gradeup

Sahi Prep Hai Toh Life Set Hai

MENSURATION-2D

Part – 2

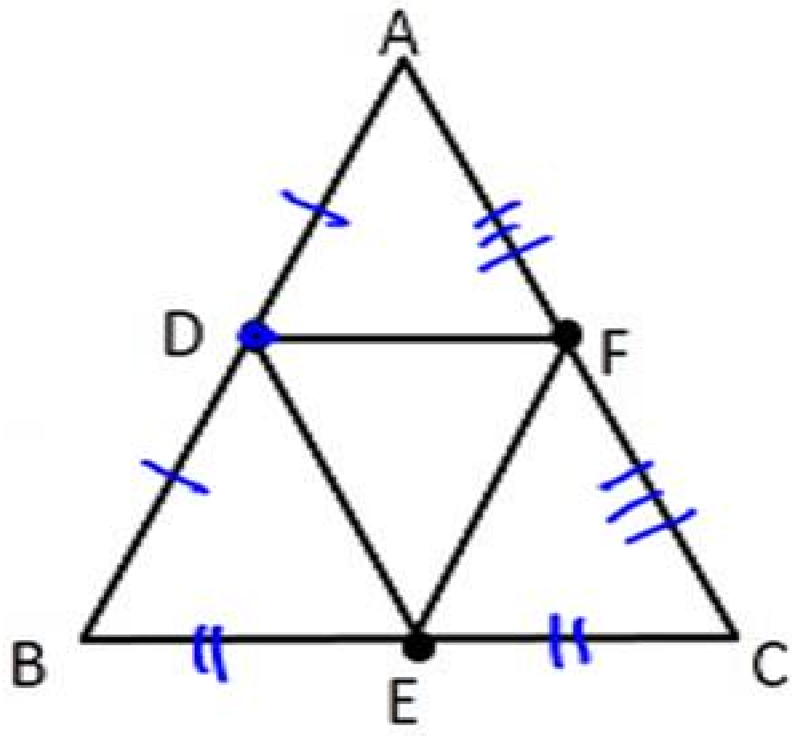
gradeup

* Area of Δ formed by joining mid pts of all sides of Δ is $\frac{1}{4}$th of the original Δ

** Perimeter of Δ formed by joining mid pt of all sides of Δ is $\frac{1}{2}$ of the original triangle.

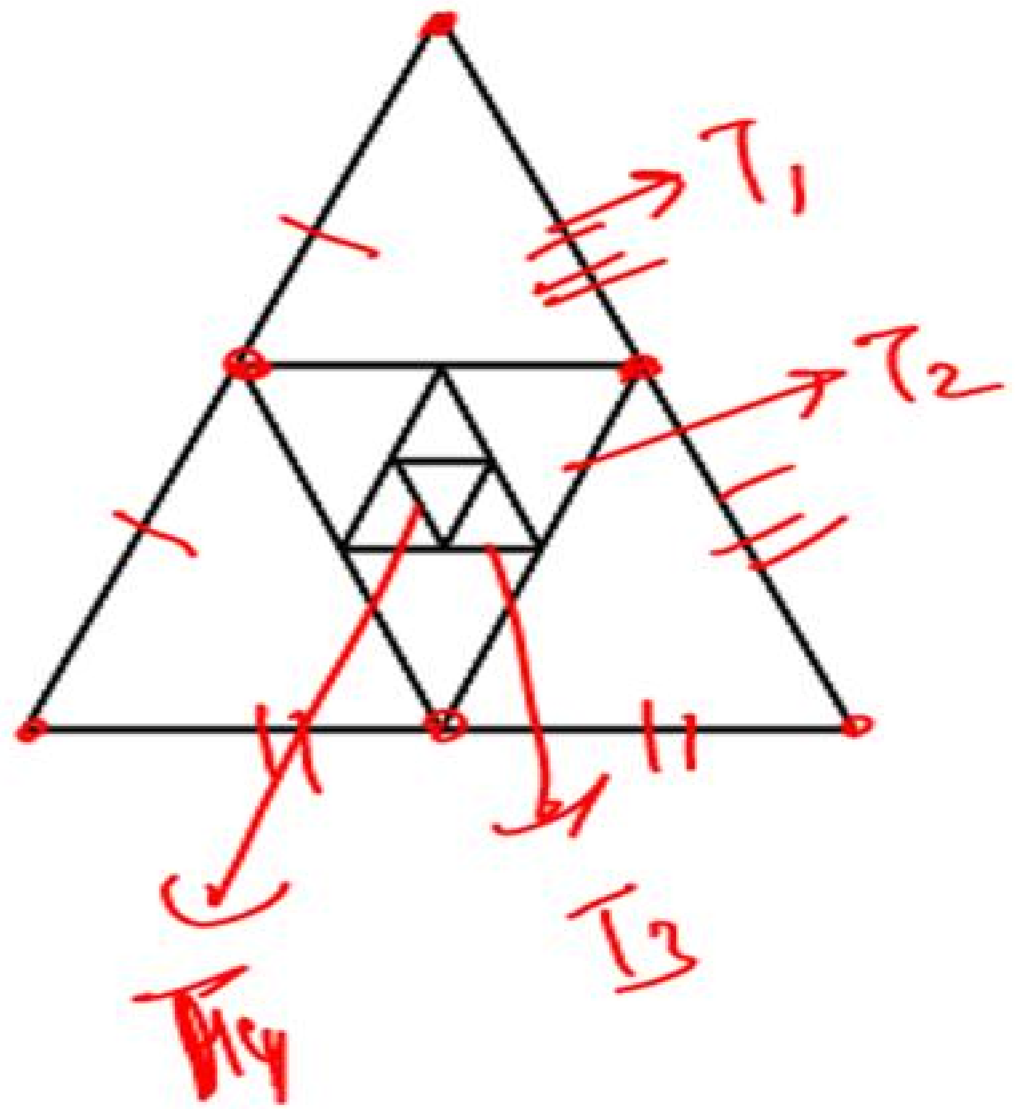
If D, E & F are midpoints of the sides AB, BC, CA

Then,



Area of $\Delta DFE = \frac{1}{4}$ (Area of ΔABC)

USAGE OF GP or Ratio



$(i)\ \dfrac{Area\ of\ T_3}{Area\ of\ T_6} = \dfrac{64}{1} = 64$

$(ii)\ \dfrac{Perimeter\ of\ T_{11}}{Perimeter\ of\ T_6} = \dfrac{1}{32}$

Ans. (i) 64 : 1

(ii) 1 : 32

gradeup

Infinite GP

20, 10, 5, 2.5, 1.25, ------→

$a \to$ first term

$a = 20$

$r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$r = \frac{1}{2}$

$S_\infty = \frac{a}{1-r} \Rightarrow \frac{20}{1-\frac{1}{2}} = 40$

$\boxed{0 < r < 1}$

gradeup

$$A = 400 + 200 + 100 + 50 + \cdots$$

$$A = \frac{a}{1-r} \Rightarrow \frac{400}{1-\frac{1}{2}} \Rightarrow 800$$

$$B = 256 + 192 + 144 + 108 + 81 + \cdots$$

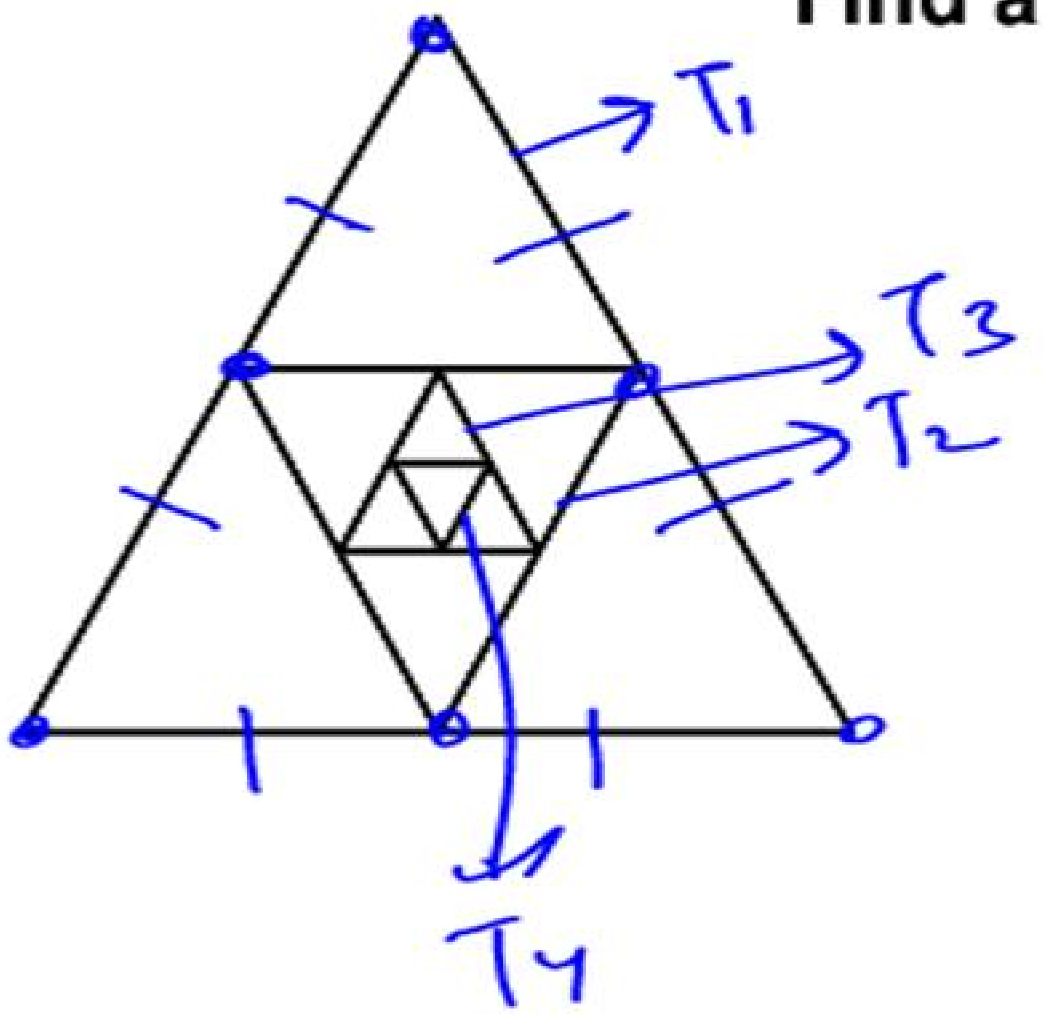
$a = 256$ $\qquad r = \frac{\cancel{192}\,3}{\cancel{256}\,4}$

$$B_\infty = \frac{256}{1-3/4} \Rightarrow \underline{\underline{1024}}$$

Equilateral Δ

(iii) If side of T_1 = 20 cm

Find area of $(T_1 + T_2 + \ldots\ldots T_\infty)$



Area of $T_1 = \frac{\sqrt{3}}{4}(20)^2$

$= 100\sqrt{3}$

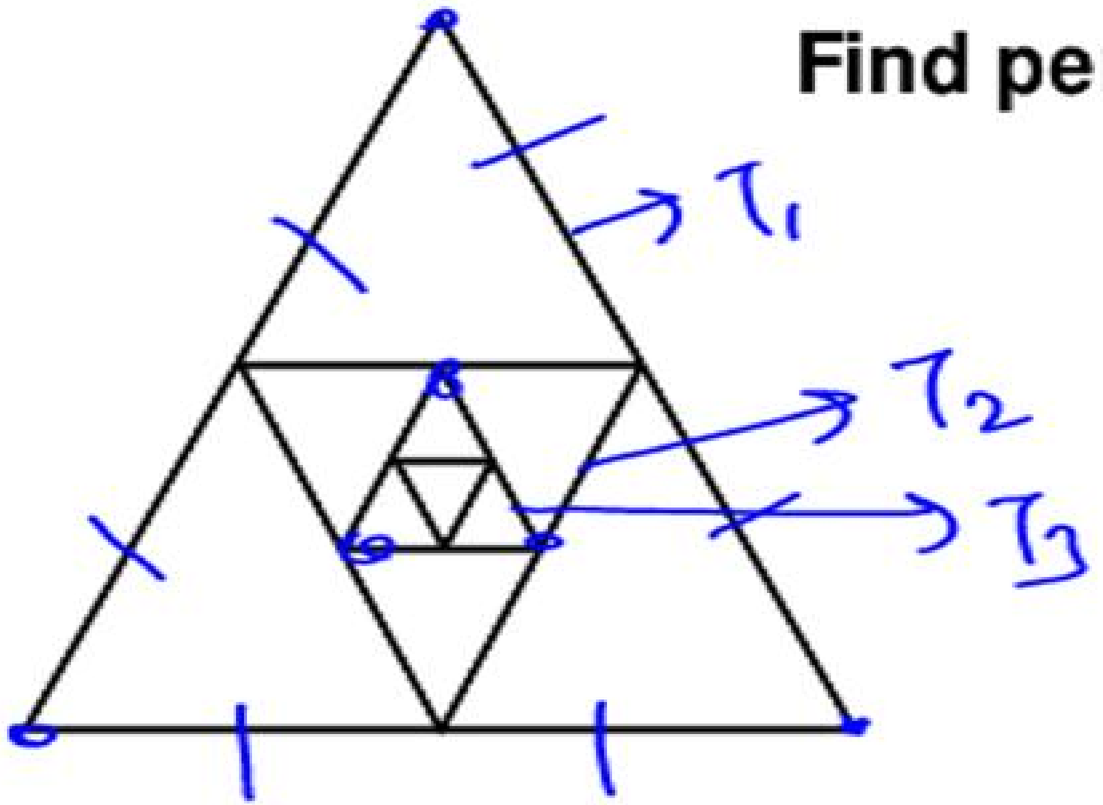
$$\frac{100\sqrt{3}}{1-\frac{1}{4}} = \frac{400\sqrt{3}}{3}\ cm^2$$

Ans. $\frac{400\sqrt{3}}{3} cm^2$

Equilateral Δ

(iv) If side of $T_1 = 20$ cm

Find perimeter of $(T_1 + T_2 + \ldots\ldots. T_\infty)$

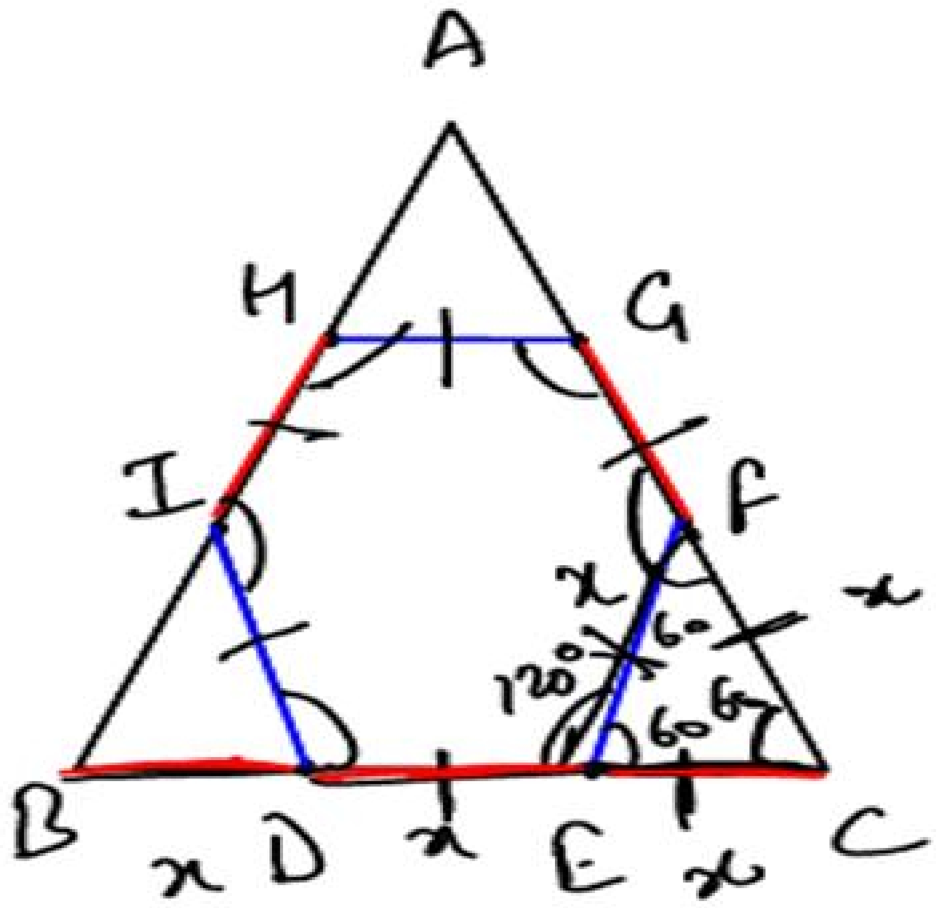


Perimeter of $T_1 = 60$

$$\frac{60}{1-\frac{1}{2}} = \underline{\underline{120}}$$

Ans. 120 cm

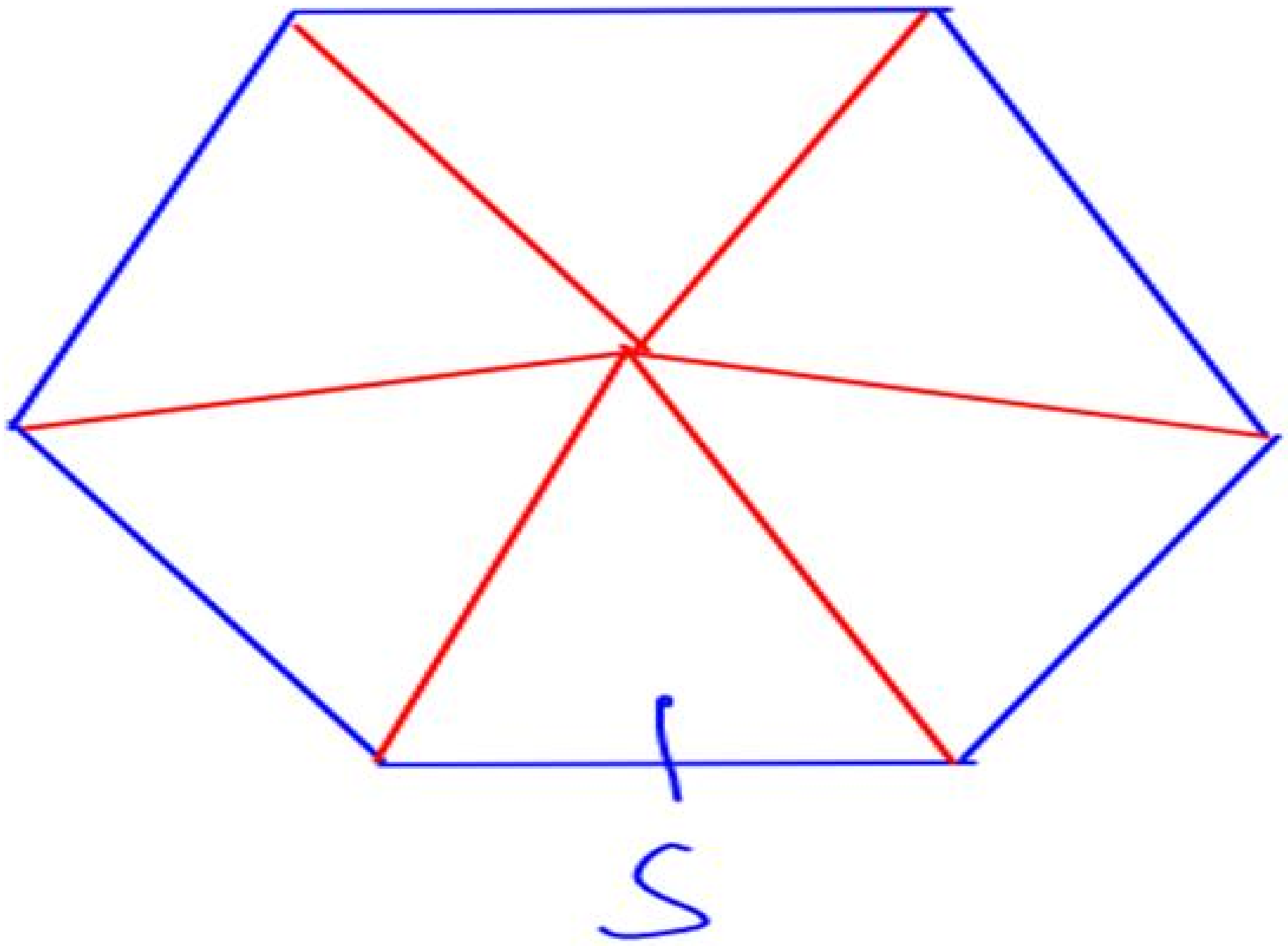
If corners of equilateral triangle are cut to form a regular hexagon.



all sides are equal

all angles are 120°

$$\frac{\text{Side of Regular Hexagon}}{\text{Side of Original } \Delta} = \frac{1}{3}$$

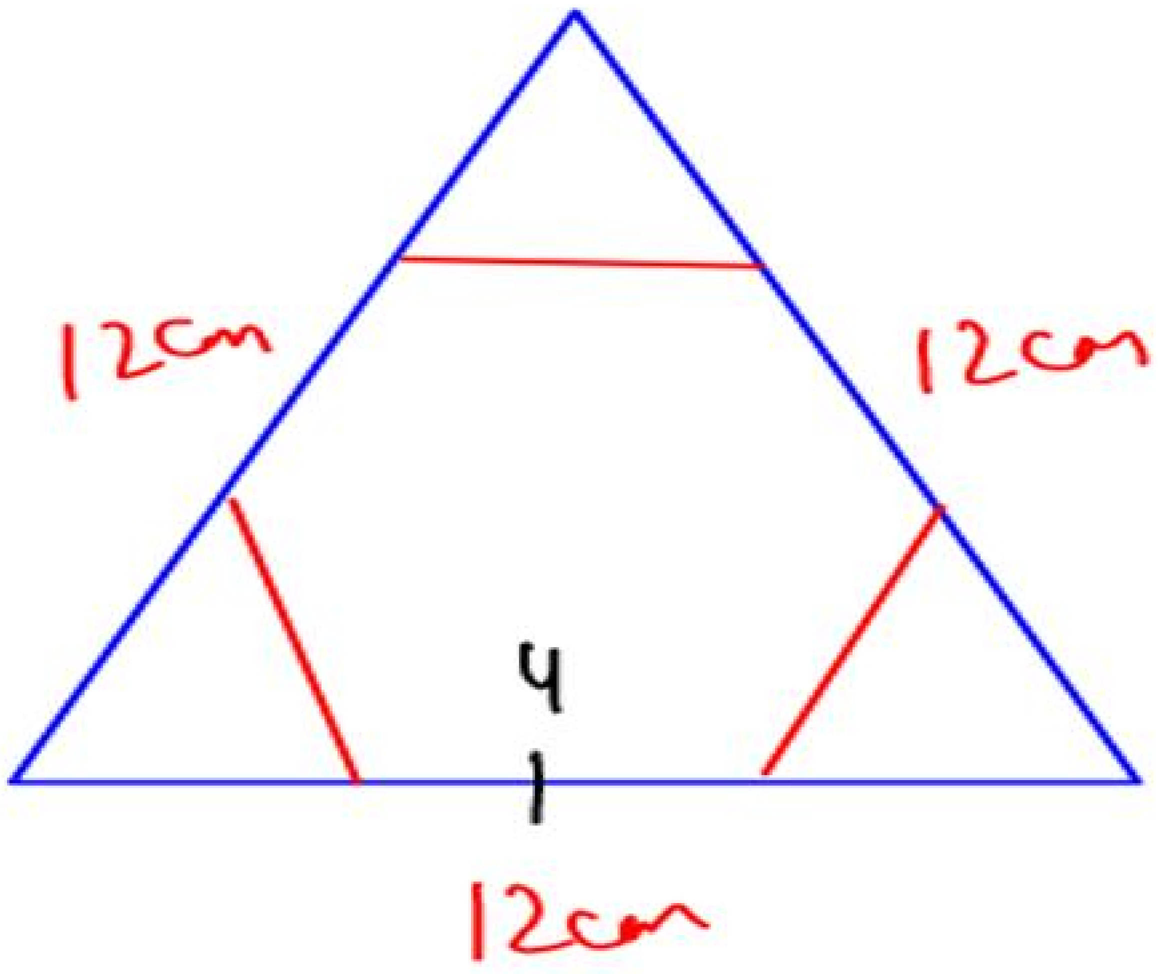


$$\overset{3}{\cancel{6}} \cdot \frac{\sqrt{3}}{\underset{2}{\cancel{4}}} s^2$$

Area of Regular Hexagon

$$= \frac{3\sqrt{3}}{2} s^2$$

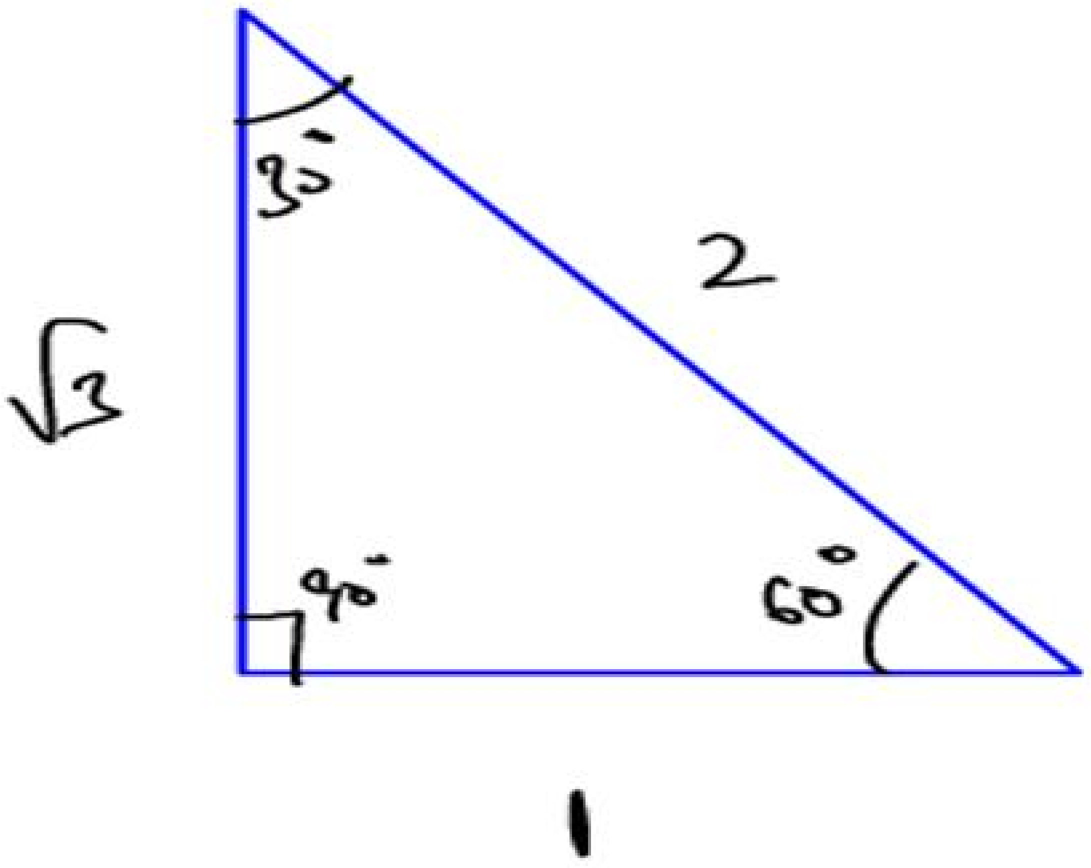
Eg. If corners of an equilateral triangle of side 12 cm are cut to form a regular hexagon. Find the area of regular hexagon.



$$\text{Area} = \frac{3\sqrt{3}}{2} \times (4)^2$$

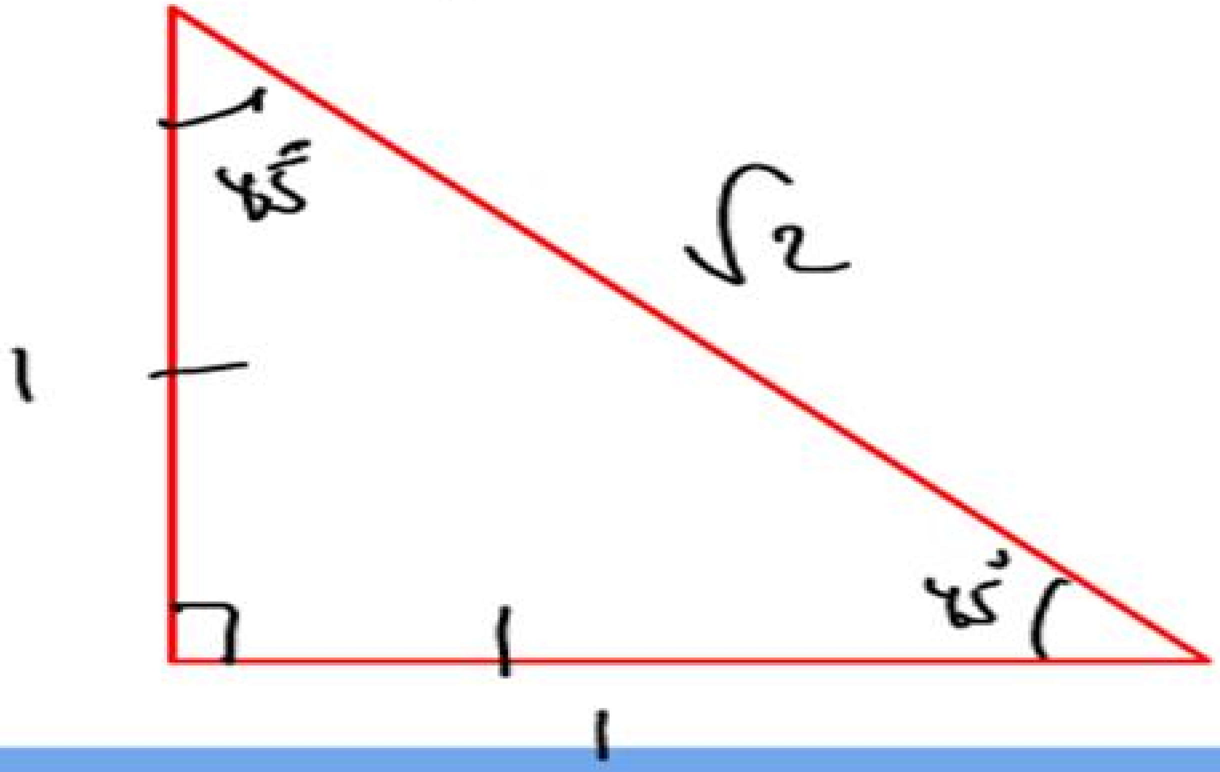
$$= \underline{\underline{24\sqrt{3}\,cm^2}}$$

Ans. $24\sqrt{3}\ cm^2$



30 – 60 – 90

$1 : \sqrt{3} : 2$

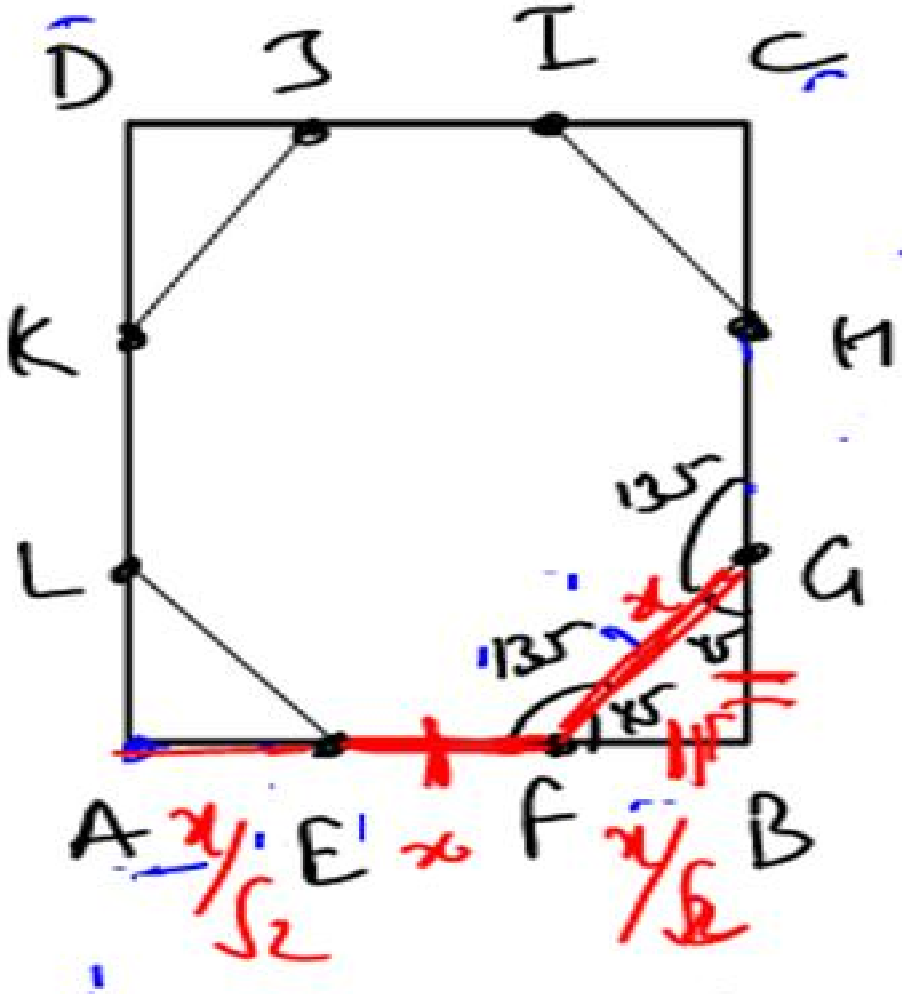


45 – 45 – 90

$1 : 1 : \sqrt{2}$

Imp

Eg. If corners of a square of side 10 cm are cut to form a REGULAR OCTAGON. Find the side of the REGULAR OCTAGON.



In Regular Octagon
All sides are equal & all angles are 135°

ΔFGB

$$AB = AE + EF + FB$$

$$10 = \frac{x}{\sqrt{2}} + x + \frac{x}{\sqrt{2}}$$

$$10 = \sqrt{2}x + x$$

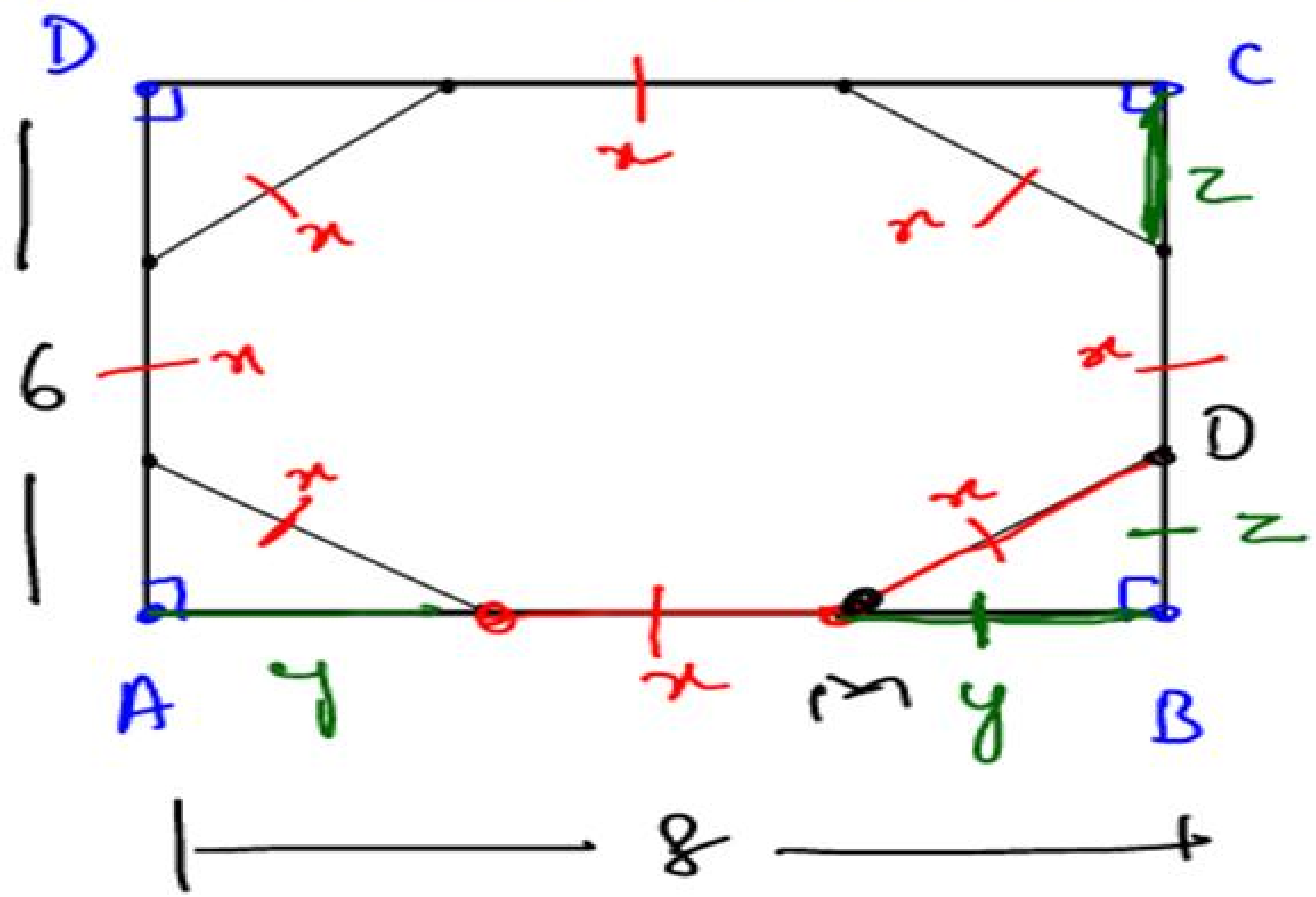
$$x = \frac{10}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = 10(\sqrt{2}-1)$$

Ans. $10(\sqrt{2}-1)$

gradeup

V.V.Imp

Eg. If corners of a rectangle of sides 6 & 8 cm, are cut to form a OCTAGON (whose all sides are equal). Find the side of the OCTAGON.



2min   PYQ of SSC

* $x + 2z = 6 \qquad x + 2y = 8$

$$y^2 + z^2 = x^2$$

$$\left(\frac{8-x}{2}\right)^2 + \left(\frac{6-x}{2}\right)^2 = x^2$$

$$64 + x^2 - 16x + 36 + x^2 - 12x = 4x^2$$

$$2x^2 + 28x - 100 = 0$$

$$x^2 + 14x - 50 = 0$$

$$x = \frac{-14 \pm \sqrt{396}}{2}$$

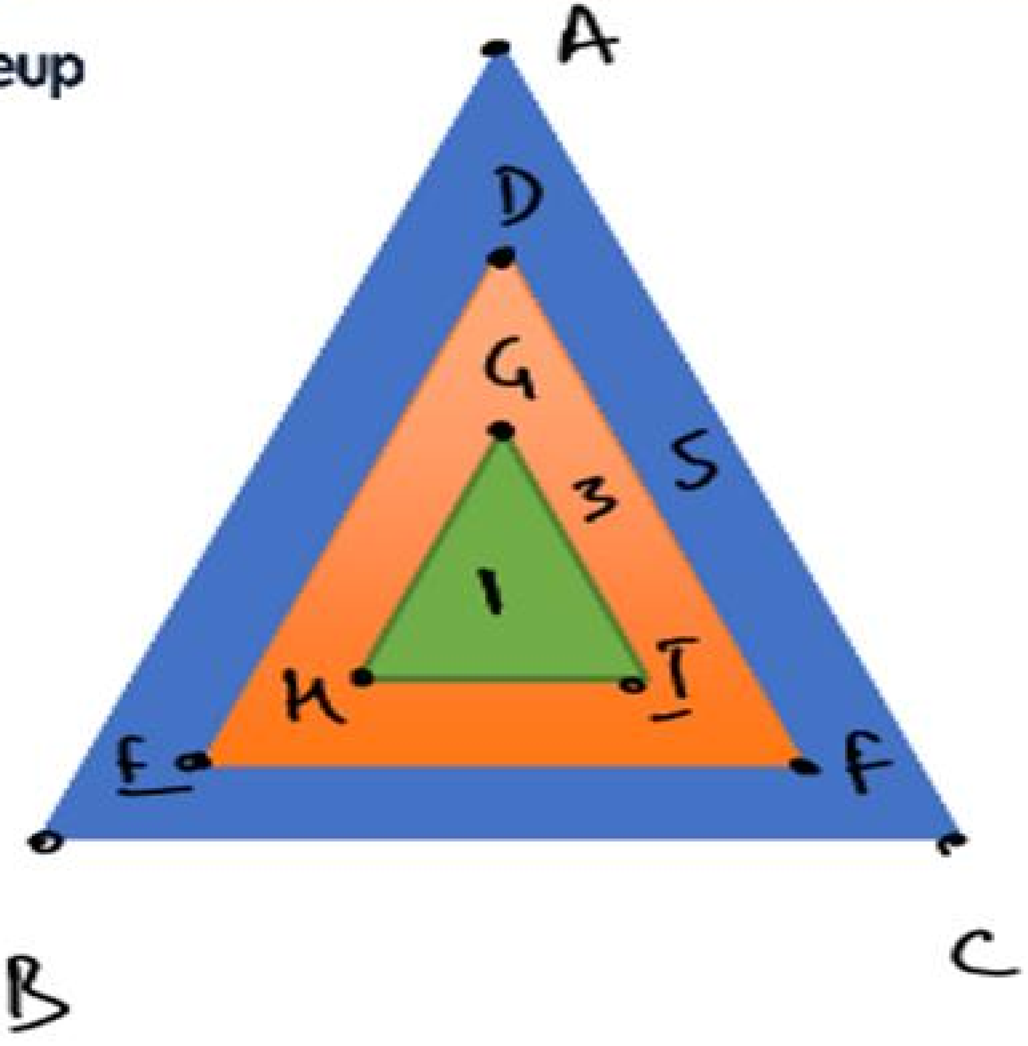
$$\boxed{x = -7 + \sqrt{99}}$$

gradeup

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ans. $-7+\sqrt{99}$



USAGE OF RATIO

All are equilateral Δ's

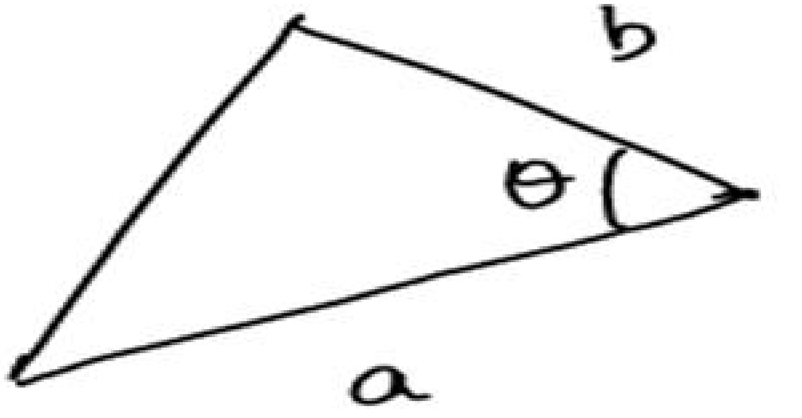
If area of Green : Orange : Blue = 1 : 3 : 5

Find the ratio of their sides.

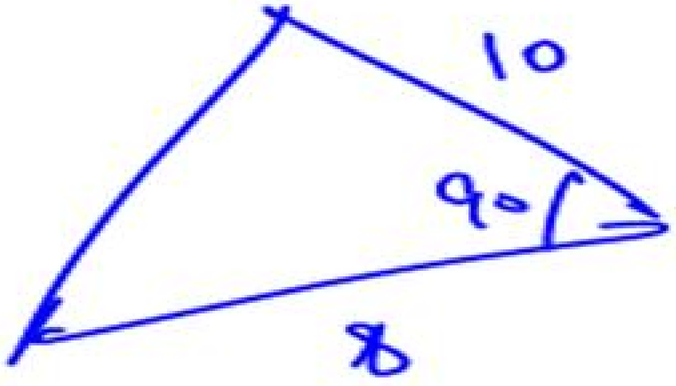
	ΔABC	:	ΔDEF	:	ΔGHI
	Green + Orange + Blue		Green + Orange		Green
Areas	9	:	4	:	1
side	3	:	2	:	1

If two sides of a triangle are given, then the maximum area is of right angle triangle.

Eg. If two sides of a triangle are 8 and 10 cm, find the maximum area of the triangle.



$\frac{1}{2} ab \sin\theta$



$\text{Area} = \frac{1}{\cancel{2}} \cdot \overset{4}{\cancel{8}} \cdot 10 \sin\theta$

$40 \cdot 1$

$= 40 \text{ cm}^2$

Ans. $40\ cm^2$

gradeup

If perimeter of a triangle is given, then the maximum area is of equilateral triangle.

Eg. If perimeter of a triangle is 60 cm. What is the maximum area of Δ?

$$3 \cdot S = 60$$

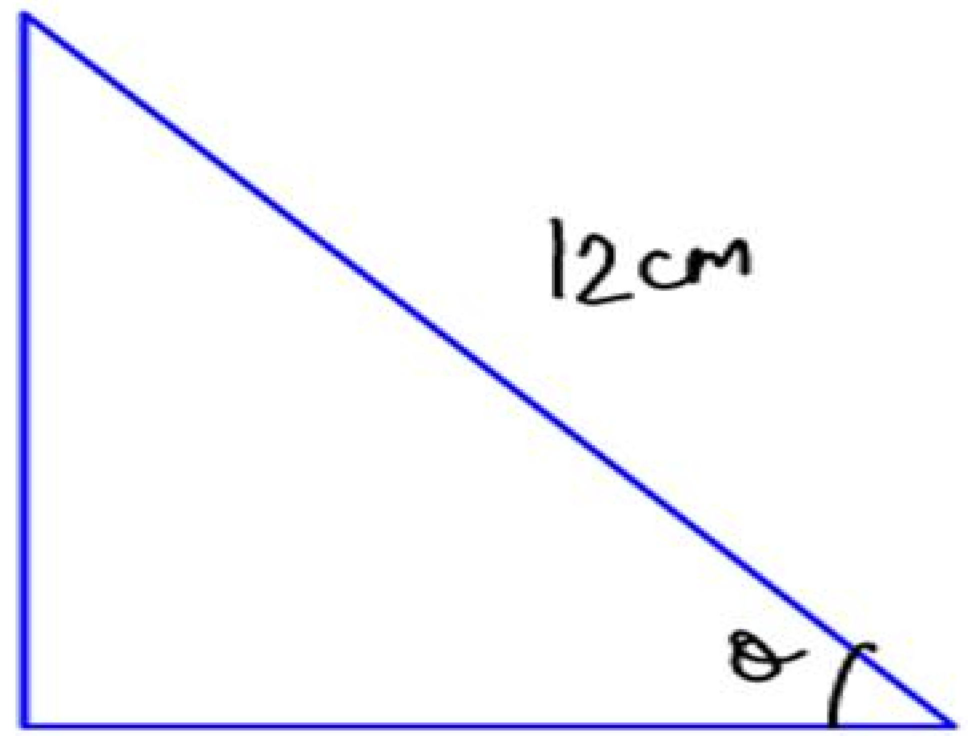
$$S = 20$$

$$\text{Area} = \frac{\sqrt{3}}{4} \cdot (20)^2 \Rightarrow 100\sqrt{3}$$

Ans. $100\sqrt{3}\ cm^2$

gradeup

Eg. If hypotenuse of a right angle triangle is 12 cm. Find the maximum area of a triangle.



Area of Right angle Δ

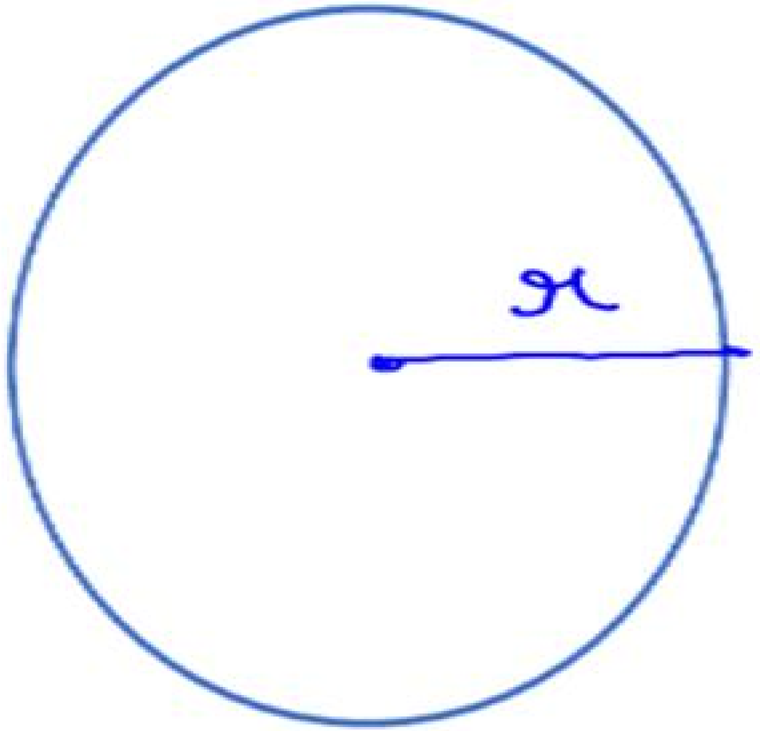
$$= \frac{H^2}{4}\sin 2\theta$$

For Max Area of Right angle Δ

$$= \frac{H^2}{4} \Rightarrow \underline{\underline{36\,cm^2}}$$

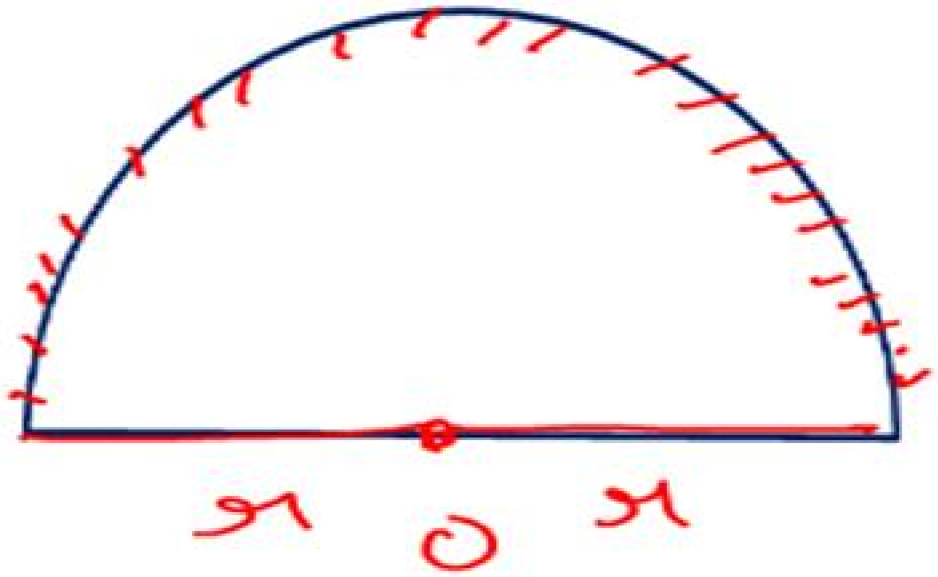
Ans. 36 cm^2

BASICS OF CIRCLE



Area of Circle = πr^2

Circumference of Circle = $2\pi r$



$$\text{Area of Semi-Circle} = \frac{\pi r^2}{2}$$

$$\text{Circumference of Semi-Circle} = \pi r + 2r$$

gradeup

Eg. If circumference of a semi-circle is 72 cm. Find its area.

(Take π = 22/ 7)

$$\pi r + 2r = 72$$

$$r\left[\frac{22}{7} + 2\right] = 72$$

$$r = 14$$

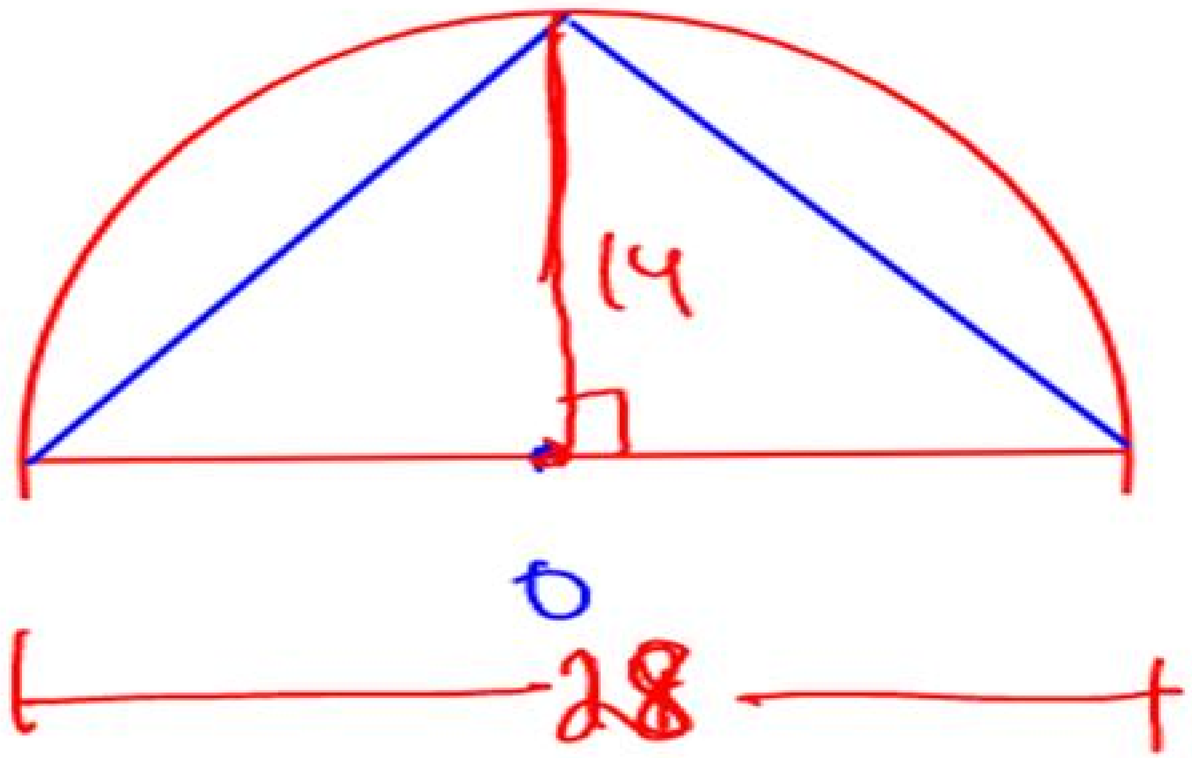
Area $\rightarrow \frac{1}{\cancel{2}} \times \frac{22}{\cancel{7}} \times \cancel{14} \times 14$

$= 308\ cm^2$

Ans. $308\ cm^2$

gradeup

Eg. Find the area of the largest triangle that can be drawn inside a semi-circle of radius 14 cm.

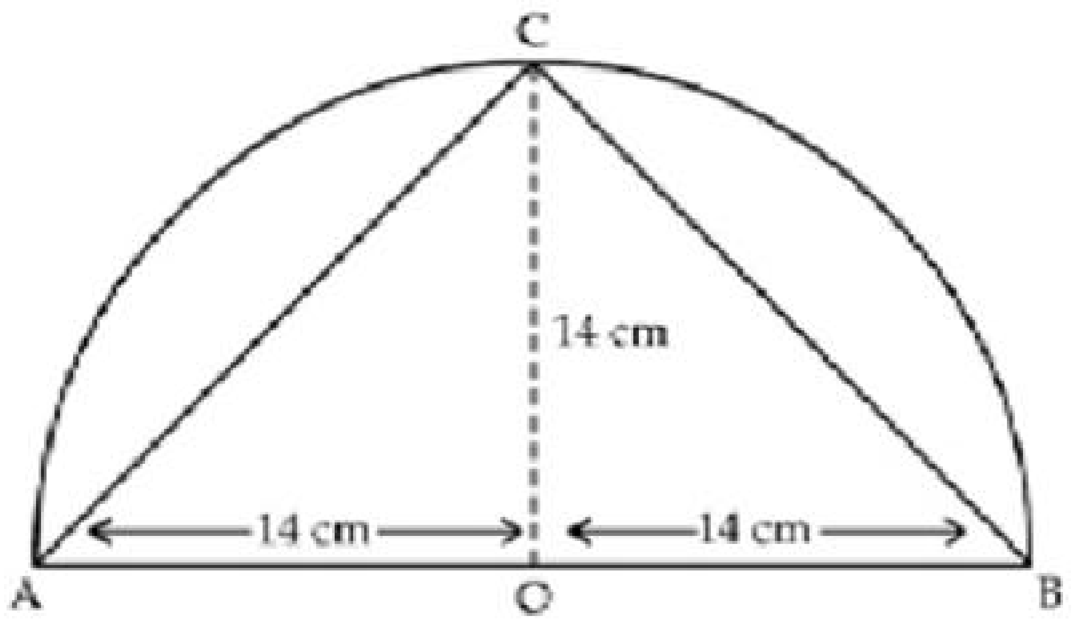


Area of $\Delta \Rightarrow \frac{1}{2} \times B \times H$

$= \frac{1}{\cancel{2}} \times \overset{14}{\cancel{28}} \times 14$

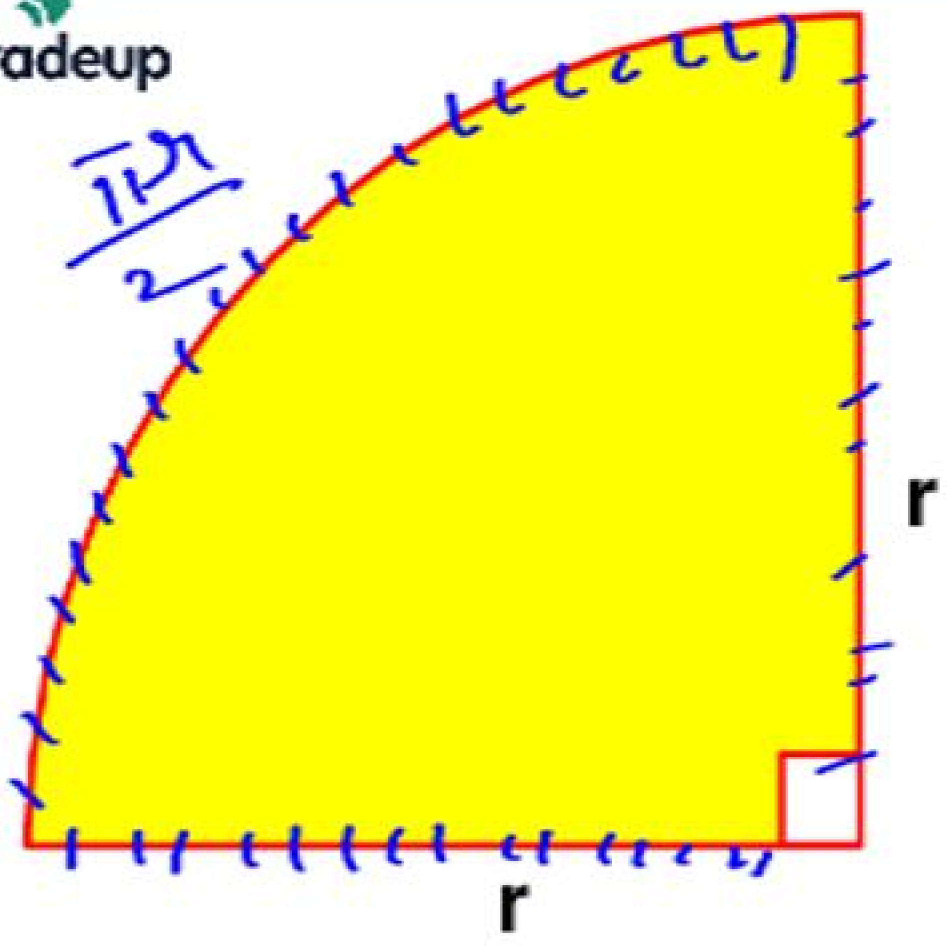
$= 196\ cm^2$

Area of largest Δ drawn inside a semicircle of Radius R is R^2


C
14 cm
14 cm
14 cm
A
O
B

Ans. $196\ cm^2$

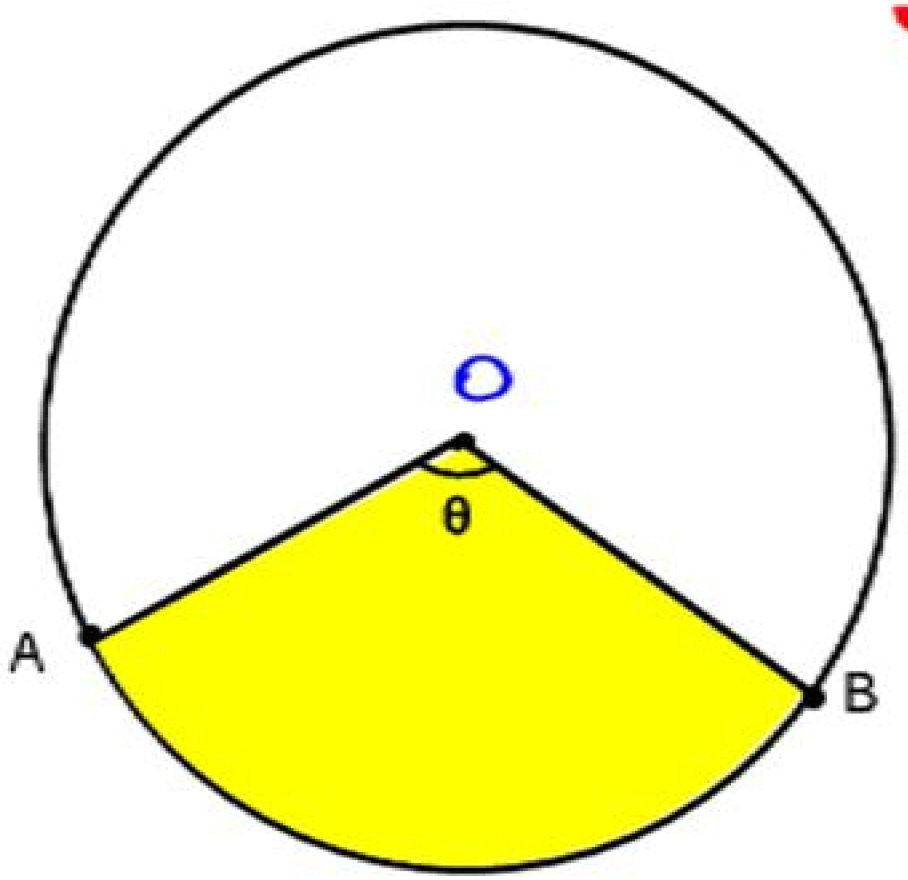
gradeup



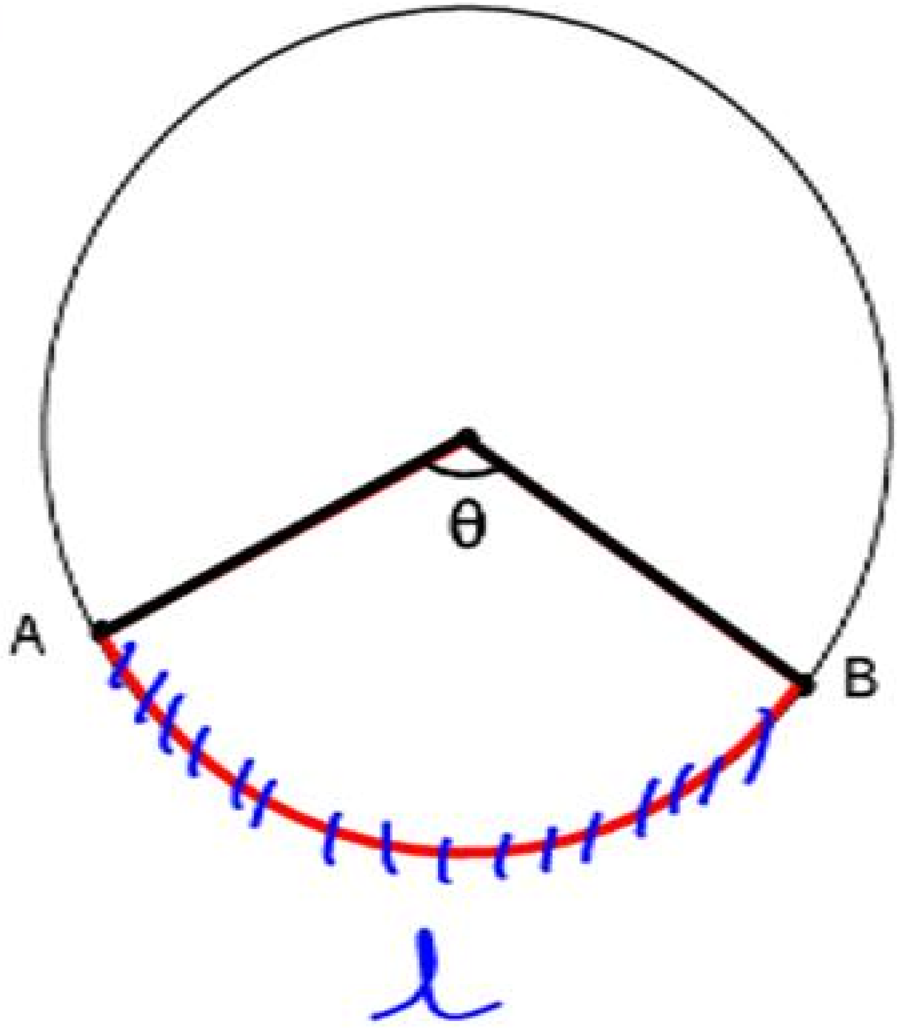
Area of Quadrant of a Circle = $\boxed{\frac{1}{4}\pi r^2}$

Circumference of Quadrant of a Circle = $\boxed{\frac{\pi r}{2} + 2r}$

SECTOR OF A CIRCLE



AREA OF SECTOR = $\boxed{\dfrac{\pi r^2 \theta}{360^\circ}}$



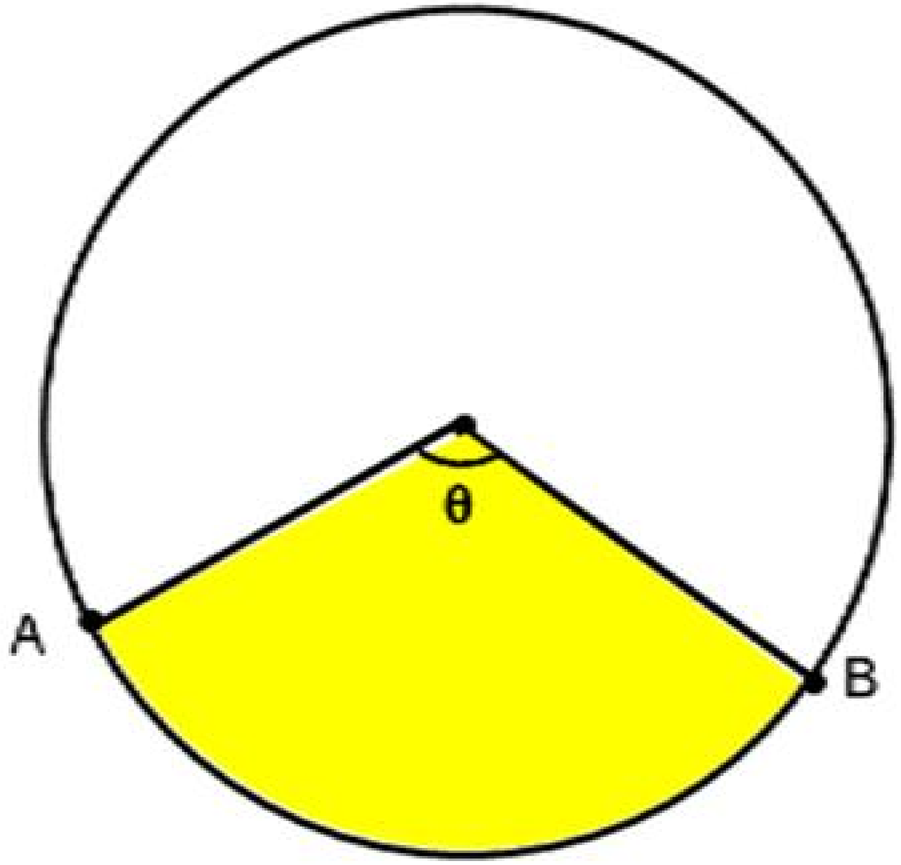
Length of the Arc AB (l) = $\boxed{\dfrac{2\pi r\theta}{360^\circ}}$

$360^\circ \longrightarrow 2\pi r$

$1^\circ \longrightarrow \dfrac{2\pi r}{360^\circ}$

$\theta \longrightarrow \dfrac{2\pi r\theta}{360^\circ}$

gradeup



Area of Sector = $\frac{1}{2}lr$

$$A = \frac{\pi r^2 \theta}{360} \quad -(1)$$

$$l = \frac{2\pi r\theta}{360} \quad -(2)$$

$$\frac{A}{l} = \frac{r}{2}$$

$$\boxed{A = \frac{1}{2} l r}$$

Eg. If length of the arc = 6 cm and radius of circle = 5 cm.

Find area of sector of a circle.

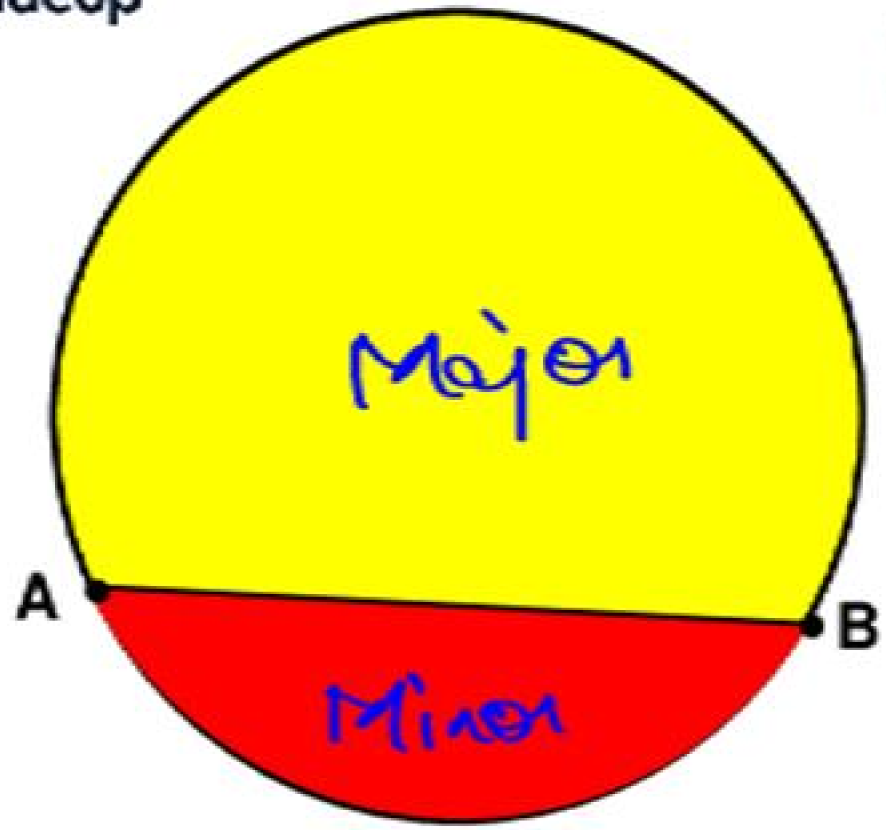
$$\text{Area of sector} = \frac{1}{2} l \cdot r$$

$$= \frac{1}{2} \cdot 6 \cdot 5$$

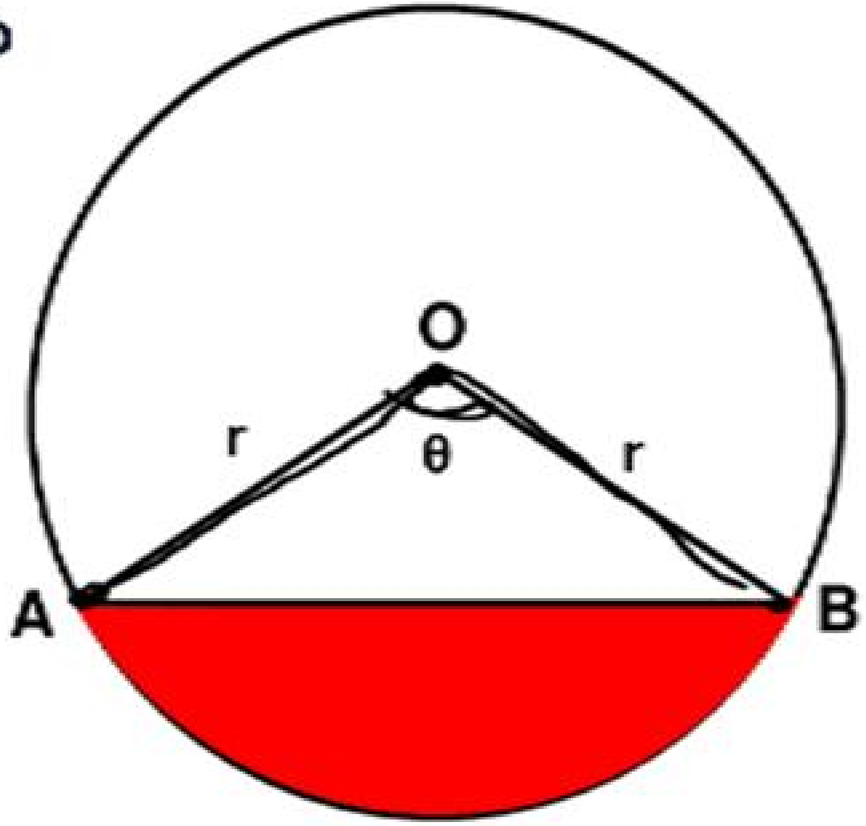
$$= \underline{\underline{15 \text{ cm}^2}}$$

Ans. $15\ cm^2$

SEGMENT OF A CIRCLE



Chord of a circle divides a circle in 2 segments.



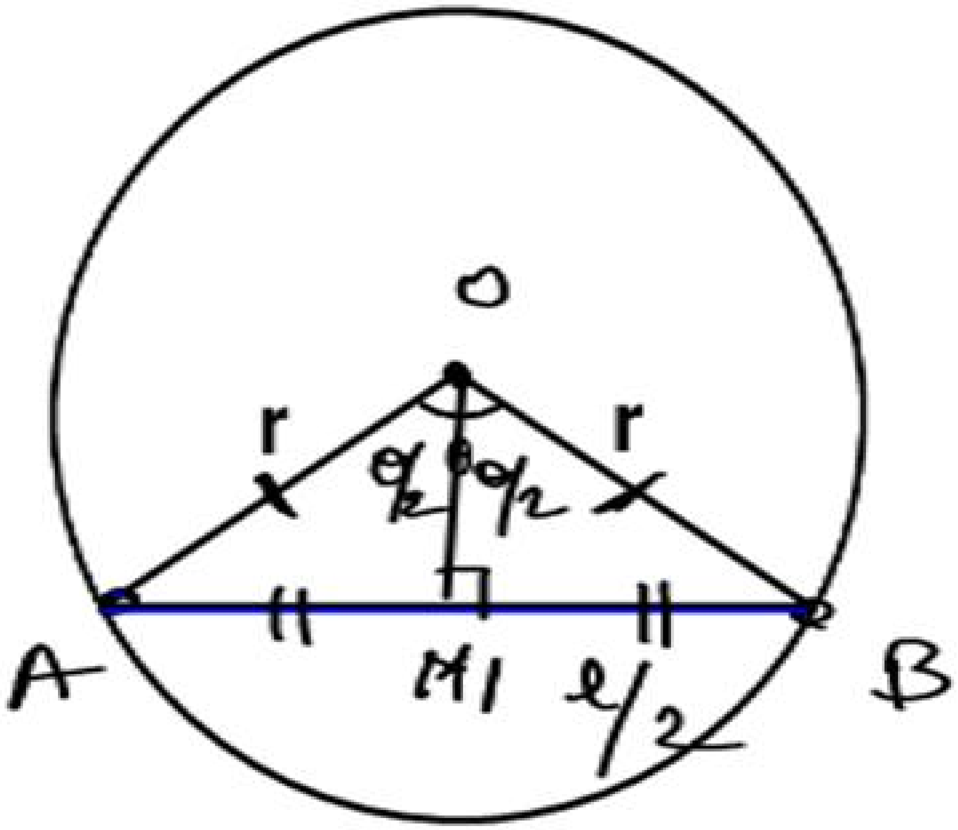
AREA OF SEGMENT

= Area of Sector − Area of ΔAOB

$$\boxed{\frac{\pi r^2\theta}{360^\circ} - \frac{1}{2}r^2\sin\theta}$$

By Default → It is always Minor segment

LENGTH OF CHORD OF A CIRCLE

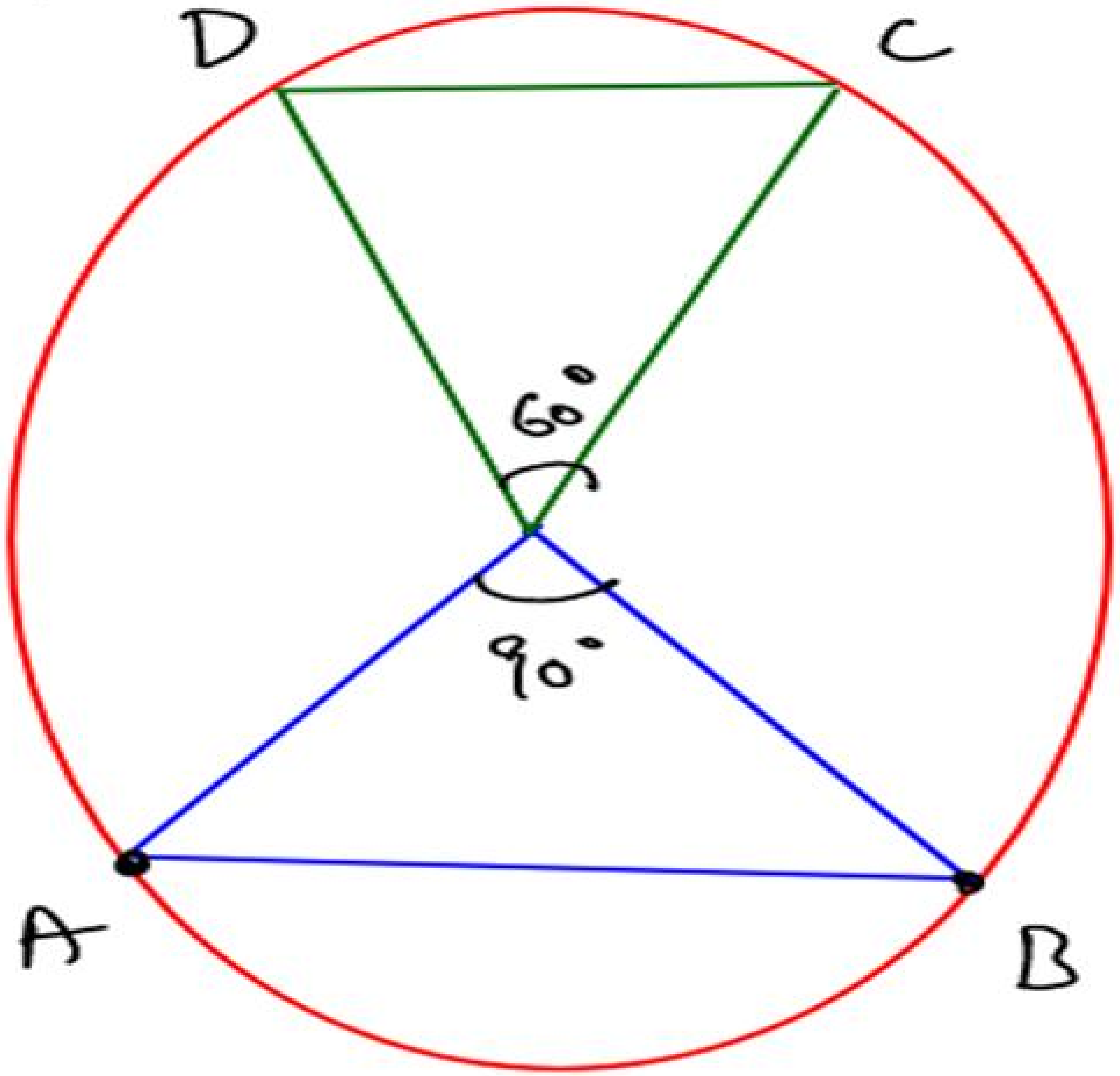


$$\sin\theta/2 = \frac{MB}{OB}$$

$$\sin\theta/2 = \frac{l}{2r}$$

$$l = 2r\sin\theta/2$$

$$\boxed{\text{Length of chord} = 2 \cdot r \sin\theta/2}$$



Find $\frac{AB}{CD} = ??$

$AB = 2r \sin 45$

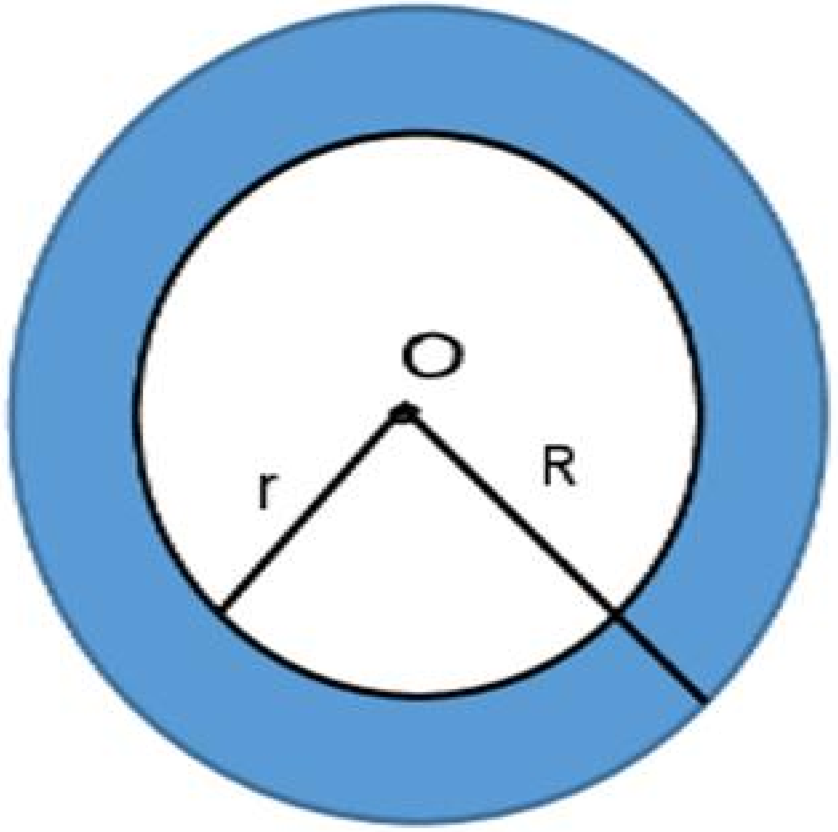
$CD = 2r \sin 30$

$\sin 30 = \frac{1}{2}$ $\sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\frac{AB}{CD} = \frac{1\ \cancel{2}\ \sqrt{2}}{\cancel{\sqrt{2}}\ 1}$$

$$\boxed{\frac{AB}{CD} = \frac{\sqrt{2}}{1}}$$

AREA ENCLOSED BY TWO CONCENTRIC CIRCLES



If R and r are radii of two concentric circles, then

Area enclosed by the two circles $= \pi R^2 - \pi r^2$

$$= \pi\left(R^2 - r^2\right)$$

$$= \pi\left(R + r\right)\left(R - r\right)$$

gradeup

Some useful results:

Imp

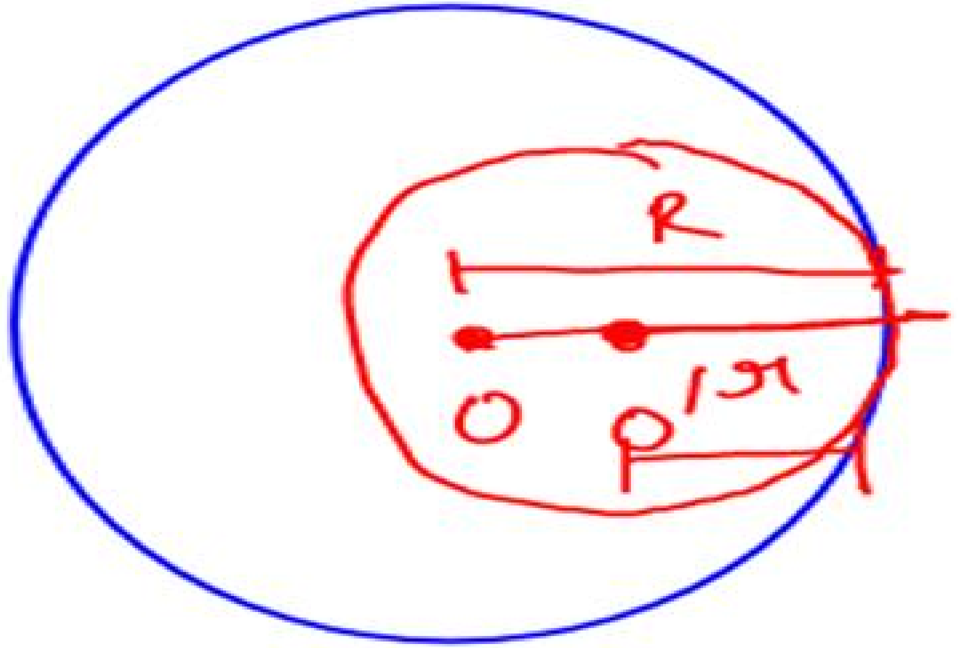
(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

(ii) If two circles touch externally, then the distance between their centres is equal to the sum of their radii.

(iii) Distance moved by a rotating wheel in one revolution is equal to the circumference of the wheel. $(2\pi r)$

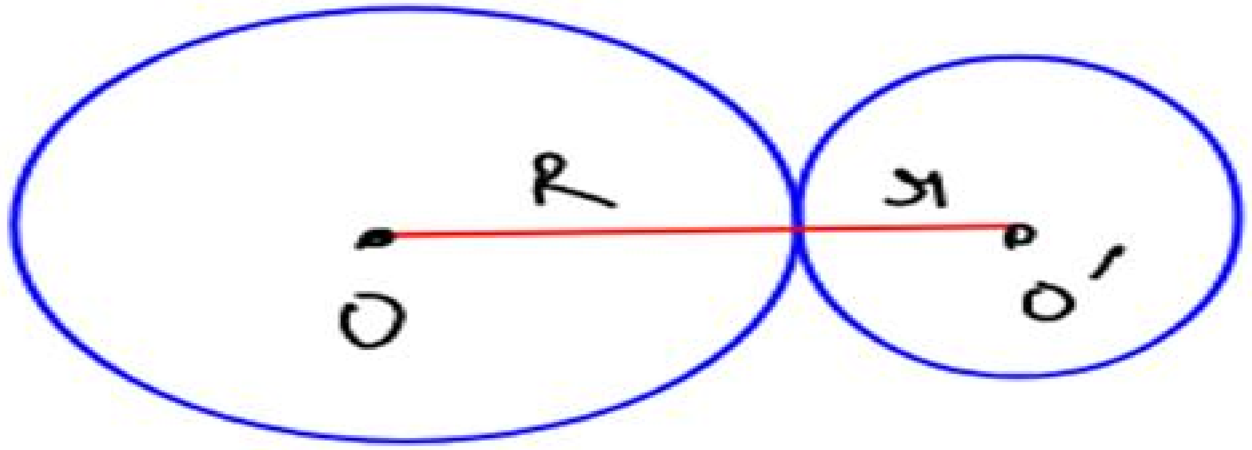
(iv) the number of revolutions completed by a rotating wheel in one minute = $\dfrac{\textit{Distance moved in one minute}}{\textit{Circumference}}$

Circles Touching Internally



$$OO' = R - r$$

Circles Touching Externally



$$OO' = R + r$$

gradeup

Q. Two circles touch externally. The sum of their areas is 130 π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

$\cancel{\pi}(R^2 + r^2) = 130\cancel{\pi}$

$R^2 + r^2 = 130$

$R + r = 14$

$R = 11$

$r = 3$

$R^2 + (14 - R)^2 = 130$

$2R^2 - 28R + 196 = 130$

$R^2 - 14R + 33 = 0$

$R = 11, \cancel{3}$

Ans. $R = 11$ cm and $r = 3$ cm.

Q. A car has wheels which are 80 cm in diameter. How many complete revolutions does each wheel make in 10 min. when the car is travelling at a speed of 66 km per hour?

$R = 40 cm$

$$D = S \cdot T$$

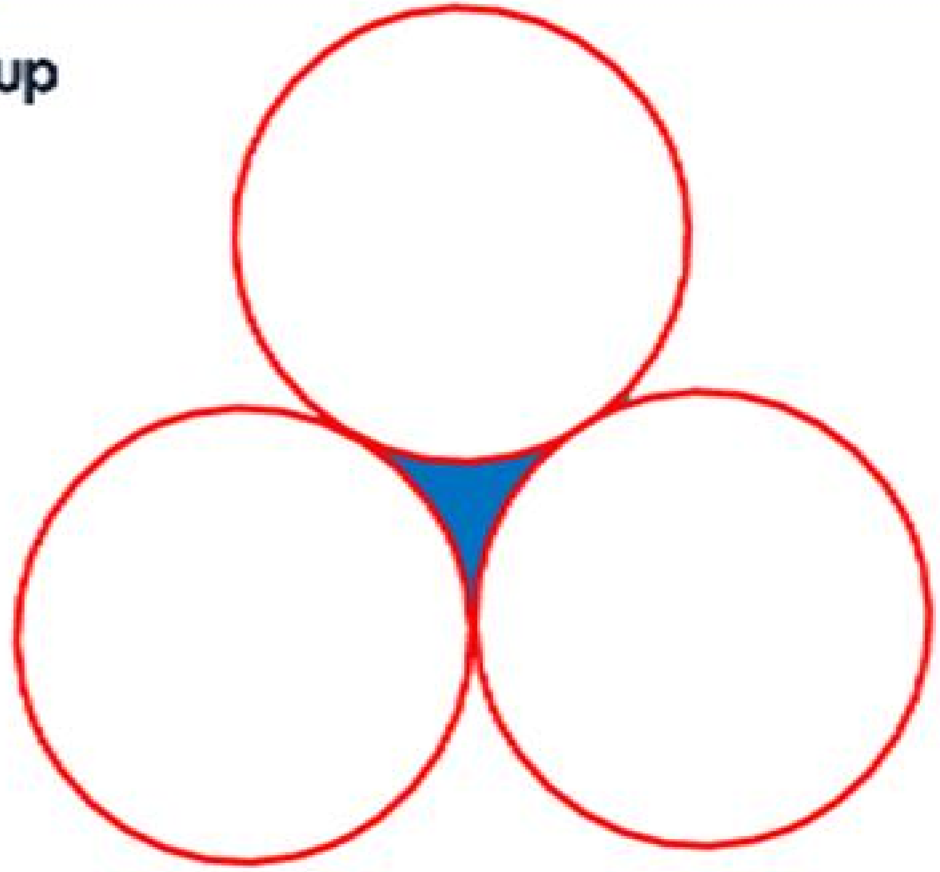
$$= 66 \cdot \frac{1\cancel{0}}{6\cancel{0}} = 11 Km$$

Distance covered in 1 Revolution $= 2 \times \frac{22}{7} \times 40$

No of Revolution $\Rightarrow \frac{\cancel{11} \times 1000 \times \cancel{100}}{\cancel{2} \times \cancel{22} \times \cancel{40}} \times 7$

$\Rightarrow 4375$

Ans. 4375

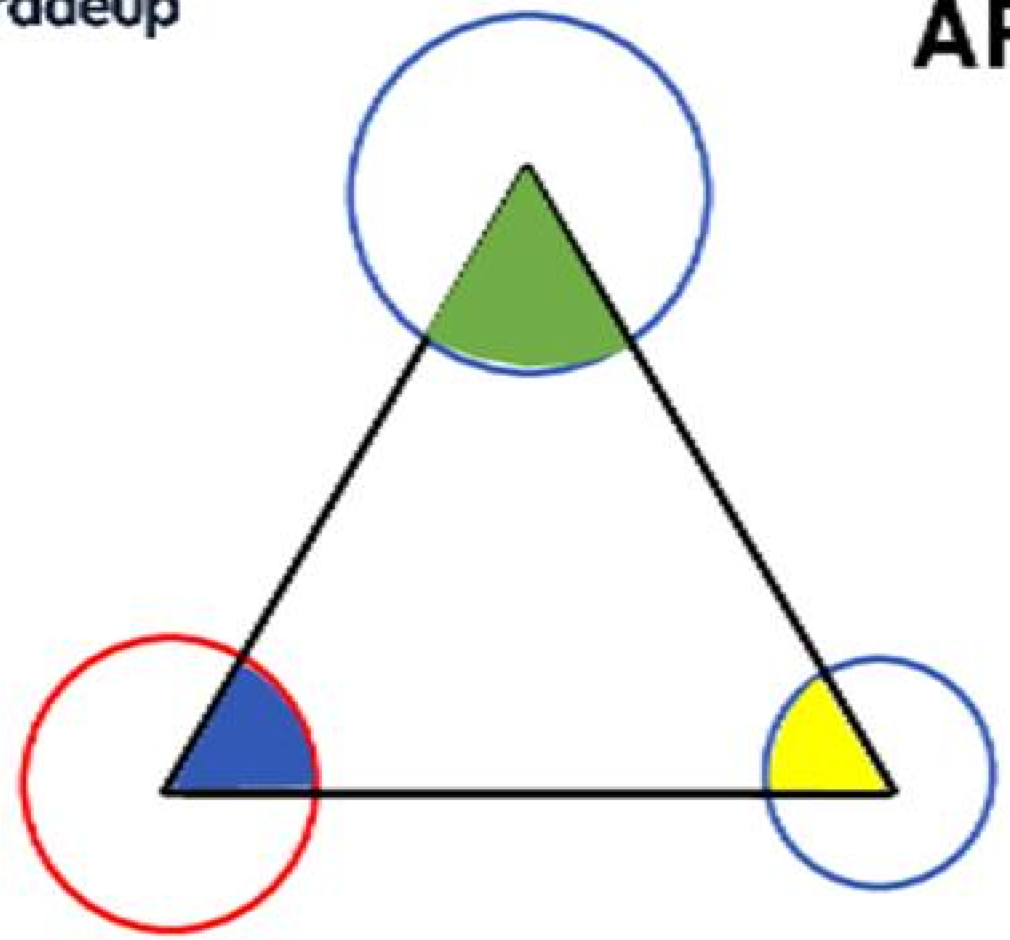


Eg. Radius of each of the circle is 10 cm. Find the area of shaded region.

gradeup

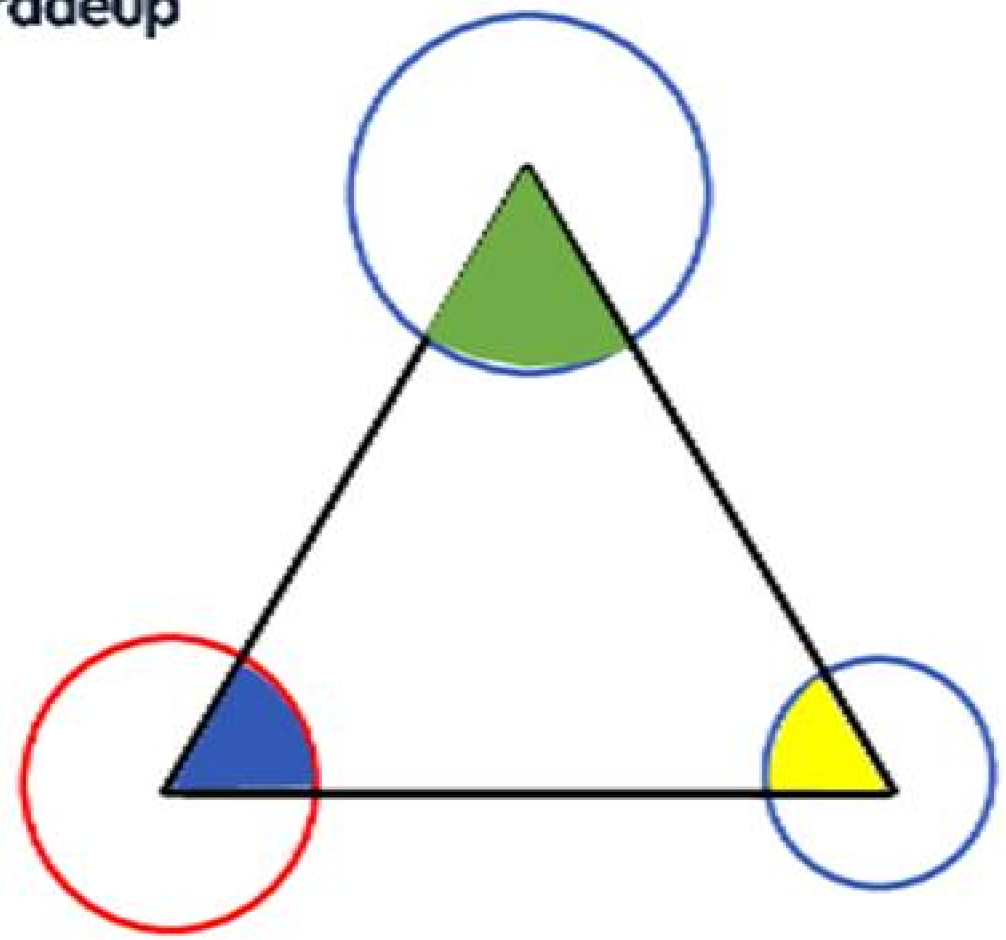
Ans. $\left(100\sqrt{3}-50\pi\right) cm^2$

AREA GRAZED BY COW



An equilateral triangle whose side is 20 cm. Find the area grazed by the cows if they are attached by rope of length 6 cm, 5 cm and 4 cm on the 3 vertices.

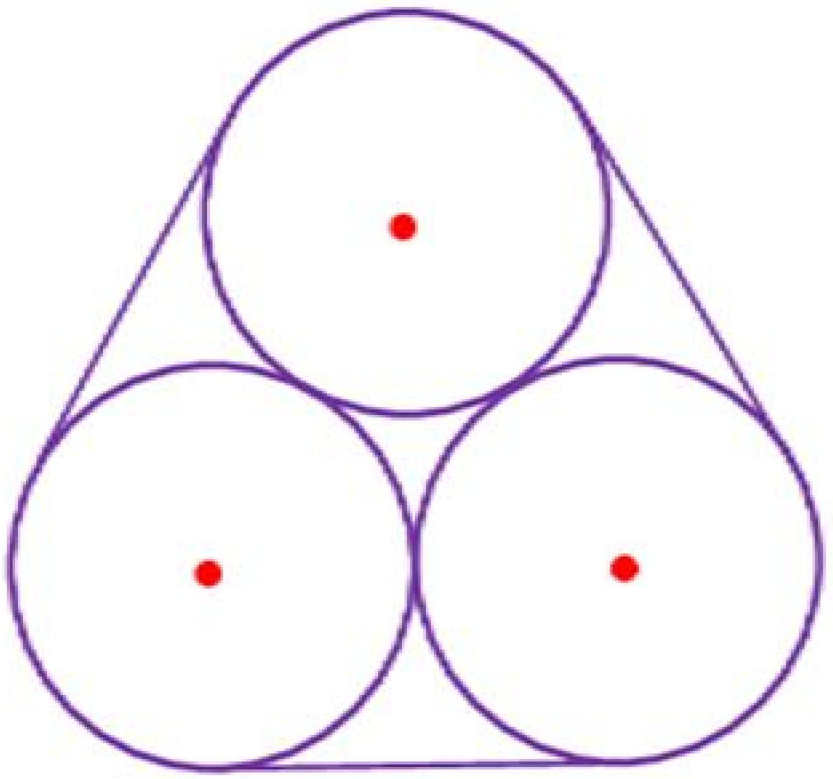
Ans. $\frac{121}{3} cm^2$



Eg. A triangle whose sides are 20 cm, 18 cm and 16 cm. Find the area grazed by the cows if they are attached by rope of length 6 cm on all the 3 vertices.

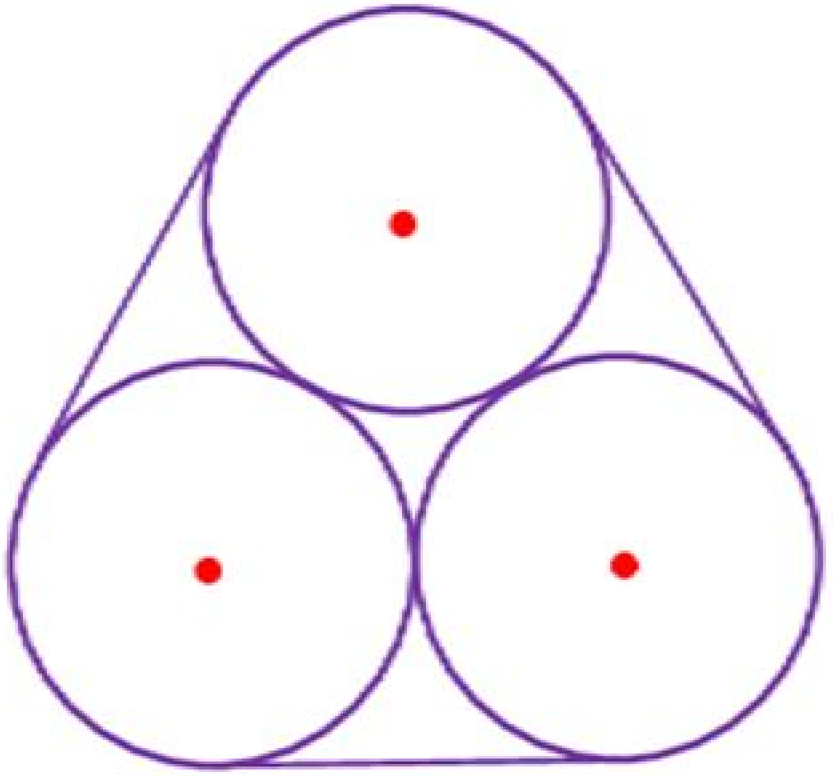
Ans. $\frac{396}{7} cm^2$

LENGTH OF RUBBER BAND

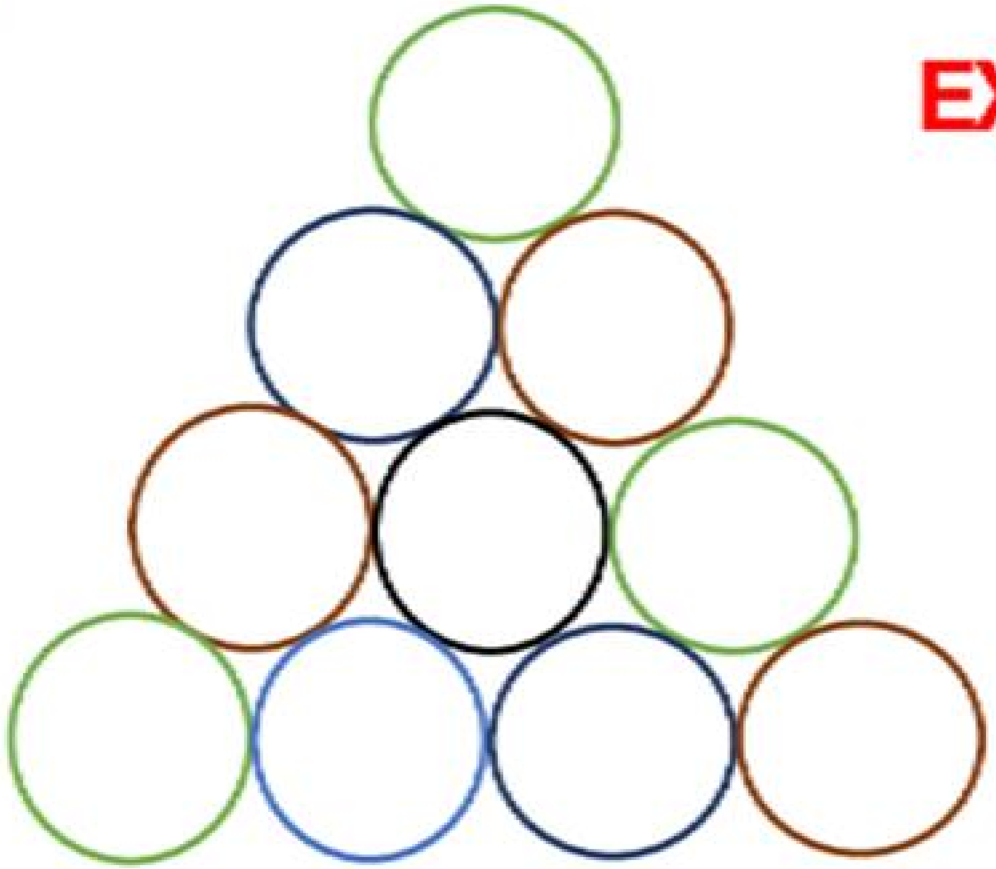


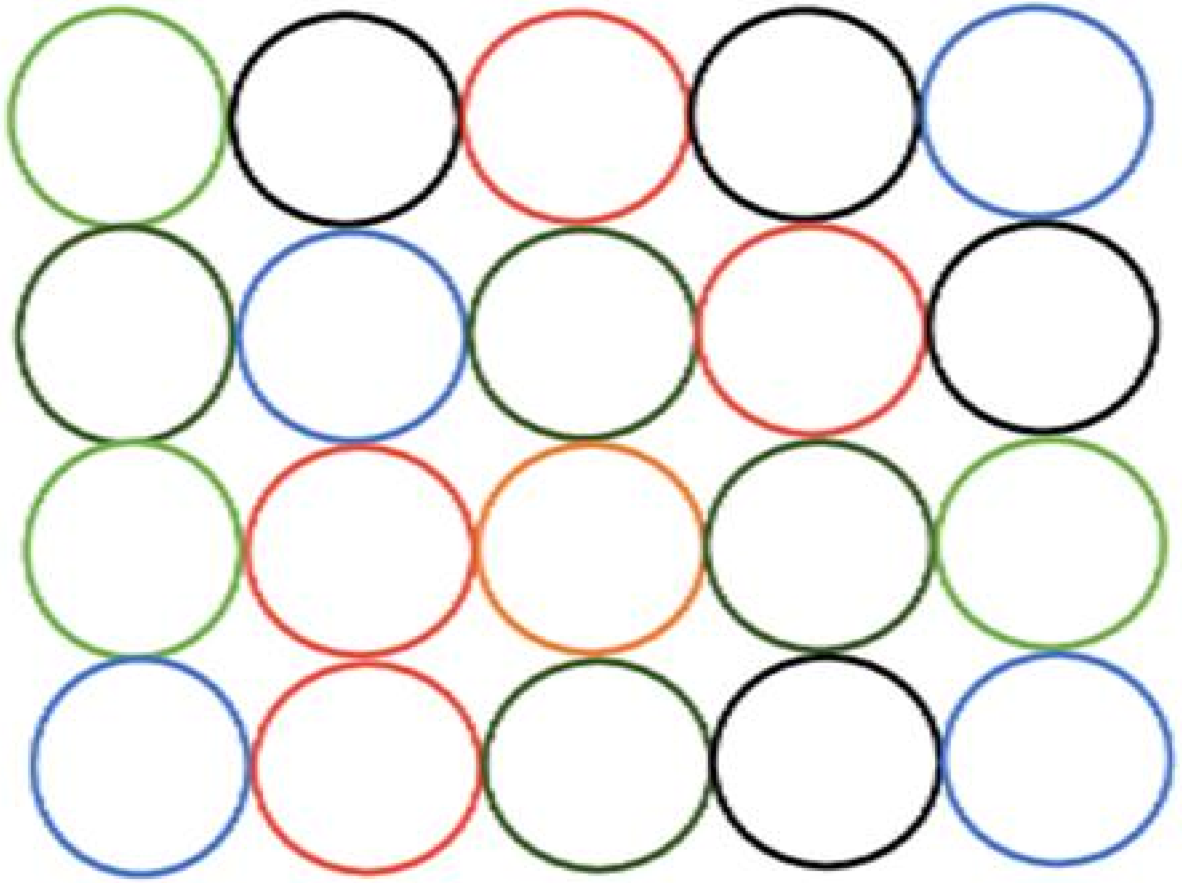
If the radius of each circle is 10 cm.

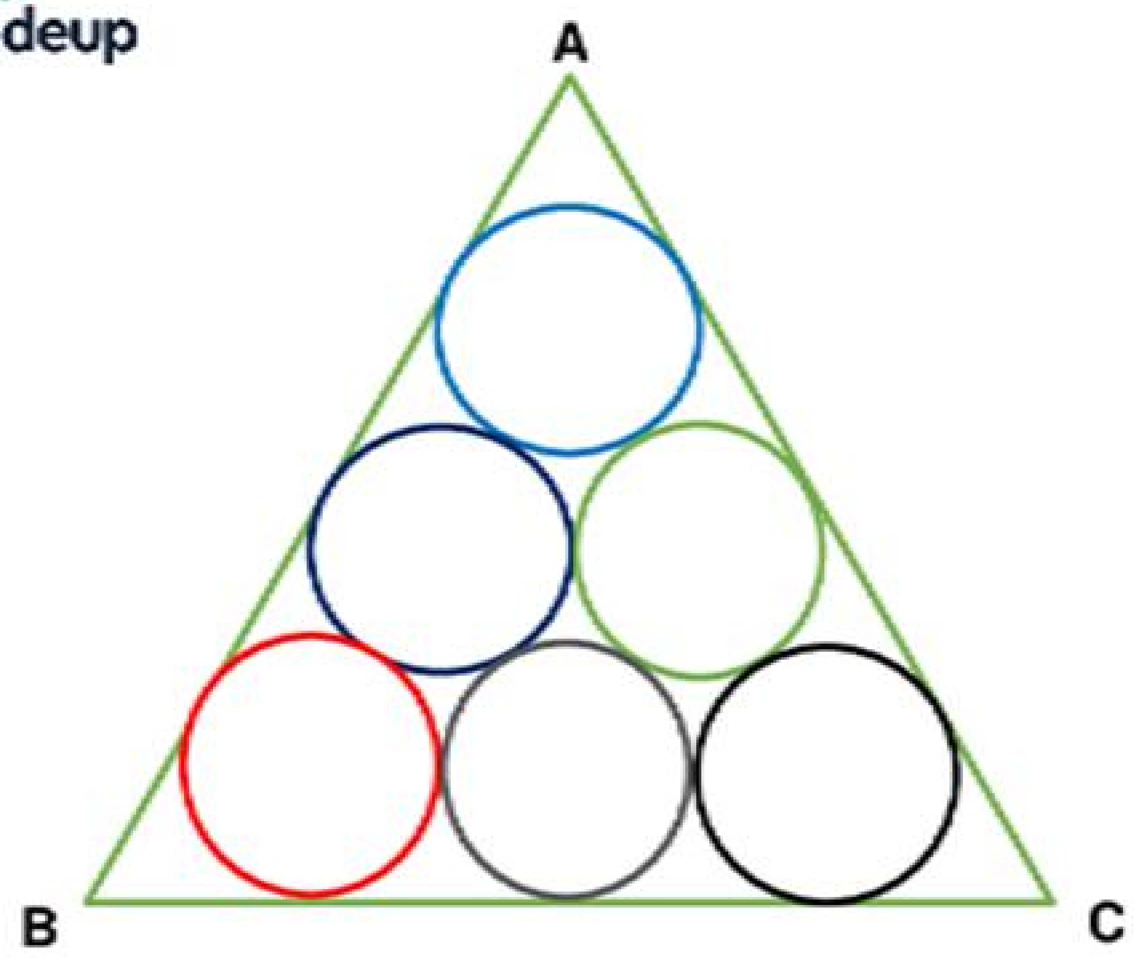
Find the length of rubber band.



EXTENSION OF RUBBER BAND QUESTIONS





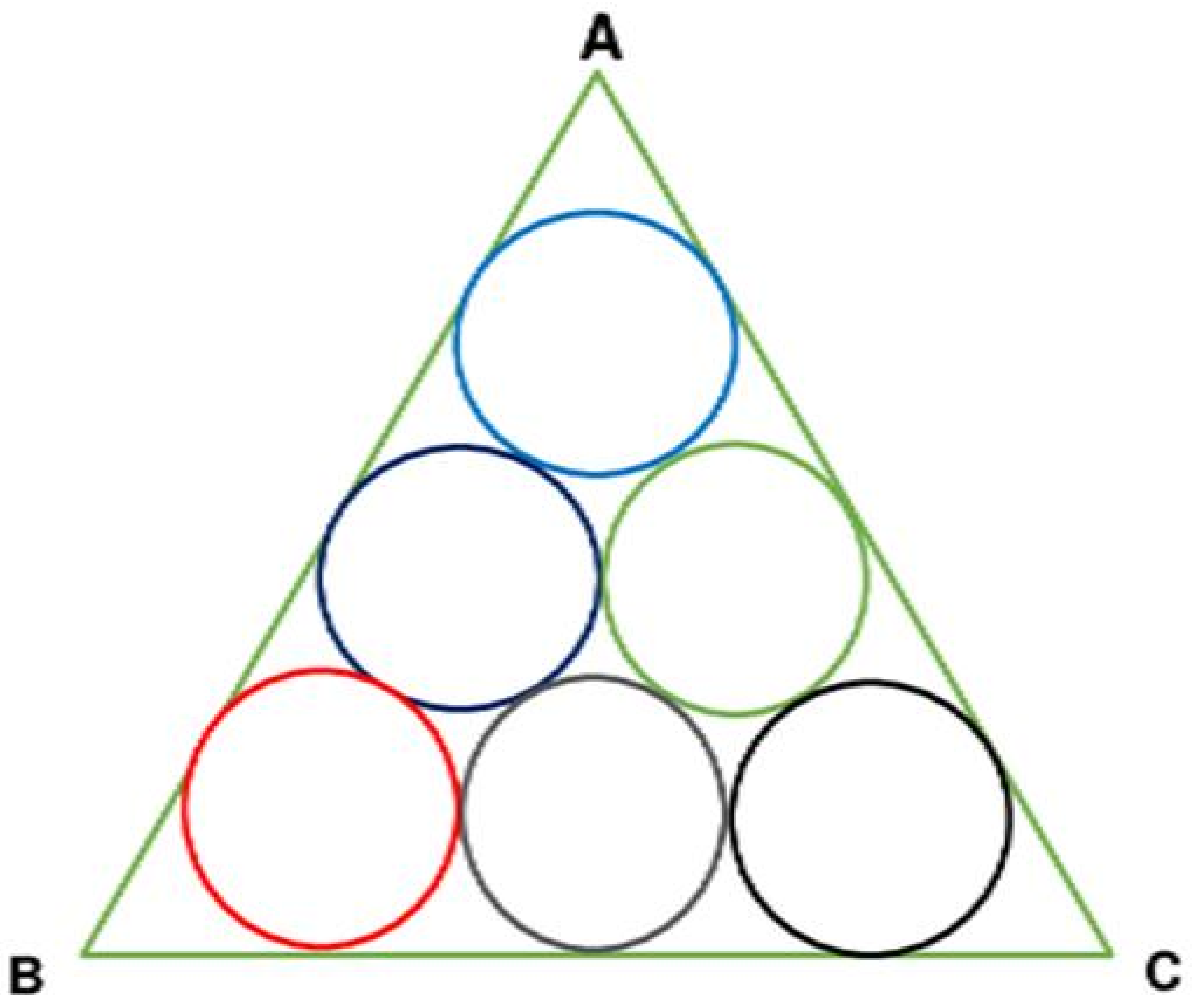


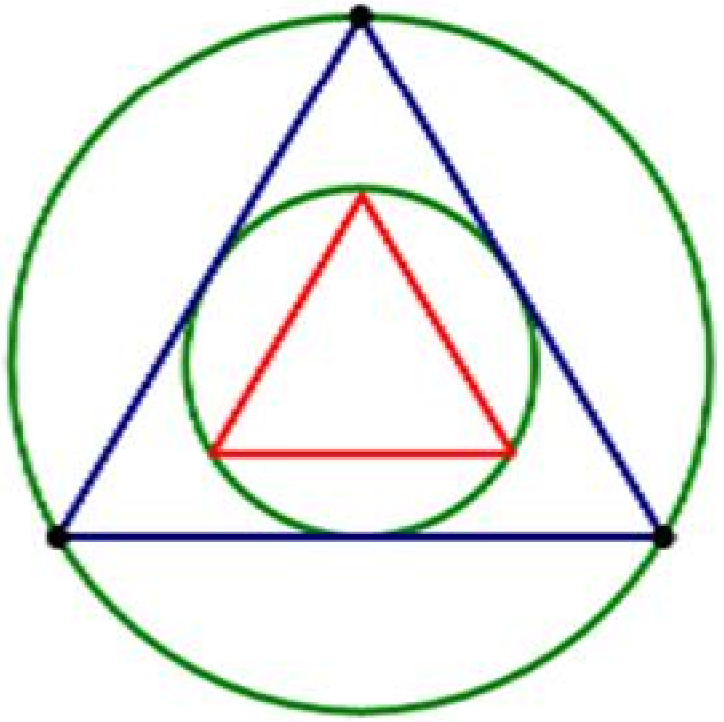
Equilateral Δ

If radius of each circle is 10 cm.

Find the side of equilateral Δ.

gradeup



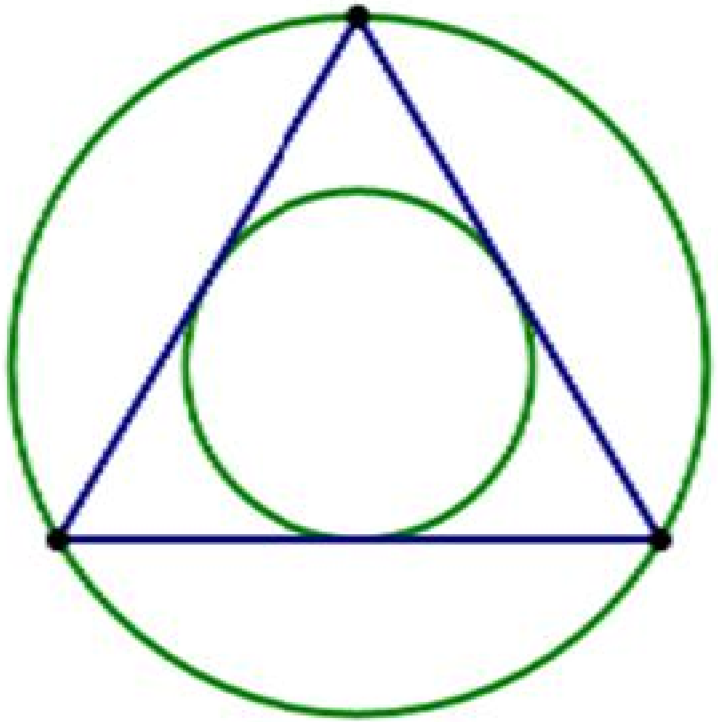


Equilateral Δ

If r = inradius

R = circumradius

S = Side of equilateral triangle



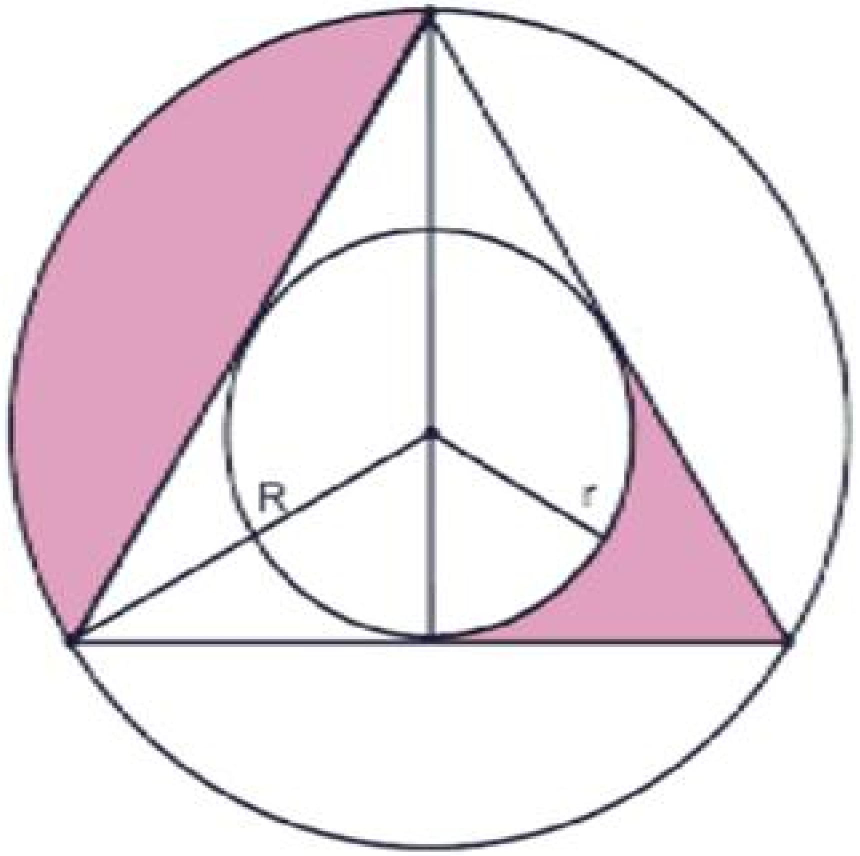
Equilateral Δ

(i) Find : $\frac{r}{R}$

(ii) $\frac{\text{Area of incircle}}{\text{Area of circumcircle}}$

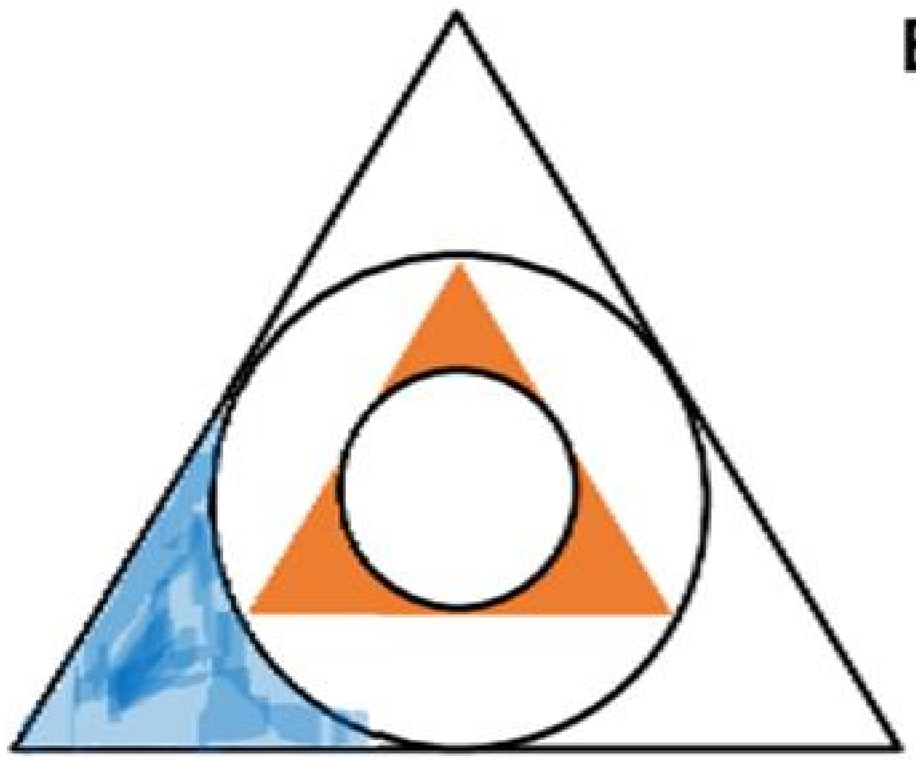
(iii) Find the ratio of r : S : R

Eg. If side of an equilateral triangle is 12 cm. Find the area of shaded region.



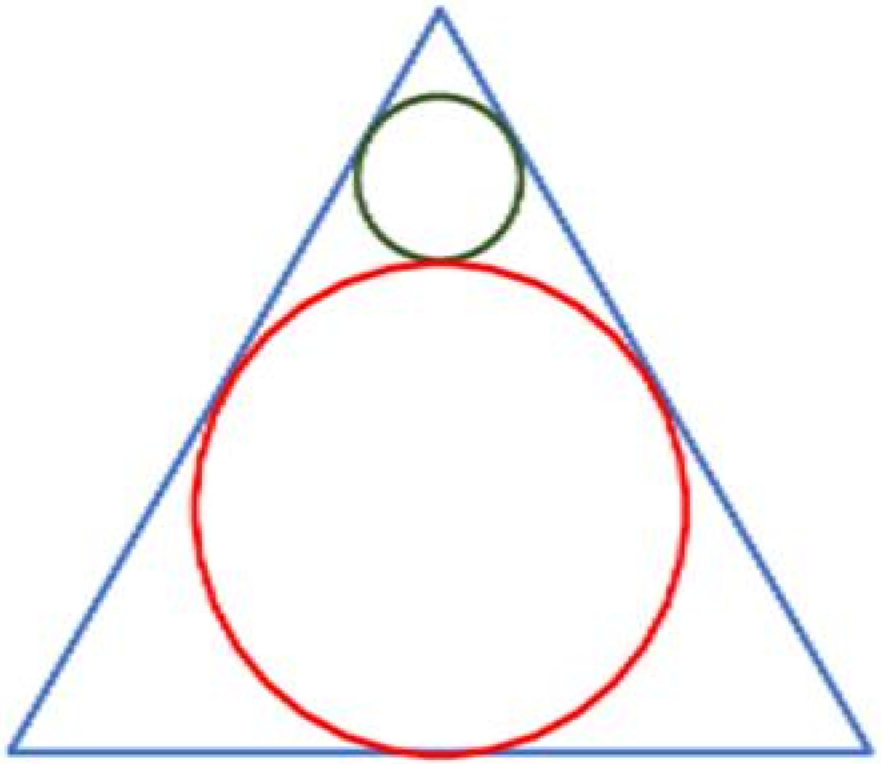
Ans. 12π

Equilateral Δ



Eg. Find : $\dfrac{\textit{Area of Orange region}}{\textit{Area of Blue region}}$

Ans. 3 : 4

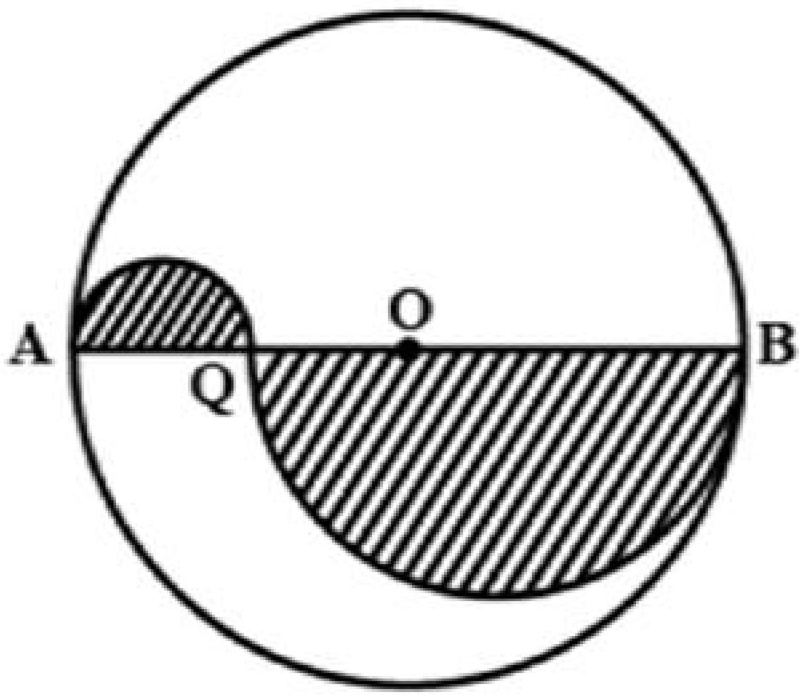


Equilateral Δ

Eg. Find : $\frac{r}{R}$

Ans. 1 : 3

gradeup



Diameter of bigger circle (AB) = 12 cm

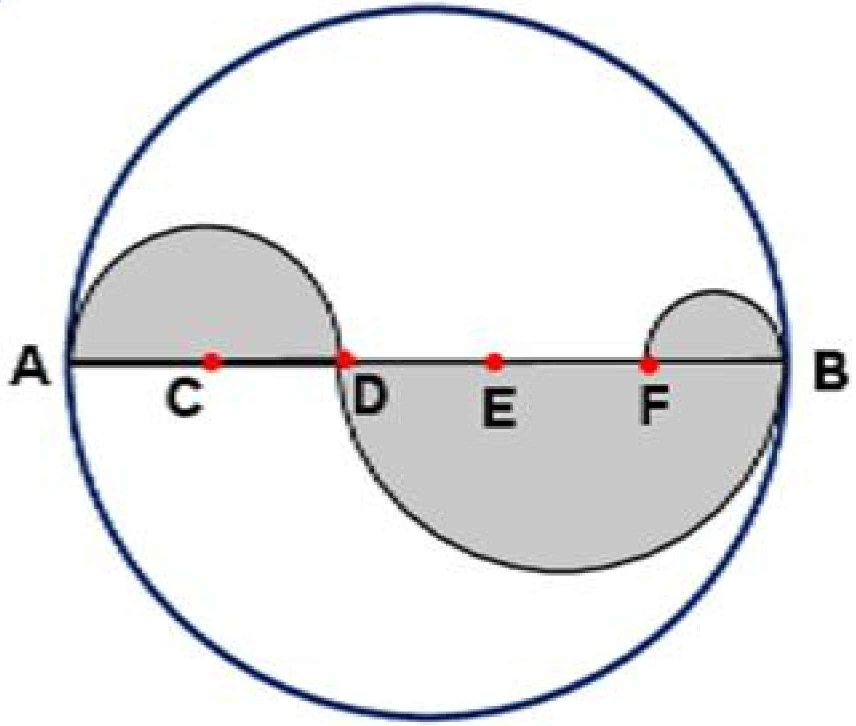
If AQ = QO and O is centre of the bigger circle

Semi-circles are drawn taking AQ and QB as diameter as shown in the figure.

Find : $\dfrac{\textit{Area of shaded part}}{\textit{Area of complete circle}}$

gradeup

Ans. 5 : 16

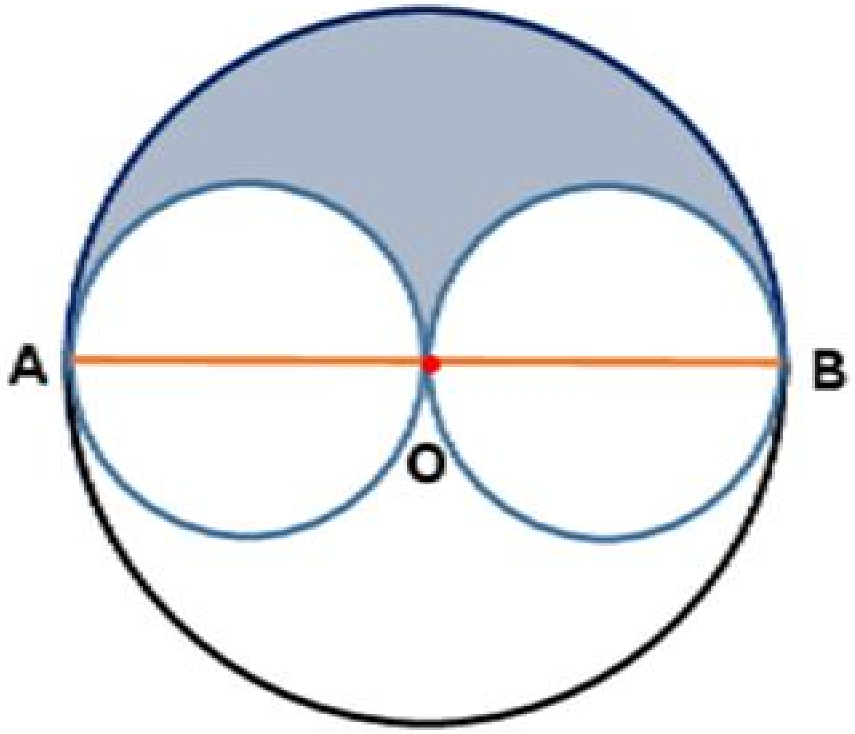


AB = Diameter of bigger circle

AC = CD = DE = EF = FB

Find : $\dfrac{\textit{Area of shaded part}}{\textit{Area of complete circle}}$

Ans. 7 : 25



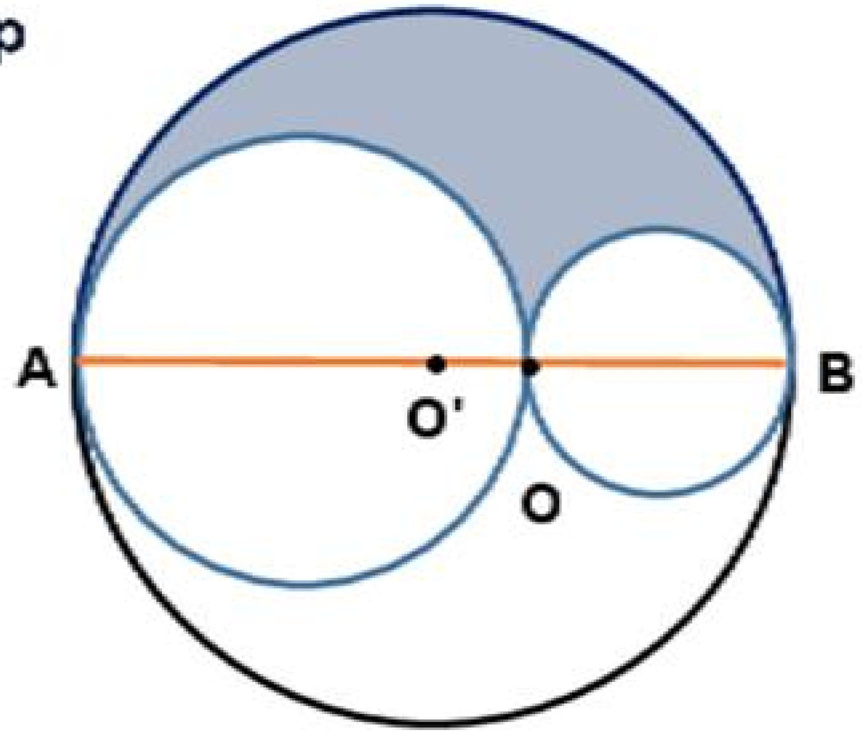
Eg. O is centre of larger circle.

AB is diameter

AB = 20 cm

Find the area of shaded part.

Ans. 25π



O' is the centre of the larger circle

AB = 20 cm

AO and OB are diameters of smaller circle

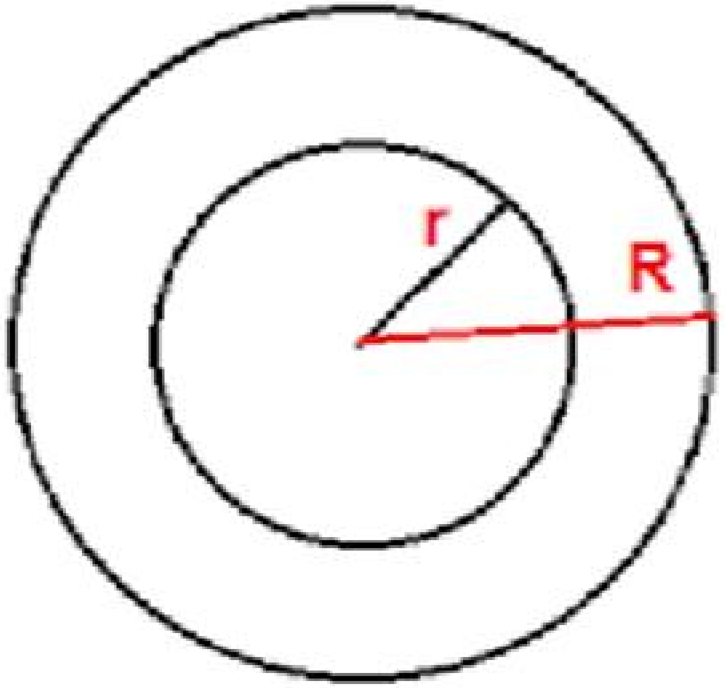
(i) Find the area of shaded region.

(ii) Find circumference of shaded region.

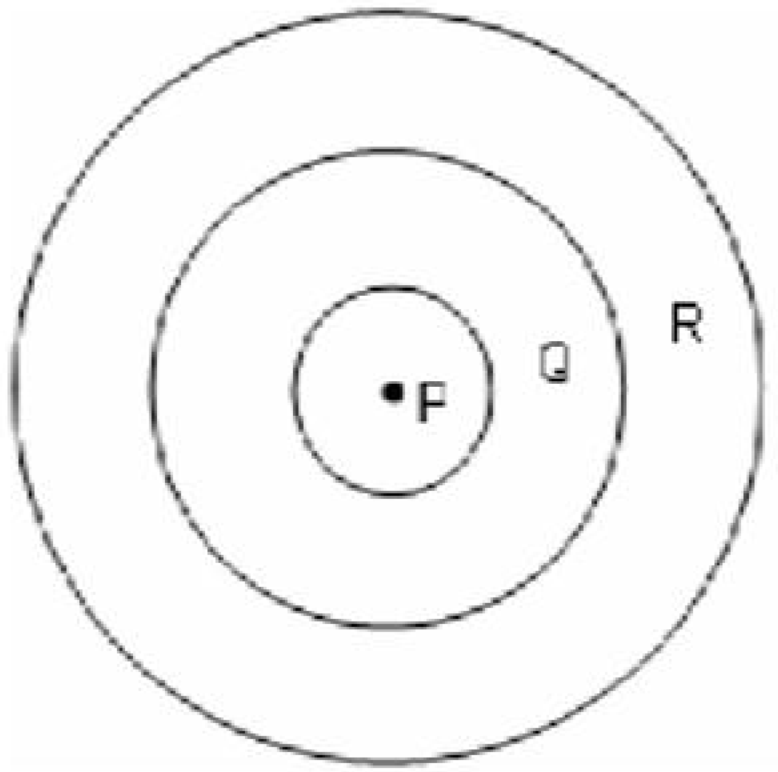
Ans. (i) Can't be determined

(ii) 20π

CONCENTRIC CIRCLES



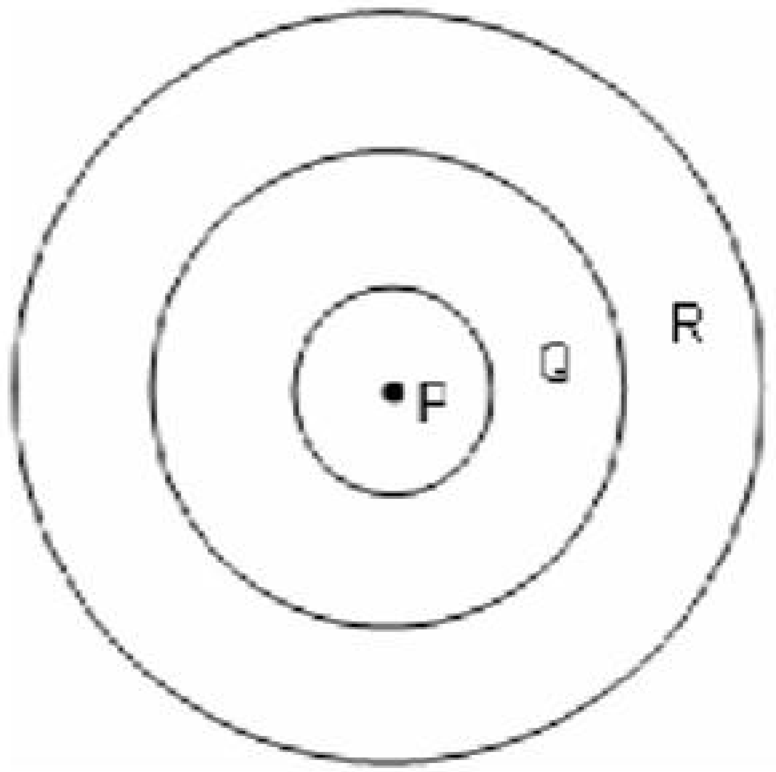
Circles with the same centre.



Radius of circles are in the ratio 1 : 2 : 3

Find the ratio of areas of region P : Q : R

gradeup



If area of region P, Q and R are equal and radius of the largest circle is 12 cm.

Find the radius of the smallest circle.

gradeup

Ans. $4\sqrt{3}$

Eg. If radius of a circle is reduced by n, then its area becomes half of its original area. Find the original radius of circle in terms of n.

Ans. $R = \frac{\sqrt{2}n}{\sqrt{2}-1}$