Factors

NUMBER OF FACTORS OF A NUMBER
If N is a composite number such that N = $a^p b^q c^r$ …… where a, b, c are prime factors of N and p, q, r …… are positive integers, then the number of factors of N is given by the expression.
= (p + 1) (q + 1) (r + 1) …...
Example 1: Find the number of factors of 140
Solution:
$140 = 2^2 \times 5^1 \times 7^1$
Hence 140 has (2 + 1) (1 + 1) (1 + 1), i.e., 12 factors.
Please note that the figure arrived at by using the above formula includes 1 and the given number N also as factors. So, if you want to find the number of factors the given number has excluding 1 and the number itself, we find out (p + 1) (q + 1) (r + 1) and then subtract 2 from that figure.
In the above example, the number 140 has 10 factors excluding 1 and 140 itself.
Product of all the factors of a number:
From the above given example of number 48. The total number of factors will be 10 i.e. 1, 2, 3, 4, 6, 8, 12, 16, 24, 48. If we try to find the product of all the factors of 48 then we have to multiply all the factors. But we can see that
Multiplication of 1 and 48 = 48;
Multiplication of 2 and 24 = 48;
Multiplication of 3 and 16 = 48;
Multiplication of 4 and 12 = 48;
Multiplication of 6 and 8 = 48
Thus, Multiplication of a factor from the beginning and a corresponding factor from the end, results into the number itself. Here, in the example 48 has total 10 factors and in the product, we have to multiply 48 to itself total 5 times (10 ÷ 2 = 5 times).
So, in general case
Product of all the factors of a number = $\boldsymbol{Number^{\left(\frac{No. of\ total\ factors}{2}\right)}}$
So, Product of all the factors of 48 = $48^{\left(\frac{10}{2}\right)} = 48^5$
Number of odd factors of a number: The total number of odd factors of any given number can be found by calculating all the possible combinations of powers of odd prime numbers as base such as 3, 5, 7…… etc.
Example 2: Find the number of odd factors of 360.
Solution:
$360 = 2^3 \times 3^2 \times 5^1$
Thus, Total number of factors = (3+1)(2+1)(1+1) = 4 x 3 x 2 = 24
Now, Total number of odd factors = (2+1)(1+1) = 3 x 2 = 6 [taking only powers of 3 and 5 in consideration and excluding powers of 2 as 2 makes the number even]
Number of even factors of a number: The total number of even factors of any given number can be found by calculating all the factors and subtracting the number of odd factors from it.
By another method, Number of even factors can be found by keeping the power of 2 anfd ignoring 2^0 for our calculations and finding the total number of factors as usual.
Example 3: Find the number of even factors of 360.
Solution:
$360 = 2^3 \times 3^2 \times 5^1$
Total number of factors = (3+1)(2+1)(1+1) = 4 x 3 x 2 = 24
Total number of odd factors = (2+1)(1+1) = 3 x 2 = 6
Thus, Total number of even factors = 24 – 6 = 18
Alternate Method:
$360 = 2^3 \times 3^2 \times 5^1$
Total number of even factors = (3+0)(2+1)(1+1) = 3 x 3 x 2 = 18 [keeping the power of 2 intact]
Number of factors of a number that are perfect squares:

A perfect square or simply square is obtained when a number is multiplied to itself. Thus, to find the total number of factors that are perfect square, we have to look for the power 0, 2, 4, 6,….. occurring in the factorization.

Example 4: Find the total number of factors of 90 that are perfect squares.

Solution:

$90 = 2^1 \times 3^2 \times 5^1$

Total number of factors of 90 = 2 x 3 x 2 = 12

The factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45 and 90.

Here we can clearly see that only 1 and 9 are factors that are perfect squares also.

But, in the factorization of 90, 2 with power 0; 3 with power 0 and 2; 5 with power 0 will produce perfect squares.

Thus, Number of factors of 90 that are perfect squares = 1 x 2 x 1 = 2

SUM OF ALL THE FACTORS OF A NUMBER:

If a number $N = a^p b^q c^r$ …… where a, b, c…... are prime numbers and p, q, r…... are positive integers, then,

The sum of all the factors of N (including 1 and the number itself) is:

$$\left(\frac{a^{p+1}-1}{a-1}\right)\left(\frac{b^{q+1}-1}{b-1}\right)\left(\frac{c^{r+1}-1}{c-1}\right)\ldots\ldots\ldots\ldots$$

The above can be verified by an example.

Consider the number 48, when resolved into prime factors, $48 = 2^4 \times 3^1$. Here a = 2, b = 3, p = 4, q = 1.

Hence, sum of all the factors:

$$= \left(\frac{2^{4+1}-1}{2-1}\right)\left(\frac{3^{1+1}-1}{3-1}\right) = \frac{31}{1} \times \frac{8}{2} = 124$$

The list of factors of 48 are:

1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

If these factors are added, the sum is 124 and tallies with the above result.

SUM OF EVEN FACTORS OF A NUMBER:

To find the sum of even factors of a number, we will omit the factor 2^0.

Example 5: Find the sum of even factors of 2520.

Solution:

$2520 = 2^3 \times 3^2 \times 5^1 \times 7^1$

Sum of the even factors = $(2^1+2^2+2^3)(3^0+3^1+3^2)(5^0+5^1)(7^0+7^1)$

= (2+4+8)(1+3+9)(1+5)(1+7) = 14 x 13 x 6 x 8 = 8736

SUM OF ODD FACTORS OF A NUMBER:

To find the sum of odd factors of a number, we will omit all the powers of 2 altogether. Taking into consideration all other factors with their powers.

Example 6: Find the sum of odd factors of 2520.

Solution:

$2520 = 2^3 \times 3^2 \times 5^1 \times 7^1$

Sum of the odd factors = $(3^0+3^1+3^2)(5^0+5^1)(7^0+7^1)$ [Not taking powers of 2 into consideration]

= (1+3+9)(1+5)(1+7) = 13 x 6 x 8 = 624

SUM OF PERFECT SQUARE FACTORS OF A NUMBER: When we find the sum of factors that are perfect squares, we only consider the powers that make perfect squares i.e. 0, 2, 4…… and follow the usual method.

Example 7: Find the sum of factors of 2520 that are perfect squares.

Solution:

$2520 = 2^3 \times 3^2 \times 5^1 \times 7^1$

Sum of factors that are perfect squares = $(2^0 + 2^2)(3^0 + 3^2)(5^0)(7^0)$

= (1 + 4)(1 + 9)(1)(1) = 5 x 10 x 1 x 1 = 50