Circle

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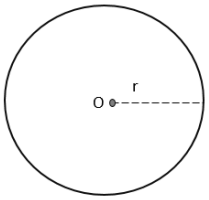
Definition: A plane 2- dimensional shape bounded by a single curved line, every point of which is equally distant from the point at the center of the shape.

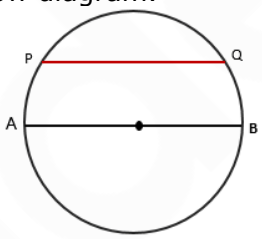


Circle

- The Distance from the center is called the radius of the Circle. Normally, it is denoted by 'r'.
- Every point on the circumference of the circle is equidistant from its center.
- The Circumference of the Circle is given as $P = 2\Pi r$
- The Ares bounded by the Circumference of the Circle is $A = \Pi r^2$
- Area of Circle in terms of its Diameter = $A = \frac{\Pi d^2}{4}$
- Area of Circle in terms of its Perimeter = $A = \frac{p^2}{4\Pi}$

Chord and Tangent

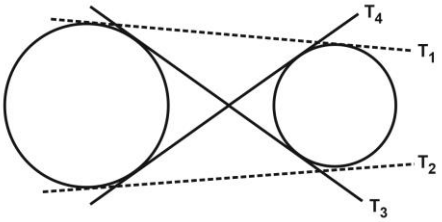
Chord: Any line that joins two arbitrary points on the circumference of the Circle is called Chord. Ex. PQ is a chord in the below diagram.



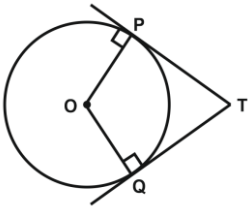
- The line joining Center of the Circle to any two arbitrary points A and B on the circumference of Circle is called Diameter of the Circle.
- Diameter of a Circle is also the biggest possible Chord of any Circle. It is Normally denoted by 'd'.

Also, $d = 2r$

Tangent: Any line that touches the circle at only one point is called Tangent to the Circle. It is normally drawn from any arbitrary point outside the circle.



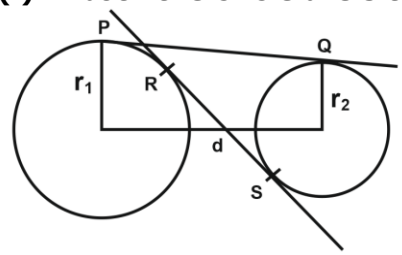
- Maximum four tangents can be drawn between two Circles.



- Maximum two tangents can be drawn from any point outside the circle and both will be equal in length. i.e. TP = TQ

• The line joining the center of the circle to the tangency point is perpendicular to the tangent. i.e. ∠TPO = ∠TQO = 90°

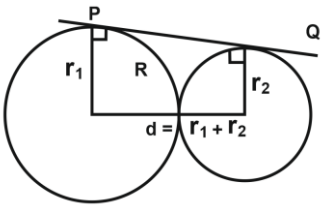
(i) If both the circle are situated at a distance 'd' –



Length of Direct Common Tangent PQ = $\sqrt{d^2-(r_1-r_2)^2}$

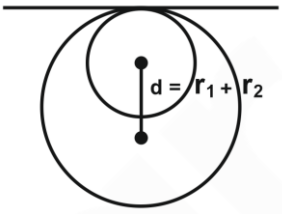
Length of Transverse Tangent RS = $\sqrt{d^2-(r_1+r_2)^2}$
Where, d = distance between centres of two circles, r_1, r_2 = radius of the two circles

(ii) If both the circles are touching each other–



In such case, there will be no Transverse Tangent.
And the Length of Direct Common Tangent PQ = $2\sqrt{r_1 r_2}$

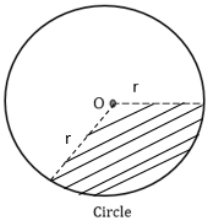
(iii) If one circle is placed inside the other circle–



In such case there will be no tangent and the length of both type of tangent will be zero.

Sector and Segment

Definition: The area between two radii and the connecting arc of a circle is called a sector.



Area of a Sector- If the angle between both the radius is 'θ'.
Then,

Length of an arc = $2\pi r\theta/360°$, where θ is in degrees.

Or

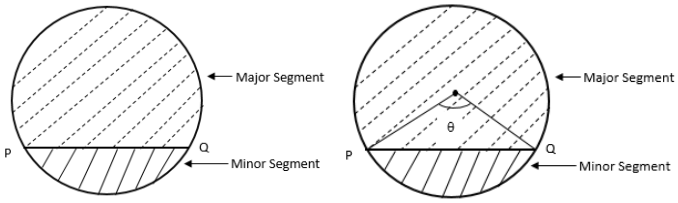
Length of an arc = $r \times \theta$

the area of sector $= \frac{1}{2} \times length\ of\ the\ arc \times radius = \frac{\Pi r^2}{360^\circ} \times \theta$

Minor Segment and Major Segment:

Definition: The minor segment is the region enclosed by the chord and the minor arc intercepted by the chord.

The major segment is the region enclosed by the chord and the major arc intercepted by the chord.

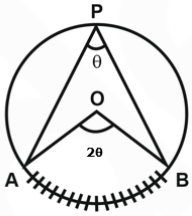


Area of Minor Segment $= \frac{\Pi r^2}{360^\circ} \times \theta - \frac{1}{2} r^2 . Sin\theta$

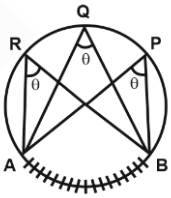
Area of Major Segment = Area of Circle – Area of Minor Segment

Properties of Circle

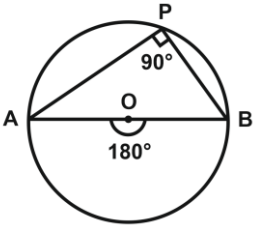
Property 1: Angle subtended by any arc on the center will be double of the angle subtended by same arc on remaining circumference.



Property 2: Angle subtended by a similar arc at different point on the circumference will be same.



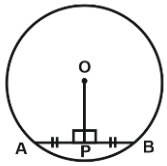
Property 3: Angle subtended by Diameter on the circumference will be a right angle.



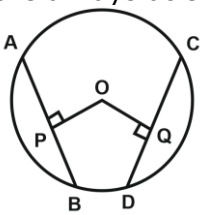
Property 4: Perpendicular drawn from center at any chord bisects it.

Or

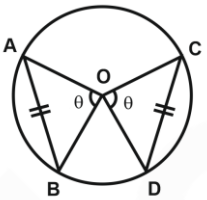
Any line joining the mid-point of Chord to center will be perpendicular to the chord.



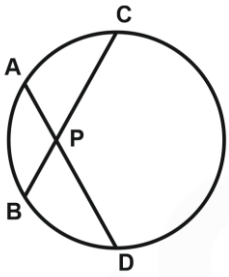
Property 5: Equal Chord of a Circle are always at equal distance from the center.

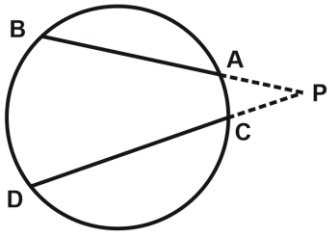


Property 6: Equal Chords subtend equal angles at Center.



Property 7:

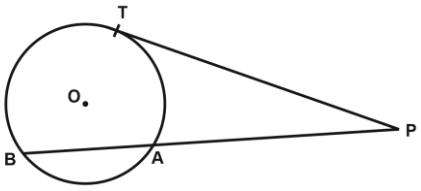




If two Chords AB and AC intersect each other at P (either inside or outside the Circle) than,

$$PA \times PB = PC \times PD$$

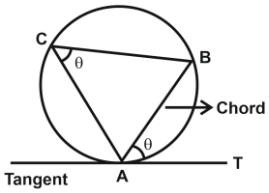
Property 8:



If Chord AB of a circle is extended till a point P outside the circle and A tangent PT is drawn from point P to the Circle. Than,

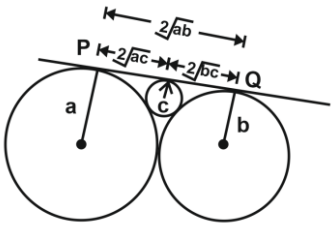
$$PT^2 = PA \times PB$$

Alternate-Segment Theorem:



For any circle, the angle between a tangent and a chord through the point of contact of the tangent is equal to the alternate segment.

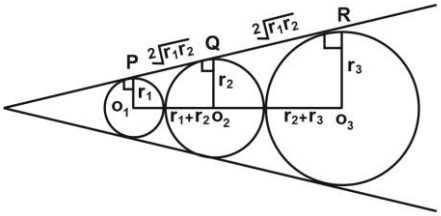
Special cases:
(i)



If three circle are placed as shown in the diagram above and a, b and c are their radii then,

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$

(ii)



If three circle are placed as shown in the diagram above and r_1, r_2 and r_3 are their radii then,

$$r_2^2 = r_1 r_3$$