gradeup

Sahi Prep Hai Toh Life Set Hai

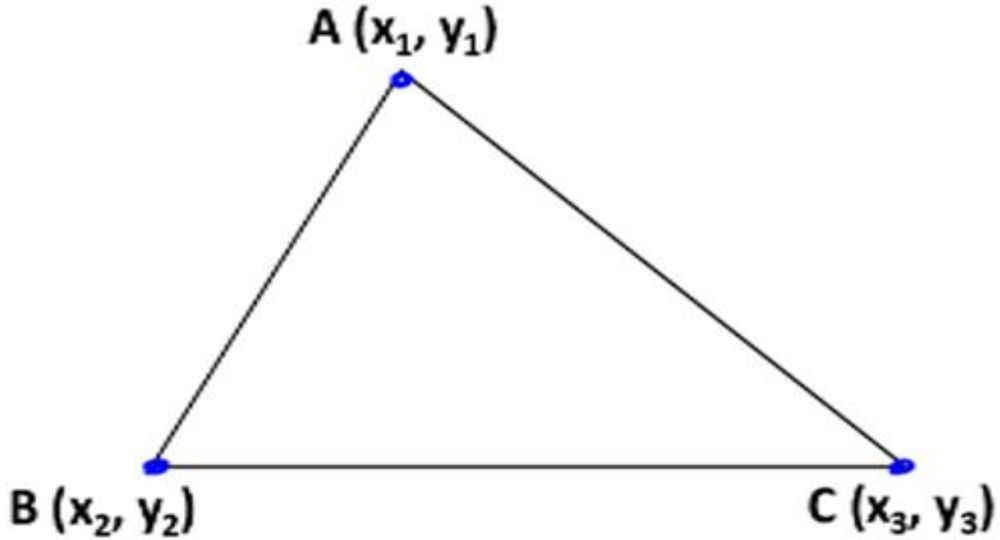
COORDINATE GEOMETRY

AREA OF TRIANGLE

If the coordinates of the three vertices of triangle ABC are :

$A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$



Eg. **Find the area of a triangle whose vertices are:**
A (3, 2), B (11, 8) & C (8, 12).

$x_1\ y_1 \quad x_2\ y_2 \quad x_3\ y_3$

$$\text{Area} = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$$

$$= \frac{1}{2}\left| 3(-4) + 11(10) + 8(-6) \right|$$

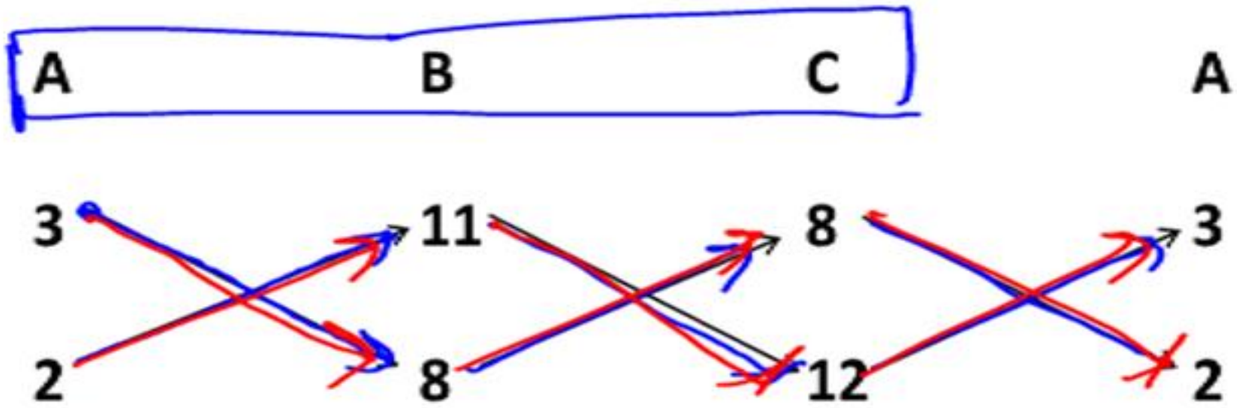
$$= \frac{1}{2}\left| -12 + 110 - 48 \right| = 25$$

Shortcut for calculating area of polygon:

A (3, 2)

B (11, 8)

C (8, 12)

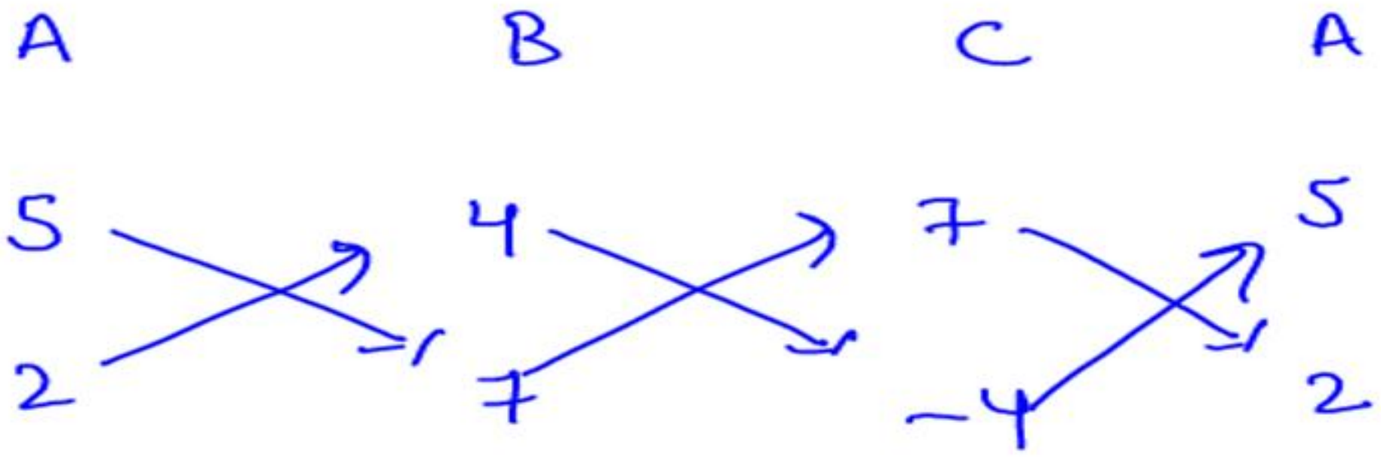


$$\frac{1}{2}\left|(3\cdot 8)+(11\cdot 12)+(8\cdot 2)-\left[(2\cdot 11)+(8\cdot 8)+(12\cdot 3)\right]\right| = \frac{1}{2}\left|172-122\right|$$

$$\frac{1}{2}\left|172-122\right| = 25$$

Ans. 25

Eg. **Find the area of a triangle formed by the points :**
A (5, 2), B (4, 7) & C (7, –4).

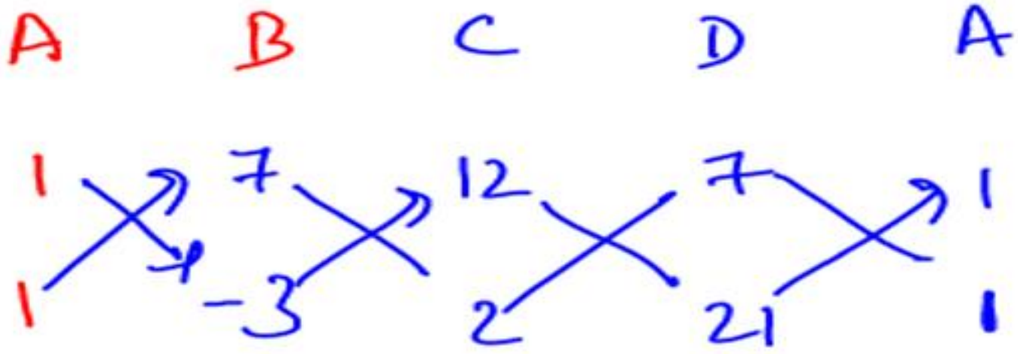


$$\frac{1}{2}\left|\left[(5\cdot 7)+4(-4)+7(2)\right]-\left[2\cdot 4+7\cdot 7-4\cdot 5\right]\right|$$

$$=\frac{1}{2}\left|\,33-37\,\right| \quad = \boxed{2}$$

Ans. 2

Eg. Find the area of the quadrilateral ABCD whose vertices are respectively A (1, 1), B (7, –3), C (12, 2) and D (7, 21).

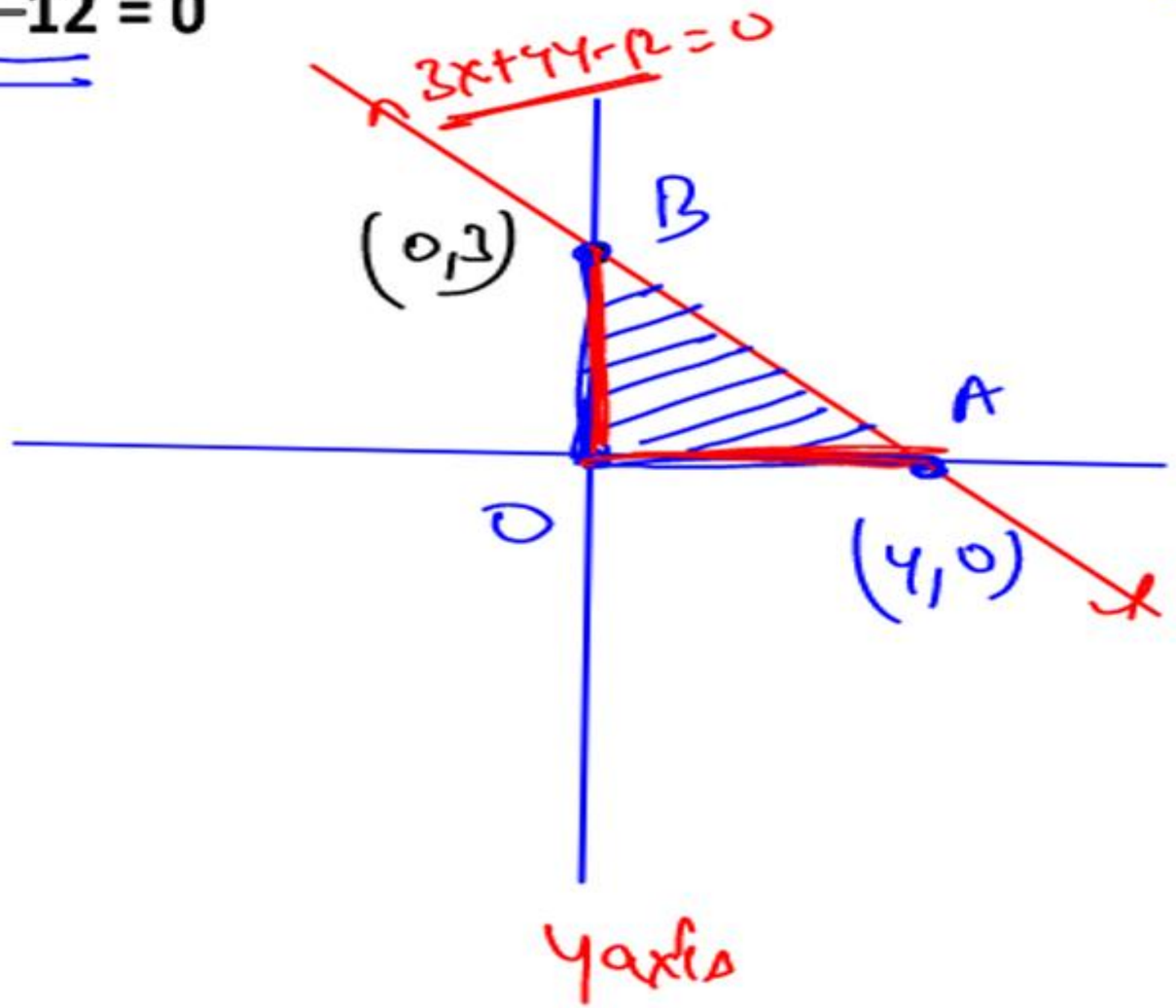


$$\frac{1}{2}\left|(-3+14+252+7)-(7-36+14+21)\right|$$

$$= \frac{1}{2}\left|270-6\right| = \underline{132}$$

Ans. 132

Eg. Find the area of triangle bounded by x-axis, y-axis and $3x+4y-12=0$



$3x+4y=12$

Xaxis

x	0	4
y	3	0

Area of $\triangle OAB$

$= \frac{1}{2} \times 4 \times 3$

$= 6$

Ans. 6

Shortcut for previous question:

$$\begin{cases} x\text{axis} \\ y\text{axis} \\ ax+by+c=0 \end{cases}$$

Area of Δ formed

$$= \frac{1}{2}\left|\frac{c^2}{ab}\right|$$

eg

x axis

y axis

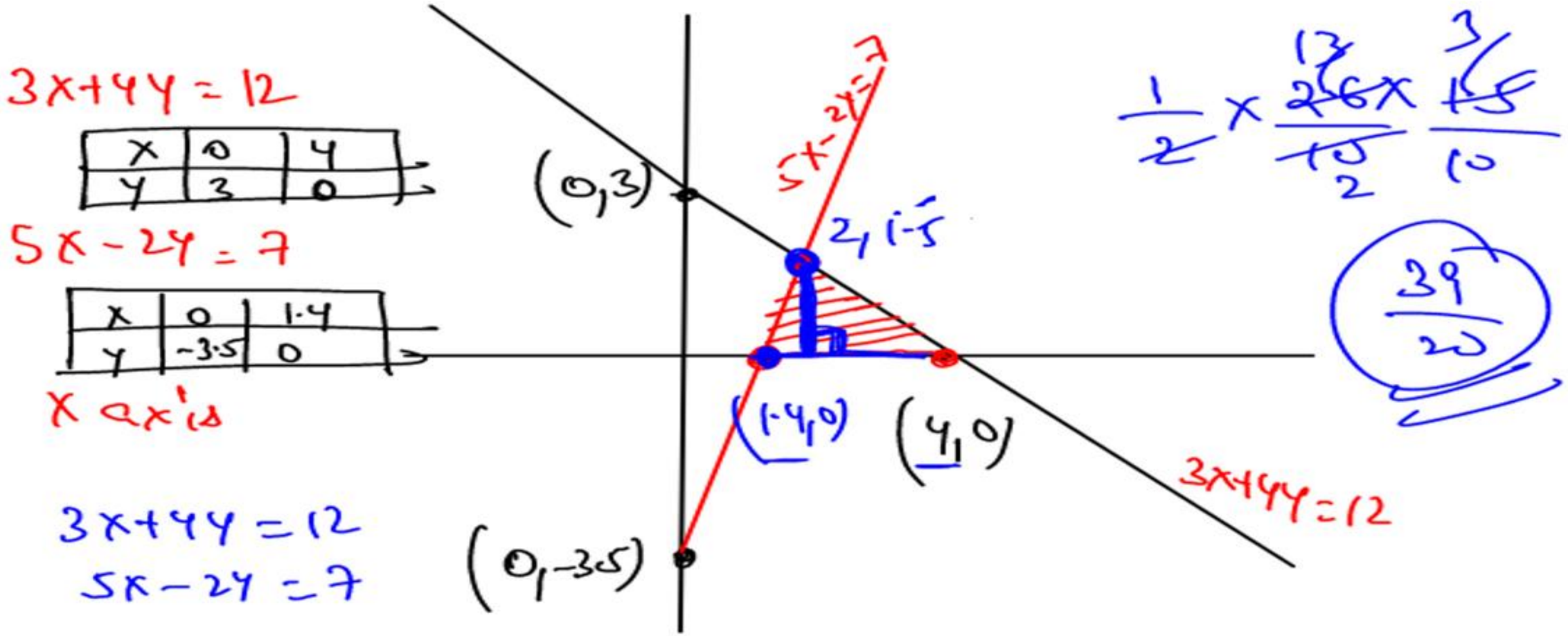
$3x+4y-12=0$

Area $\rightarrow \frac{1}{2} \times \frac{12 \times 12}{3 \times 4}$

$= 6$

Eg. Find the area of the triangle formed by $3x + 4y = 12$, $5x - 2y = 7$ and x-axis:

Time → 2min



$\frac{1}{2} \times \frac{13}{2} \times \frac{3}{10}$

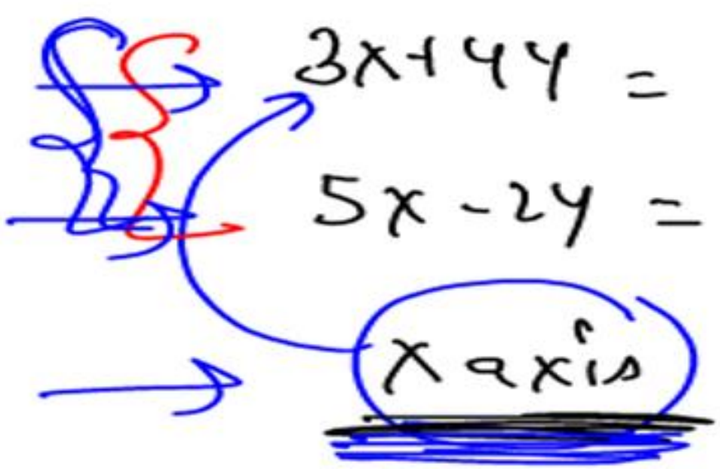
$\frac{39}{20}$

$3x + 4y = 12$

$5x - 2y = 7$

$x = 2 \quad y = 1.5$

Ans. 39/20



$$\begin{cases} x = 4 \\ x = 7/5 \end{cases}$$

$$\begin{cases} 3x+4y=12 \\ 5x-2y=7 \end{cases} \quad \left(2, \frac{3}{2}\right)$$

$$\text{Area} = \frac{1}{2} \times \left(4 - \frac{7}{5}\right) \frac{3}{2}$$

$$\frac{1}{2} \times \frac{13}{5} \times \frac{3}{2} = \frac{39}{20}$$

eg Find area of Δ formed by

$2x + 3y = 12$

$5x - 7y = 1$

& y axis