gradeup

gradeup

Sahi Prep Hai Toh Life Set Hai

TRIANGLE- 4

* Centroid

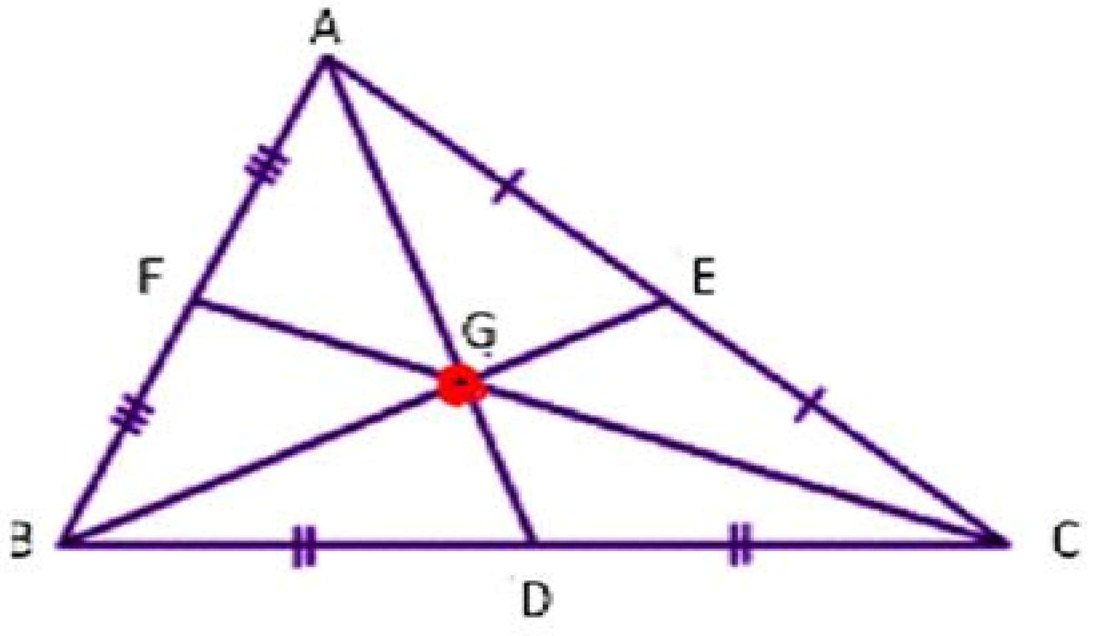
* Incentre

* Circumcentre

* General pt about all centres

CENTROID

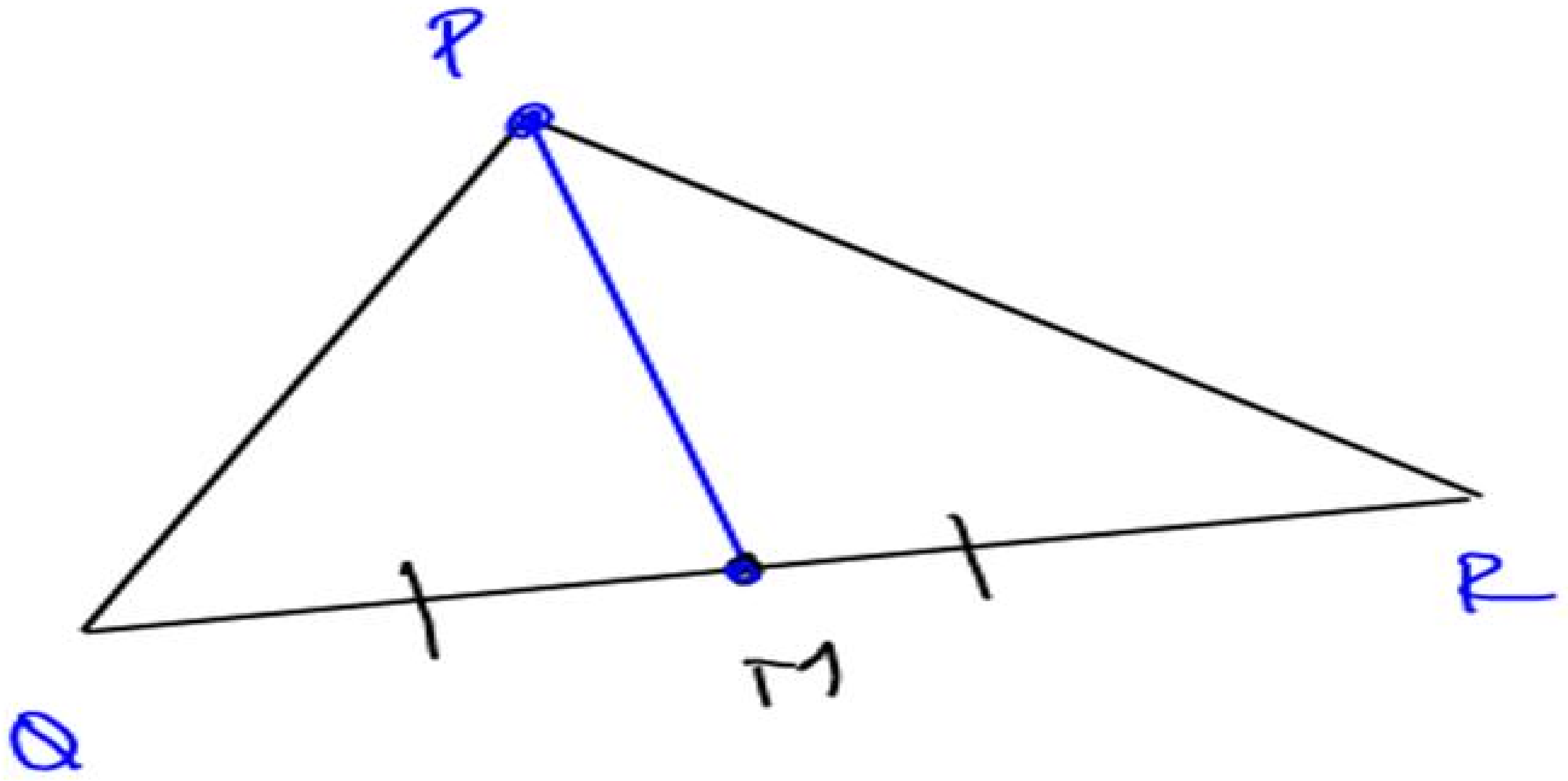
Def: Meeting point of all medians.



Here, G is the centroid of ΔABC.

Median

The line segment which joins one vertex to the mid point of the opposite side.



If QM = MR

→ PM is Median

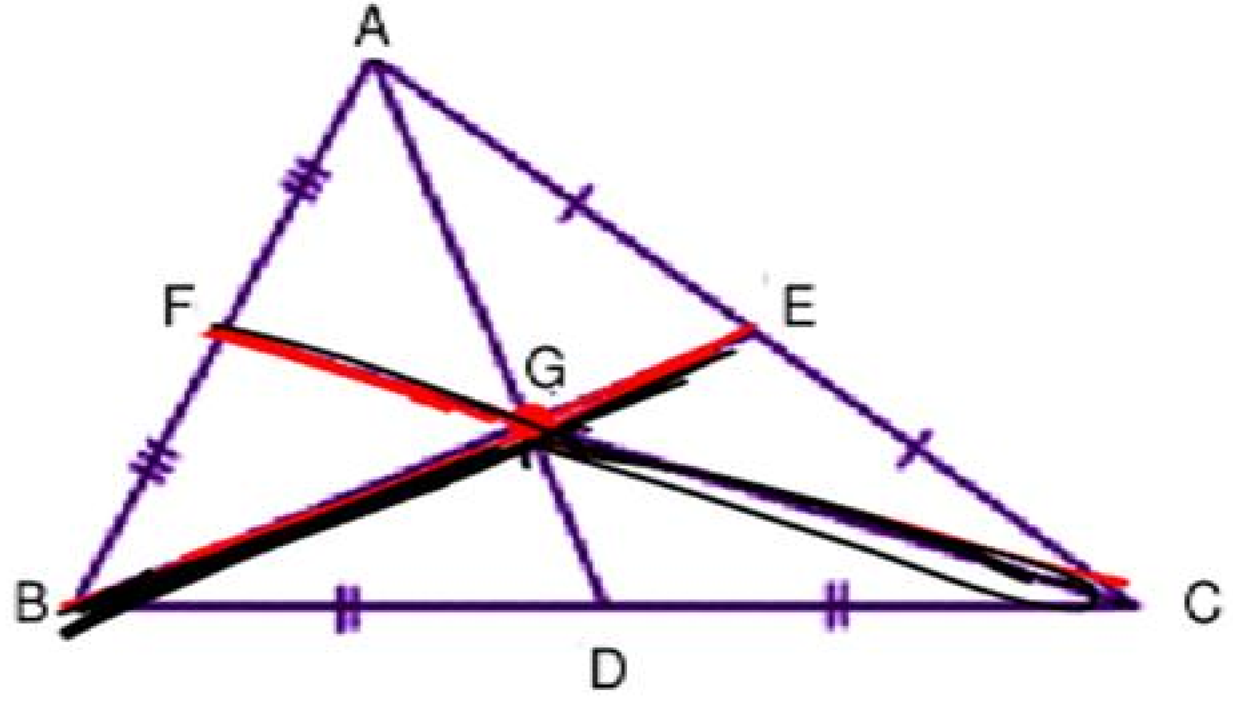
gradeup

1. Centroid divides the median in 2 : 1.

AG : GD = 2 : 1

BG : GE = 2 : 1

CG : GF = 2 : 1



eg If AG = 10cm GD = 5cm

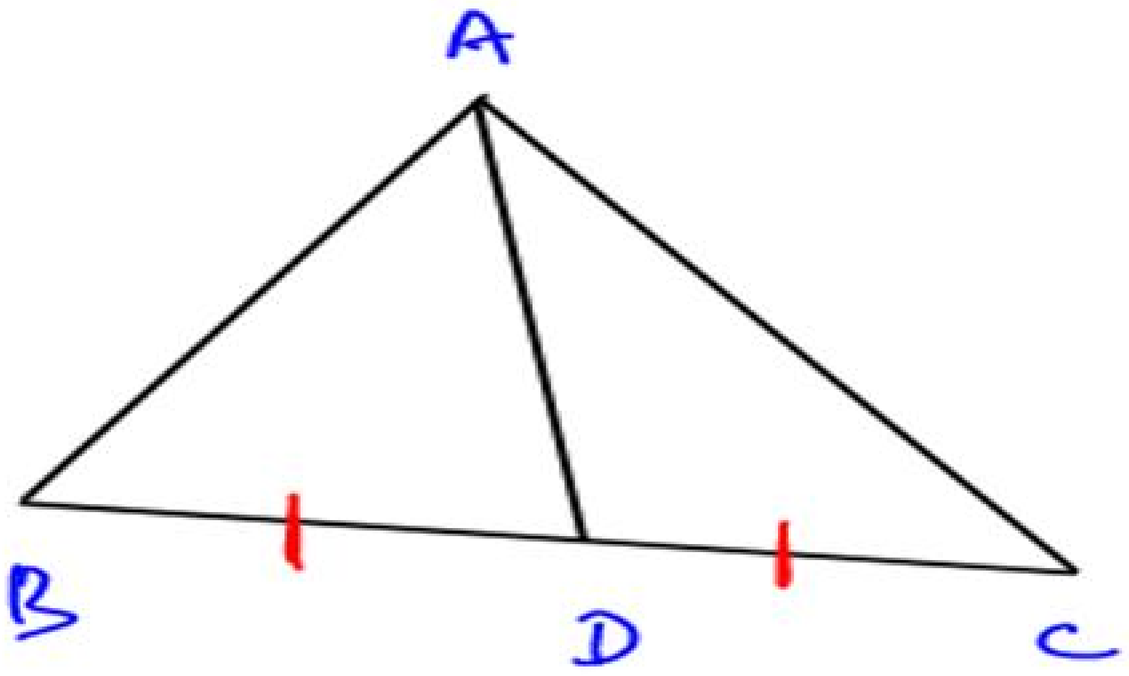
BG = 12cm BE = 18

FG = 8cm GC = 16

If BG = 20cm GC = ??

Can't be determined

(i) Median divides a triangle in two equal areas.



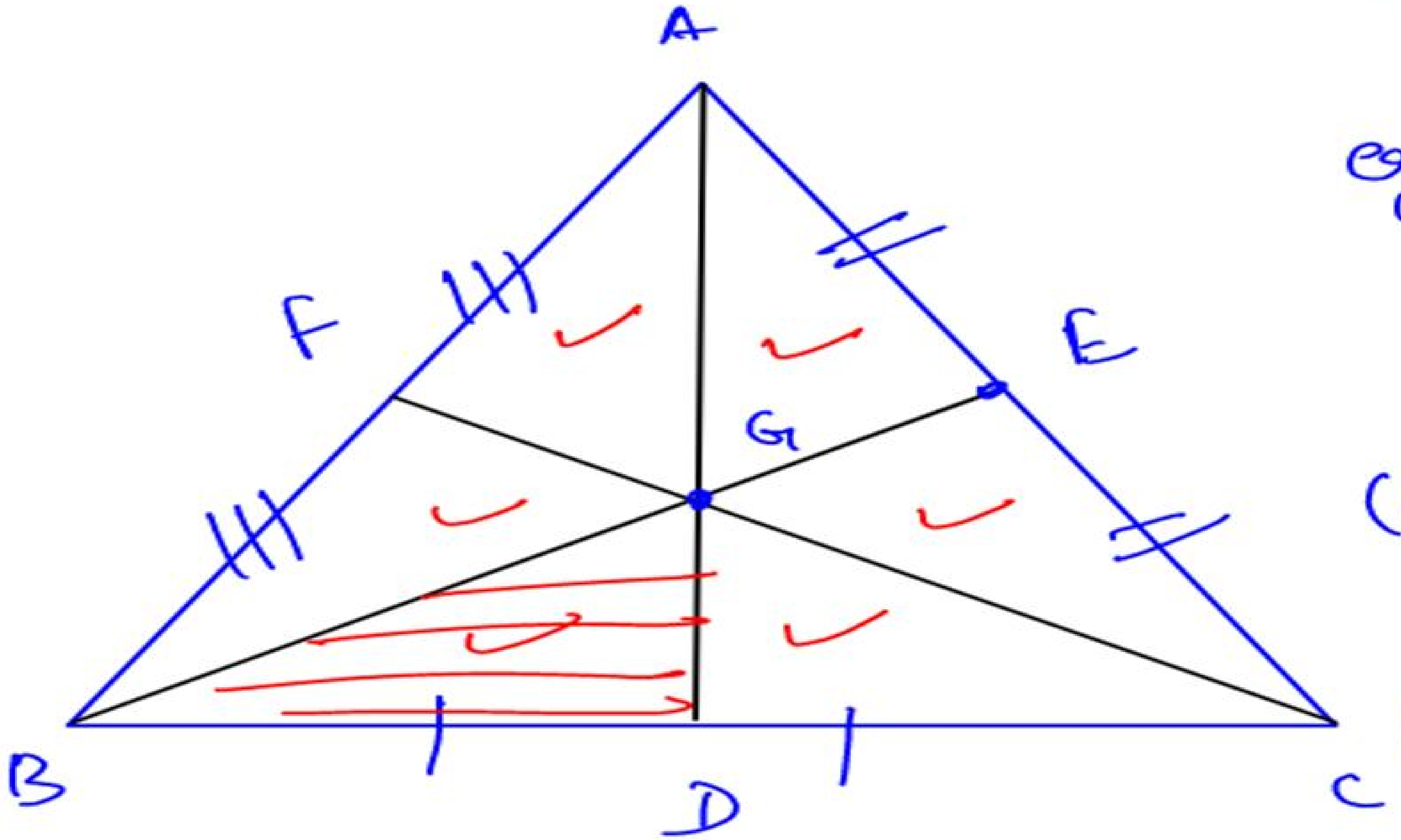
ABC is a Δ

& AD is the median

area of ΔABD = area of ΔACD

Reason :- Both Δ have same base & same height

(ii) Medians of a triangle divides a triangle in six equal areas.



eg If area of $\triangle BGD$

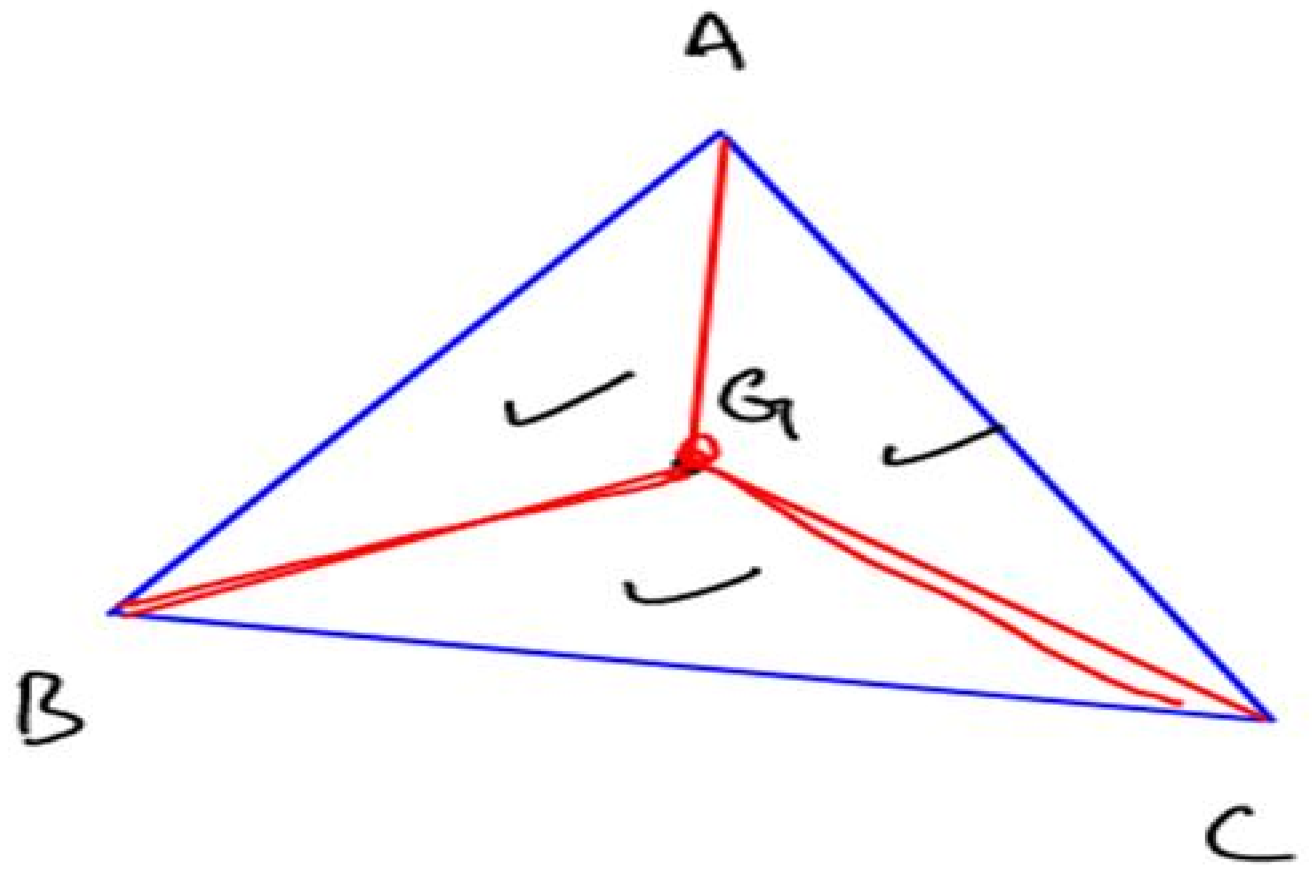
$= 10 cm^2$

(i) area of $\triangle BGC$

$\longrightarrow 20 cm^2$

(ii) area of $\triangle ABC$

$\longrightarrow 60 cm^2$



G is centroid of ΔABC

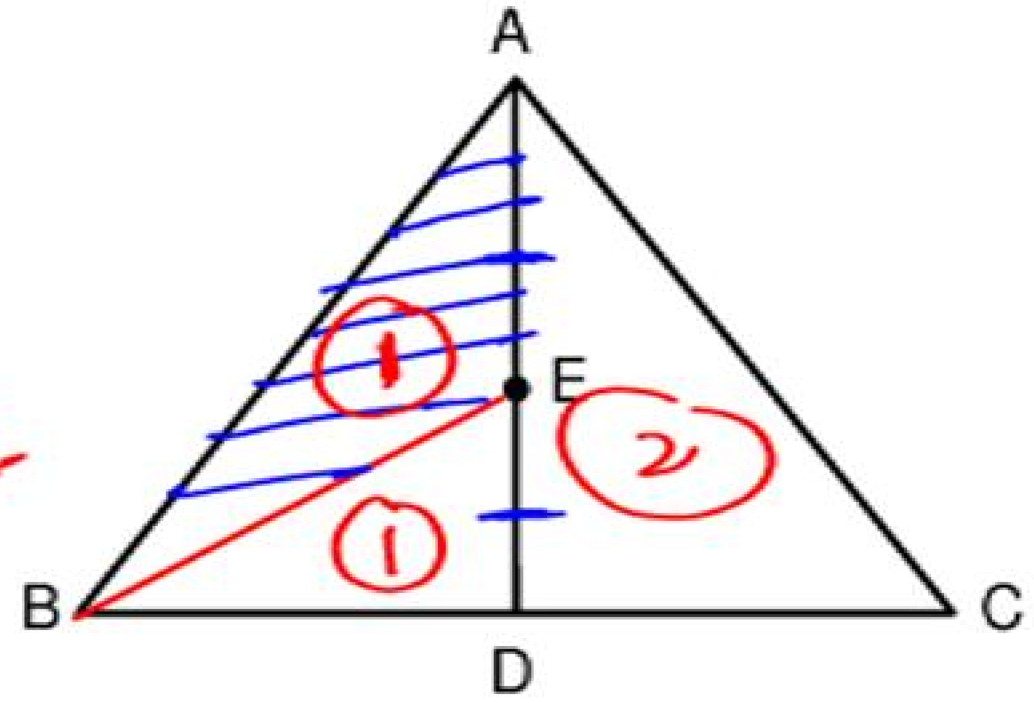
area of

$[\Delta ABG = \Delta BCG = \Delta CAG]$

Eg1. Given : AD is the median of ΔABC.

E is the mid-point of AD.

Find : $\dfrac{\text{Area of } \Delta ABE}{\text{Area of } \Delta ABC} \rightarrow \dfrac{1}{4}$



$\Delta ABD \longrightarrow$ BE is the median

$\dfrac{\text{area of } \Delta ABE}{\text{area of } \Delta ABD} = \dfrac{1}{2}$

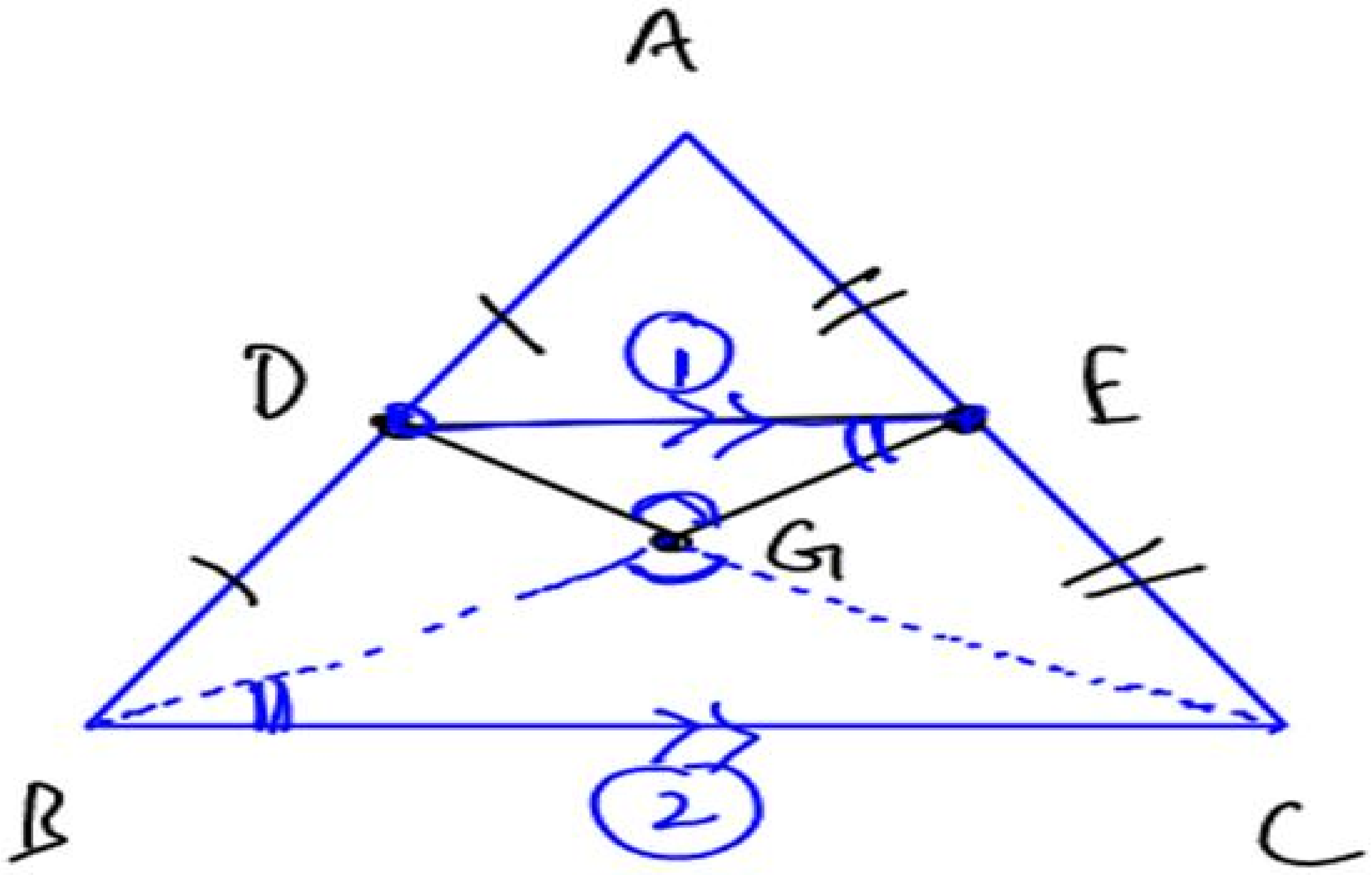
gradeup

Eg2. In a ΔABC,

v. imp

D, E are mid points of AB and AC and G is the centroid of ΔABC.

Find : $\boxed{\dfrac{\text{Area of } \Delta DEG}{\text{Area of } \Delta ABC} = \dfrac{1}{12}}$ I^{st}



ΔDGE & ΔCGB

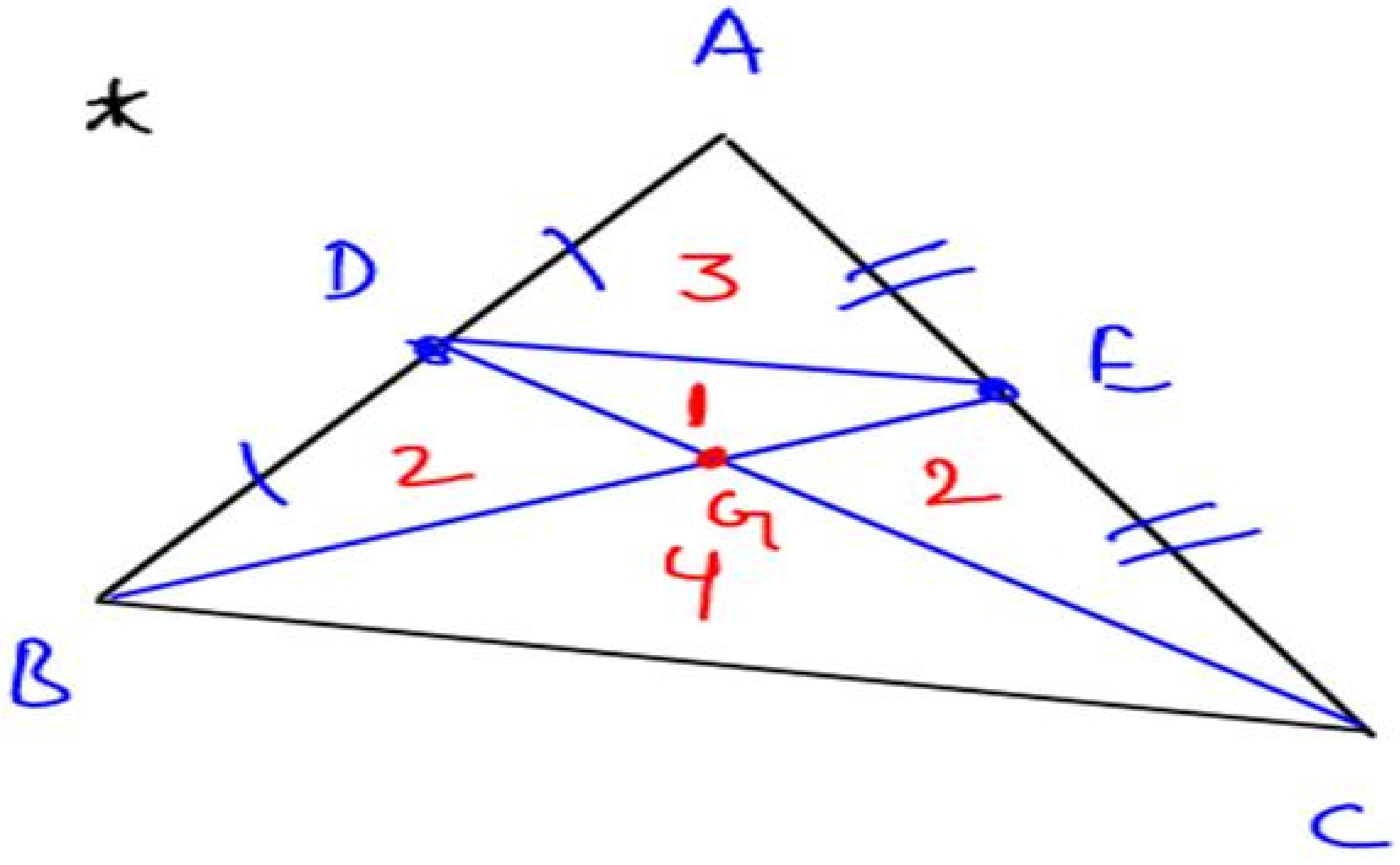
$\Delta DGE \sim \Delta CGB$

$$\frac{\text{area of } \Delta DGE}{\text{area of } \Delta CGB} = \frac{1}{4}$$

$$\text{area of } \Delta DGE = \frac{1}{4}\,\Delta CGB$$

$$= \frac{1}{4} \cdot \frac{1}{3}\,\Delta ABC$$

$$= \frac{1}{12}\,\Delta ABC$$



G → centroid

If area of ΔABC

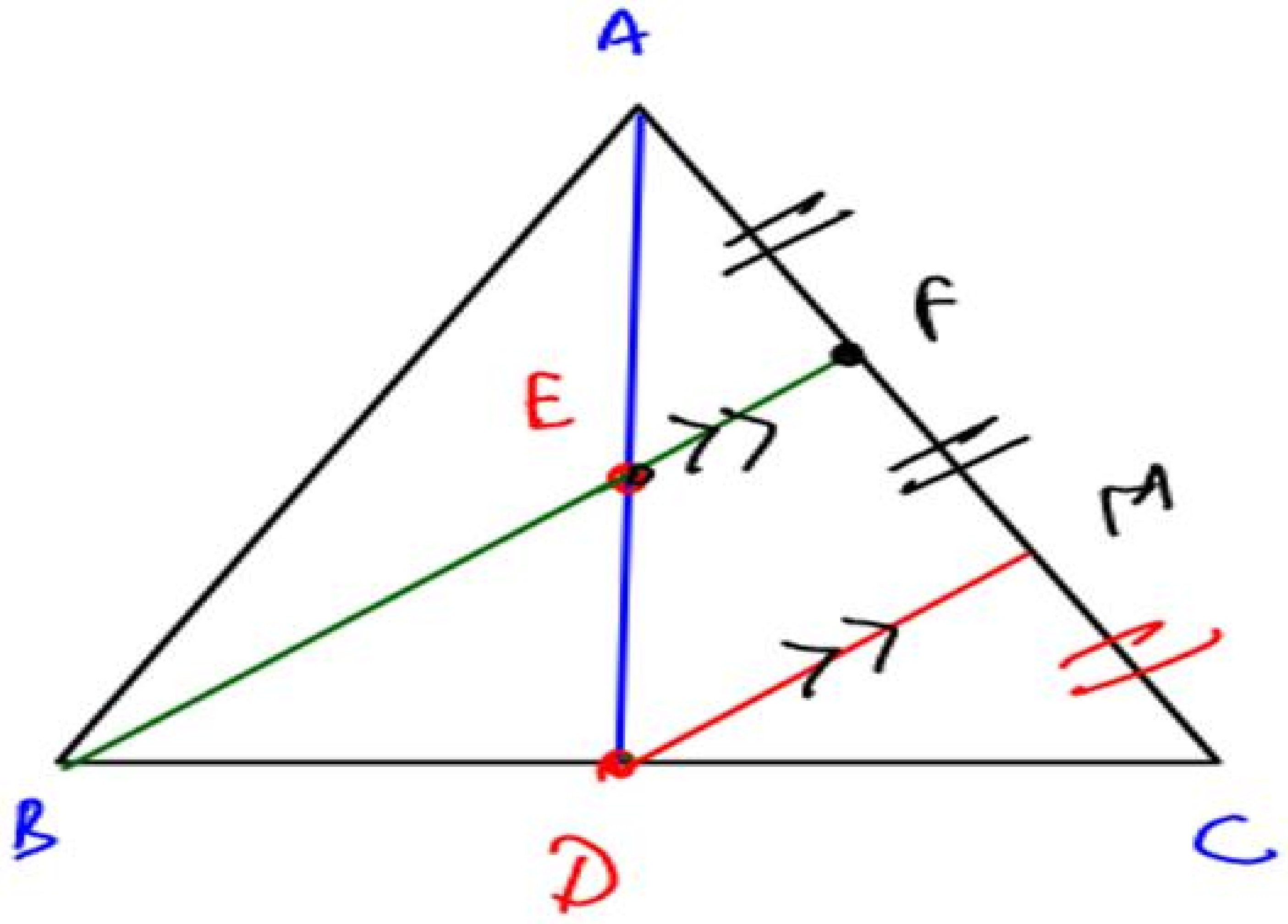
$= 12$ units

V. Imp

Eg3. In a ΔABC, AD is the median, E is the mid point of AD. On producing BE it meets AC at F.

Find : (i) AF : FC $\Rightarrow 1:2$

(ii) $\dfrac{\text{Area of } \Delta AEF}{\text{Area of } \Delta ABC} = \dfrac{1}{12}$



Const ÷ Draw DM || BF meeting AC at M

ΔADM

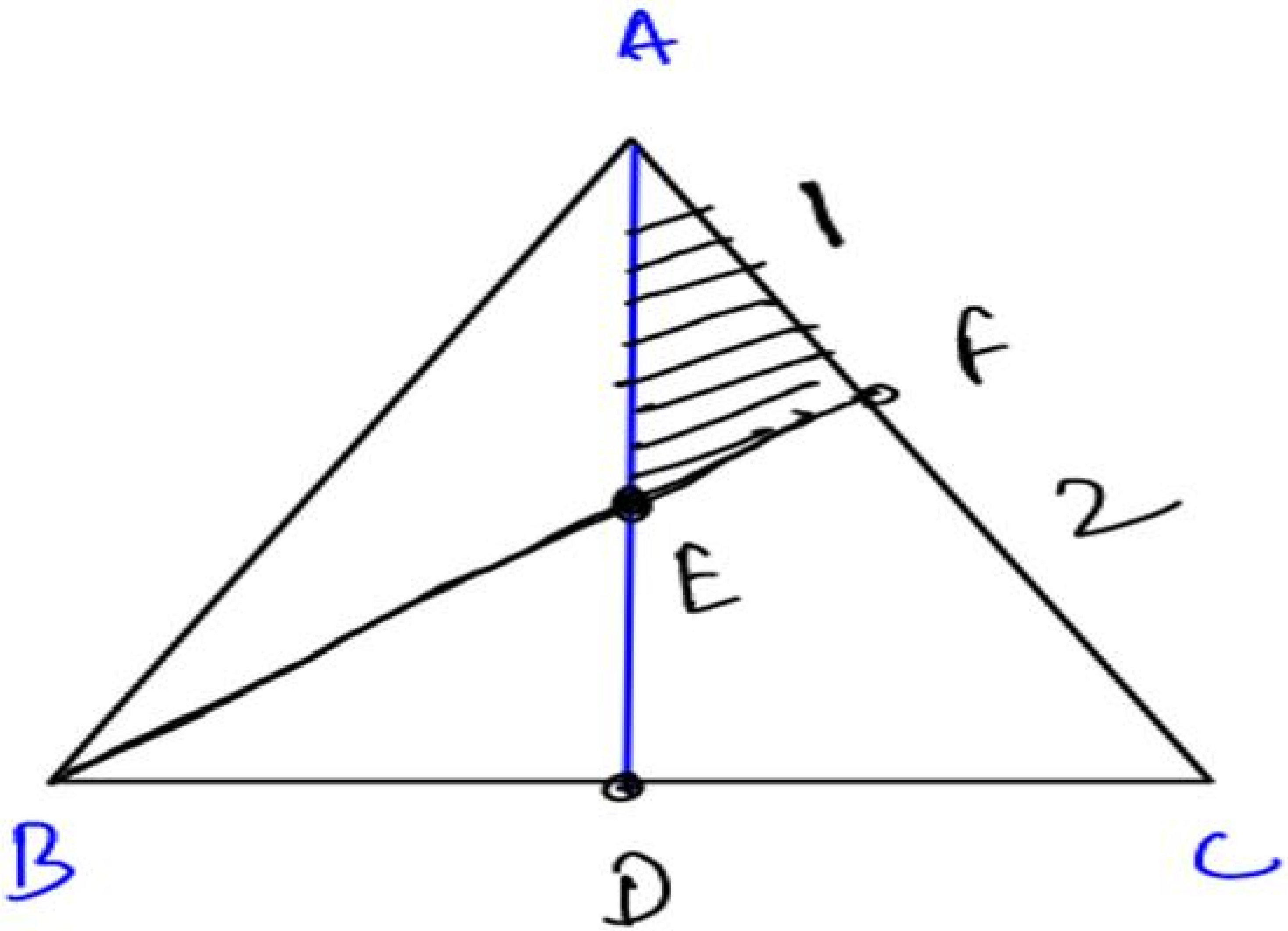
→ E is m.p of AD & EF is || to DM

So AF = FM [Converse of m.p Theorem]

ΔBFC D is m.p of BC

DM || BF

FM = MC [Converse of mid pt]



$$\frac{\text{area of } \Delta ABF}{\text{area of } \Delta ABC} = \frac{1}{3}$$

$$\frac{\text{area of } \Delta ABE}{\text{area of } \Delta ABC} = \frac{1}{4}$$

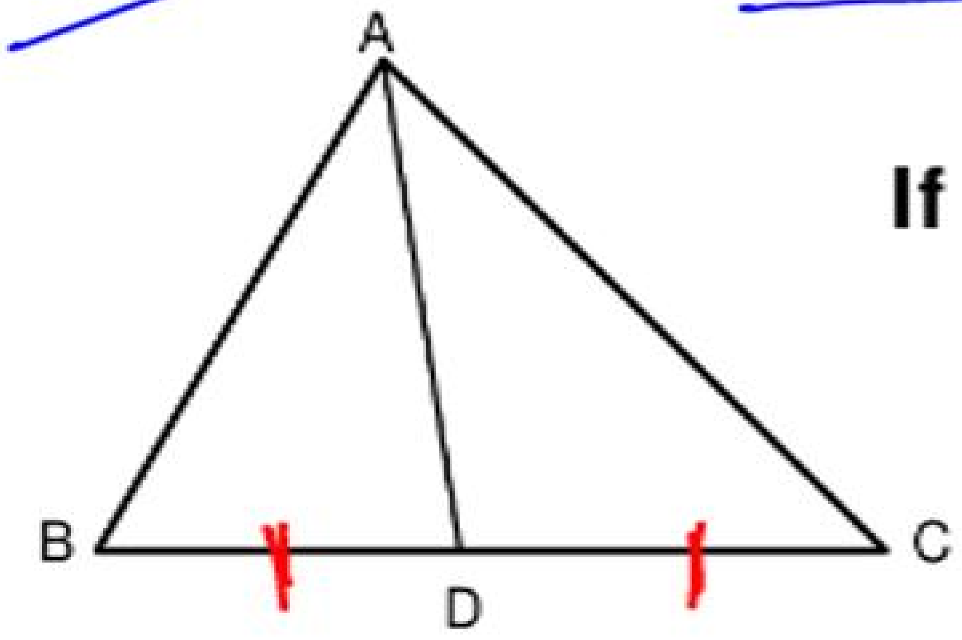
$$\text{area of } \Delta AEF = \text{area of} (\Delta ABF - \Delta ABE)$$

$$= \frac{1}{3} \Delta ABC - \frac{1}{4} \Delta ABC$$

$$= \frac{1}{12} \Delta ABC$$

APOLLONIUS THEOREM

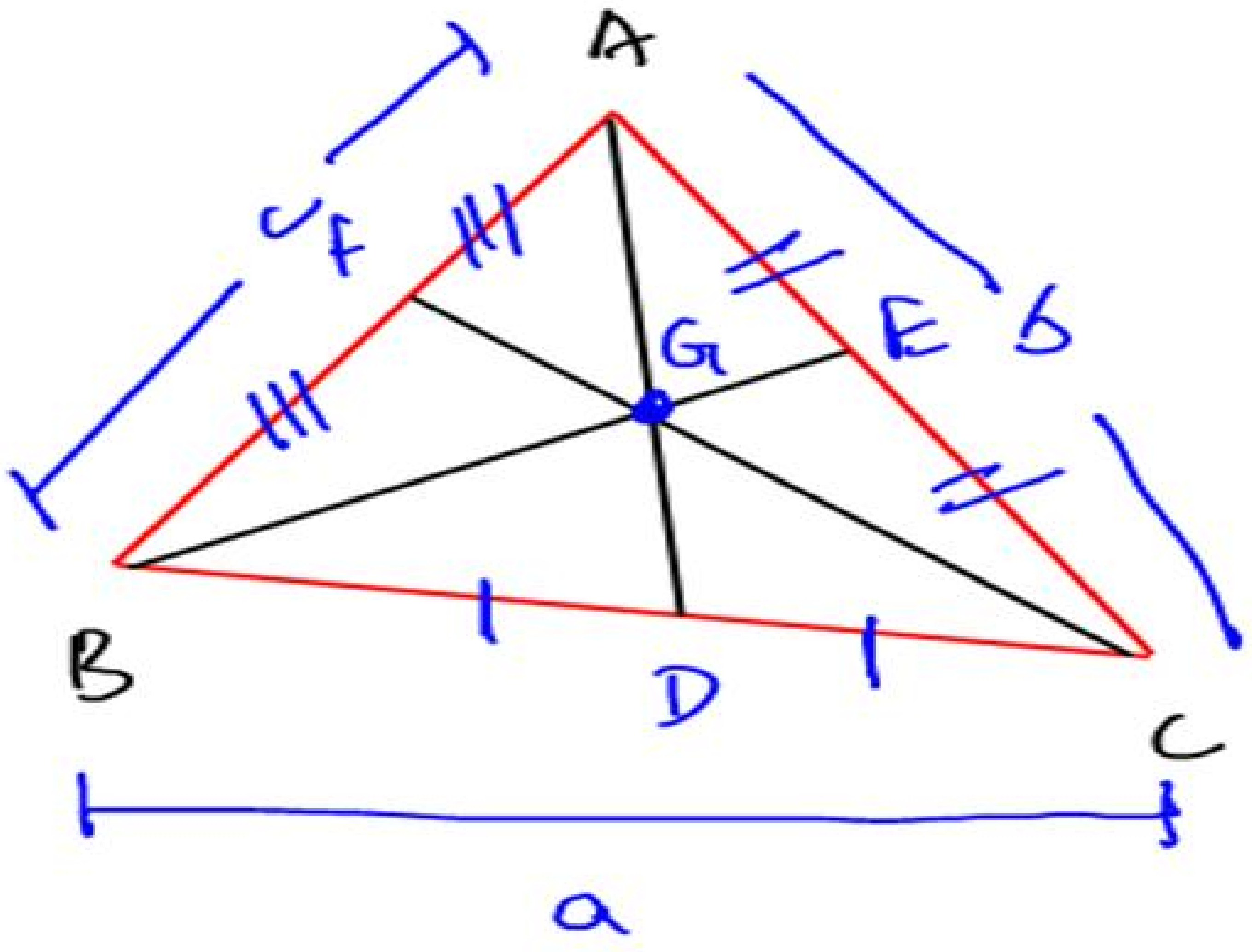
V. imp



If AD is median of ΔABC:

$$AB^2 + AC^2 = 2\left(AD^2 + BD^2\right)$$

Use: When you have to find length of medians.



$$c^2 + b^2 = 2\left(AD^2 + \frac{a^2}{4}\right)$$

$$2(c^2+b^2) = 4(AD^2) + a^2 \quad \text{— (1)}$$

$$c^2 + a^2 = 2\left(BE^2 + \frac{b^2}{4}\right)$$

$$2(c^2+a^2) = 4BE^2 + b^2 \quad \text{— (2)}$$

$$a^2 + b^2 = 2\left(CF^2 + \frac{c^2}{4}\right)$$

$$2(a^2+b^2) = 4CF^2 + c^2 \quad \text{— (3)}$$

$$\boxed{3(a^2+b^2+c^2) = 4(AD^2+BE^2+CF^2)}$$

V. Imp

$$3\left(AB^2 + BC^2 + CA^2\right) = 4\left(AD^2 + BE^2 + CF^2\right)$$

AB, BC and CA are sides of Δ.

AD, BE and CF are medians of Δ.

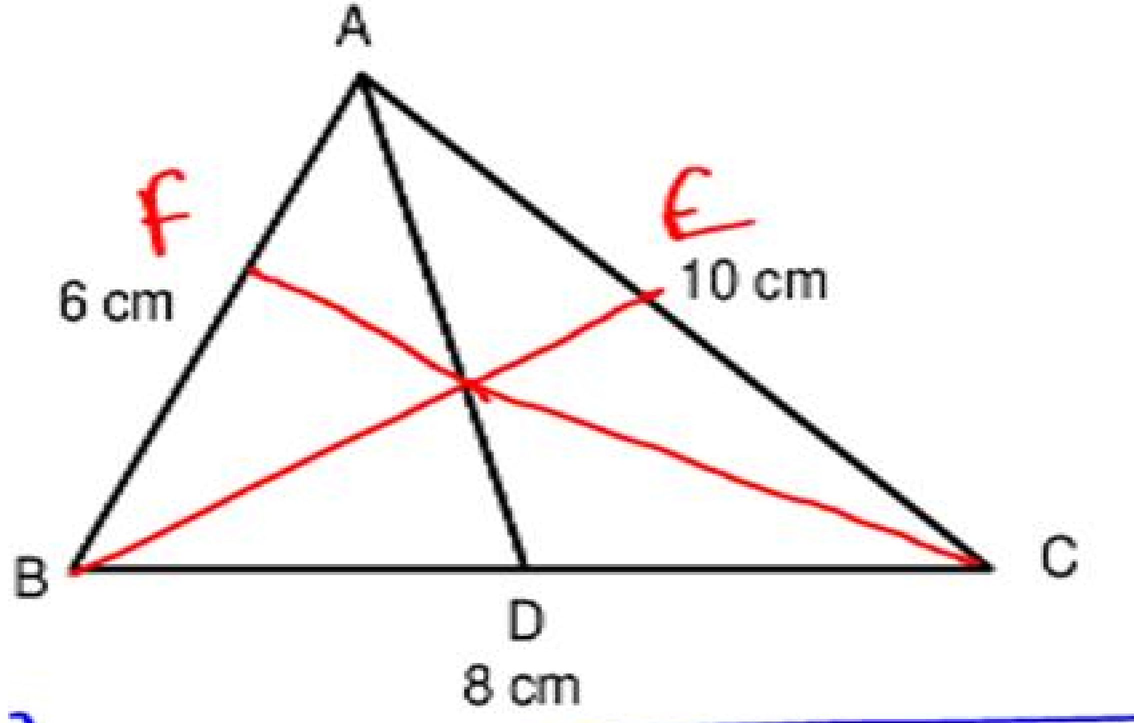
Eg4. In the given figure, if the sides of the triangle are 6, 8 and 10 cm.

Find the length of :

(i) median AD

(ii) median BE

(iii) median CF



for AD

$$6^2 + 10^2 = 2(AD^2 + 4^2)$$

$$68 - 16 = AD^2$$

$$\boxed{AD = \sqrt{52}}$$

for BE

$$6^2 + 8^2 = 2(BE^2 + 5^2)$$

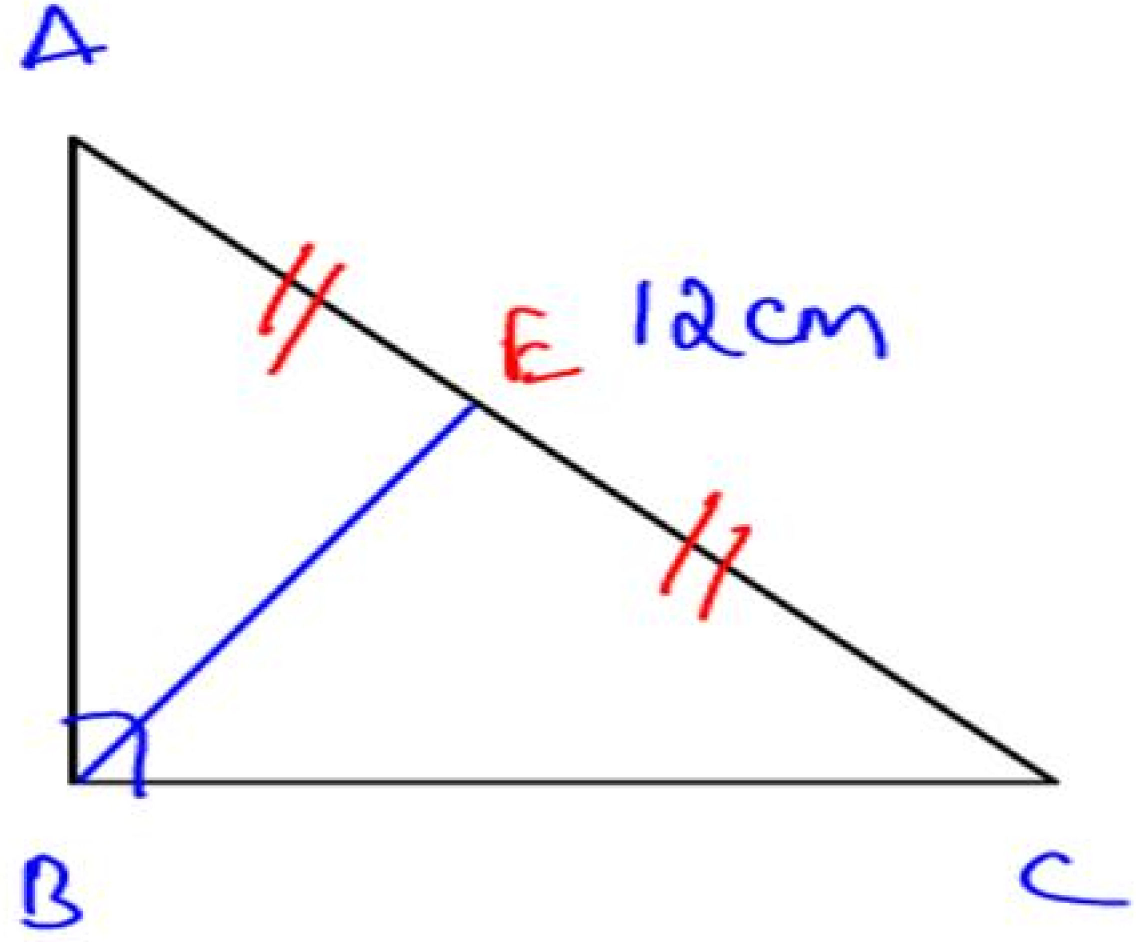
$$BE = 5$$

for CF

$$8^2 + 10^2 = 2(CF^2 + 3^2)$$

$$CF = \sqrt{73}$$

eg



length of median $BE = \frac{1}{2} \cdot 12$

$= 6\text{ cm}$

Observations drawn from previous example.

1. Median drawn to the smallest side is largest and median drawn to the largest side is smallest.
2. Median drawn to the hypotenuse is half of the hypotenuse.

gradeup

FOR ALL TRIANGLES:

V. V Imp

If the length of the medians are M_1, M_2 & M_3 then,

$$\textbf{\textit{Area of }} \Delta = \frac{\mathbf{4}}{\mathbf{3}} \times \left(\textbf{\textit{Area of }} \Delta \textbf{\textit{ considering medians as sides}}\right)$$

gradeup

Eg5. If the length of the medians are 9, 12 & 15 cm, then find the area of triangle.

$a = 9 \quad b = 12 \quad c = 15$

$S = 18$

$\text{Area} = \sqrt{18(9)(6)(3)}$

$= 54$

$\text{Area of } \Delta = \frac{4}{\cancel{3}} \times \overset{18}{\cancel{54}} = \underline{\underline{72 cm^2}}$

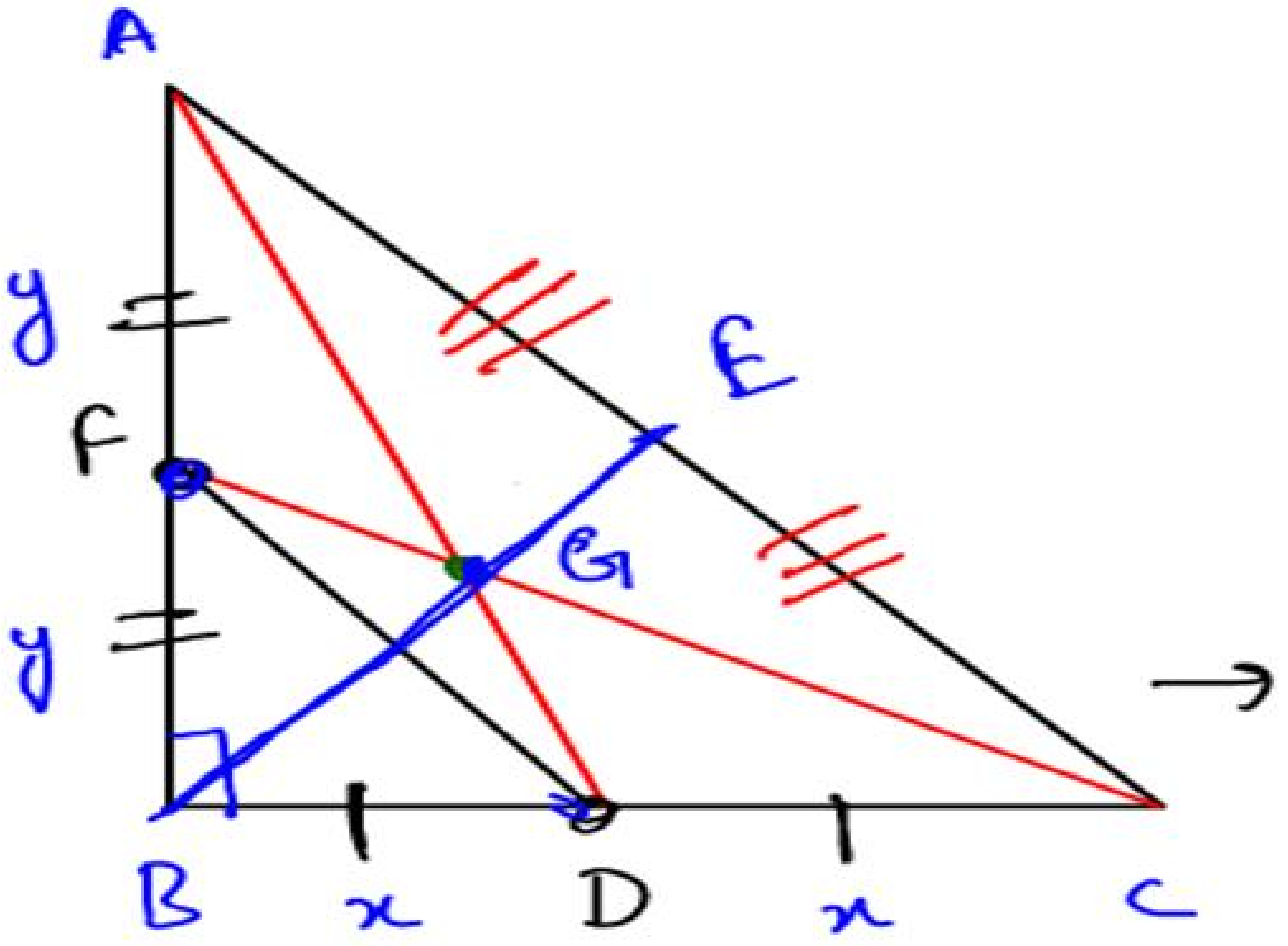
gradeup

In a right angle ΔABC, right angled at B. AD, BE and CF are medians of triangle, then:

ans (i) $4\,(AD^2 + CF^2) = 5\,(AC)^2$

(ii) $AD^2 + CF^2 = 5\,(DF)^2$

(ii) $AD^2 + CF^2 = 5\,(BE)^2$



$\underline{\Delta ABD}$ $\quad AD^2 = 4y^2 + x^2$

ΔBFC $\quad CF^2 = 4x^2 + y^2$

$AD^2 + CF^2 = 5(x^2 + y^2)$

$4\left[AD^2 + CF^2\right] = 5\left((2x)^2 + (2y)^2\right)$

$= 5\left(BC^2 + AB^2\right)$

$\rightarrow \quad 4\left[AD^2 + CF^2\right] = 5AC^2$

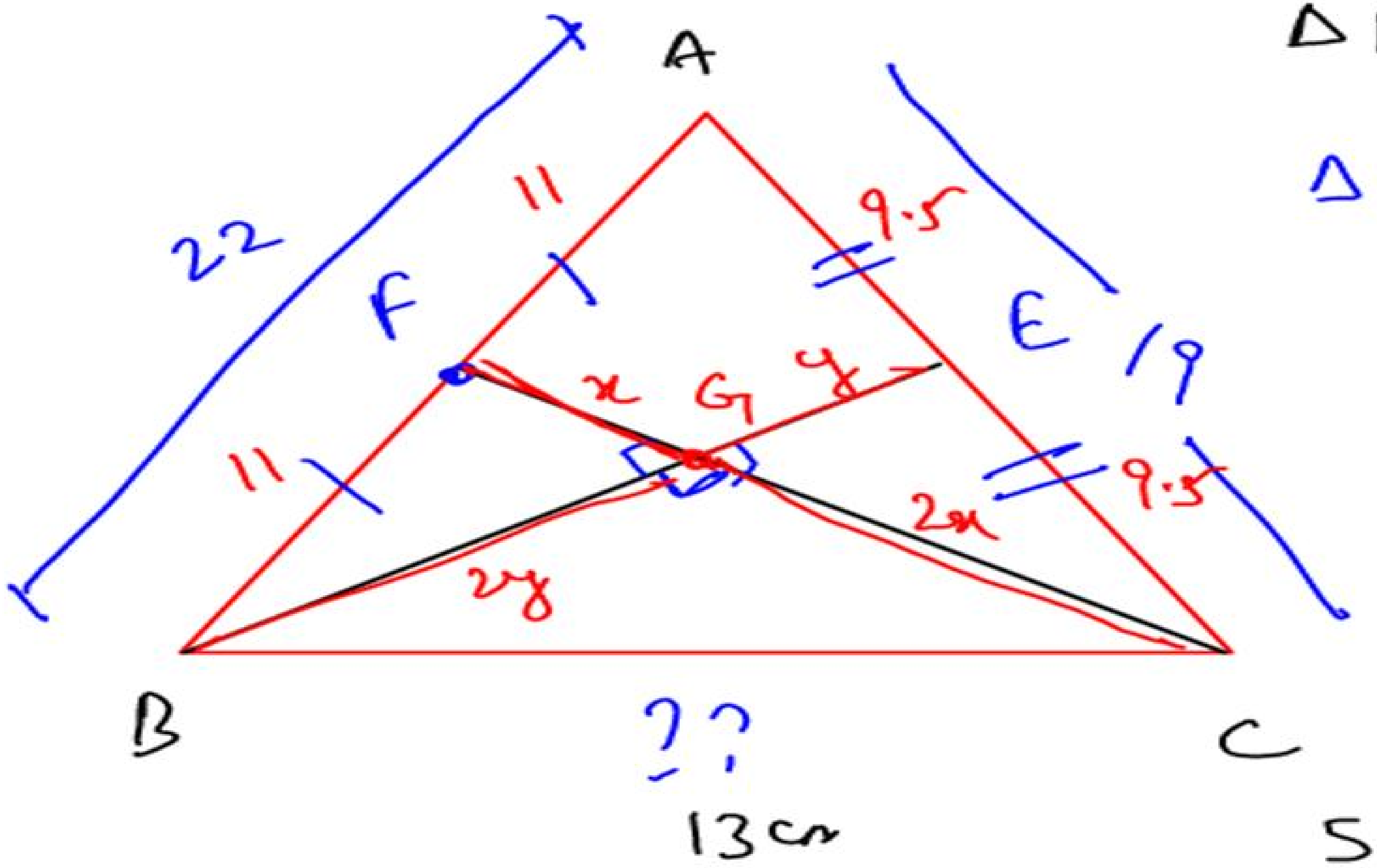
$\Rightarrow \quad AD^2 + CF^2 = 5\left(\frac{AC}{2}\right)^2$

$AD^2 + CF^2 = 5DF^2$

gradeup

Eg6. In a ΔABC, medians BE and CF are $\perp$ to each other, if AB = 22 cm and AC = 19 cm. Find the length of BC.

V. imp



ΔFGB $\quad x^2+4y^2 = 121$

ΔEGC $\quad 4x^2+y^2 = 90.25$

$5(x^2+y^2) = 211.25$ — ①

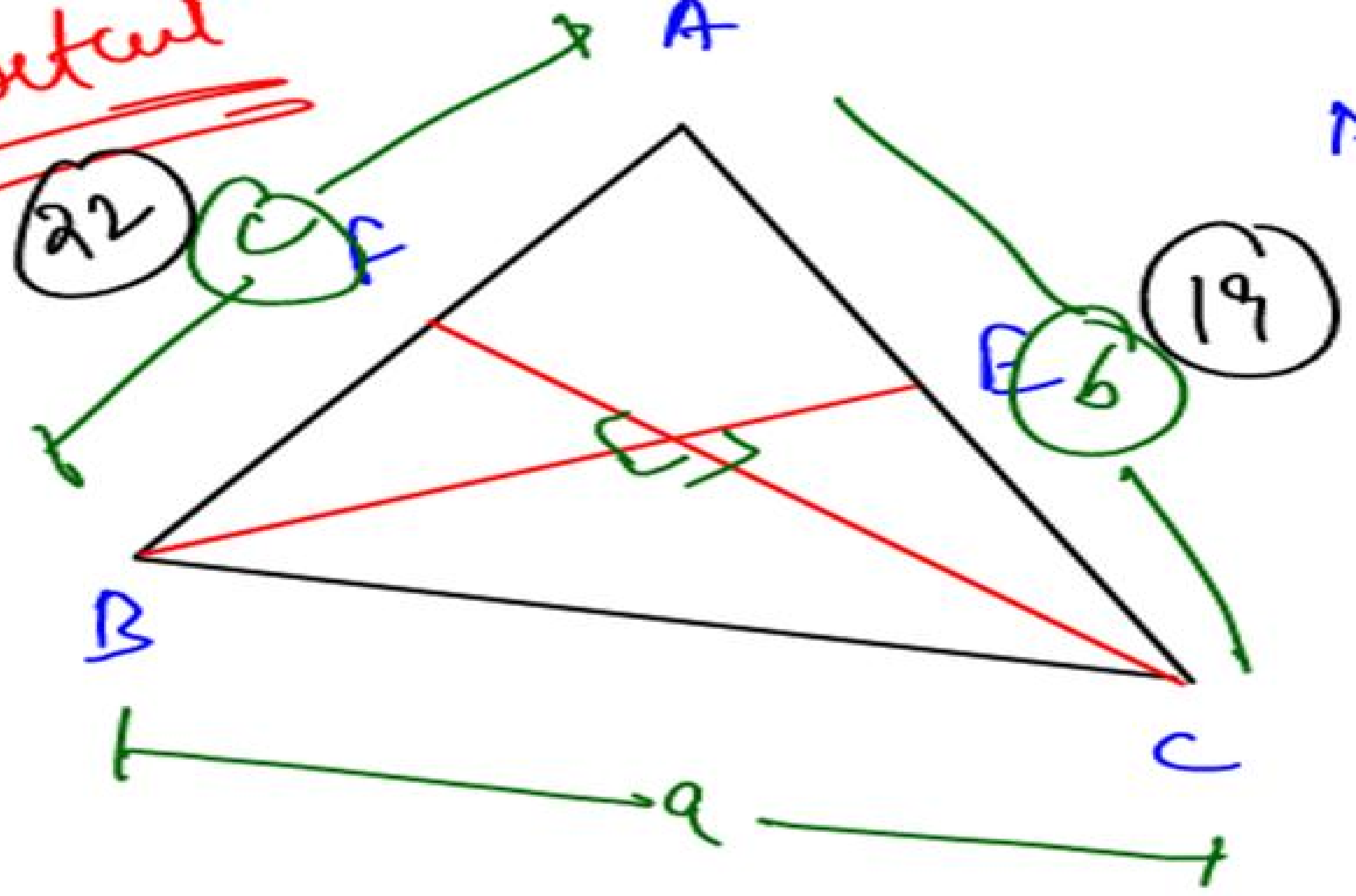
ΔBGC $\quad 4x^2+4y^2 = BC^2$

$4(x^2+y^2) = BC^2$ — ②

$\frac{5}{4} = \frac{211.25}{(BC)^2}$ $\quad BC^2 = 169$

$BC = 13$ cm

Shortcut



Medians $\underline{BE}$ & $\underline{CF}$ are $\perp^{nd}$ to each other

$$\boxed{b^2 + c^2 = 5a^2}$$

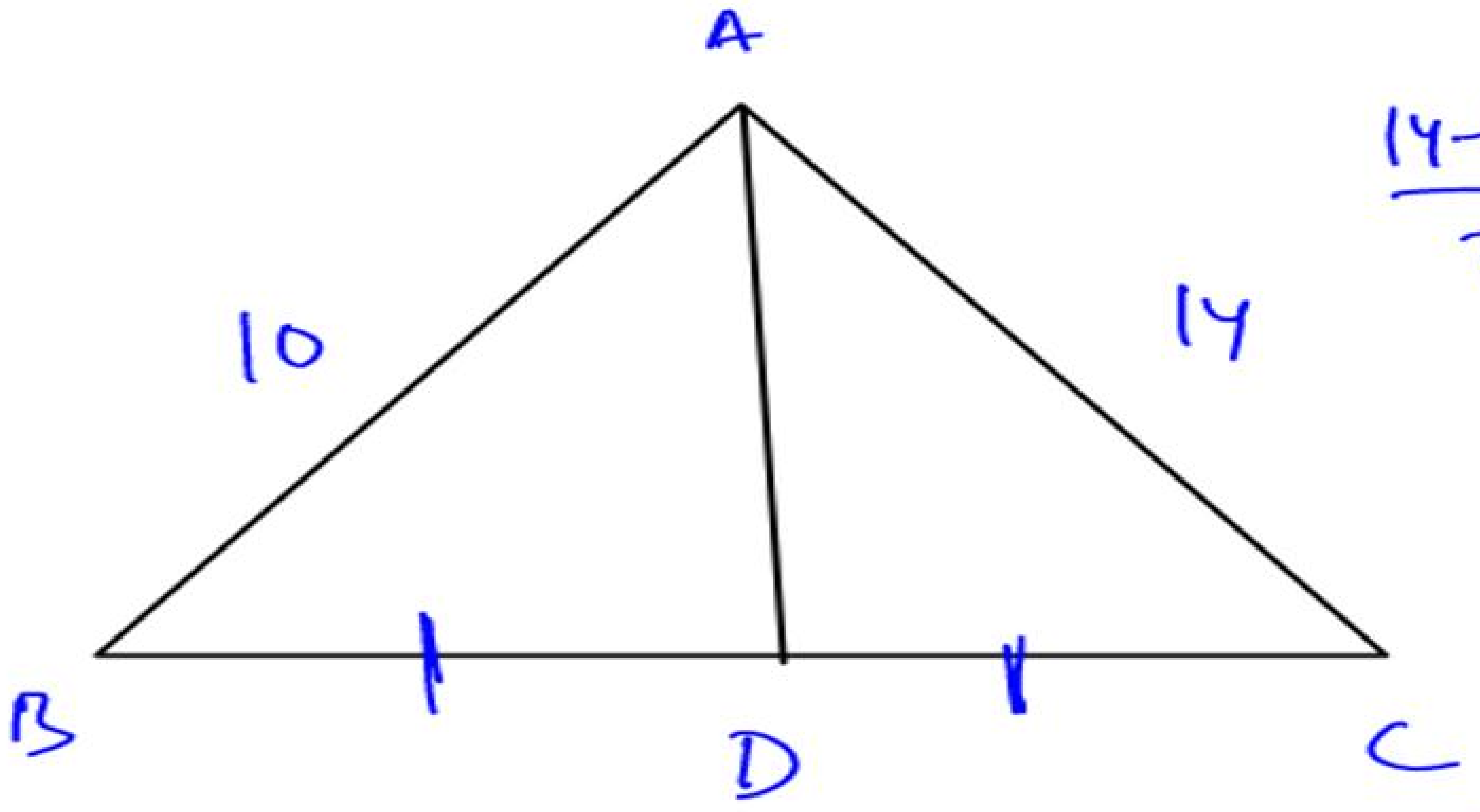
$$22^2 + 19^2 = 5a^2$$

$$\frac{484 + 361}{5} = a^2 \qquad a^2 = 169$$

$$a = 13$$

In a ΔABC, $AB = 10$ cm, $AC = 14$ cm

How many values of median AD are possible, if the length of median AD is a natural number?



$$\frac{14-10}{2} < AD < \frac{14+10}{2}$$

$$2 < AD < 12$$

$3, 4, 5, \ldots 11$

$\boxed{\text{9 values are possible}}$

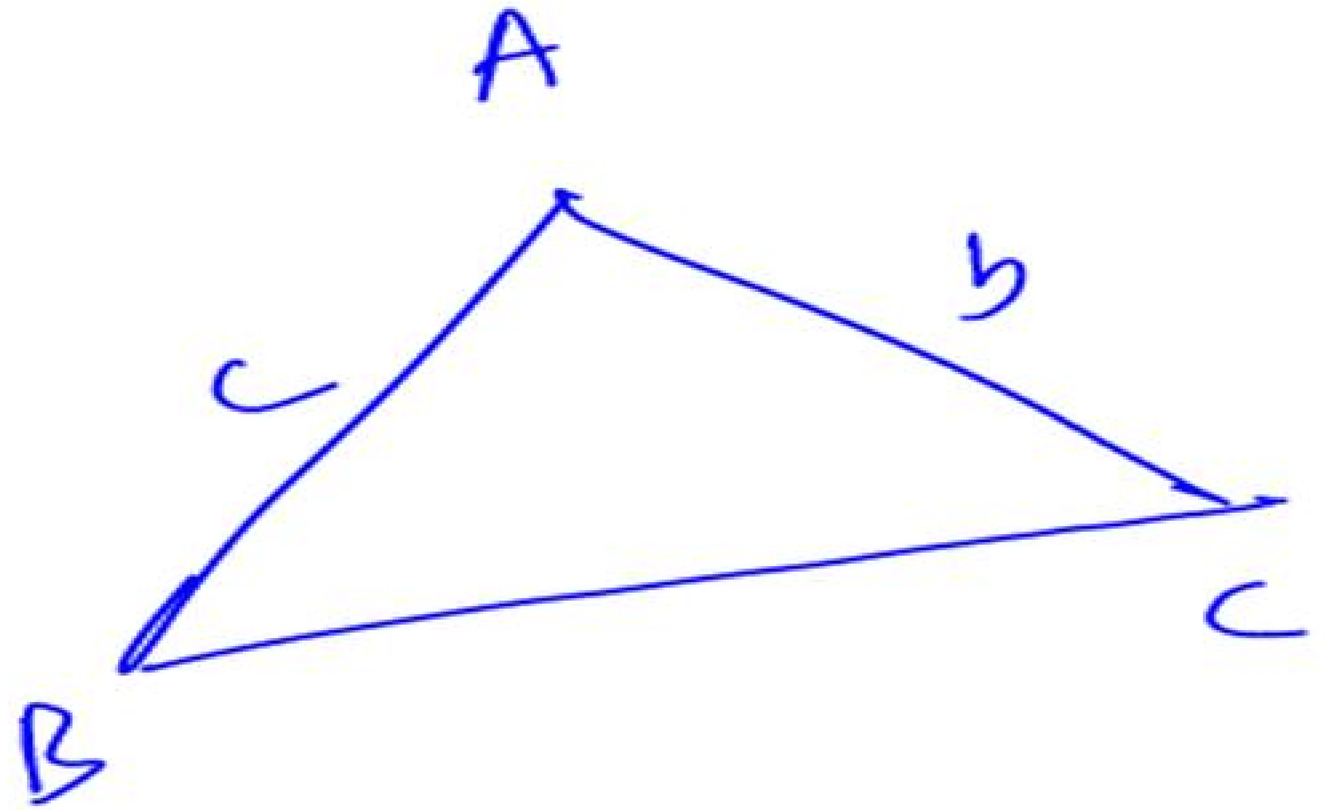
In a ΔABC, AD is the median

AB = c

BC = a

CA = b

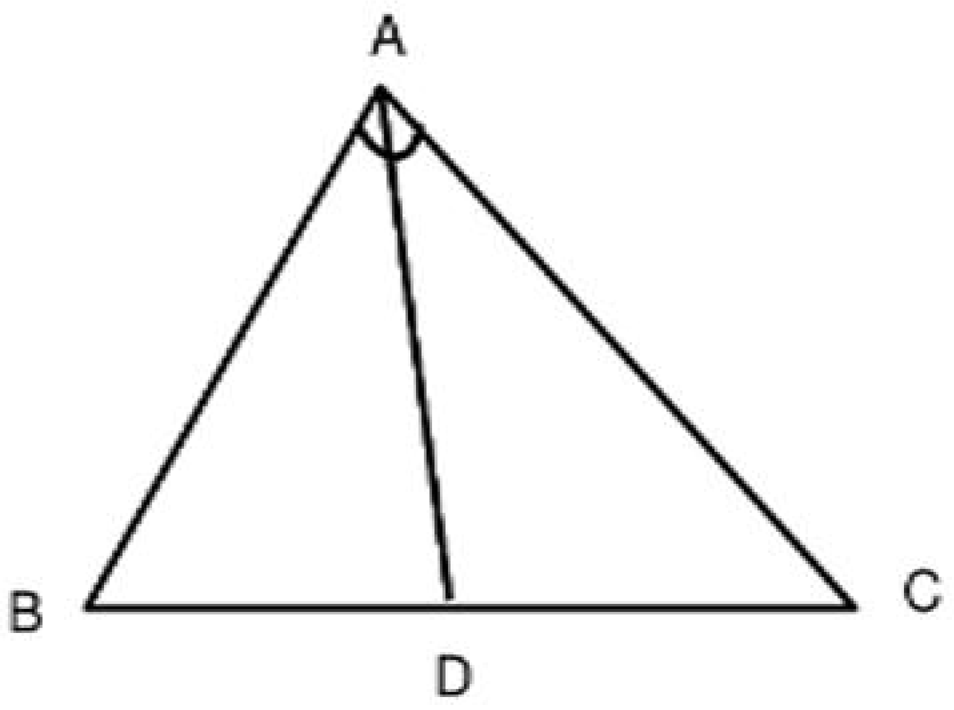
$$\frac{|b-c|}{2} < AD < \frac{b+c}{2}$$



$$\frac{3}{4}(Perimeter) < (AD + BE + CF) < Perimeter$$

Where, AD, BE and CF are medians of the triangle.

INTERNAL ANGLE BISECTOR THEOREM



Given AD is angle bisector of $\angle BAC$.

$$\frac{AB}{AC} = \frac{BD}{DC}$$