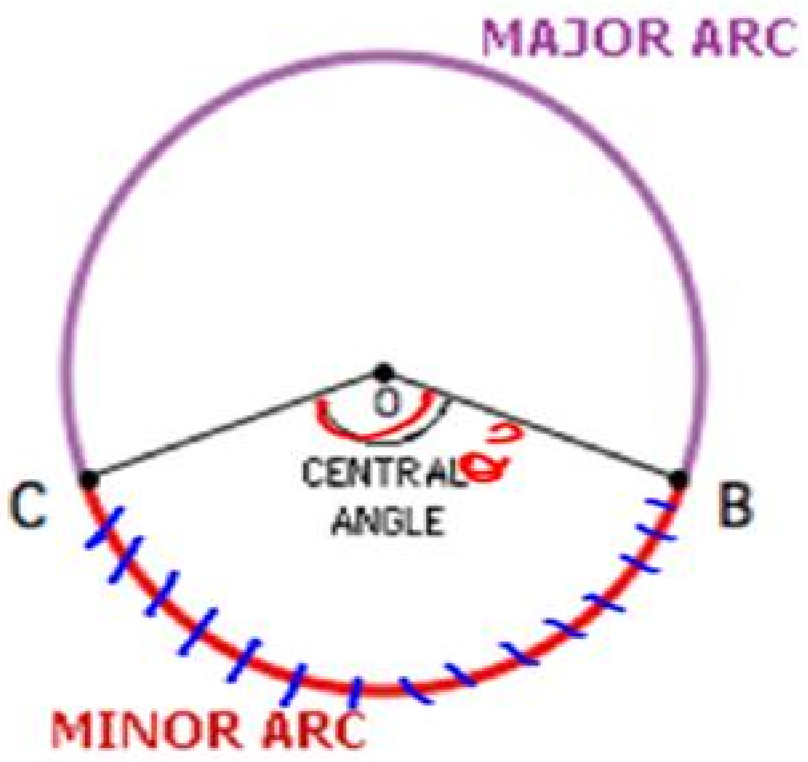
Arc



$\overset{\frown}{AB}$

$AB \rightarrow$ chord

Minor Arc & Major Arc



$$\text{Length of Arc} = \frac{2\pi r\theta}{360}$$

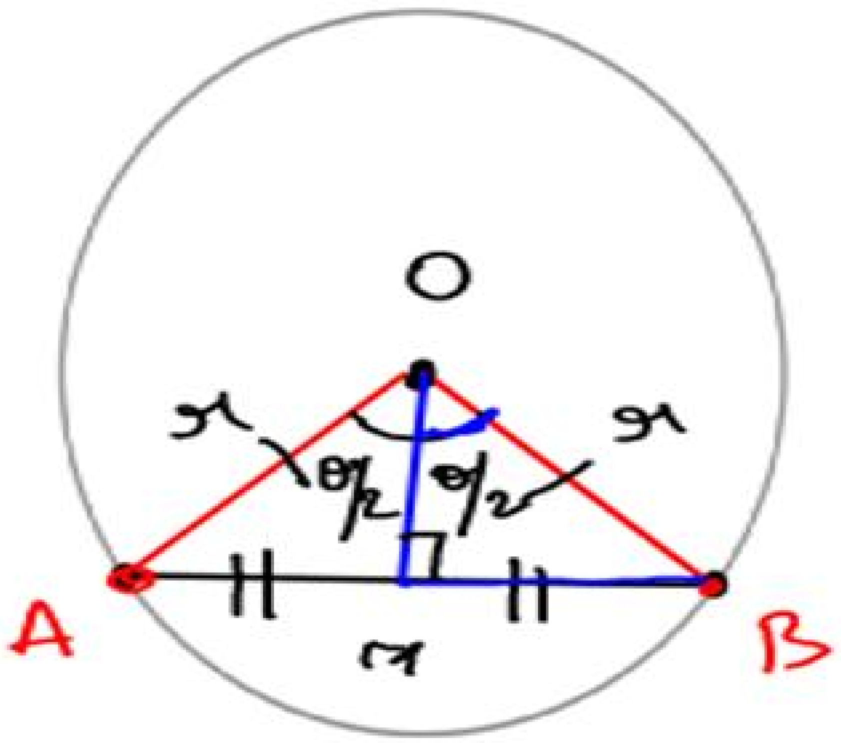
If nothing is given in the question, it is always considered as minor arc.

$360^\circ \rightarrow 2\pi r$

$1^\circ \rightarrow \frac{2\pi r}{360^\circ}$

$\theta^\circ \rightarrow \frac{2\pi r\theta}{360^\circ}$

Chord



ΔOMB

$$\sin\theta/2 = \frac{MB}{r}$$

$$MB = r\sin\theta/2$$

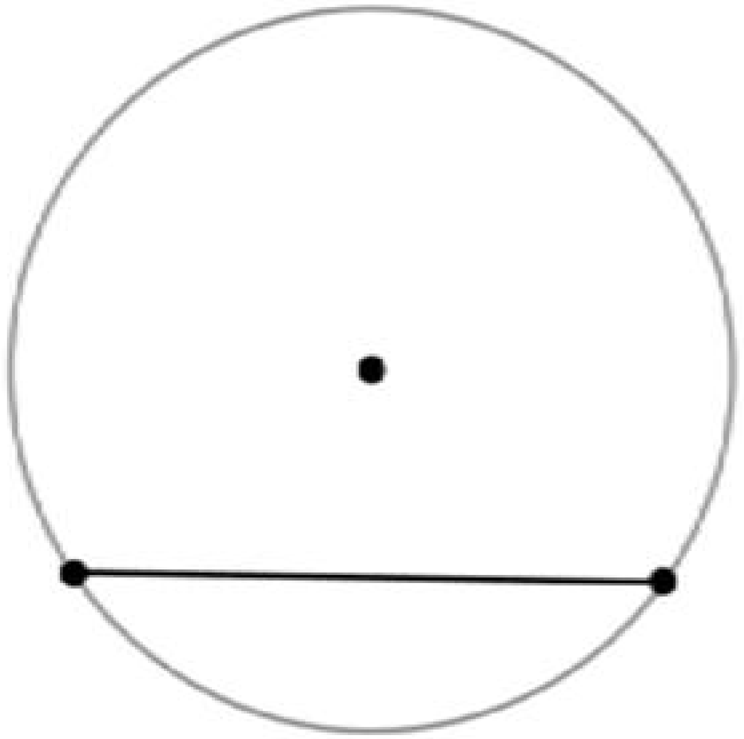
$$2MB = 2r\sin\theta/2$$

$$AB = 2r\sin\theta/2$$

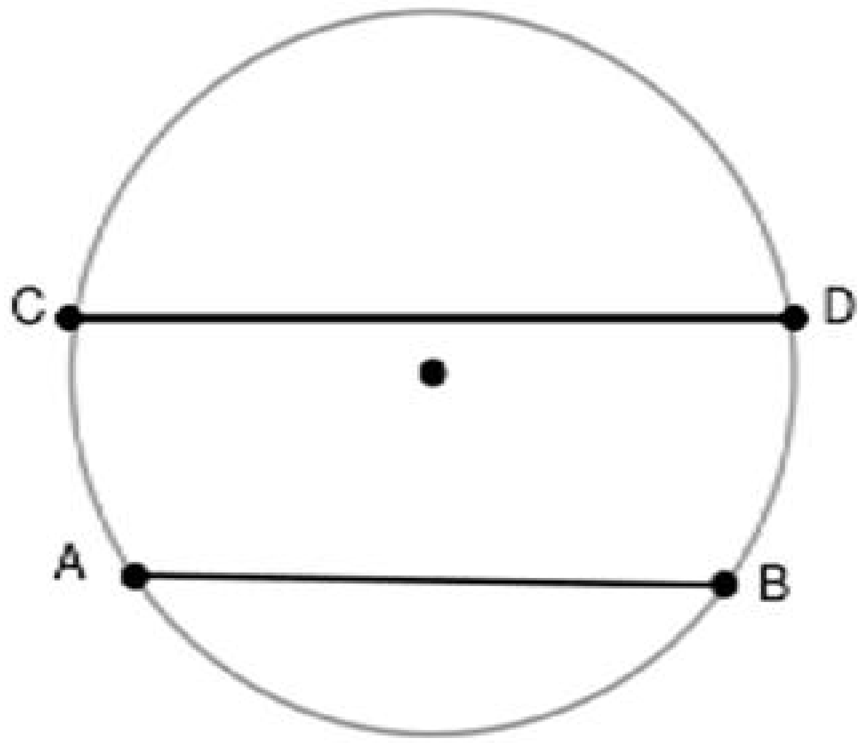
Length of chord = $\boxed{2r\sin\theta/2}$

$\theta \rightarrow$ Central Angle

Length of a chord of a circle



Chord, which is more closer to the centre is larger.



CD > AB

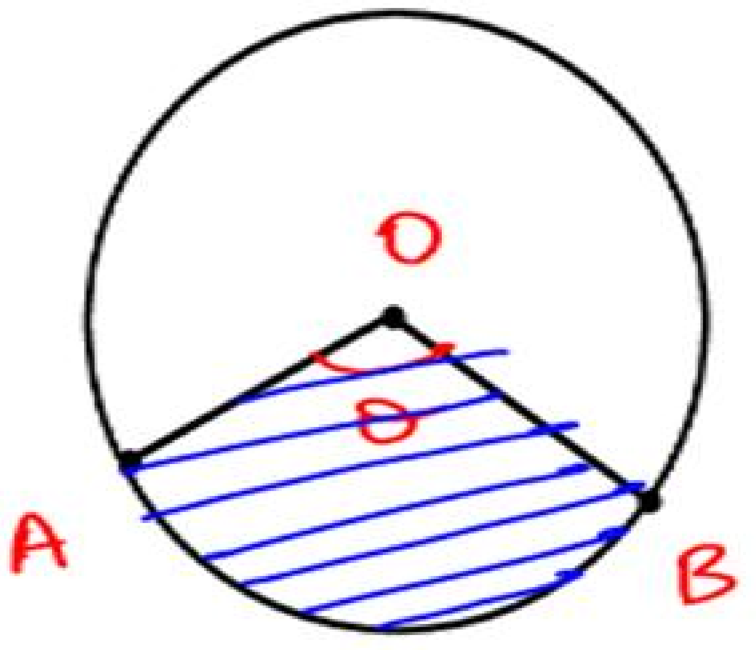
If the chord passes through centre is largest chord

↳ Diameter of circle

$2r$

gradeup

SECTOR OF A CIRCLE



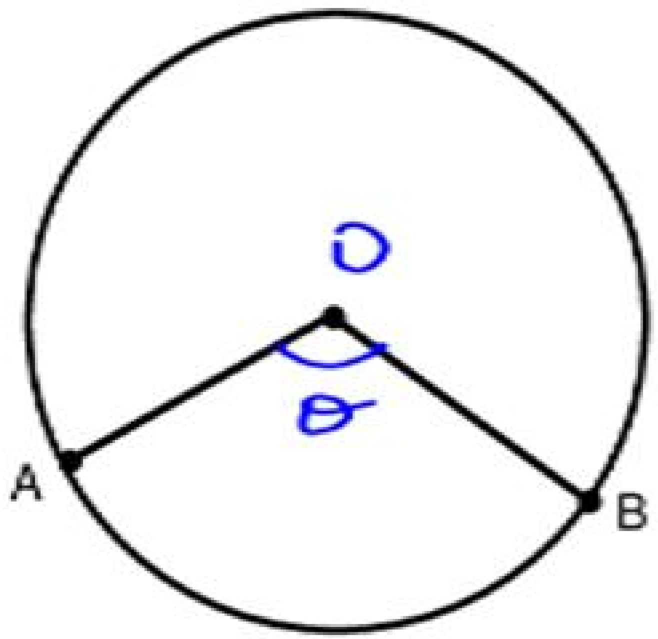
AREA OF SECTOR = $\dfrac{\pi r^2\theta}{360^\circ}$

$360^\circ \longrightarrow \pi r^2$

$1^\circ \longrightarrow \dfrac{\pi r^2}{360^\circ}$

$\theta^\circ \longrightarrow \dfrac{\pi r^2 \theta}{360^\circ}$

Length of the Arc AB (l) = $\dfrac{2\pi r\theta}{360^\circ}$



Area of sector = $\dfrac{1}{2}lr$

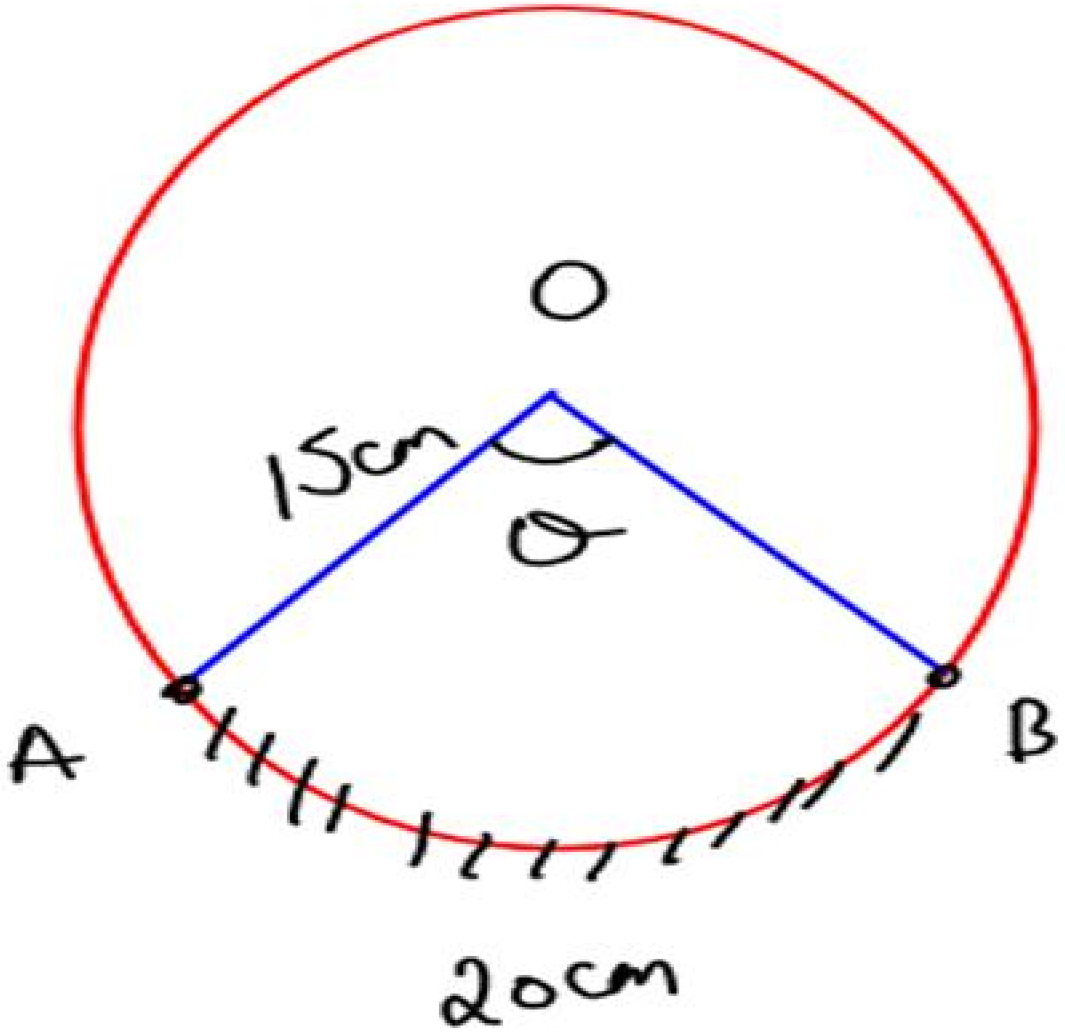
$$A = \frac{\pi r^2 \theta}{360} \quad \text{—(1)}$$

$$l = \frac{2\pi r\theta}{360} \quad \text{—(2)}$$

$$\frac{A}{l} = \frac{r}{2}$$

$$\boxed{A = \frac{1}{2} l r}$$

Eg



$l = 20\,cm$

$r = 15\,cm$

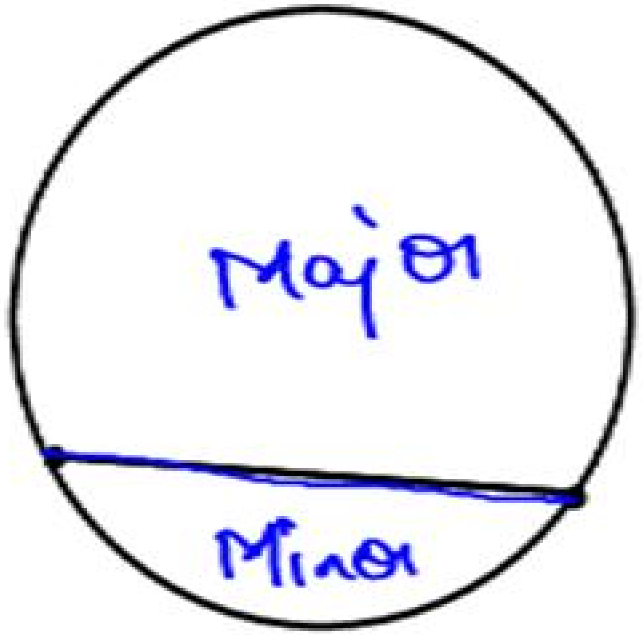
find the area of sector of circle ??

Sol^n

$$\frac{1}{\cancel{2}} \cdot \cancel{20}^{10} \cdot 15$$

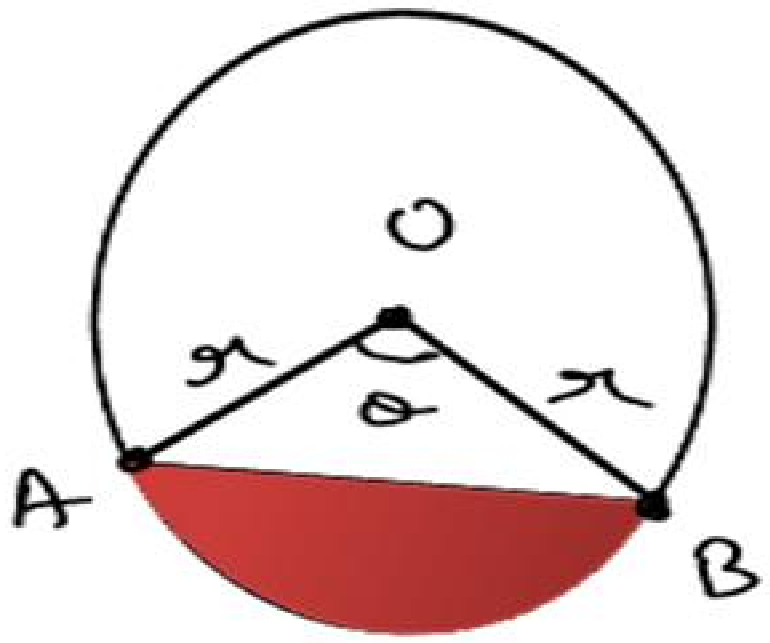
$150\,cm^2$

SEGMENT OF A CIRCLE



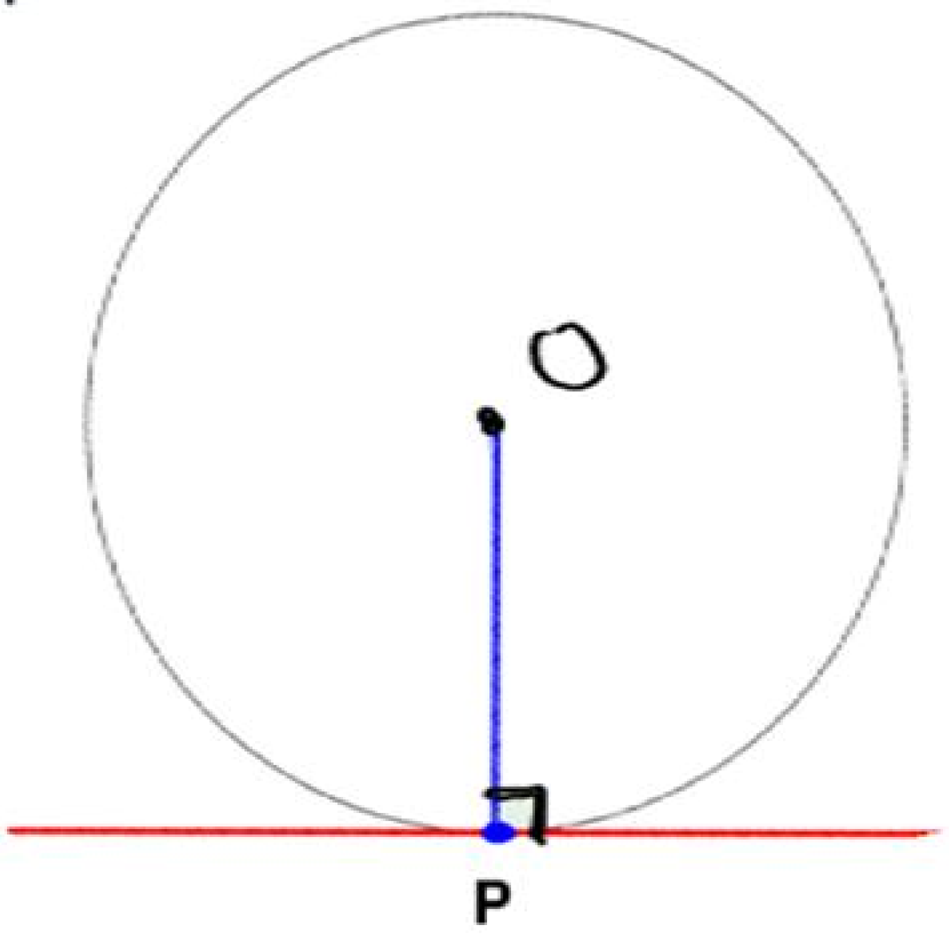
Chord of a circle divides a circle in 2 segments and they are alternate segments to each other.

AREA OF SEGMENT = Area of sector – Area of ΔAOB



$$\frac{\pi r^2\theta}{360^\circ} - \frac{1}{2}r^2\sin\theta$$

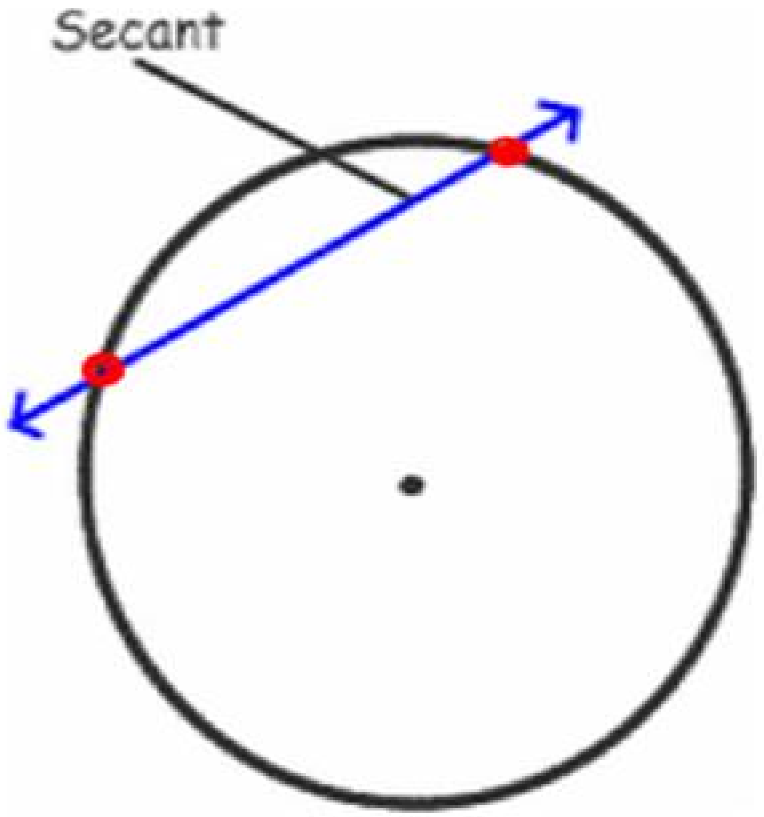
TANGENT



A **tangent** is a line that touches the **circle** at exactly one point.

P → Point of contact

SECANT

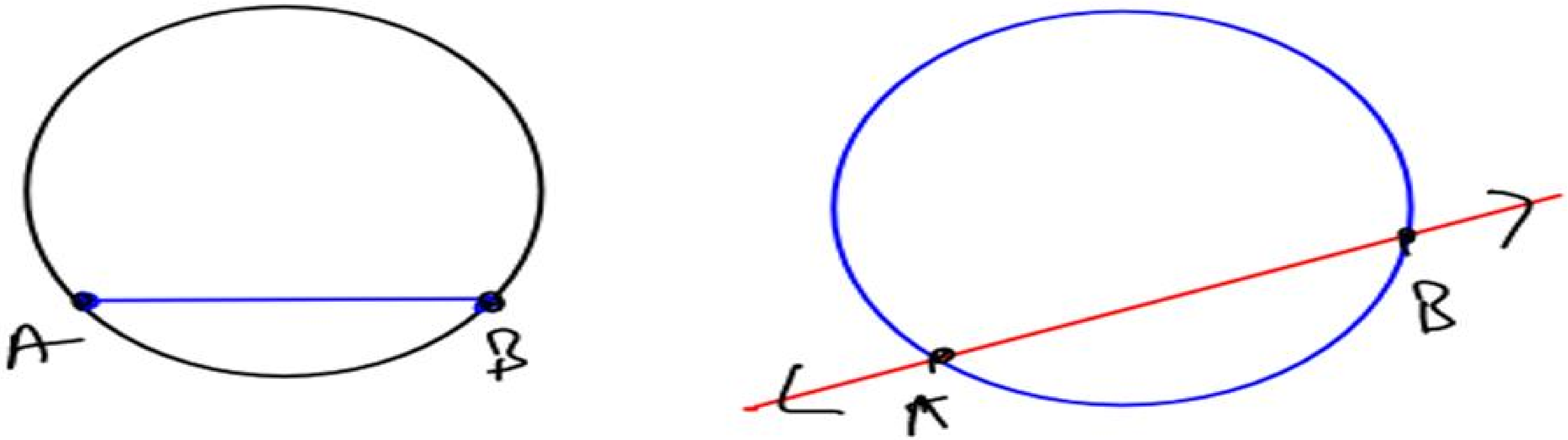


A secant is a line which intersect the circle at 2 distinct points.

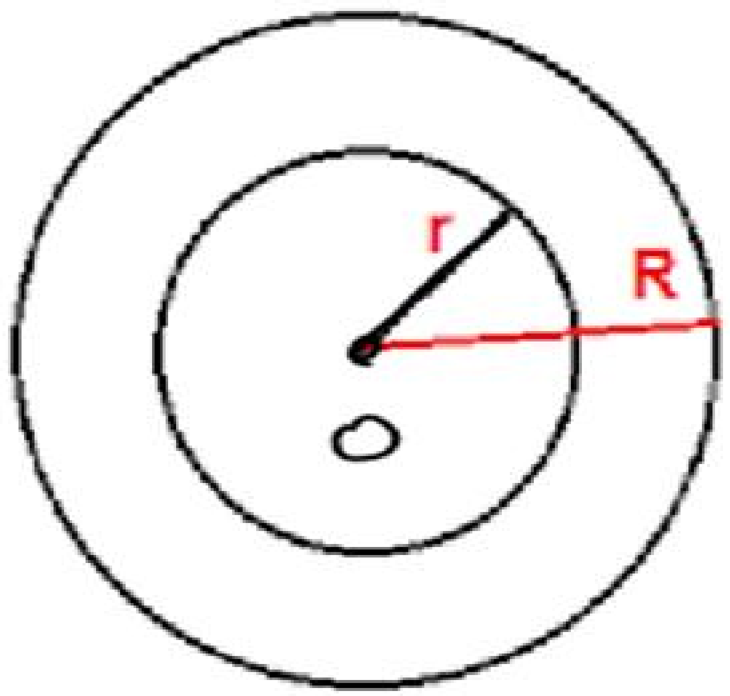
DIFFERENCE BETWEEN A CHORD AND A SECANT

Chord → line segment

Secant → line



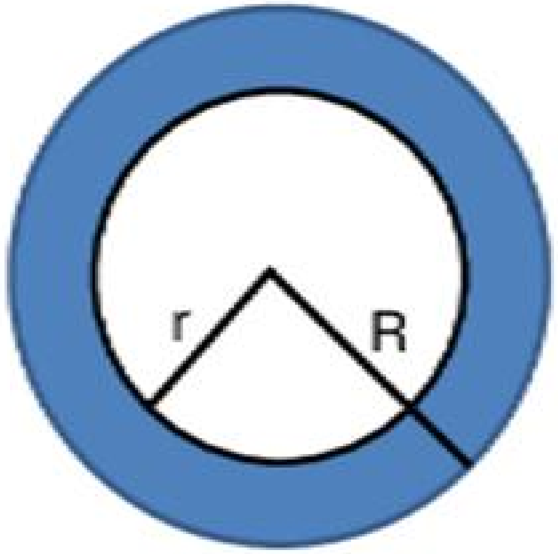
CONCENTRIC CIRCLES



Circles with the same centre.

AREA ENCLOSED BY TWO CONCENTRIC CIRCLES

If R and r are radii of two concentric circles, then



Area enclosed by the two circles $= \pi \boldsymbol{R}^2 - \pi \boldsymbol{r}^2$

$$= \pi\left(\boldsymbol{R}^2 - \boldsymbol{r}^2\right)$$

$$= \pi\left(\boldsymbol{R} + \boldsymbol{r}\right)\left(\boldsymbol{R} - \boldsymbol{r}\right)$$

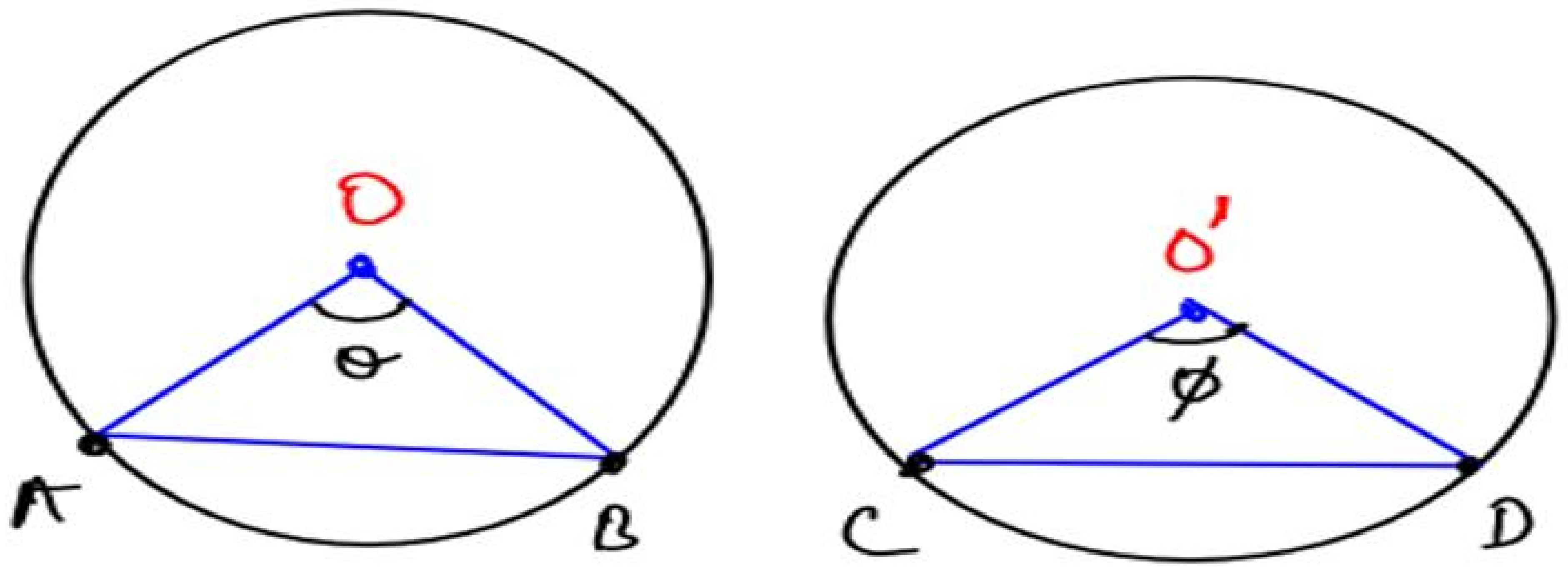
Congruent Circle : Circle having same radius.

2 circles are congruent → same radius

BASIC THEOREMS RELATED TO CIRCLE

1. In same circle / Congruent circle

(i) Equal chords of a circle subtends equal angle at the centre.



$\theta = \phi$

$AB = CD$

$\overset{\frown}{AB} = \overset{\frown}{CD}$

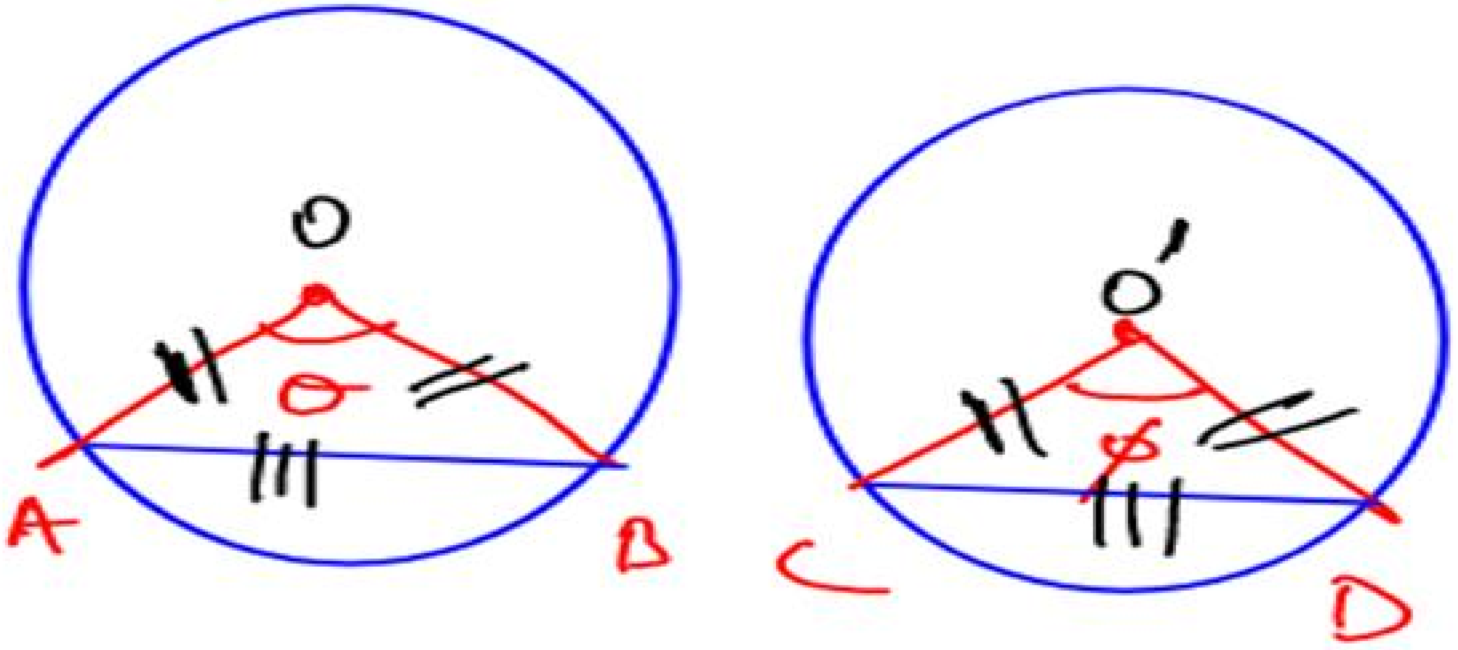
* If out of these 3 (Central Angle, chord & Arc), any one is equal then the remaining 2 will be equal.

gradeup

2 circles are congruent

Given : $AB = CD$

To prove $\theta = \phi$



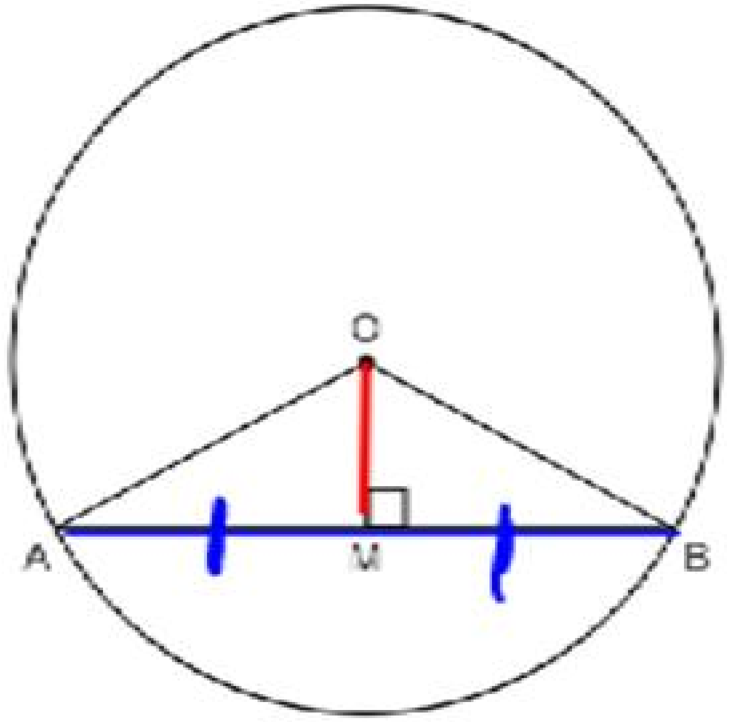
Proof

$\Delta AOB \cong \Delta CO'D$

(By SSS)

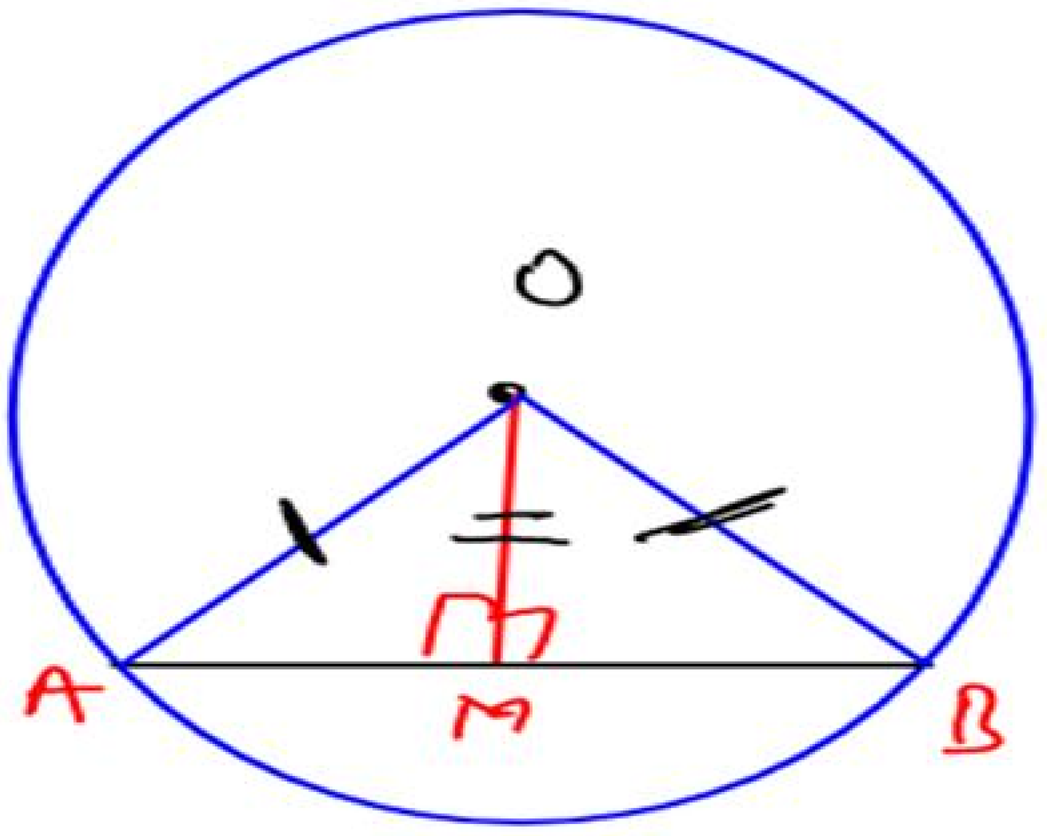
$\boxed{\angle AOB = \angle CO'D}$

2. Perpendicular dropped from the centre of a circle bisect the chord.



Given, OM $\perp$ AB

$\boxed{AM = MB}$



Give A circle with centre O

$OM \perp AB$

To prove $AM = MB$

Proof

ΔAOM & ΔBOM

$AO = BO$

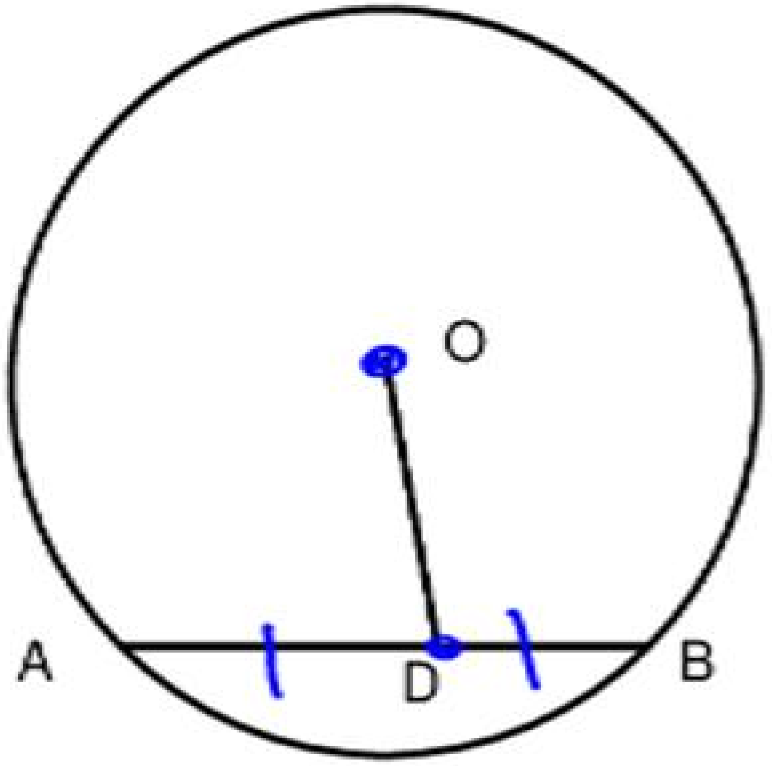
$OM = OM$

$\angle AMO = \angle BMO$

$\Delta AOM \cong \Delta BOM$ [RHS]

Converse

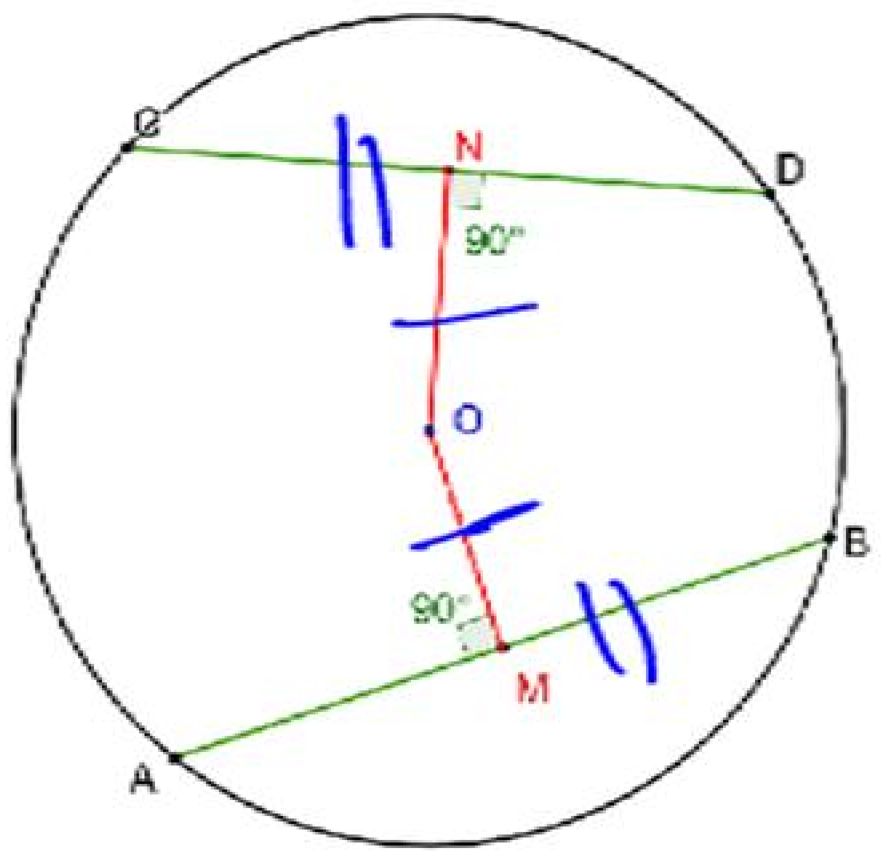
The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.



Given $AD = DB$

$\angle ODA = \angle ODB = 90^\circ$

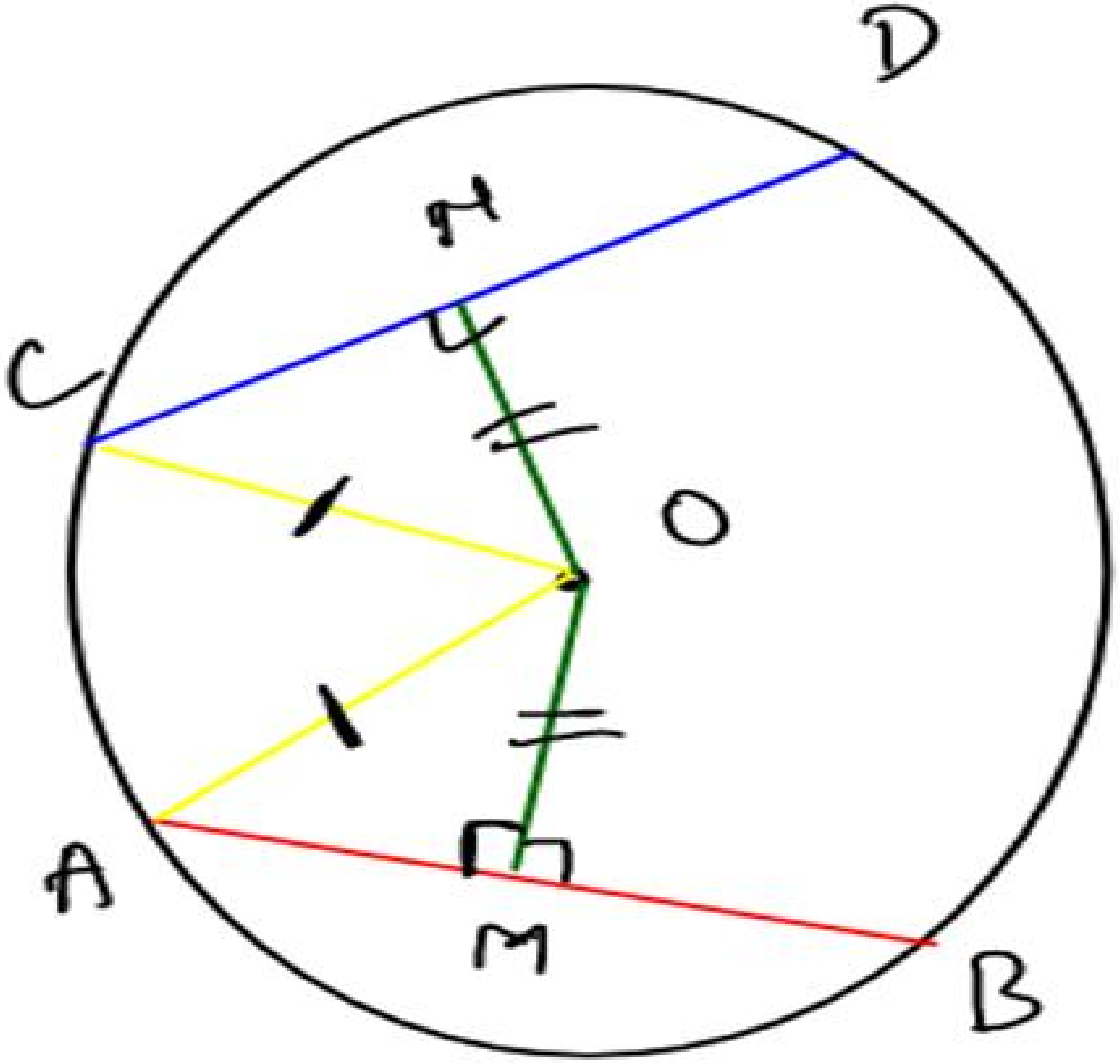
3. Chords equidistant from the centre of the circle are equal.



Given, ON = OM

AB = CD

gradeup



Given $OM = ON$

To prove $AB = CD$

Proof ΔAOM & ΔCON

$AO = CO$ (Radius)

$\angle AMO = \angle CNO$ (90°)

$OM = ON$

$\Delta AOM \cong \Delta CON$

(RHS)

$AM = CN$

$2AM = 2CN$

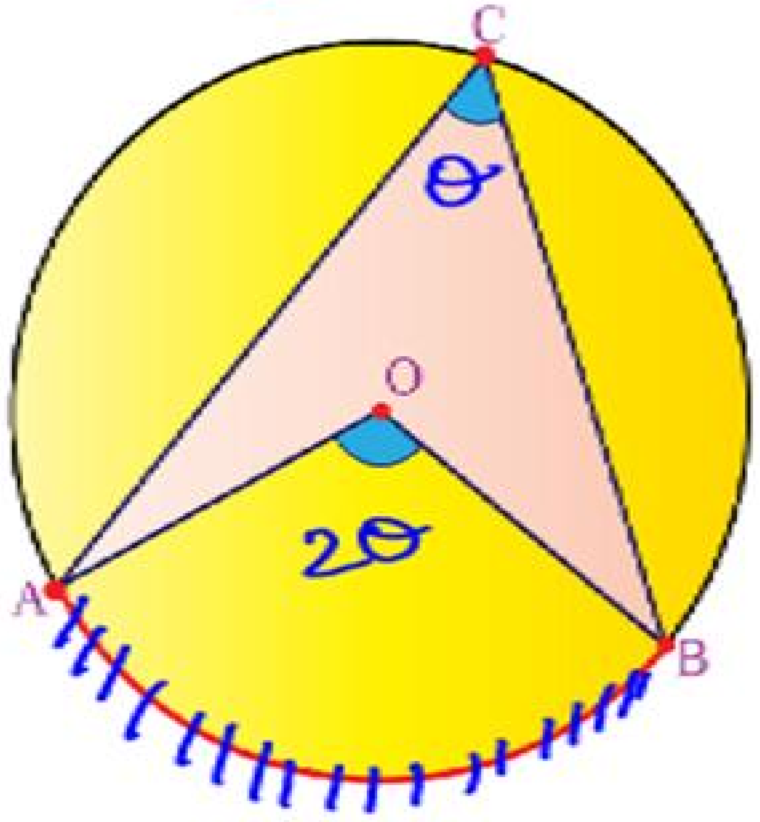
$\boxed{AB = CD}$

Converse :

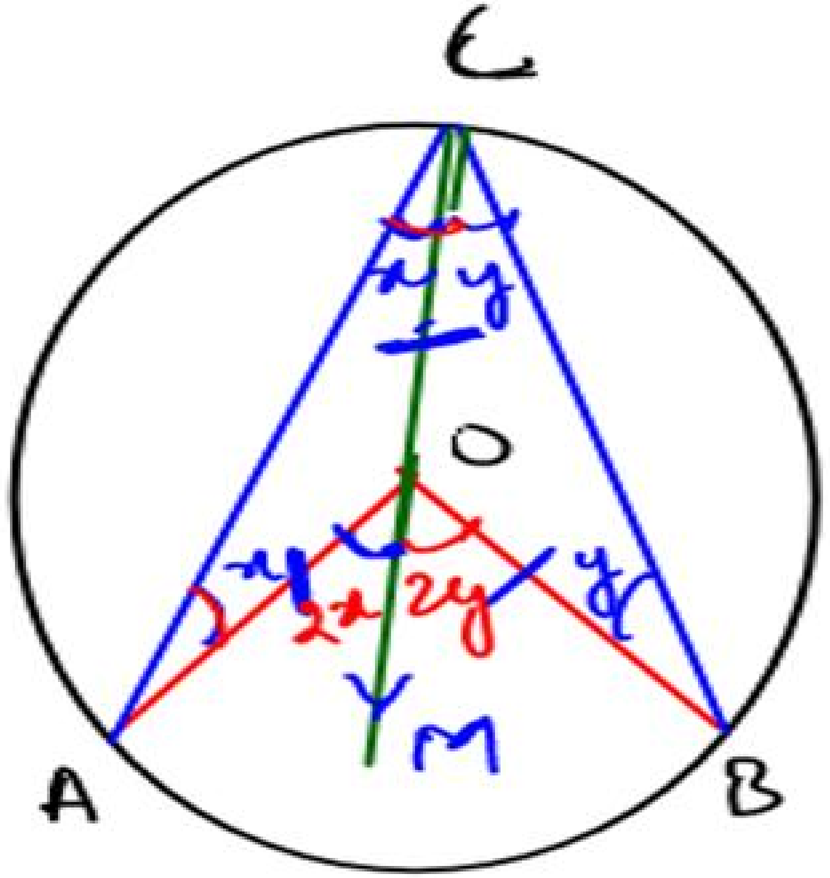
If 2 chords are equal then their distance from centre is also equal.

4 (i) . Angle made by an arc at the centre of circle is twice of the angle made by the same arc on the circumference of the circle (except the arc).

V. Aub



$\angle AOB = 2 \cdot \angle ACB$



To prove $\angle AOB = 2\angle ACB$

Proof

ΔAOC

ΔBOC

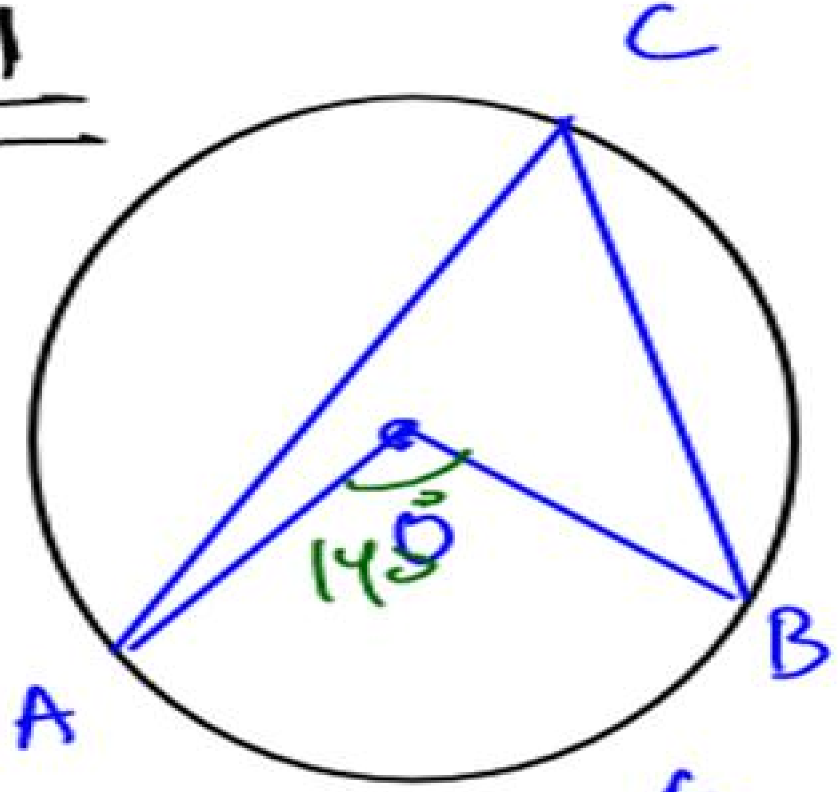
$\angle AOM = 2x$

$\angle BOM = 2y$

$\angle AOM + \angle BOM$

$= 2x + 2y \longrightarrow \boxed{\angle AOB = 2\angle ACB}$

eg 1

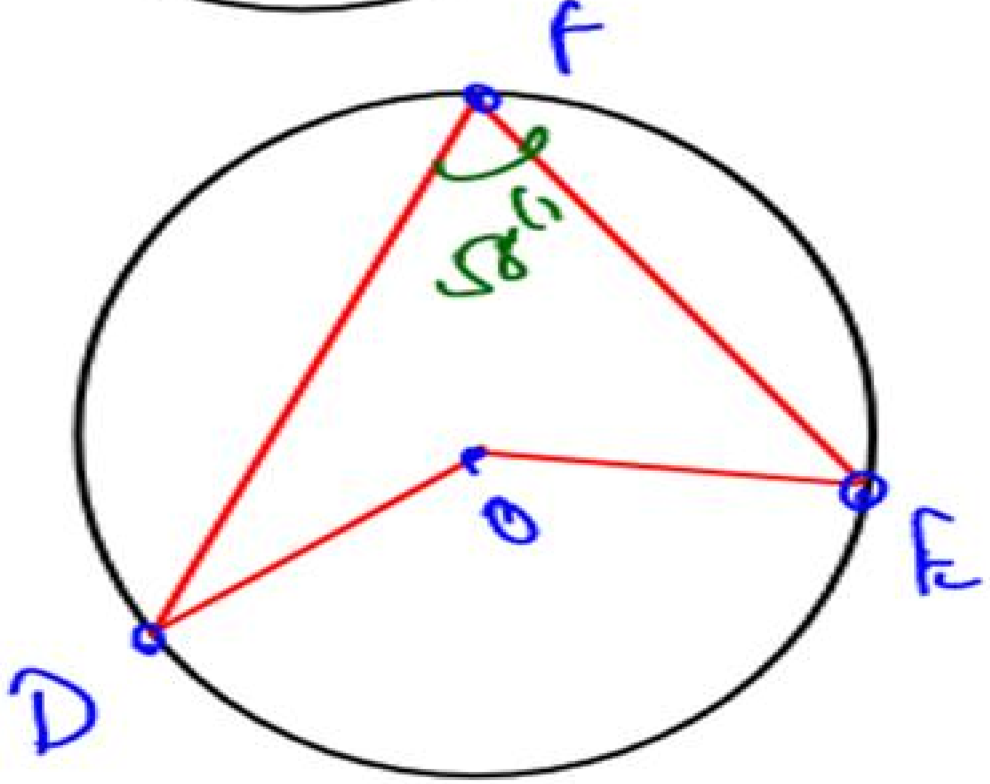


If $\angle AOB = 140°$

$\angle ACB = ??$

$70°$

eg 2

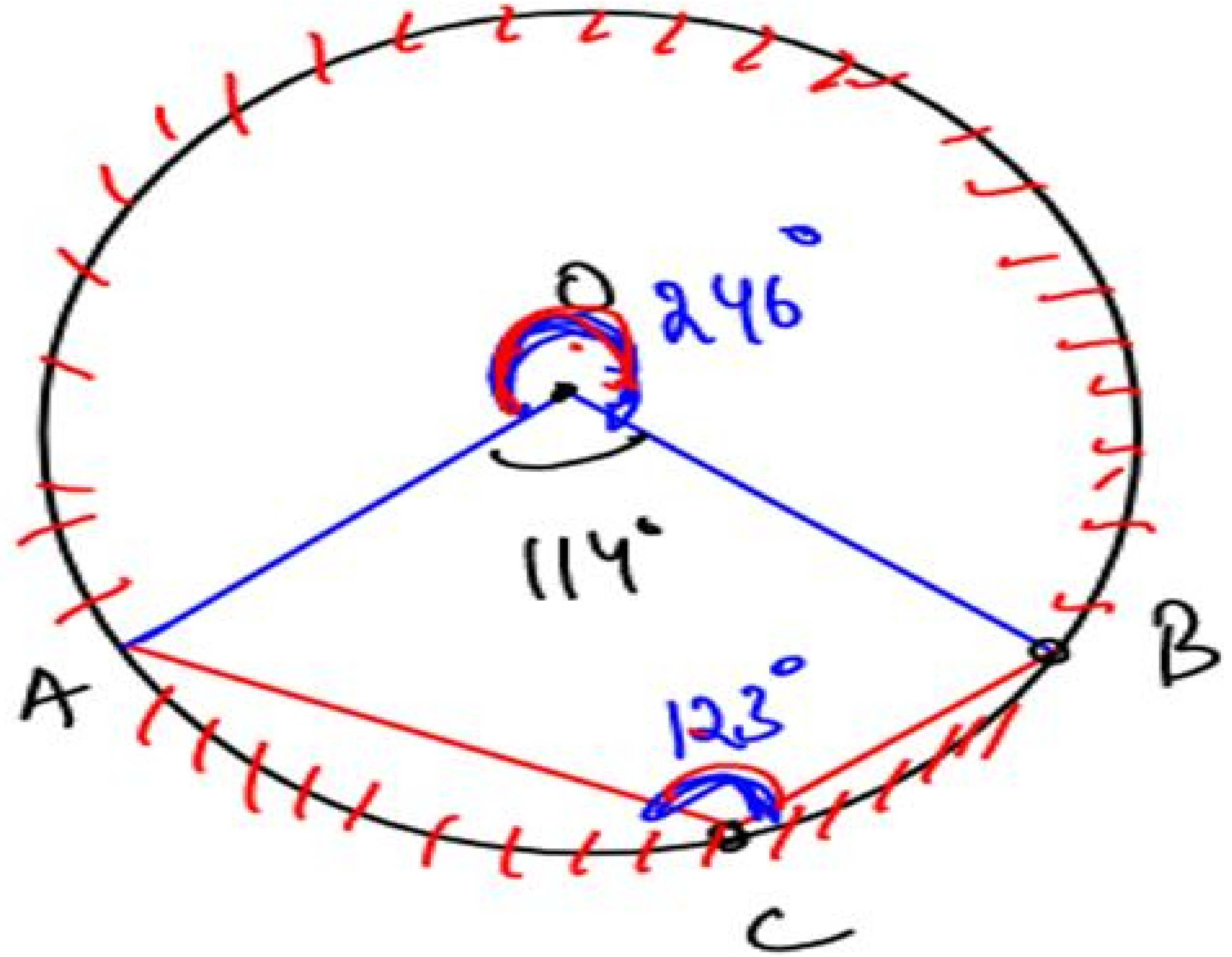


If $\angle DFE = 58°$

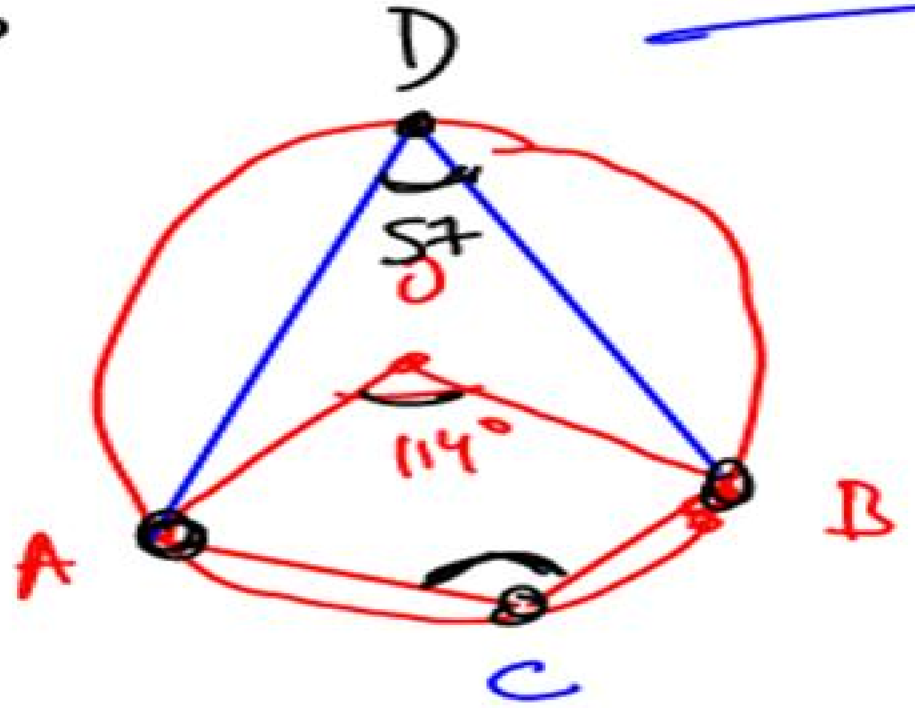
$\angle DOE \rightarrow 116°$

eg

find $\angle ACB = ??$



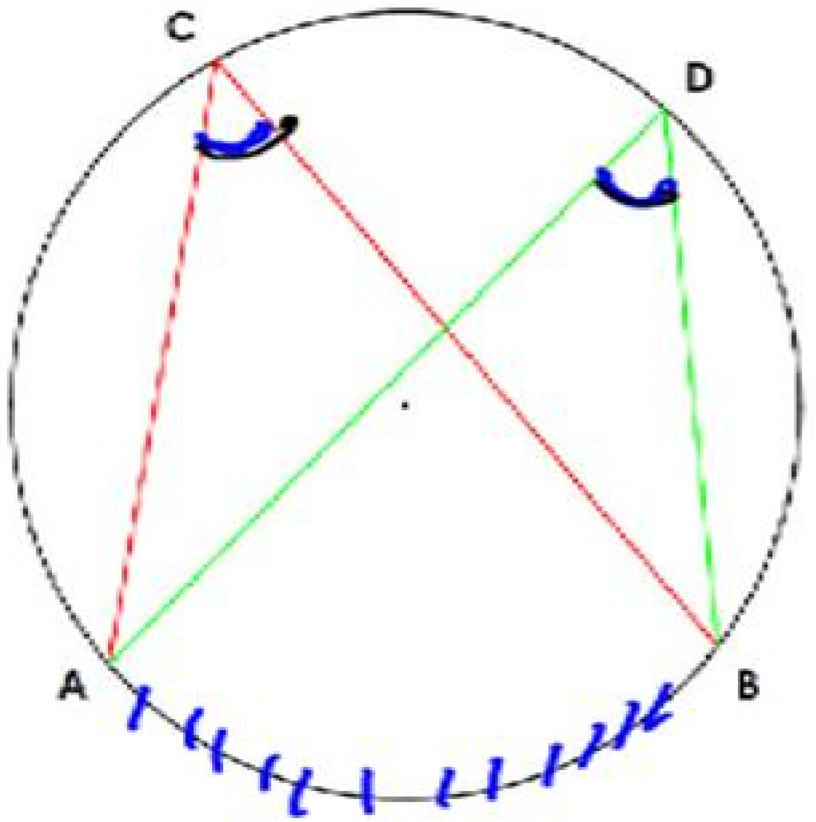
$\angle ACB = 123^\circ$



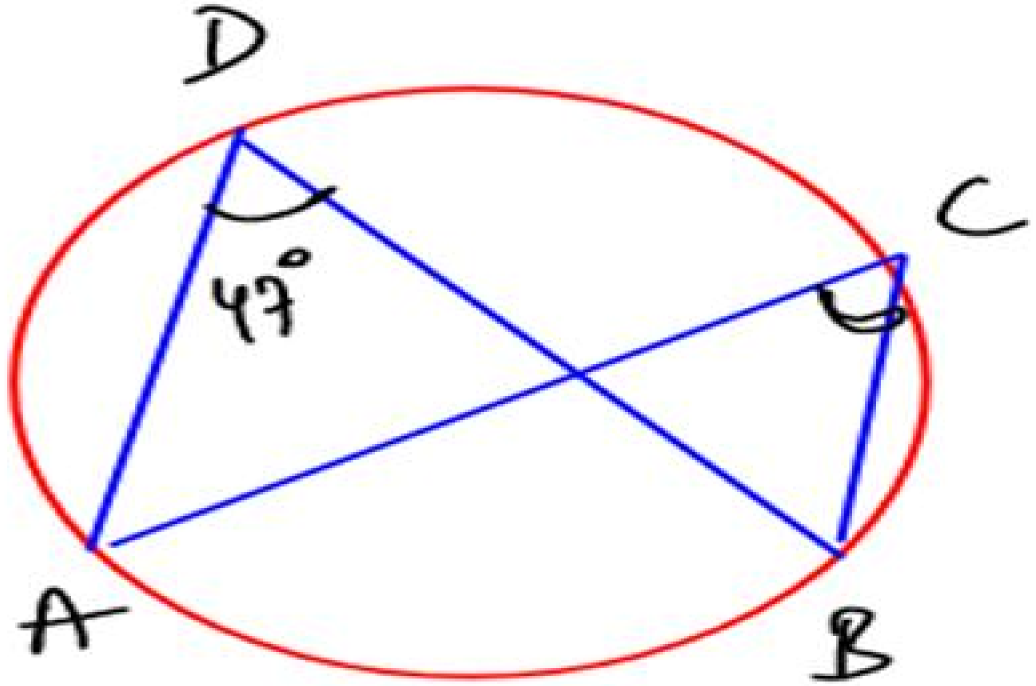
$57 + \angle C = 180$

$\angle C = 123^\circ$

4 (ii) . Angles in the same segment of a circle are equal.

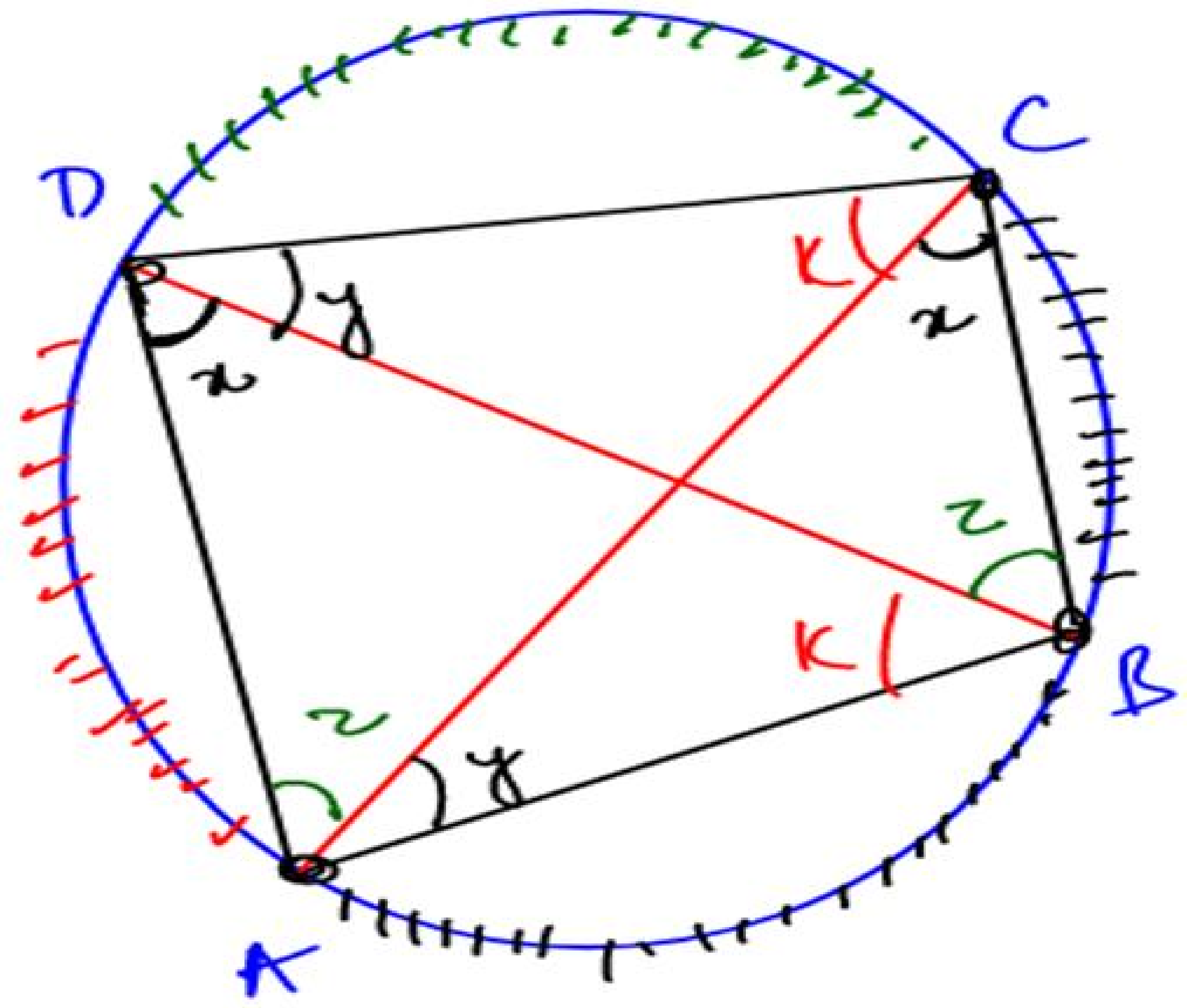


$\angle ACB = \angle ADB$



$\angle ADB = 47^\circ$

$\angle ACB \longrightarrow 47^\circ$



* If all the vertices of a Quadrilateral lie on a single circle $\longrightarrow$ Cyclic Quad

**

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$(y+z) + (z+K) + (K+x) + (x+y) = 360$

$2(\underline{x+y} + z + K) = 360$

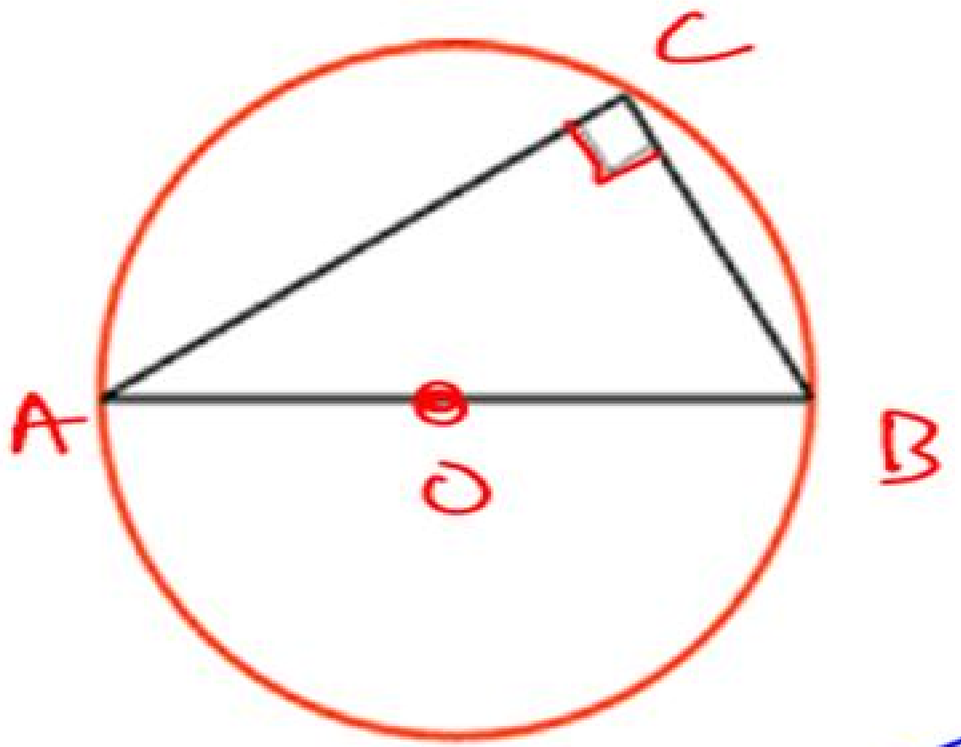
$2(\angle D + \angle B) = 360$

$\angle D + \angle B = 180^\circ$

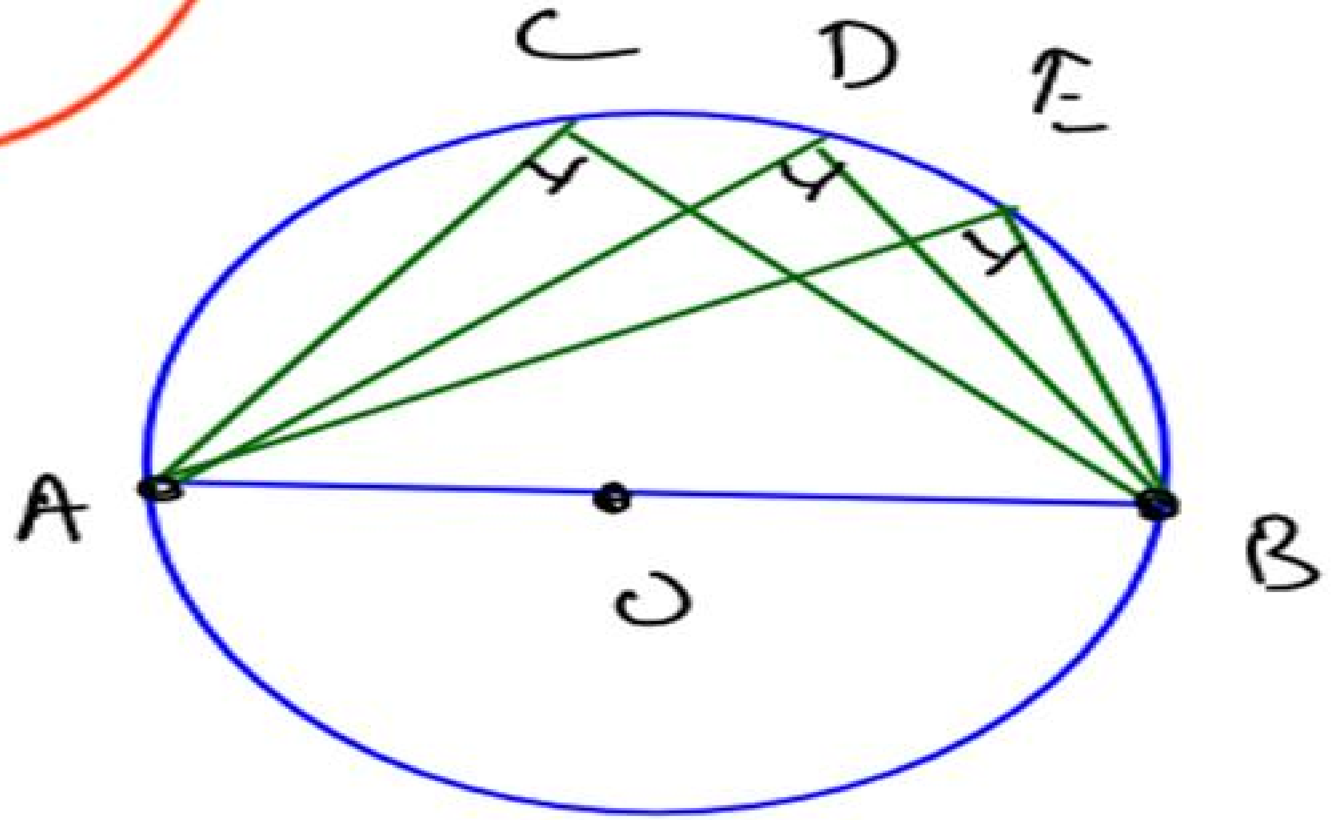
$$\boxed{\begin{aligned} \angle A + \angle C &= 180^\circ \\ \angle B + \angle D &= 180^\circ \end{aligned}}$$

gradeup

4 (iii). Angles in a semi-circle is always a right angle.



Hint → AB is the diameter

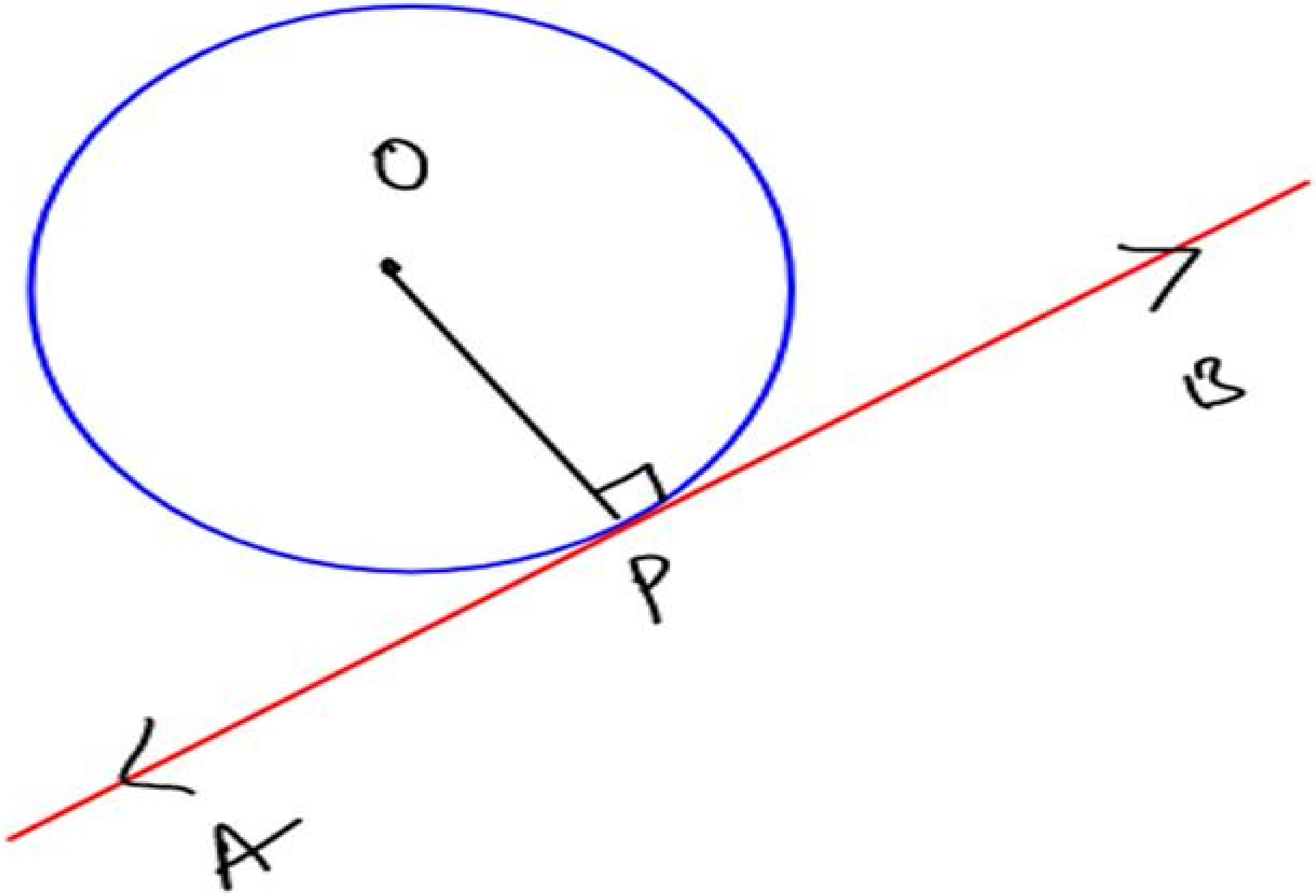


$\angle ACB = \angle ADB = \angle AEB$

$= 90^\circ$

gradeup

5. Tangent



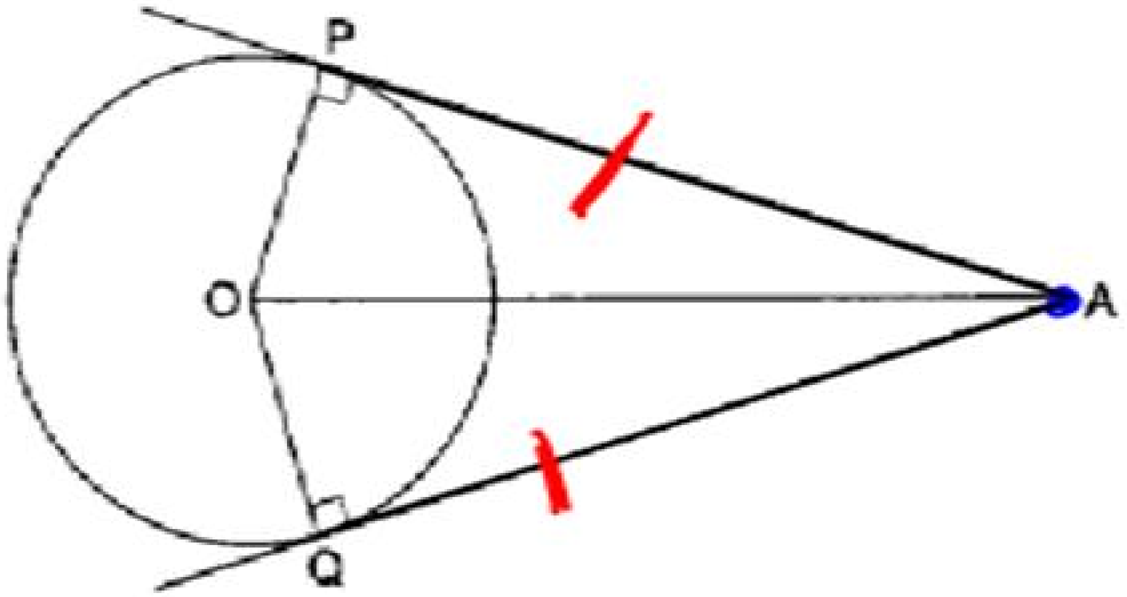
OP → radius

APB → Tangent

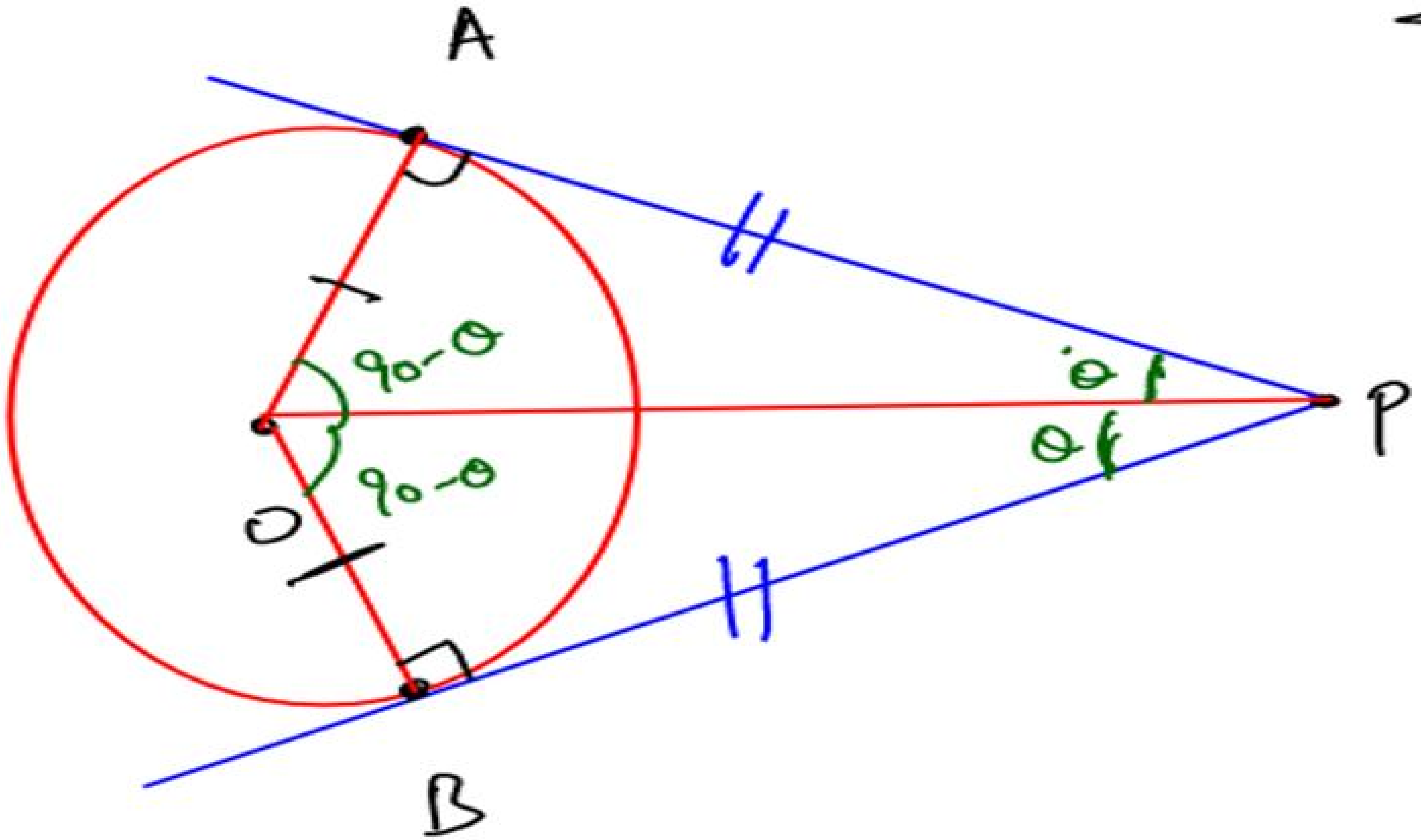
P → Point of contact

$\angle OPB = 90^\circ$

Tangents drawn from an external point are always equal.



gradeup



To prove : PA = PB

Proof

$\triangle APO$ & $\triangle BPO$

AO = BO (Radius)

$\angle OAP = \angle OBP = 90°$

OP = OP (Common)

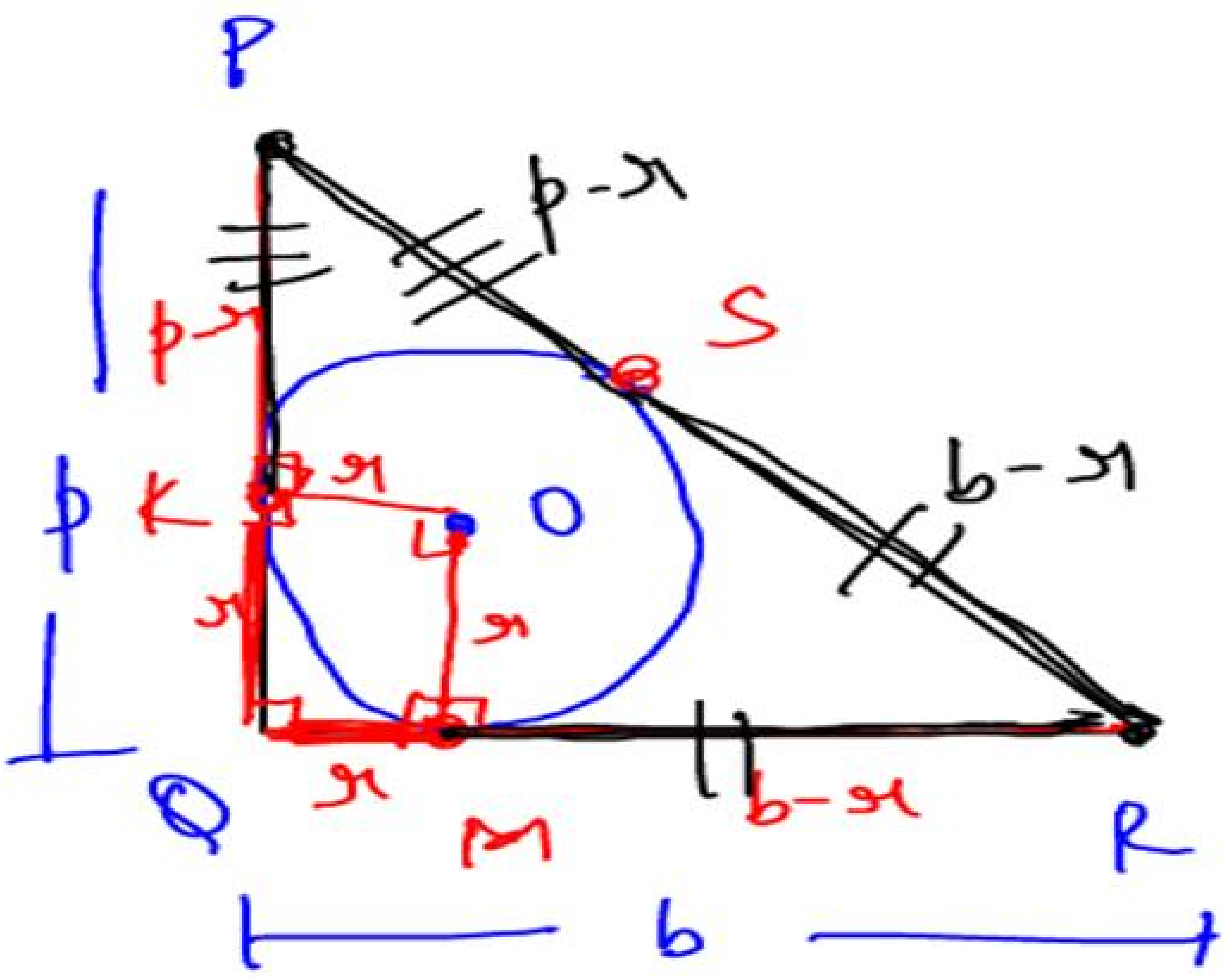
$\triangle APO \cong \triangle BPO$ (RHS)

$\angle APO = \angle BPO$

gradeup

Inradius of a Right angle Δ

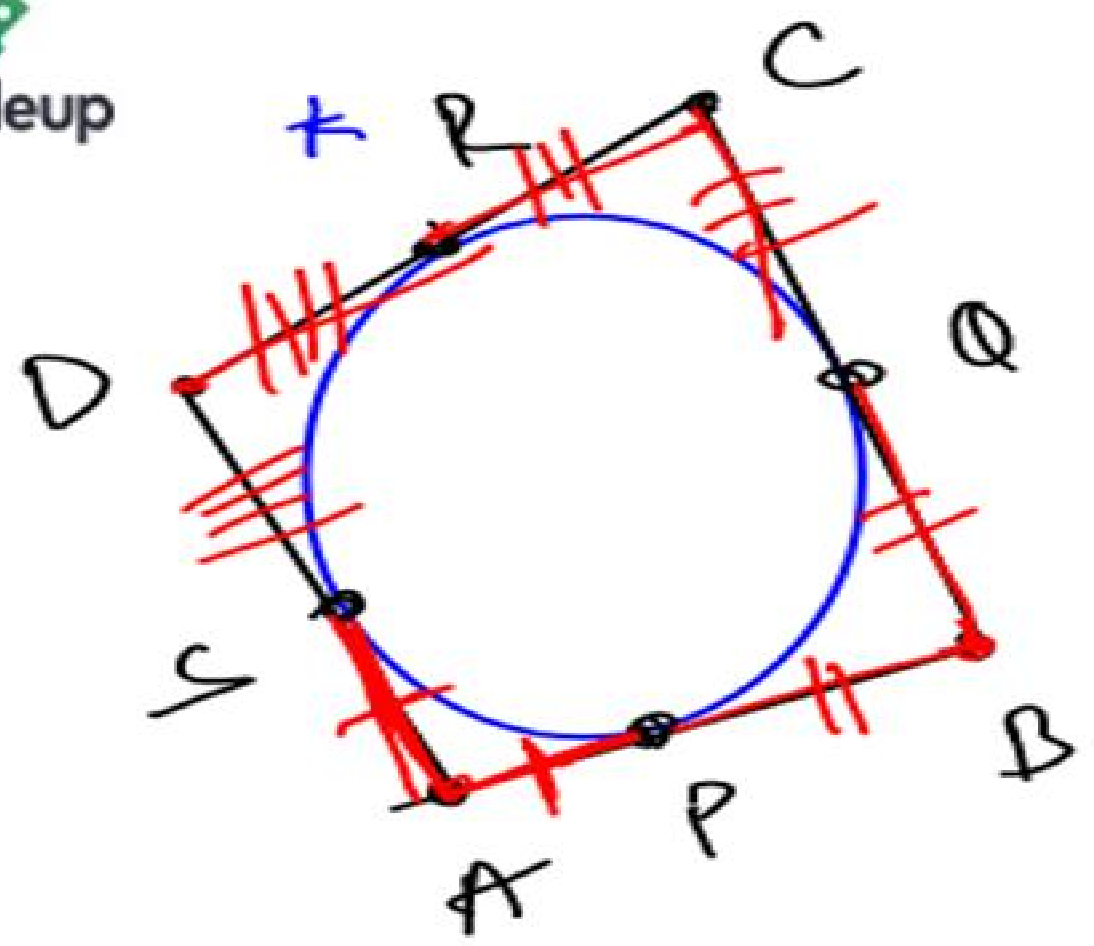
$$= \frac{b + p - h}{2}$$



$$p - r + b - r = h$$

$$p + b - h = 2r$$

$$\boxed{r = \frac{p + b - h}{2}}$$

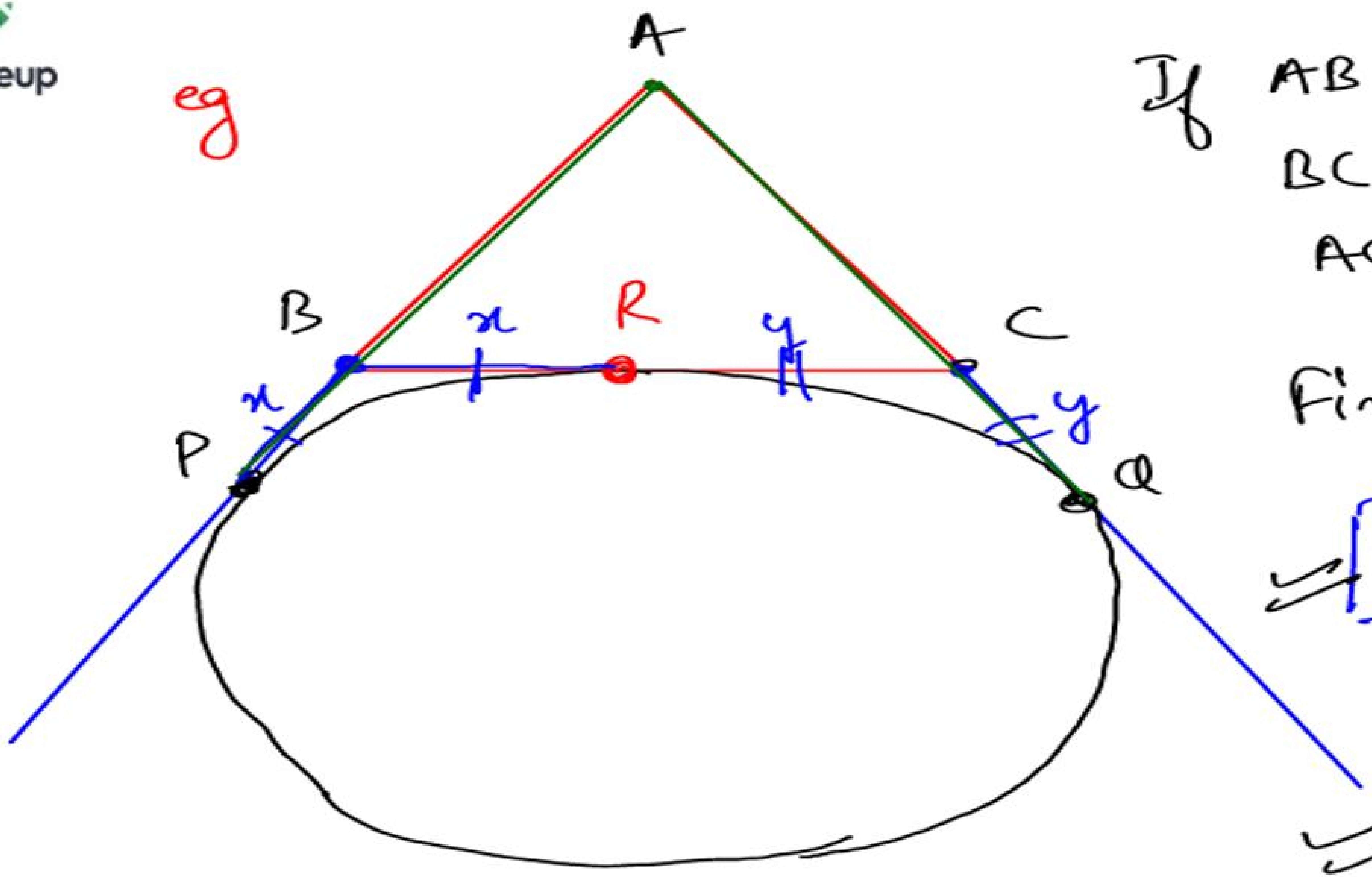


To prove

$AB + CD = BC + AD$

gradeup

eg



If $AB = 13$ cm

$BC = 17$ cm

$AC = 20$ cm

Find $AP = ??$

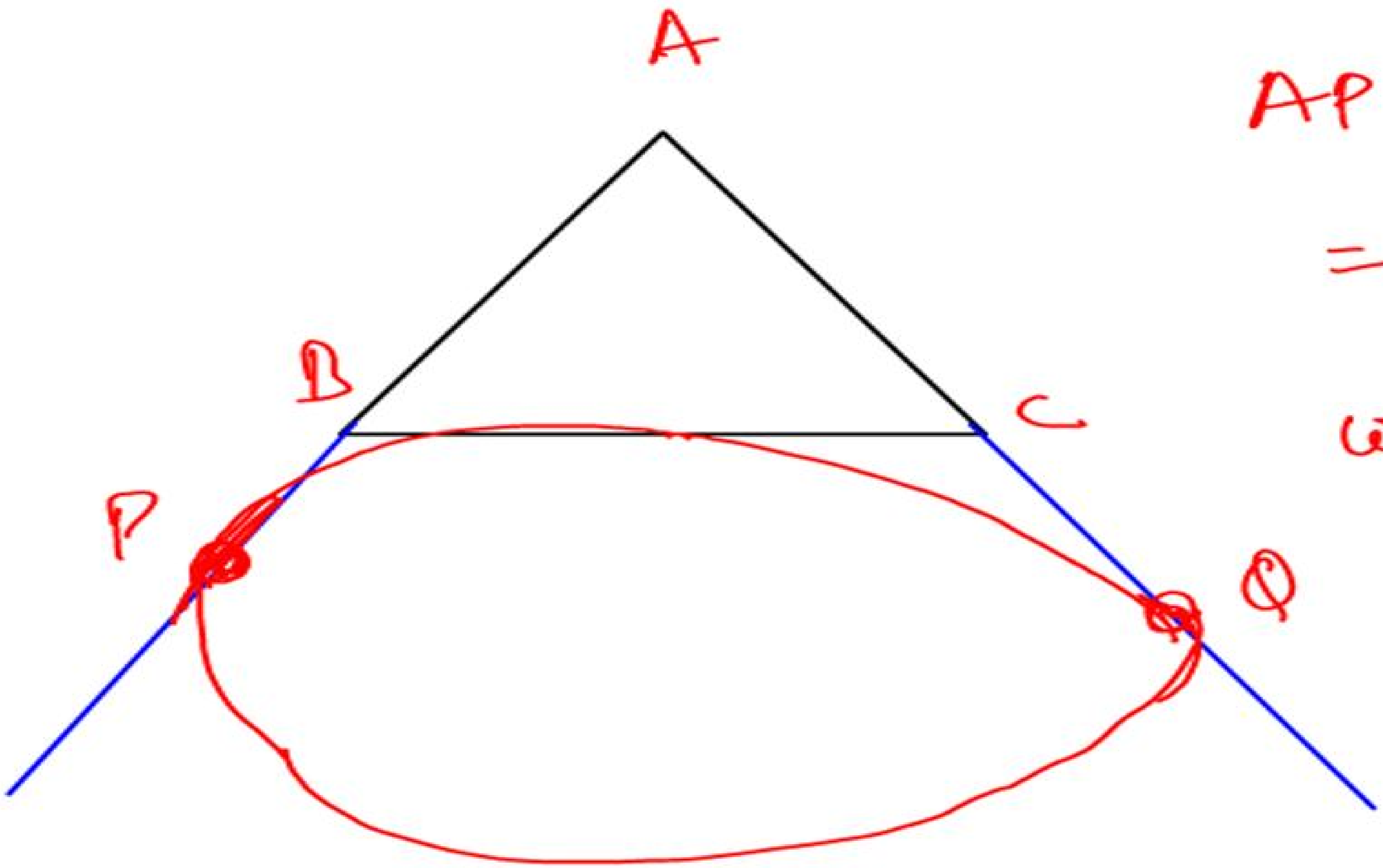
$$\boxed{x + y = 17}$$

$$13 + x = 20 + y$$

$$\boxed{x - y = 7}$$

$$AP = AB + x$$
$$= 13 + 12 = 25$$

$\underline{x = 12}$ $\underline{y = 5}$



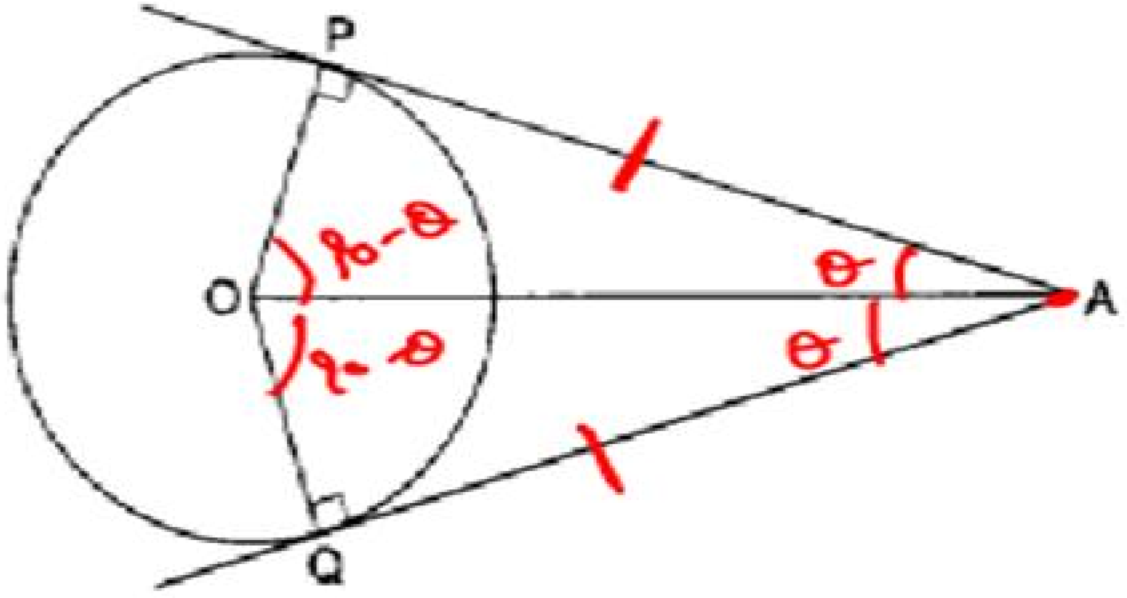
$AP = AQ$

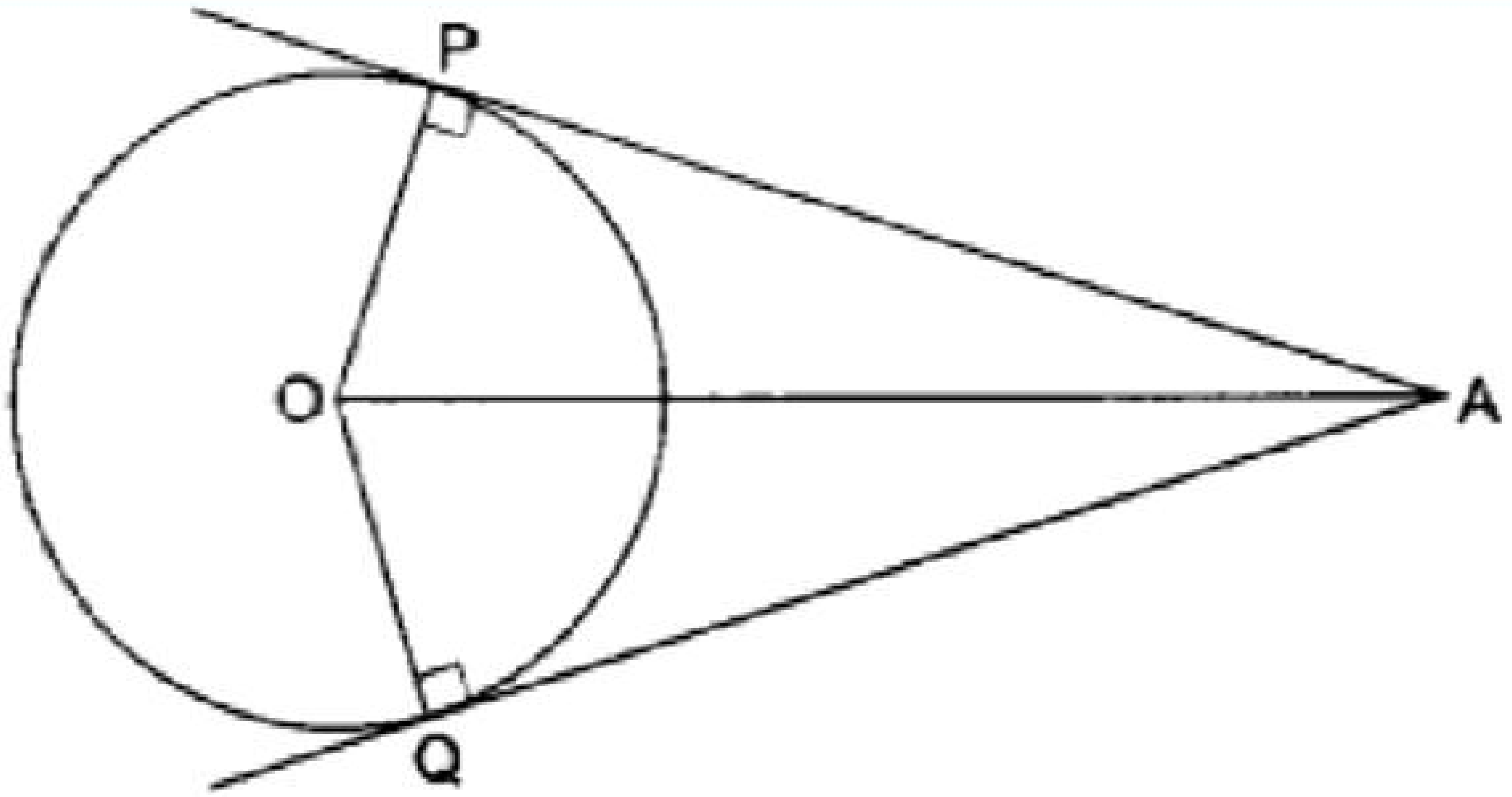
$= s$

where

$s \to$ semiperimeter

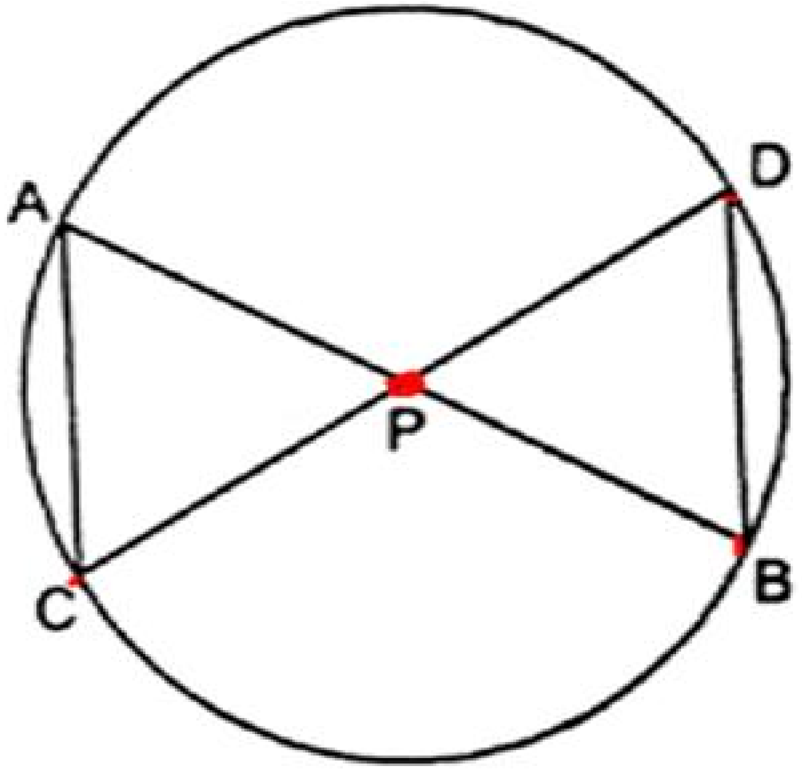
of ΔABC


P
O
A
Q
90-θ
90-θ
θ
θ


P
O
A
Q

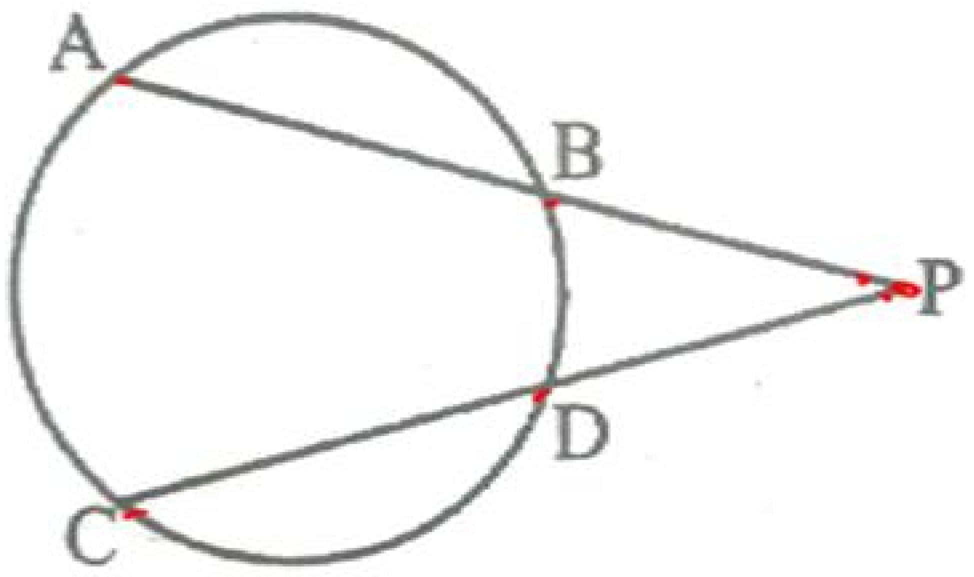
gradeup

6 (i). If 2 chords AB and CD intersect each other at P.

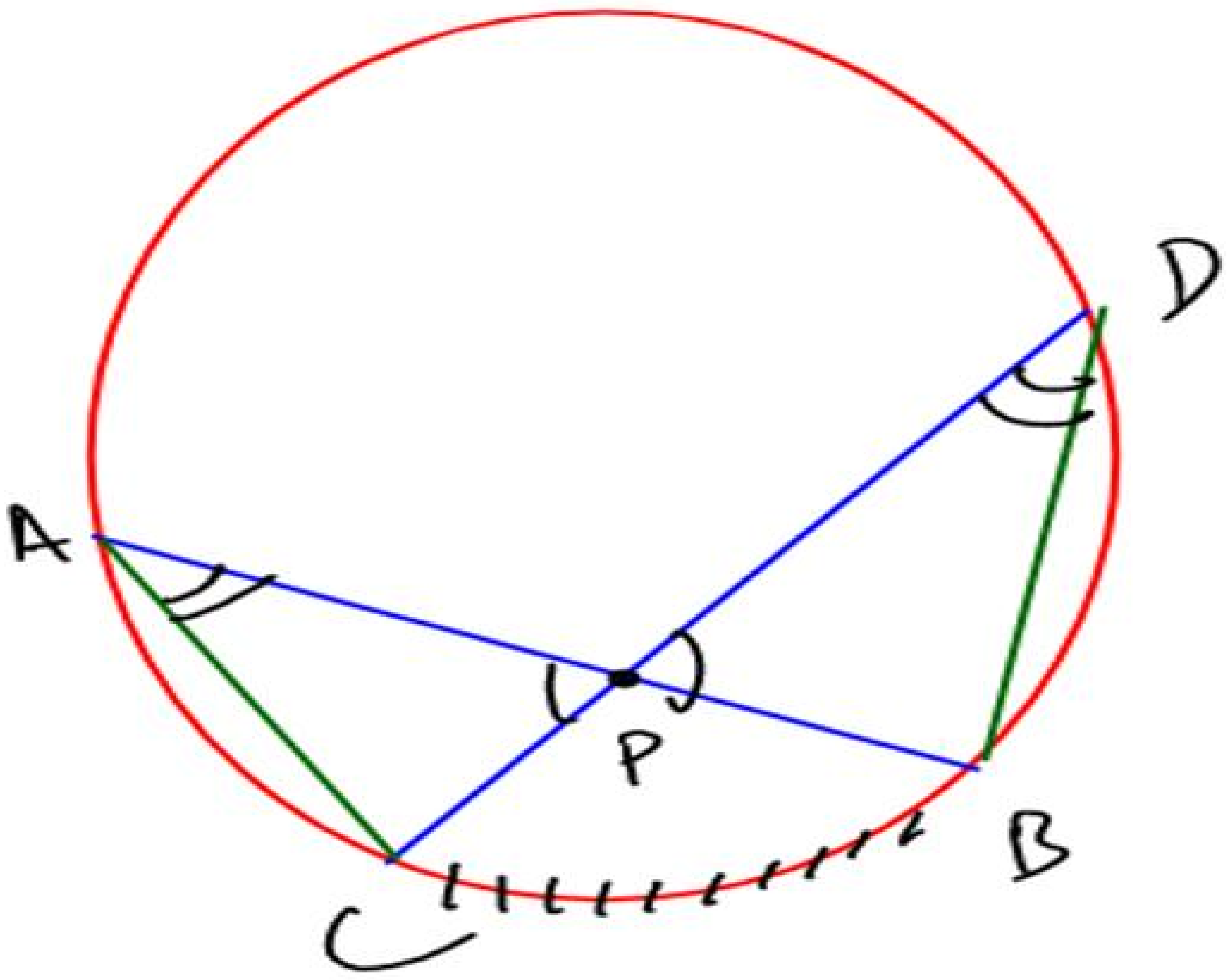


$$AP \cdot BP = CP \cdot DP$$

6 (ii). If 2 chords AB and CD intersect each other externally at P.



$$AP \cdot BP = CP \cdot DP$$



To prove $(AP)(BP) = (CP)(DP)$

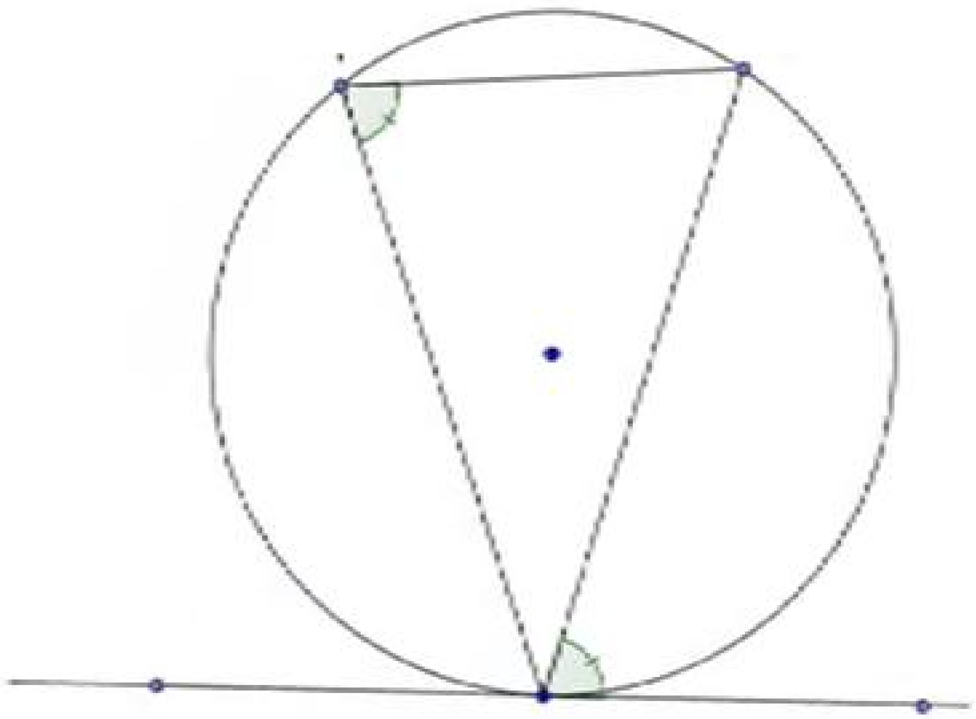
Proof →

$\Delta APC \sim \Delta DPB$ (AA)

$$\frac{AP}{DP} = \frac{CP}{BP}$$

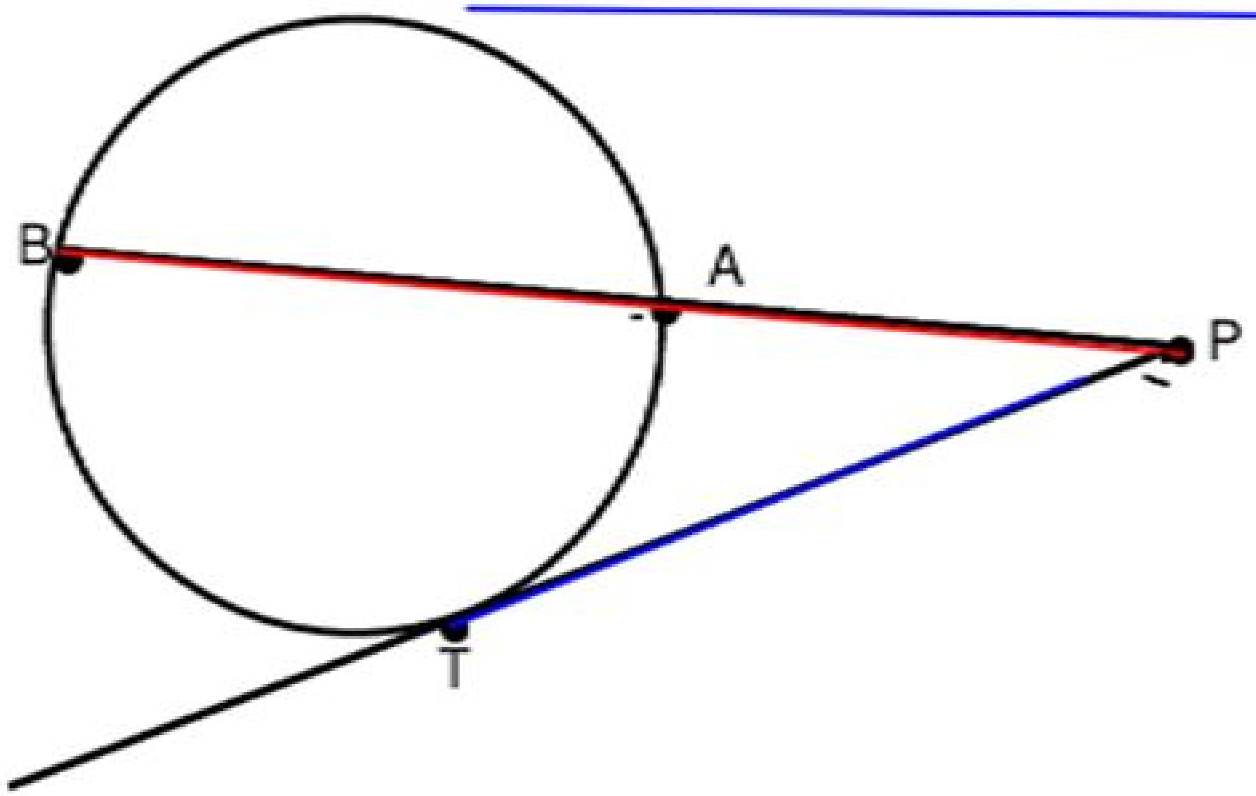
$$(AP)(BP) = (CP)(DP)$$

7. Alternate segment theorem



Angle made by a chord with the tangent of a circle is always equal the angle made by the same chord in alternate segment.

8. Tangent – Secant Theorem



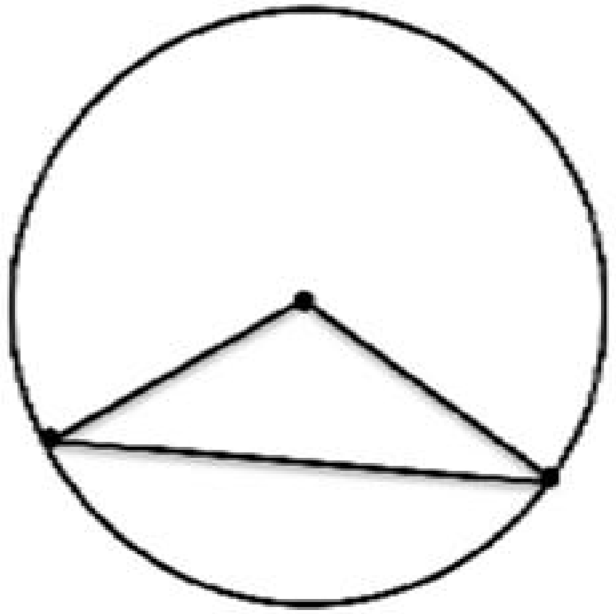
$$PT^2 = PA \times PB$$

Whenever you do questions on circles, focus on :

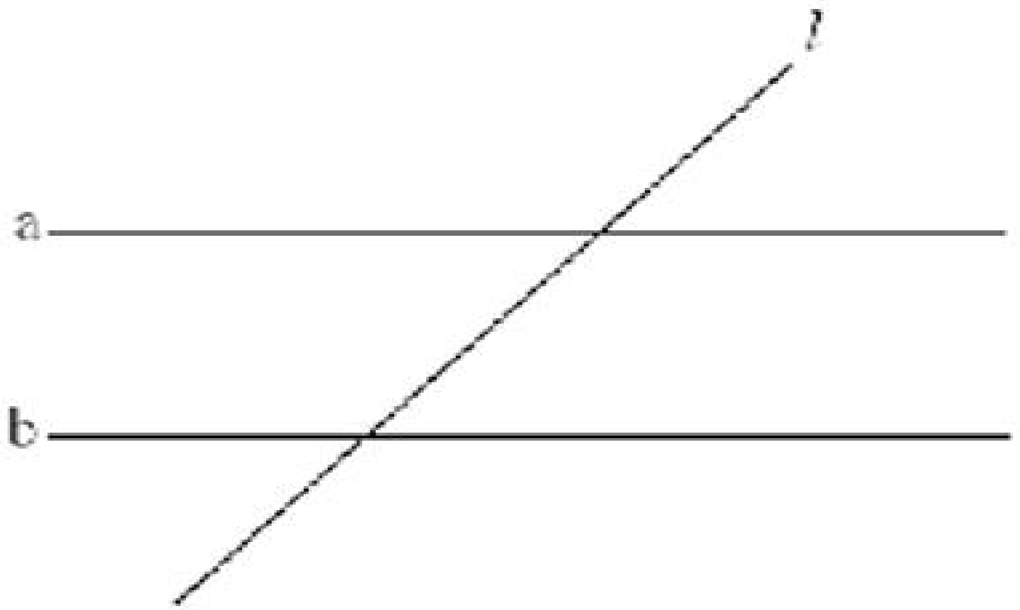
(i) Central angle $\rightleftarrows$ Circumference

(ii) Diameter $\longrightarrow$ Angle in a semi-circle

(iii)

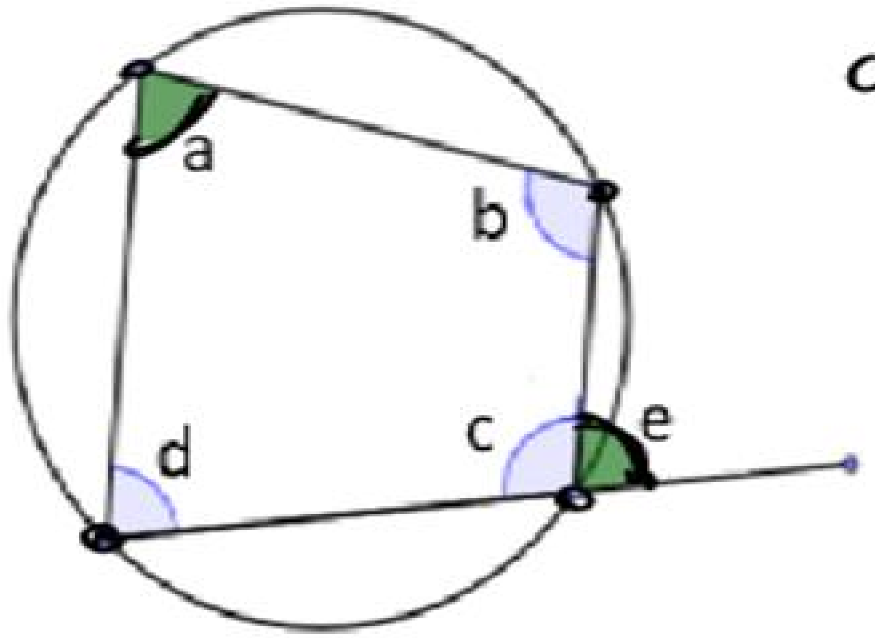


(iv)



Cyclic Quadrilateral

A cyclic quadrilateral has all its vertices on the circumference of the circle.



$a + \cancel{d} = \cancel{d} + e$

$a = e$

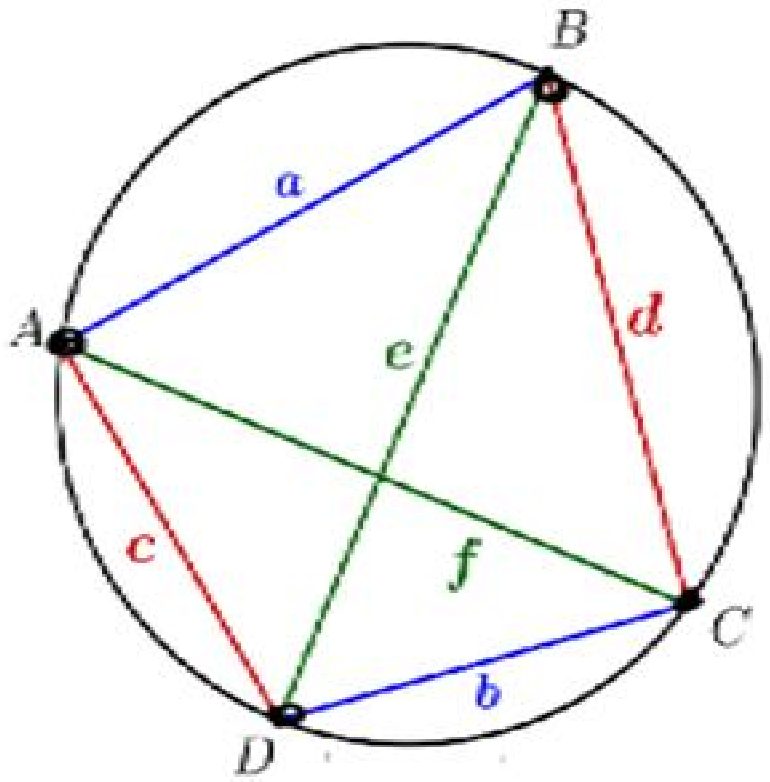
Opposite angles add up to 180°

$\boxed{\begin{array}{l}\angle a + \angle c = 180^\circ \\ \angle b + \angle d = 180^\circ\end{array}}$

Exterior angle is equal to the interior opposite angle

$\boxed{\angle a = \angle e}$

Ptolemy's Theorem



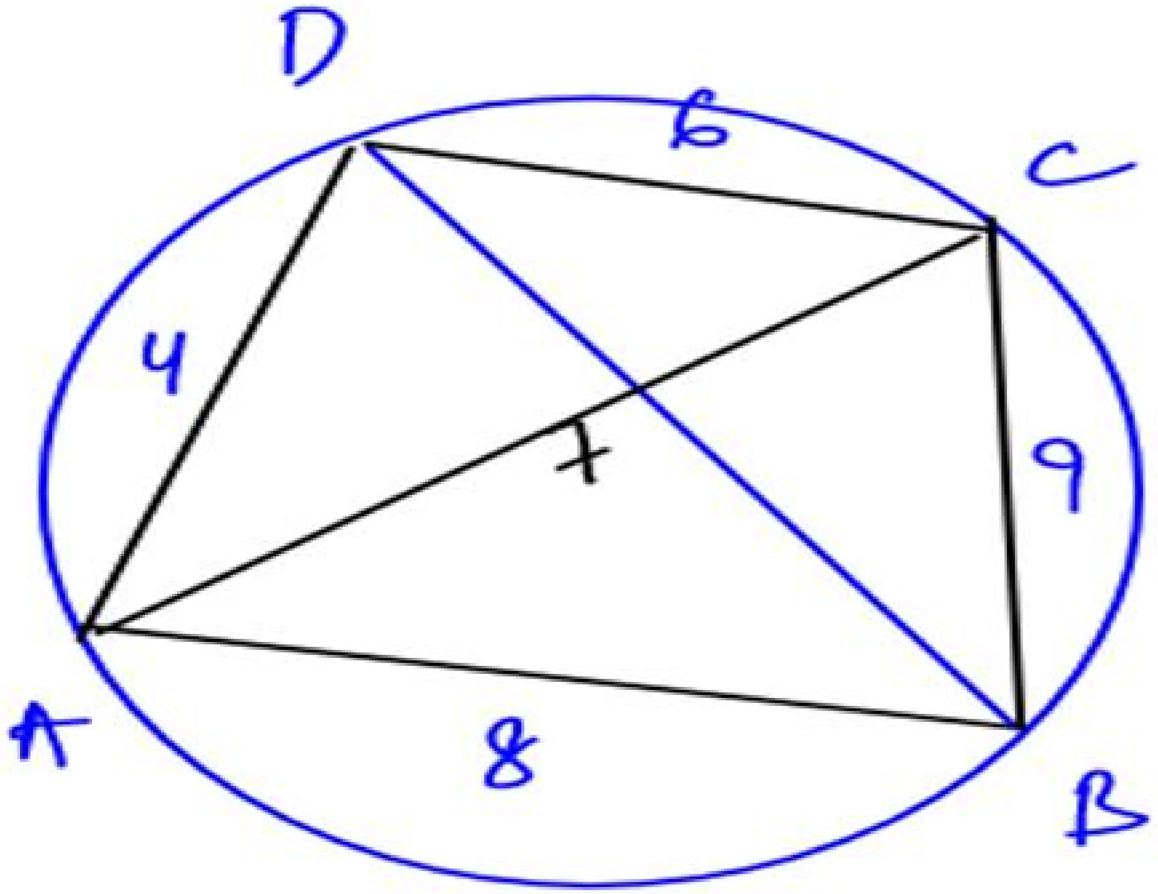
$$AC \times BD = AB \times CD + BC \times AD$$

$$\boxed{e \cdot f = a \cdot b + c \cdot d}$$

Eg. In a cyclic quadrilateral ABCD, AB = 8 cm, BC = 9 cm, CD = 6 cm and DA = 4 cm. If the value BD is 7 cm, the value of AC is:

(a) 10 (b) 12

(c) 14 (d) 16



$(AC)(BD) = 6 \cdot 8 + 4 \cdot 9$

$AC \cdot 7 = 84$

$AC = 12$

Ans. (b)

If the sides of a cyclic quadrilateral is a, b, c and d.

Area of cyclic quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

Where, s is semi-perimeter of cyclic quadrilateral.

Eg. Find the area of a cyclic quadrilateral whose sides are 5 cm, 2 cm, 5 cm and 8 cm.

(a) 10 cm^2 (b) 20 cm^2 (c) 40 cm^2 (d) 25 cm^2

$$S = \frac{5+2+5+8}{2} = 10$$

Area $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

$$= \sqrt{5 \cdot 8 \cdot 5 \cdot 2}$$

$$= 5 \cdot 4 = 20\,cm^2$$