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Sahi Prep Hai Toh Life Set Hai

QUADRILATERAL

Part-2

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Agenda

Rhombus →

Rectangle →

Square →

Trapezium →

(46 - 48) min

(46 - 48) min

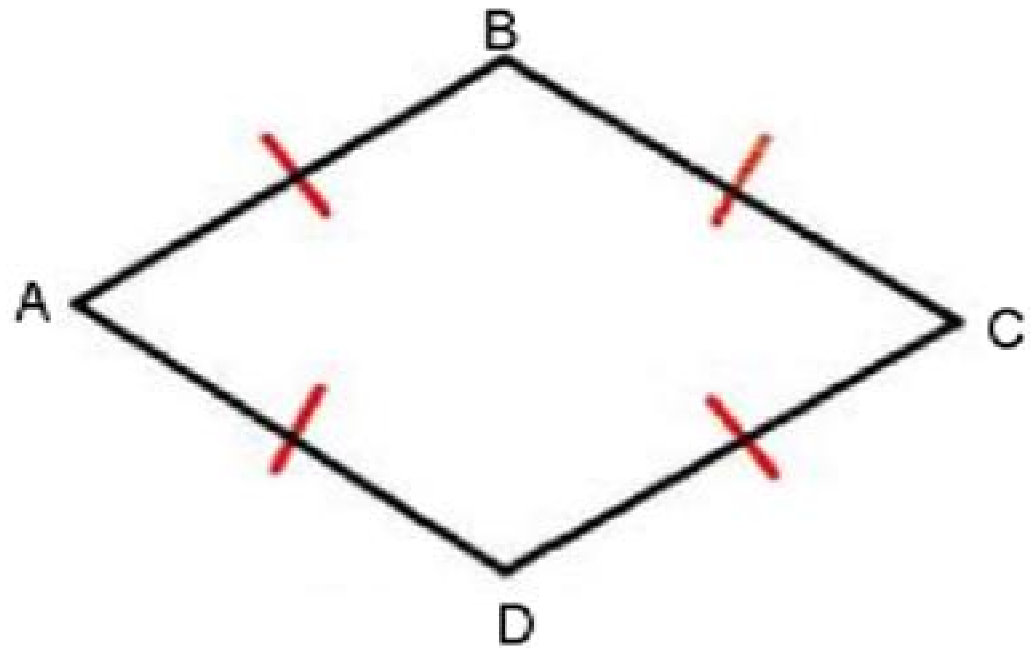
→ Isosceles Trapezium

Kite

Tomm

RHOMBUS

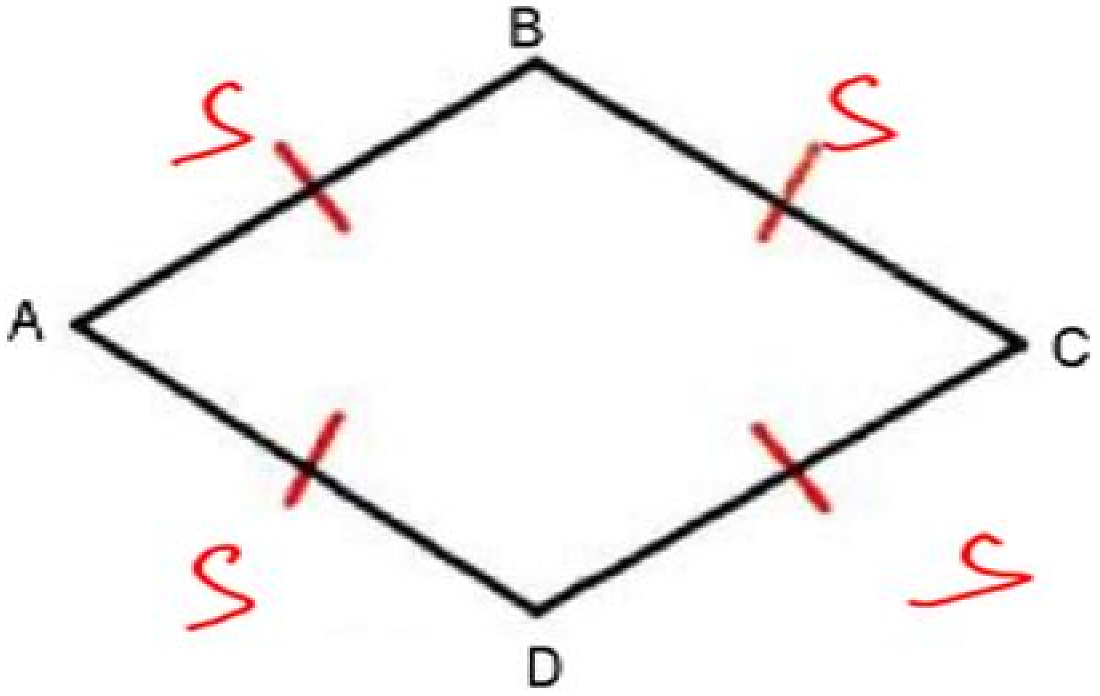
Def: Rhombus is a parallelogram in which adjacent sides are equal.



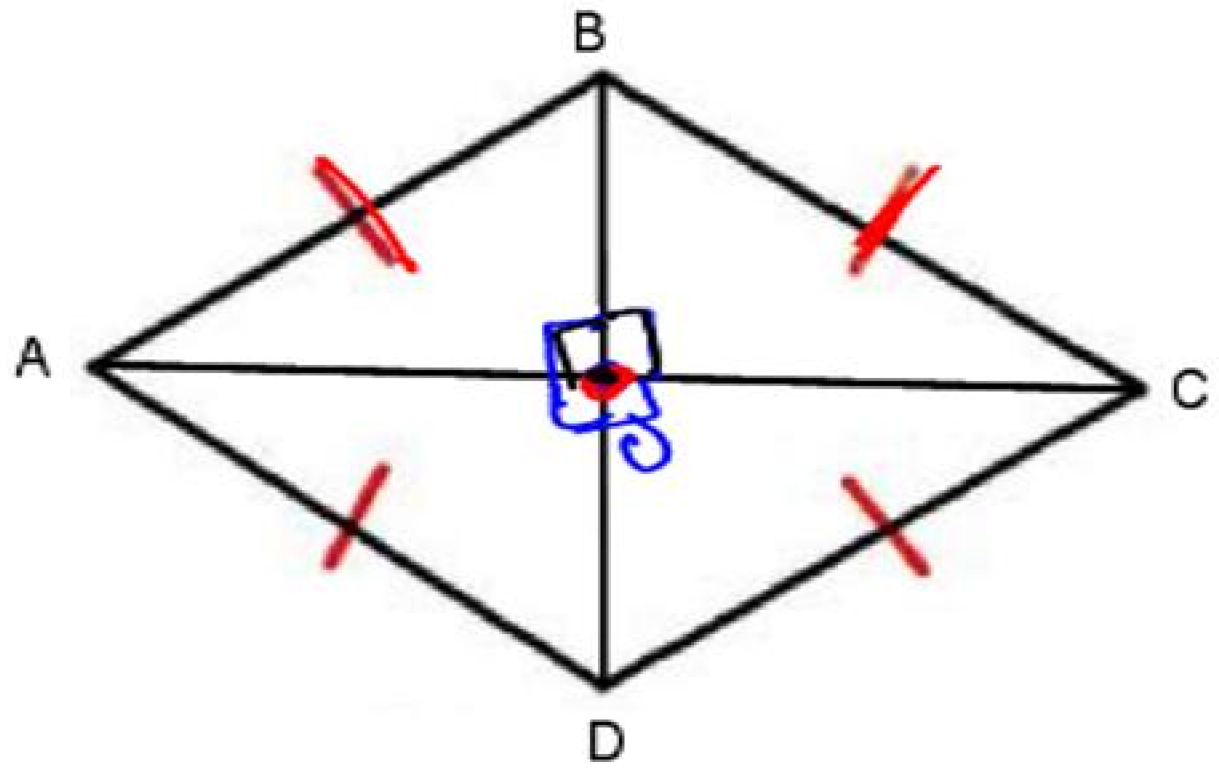
Def → Parallelogram + adjacent sides are equal

PROPERTIES OF RHOM BUS

1. All sides of rhombus are equal.



2. (i) Diagonals of a rhombus bisect each other at 90°.



$\angle AOB = \angle BOC = \angle COD = \angle DOA$

$= 90^\circ$

Reason

$\triangle ABC$

$AB = BC$

In Isosceles

Median → Altitude

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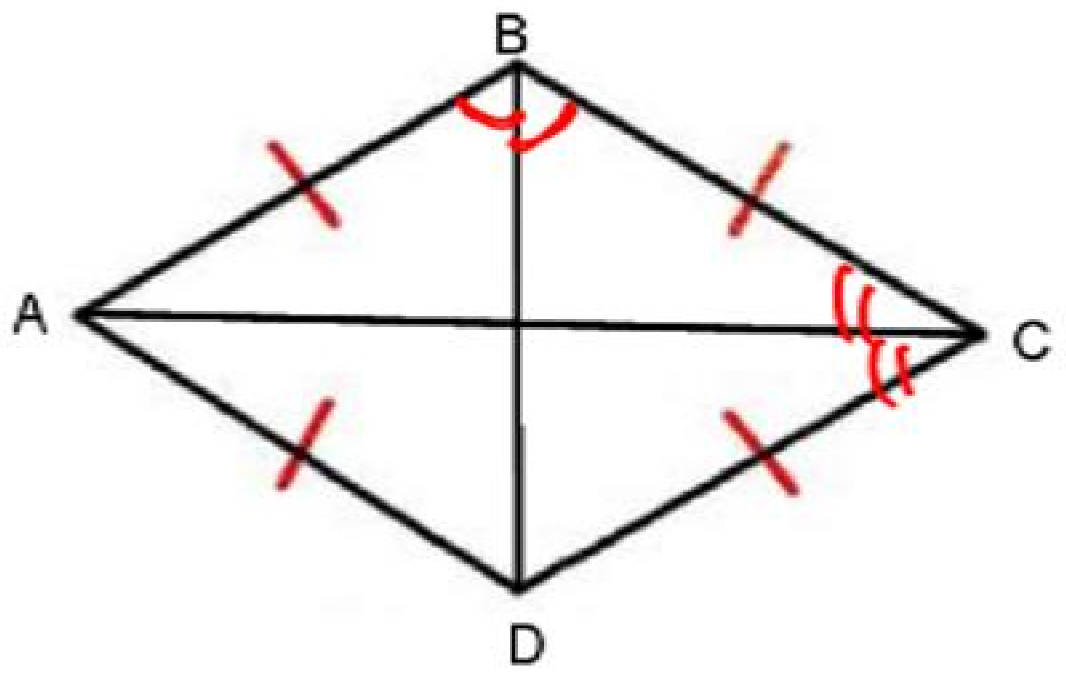
(ii) Diagonals of a rhombus are angle bisector.



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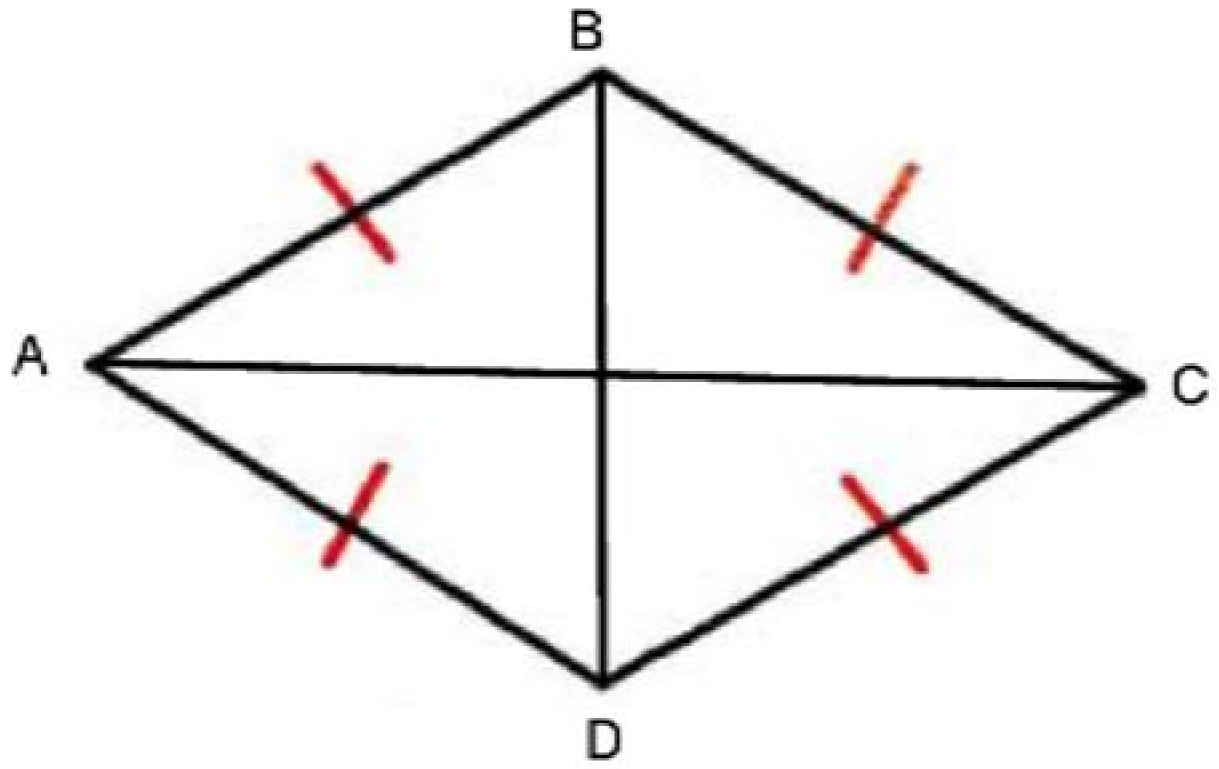
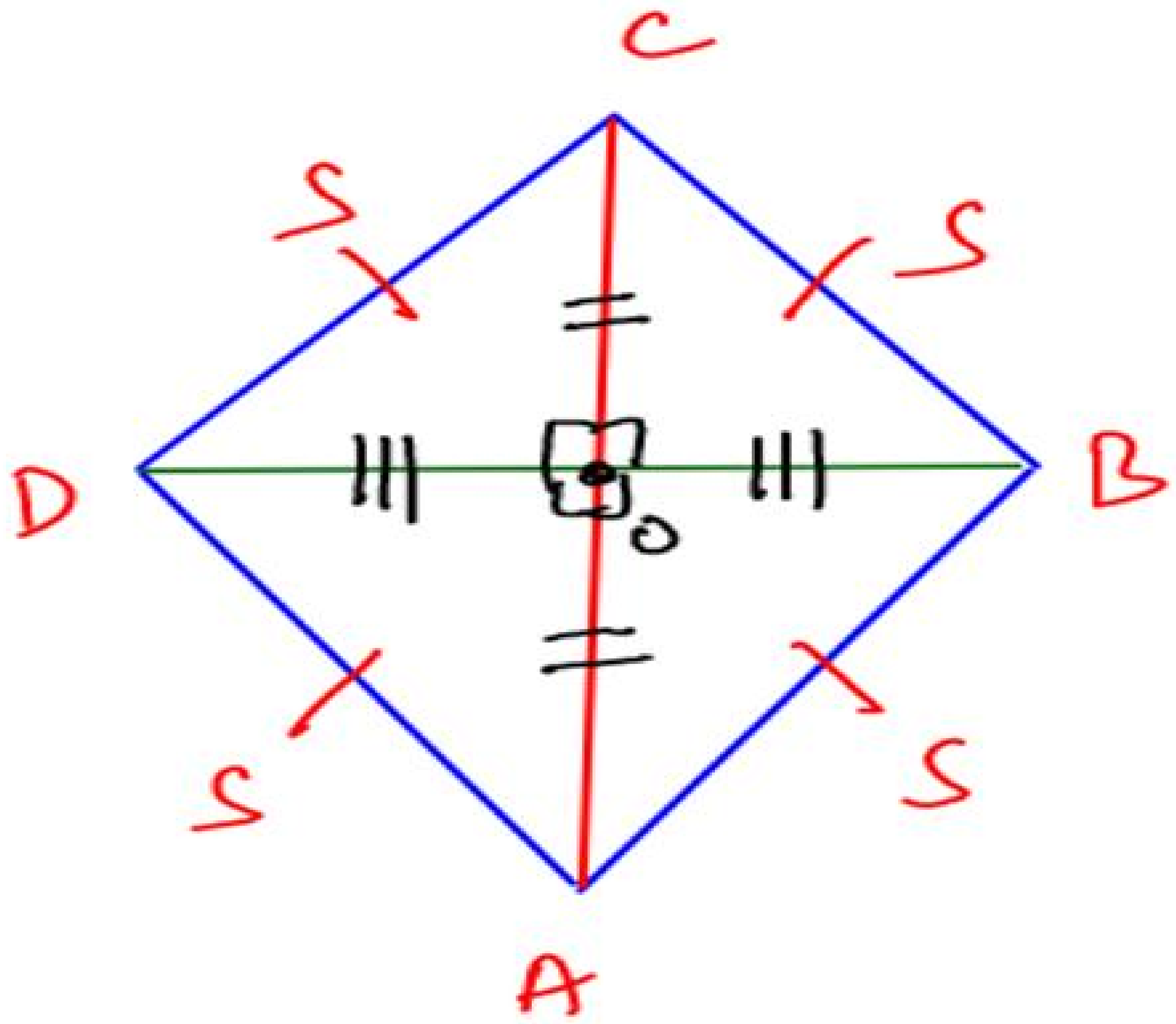
3. Diagonals of a rhombus need not be equal.



$D_1 \neq D_2$



$$\Delta DCO \cong \Delta BCO \cong \Delta BAO \cong \Delta DAO$$

(By SSS)

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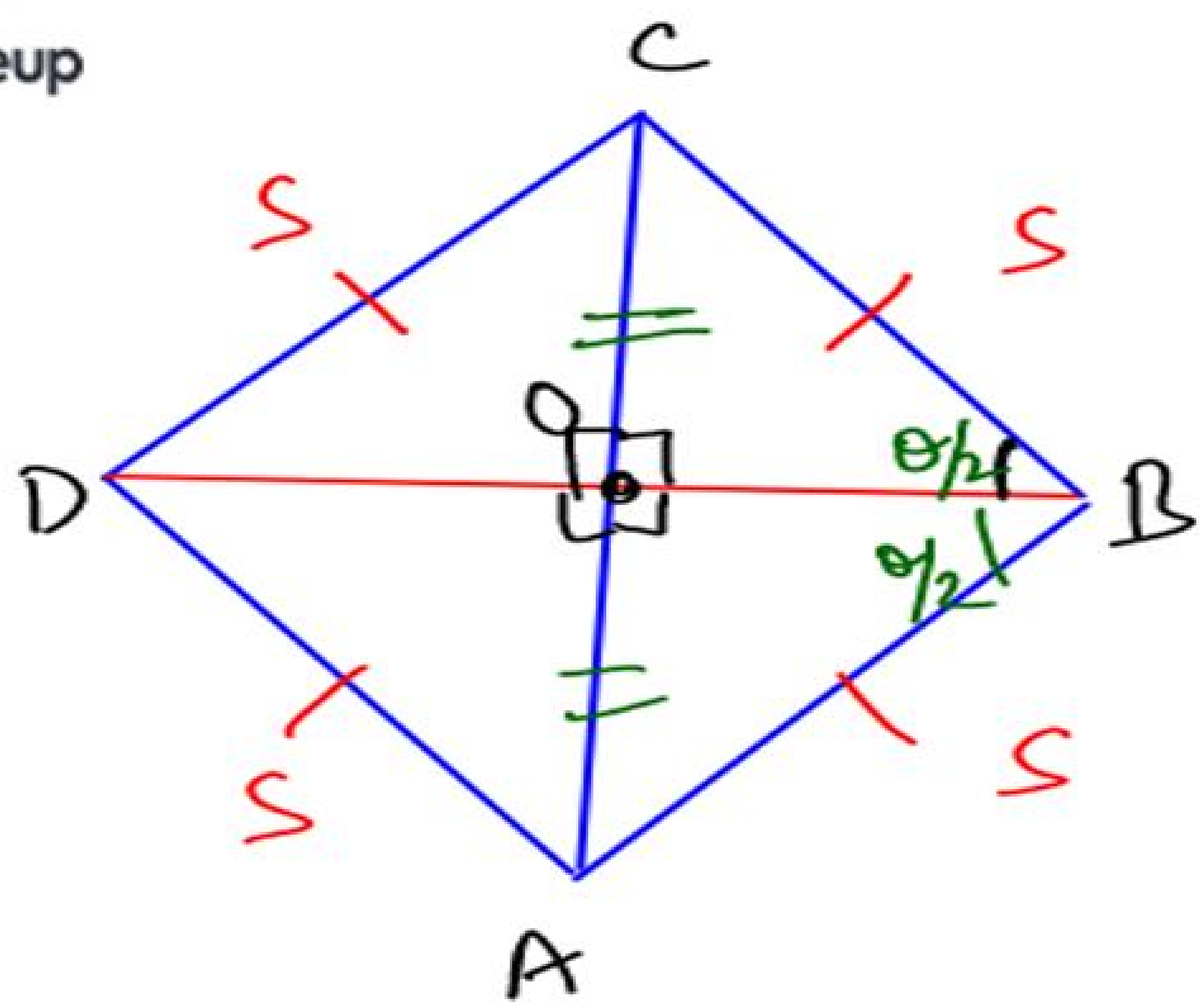

ABCD is a Rhombus

Let θ is one Acute Angle of Rhombus

$0 < \theta < 90$

$0 < \frac{\theta}{2} < 45$

$\cos\theta/2 > \sin\theta/2$

$\sin\theta/2 = \frac{CO}{S}$

$CO = S\sin\theta/2$

$\boxed{AC = 2S\sin\theta/2}$ Shorter Diagonal

$\cos\theta/2 = \frac{BO}{S}$

$BO = S\cos\theta/2$

$BD = 2S\cos\theta/2$ Longer Diagonal

4. ABCD is a rhombus and one of the angle of rhombus is θ, where $0° < \theta < 90°$

Length of longer diagonal = $2s\cos\frac{\theta}{2}$

Length of shorter diagonal = $2s\sin\frac{\theta}{2}$

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Eg. If perimeter of rhombus is 40 cm and one of its angle is 120°. Find the length of longer diagonal.

$4S = 40$

$S = 10\,cm$

$120^\circ \longrightarrow 60^\circ$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\sin 30 = \frac{1}{2}$

$\cos 60 = \frac{1}{2}$

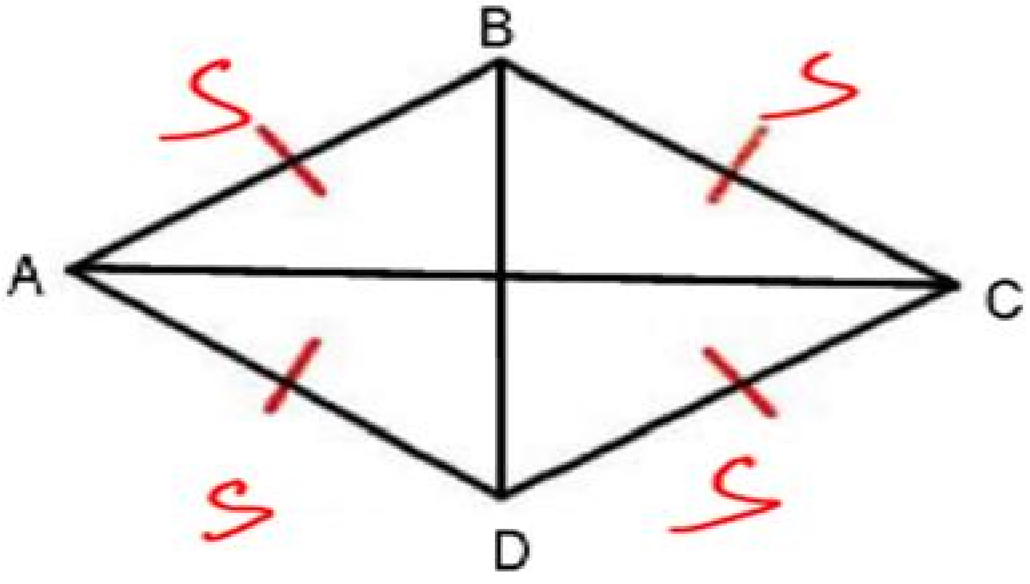
$\cos 30 = \frac{\sqrt{3}}{2}$

Longer Diagonal $= 2S \cos \theta/2$

$= 2 \cdot 10 \cdot \cos 30$

$= \cancel{20} \cdot \frac{\sqrt{3}}{\cancel{2}} = \underline{10\sqrt{3}\,cm}$

5. ABCD is a rhombus and D_1 and D_2 are the diagonals of rhombus and S is the side of rhombus.



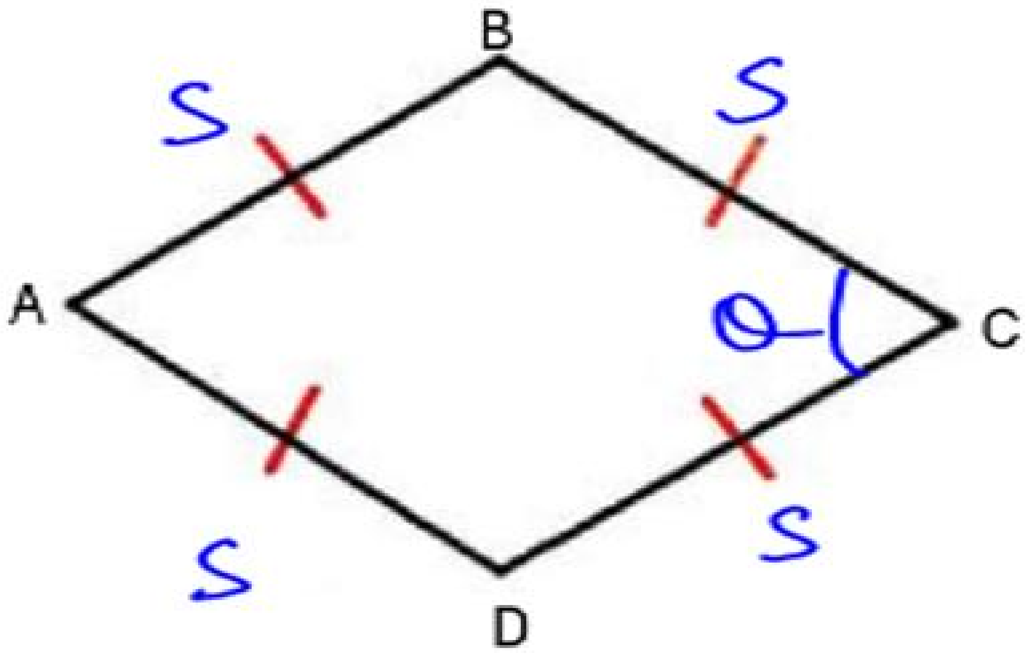
$$D_1^2 + D_2^2 = 4S^2$$

Parallelogram $\rightarrow$ $D_1^2 + D_2^2 = 2(x^2 + y^2)$

$= 2(2S^2)$

$$\boxed{D_1^2 + D_2^2 = 4S^2}$$

6.



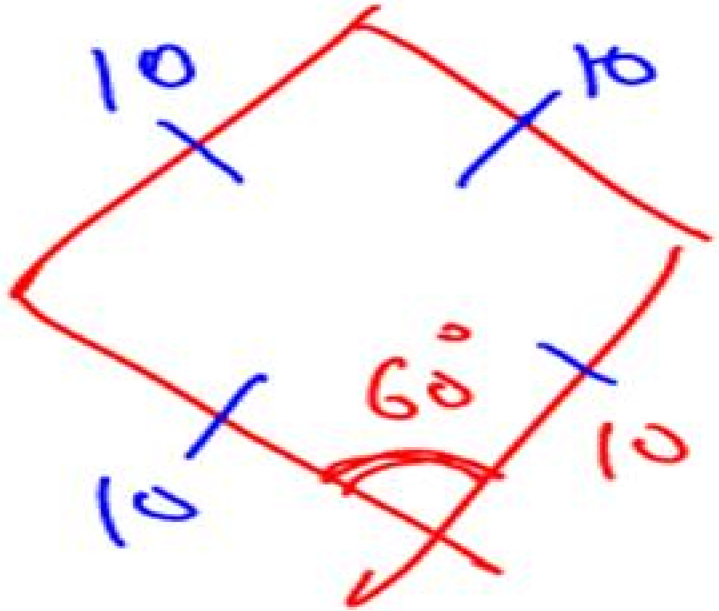
Perimeter of Rhombus = 4S

Area of Rhombus = $\frac{1}{2} D_1 D_2$

$= S^2 \cdot \sin\theta$

Where, θ is one of the angle of rhombus.

eg



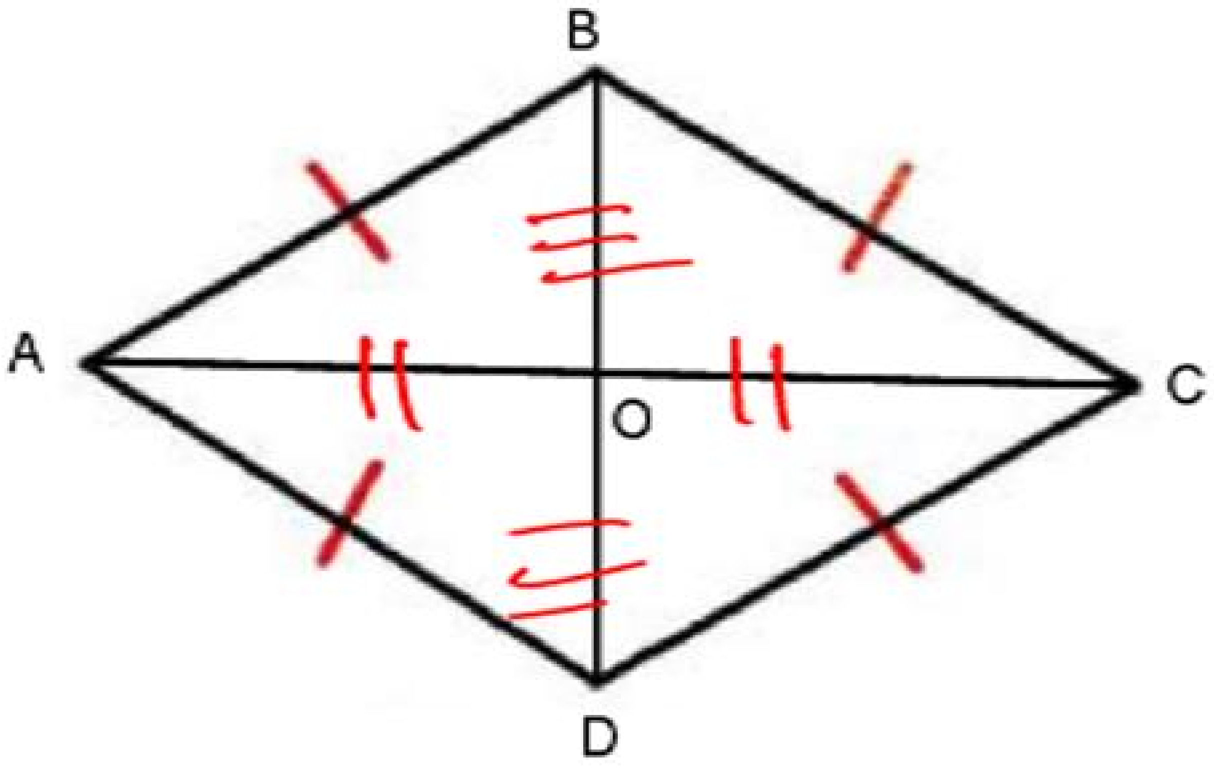
Area of Rhombus

→ $S^2 \sin\theta$

$\cancel{100}^{50} \cdot \frac{\sqrt{3}}{\cancel{2}}$

$50\sqrt{3}\ cm^2$

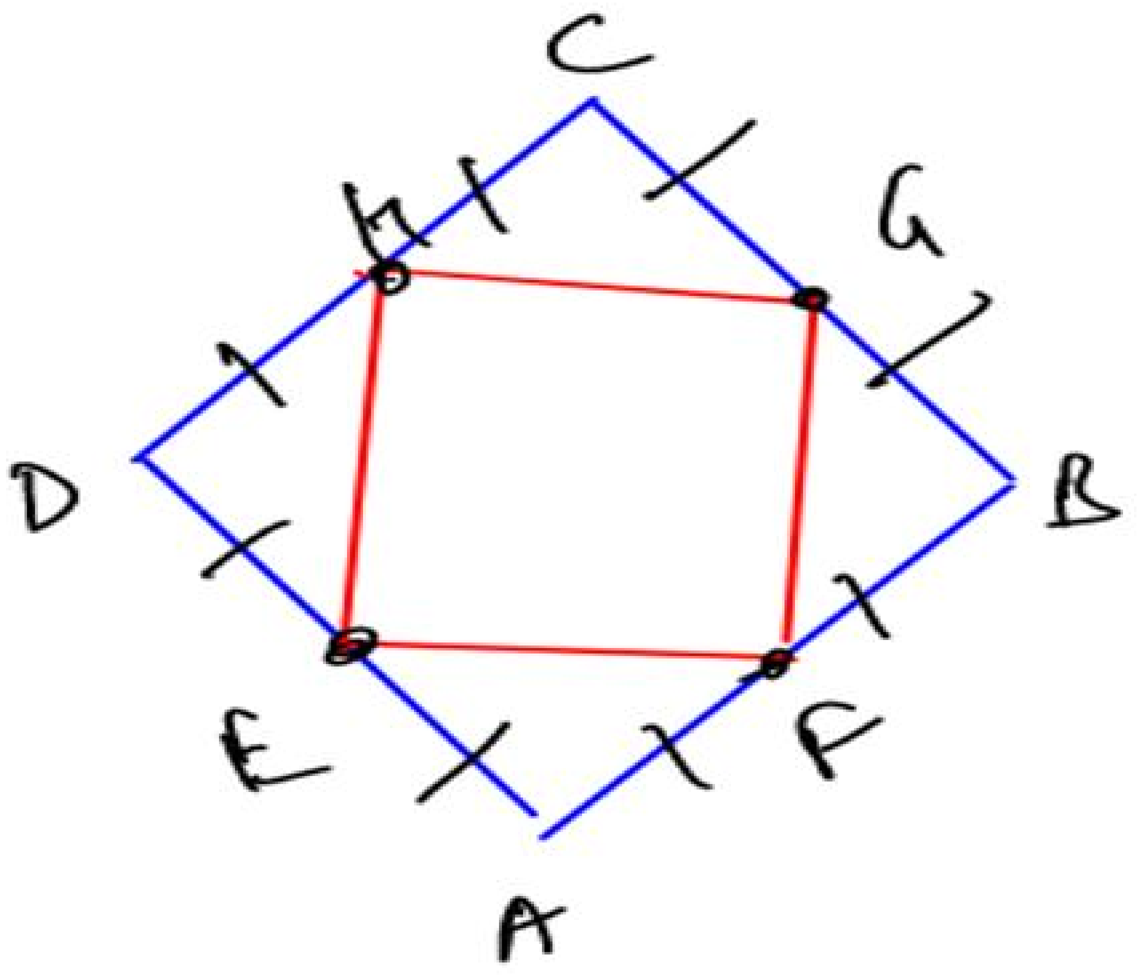
7.



$\Delta AOB \cong \Delta BOC \cong \Delta COD \cong \Delta DOA$

$\Delta BAO \cong \Delta BCO \cong \Delta DCO \cong \Delta DAO$

8. Figure formed by joining the mid point of all sides of a rhombus is RECTANGLE.

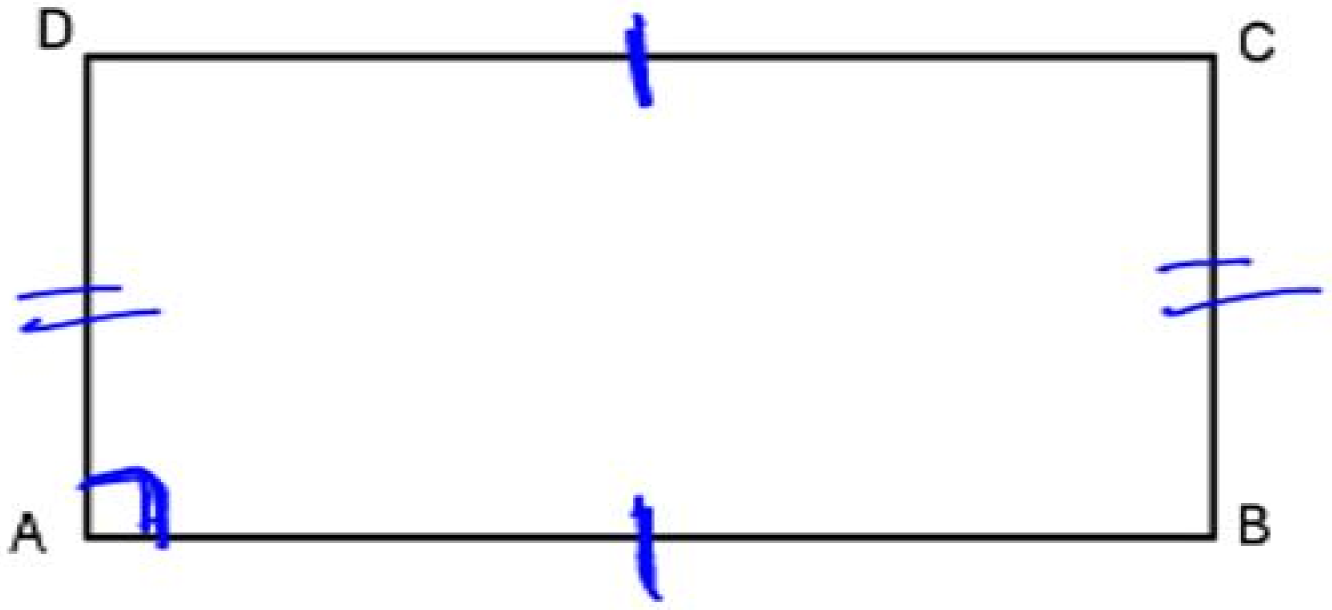


ABCD → Rhombus

EFGH → Rectangle

RECTANGLE

Def: A parallelogram in which one angle is 90°.

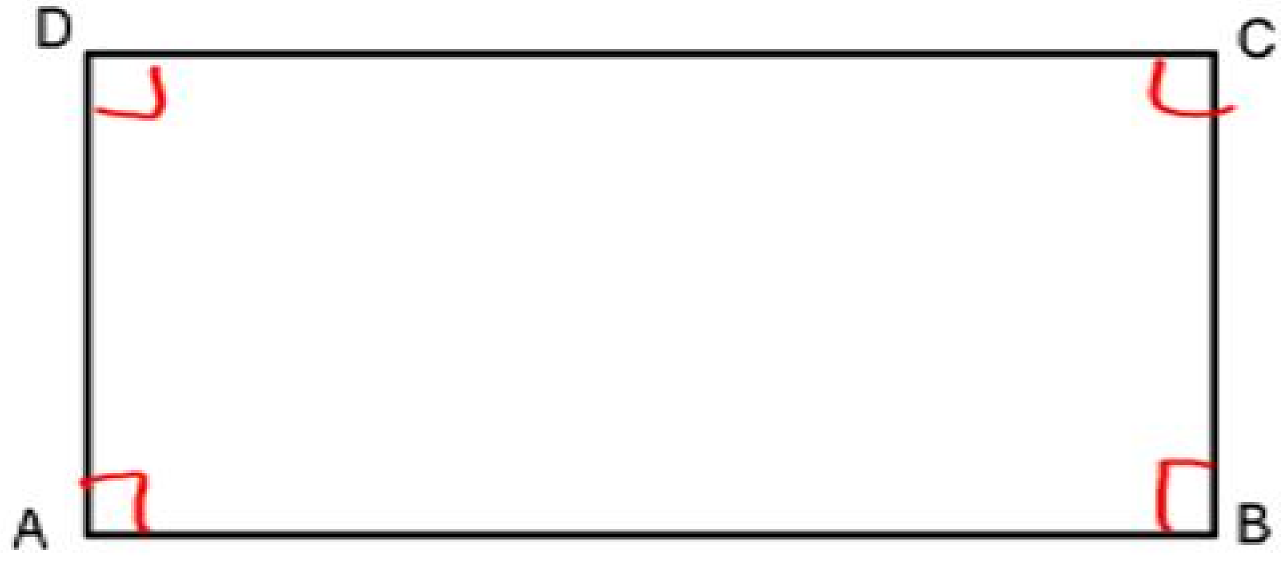


Parallelogram

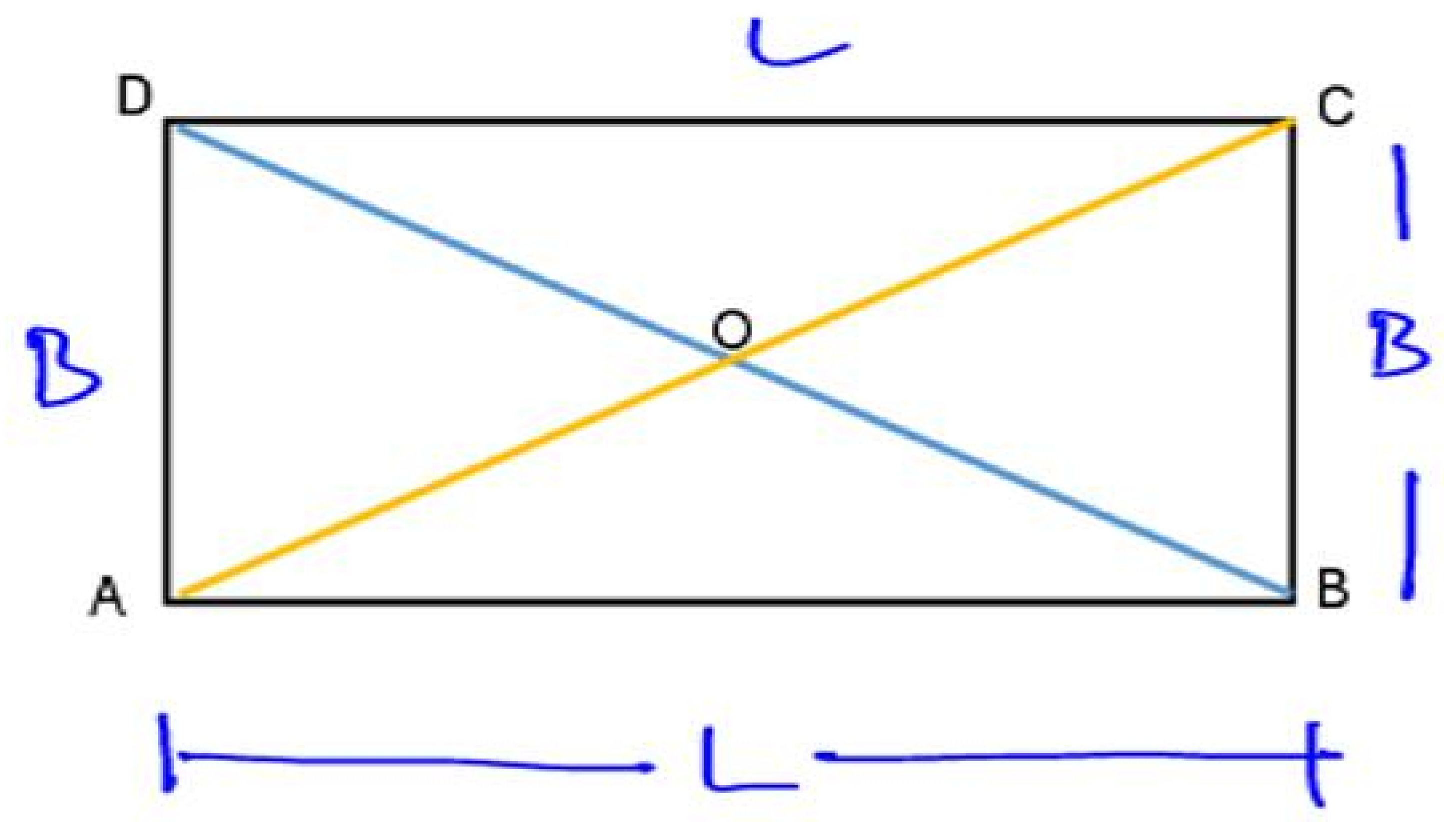
+ One Angle = 90°

PROPERTIES OF RECTANGLE

1. All angles of a rectangle are right angle.



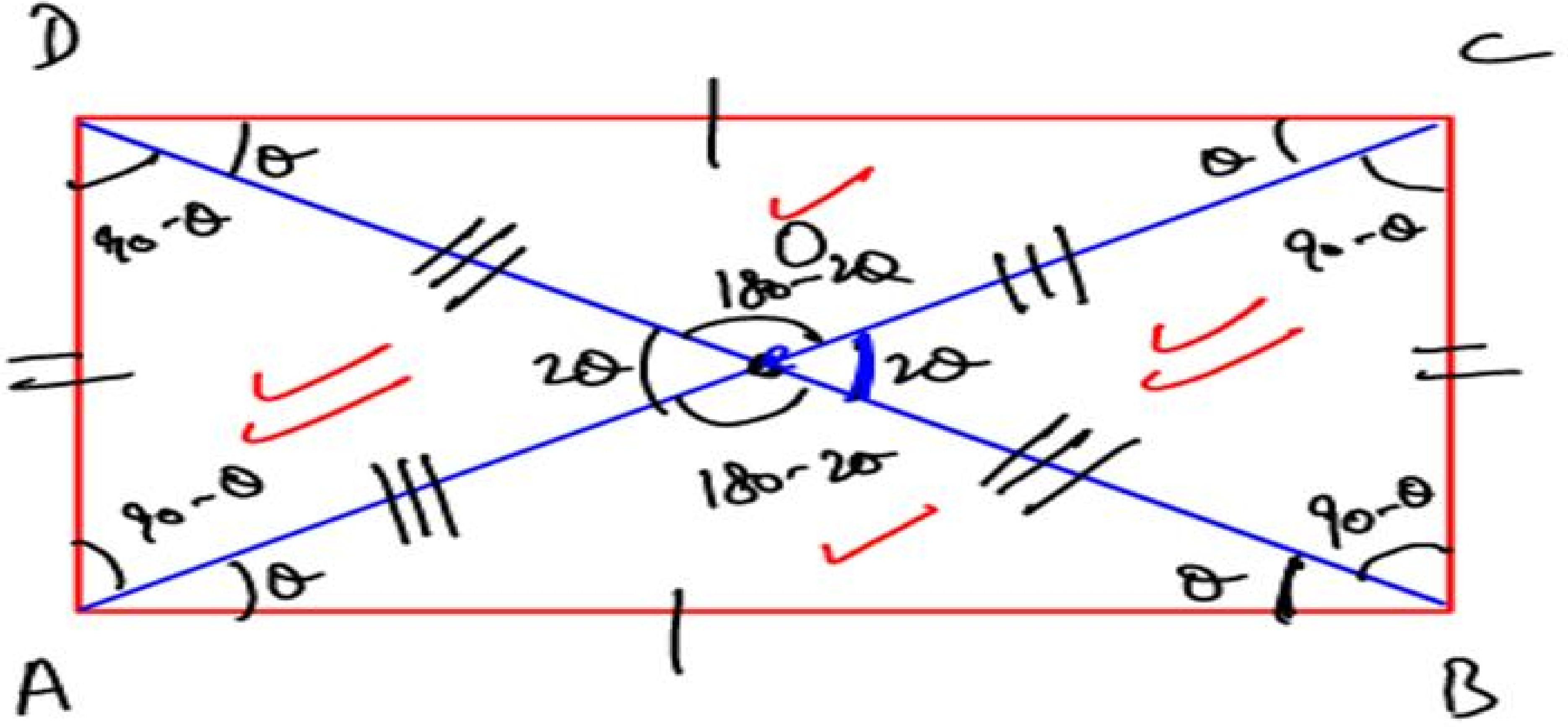
2. Diagonals of a rectangle are equal.



$$AC = \sqrt{L^2 + B^2}$$

$$BD = \sqrt{L^2 + B^2}$$

$$\boxed{D_1 = D_2}$$



$D_1 = D_2$

Diagonals are NOT Angle Bisector

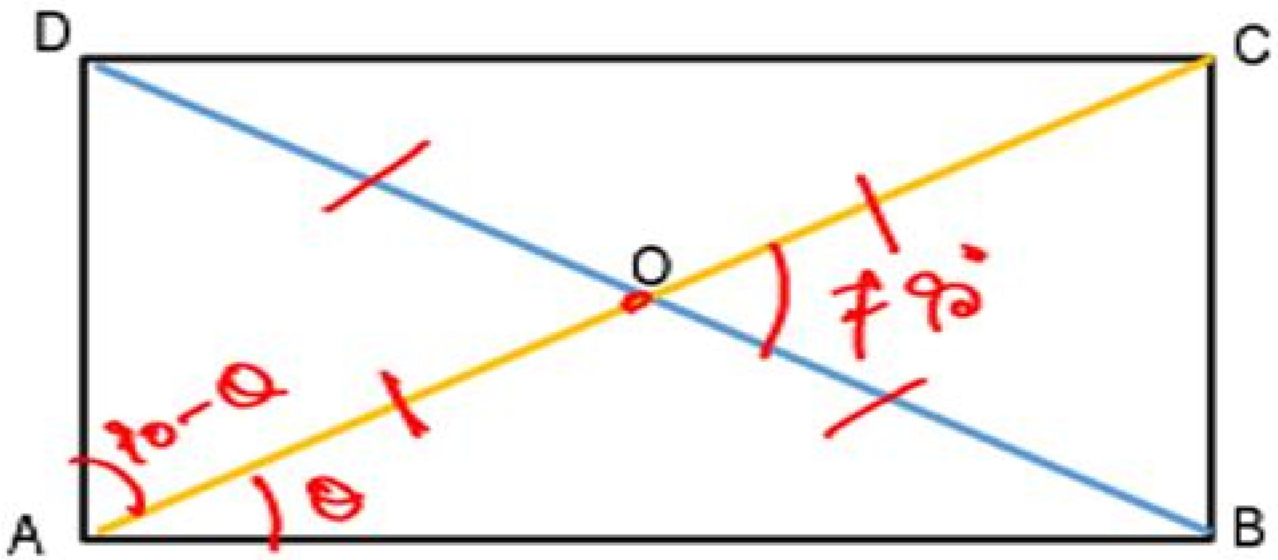
Diagonals bisect each other but not at 90°

$\Delta AOB \cong \Delta COD$

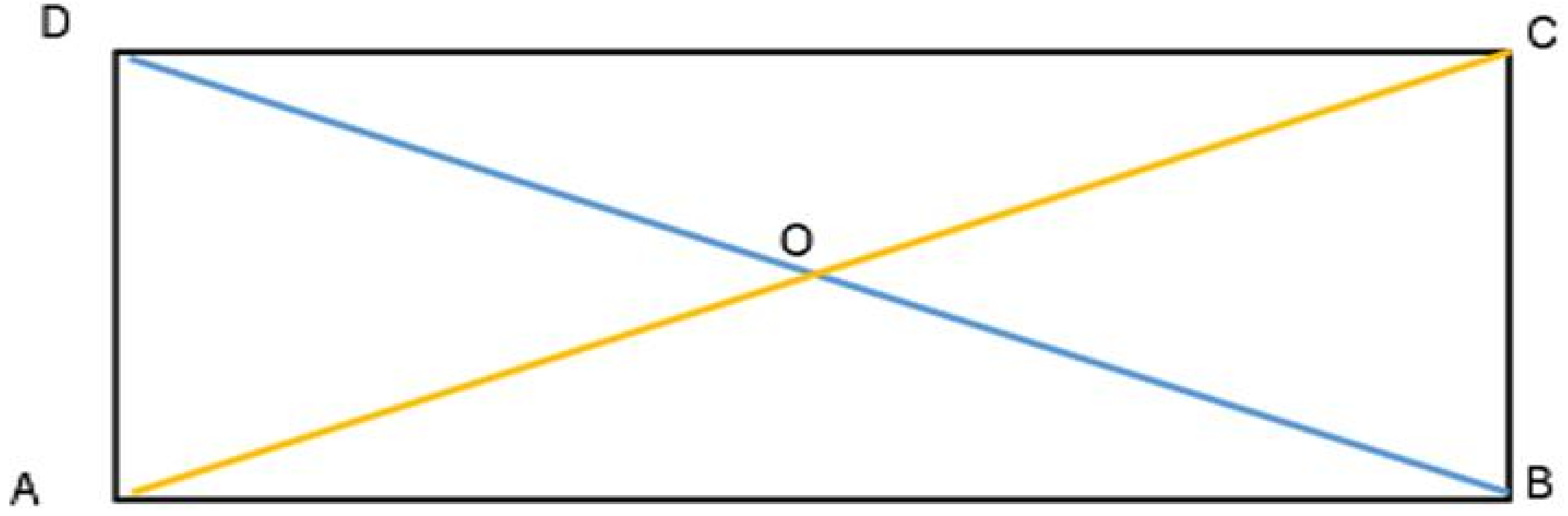
$\Delta DOA \cong \Delta BOC$

3. (i) Diagonals of a rectangle bisect each other but not necessarily at 90°.

(ii) Diagonals of a rectangle need not be angle bisector.



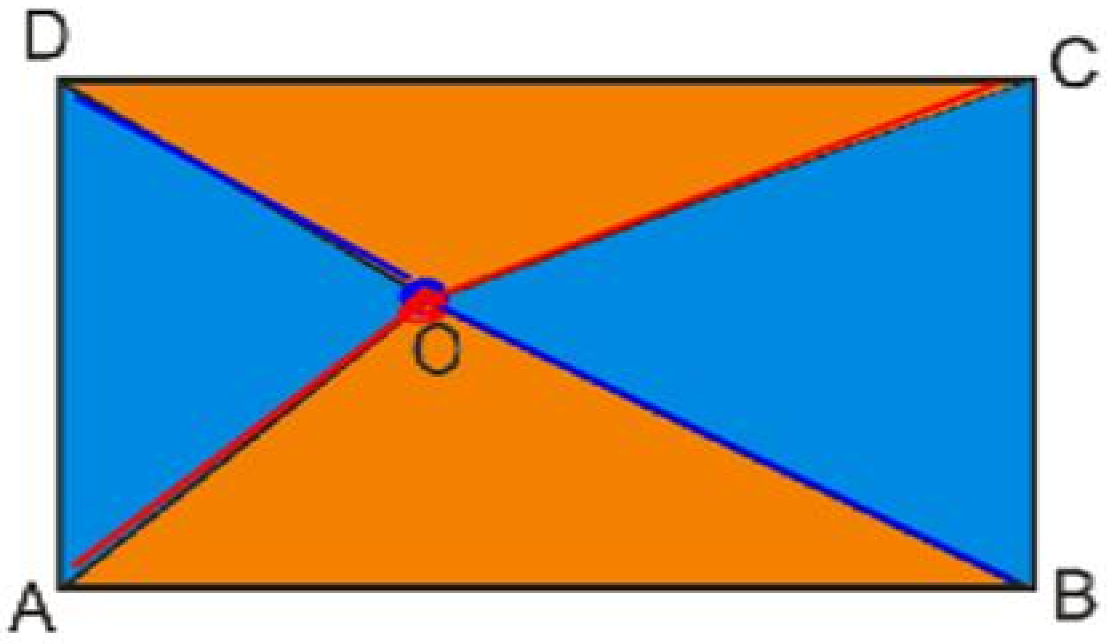
UNDERSTANDING OF A RECTANGLE FIGURE



V.imp

4. If O is any point in the interior of rectangle ABCD, then

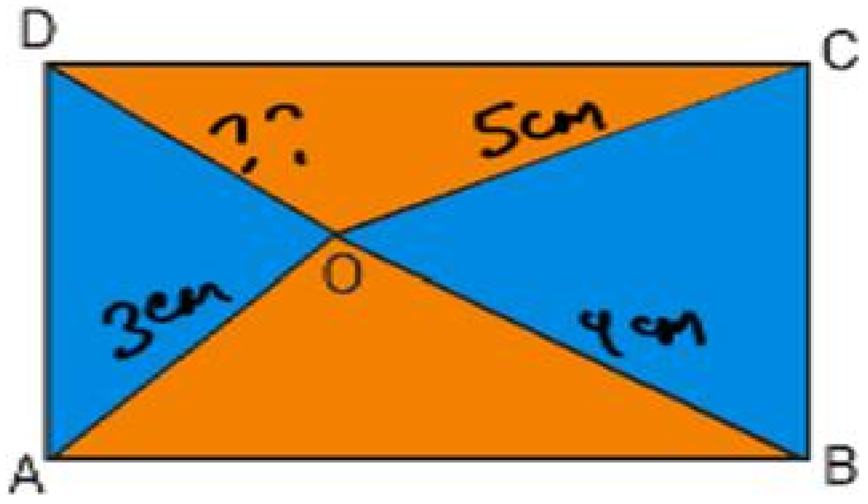
$$(OA)^2 + (OC)^2 = (OB)^2 + (OD)^2$$



Reason :- Pythagoras

PYQ of SSC

Eg. ABCD is a rectangle and O is only point in the interior of rectangle ABCD. If OA = 3 cm, OB = 4 cm, OC = 5 cm, find the value of OD = ??



$$(AO)^2 + (CO)^2 = (BO)^2 + (DO)^2$$

$$9 + 25 = 16 + (DO)^2$$

$$18 = (DO)^2$$

$$DO = 3\sqrt{2}\text{ cm}$$

imp

5. Perimeter of rectangle (P) $= 2(L + B)$

Area of rectangle (A) $= L \cdot B$

Diagonal of rectangle (D) $= \sqrt{L^2 + B^2}$

Important relationship between P, A & D of rectangle.

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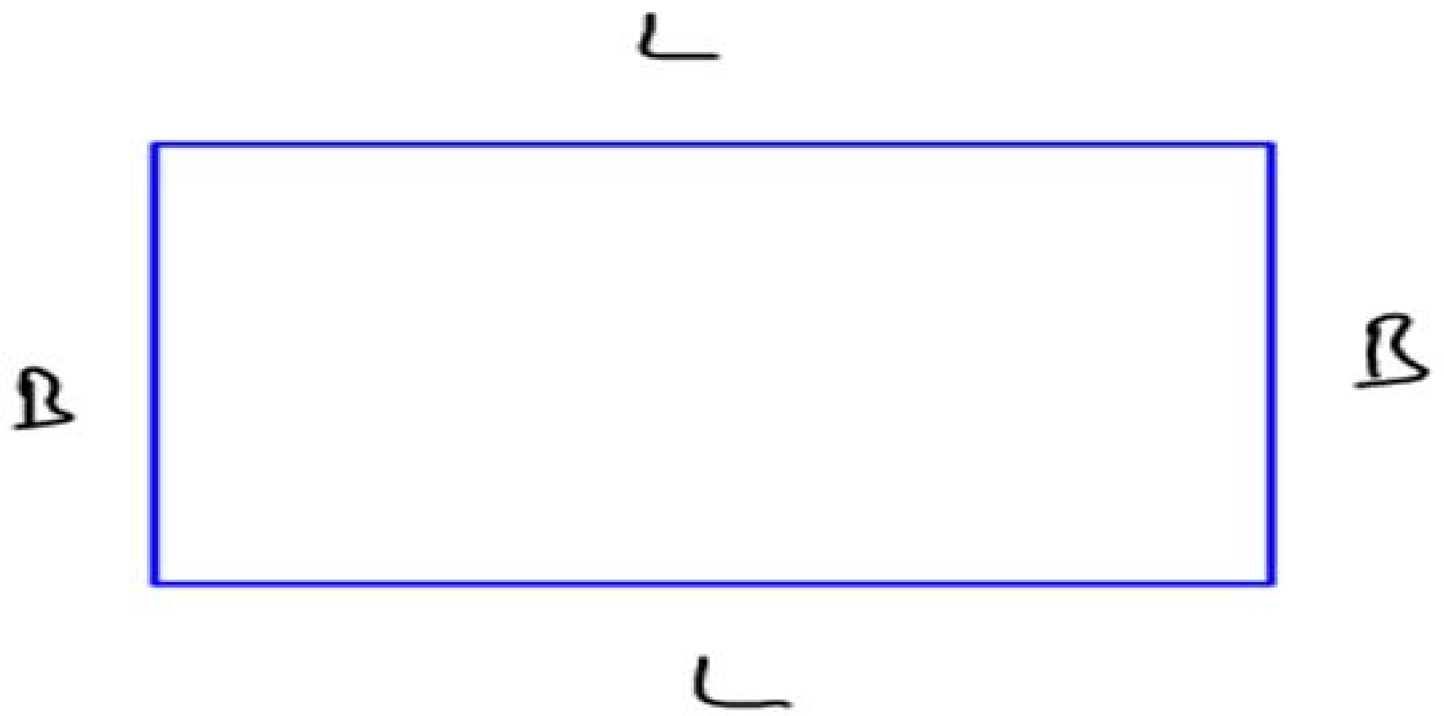
$$P^2 = 4\left(D^2 + 2A\right)$$



Eg. If diagonal of rectangle is 14 cm and its area is 68 cm^2. Find its perimeter.

$$\boxed{P^2 = 4(D^2 + 2A)}$$

$$P^2 = 4(196 + 2\cdot 68)$$

$$P^2 = 4(332)$$

$$P = \sqrt{4\cdot 4\cdot 83}$$

$$\boxed{P = 4\sqrt{83}\ \text{cm}}$$

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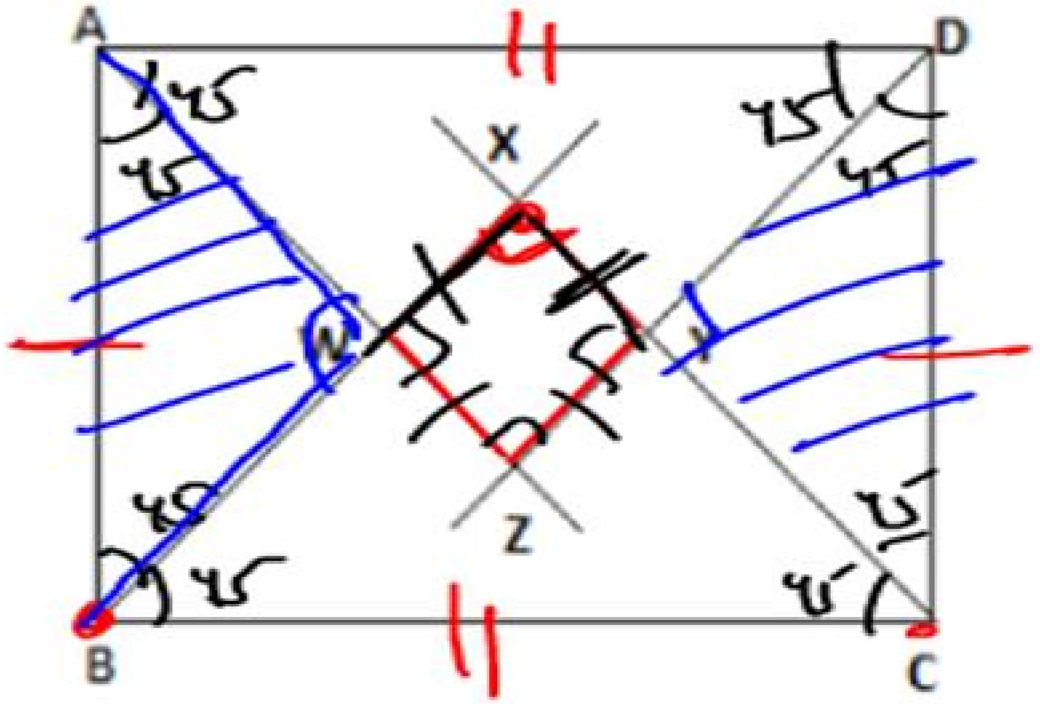
6. Angle bisectors of a rectangle forms a square.



WXYZ → Square

Reason

ΔBXC (Isosceles Δ)

$BX = XC$ — (1)

ΔAWB (Isosceles Δ)

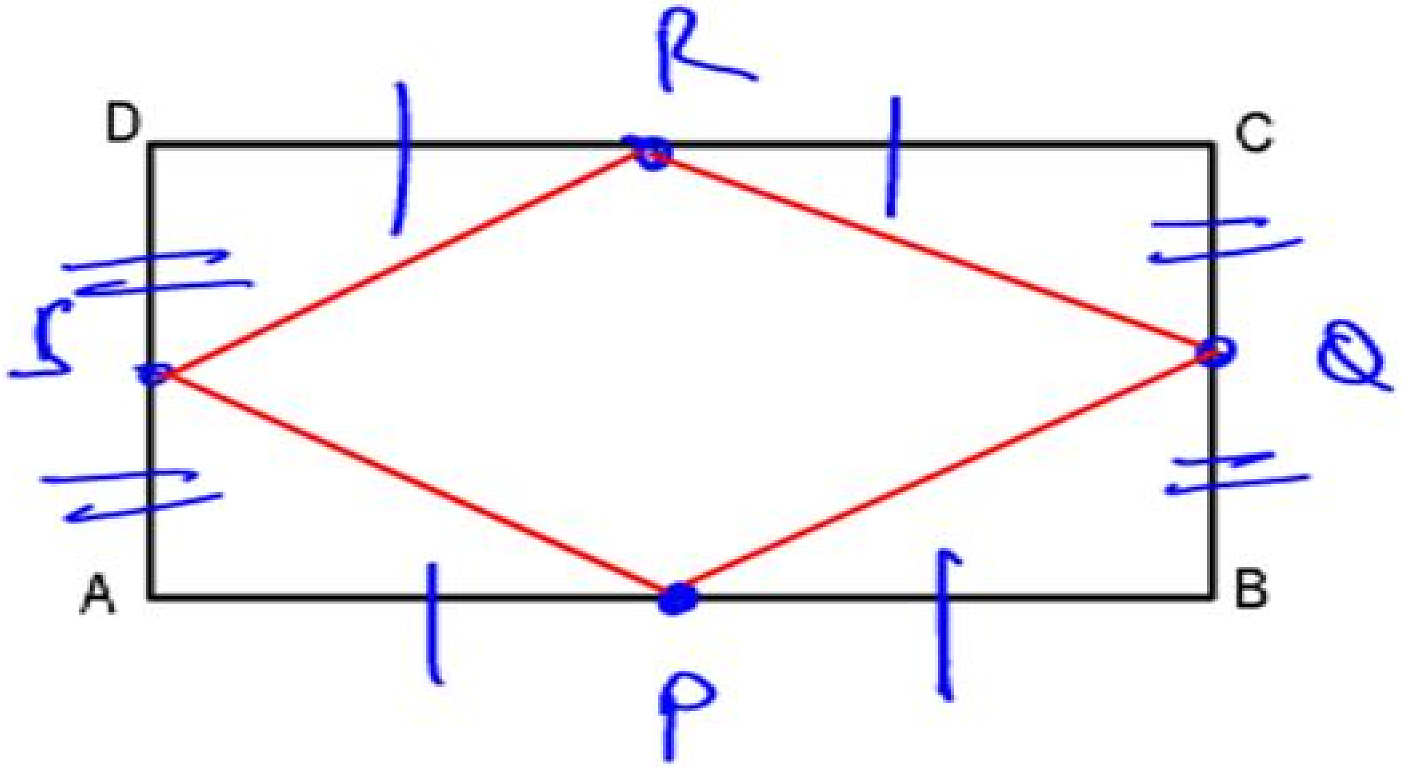
$BW = AW$

$BW = CY$ — (2)

(1) − (2)

$XW = XY$

7. Figure formed by joining the mid-point of all sides of a rectangle is rhombus.



SQUARE

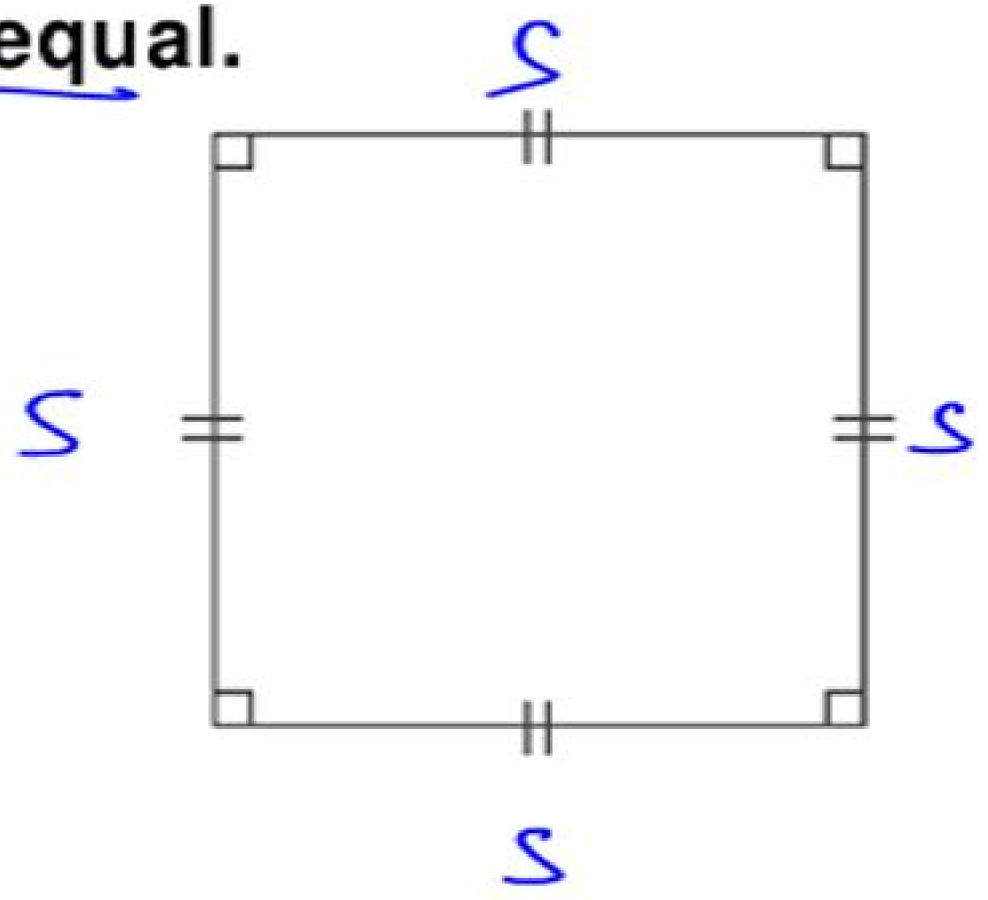
Def :

(1) Quadrilateral + all sides are equal + all angles are equal.

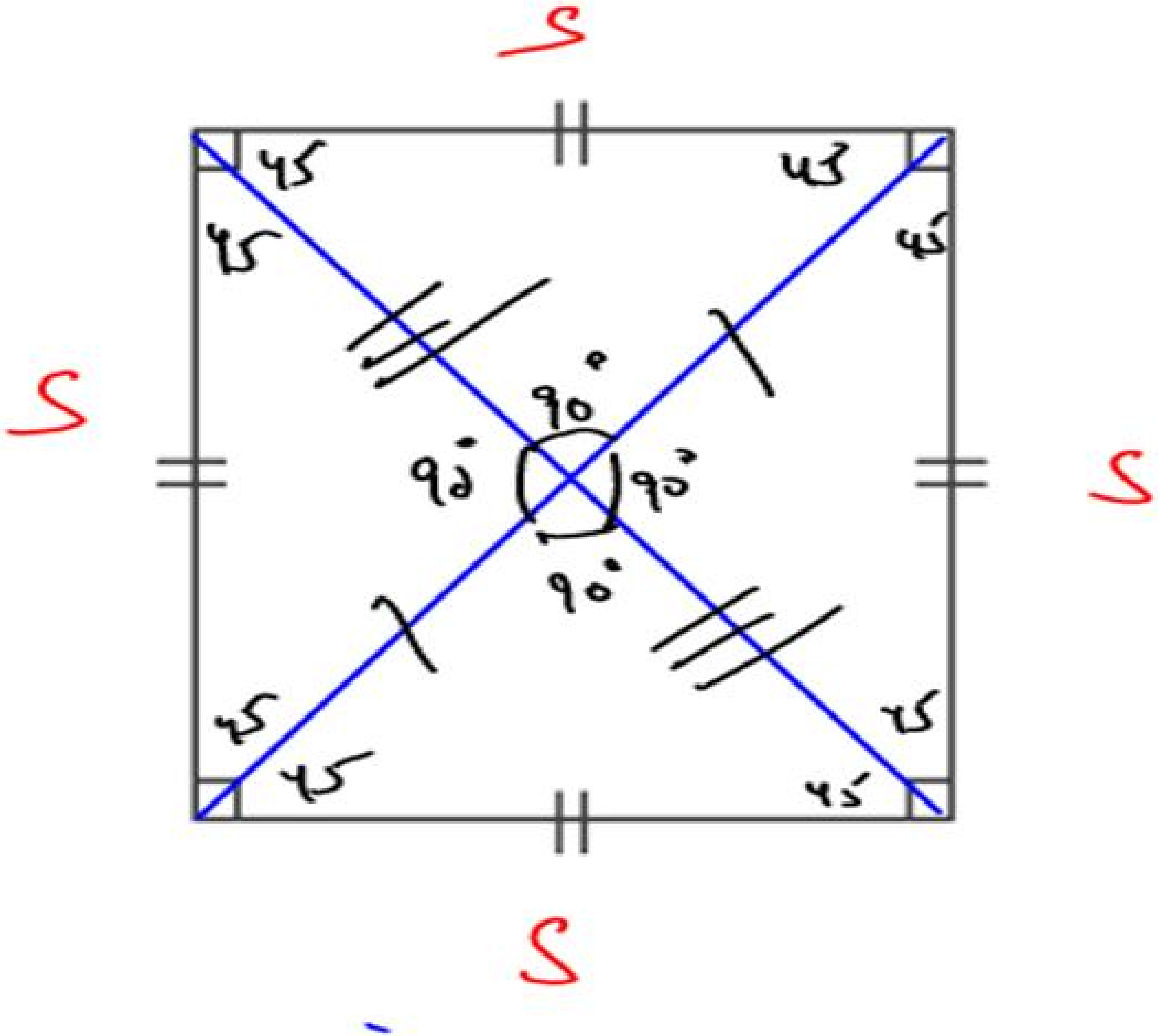
(2) Regular polygon of 4 sides.

(3) Rectangle in which adjacent sides are equal.

(4) Rhombus + one angle = $90°$



DETAILED ANALYSIS OF SQUARE FIGURE



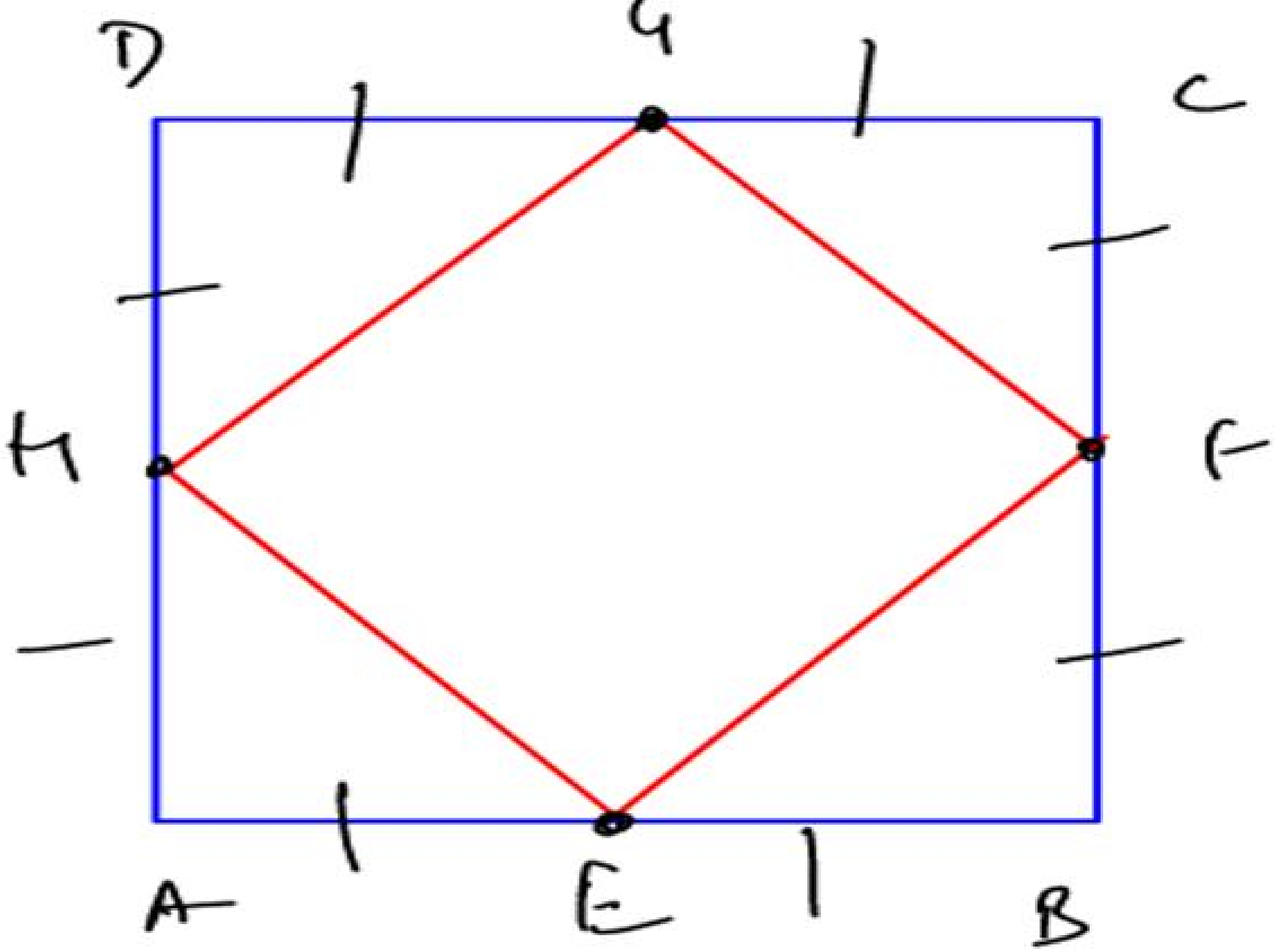
Perimeter = 4S

$$\text{Area} = S^2$$

$$= \frac{(\text{Diagonal})^2}{2}$$

$$\text{Diagonal} = \sqrt{2}\cdot S$$

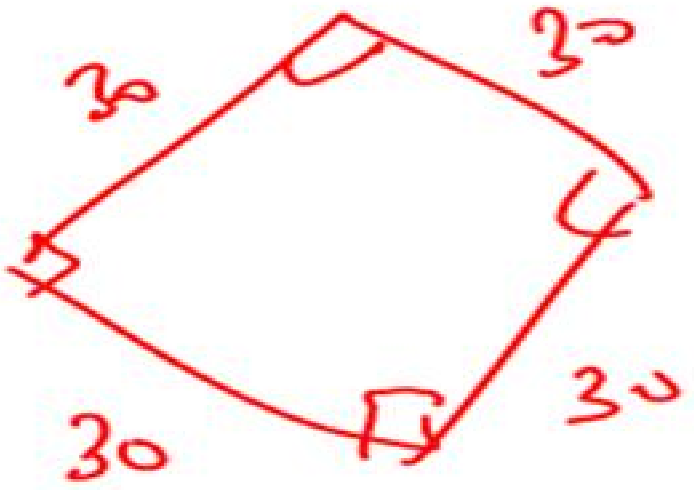
Figure formed by joining the mid-points of all sides of a square is a square.



For a given perimeter of a quadrilateral, square will have maximum area.

Eg. A quadrilateral whose perimeter = 120 cm

Find maximum area of quadrilateral .



$$\text{Area} = (30)^2$$
$$= 900\ \text{cm}^2$$

Property	Rhombus	Rectangle	Square
Diagonals bisect each other	✓	✓	✓
Diagonals bisect each other at 90°	✓	✗	✓
Diagonals are angle bisector	✓	✗	✓
Diagonals are equal	✗	✓	✓

Figure formed by joining mid-points of all sides of a:

Quadrilateral	$\rightarrow$	Parallelogram
Parallelogram	$\rightarrow$	Parallelogram
Rhombus	$\rightarrow$	Rectangle
Rectangle	$\rightarrow$	Rhombus
Square	$\rightarrow$	Square

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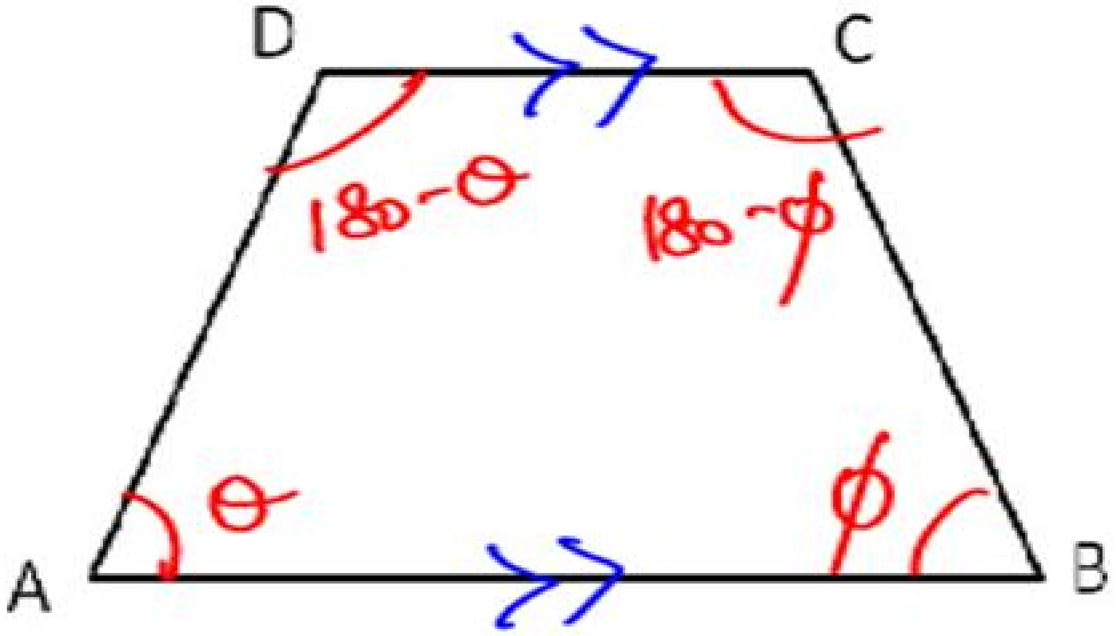
TRAPEZIUM

Def: A quadrilateral in which one pair of side is parallel.



Def → Quad + one pair of side is ||

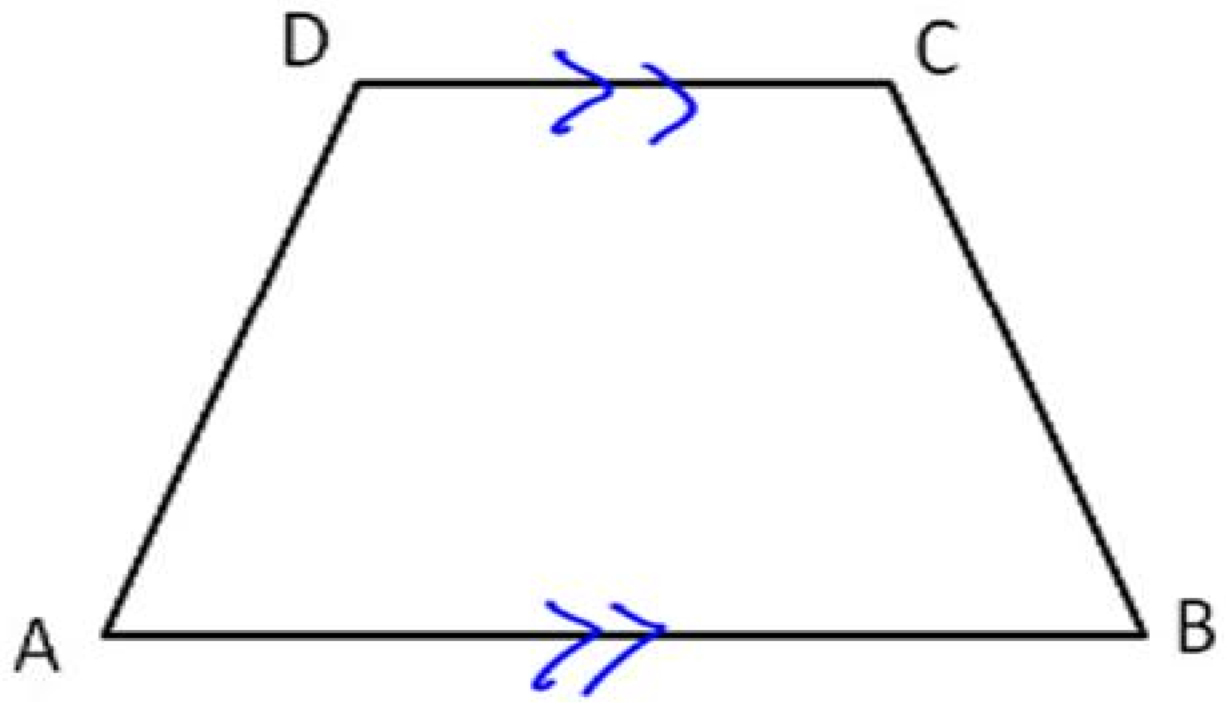
* If ABCD is a trapezium

AB || CD

$$\left\{ \begin{array}{l} \angle A + \angle D = 180^\circ \\ \angle B + \angle C = 180^\circ \end{array} \right.$$

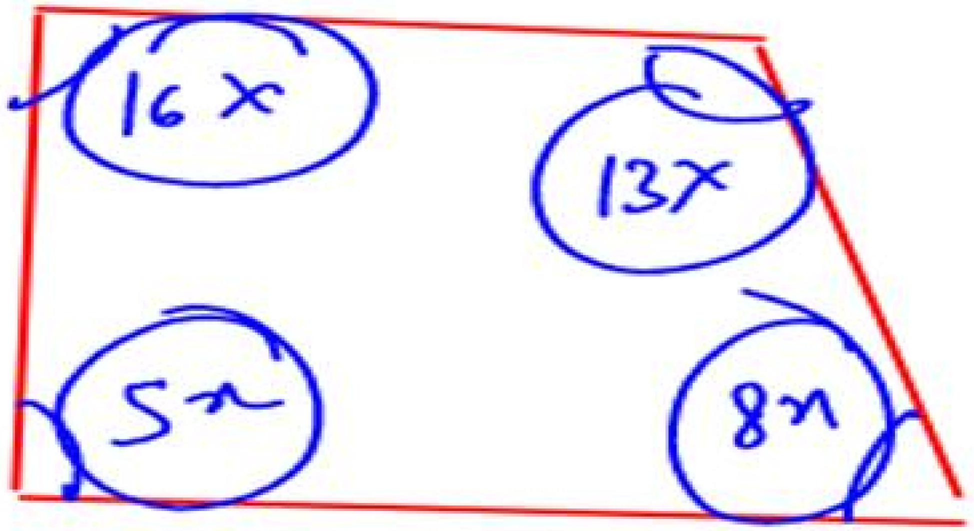
1. In a trapezium ABCD, if AB || CD, then

$\angle A + \angle D = \angle B + \angle C = 180°$



Eg. If 4 angles of a quadrilateral are in the ratio 5 : 8 : 13 : 16, then what can be the name of the quadrilateral?

(a) Parallelogram
(b) Rectangle
(c) Trapezium
(d) None of these



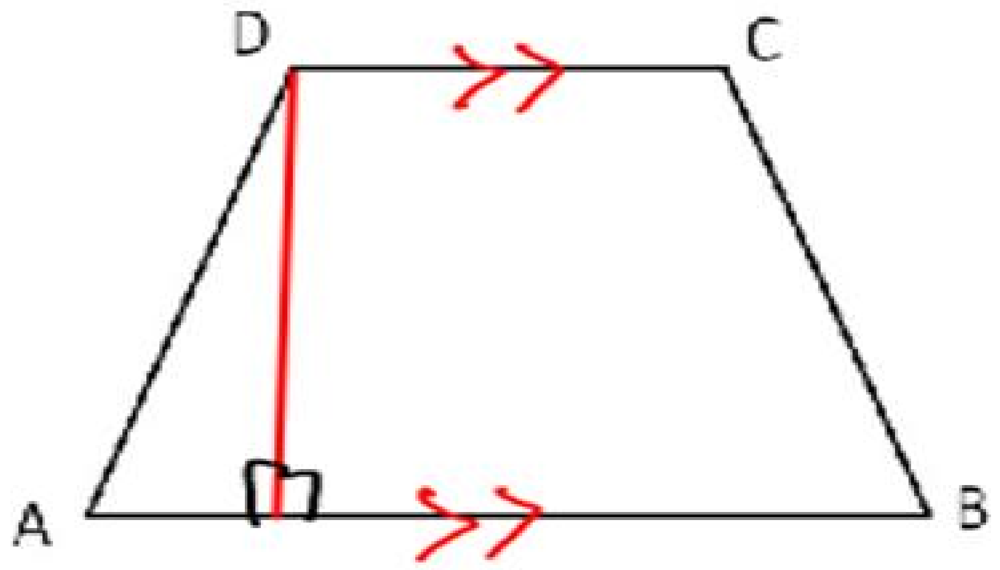
$5x + 16x = 21x$

$8x + 13x = 21x$

→ Trapezium

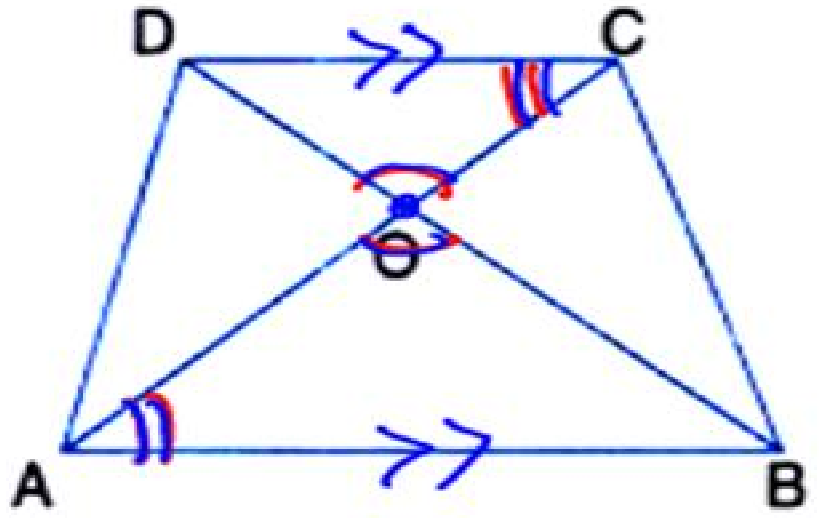
2. Area of trapezium = ½ × (Sum of parallel sides) × Distance between them

$$= \frac{1}{2} \times (AB + CD) \times H$$



3. If diagonals AC and BD of a trapezium intersect each other at O, where AB| | CD, then ΔAOB ~ ΔCOD.

v. imp



$\Delta AOB \sim \Delta COD$

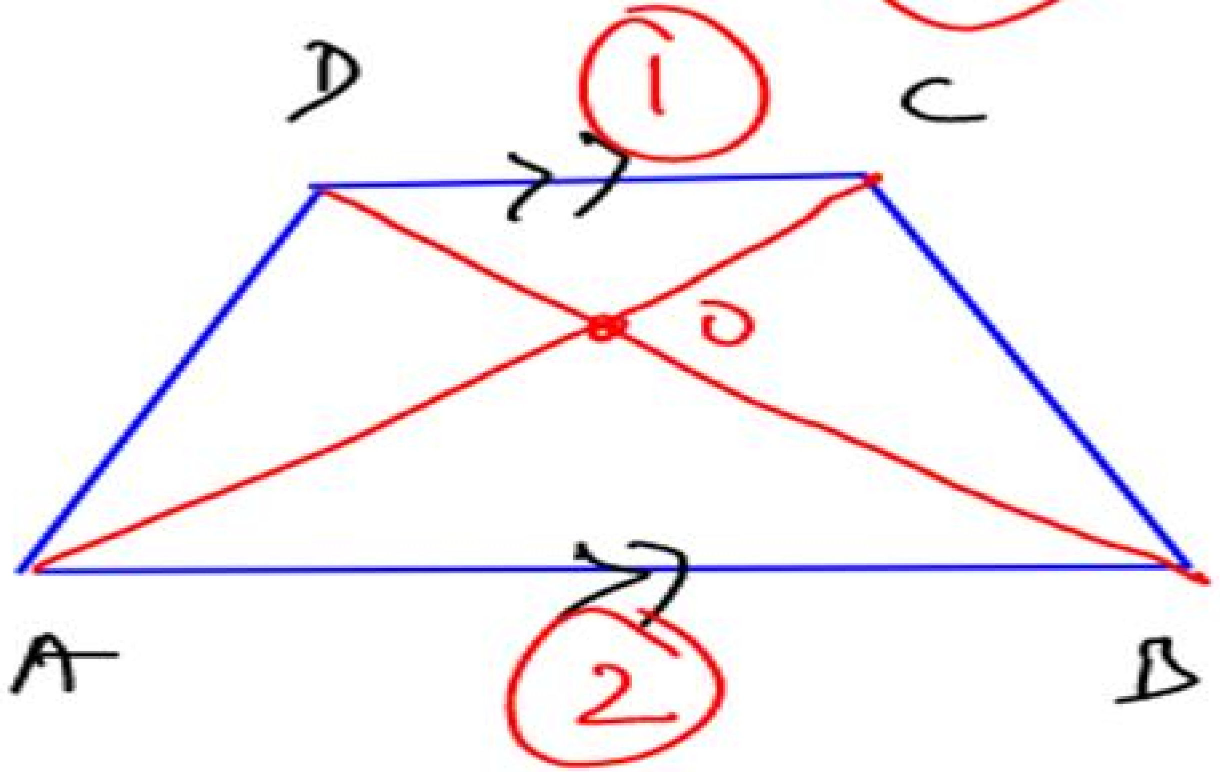
(i) $\dfrac{AO}{CO} = \dfrac{BO}{DO} = \dfrac{AB}{CD}$

(ii) $\dfrac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \left(\dfrac{AB}{CD}\right)^2$

Eg. In a trapezium ABCD (AB || CD), diagonals AC & BD intersect each other at O and AB = 2 CD.

Find : $\dfrac{Area\ of\ \triangle AOB}{Area\ of\ \triangle COD}$. $= \dfrac{4}{1}$

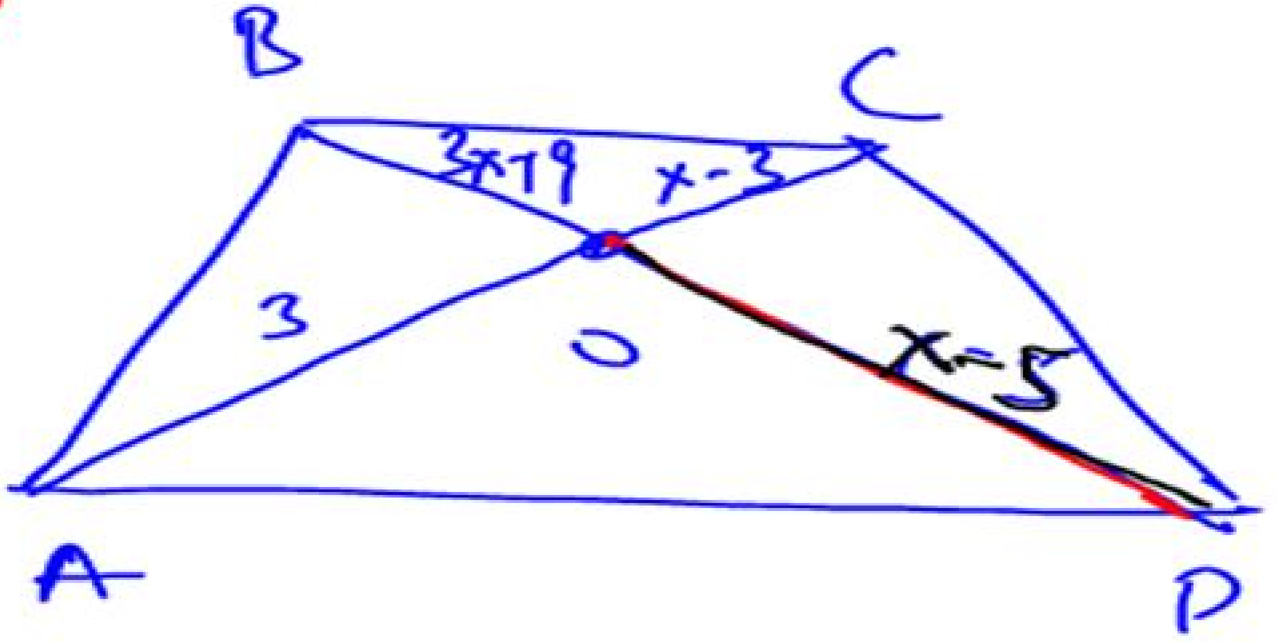
$\dfrac{AB}{CD} = \dfrac{2}{1}$



Eg. ABCD is a trapezium where AD || BC. The diagonals AC and BD intersect each other at a point O. If AO = 3, CO = x – 3 , BO = 3x – 19 and DO = x – 5, the value of x is:

(a) -8, 9 (b) 8, -9

(c) -8, -9 (d) 8, 9



$x > 5$

Concept

$$\frac{3}{x-3} = \frac{x-5}{3x-19}$$

$$9x - 57 = x^2 - 8x + 15$$

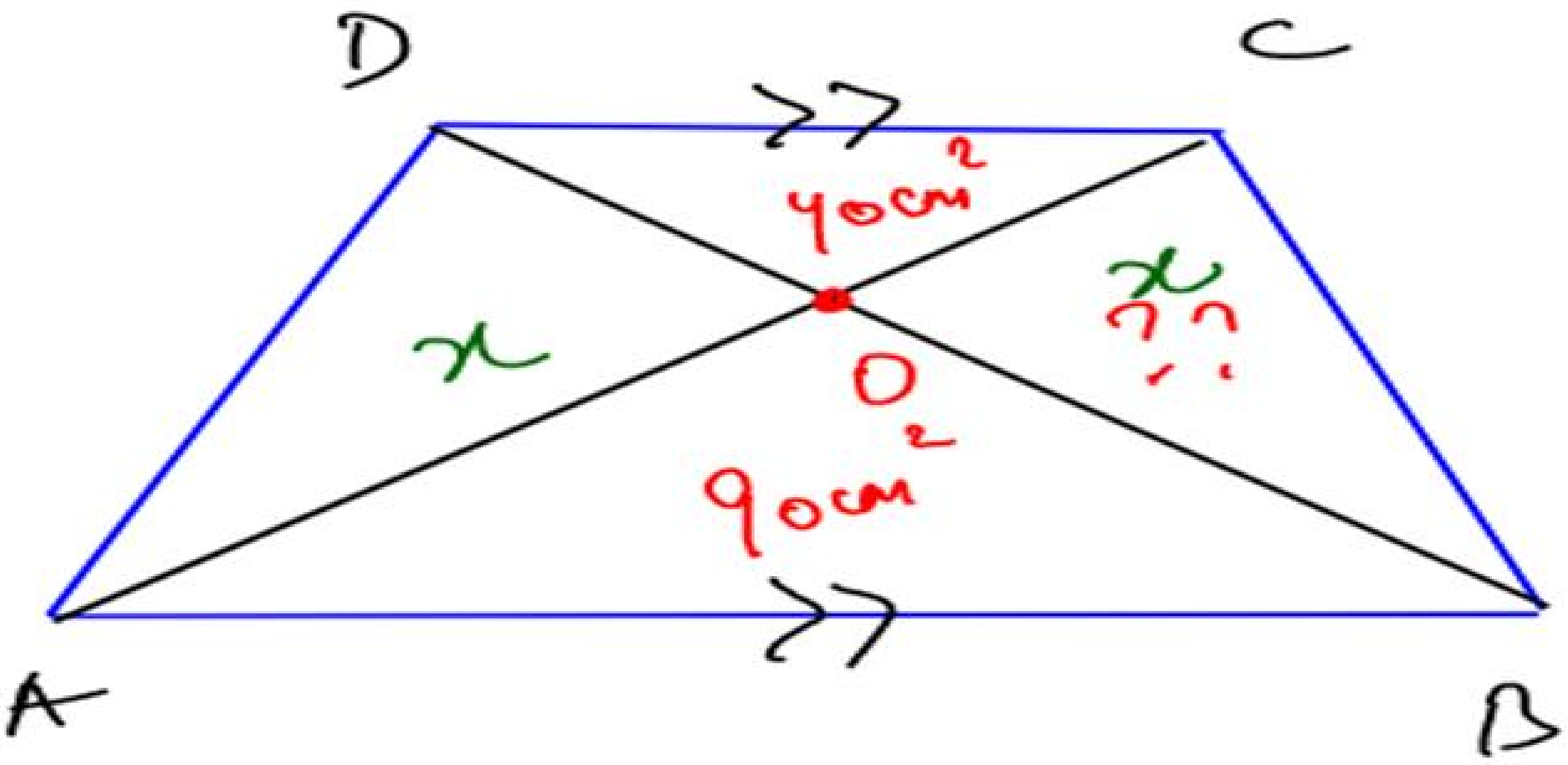
$$x^2 - 17x + 72$$

$$x = 8, 9$$

V-imp

Eg. ABCD is a trapezium, AB || CD and AC and BD intersect each other O. If area of $\Delta AOB = 90\ cm^2$ and area of $\Delta COD = 40\ cm^2$. Find area of ΔBOC.

Time → 90sec



Ist

In Quad

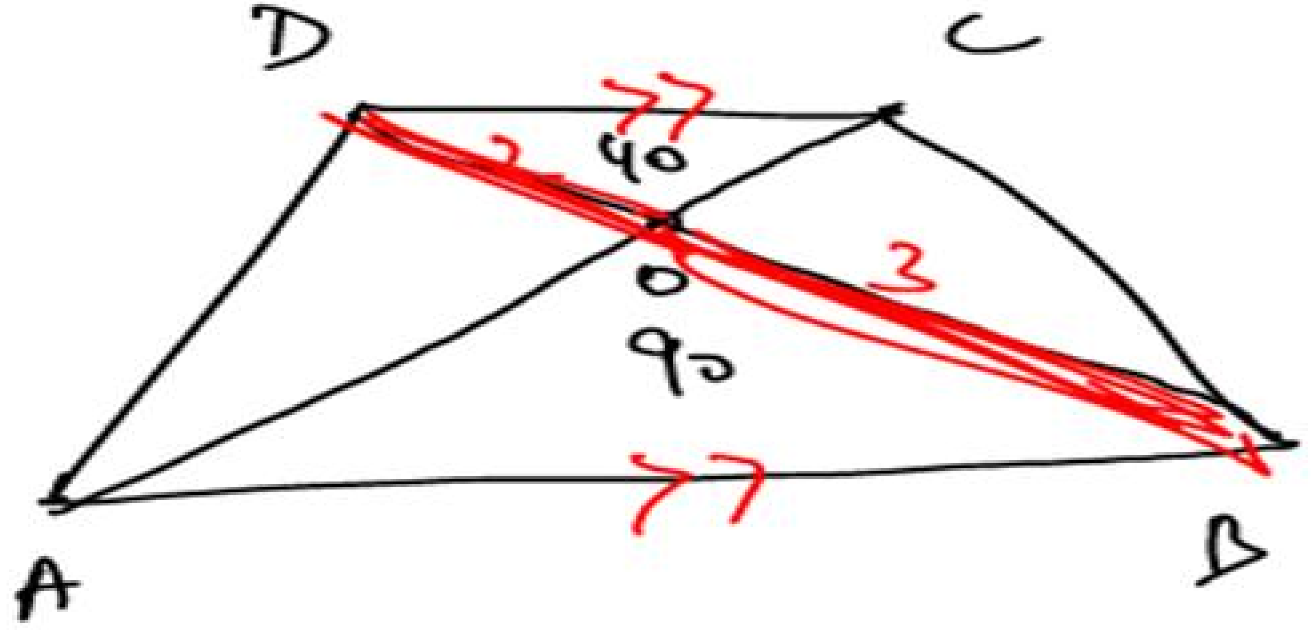
area of $\Delta ABD = \Delta ABC$

area of $\Delta AOD = \Delta BOC$

$40 \cdot 90 = x \cdot x$

$x^2 = 3600$

$x = 60$



$$\frac{\text{area of } \Delta AOB}{\text{area of } \Delta COD} = \frac{90}{40}$$

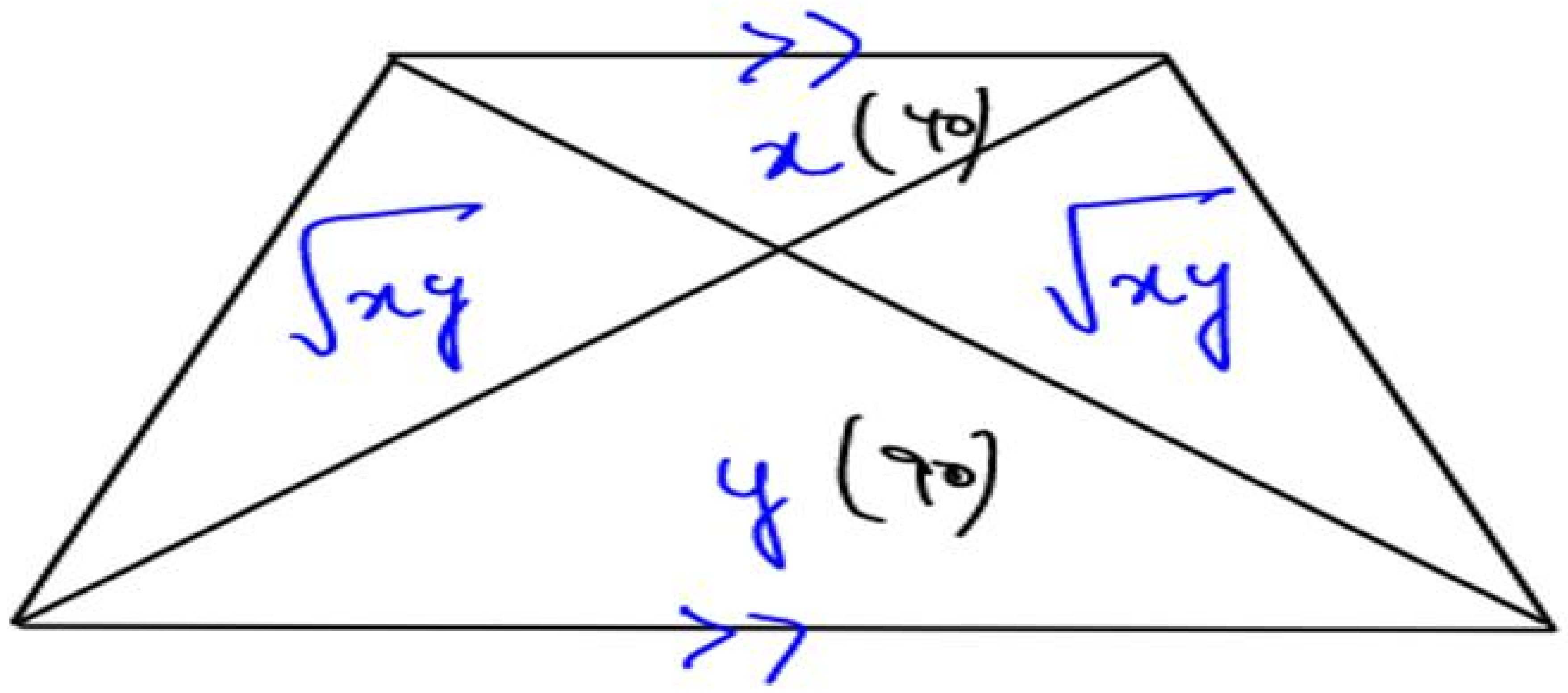
$$\left(\frac{BO}{DO}\right)^2 = \frac{9}{4}$$

$$\frac{BO}{DO} = \frac{3}{2}$$

ΔBCD

$2 \rightarrow 40\,cm^2$

$3 \rightarrow 60\,cm^2$

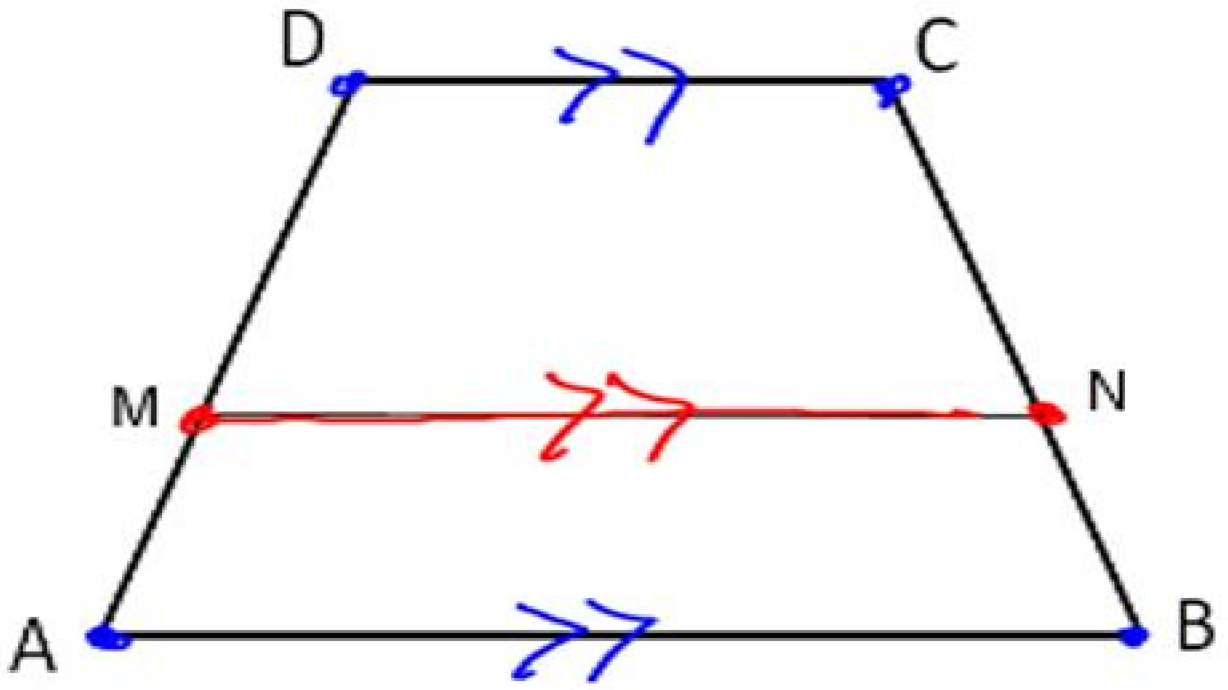


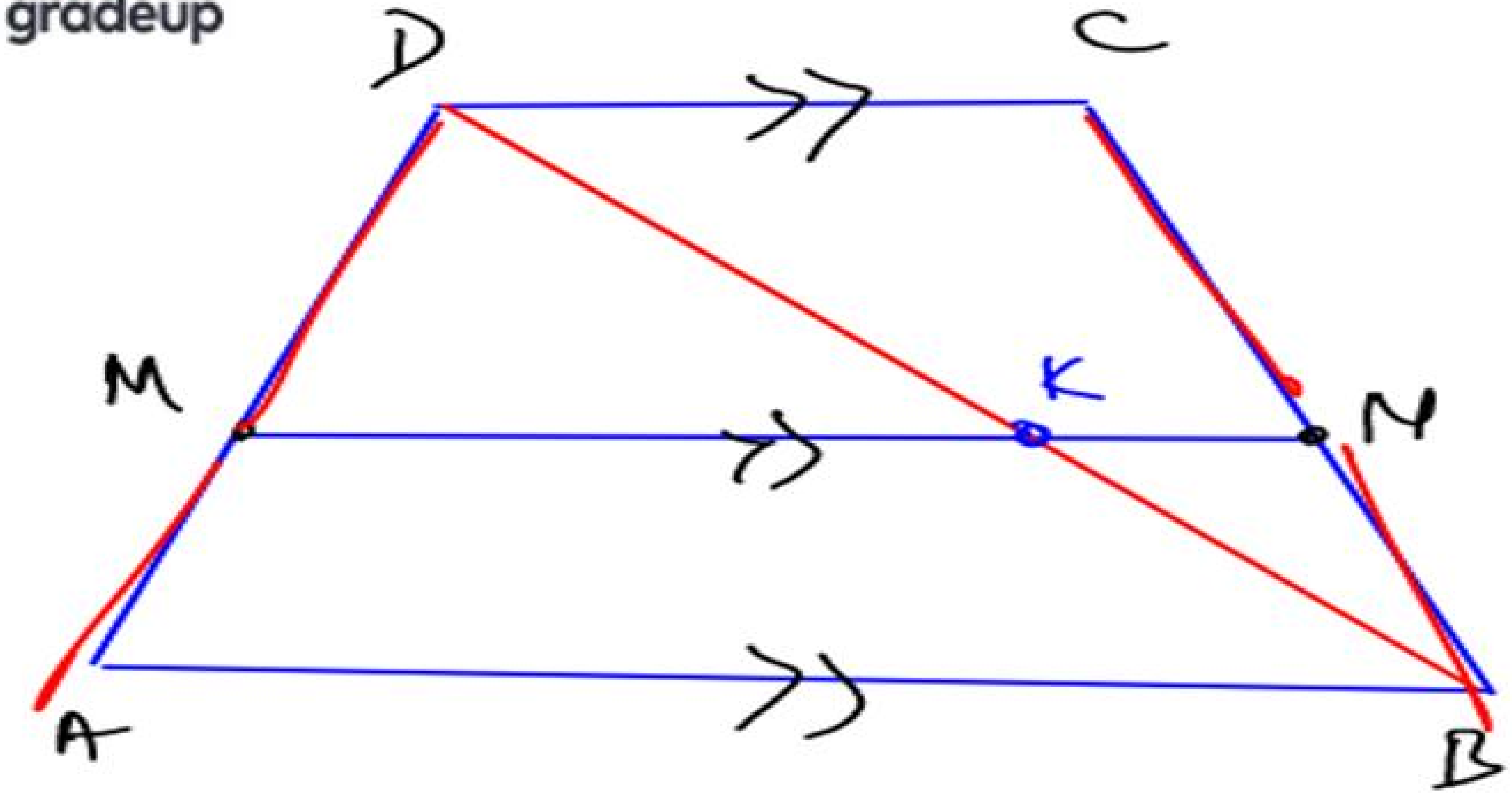
x (40)
√xy
√xy
y (90)

4. ABCD is a trapezium where AB || CD.

M, N are points on AD and BC in such a way that MN || AB.

then $\dfrac{DM}{MA} = \dfrac{CN}{NB}$





Given ABCD is a trapezium

$AB \parallel CD$

$MN \parallel AB$

To prove $\dfrac{DM}{MA} = \dfrac{CN}{NB}$

Const → Join DB meeting MN at K

Proof In ΔDAB

$MK \parallel AB$

$$\boxed{\frac{DM}{MA} = \frac{DK}{KB}} \quad — (1)$$

In ΔBCD $\quad KN \parallel CD$

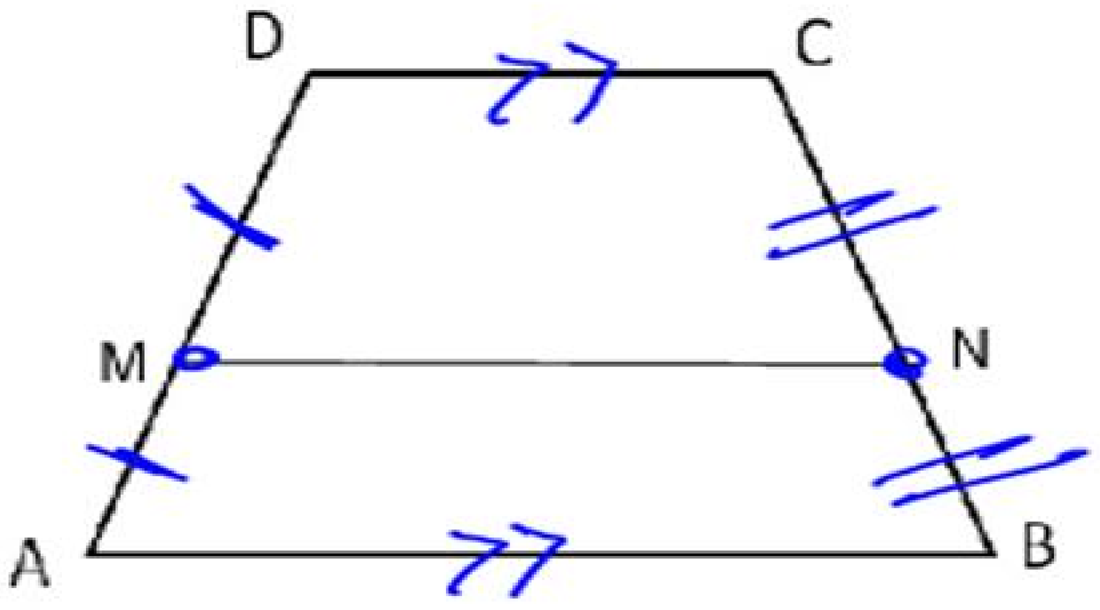
$$\frac{BN}{NC} = \frac{BK}{KD}$$

$$\boxed{\frac{NC}{NB} = \frac{KD}{KB}} \quad — (2)$$

5. ABCD is a trapezium where AB || CD.

M, N are mid-points on AD and BC

v. imp

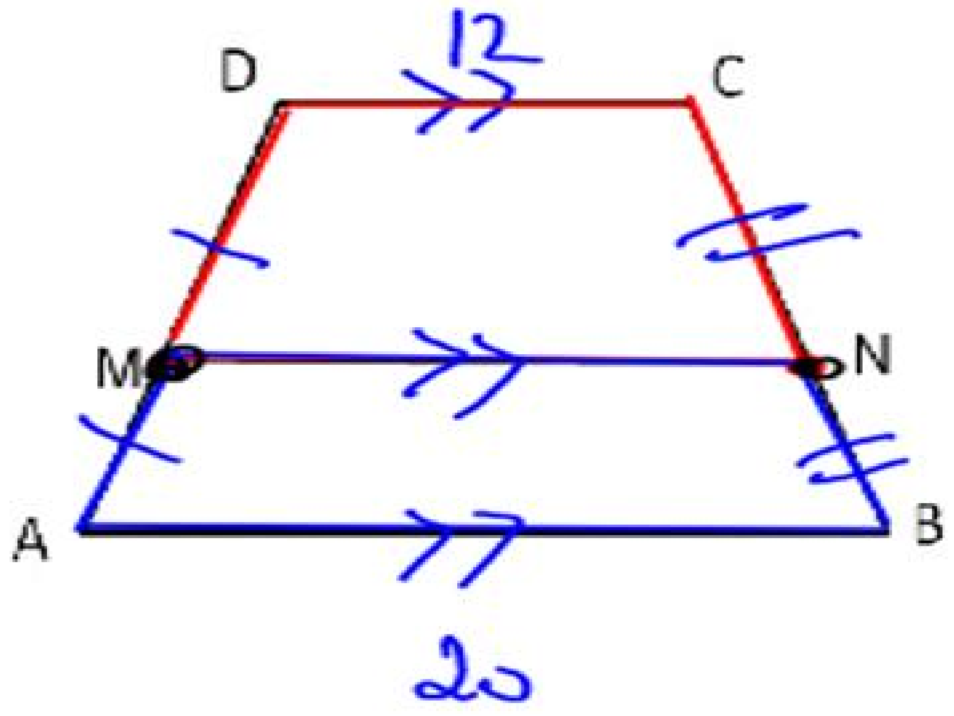


then (i) MN || AB

$$\boxed{\text{(ii) } MN = \frac{1}{2}(AB + CD)}$$

MN $\longrightarrow$ Median of Trapezium ABCD

Eg. ABCD is a trapezium where AB || CD.
M , N are mid-points on AD and BC.
If AB = 20 cm and CD = 12 cm.
Find Area of DCNM : Area of MNBA



$$MN = \frac{1}{2}(12+20) = 16$$

$$\frac{\text{Area of DCNM}}{\text{Area of MNBA}} = \frac{\cancel{\frac{1}{2}}(DC+NM)\times \cancel{H/2}}{\cancel{\frac{1}{2}}(MN+AB)\times \cancel{H/2}}$$

$$= \frac{\cancel{28}\ 7}{\cancel{36}\ 9}$$

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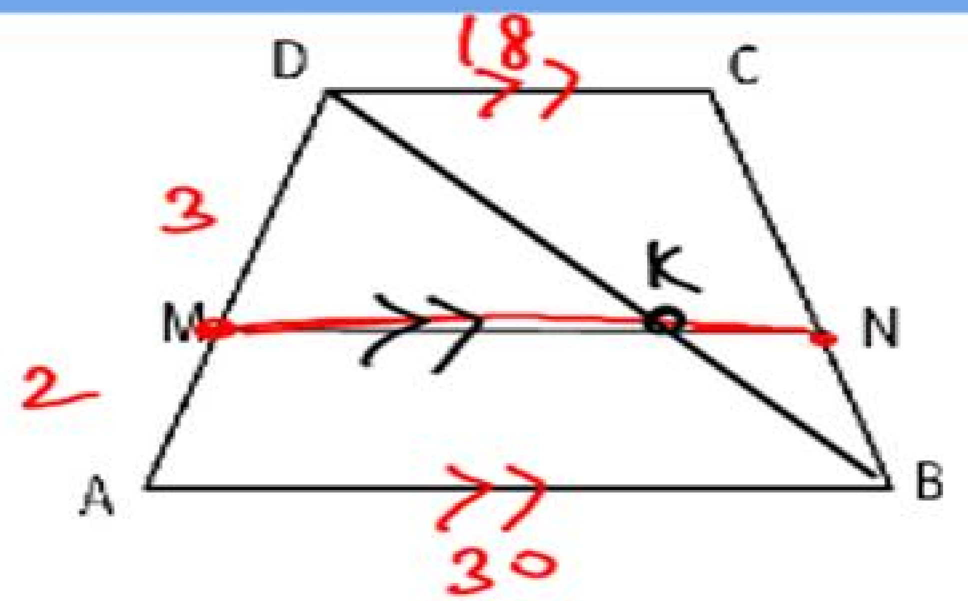
Eg. ABCD is a trapezium where AB || CD.
M, N are points on AD and BC in such a way that MN || AB.
If DM : MA = 3 : 2, DC = 18 cm, AB = 30 cm.
Find the value of MN.



$\underline{\underline{I^{st}}}$ Detailed App

In ΔDAB

$MK \parallel AB$

$$\frac{DM}{DA} = \frac{MK}{AB}$$

$$\frac{3}{5} = \frac{MK}{30}$$

$$\boxed{MK = 18}$$

In ΔBCD

$NK \parallel CD$

$$\frac{BN}{BC} = \frac{KN}{CD}$$

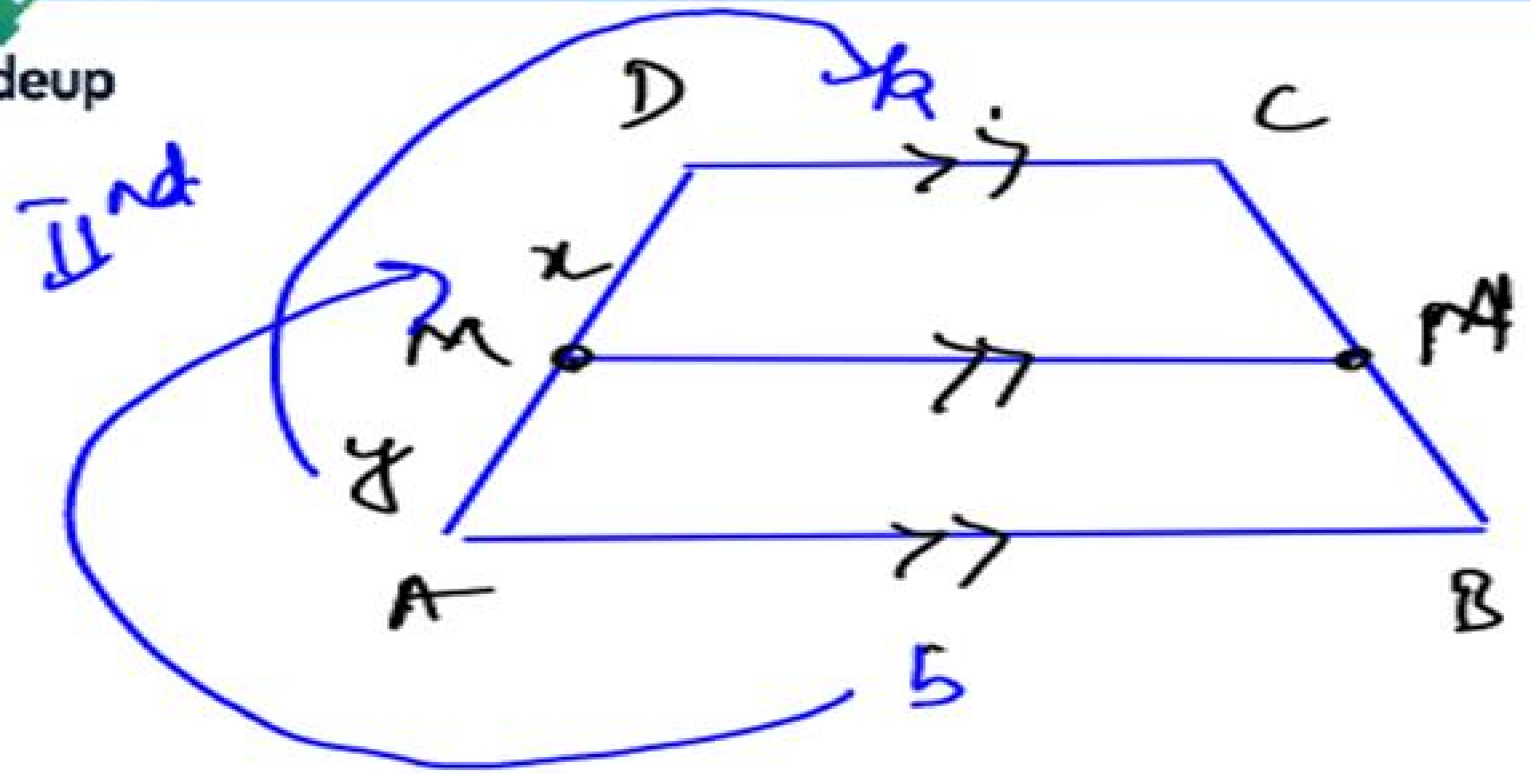
$$\frac{2}{5} = \frac{KN}{18}$$

$$\boxed{KN = 7.2}$$

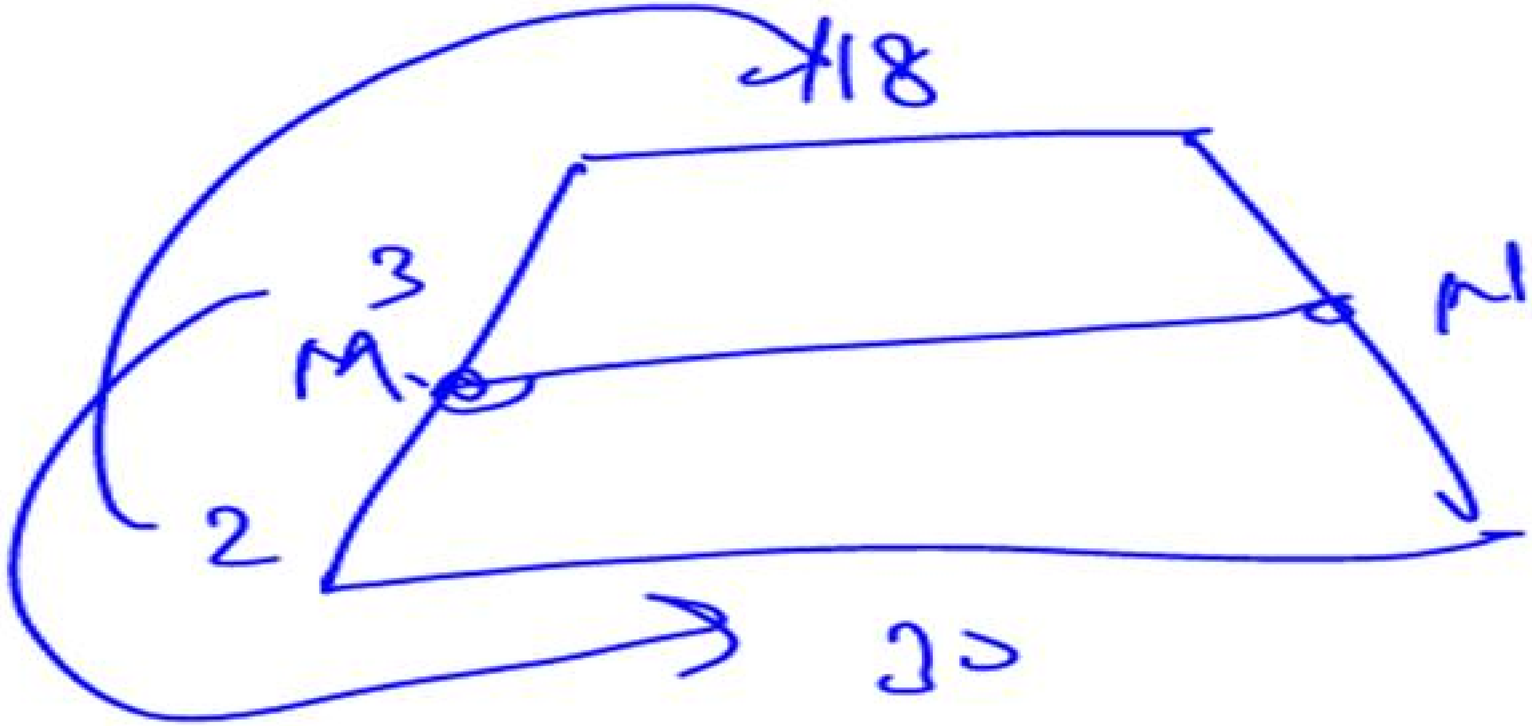
$MN = MK + KN$

$= 18 + 7.2$

$= \underline{\underline{25.2 \text{ cm}}}$

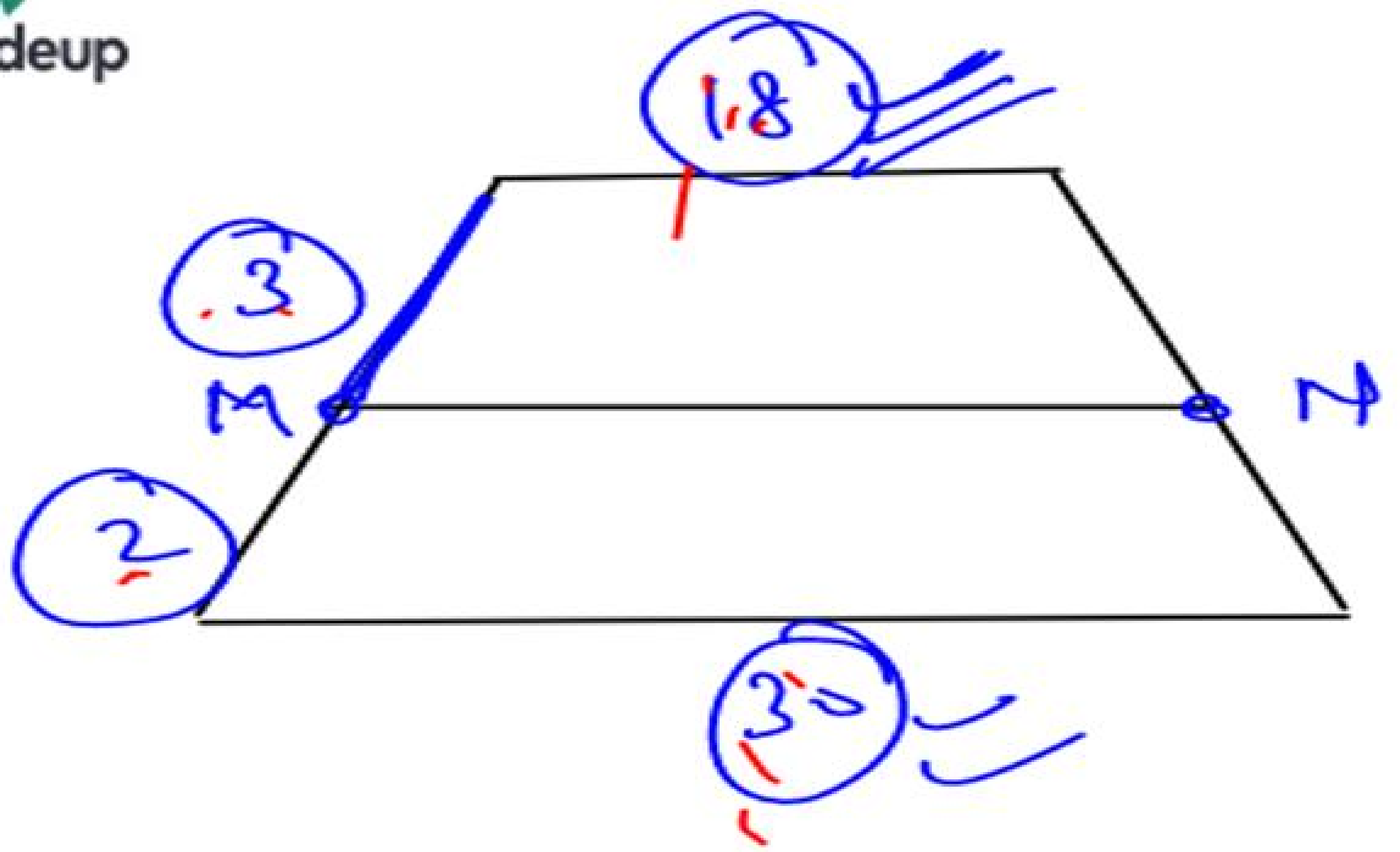


$$\boxed{MN = \frac{ay + bx}{x+y}}$$



$$MN = \frac{2\cdot 18 + 3\cdot 30}{3+2}$$

$$= \frac{126}{5} = \underline{\underline{25.2}}$$



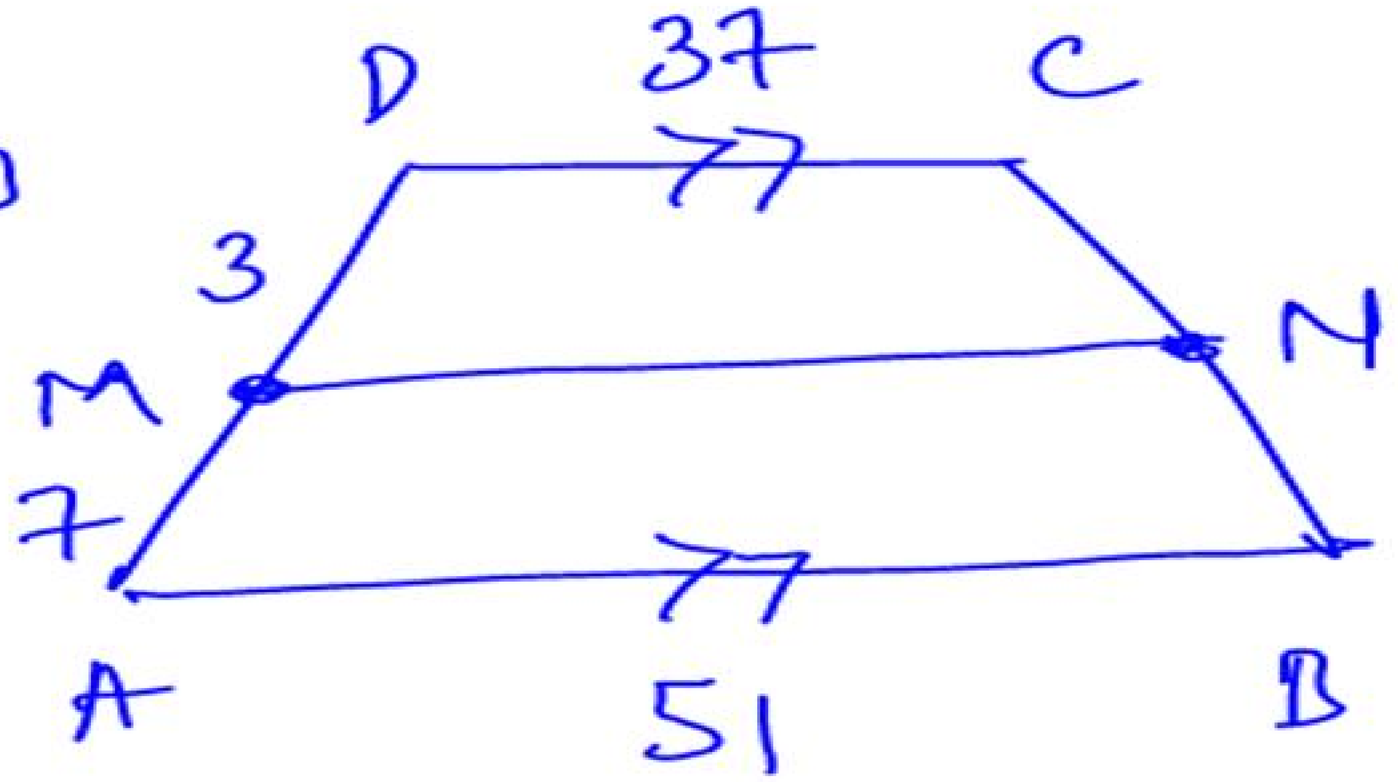
Gap

$\frac{3}{5} \cdot 12$

$= 7.2$

$18 + 7.2 = 25.2$

eg



MN = ??

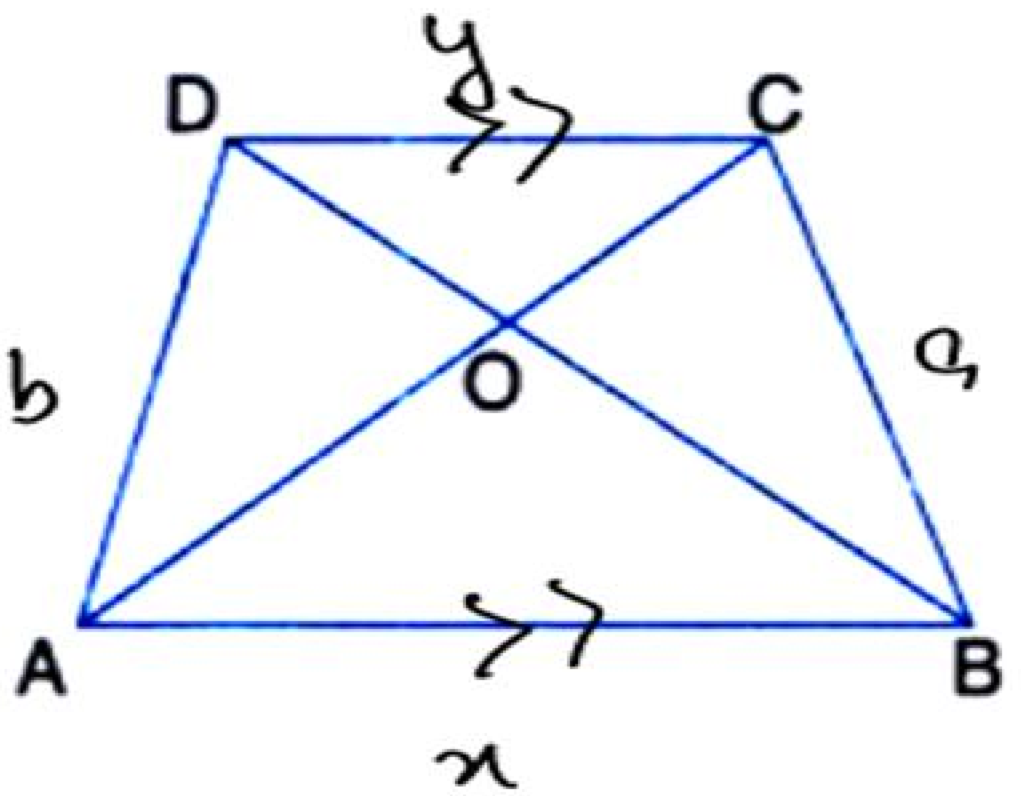
$$37 + \frac{3}{10} \times 14$$

41.2

gradeup

6. $(AC)^2 + (BD)^2 = (AD)^2 + (BC)^2 + 2(AB)(CD)$

Sum of square of diagonals = Sum of squares of non-parallel sides + 2 (product of parallel sides)

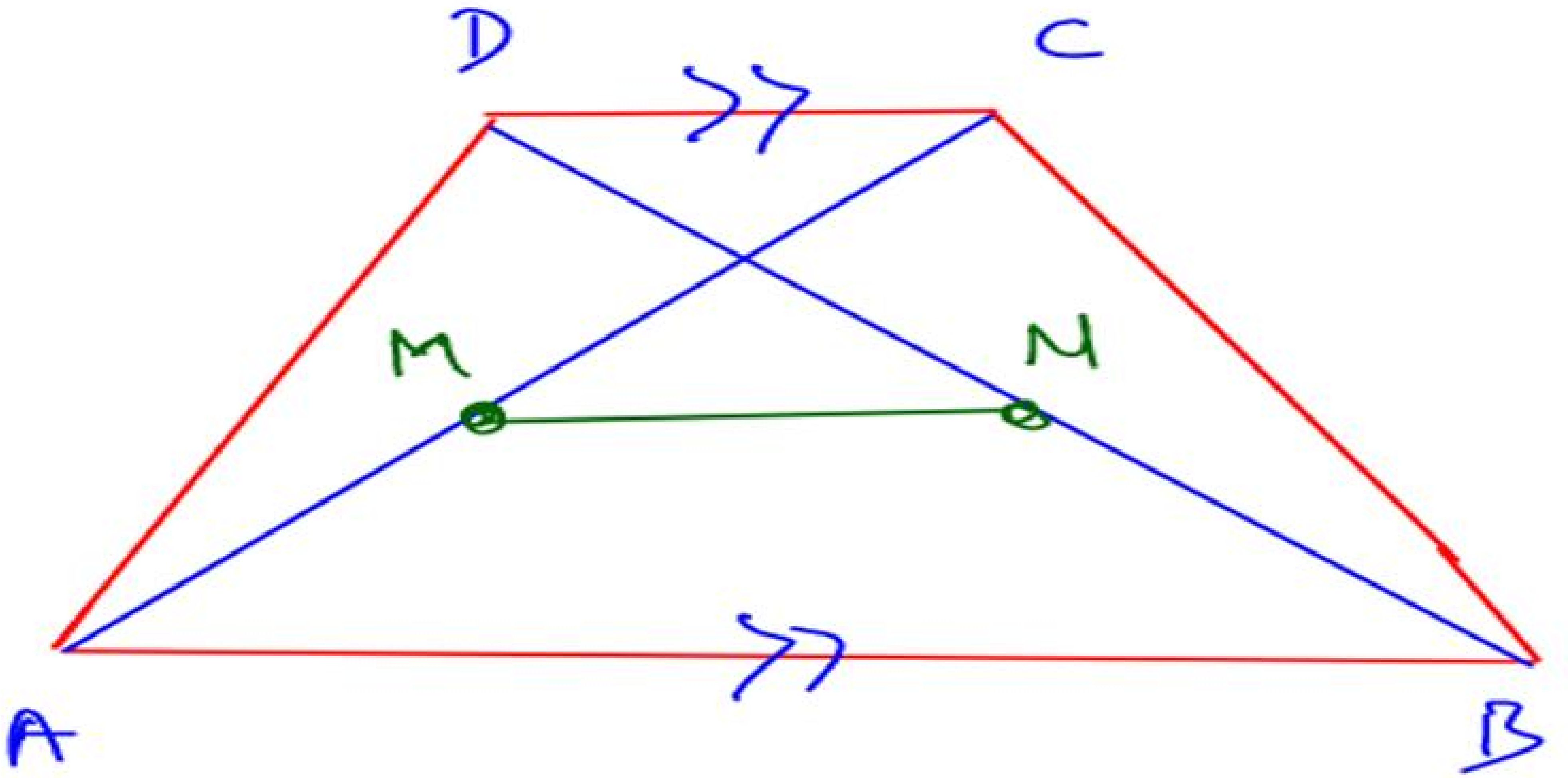


$$\boxed{D_1^2 + D_2^2 = a^2 + b^2 + 2xy}$$

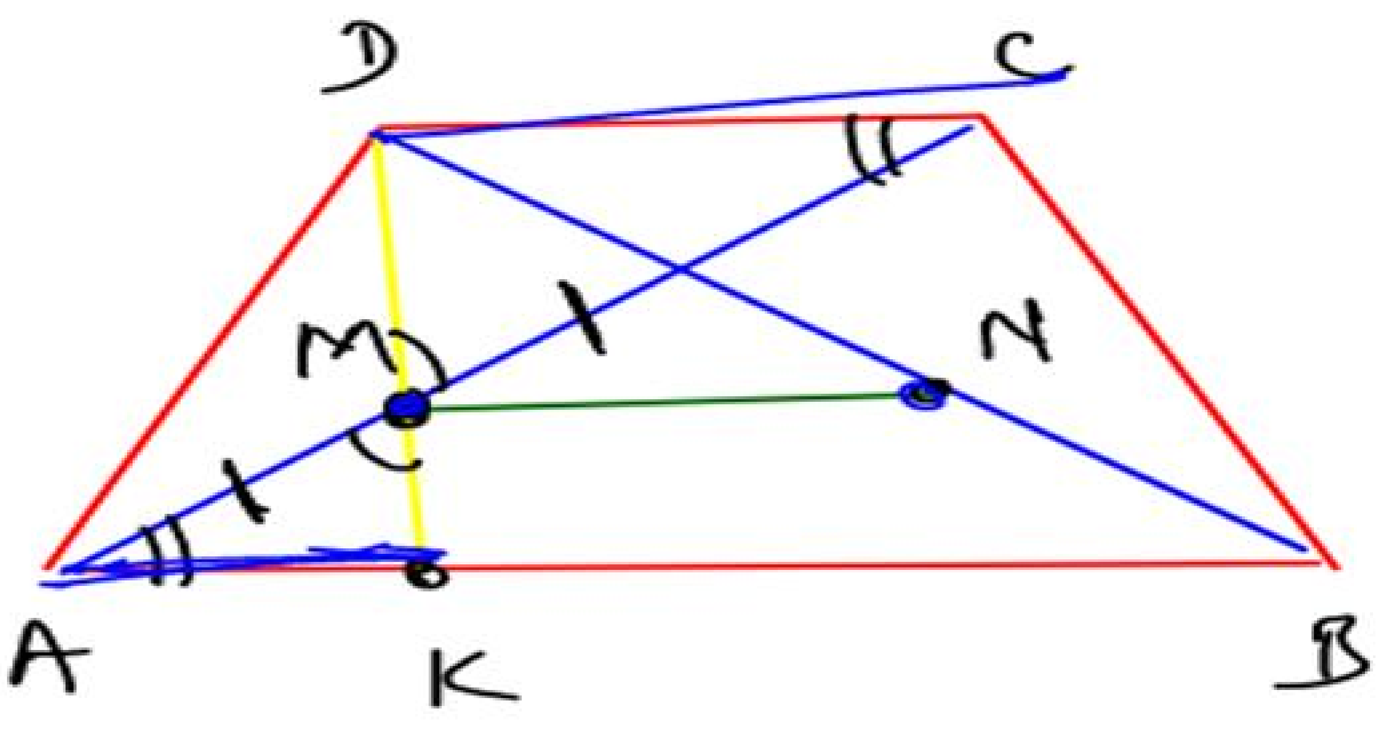
7. ABCD is a trapezium, where AB || CD. M, N are mid-points of AC and BD,

v. imp

then $\boldsymbol{MN = \frac{1}{2}|AB - CD|}$



$$MN = \frac{1}{2}\left|(AB - CD)\right|$$



Given ABCD is a trapezium

AB ∥ CD

M, N are mid pts of

AC & BD

To prove $MN = \frac{1}{2}(AB - CD)$

Const → Join DK

Proof :-

① ΔDMC & ΔAMK

$\Delta DMC \cong \Delta KMA$ (ASA)

DM = KM

M is mid pt of DK

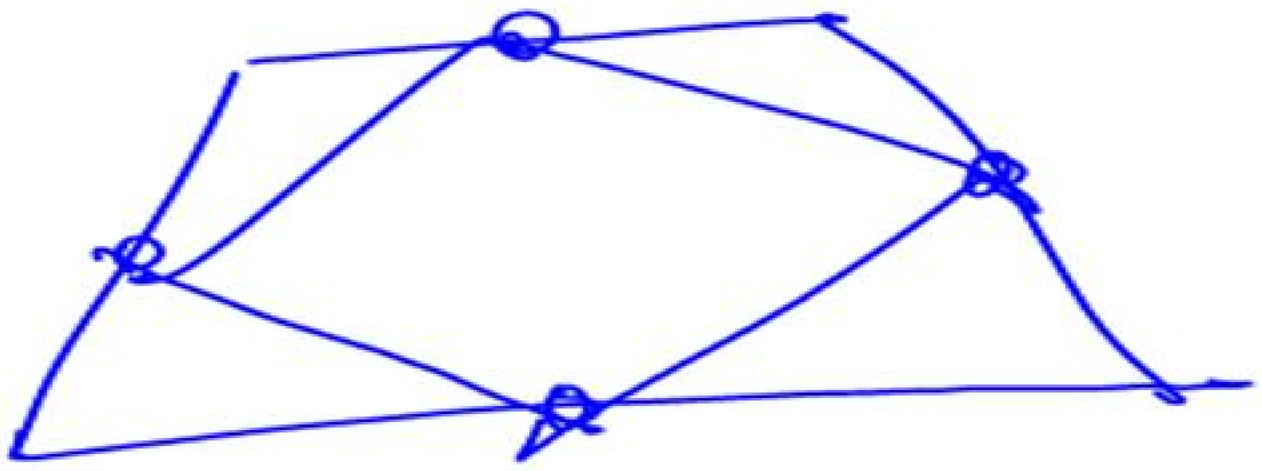
② ΔDKB

$MN = \frac{1}{2}(KB)$

$= \frac{1}{2}(AB - AK)$

$\frac{1}{2}(AB - DC)$

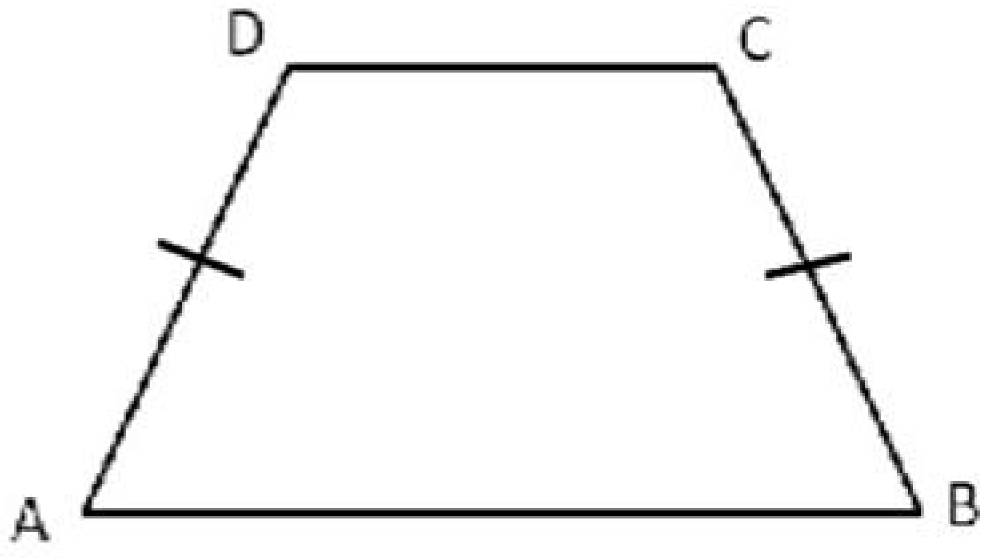
8. Figure formed by joining mid-point of all sides of the trapezium is a parallelogram.



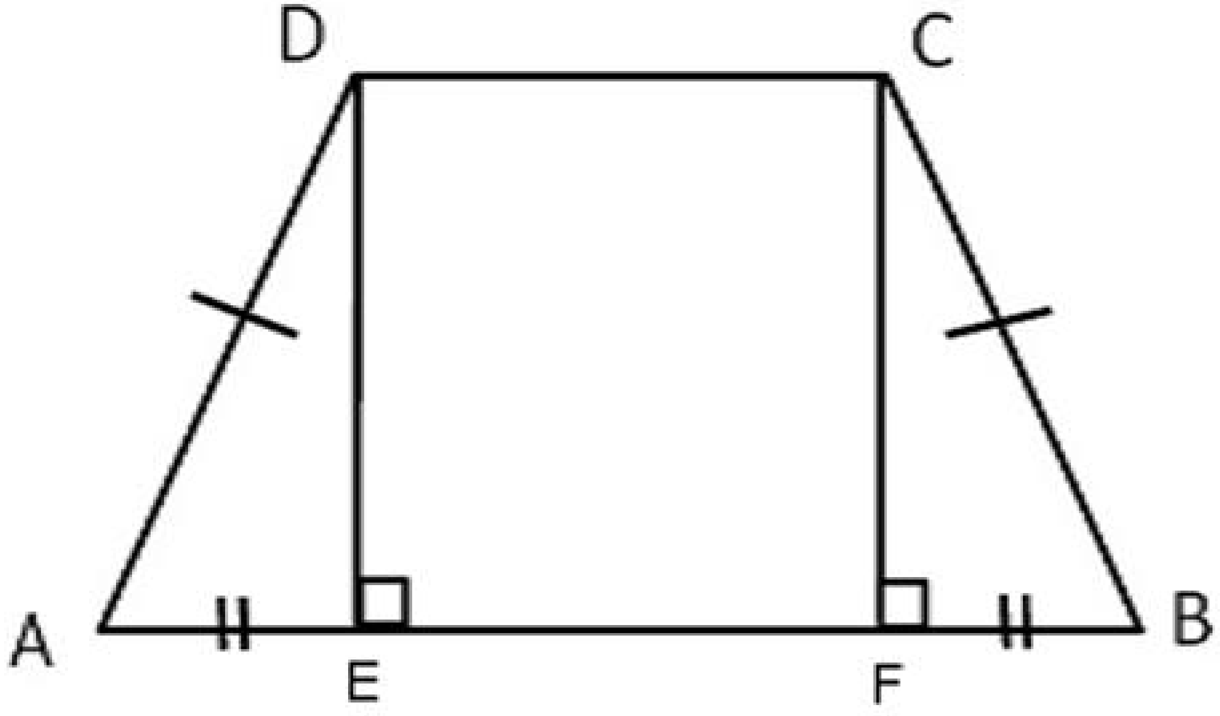
→ Parallelogram

ISOSCELES TRAPEZIUM

Def: A trapezium in which non-parallel sides are equal.



AD = BC



In Isosceles trapezium where AB || CD

(1) AD = BC

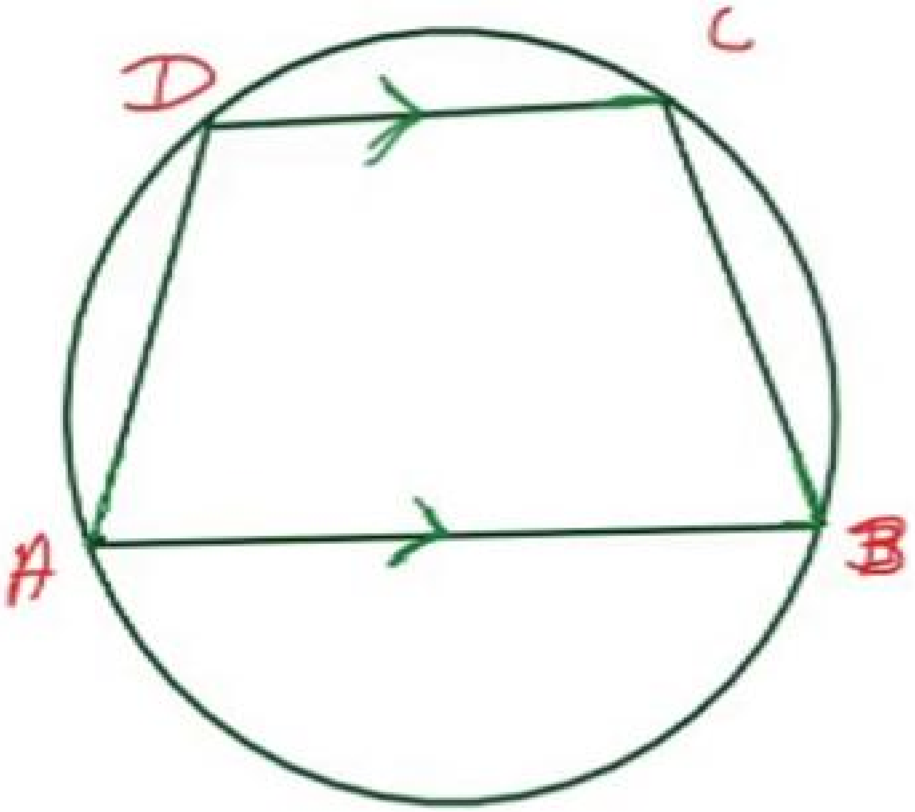
(2) AE = BF

(3) AC = BD

(4) $\angle D = \angle C$

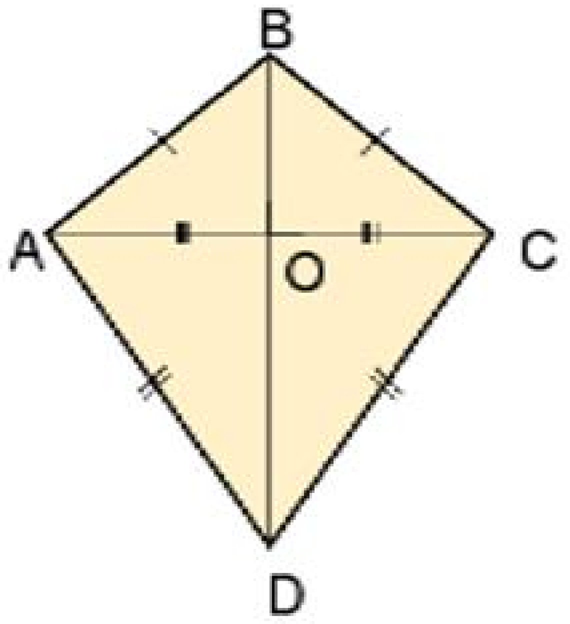
(5) $\angle A = \angle B$

Cyclic trapezium is always an Isosceles Trapezium.



KITE

Kite is a quadrilateral in which two pairs of adjacent sides are of equal length and the diagonals intersect each other at right angles.



(1) $AB = BC$

$AD = CD$

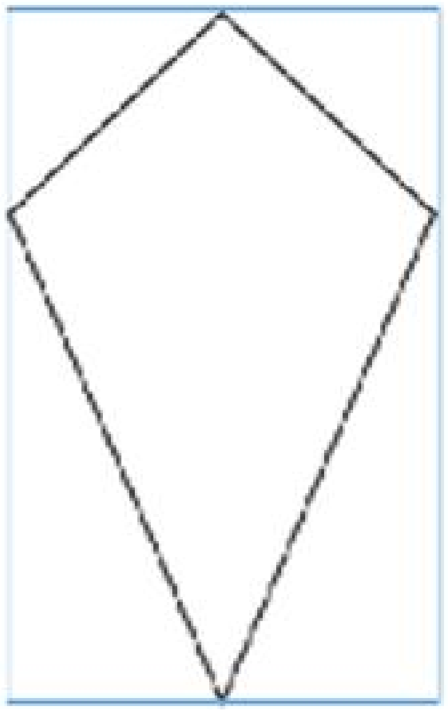
(2) $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$

(3) $AO = OC$ (The longer diagonal bisects the shorter diagonal.)

(4) $\angle A = \angle C$

$$\textit{Area of Kite} = \frac{1}{2} D_1 D_2$$

Eg. The area of the rectangle is 80 cm^2, what is the area of the kite?



Eg. HATS is a kite with diagonals that intersect at C. $\angle TSC = 32°$. Find $\angle SHC$.