BYJU'S EXAM PREP

The Most Comprehensive
Preparation App For All Exams

MENSURATION-3D

Part-1

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MENSURATION - 3D

- Cube/Cuboid → 1
- Cylinder/Hollow Cylinder
- Cone/Frustum
- Sphere/Hemisphere
  → 2
- Combination of Figures
- Optimization of Figures
  → 1
- Prism & Pyramid
  Tetrahedron
  → 1

Extra → 1

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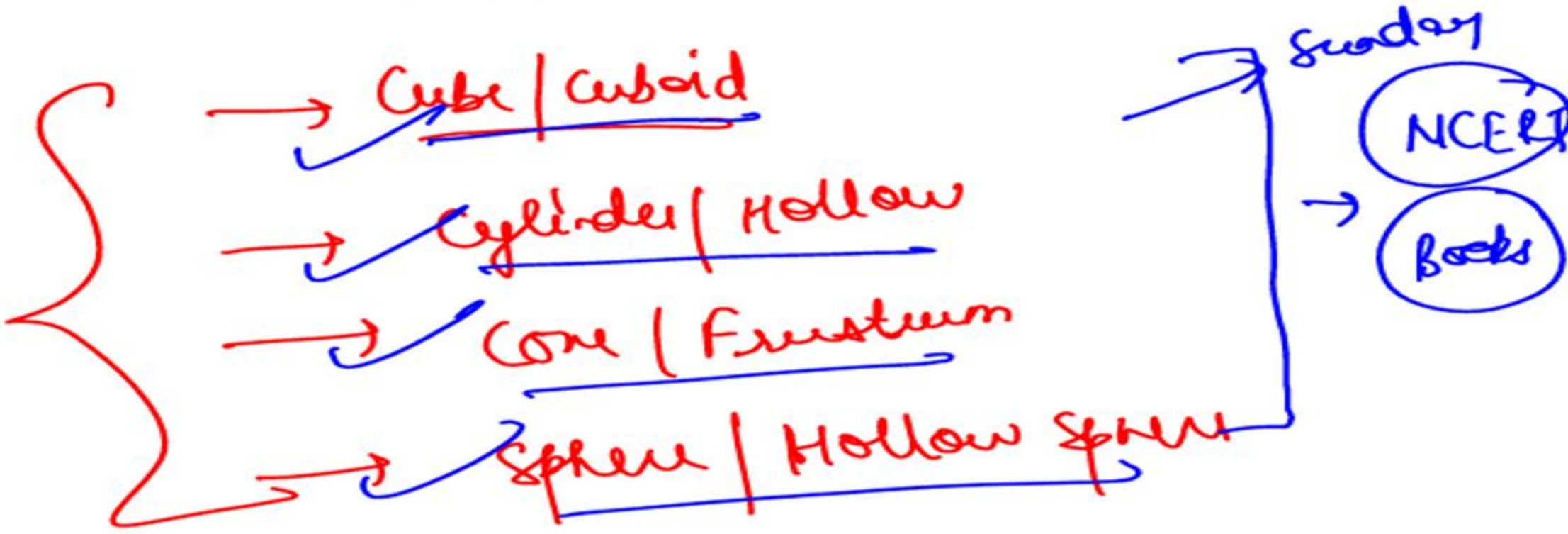
For Basics of Mensuration - 3D

NCERT & RD sharma (9th | 10th)



Units :

$1\ cm^3 = 1\ ml$

$1\ m^3 = 1000\ litres$

$1m^3 = (1m)(1m)(1m)$

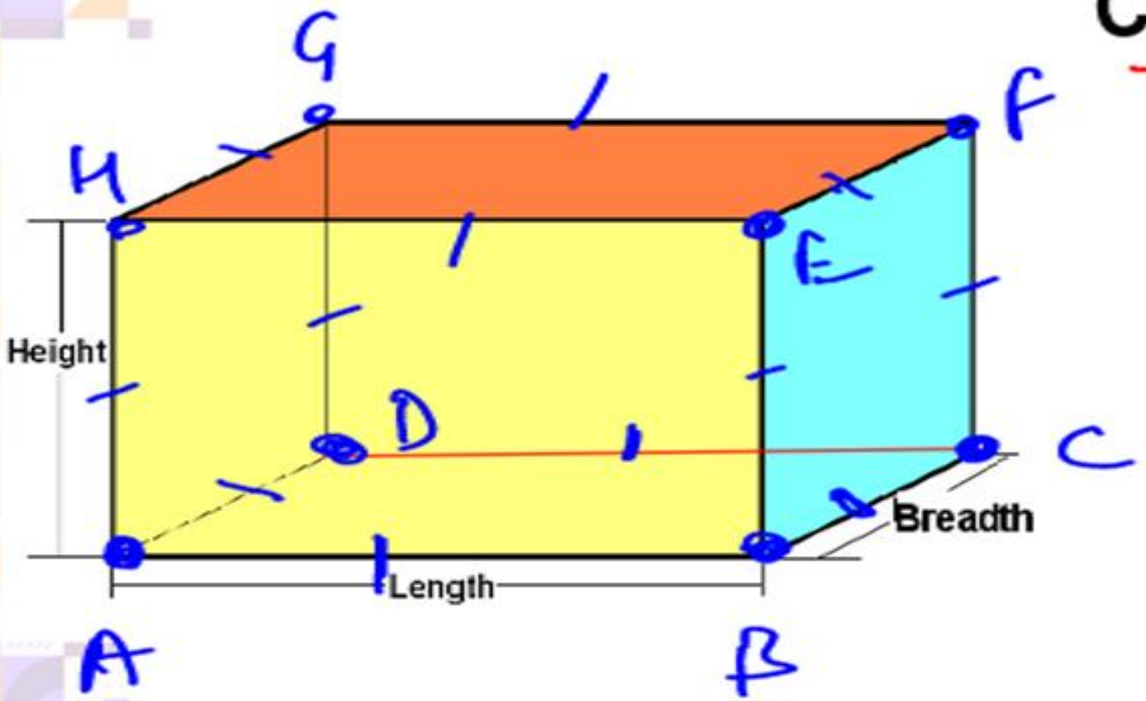
$= (100cm)(100cm)(100cm)$

$= 10^6\ cm^3$

$1m^3 = 1000\ litres = 10^6\ ml$

$\Rightarrow \frac{10^6}{10^3}\ litres$

CUBOID



Faces = 6
Vertices = 8
Edges = 12

l = length
b = breadth
h = height

Cuboid

$f + v - E$

$f \rightarrow$ faces 6

$v \rightarrow$ vertices 8 $6+8-12$

$E \rightarrow$ Edge 12 $= 2$

POLYHEDRON

Def: A polyhedron is a 3-dimensional solid made by joining polygons.

The word 'polyhedron' comes from two Greek words, poly meaning many, and hedron referring to surface

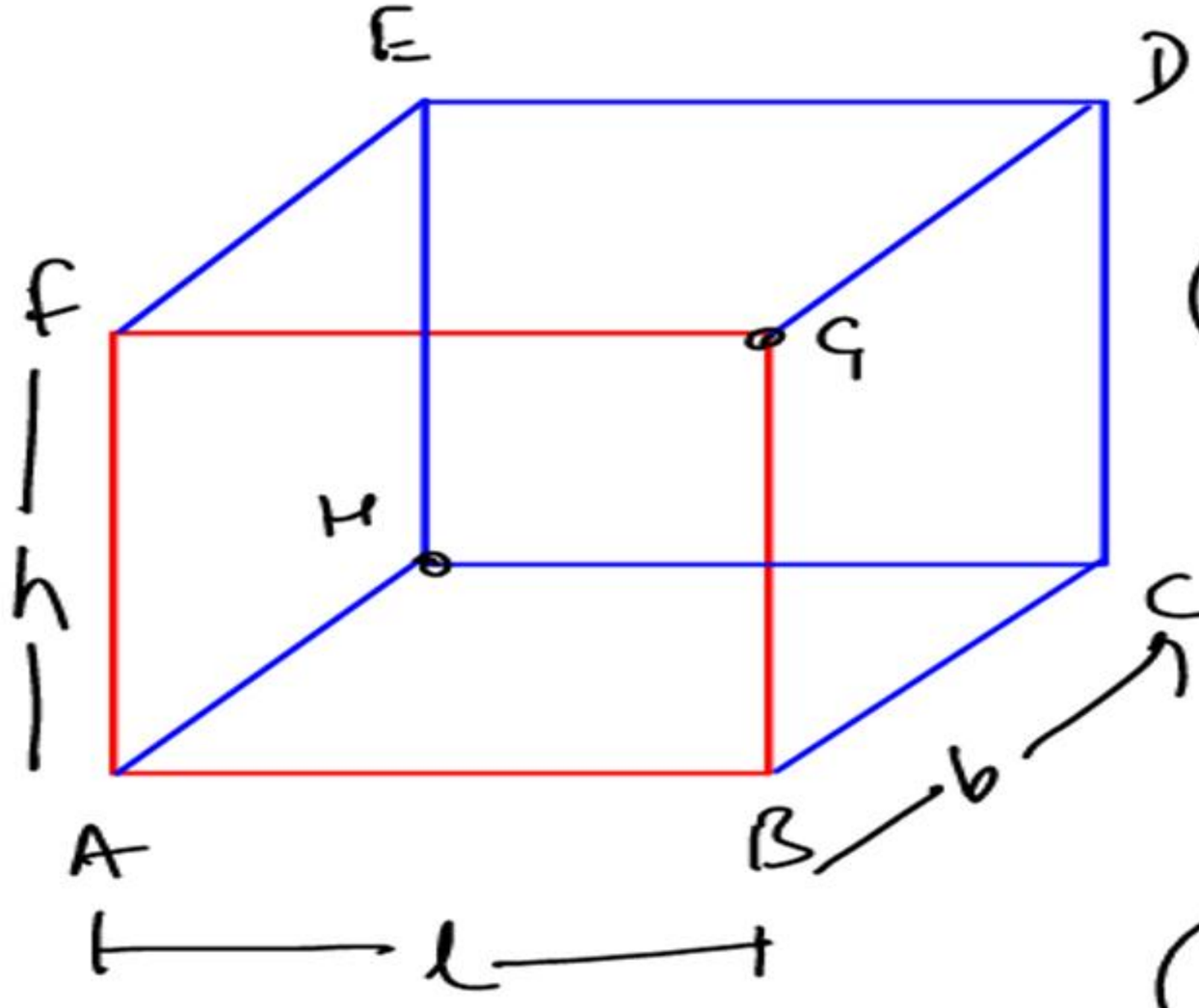
Made up of Polygon

+

Atleast 4 faces

Euler formula : $F + V - E = 2$

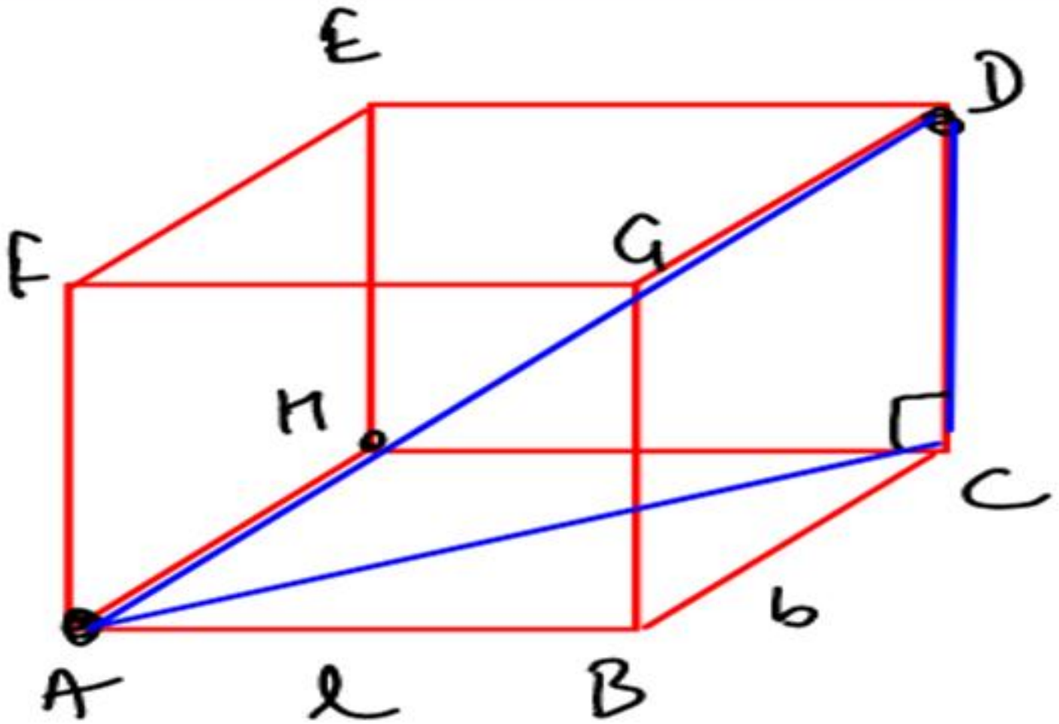
CSA/ LSA Area of 4 walls	= 2 $(l$ + b) h (Perimeter of Floor x Height)
TSA	= 2 (lb + bh + hl)
Volume of Cuboid	= L x B x H



lb
Bottom → 1
Top → 2
Left → 3
Right → 4
bh
lh
Front → 5
Back → 6

Diagonal of Cuboid = $\sqrt{l^2 + b^2 + h^2}$

Length of longest rod that can be placed in the room.



ΔACD

$AD^2 = AC^2 + CD^2$

$AD^2 = l^2 + b^2 + h^2$

$AD = \sqrt{l^2 + b^2 + h^2}$

Eg. Diagonal of cuboid = $10\sqrt{2}$ cm

l + b + h = 24 cm

TSA of cuboid = ??

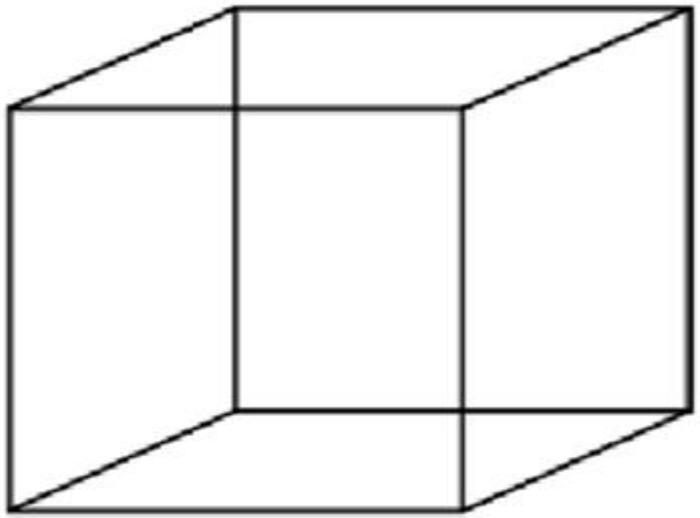
$$(l+b+h)^2 = l^2+b^2+h^2+2(lb+bh+hl)$$

$$576 = D^2 + TSA$$

$$576 = 200 + TSA$$

$$TSA = 376 \text{ cm}^2$$

Ans. 376 cm^2



Floor / top = l b

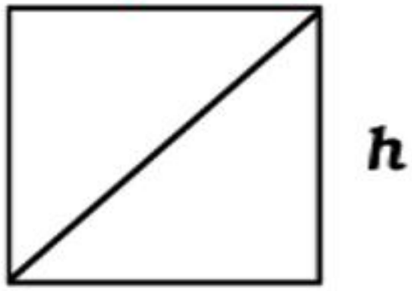
Left / Right = bh

Front back = l h

Top | Bottom

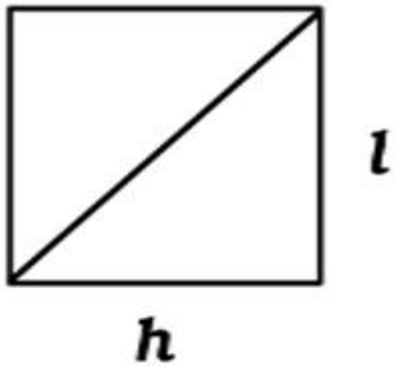
$\sqrt{l^2 + b^2}$

Left | Right

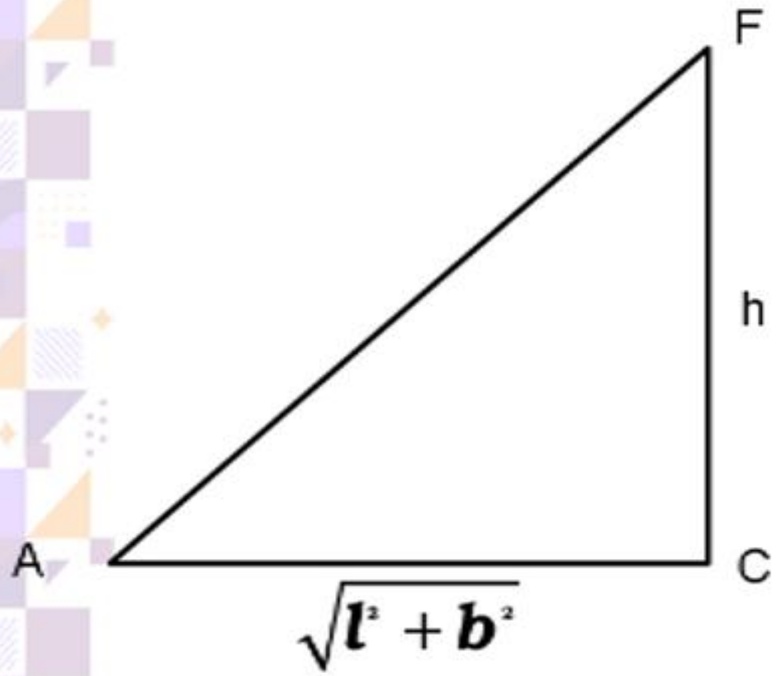


$\sqrt{b^2 + h^2}$

Front | Back



$\sqrt{h^2 + l^2}$



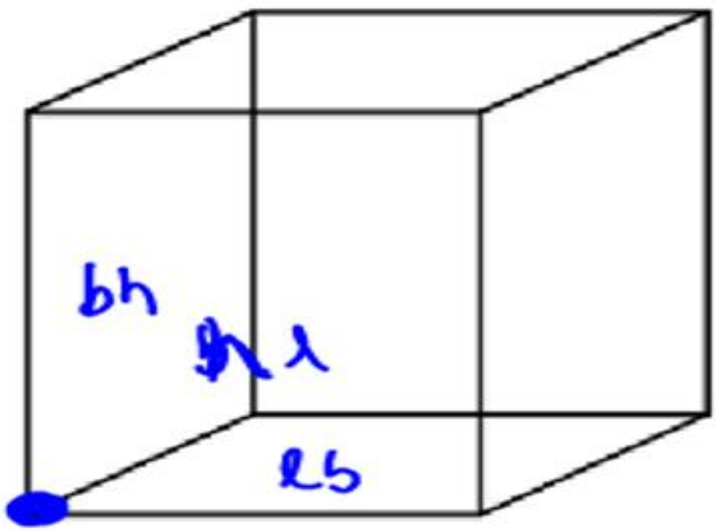
$$AF^2 = l^2 + b^2 + h^2$$

3 Adjacent Faces

$A_1 = l \cdot b$

$A_2 = b \cdot h$

$A_3 = h \cdot l$



Eg. Area of 3 adjacent faces of a cuboid are p, q and r. Find the volume of cuboid?

$l \cdot b = p$ — (1)

$b \cdot h = q$ — (2)

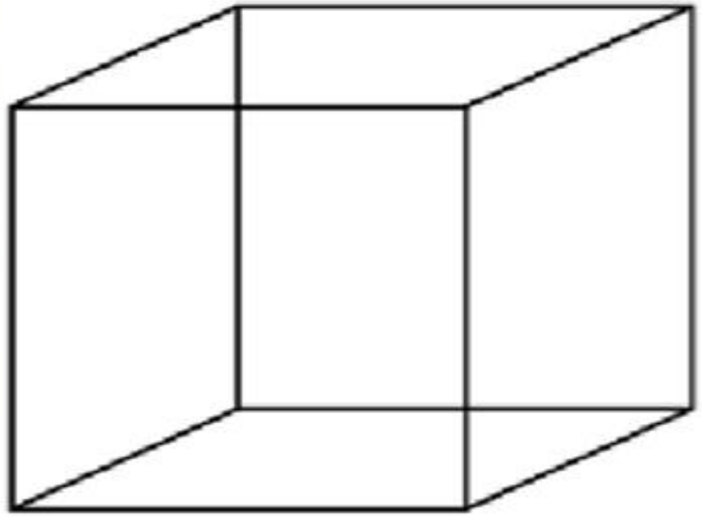
$h \cdot l = r$ — (3)

$l^2 b^2 h^2 = p \cdot q \cdot r$

$V^2 = pqr$

$V = \sqrt{pqr}$

CUBE



Def: A cuboid in which $l = b = h$

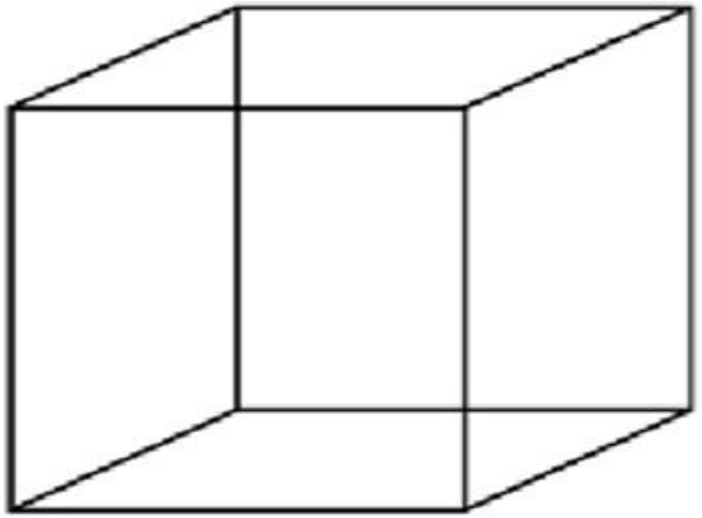
LSA = $4S^2$

TSA = $6S^2$

Volume = S^3

Diagonal = $\sqrt{3}S$

Eg. If side of cube is 5 cm. Find the total surface area (TSA) of cube.

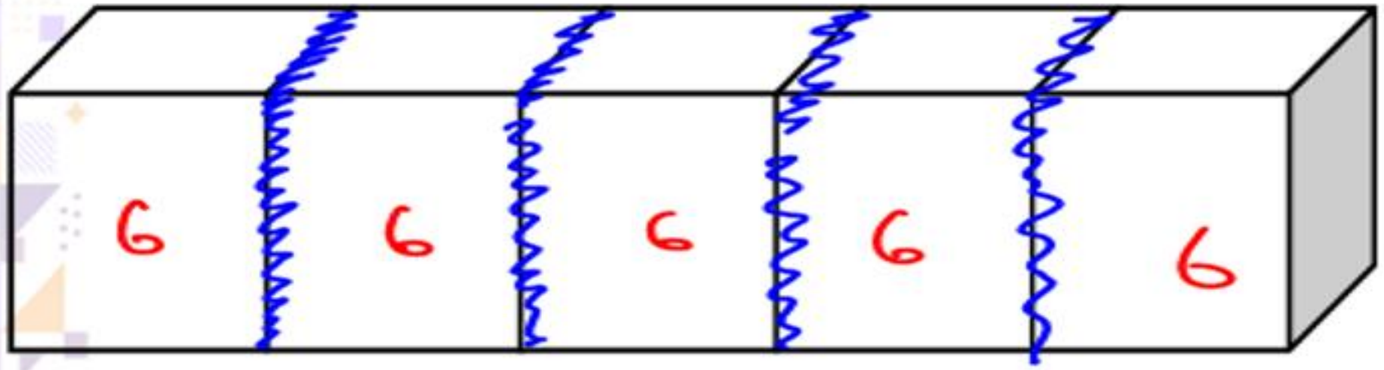


$$TSA = 6 \cdot 5^2$$

$$= 150 cm^2$$

Ans. 150 cm^2

Eg. If all cubes are of side 10 cm. Find the total surface area (TSA) of this figure.



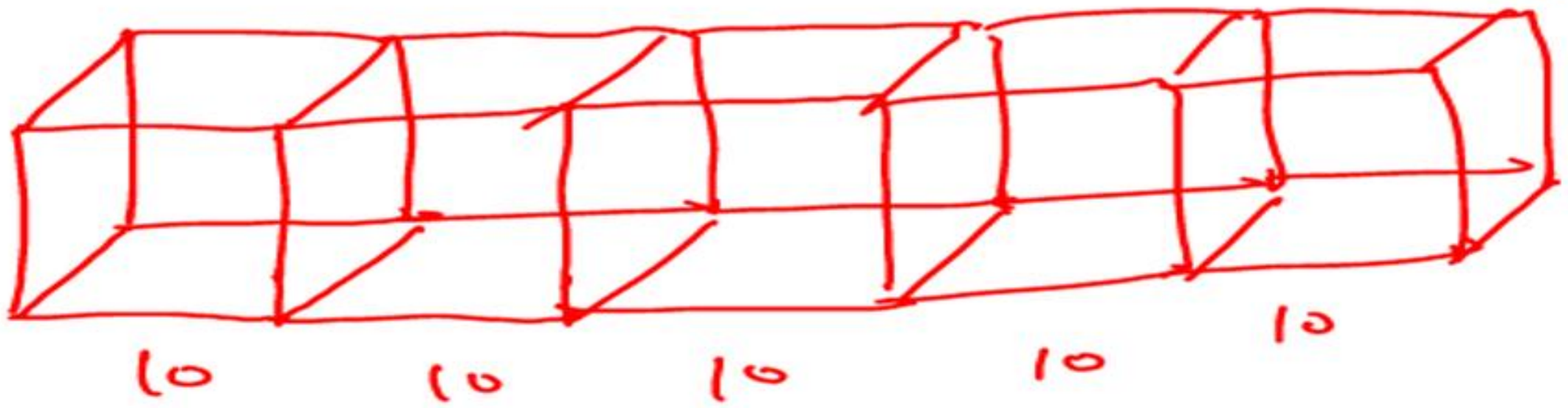
1st

30 Faces − 8 Faces

= 22 faces

= 22×10^2 = 2200 cm^2

II nd



$l = 50$

$b = 10$

$h = 10$

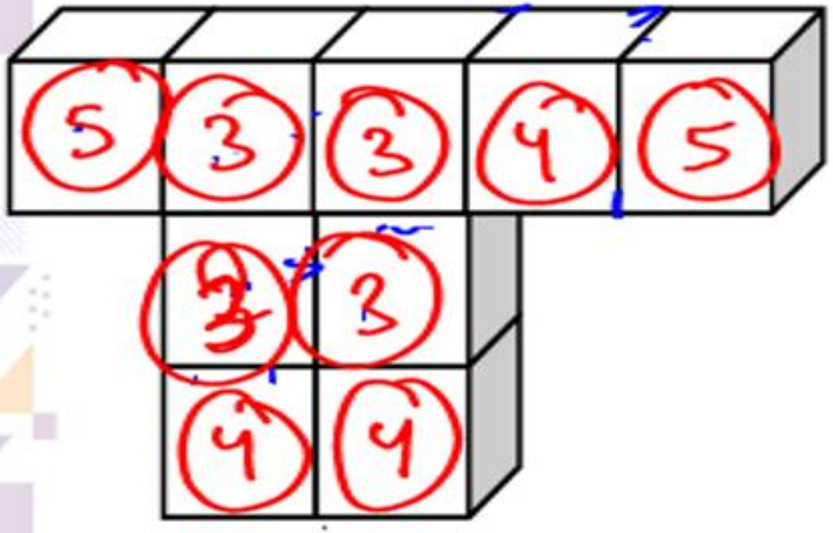
$TSA = 2(lb + bh + hl)$

$= 2(500 + 100 + 500)$

$= 2200 \text{ cm}^2$

Ans. 2200 cm^2

Eg. Find TSA of given figure of side all smaller cubes is 10 cm.



I st

54 − 20

$= 34 \times 10^2$

$= 3400\ cm^2$

II nd

34 faces

$34 \times 10^2 = 3400\ cm^2$

Ans. 3400 cm^2

IF VOLUME OF ALL CUBOIDS IS A CONSTANT THEN CUBE WILL HAVE MINIMUM SURFACE AREA.

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I $l = 36$, $b = 12$, $h = 4$

$36 \cdot 12 \cdot 4 = 1728$

$2[432 + 48 + 144]$

$2[624]$

1248

II $l = 18$, $b = 12$, $h = 8$

$18 \cdot 12 \cdot 8 = 1728$

$2[216 + 96 + 144]$

$2[456]$

912 cm²

III $l = 12$, $b = 12$, $h = 12$

$12 \cdot 12 \cdot 12$

$= 1728$

$6 \cdot 12^2$

$= 864$ cm²

V. Imp

Eg. 64 small cubes of 1 cm^3 are to be arranged in a cuboidal shape in such a way that the surface area will be minimum. What is the length of diagonal of the larger cuboid ?

(a) $8\sqrt{2}$ cm (b) $\sqrt{273}$ cm

(c) $4\sqrt{3}$ cm (d) 4 cm

$64 \times 1 cm^3 = 64 cm^3$

$s^3 = 64$

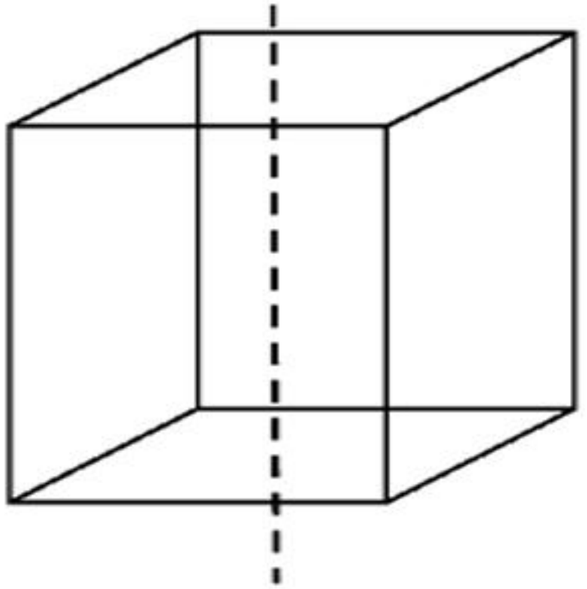
$s = 4cm$

$\sqrt{3}s \Rightarrow \sqrt{3} \cdot 4$

$= 4\sqrt{3}$ cm

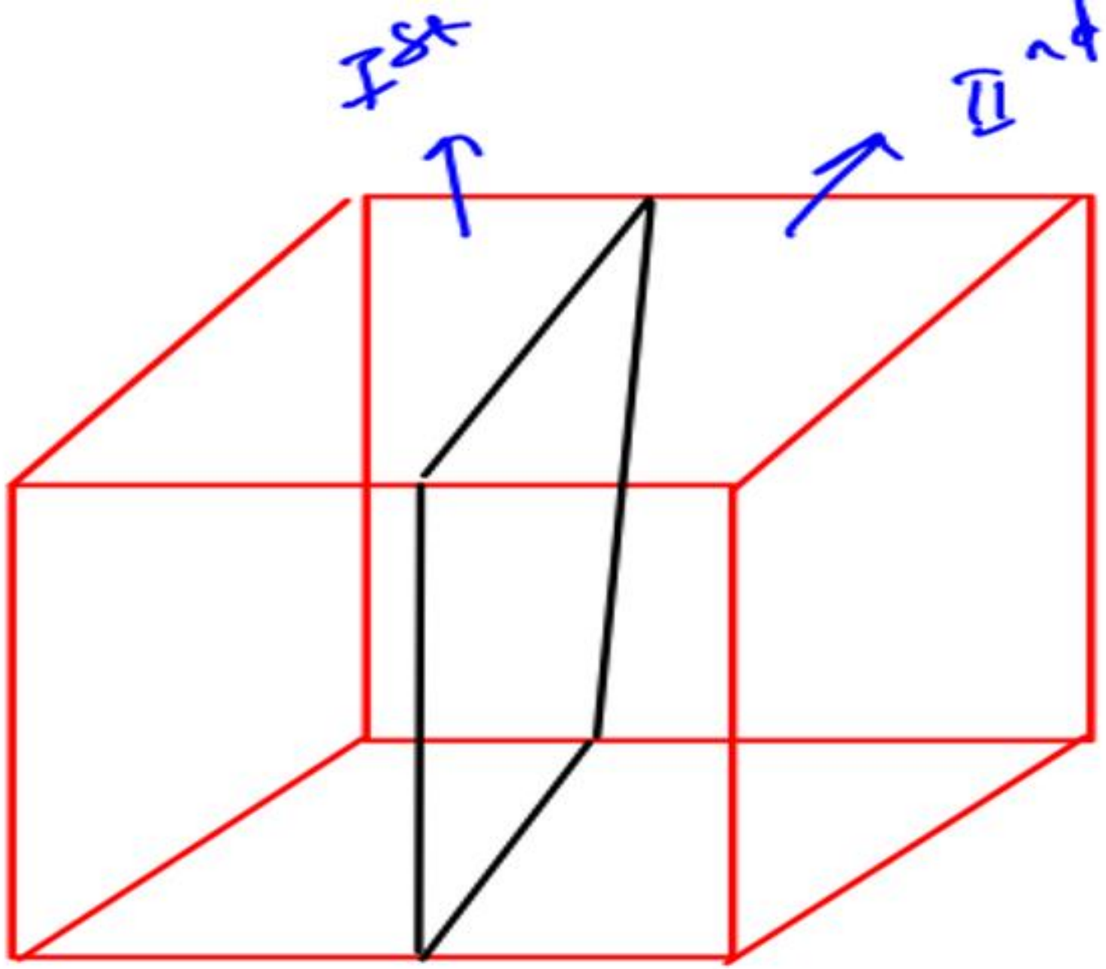
Ans. (c)

CUTTING OF CUBE



2 Parts

* If nothing is mention about how cutting is done it is assumed that the cutting has resulted into symmetrical figures.



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Let Cube whose side is S

Case I

1 cut

2 part

Cube is cut into 2 parts

→ One of the dimension becomes half

→ $\frac{S}{2}, S, S$

Case II

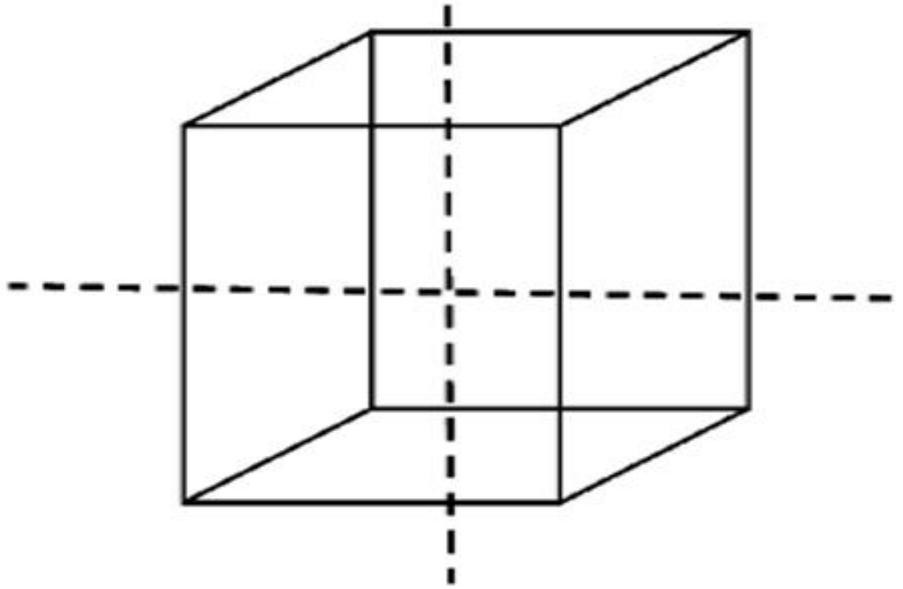
Two of the dimension become half

$\frac{s}{2}, \frac{s}{2}, s$

2 cuts

2^2

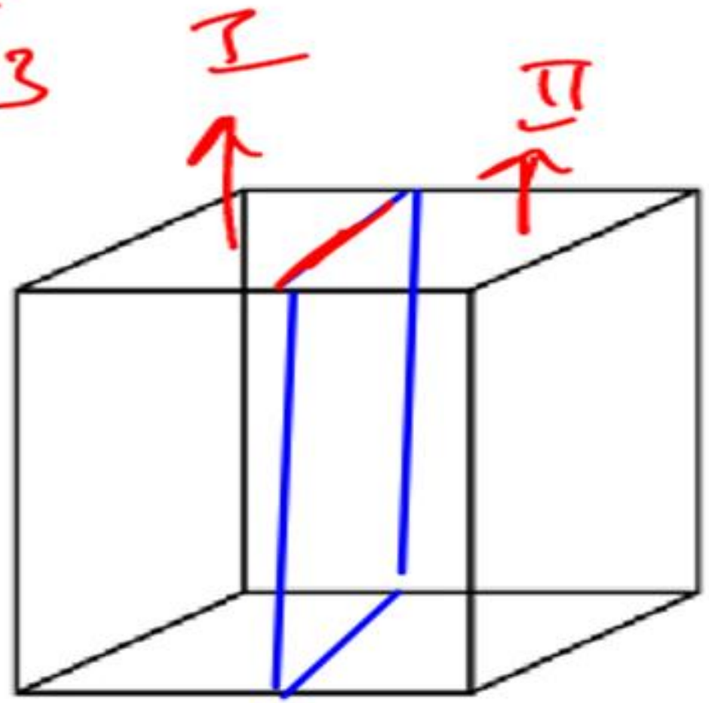
= 4 parts



4 Parts

CUTTING OF CUBE IN 2 EQUAL PARTS

Find: $\frac{\text{TSA of } 1^{st} \text{ part}}{\text{TSA of original cube}}$ = $\frac{x^2}{6x^2}$ = $\frac{1}{3}$



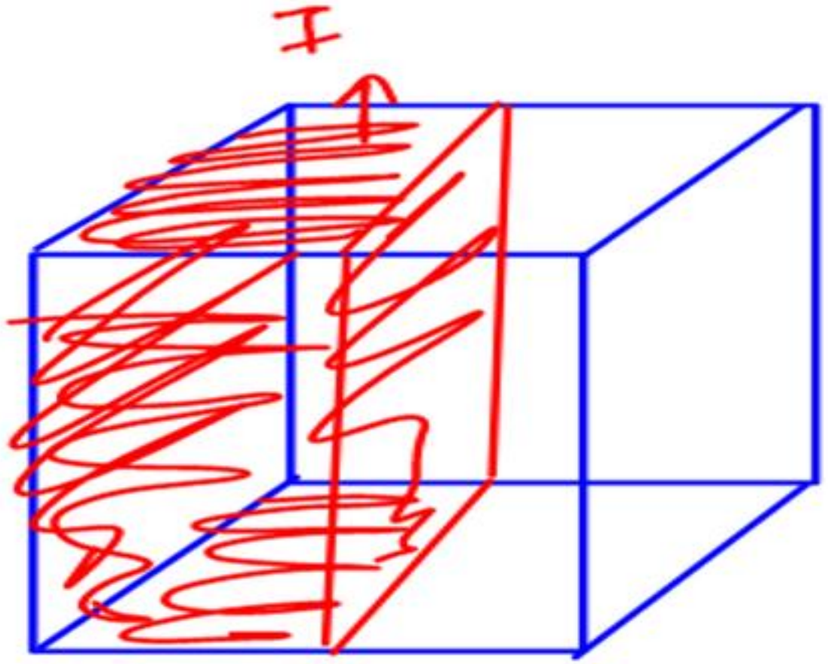
1st Approach

TSA = 6

6 + 2 → 8 faces

$\frac{4}{6} = \frac{2}{3}$

2nd Approach



$$\frac{1}{2}+\frac{1}{2}+1+1+\frac{1}{2}+\frac{1}{2}$$

$$= \frac{6}{3} = 2$$

3rd

1 cut → 2 parts

$\frac{s}{2}, s, s$

$$TSA = 2\left[\frac{s^2}{2} + s^2 + \frac{s^2}{2}\right]$$

$$= 4s^2$$

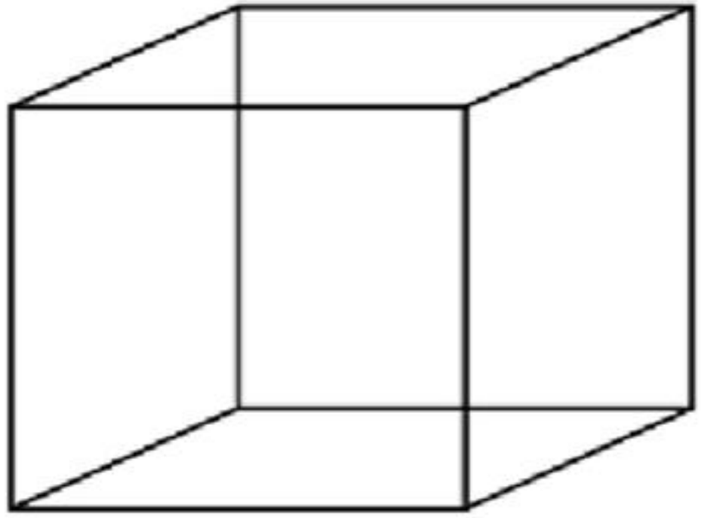
$$\frac{4s^2}{6s^2} = \frac{2}{3}$$

CUTTING OF CUBE IN 4 EQUAL PARTS

Find: TSA of 1st part

TSA of Original Cube

Parts



I^st

6 + 2 + 2

= 10 faces

$\frac{2-5}{6} = \frac{5}{12}$

IInd

$\frac{S}{2} \times \frac{S}{2} \times S$

$TSA = 2\left[\frac{S^2}{4} + \frac{S^2}{2} + \frac{S^2}{2}\right]$

$= 2\left[\frac{S^2 + 2S^2 + 2S^2}{4}\right] = \frac{5S^2}{2}$

$\frac{5S^2}{2 \cdot 6S^2} \Rightarrow \boxed{5/12}$

Eg. A cuboid of size 50 cm x 40 cm x 30 cm is cut into 8 identical parts by 3 cuts. What is the total surface area (in cm^2) of all the 8 parts?

(a) 11750
(b) 14100
(c) 18800
(d) 23500

I st

$l = 25 \quad b = 20 \quad h = 15$

$8 \times 2[500 + 300 + 375]$

$16\,[1175]$

Ans. (c)

Object 1 —Melted→ Object 2

It means their volumes are same.

WHEN TO USED FORMULA FOR VOLUME AND SURFACE AREA

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Amount of Air / Water inside the container → Volume

Amount of paint required to paint the object → Surface Area

OPEN BOX

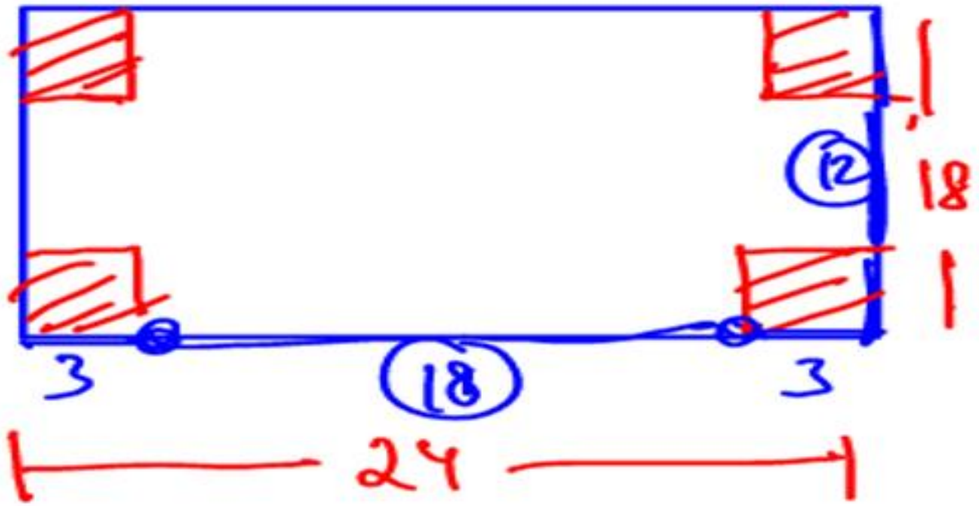
Cuboid ⟶ 6 faces

Open Box ⟶ 5 faces

Eg. A square of side 3 cm is cut off from each corner of a rectangular sheet of length 24 cm and breadth 18 cm and the remaining sheet is folded to form an open rectangular box. The surface area of the box is

(a) 468 cm^2 (b) 396 cm^2

(c) 612 cm^2 (d) 423 cm^2



I st

$l = 18$

$b = 12$

$h = 3$

$18 \cdot 12 + 2 \cdot 3 (18+12)$

$= 216 + 180 = 396 cm^2$

II nd

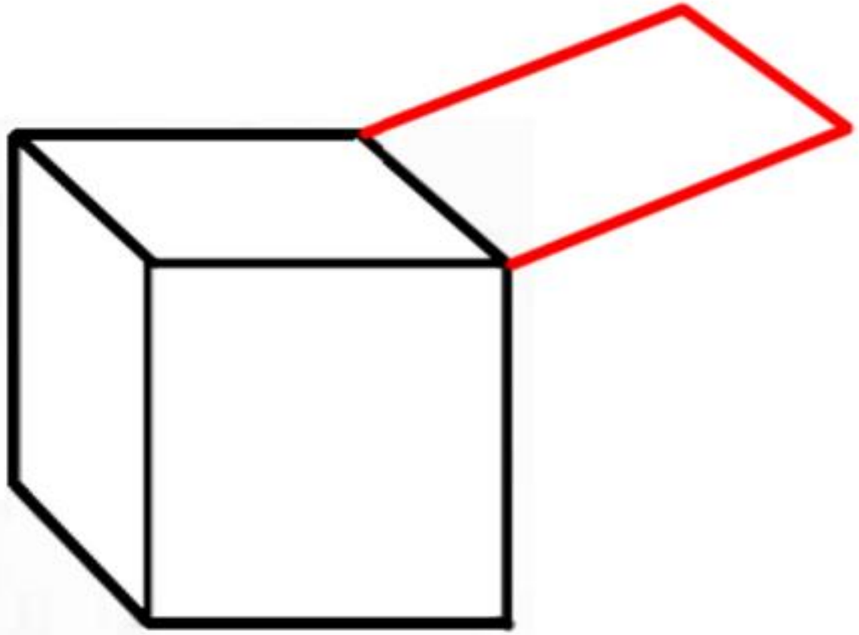
$18 \cdot 24 - 4 \cdot 3^2$

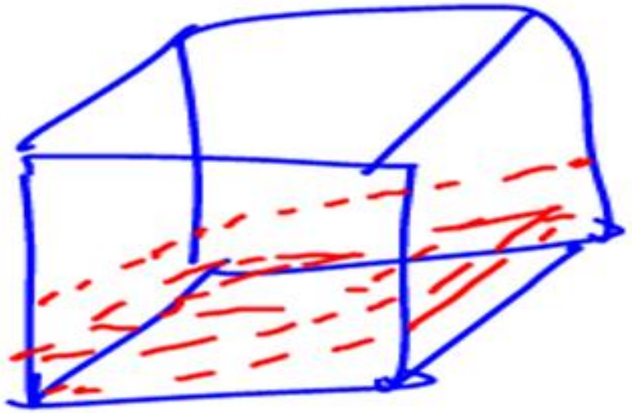
$432 - 36 = 396 cm^2$

Ans. (b)

TSA of an open box = l b + 2bh + 2hl

Area of wet surface = l b + 2bh + 2hl





Eg. A cistern 6 m long and 4 m wide, contains water up to a depth of 1m25 cm. The total area of the wet surface is

(a) 55 m^2 (b) 53.5 m^2

(c) 50 m^2 (d) 49 m^2

$l = 6 \quad b = 4 \quad h = 1.25$

$24 + 2 \times 1.25 [10]$

$24 + 25$

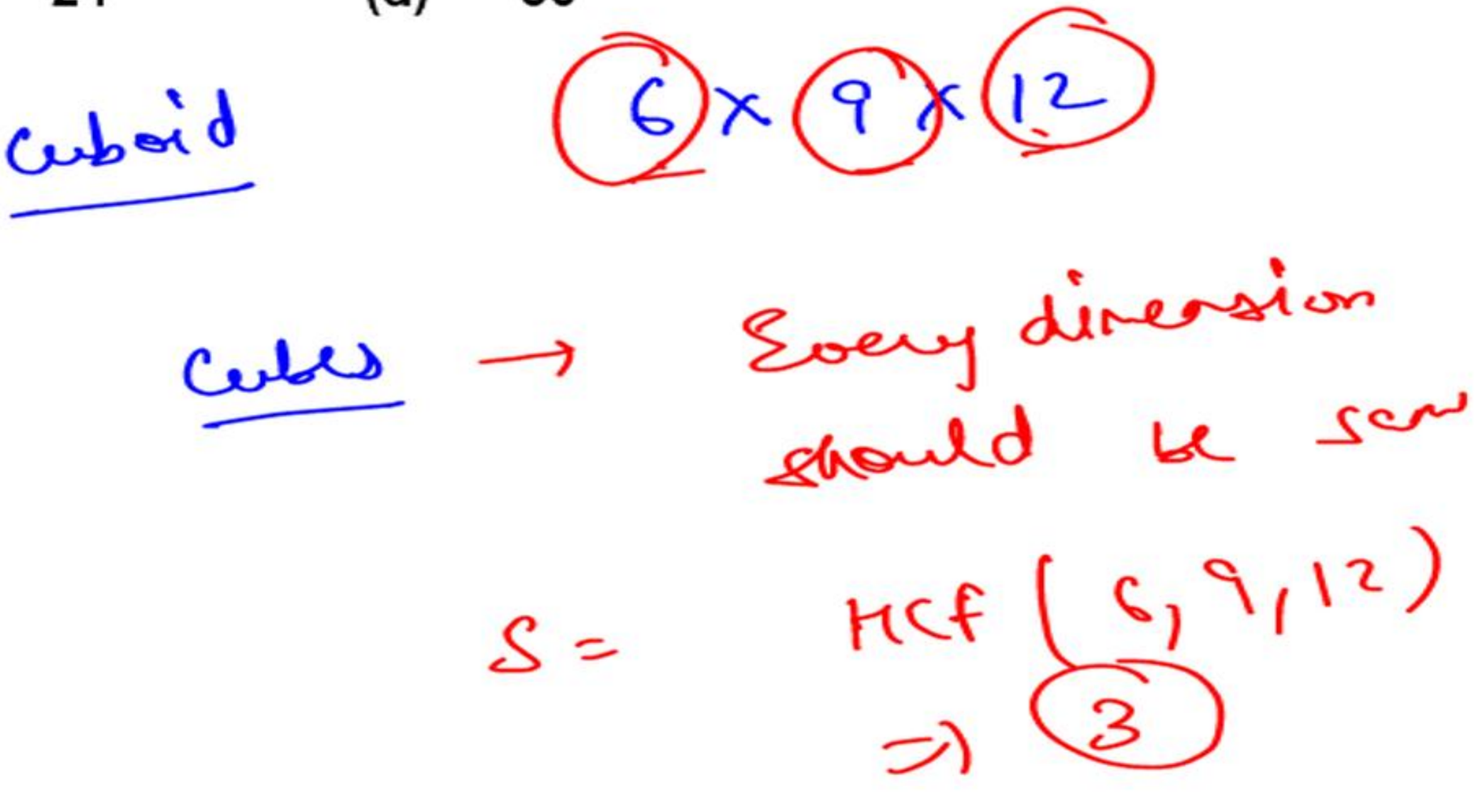
$= 49 \text{ cm}^2$

Ans. (d)

Eg. A cuboidal block of 6 cm × 9 cm × 12 cm is cut up into exact number of equal cubes. The least possible number of cubes will be

(a) 6 (b) 9

(c) 24 (d) 30



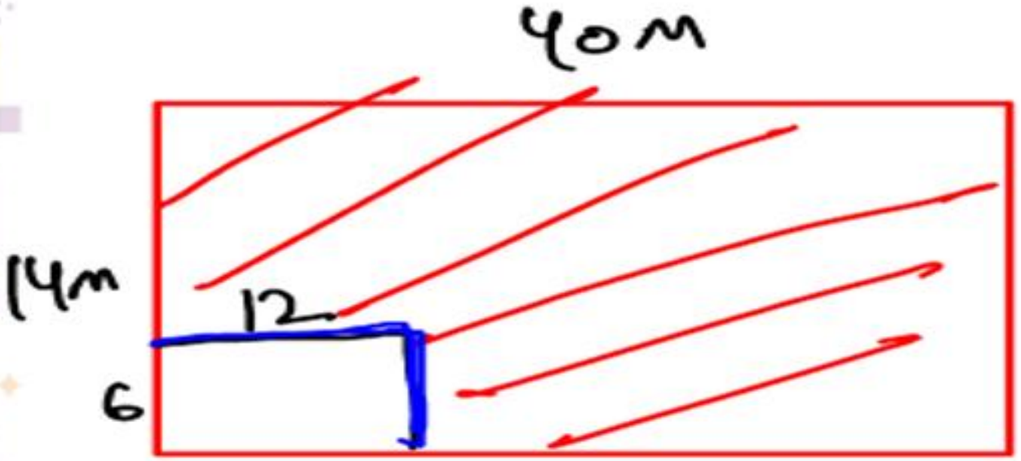
Ans. (c)

For least no. of cubes
Size of cube should be max.
So, l = b = h = HCF of 6, 9, 12
So, side of cube = 3 cm

$$\text{No. of cubes} = \frac{6 \times 9 \times 12}{3 \times 3 \times 3} = 24$$

Eg. A rectangular field is 40 m long and 14 m broad. In one corner of it, a pit 12 m long, 6 m wide and 5 m deep has been dug out and the earth taken out of it has been evenly spread over the remaining part of the field. Find the rise in level of the field.

(a) 73.77 cm (b) 72.12 cm
(c) 70 cm (d) 75 cm



$12 \cdot 6 \cdot 5 = (40 \cdot 14 - 12 \cdot 6) \cdot h$

$\frac{12 \cdot 6 \cdot 5}{488} = h$

$\frac{45}{61} = h$

0.73

Ans. (a)

Eg. A wooden box measures 20 cm by 12 cm by 10 cm. Thickness of wood is 1 cm. Volume of wood to make the box (in cubic cm) is

(a) 960 (b) 519

(c) 2400 (d) 1120

Ans. (a)

Eg. A cistern of capacity 8000 litres measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is :

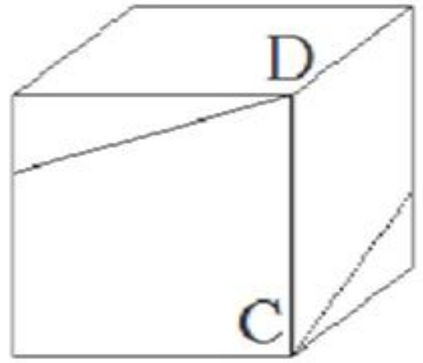
(a) 1 m (b) 1.1 m

(c) 1 dm (d) 90 cm

Ans. (c)

Eg. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale).The length of the string, in cm, is

(a) $\sqrt{2}n$ (b) $\sqrt{17}n$ (c) n (d) $\sqrt{13}n$



Ans. (b)

Eg. A solid cube has side 8 cm. It is cut along diagonals of top face to get 4 equal parts. What is the total surface area (in cm^2) of each part.

$(a) 96 + 64\sqrt{2}$

$(b) 80 + 64\sqrt{2}$

$(c) 96 + 48\sqrt{2}$

$(a) 80 + 48\sqrt{2}$

Ans. (a)

PRACTICE QUESTIONS

Q1. 125 identical cubes are cut from a big cube and all the smaller cubes are arranged in a row to form a long cuboid. What is the percentage increase in the total surface area of the cuboid over the total surface area of the cube ?

(a) $234\frac{1}{3}\%$

(b) $234\frac{2}{3}\%$

(c) 117%

(d) None of these

Ans. (b)

Q2. Surface area of three adjacent faces of a cuboid are p, q, r. Its volume is

(a) $\sqrt{pq^2 + qr^2 + rp^2}$

(b) $\left(\sqrt{pq} + \sqrt{qr} + \sqrt{rp}\right)\left(p^2 + q^2 + r^2\right)$

(c) $\sqrt{\left(p^2 + q^2 + r^2\right)(p + q + r)}$

(d) $\sqrt{pqr}$

Ans. (d)

Q3. The diagonals of the three faces of a cuboid are x, y and z, respectively. What is the volume of the cuboid ?

(a) $\frac{xyz}{2\sqrt{2}}$

(b) $\frac{\sqrt{(y^2+z^2)(z^2+x^2)(x^2+y^2)}}{2\sqrt{2}}$

(c) $\frac{\sqrt{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}}{2\sqrt{2}}$

(d) None of these

Ans. (c)

Q4. A field in the form of a rectangle having length 20 m and breadth 25 m. There is a square pit outside the field having dimension 15 m x 15 m. This pit is to be filled uniformly upto a height of 4 m with the soil taken out by digging the rectangular field. Find out the depth upto which the rectangular field must be dug if the soil is to fill the pit?

$(a)\ \frac{9}{5}\ m$ $(b)\ \frac{9}{2}\ m$ $(c)\ \frac{9}{7}\ m$ $(d)\ \frac{9}{4}$

Ans. (a)

Q5. The paint in certain container is sufficient to paint an area equal to 5.875 m^2. How many bricks of dimensions 12.5 cm × 10 cm × 7.5 cm can be painted out of this container ?
(a) 225 (b) 180 (c) 150 (d) 100

Ans. (d)

Q6. A cuboidal water tank is filled with water. When 64 bucket of water is drawn off the tank, then 1/3rd of the water tank remains filled. Length of each side of the tank is 1.2 metre. Considering the measure of each bucket equal, find the volume (in ltr.) of the water in each bucket.

(a) 18 (b) 16

(c) 12 (d) 15

Ans. (a)

Q7. A rectangular tank is 50 metres long and 29 metres deep. If 1000 cubic metres of water be drawn off the tank, the level of the water in the tank goes down by 2 metres. How many cubic metres of water can the tank hold? And also find the breadth of the tank.

(a) 1400 m^3, 10m
(b) 14500 m^3, 15m
(c) 14000 m^3, 15m
(d) 14500 m^3, 10 m

Ans. (d)

Q8. If S is the total surface area of a cube and V is its volume, the which one of the following is correct ?

(a) $V^3 = 216\ S^2$

(b) $S^3 = 216\ V^2$

(c) $S^3 = 6V^2$

(d) $S^2 = 36V^3$

Ans. (b)

Q9. Three solid iron cubes of edges 4 cm, 5 cm and 6 cm are melted together to make a new cube. 62 cm^3 of the melted material is lost due to improper handling. The area (in cm^2) of the whole surface of the newly formed cube is

(a) 294 (b) 343

(c) 125 (d) 216

Ans. (a)

Q10. 2 cm of rain has fallen on a square km of land. Assuming that 50% of the raindrops could have been collected and contained in a pool having a 100 m × 10 m base, by what level would the water level in the pool have increased ?

(a) 1 km
(b) 10 m
(c) 10 cm
(d) 1 m

Ans. (c)

Q11. A cuboidal water tank has 216 litres of water. Its depth is 1/3 of its length and breadth is 1/2 of 1/3 of the difference of length and depth. The length of the tank is

(a) 72 dm
(b) 18 dm
(c) 6 dm
(d) 2 dm

Ans. (b)

Q12. A rectangular sheet of metal is 40 cm by 15 cm. Equal square of side 4 cm are cut off at the corners and the remainder is folded up to form an open rectangular box. The volume of the box is

(a) 896 cm^3
(b) 986 cm^3
(c) 600 cm^3
(d) 916 cm^3

Ans. (a)

Q13. If the sum of three dimensions and the total surface area of a rectangular box are 12 cm and 94 cm^2 respectively, then the maximum length of a stick that can be placed inside the box.

(a) $\mathbf{5\sqrt{2}}$ cm

(b) 5 cm

(c) 6 cm

(d) $\mathbf{2\sqrt{5}}$ cm

Ans. (a)