The Most Comprehensive
Preparation App For All Exams

MENSURATION-3D

Part-3

Sphere

Hemisphere

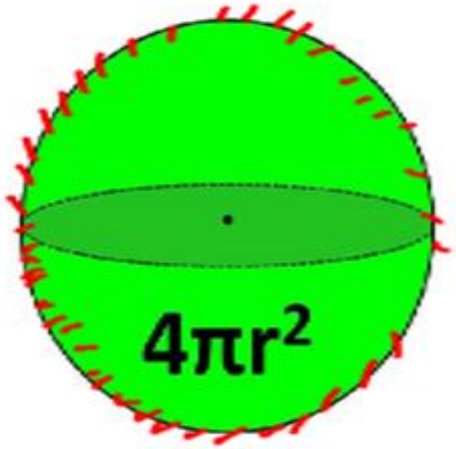
$1/4^{th}$, $1/8^{th}$

Cutting of Sphere

Spherical Shells

→ $(70-75)_{Min}$

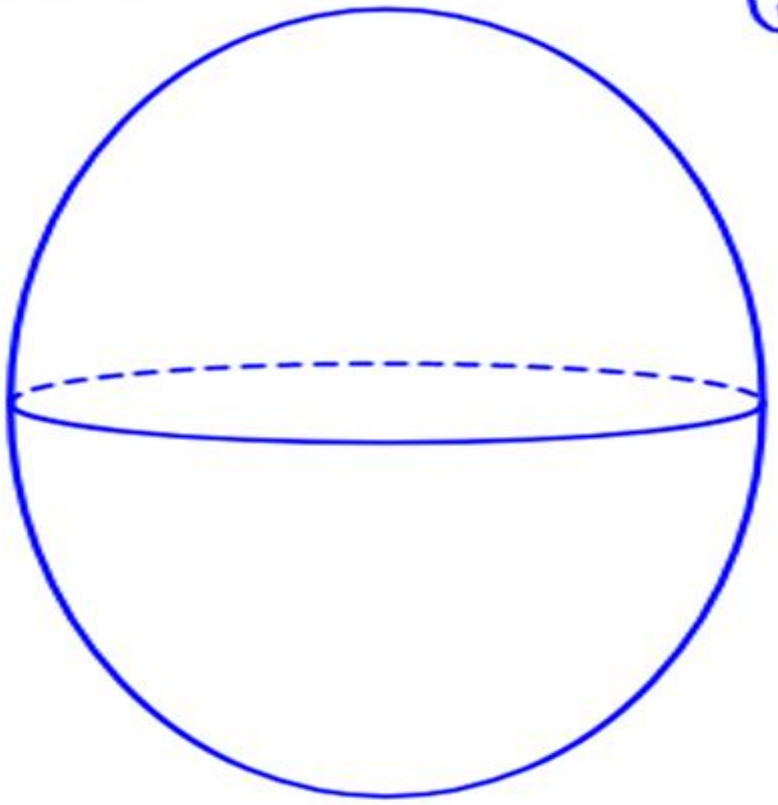
SPHERE

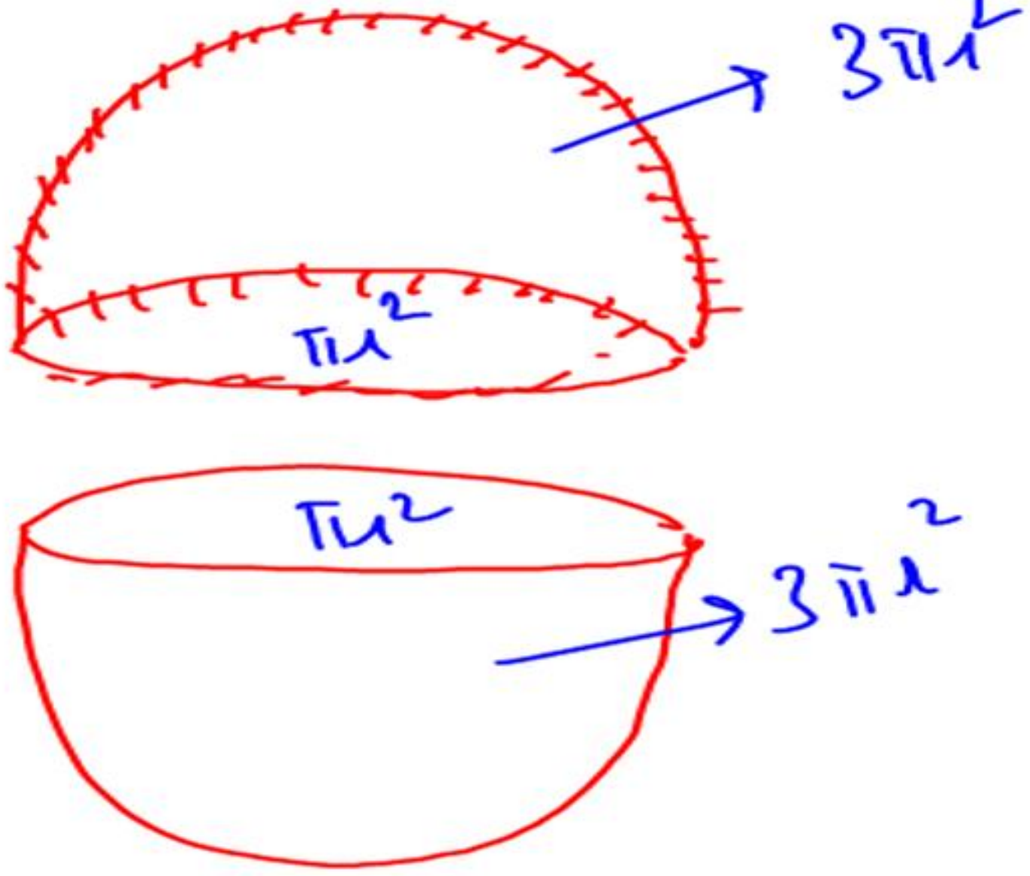


$$\text{CSA/ TSA} = 4\pi R^2$$

$$\text{Volume} = \frac{4}{3}\pi R^3$$

$4\pi r^2 + 2\pi r^2$





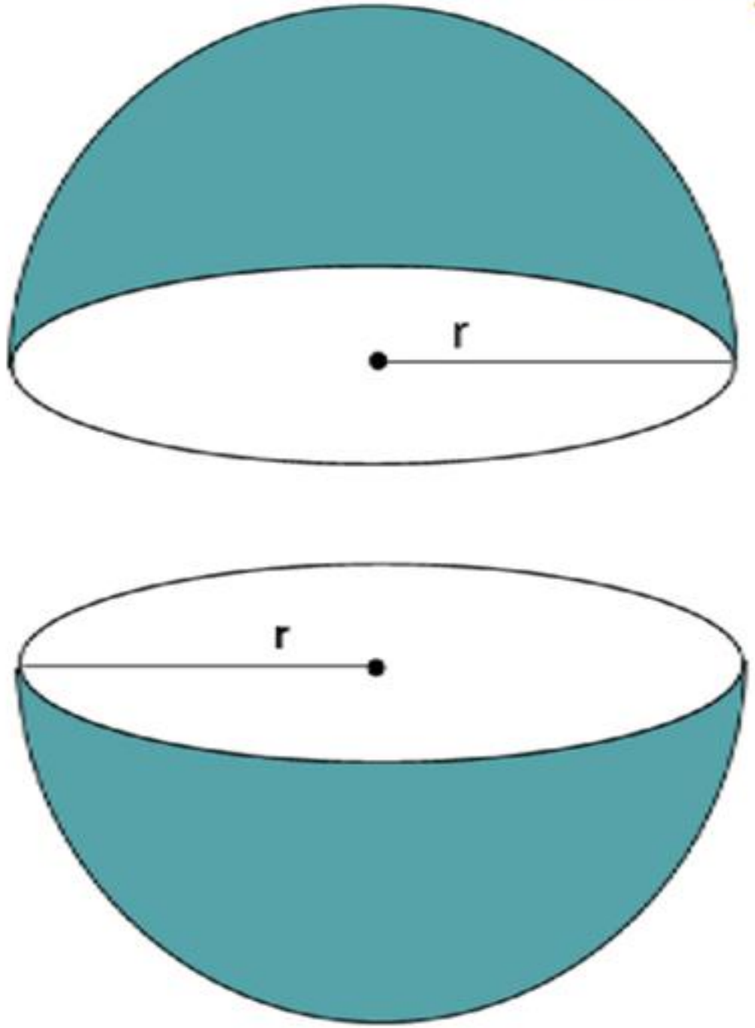
$CSA = 2\pi R^2$ $\quad TSA = 3\pi x^2$

HEMISPHERE

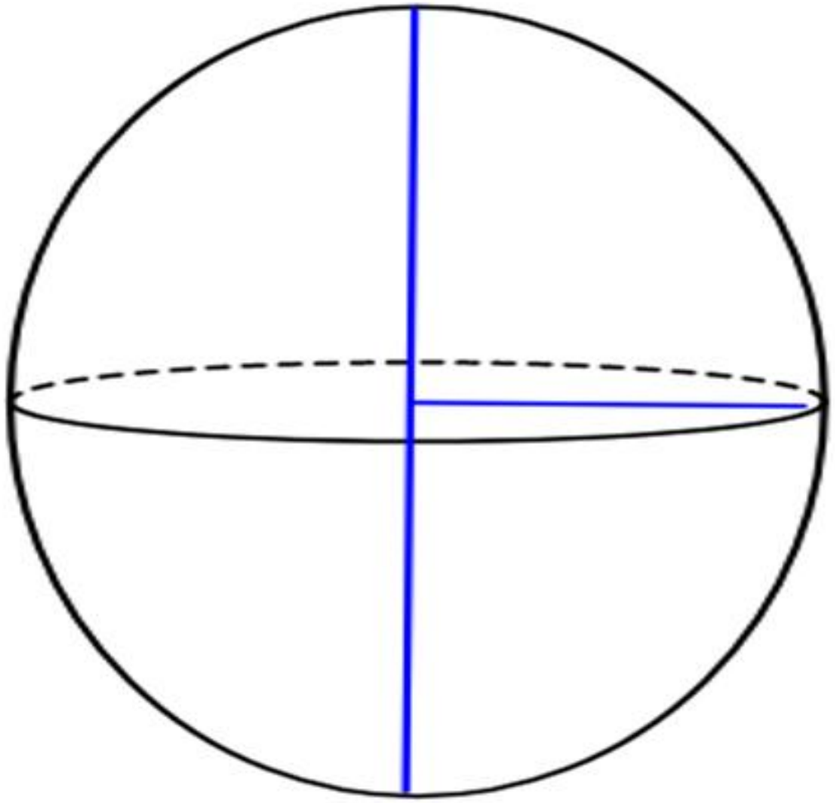
CSA = $2\pi R^2$

TSA = $3\pi R^2$

Volume = $\frac{2}{3}\pi R^3$



One-fourth ($\frac{1}{4}^{th}$) OF A SPHERE



Sphere → $4\pi R^2$

Hemisphere (1 cut) → 2 parts → $\frac{4\pi R^2 + 2\pi R^2}{2}$

$= 3\pi R^2$

$\frac{1}{4}^{th}$ sphere (4 parts) → 2 cuts → $\frac{4\pi R^2 + 2 \cdot 2\pi R^2}{4}$

$= 2\pi R^2$

	Sphere	Hemisphere	$\frac{1}{4}^{th}$ sphere
CSA	$4\pi r^2$	$2\pi r^2$	πr^2
TSA	$4\pi r^2$	$3\pi r^2$	$2\pi r^2$
Volume	$\frac{4}{3}\pi r^3$	$\frac{2}{3}\pi r^3$	$\frac{1}{3}\pi r^3$

3 cuts ($\frac{1}{8}^{th}$ of a sphere)

TSA = ??

$$\frac{4\pi R^2 + 3 \cdot 2\pi R^2}{8}$$

$$\frac{10\pi R^2}{8} \Rightarrow \frac{5}{4}\pi R^2$$

	Sphere	½ sphere (Hemisphere)	1/4th of Sphere	1/8th of Sphere	1/2n of a Sphere (n cuts)
CSA	$4\pi R^2$	$2\pi R^2$	πR^2	$\frac{\pi R^2}{2}$	$\frac{4\pi R^2}{2^n} = \frac{\pi R^2}{2^{n-2}}$
TSA	$4\pi R^2$	$3\pi R^2$	$2\pi R^2$	$\frac{5}{4}\pi R^2$	$\frac{4\pi R^2 + 2n\pi r^2}{2^n}$
Volume	$\frac{4}{3}\pi R^3$	$\frac{2}{3}\pi R^3$	$\frac{1}{3}\pi R^3$	$\frac{1\pi R^3}{6}$	$\frac{\frac{4}{3}\pi R^3}{2^n}$

$$\frac{4\pi r^2 + (2\pi r^2)n}{2^n}$$

Eg. A sphere and a hemisphere have the same volume. The ratio of their curved surface area is :

(a) $2^{3/2} : 1$ (b) $2^{3/3} : 1$

(c) $4^{2/3} : 1$ (d) $2^{1/3} : 1$

75 sec

Sphere (R) Hemisphere (r)

$\frac{4}{3}\pi R^3 = \frac{2}{3}\pi r^3$

$\frac{R^3}{r^3} = \frac{1}{2}$

$\frac{R}{r} = \frac{1}{2^{1/3}}$

$\frac{2\pi R^2}{2\pi r^2} \Rightarrow 2 \cdot \left(\frac{1}{2^{2/3}}\right) \Rightarrow \frac{2^{1/3}}{1}$

Ans. (d)

Eg. If the radius of sphere is increased by 2 cm its surface area increased by 352 cm^2. The radius of sphere before change is : (Take $\pi = 22/7$)

(a) 3 cm (b) 4 cm

(c) 5 cm (d) 6 cm

Let radius of sphere $= r$

$$4\pi (r+2)^2 - 4\pi r^2 = 352$$

$$4 \cdot \frac{22}{7} \left[(r+2)^2 - r^2 \right] = 352$$

$$(2)(2r+2) = 28$$

$$\boxed{r = 6}$$

Ans. (d)

Eg. A solid metallic sphere of radius 8 cm is melted to from 64 equal small solid spheres. The ratio of the surface area of this sphere to that of a small sphere is

(a) 4 : 1 (b) 1 : 16

(c) 16 : 1 (d) 1 : 4

60sec

$$\frac{\text{Volume of Big sphere}}{\text{Volume of Small sphere}} = \frac{64}{1}$$

$$\frac{R}{r} = \frac{4}{1}$$

$$\frac{R^2}{r^2} = \frac{16}{1}$$

Ans. (c)

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \frac{(\text{Kg})}{(\text{m}^3)}$$

Kg/m^3

Eg. The ratio of weights of two spheres of different materials is 8 : 17 and the ratio of weights per 1 cm^3 of materials of each is 289 : 64. The ratio of radii of the two spheres is :

(a) 8 : 17 (b) 4 : 17

(c) 17 : 4 (d) 17 : 8

90 sec

$\frac{M_1}{M_2} = \frac{8}{17}$ $\frac{D_1}{D_2} = \frac{289}{64}$

$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$ $\text{Volume} = \frac{\text{Mass}}{\text{Density}}$

$\frac{V_1}{V_2} = \frac{M_1}{M_2} \times \frac{D_2}{D_1}$

$\frac{R_1}{R_2} = \frac{8}{17}$

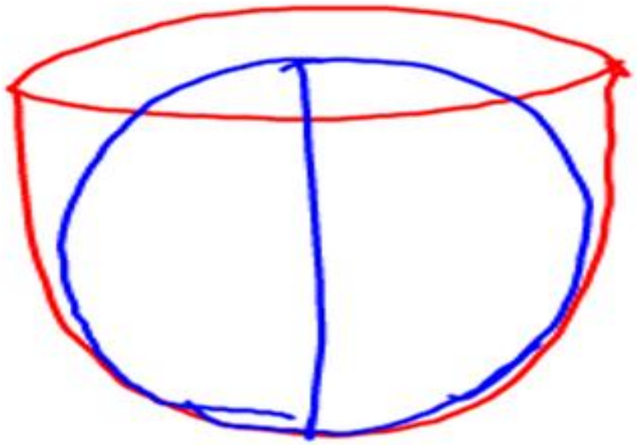
$\frac{V_1}{V_2} = \frac{8}{17} \times \frac{64}{289}$

Ans. (a)

Eg. A sphere of maximum volume is cut out from a solid hemisphere of radius r. The ratio of the volume of the hemisphere to that of the cut out sphere is :

(a) 3 : 2 (b) 4 : 1

(c) 4 : 3 (d) 7 : 4



Radius of hemisphere = R

Radius of sphere = R/2

$$\frac{\frac{2}{3}\pi R^3}{\frac{4}{3}\pi \frac{R^3}{8}} = \frac{4}{1}$$

Ans. (b)

BYJU'S ExamPrep

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

Eg. The sum of radii of two spheres is 10 cm and the sum of their volume is 880 cm³. What will be the product of their radii ?

(a) 21 (b) $26\frac{1}{3}$ (c) $33\frac{1}{3}$ (d) 70

$R_1 + R_2 = 10$

$\frac{4}{3} \times \frac{22}{7} \left[R_1^3 + R_2^3 \right] = 880$ 10

$R_1^3 + R_2^3 = 210$

$(R_1 + R_2)^3 - 3R_1 \cdot R_2 (R_1 + R_2) = 210$

$100 - 30R_1R_2 = 210$ $\quad R_1R_2 = \frac{79}{3}$

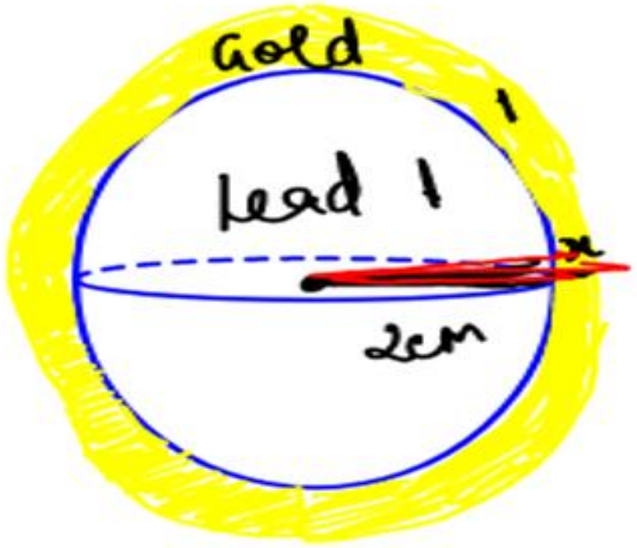
Ans. (b)

V. V. Imp

Eg. A ball of lead 4 cm in diameter is covered with gold. If the volume of the gold and lead are equal, then the thickness of gold [given $\sqrt[3]{2} = 1.259$] is approximately.

PYQ

(a) 5.038 cm
(b) 5.190 cm
(c) 1.038 cm
(d) 0.518 cm



I^st

$$\frac{(2+x)^3}{(2)^3} = \frac{2}{1}$$

$$(2+x)^3 = 2^3 \cdot 2$$

$$2+x = 2 \cdot (1.259)$$

$$x = 0.518 \text{ cm}$$

Ans. (d)

Logical Approach

Volume of Load = 8 units

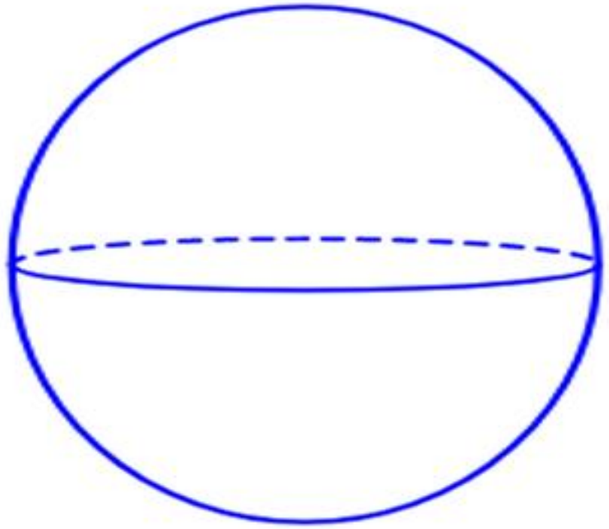
Volume of Ball = 16 units

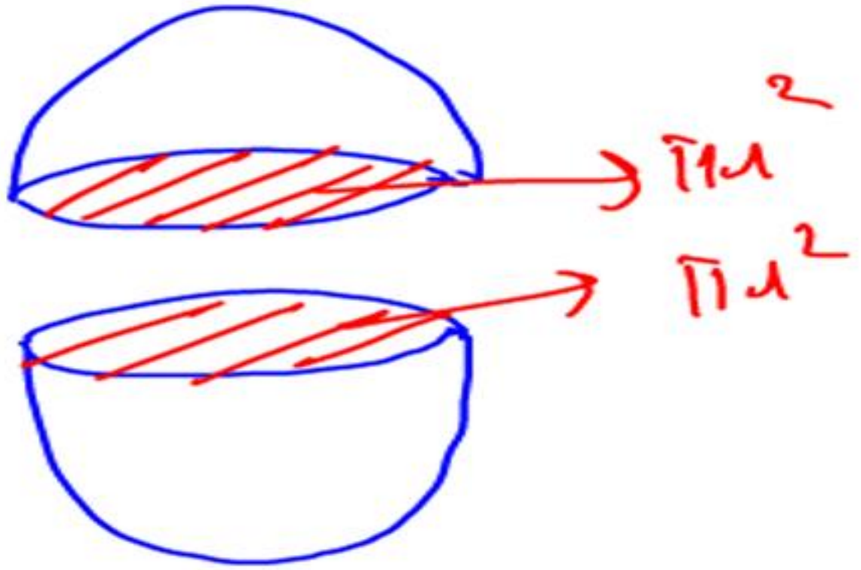
$2 < R < 3$

$0 < \text{thickness} < 1$

Eg. A sphere of diameter 14 cm is cut into two halves. Find the increase in its surface area.

(a) 207 cm^2
(b) 154 cm^2
(c) 308 cm^2
(d) 616 cm^2





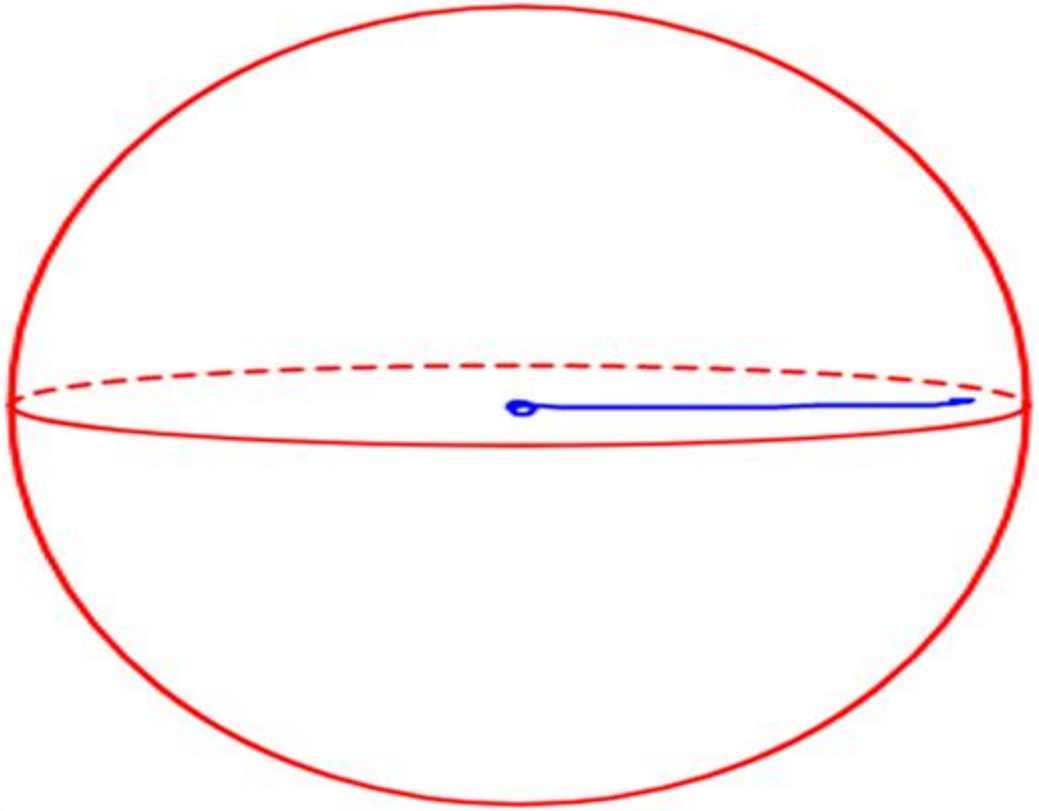
$2\pi r^2$

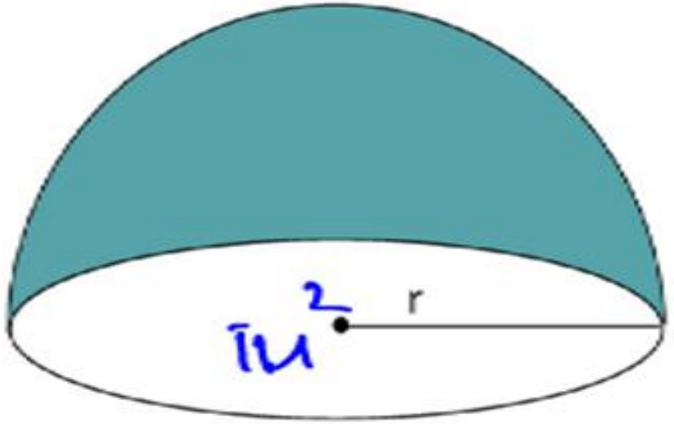
$2 \times \frac{22}{7} \times 7 \times 7$

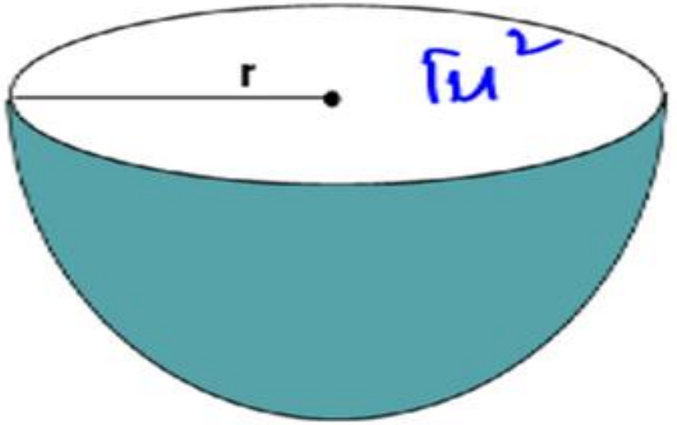
$= 308\ cm^2$

Ans. (c)

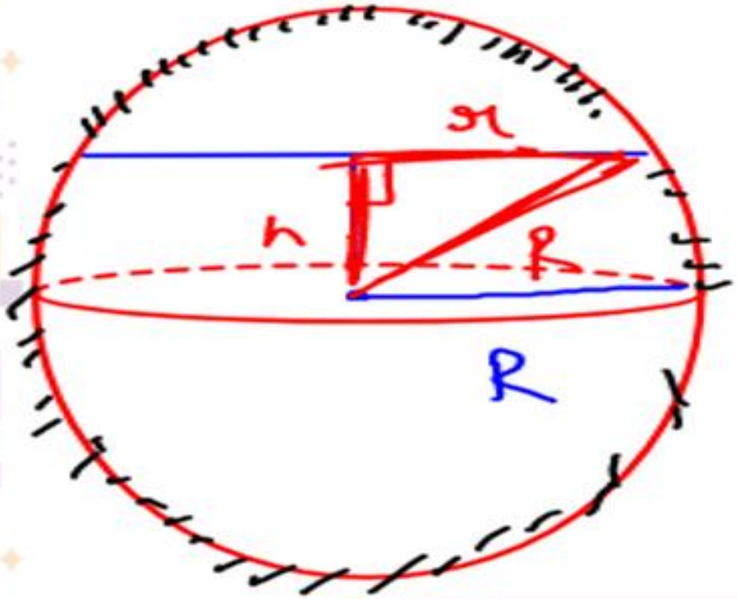
SPHERE IS CUT INTO TWO PIECES



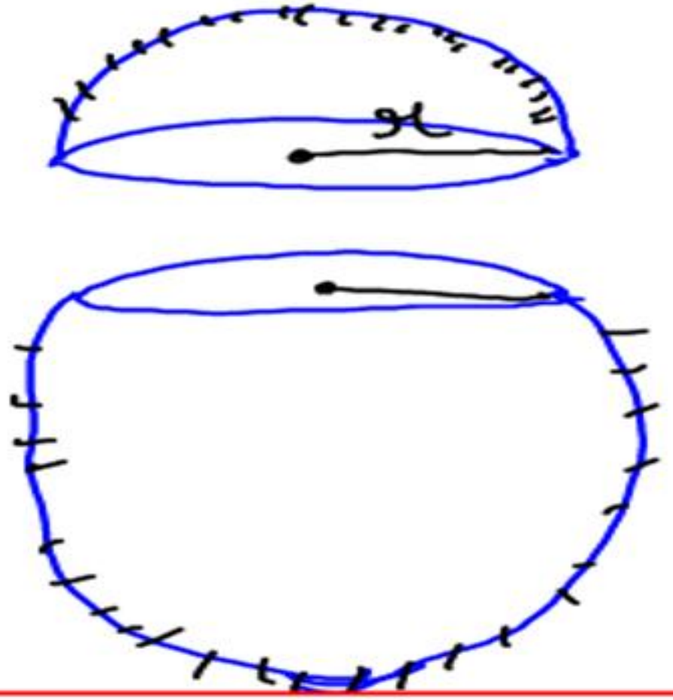




extra → $2\pi r^2$



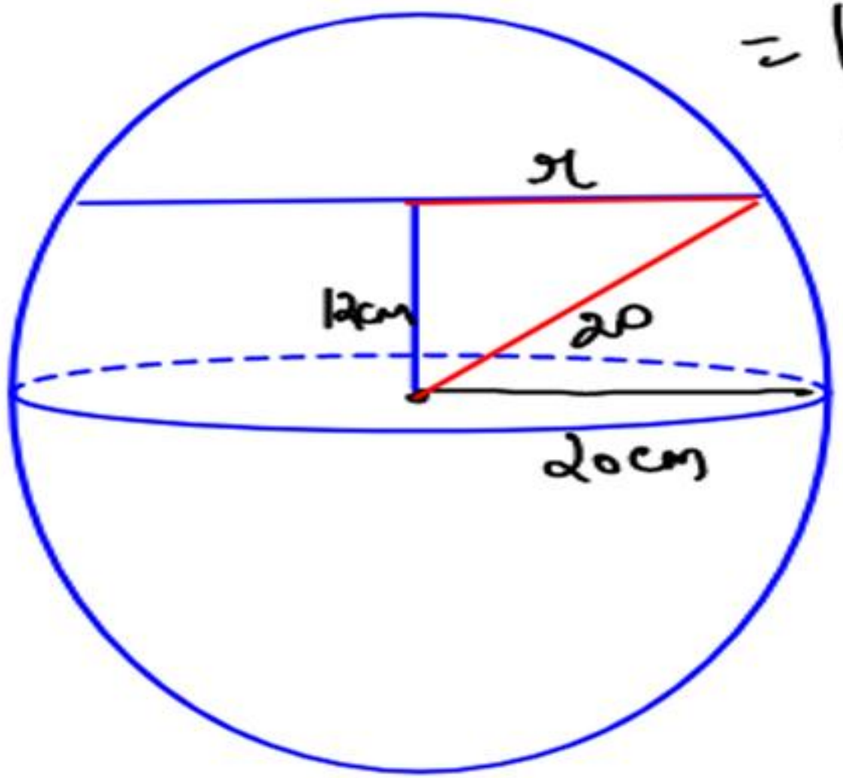
$$R^2 = r^2 + h^2$$



$$\text{extra} = 2\pi r^2$$

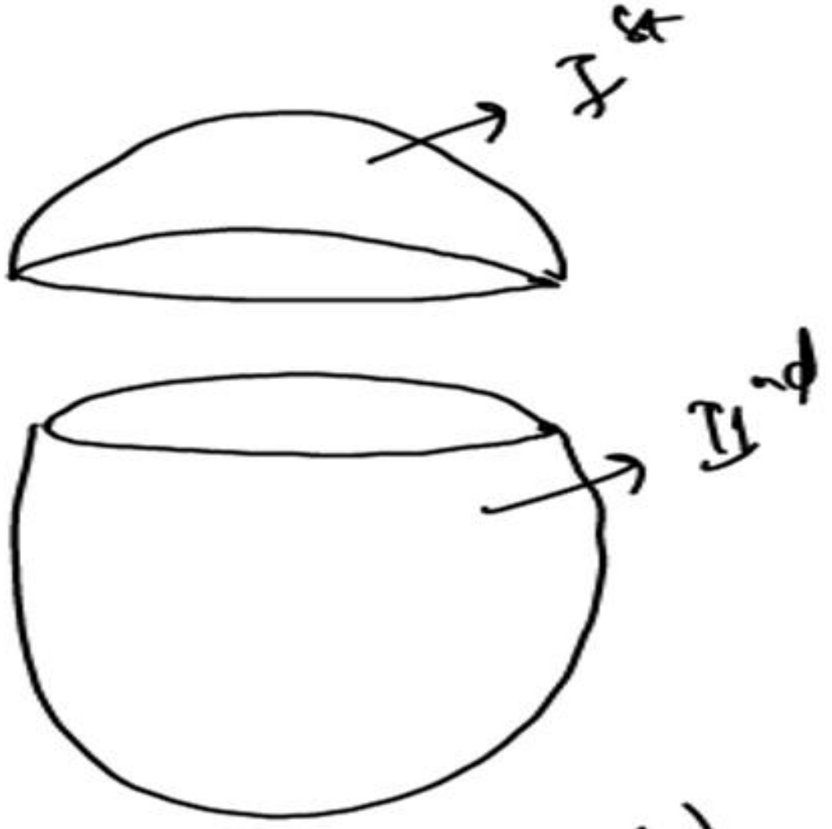
$4\pi(20)^2$

$= 1600\pi$



$x^2 = 20^2 - 12^2$

$x^2 = 256$



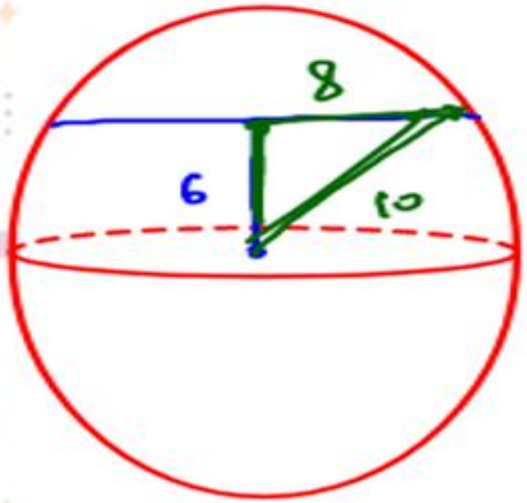
TSA of (I^{st} & II^{nd})

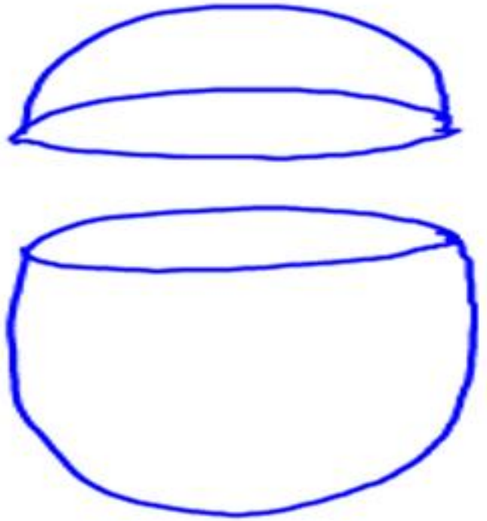
extra $\longrightarrow 2\pi(256)$

512π

TSA $= 2112\pi$ cm

Eg. Sphere of Radius is 10 cm. It is cut into 2 parts parallel to its diameter and at a distance of 6 cm from its centre. Find the TSA of the 2 parts is what % more than the TSA of original sphere?

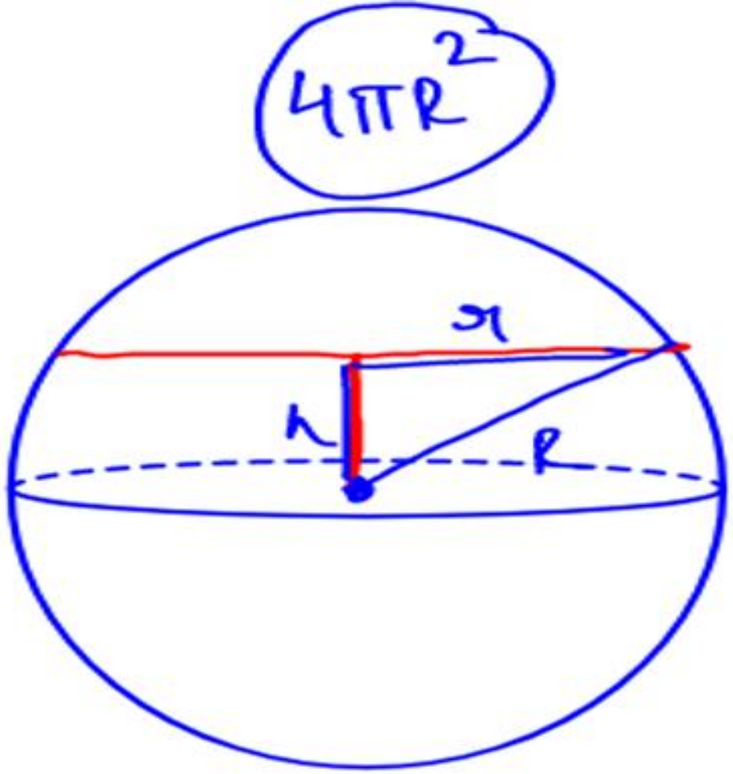


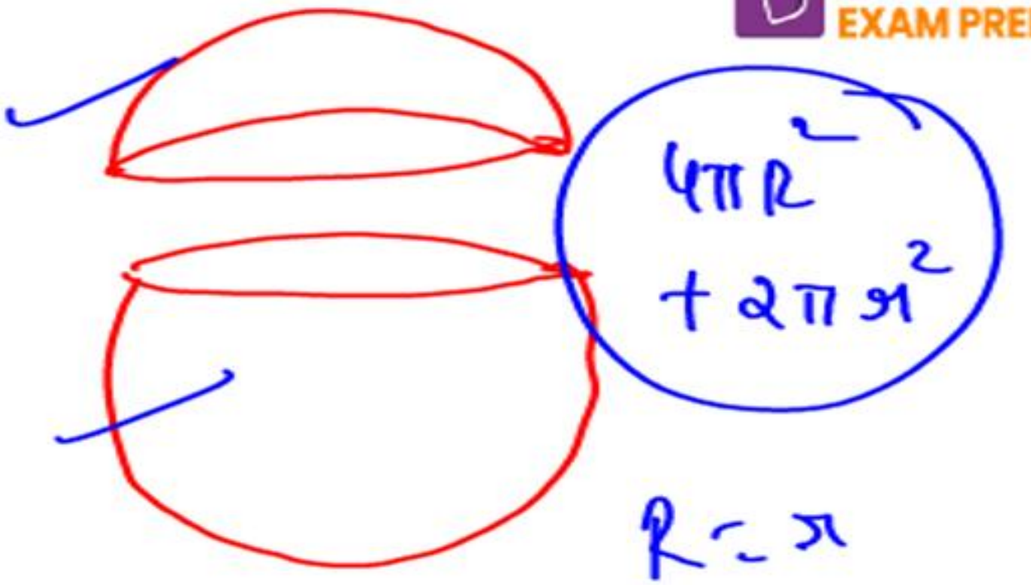


$4\pi 10^2$

extra $2\pi r^2$

$\frac{2\pi \cdot 8^2 \cdot 16}{4\pi \cdot 10^2} \times 100 = 32\%$





TSA of (Both the parts) is 50% more than the original surface Area so Find the height from the centre from where it is cut → Ocm

Eg. A spherical ball of radius 12 cm is cut at some distance (x cm) from its centre, into two different pieces. If the surface area of newly formed pieces is 29/24 of the ball's original surface area, then find value of x (in cm).

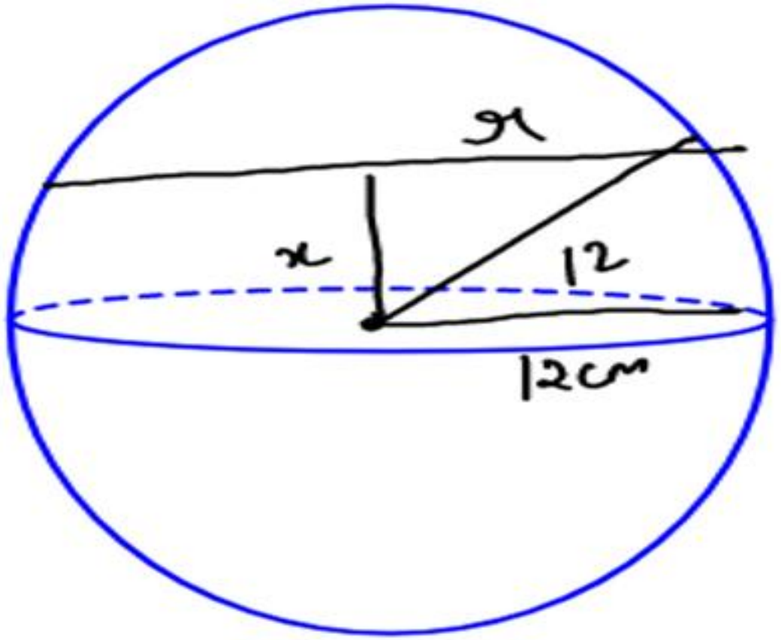
90sec

(a) $2\sqrt{21}$ cm

(b) $3\sqrt{21}$ cm

(c) $4\sqrt{21}$ cm

(d) None of these



extra : $\frac{5}{24} \times 4\pi(12)^2$

$\frac{5}{24} \cdot 4\pi(12)^2 = 2\pi r^2$

$r^2 = 60$

$r^2 + x^2 = 144$

$x^2 = 84$

$x = 2\sqrt{21}$ cm

Ans. (a)

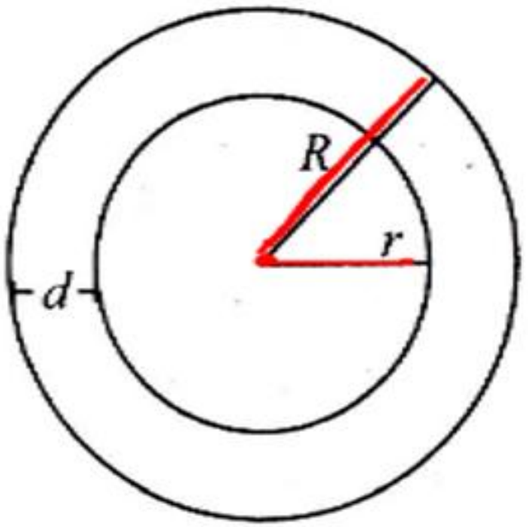
HOLLOW SPHERE

$$\text{Volume} = 4\pi (R^3 - r^3)$$

SPHERICAL AND HEMISPHERICAL SHELL

BYJU'S EXAM PREP

1. Spherical Shell



Inner radius = r

Outer radius = R

Thickness = $d = R - r$

(i) Outer surface area $= 4\pi R^2$

(ii) Inner surface area $= 4\pi r^2$

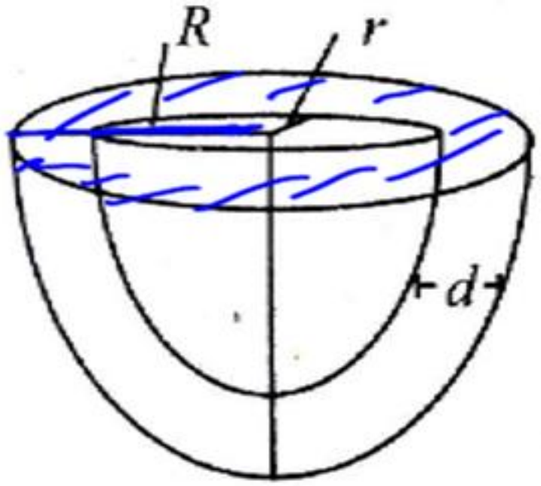
(iii) Volume of the shell $= \frac{4}{3}\pi (R^3 - r^3)$

2. Hemispherical Shell

Inner radius = r

Outer radius = R

Thickness = $d = R - r$



(i) Inner curved surface area = $2\pi r^2$

(ii) Outer curved surface area = $2\pi R^2$

(iii) Total surface area = $3\pi R^2 + \pi r^2$

$\pi(3R^2 + r^2)$

(iv) Volume = $\frac{2}{3}\pi(R^3 - r^3)$

$2\pi r^2 + 2\pi R^2 + \pi(R^2 - r^2)$

$3\pi R^2 + \pi r^2$

$\pi(3R^2 + r^2)$

Eg. The volume (in cm^3) of the material of a hemispherical shell with outer and inner radii 6 cm and 5 cm respectively is:

(a) $\frac{124\pi}{3}$

(b) $\frac{241\pi}{3}$

(c) $\frac{182\pi}{3}$

(d) $\frac{418\pi}{3}$

$$\frac{2}{3}\pi\left(6^3-5^3\right)$$

$$= \frac{182\pi}{3}\ cm^3$$

Ans. (c)

Eg. A hollow spherical metallic ball has an external diameter 6 cm and is 1/2 cm thick. The volume of the ball (in cm^3) is:

(a) $41\frac{2}{3}$ (b) $37\frac{2}{3}$ (c) $47\frac{2}{3}$ (d) $40\frac{2}{3}$

$R = 3\,cm \qquad x = \frac{5}{2}$

$$\frac{4}{3} \times \frac{22}{7}\left[27 - \frac{125}{8}\right]$$

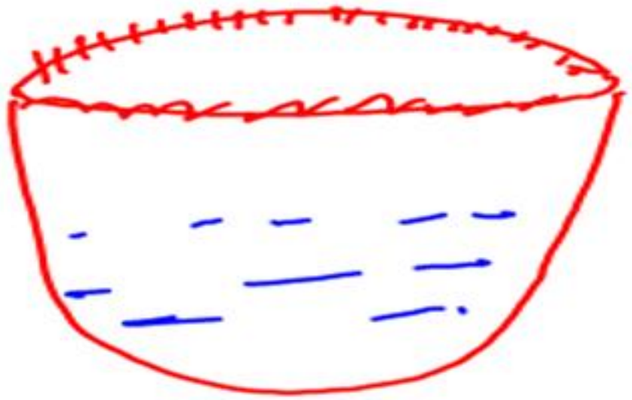
$$\frac{4}{3} \times \frac{22^{11}}{7} \times \frac{91}{8}^{13} = \frac{143}{3}$$

Ans. (c)

Eg. A hemispherical bowl is 176 cm round the brim. Supposing it to be half full, how many persons may be served from it in hemispherical glasses 4 cm in diameter at the top ?

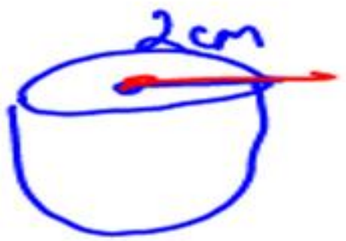
(a) 1372 (b) 1272

(c) 1172 (d) 1472



$2\pi r = 176$

$r = 28$ cm



$\frac{28}{2} \times \frac{14}{1}$

$2744 \times \frac{1}{2}$

1372

Ans. (a)

PRACTICE QUESTIONS

Q1. If the surface area of a sphere is 346.5 cm^2, then its radius (where $\pi = 22/7$) is :

(a) 7 cm
(b) 3.25 cm
(c) 5.25 cm
(d) 9 cm

Ans. (c)

Q2. By melting a solid lead sphere of diameter 12 cm, three small spheres are made whose diameters are in the ratio 3 : 4 : 5. The radius (in cm) of the smallest sphere is:

(a) 3 (b) 6

(c) 1.5 (d) 4

Ans. (a)

Q3. Find the volume of a spherical shell in which the outer and the inner diameters are 16 cm and 4 cm, respectively:

(a) 1221 cm^3

(b) 2112 cm^3

(c) 1826 cm^3

(d) 3170 cm^3

Ans. (b)

Q4. Smaller lead shots are to be prepared by using the material of a spherical lead shot of radius 1cm. Some possibilities are listed in the statements given below.

I The material is just sufficient to prepare 8 shots each of radius 0.5 cm.

II. A shot of radius 0.75 cm and a second shot of radius 0.8 cm can be prepared from the available material.

Which of the above statements is/ are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

(SSC, 2010)

Ans. (c)

Q5. The surface area of a sphere is 616 sq.cm. If its radius is changed so that the area get reduced by 75%, then the radius becomes.

(a) 1.6 cm (b) 2.3 cm

(c) 2.5 cm (d) 3.5 cm

Ans. (d)

Q6. A large solid sphere is melted and moulded to form identical right circular cones with base radius and height same as the radius of the sphere. One of these cones is melted and moulded to form a smaller solid sphere. Then the ratio of the surface area of the smaller to the surface area of the larger sphere is –

(a) $1 : 3^{\frac{4}{3}}$

(b) $1 : 2^{\frac{3}{2}}$

(c) $1 : 3^{\frac{2}{3}}$

(d) $1 : 2^{\frac{4}{3}}$

Ans. (d)

Q7. A hollow sphere of internal and external diameters 6 cm and 10 cm respectively is melted into a right circular cone of diameter 8 cm. The height of the cone is :

(a) 22.5 cm (b) 23.5 cm

(c) 24.5 cm (d) 25.5 cm

Ans. (c)

Q8. A solid metallic sphere of radius 3 decimeters is melted to form a circular sheet of 1 milimetre thickness. The diameter of the sheet so formed is :

(a) 26 metres
(b) 24 metres
(c) 12 metres
(d) 6 metres

Ans. (c)

Q9. The total number of spherical bullets, each of diameter 5 decimeter, that can be made by utilizing the maximum of a rectangular block of lead with 11 metre length, 10 metre breadth and 5 metre width is

(assume that $p > 3$)

(a) equal to 8800
(b) less than 8800
(c) equal to 8400
(d) greater than 9000

Ans. (c)

Q10. A sphere and a hemisphere have the same volume. The ratio of their radii is :

(a) $1:2$ (b) $1:8$

(c) $1:\sqrt{2}$ (d) $1:\sqrt[3]{2}$

Ans. (d)