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Sahi Prep Hai Toh Life Set Hai

TRIANGLE-4

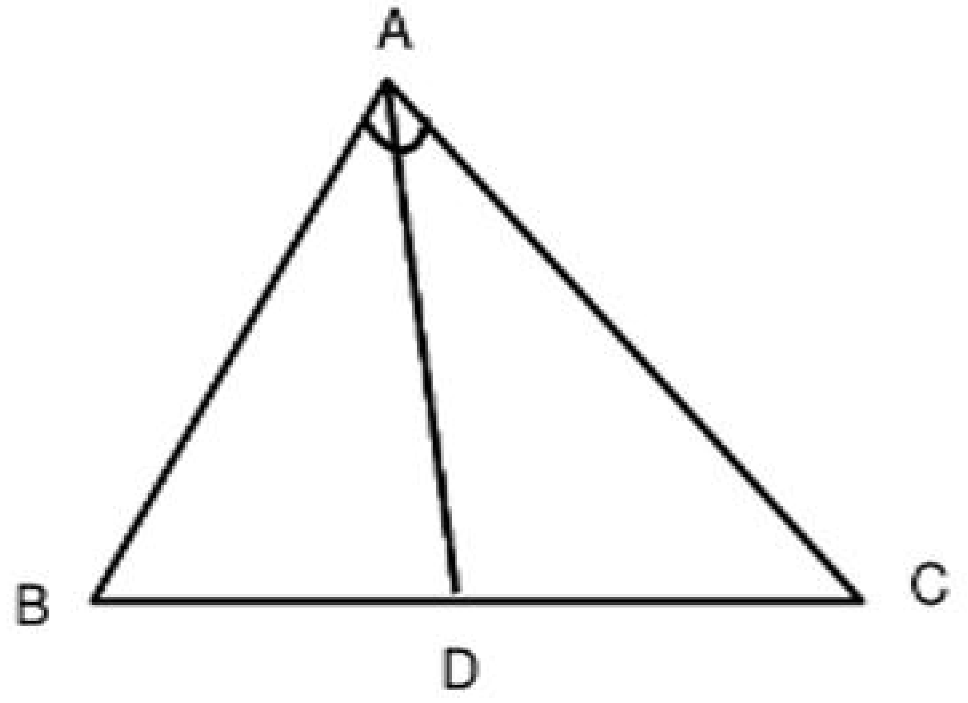
Agenda

46 — Incentre → (45-50) min

13 — Circumcentre → 12-15 min

Common Points → (8-10) min

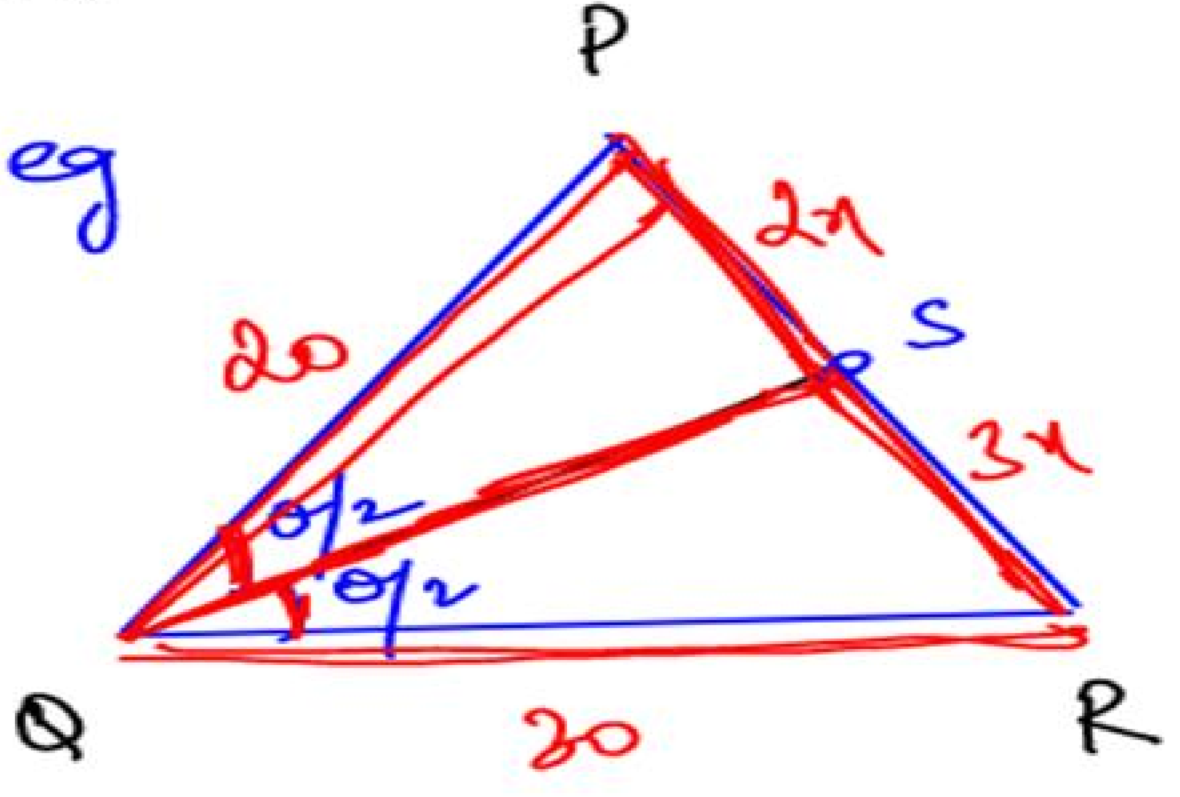
INTERNAL ANGLE BISECTOR THEOREM



Given AD is angle bisector of $\angle BAC$.

$$\frac{AB}{AC} = \frac{BD}{DC}$$

eg



2. If PQ = 20cm

PR = 25cm

3

QR = 30cm

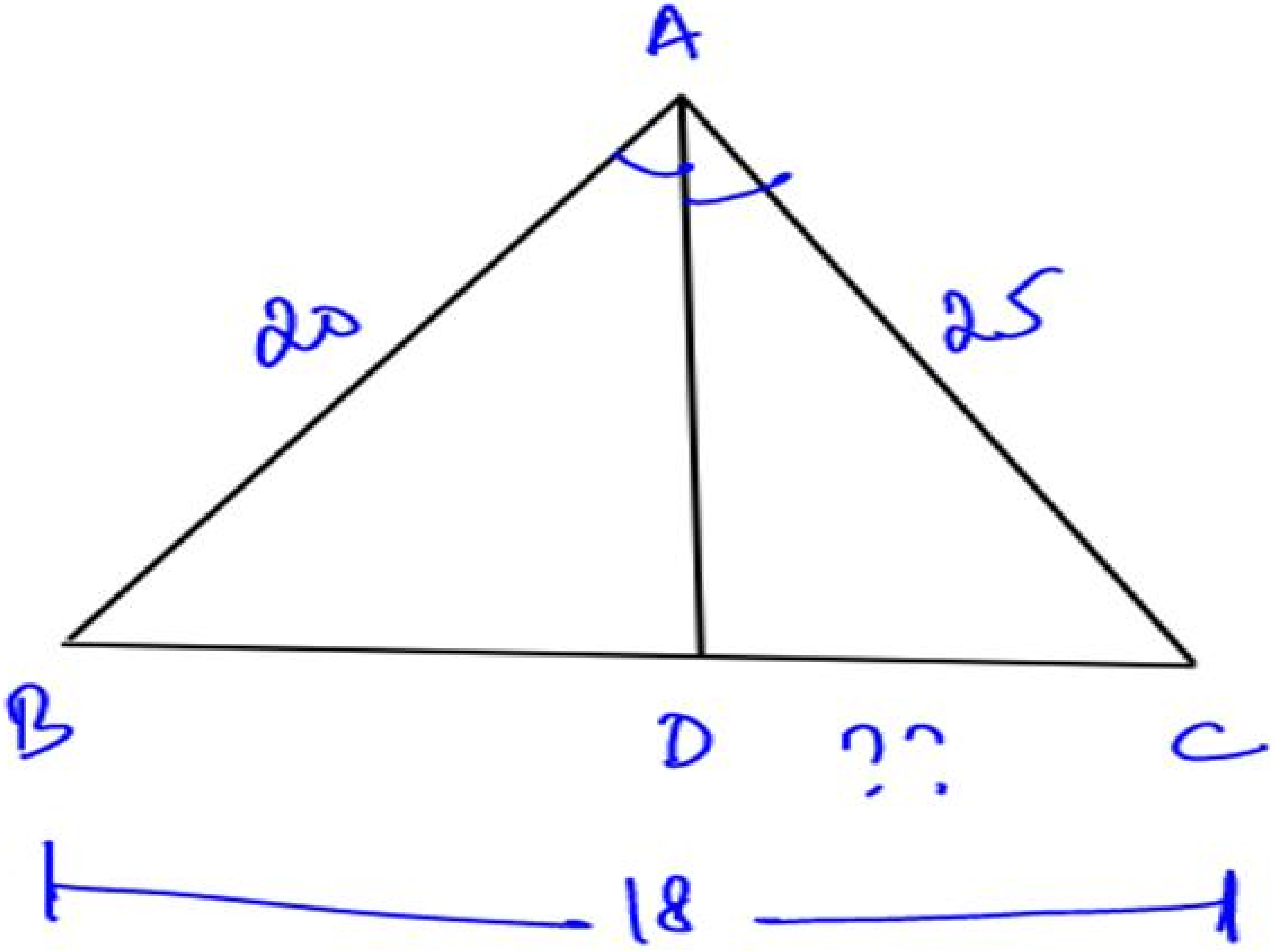
Find PS = ??

$5x = 25$

$x = 5$

PS = 10cm

E.g. In a ΔABC, AD is the angle bisector of $\angle BAC$, where D is point on BC. If AB = 20 cm, AC = 25 cm, BC = 18 cm, find the length of DC.

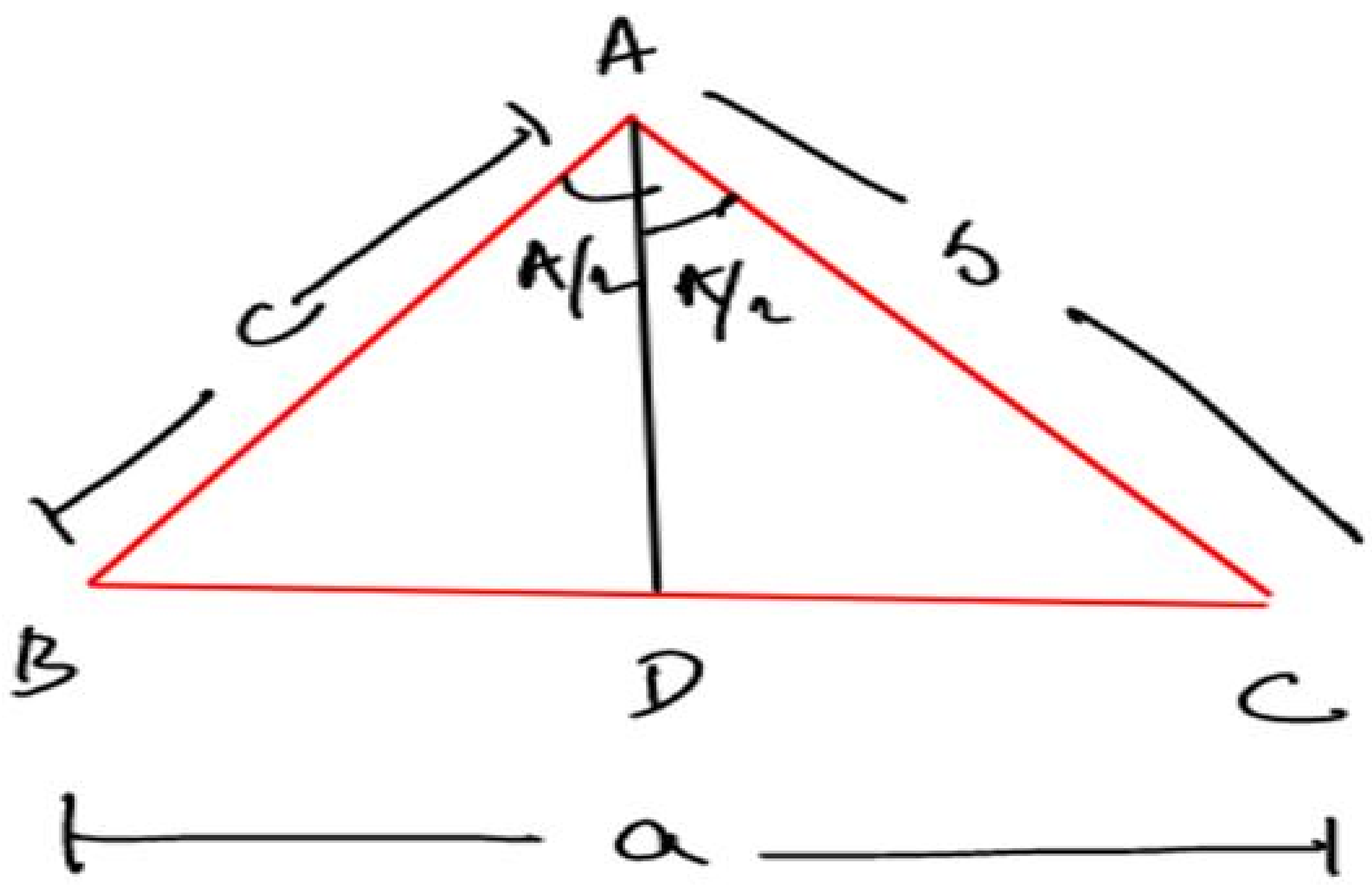


$$\frac{\overset{4}{\cancel{20}}}{\underset{5}{\cancel{25}}} = \frac{BD}{DC}$$

$$\frac{5}{\cancel{9}} \times \overset{2}{\cancel{18}}$$

$$= 10\text{ cm}$$

LENGTH OF ANGLE BISECTOR



$AD \rightarrow$ Angle Bisector

$$\text{Area of } \triangle ABC = \frac{1}{2} b \cdot c \sin A$$

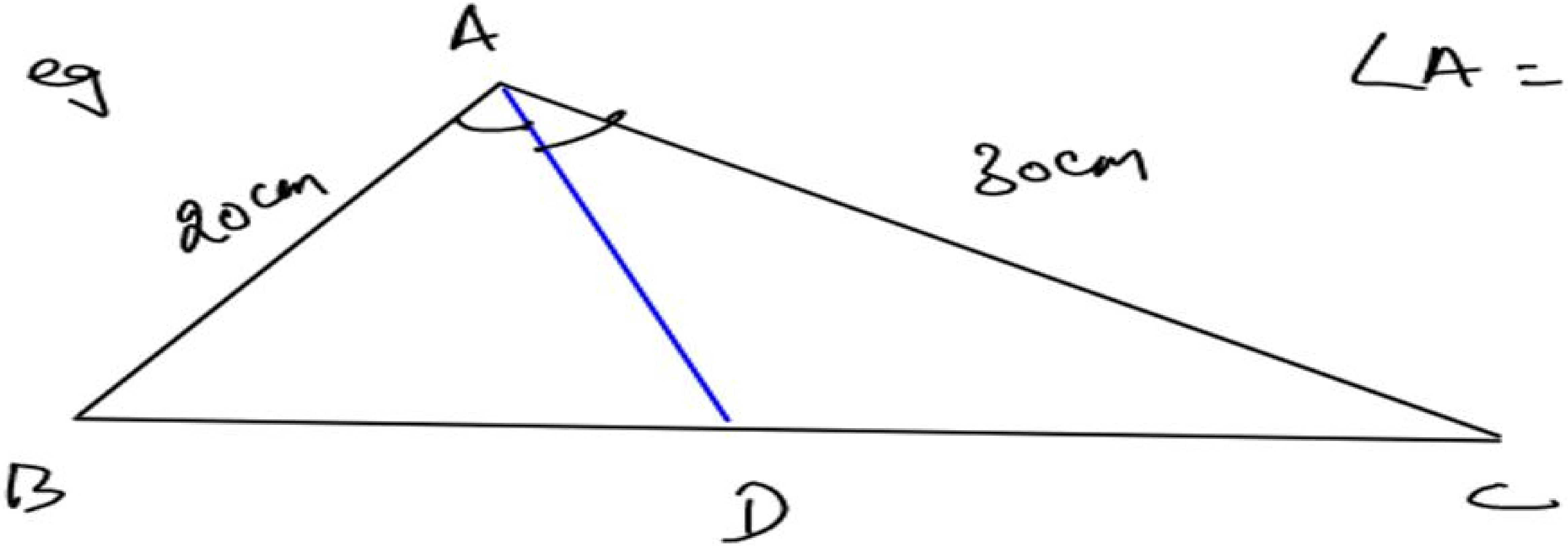
$$\text{Area of } \triangle ABD = \frac{1}{2} c \cdot AD \sin A/2$$

$$\text{Area of } \triangle ACD = \frac{1}{2} b \cdot AD \sin A/2$$

$$\cancel{\tfrac{1}{2}} AD \cancel{\sin A/2}\,(c+b) = \cancel{\tfrac{1}{2}} b \cdot c \cdot 2 \cancel{\sin A/2} \cos A/2$$

$$\boxed{AD = \frac{2bc}{b+c} \cos A/2}$$

eg



$\angle A = 60^\circ$

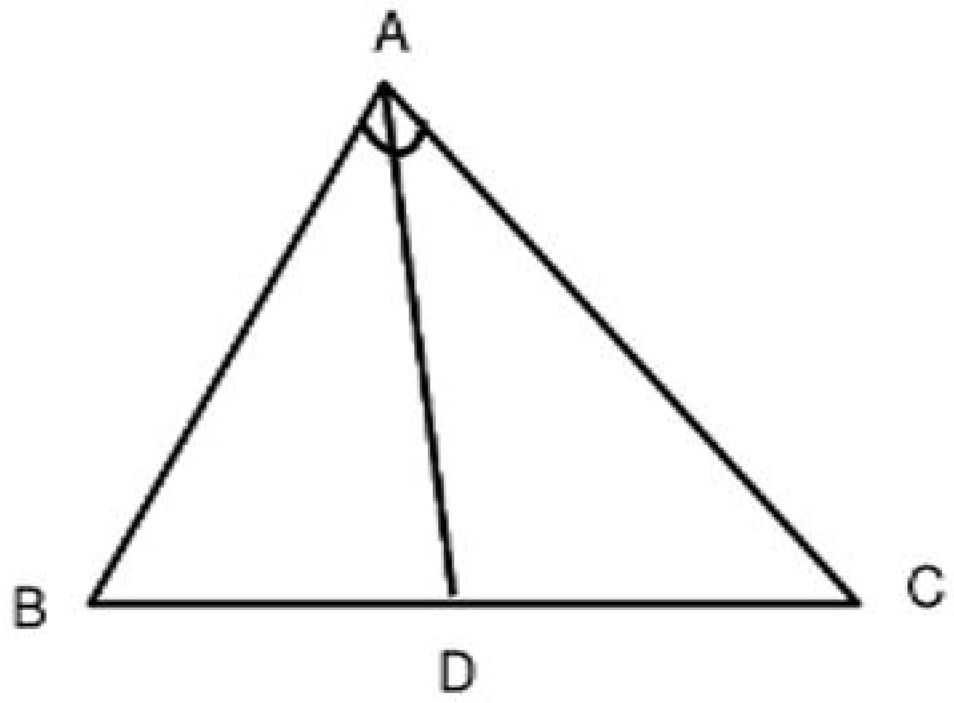
$\cos 30^\circ = \frac{\sqrt{3}}{2}$

AD → Angle Bisector

Length of AD $\Rightarrow \dfrac{\cancel{2}\cdot 20 \cdot 30}{20+30} \cdot \dfrac{\sqrt{3}}{\cancel{2}}$

$\Rightarrow \underline{\underline{12\sqrt{3}\ \text{cm}}}$

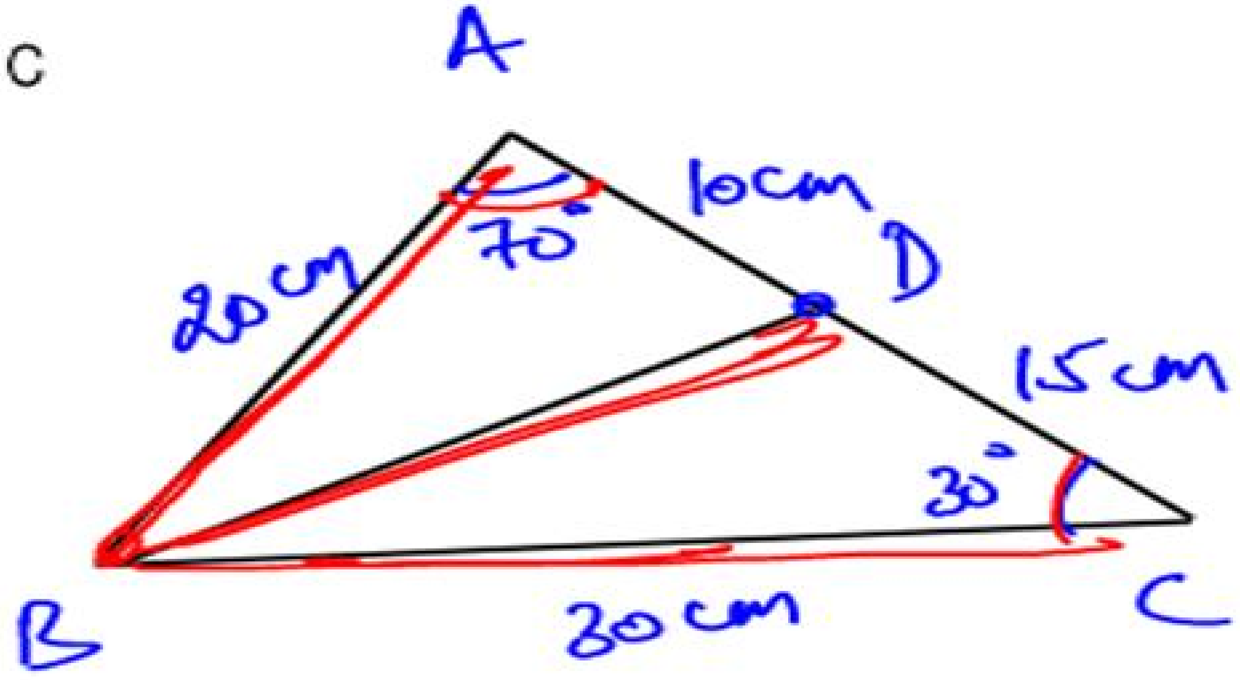
CONVERSE OF ANGLE BISECTOR THEOREM



If $\frac{AB}{AC} = \frac{BD}{DC}$

Then AD → Angle bisector of ∠BAC.

eg



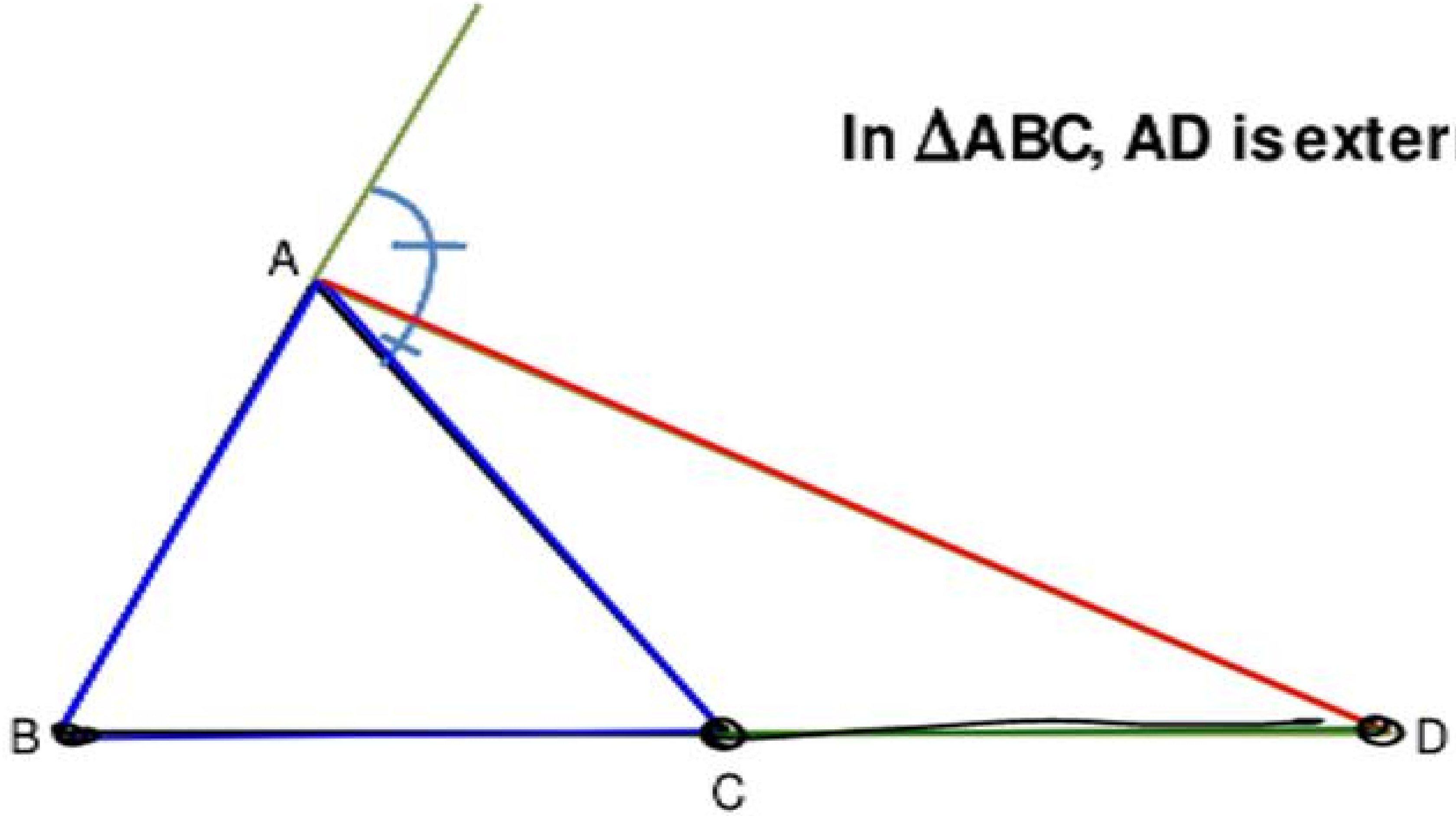
Find ∠ABD = ??

$\frac{20}{30} = \frac{10}{15}$

BD → Angle Bisector

∠ABD = 40°

EXTERNAL ANGLE BISECTOR THEOREM

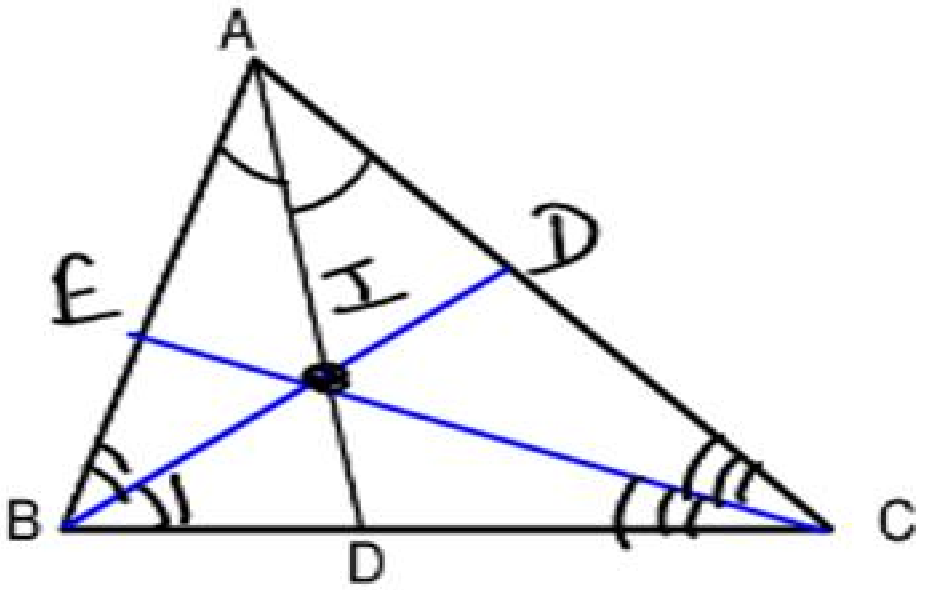


In ΔABC, AD is external angle bisector.

$$\boxed{\frac{AB}{AC} = \frac{BD}{CD}}$$

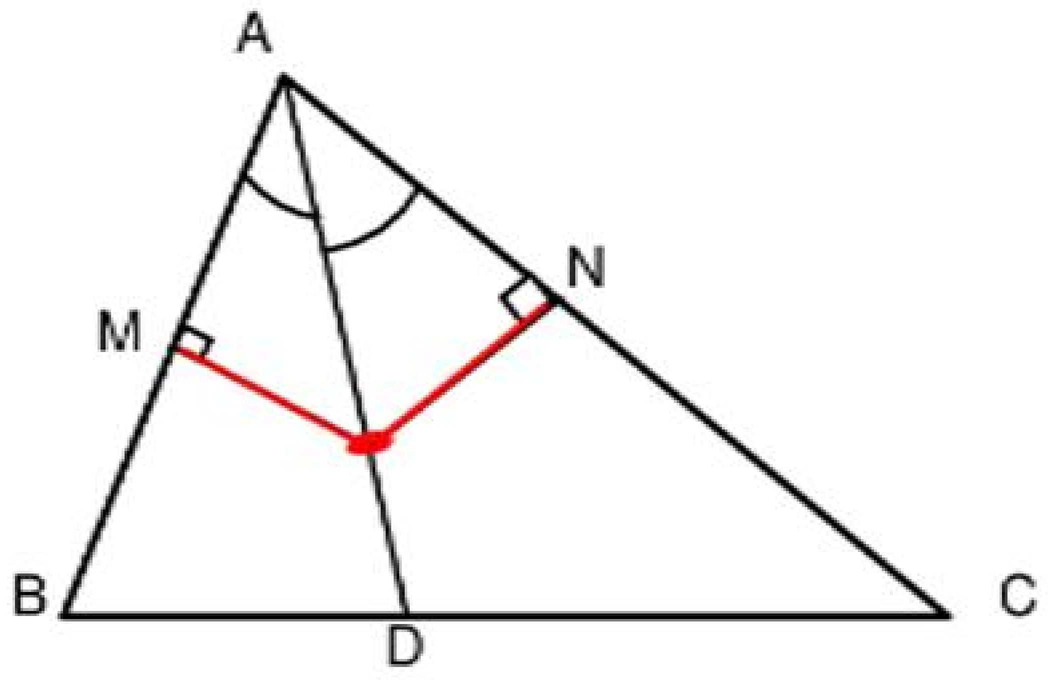
INCENTRE

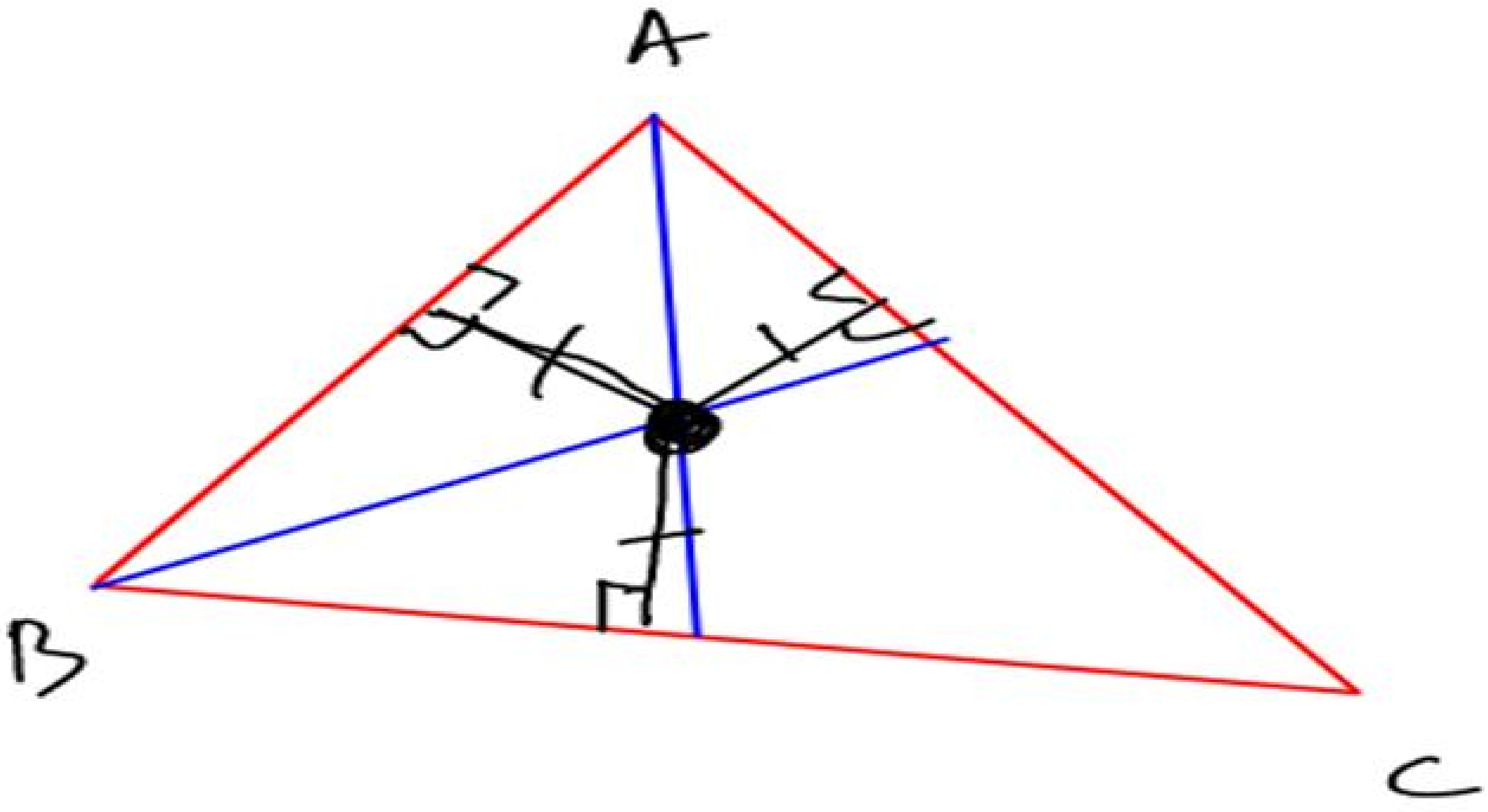
Def: Meeting point of Angle Bisector.



I → Incentre

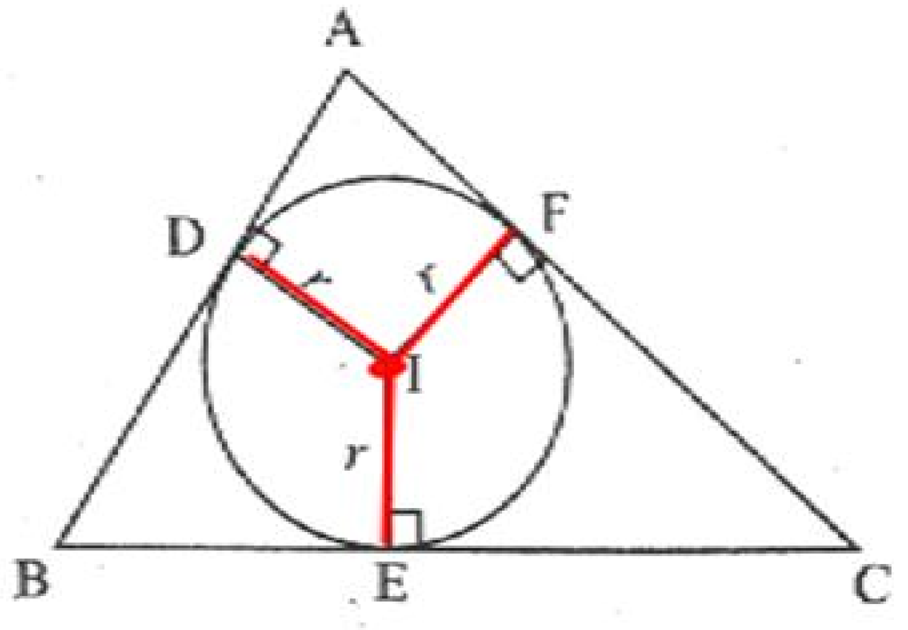
If you take any point on the angle bisector of $\angle A$, then that point is equidistant from the sides AB and AC.

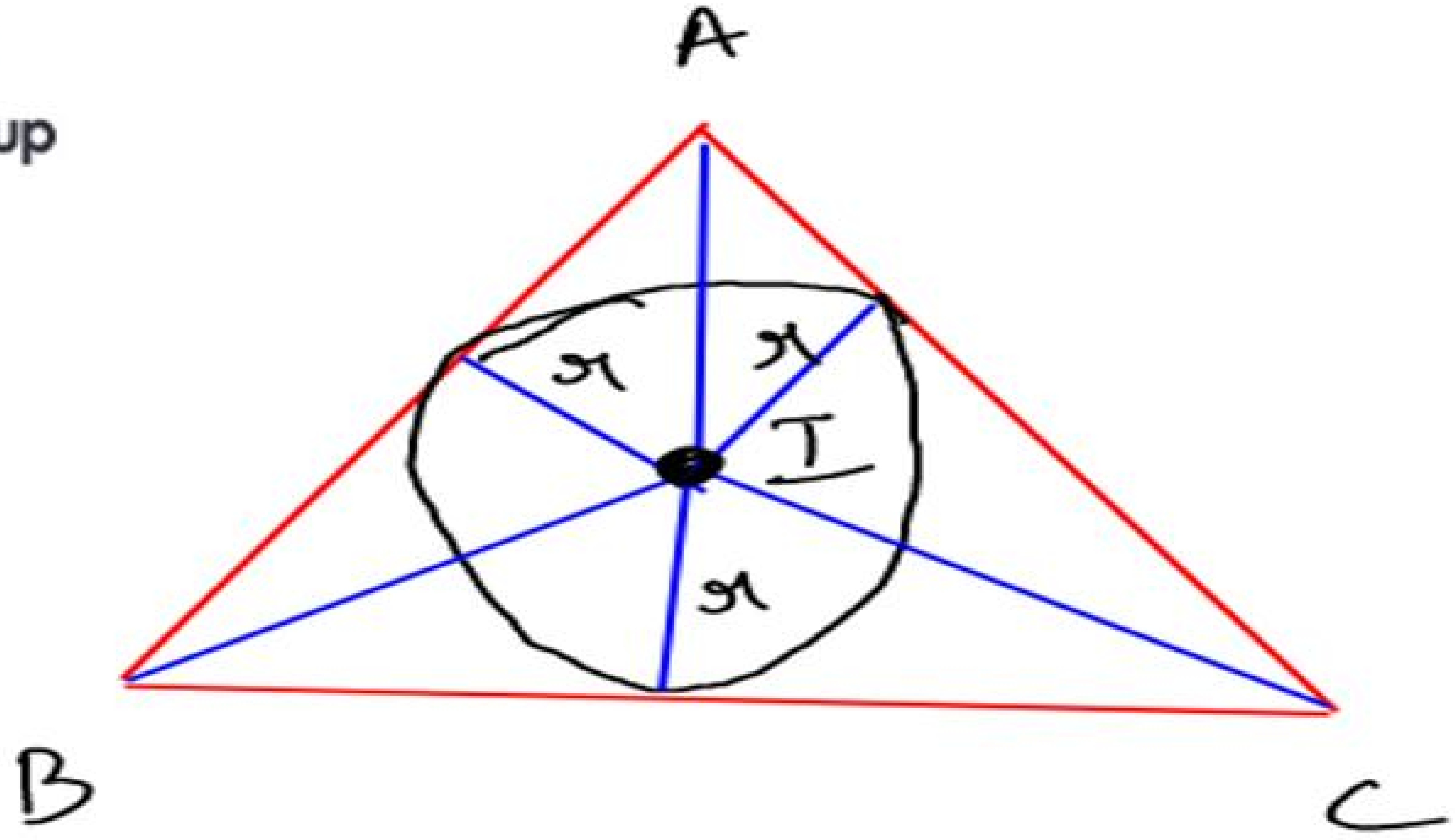




Incentre is equidistant from sides of Δ

Incentre is the centre of the circle inscribed in a triangle and it is equidistant from the sides of the triangle.





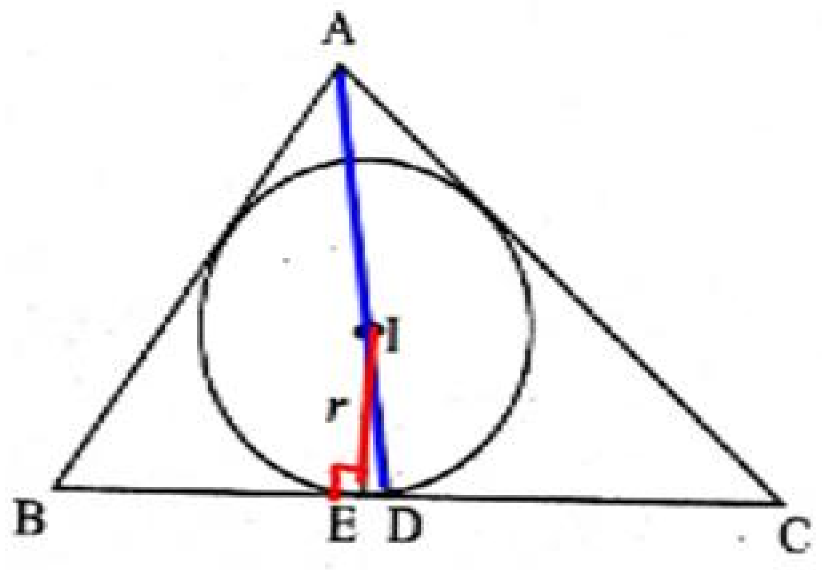
Area of ΔABC = Area of $(\Delta AIB + \Delta BIC + \Delta CIA)$

$$= \frac{1}{2} r \cdot AB + \frac{1}{2} r BC + \frac{1}{2} r AC$$

$$= r \left[S \right]$$

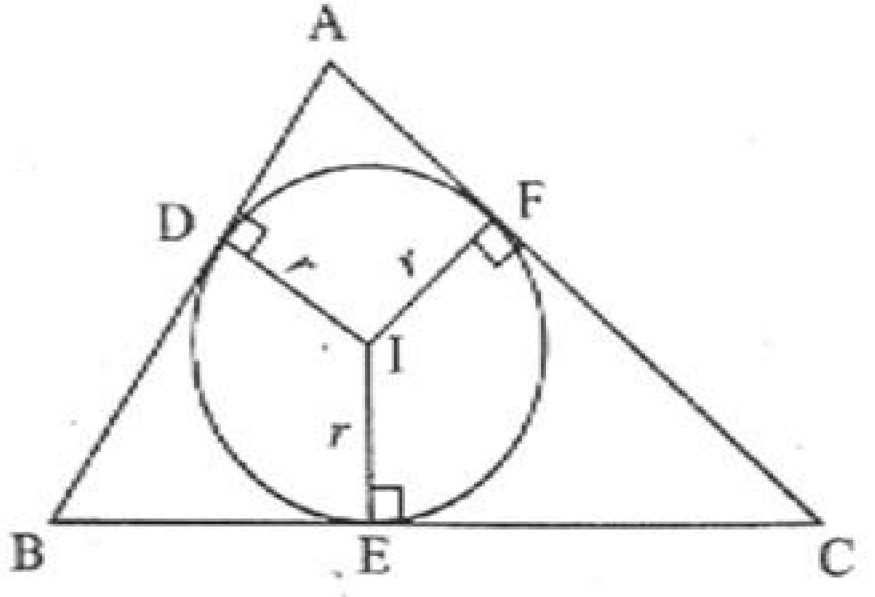
$$= r \cdot S$$

The bisector of $\angle A$ of ΔABC may or may not intersect side BC at point E where the incircle touches the side BC of the triangle and the same is true for other angle bisectors.



Area of $\Delta = r \cdot s$

Where, r is the inradius of ΔABC and s is semi-perimeter



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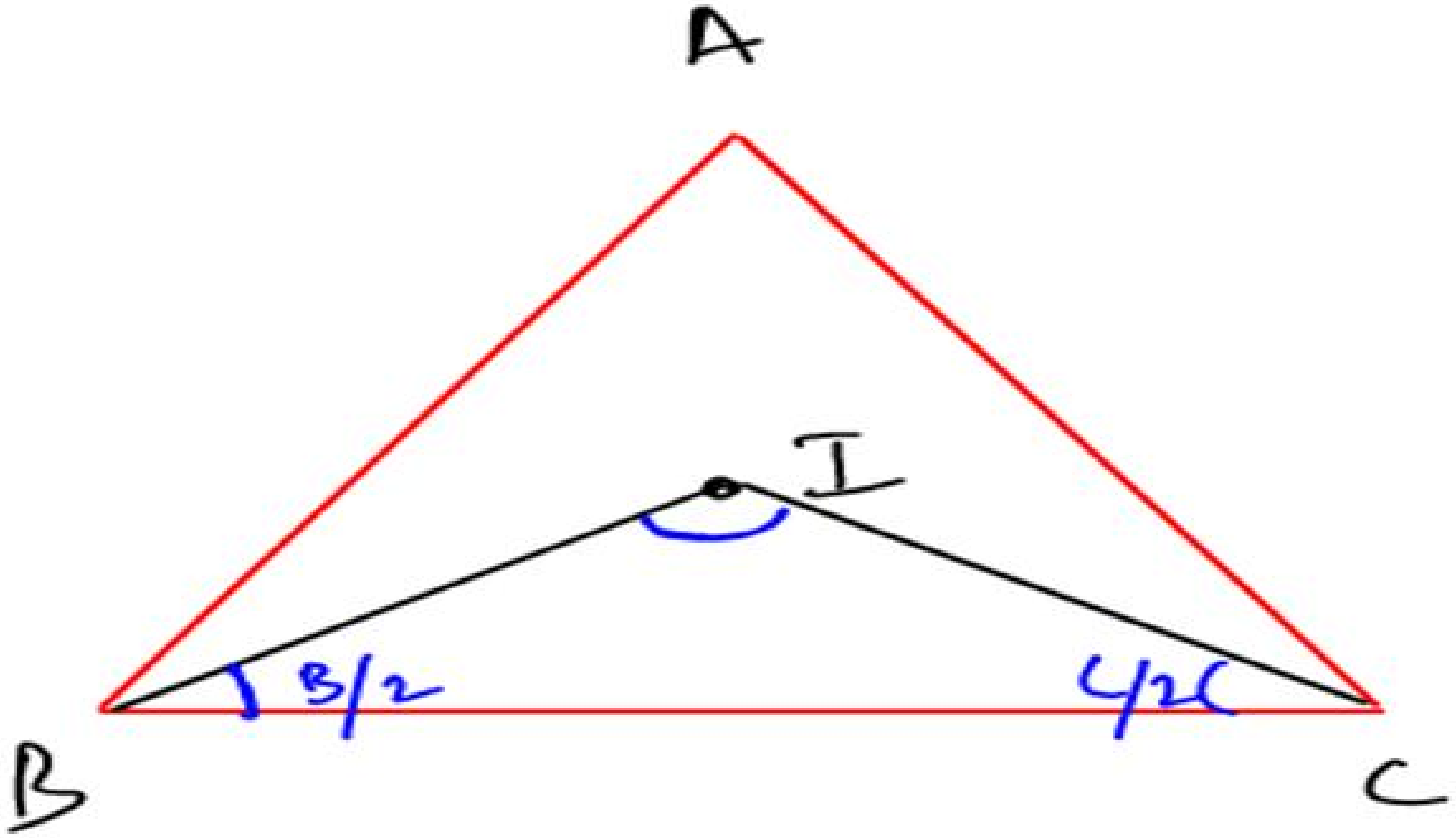
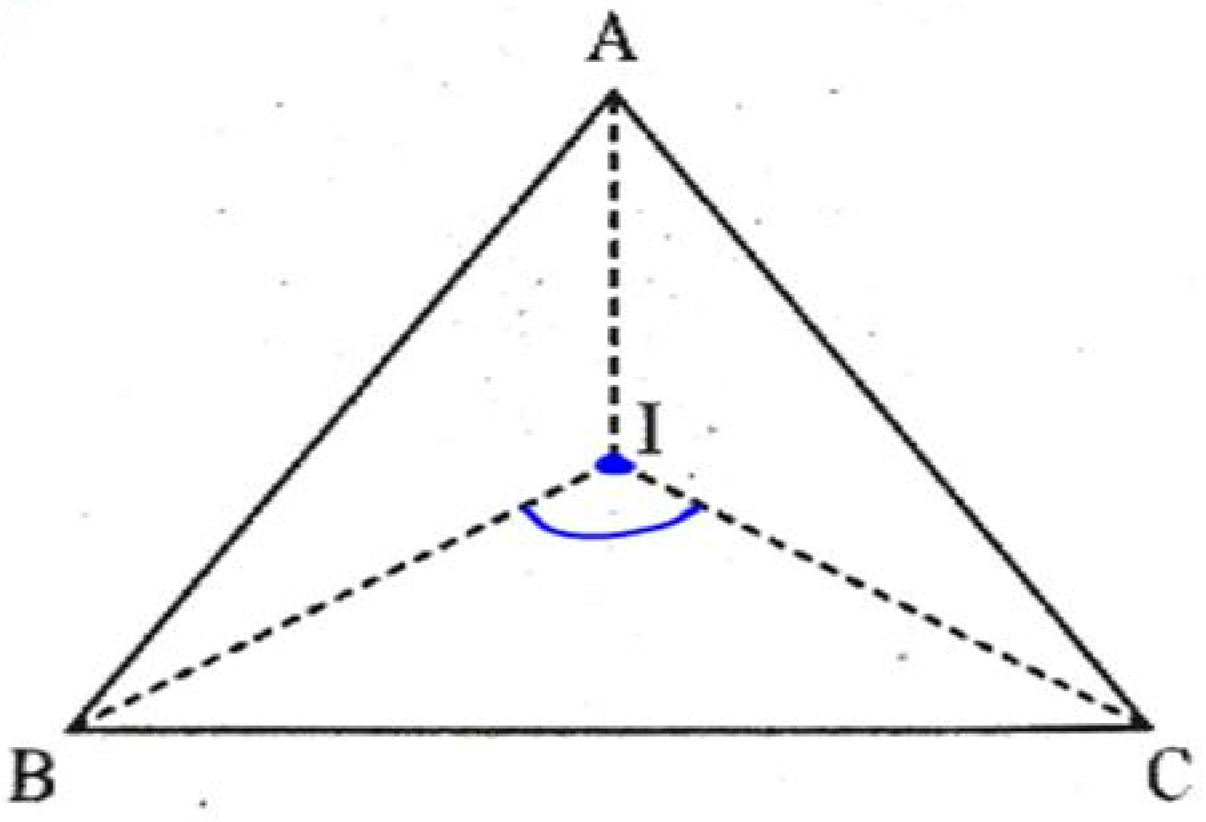


$$\frac{B}{2}+\frac{C}{2}+\angle BIC = 180^\circ$$

$$\cancel{\frac{B}{2}}+\cancel{\frac{C}{2}}+\angle BIC = 90+\frac{A}{2}+\cancel{\frac{B}{2}}+\cancel{\frac{C}{2}}$$

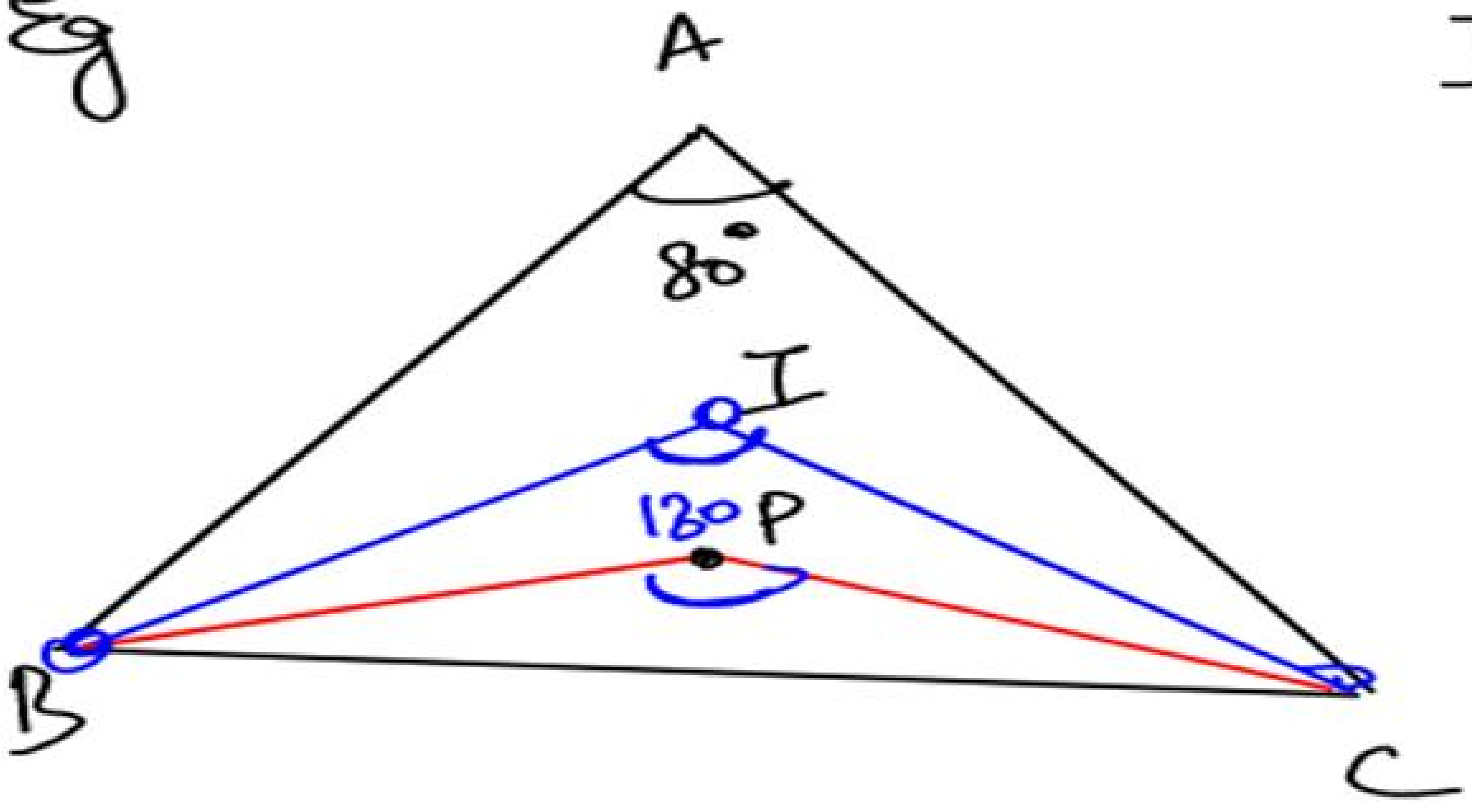
$$\boxed{\angle BIC = 90+\frac{A}{2}}$$



If I is the incentre of ΔABC,

$$\angle BIC = 90 + \frac{\angle A}{2}$$

Eg

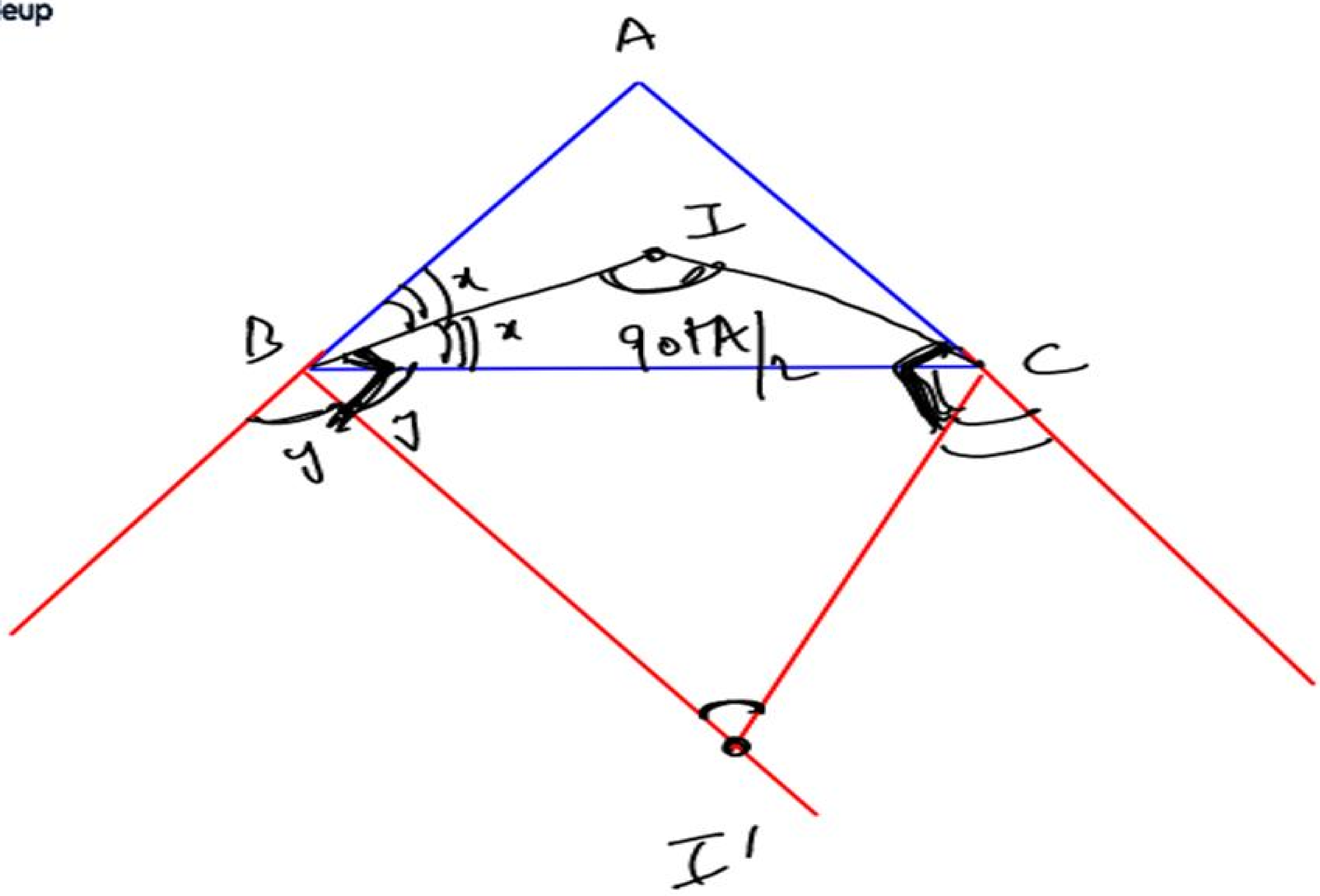


I is Incentre of ΔABC

P is Incentre of ΔBIC

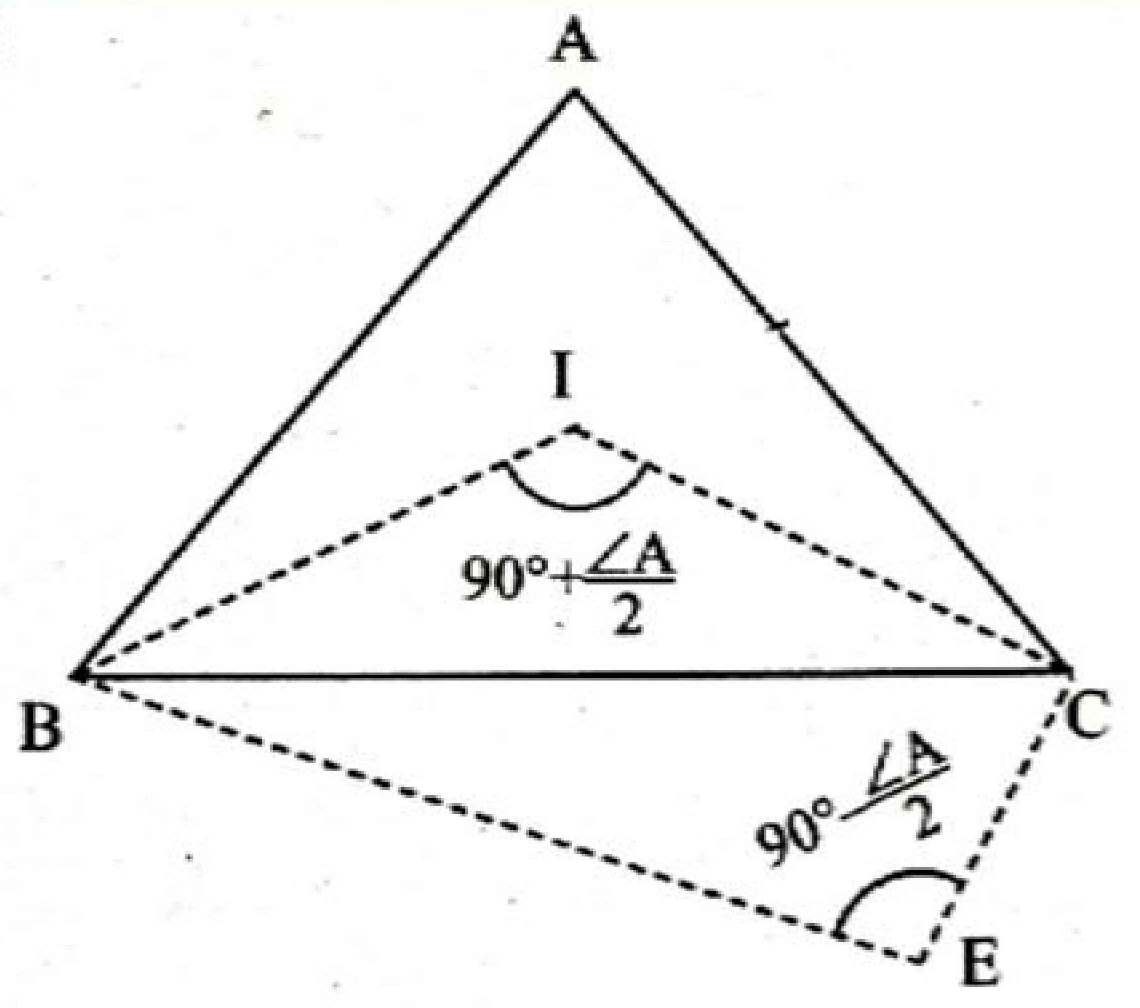
find $\dfrac{\angle BIC}{\angle BPC} = ??$

$$\frac{\cancel{130}\ 26}{\cancel{155}\ 31}$$



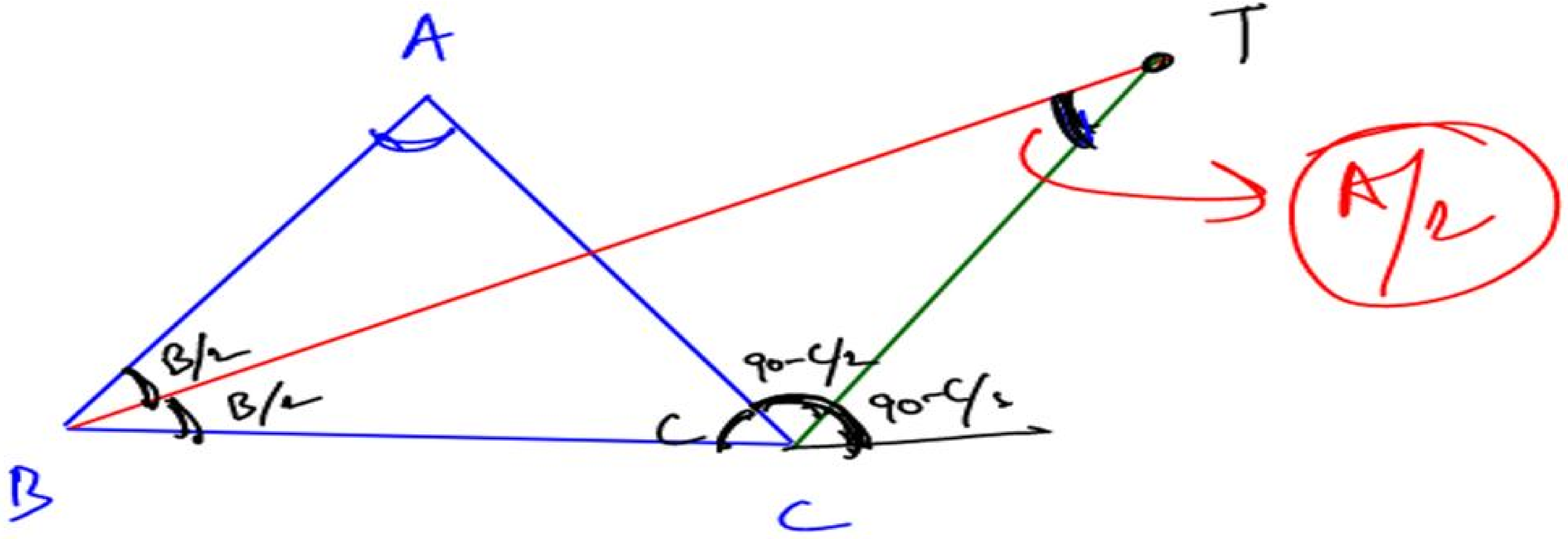
$\angle BI'C$

$= 90 - A/2$



The external angle bisectors of $\angle B$ and $\angle C$ meet at point E.

$$\angle BEC = 90 - \frac{\angle A}{2}$$



ΔBTC

$$\frac{B}{2} + C + 90 - \frac{C}{2} + \angle BTC = 180$$

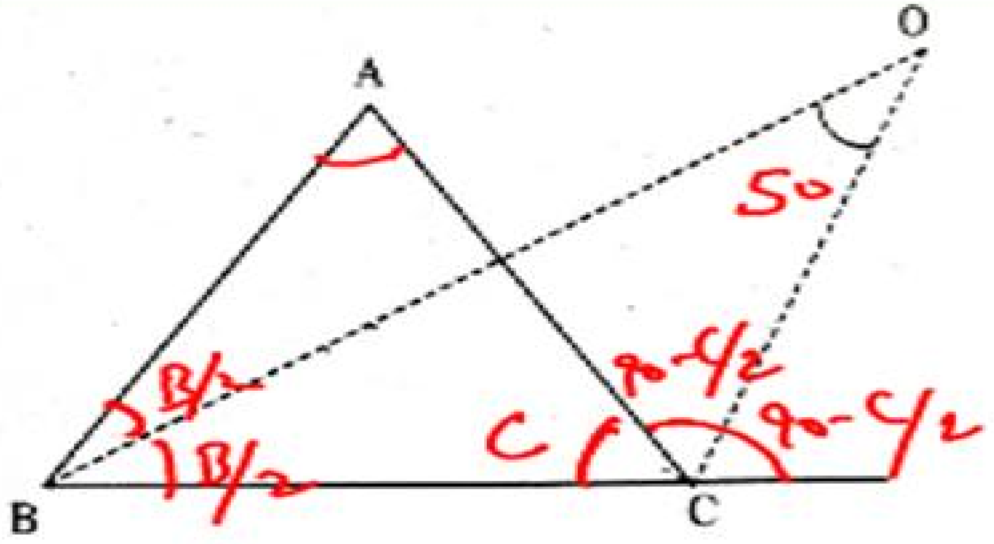
$$\frac{B}{2} + \frac{C}{2} + \angle BTC = 90$$

$$\cancel{B/2} + \cancel{C/2} + \angle BTC = A/2 + \cancel{B/2} + \cancel{C/2}$$

E.g. The bisectors of the internal angle $\angle B$ and external angle $\angle C$ of a triangle ABC intersect at O. If $\angle BOC = 50$, then $\angle A$ is :

(a) 100 (b) 60

(c) 120 (d) 90



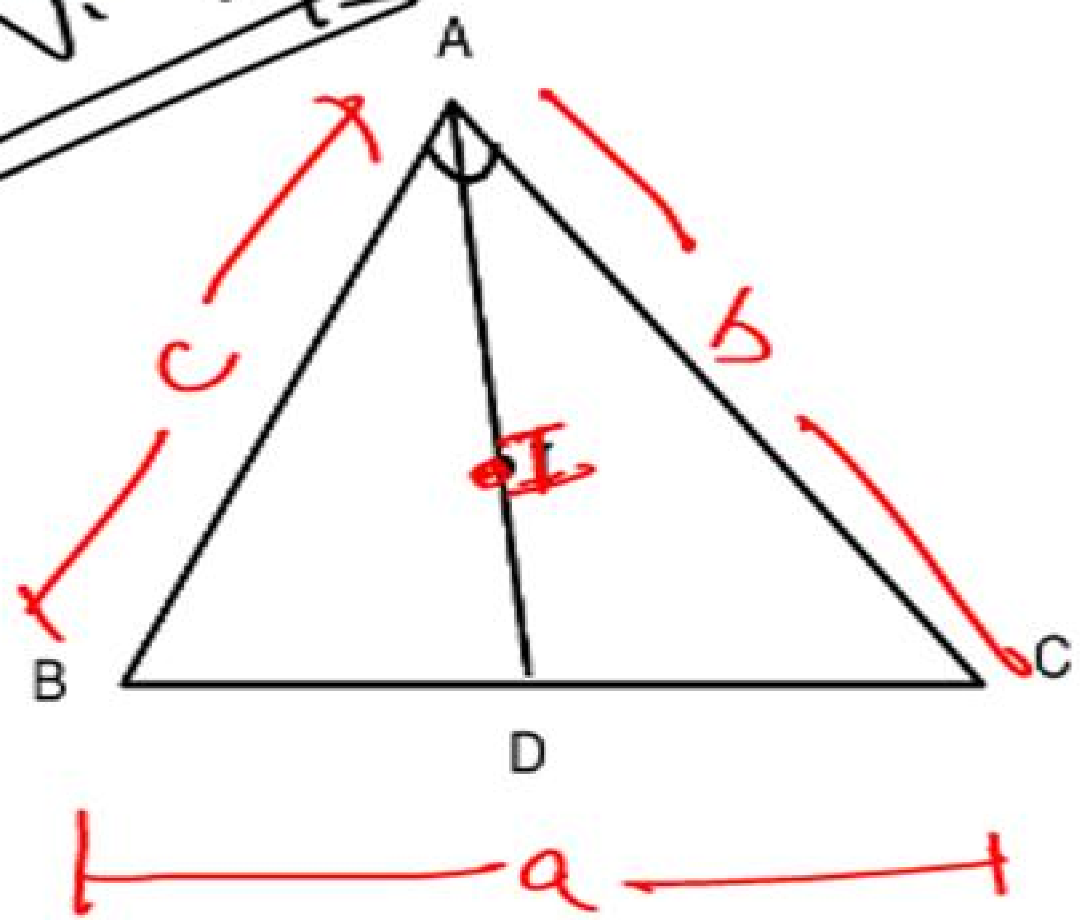
ΔBOC

$$\frac{B}{2} + 50 + 90 - \frac{C}{2} + C = 180$$

$$\frac{B}{2} + \frac{C}{2} = 40$$

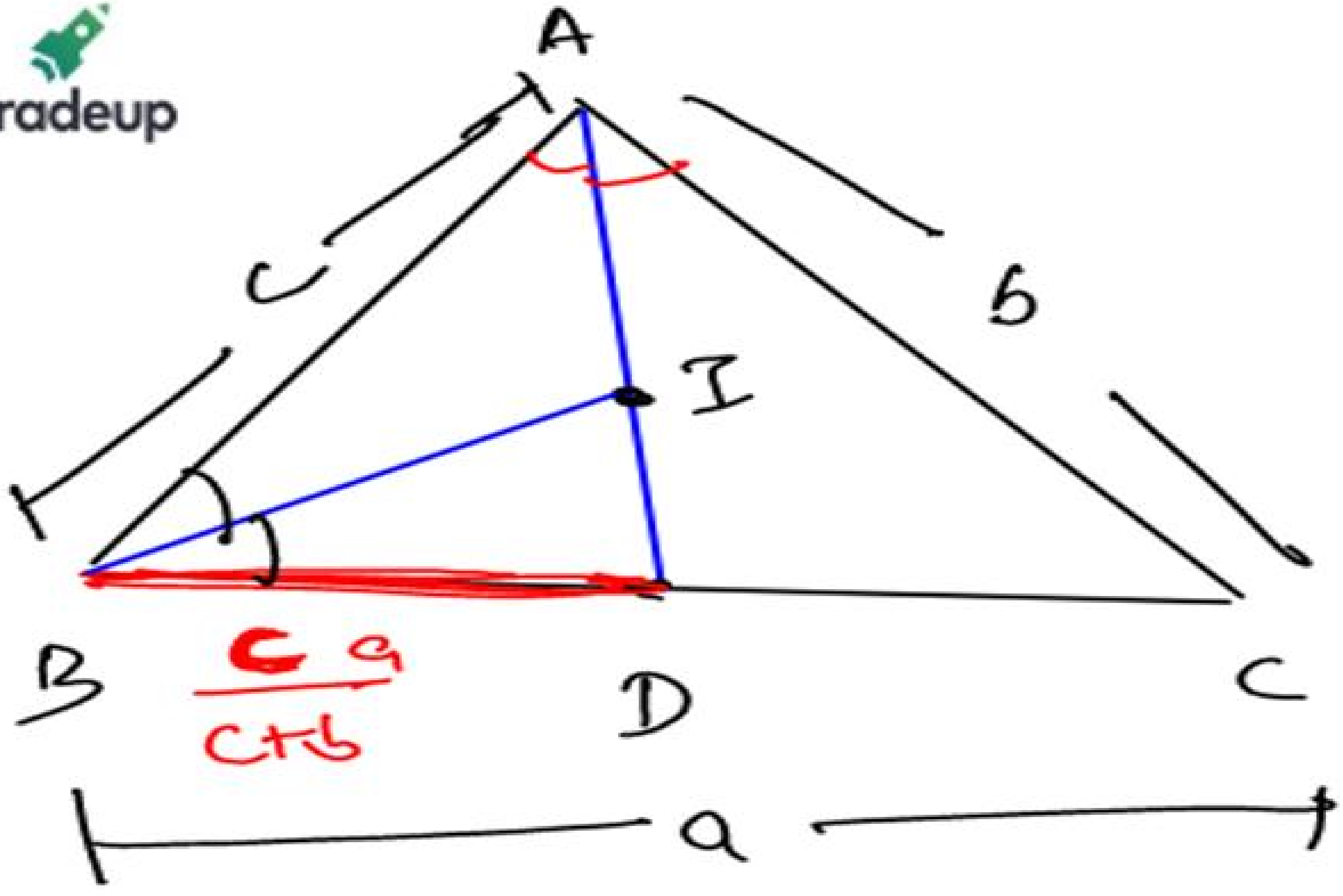
$$\boxed{B + C = 80}$$

$$\angle A = 100^\circ$$



In a ΔABC, $\mathbf{I}$ is the incentre

$$\frac{AI}{ID} = \frac{b+c}{a}$$



$AD \rightarrow$ Angle Bisector

$I \rightarrow$ Incentre

To prove $\boxed{\dfrac{AI}{ID} = \dfrac{b+c}{a}}$

$$\frac{c}{b} = \frac{BD}{DC}$$

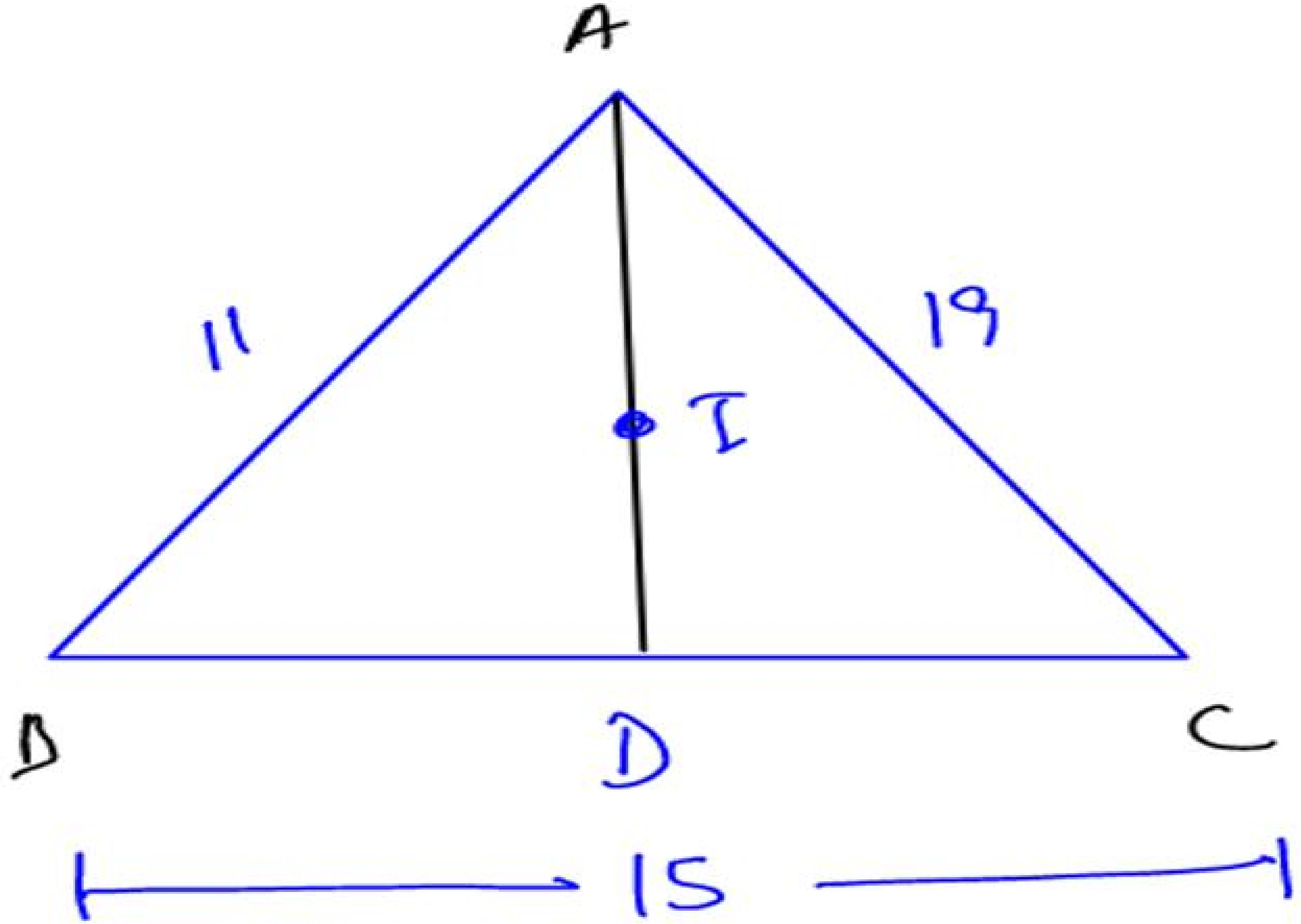
$$\boxed{BD = \frac{ca}{c+b}}$$

ΔABD

$$\frac{AB}{BD} = \frac{AI}{ID}$$

$$\frac{\cancel{c}(c+b)}{\cancel{c}a} = \frac{AI}{ID}$$

E.g. In a ΔABC, AD is the angle bisector of $\angle A$ meeting BC at D.
If AB = 11 cm, BC = 15 cm and AC = 19 cm
Find AI : ID (where I is the incentre of ΔABC)

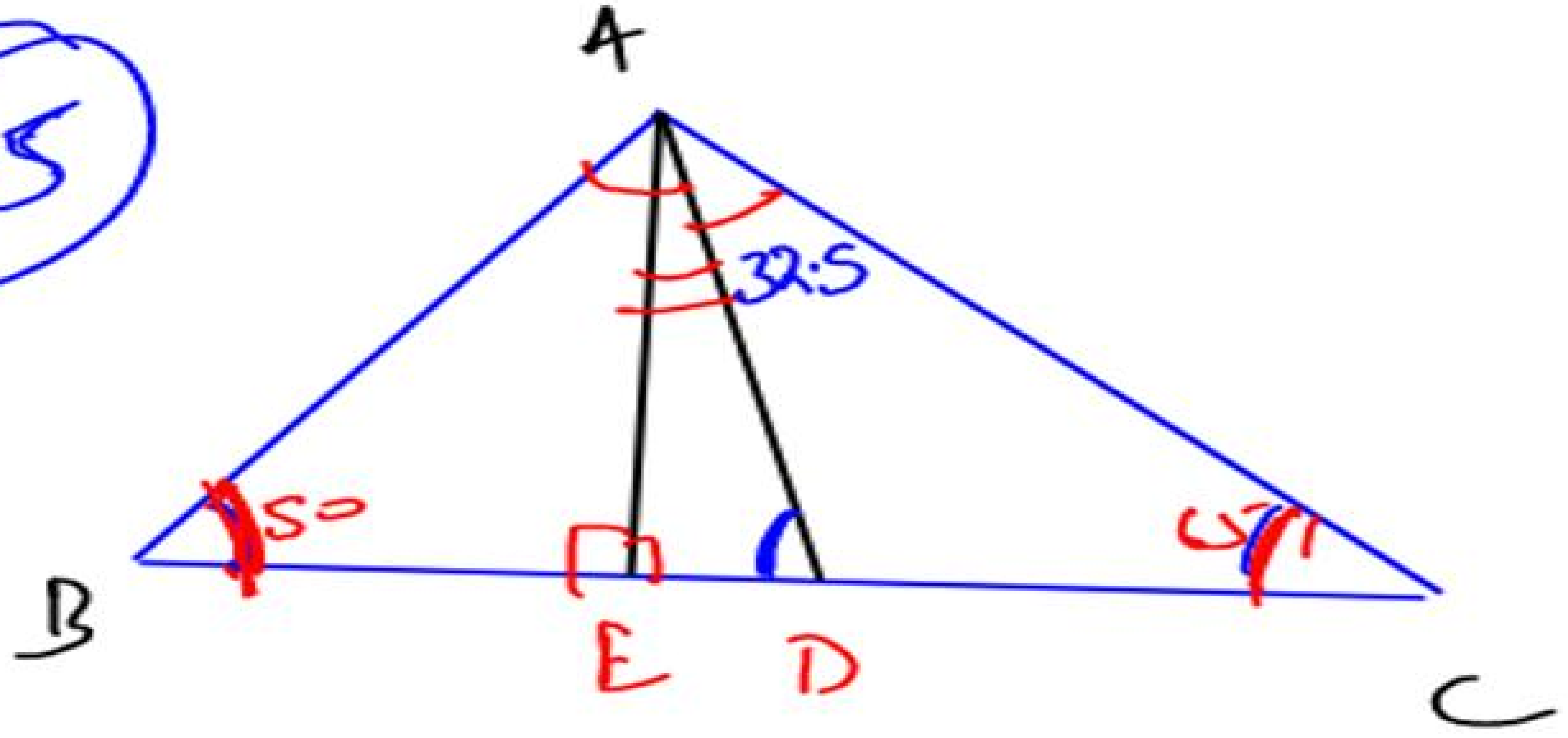


$$\frac{AI}{ID} = \frac{11+19}{15} = \frac{2}{1}$$

E.g. In a ΔABC, AE is perpendicular to BC and AD is the angle bisector of $\angle BAC$ meeting BC at E and D respectively. If $\angle B = 50$ and $\angle C = 65$.

Find $\angle EAD$.

7.5



$\angle EAD = ??$

$\angle A = 65$

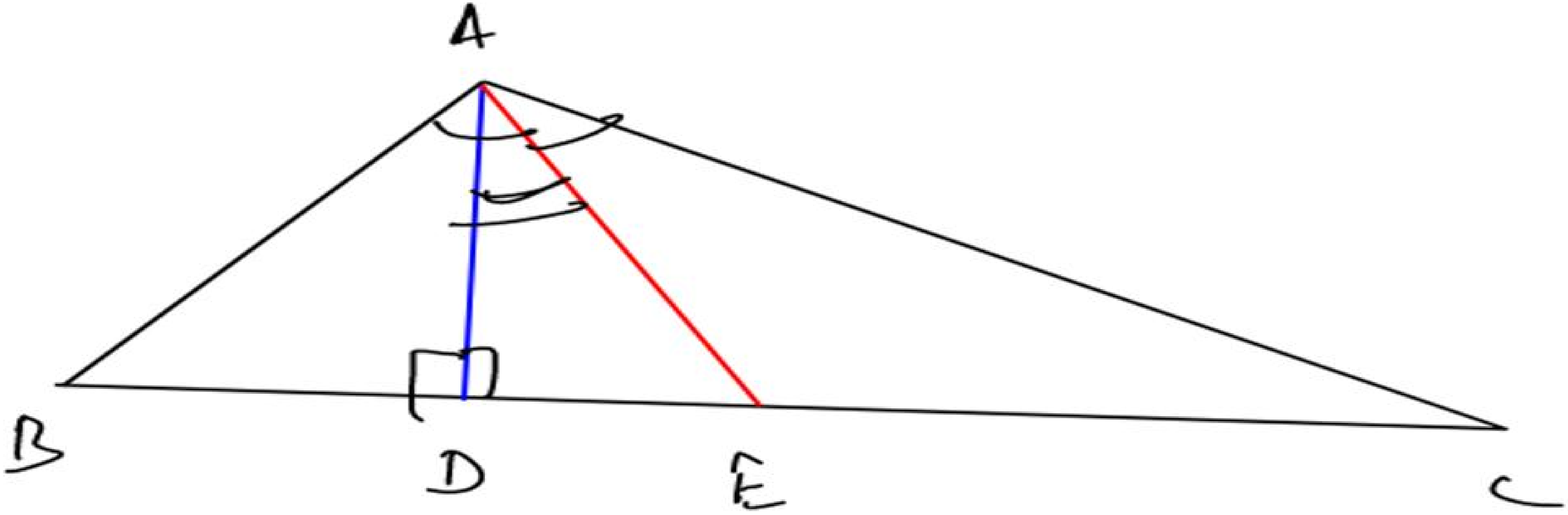
$\angle ADE = 65 + 32.5$

$= 97.5$

ΔEAD

$\angle EAD + 90 + 97.5 = 180$

$\angle EAD = -7.5$

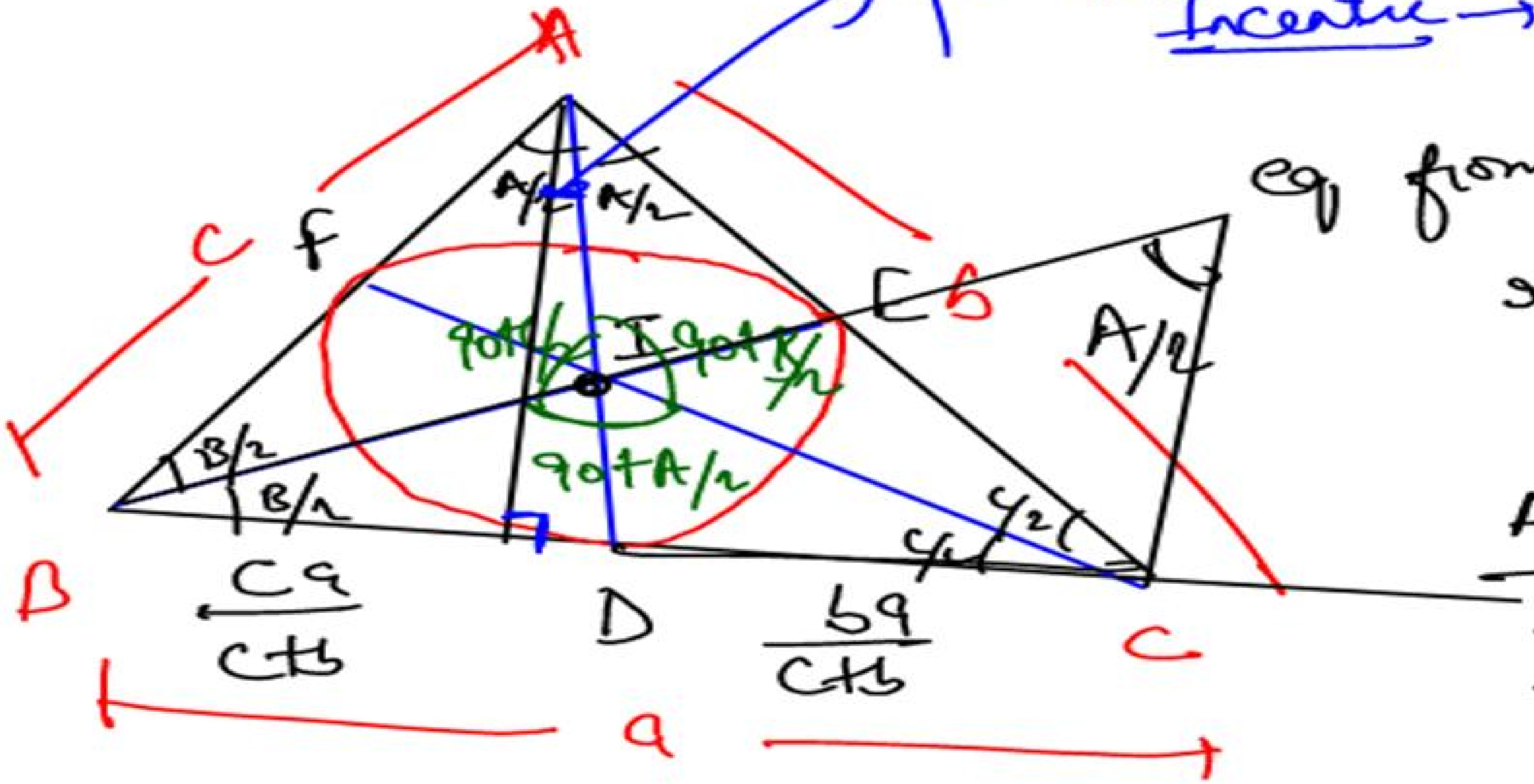


$$\angle DAE = \left| \frac{\angle B - \angle C}{2} \right|$$

$\left|\frac{\angle B - \angle C}{2}\right|$

<u>Incentre</u> → Meeting Angle Bisector

eq from sides

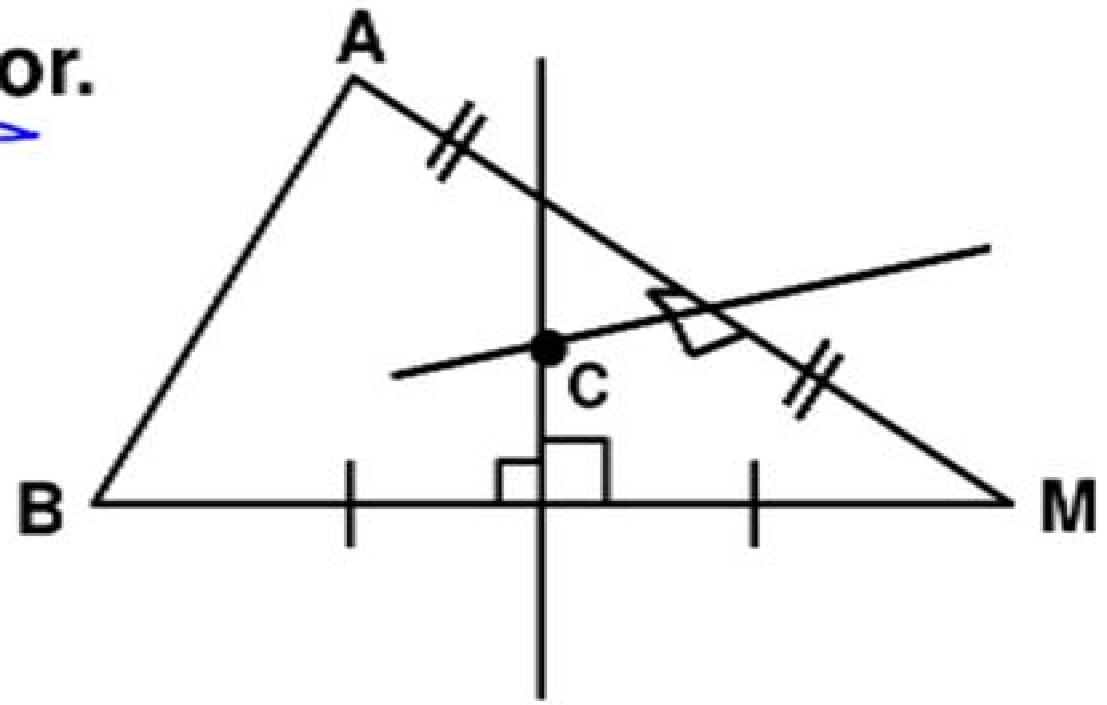


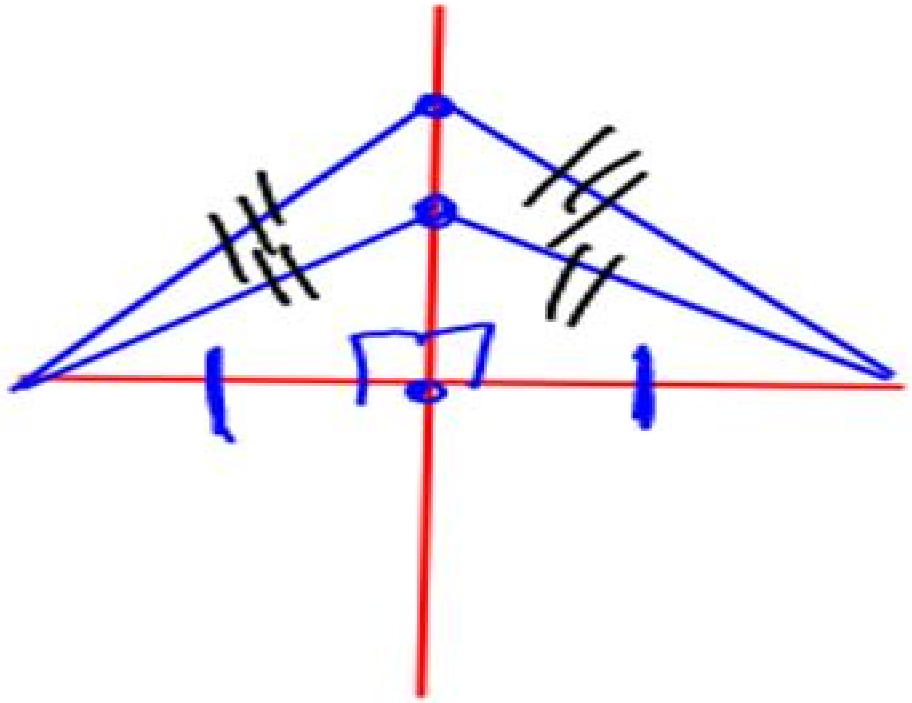
$$r = \frac{\text{Area}}{s}$$

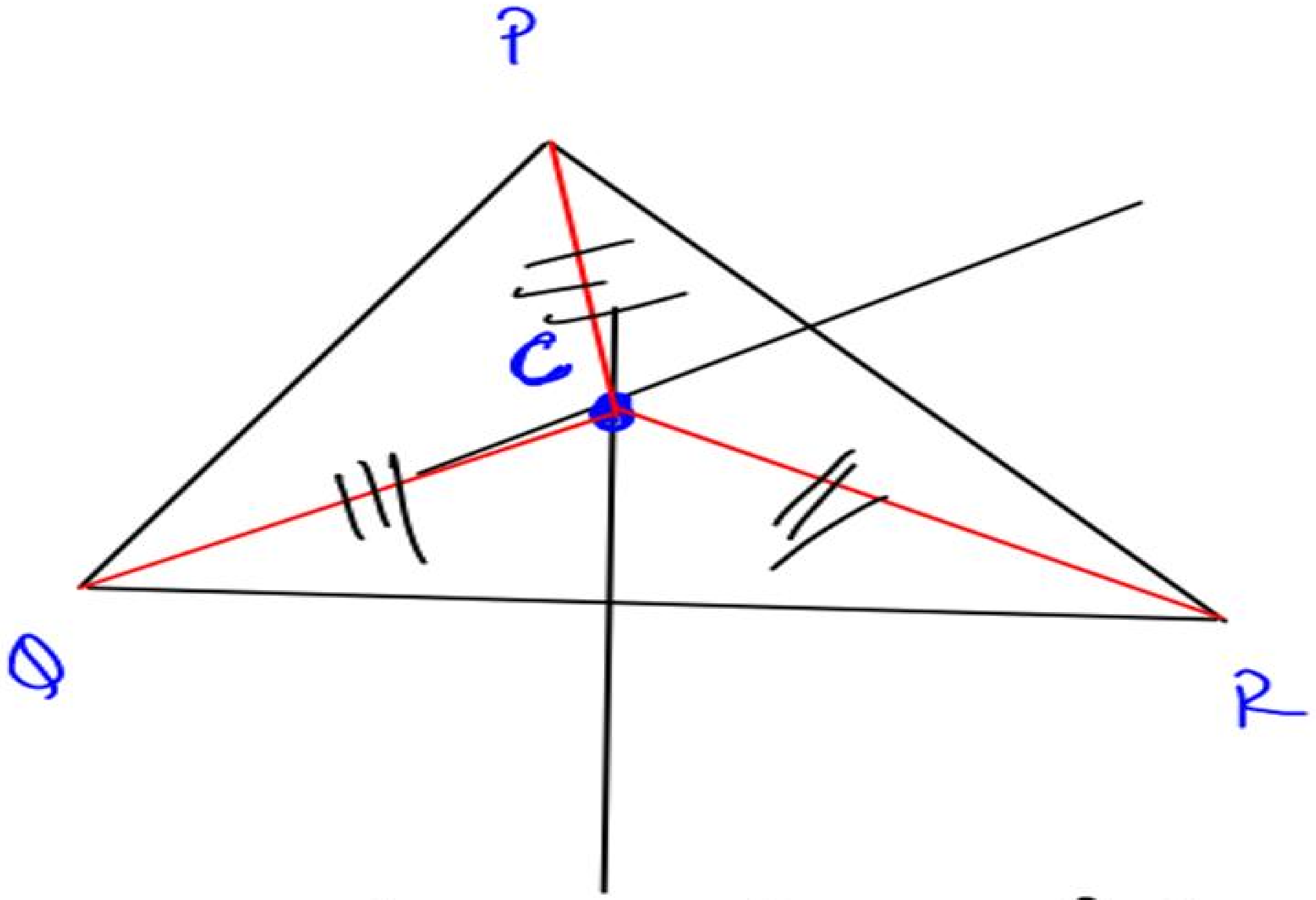
$$\frac{AI}{ID} = \frac{b+c}{a}$$

CIRCUM CENTRE

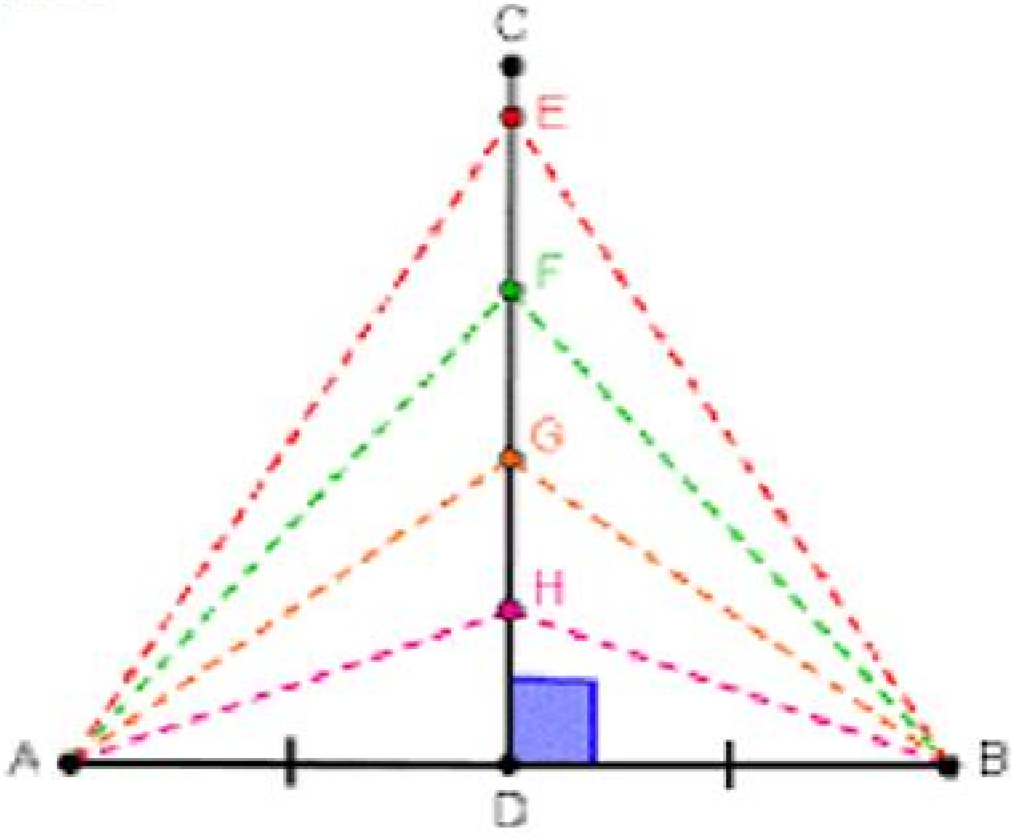
Def: Meeting point of all perpendicular bisector.





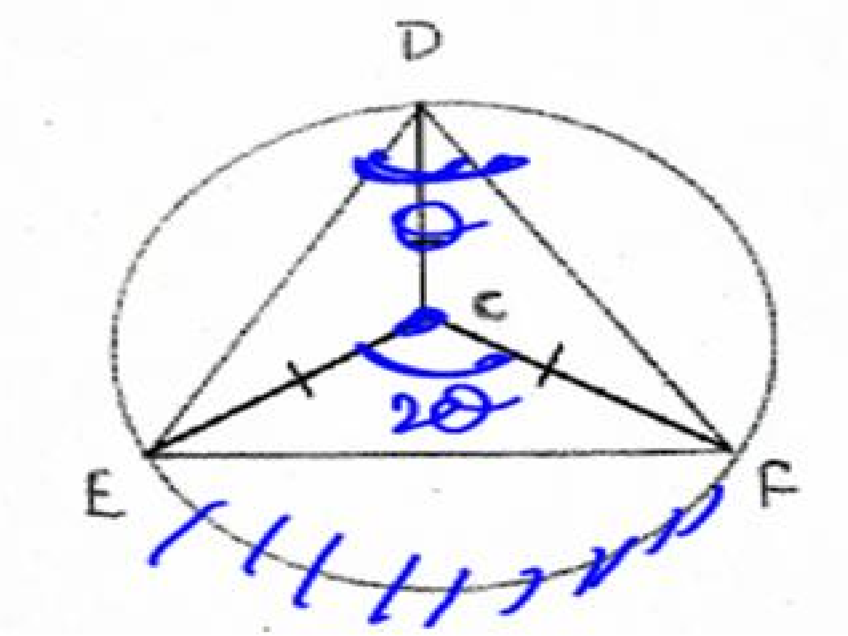


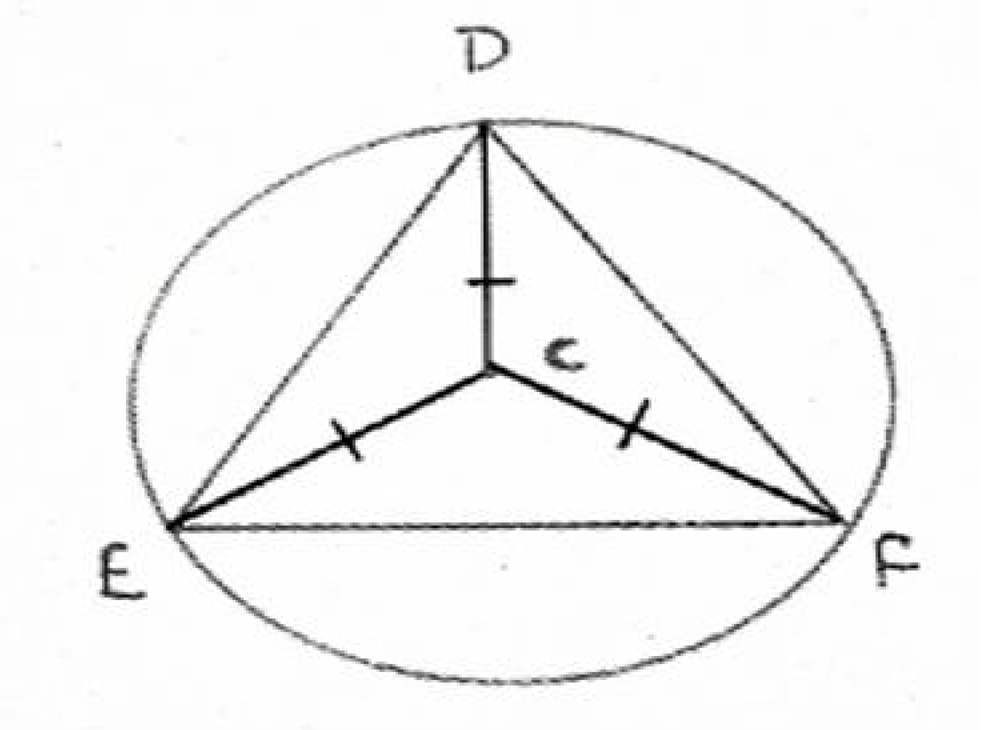
$QC = RC = PC$



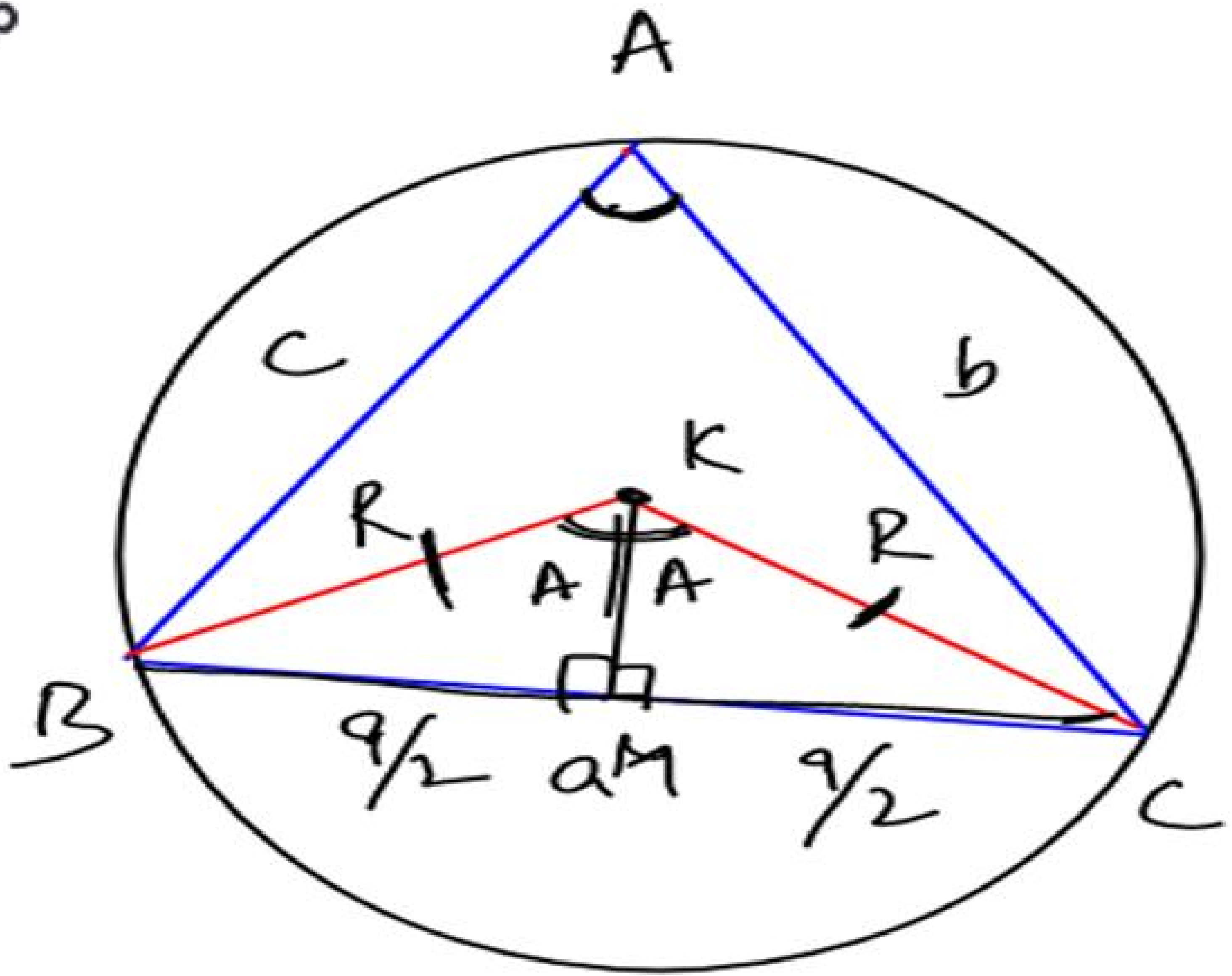
Any point on perpendicular bisector of AB, is equidistant from the end points of line segment AB.

Circumcentre is equidistant from the vertices of triangle.





CIRCUM RADIUS (R) = $\dfrac{abc}{4\Delta}$



$K \rightarrow$ circumcentre

$\angle BKC = 2A$

$\Delta\ KMC$

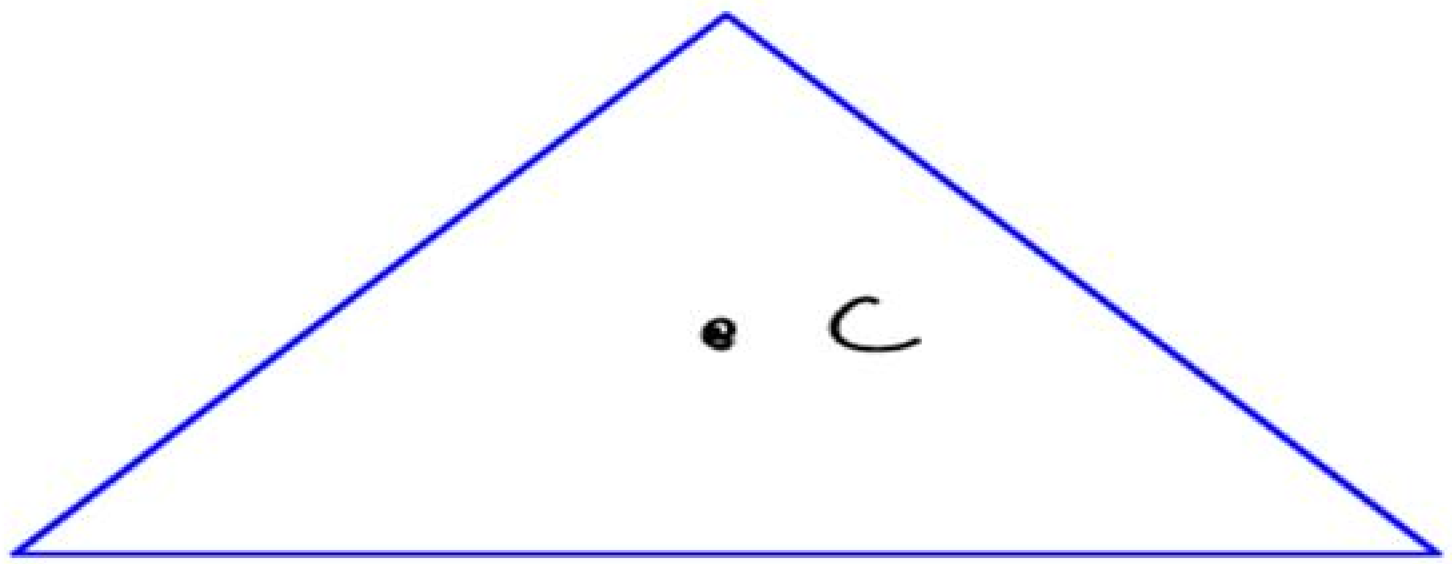
$$\sin A = \frac{a}{2R}$$

$$\frac{1}{2} bc \sin A$$

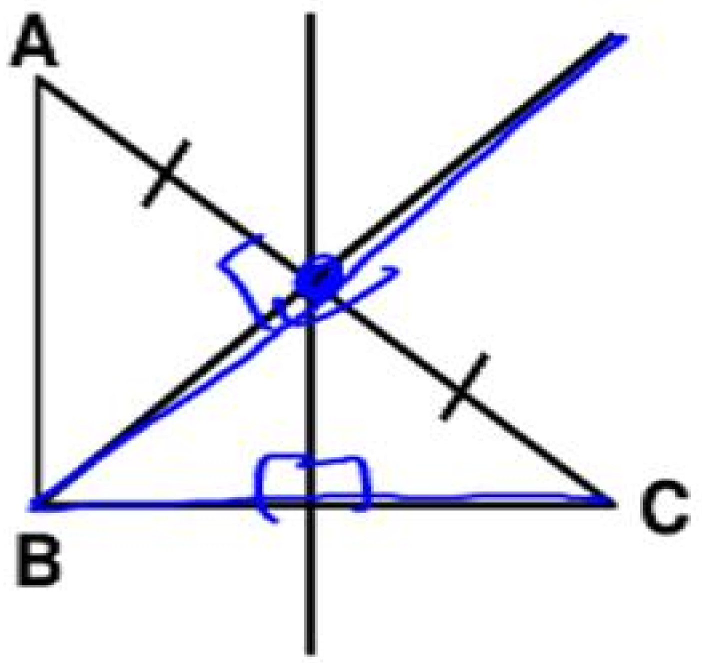
$$\frac{1}{2} \frac{bc\,a}{2R} = \frac{abc}{4R}$$

POSITION OF CIRCUM CENTRE

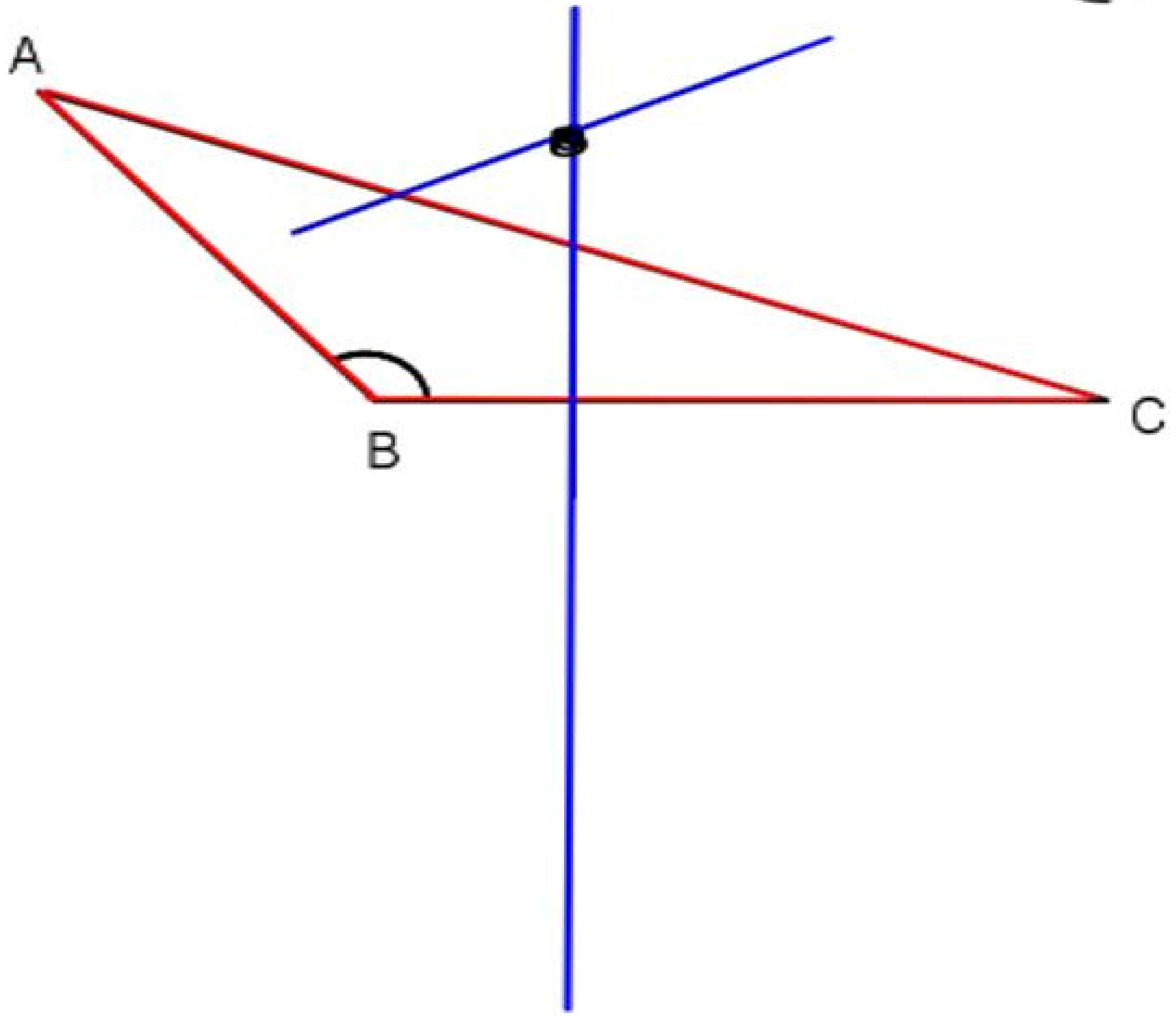
(1) Acute angle Δ → lies inside the Δ

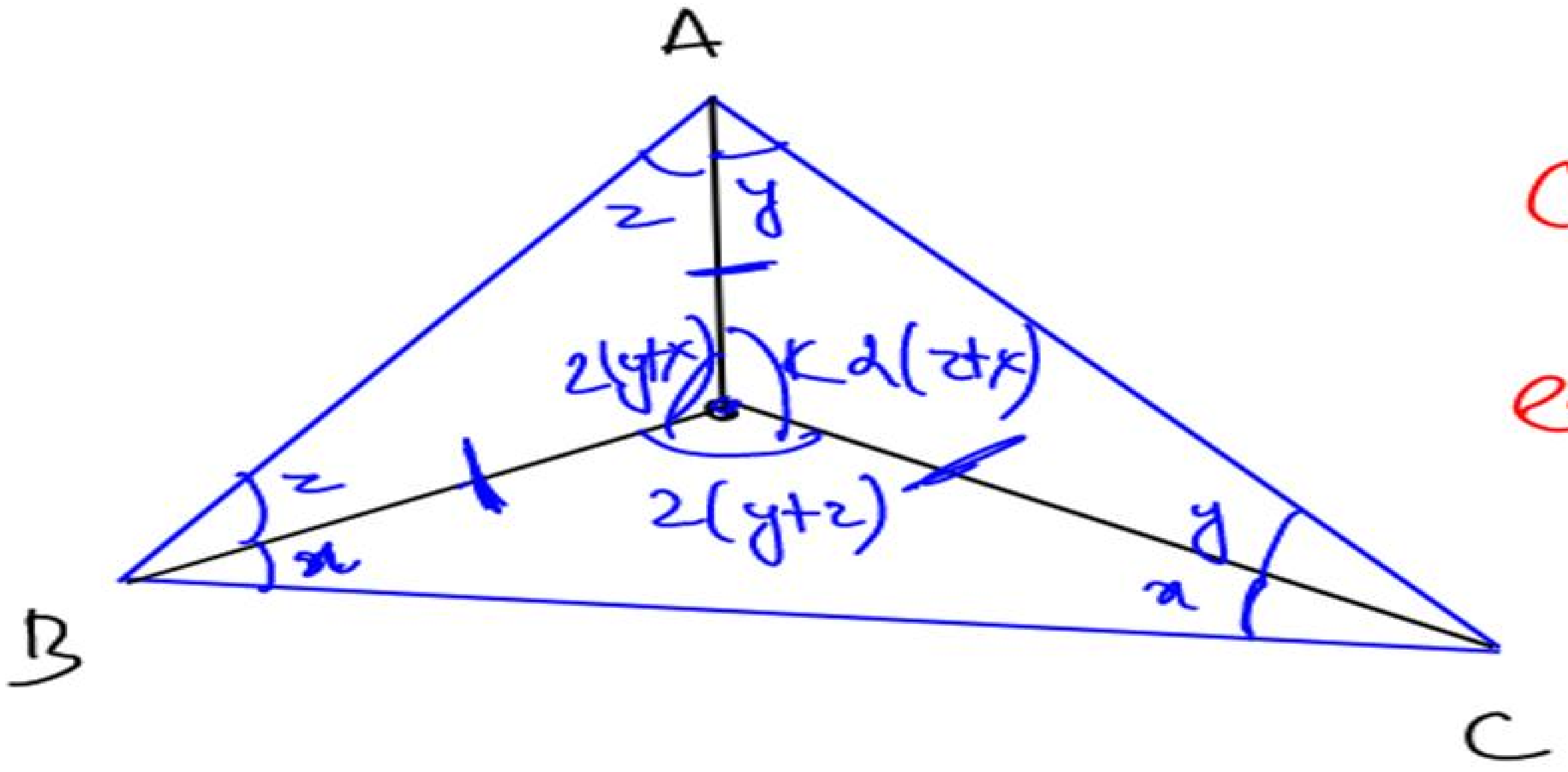


(2) Right angle Δ $\rightarrow$ mid-point of hypotenuse



(3) Obtuse angle Δ $\rightarrow$ outside the Δ





Circumcentre is equidistant from vertices of $\triangle$

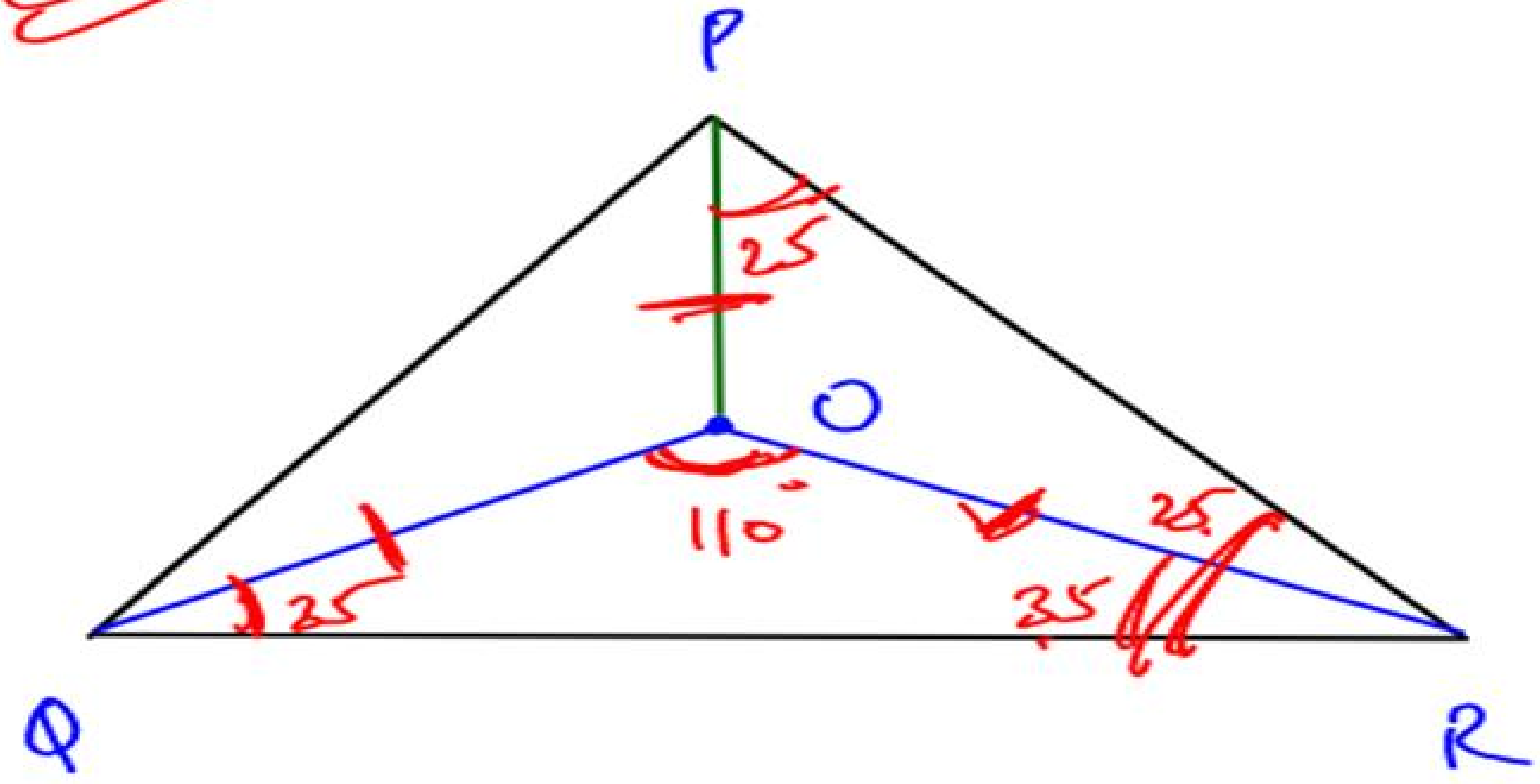
E.g. If O be the circumcentre of a triangle PQR and $\angle QOR = 110$, $\angle OPR = 25$, the measure of $\angle PRQ$ is :

(a) 65

(b) 50

(c) 60

(d) 55



(1) In all Δ's O, G & C are collinear.

O → Orthocentre

I → Incentre

G → Centroid

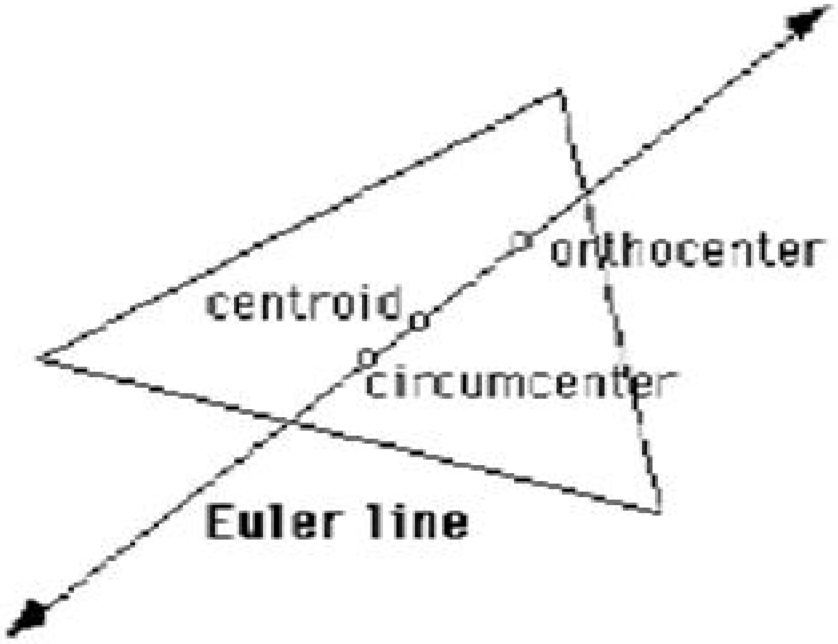
C → Circumcentre

(2) In Isosceles Δ, O, G, C and I are collinear.

(3) In Equilateral Δ, O, G, C and I are coincide.

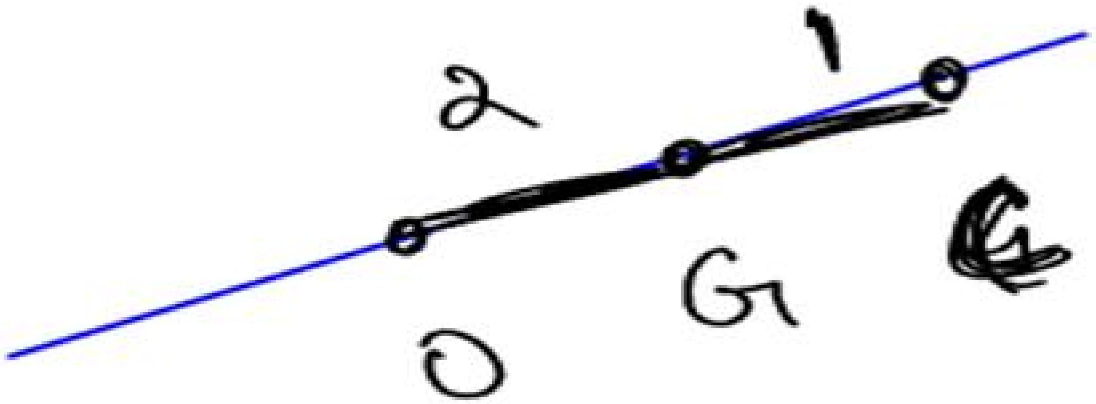
$$\frac{\text{All Mathematics}}{\text{My Knowledge}} \approx \infty$$

Euler's Line



Centroid divides the line segment which joins orthocentre & circumcentre in 2 : 1.

OG : GC = 2 : 1



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Ans

Distance between C and I

$$D = \sqrt{R^2 - 2Rr}$$

where,

R → circumradius

r → inradius

$$R^2 - 2Rr \geq O$$

$$R^2 \geq 2Rr$$

$$R \geq 2r$$

$$\frac{R}{r} \geq 2$$

Equilateral Δ

$$\boxed{\frac{R}{r} = 2}$$

Distance between G and C

$$D^2 = R^2 - \frac{1}{9}\left(a^2 + b^2 + c^2\right)$$

not important

where,

R → circumradius

a, b, c are sides of Δ