



gradeup

Sahi Prep Hai Toh Life Set Hai

TRIGONOMETRY

(Measurement of Angles)

Syllabus of Maths

I Number System

II Arithmetic

III Algebra

IV Trigonometry

V Measurement & Geometry

VI DI

* It doesn't
matter with
which topic
you are starting

* { Coordinate Geometry Part 2.
(Back up Monday)

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195 SSC Mains 2015
Rank 11th
 99.63 percentile CAT
 99.8 XAT
 99.98 MAT

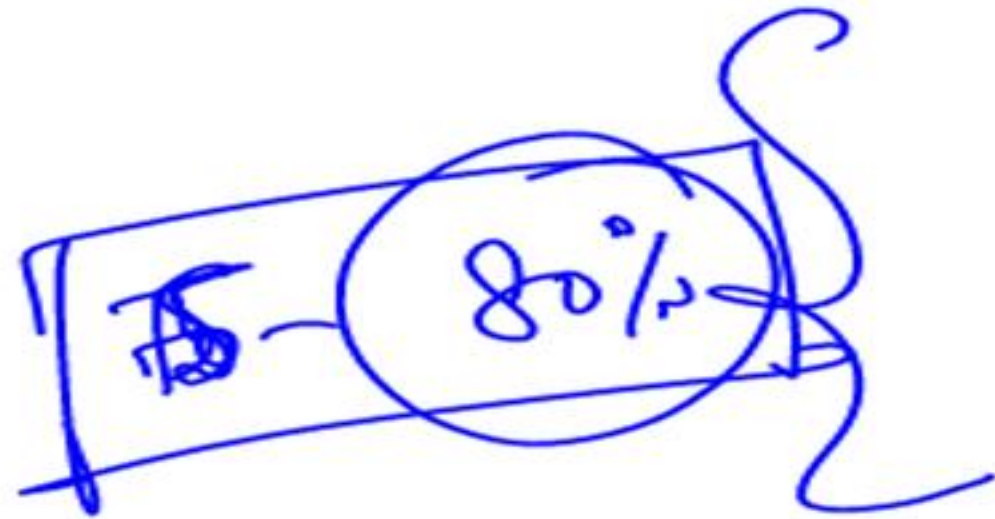
Trigonometry

V. V. and

- Measurement of Angles → ① (2-3)°
- Trigonometric Ratios → ① (10-15)°
- Trigonometric Identities → (45-55)°
- Maximum and Minimum of Trigonometric function → ① ≈ 5°
- Practice Questions → ②
- Transformation Formulas
- Trigonometric Ratios of Multiples and Sub-multiples angles → ② (10°-15°)
- Height & Distance ② → 10-15°
- Doubt class → ①

For Basics

NCERT | RD Sharma



Trigonometric Ratios
Trigonometric Identities
Height & Distance

10th class
NCERT

Compound Angles
Multiple Angles
Measurement of

TRIGONOMETRY

The word 'Trigonometry' is derived from the Greek words :

(i) Trigonon and (ii) Metron

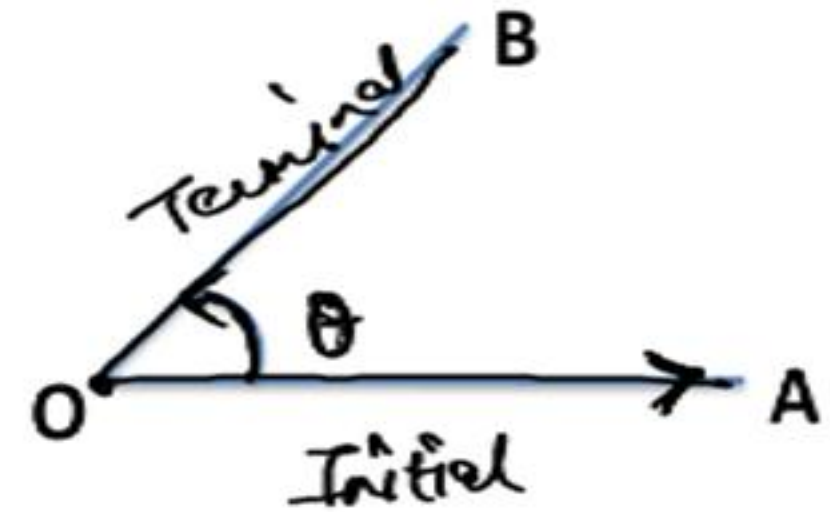
The word trigonon means a triangle and the word metron means a measure.

In broader sense trigonometry is a branch of Mathematics which deals with measurement of sides and the angles of triangle.

ANGLE

An angle is considered as a figure obtained by rotating a given ray about its end point.

The measurement of an angle is the amount of rotation from initial side to terminal side.



* Angle is always
Anti-clockwise
measured in

MEASUREMENT OF ANGLES

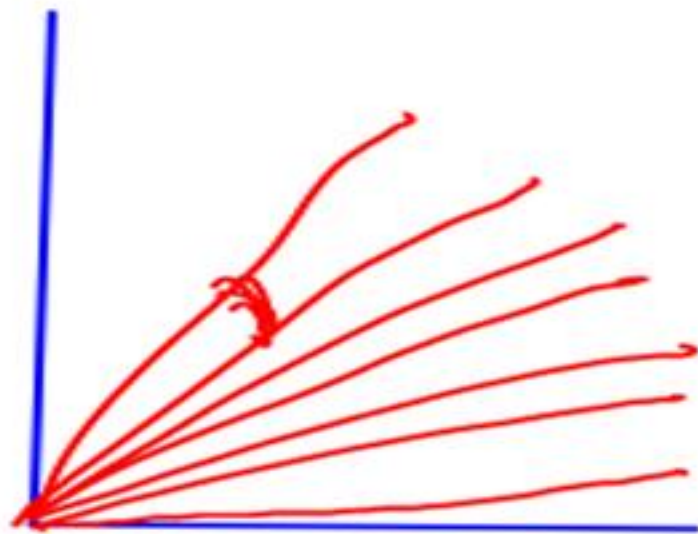
There are three system in which we generally measure angles:

- Sexagesimal System
- Centesimal System
- Circular System

SEXAGESIMAL SYSTEM

This system is also commonly called as the English system. The principal unit of this system is degree ($^{\circ}$). A right angle when divided into 90 equal parts and each of the part is called a degree.

This 1° is further divided into 60 equal parts wherein each such part is called a minute denoted by $1'$. One minute is further divided into 60 equal parts and each such part is called a second.



1 Right angle = 90°
 $1^{\circ} = 60'$ (minutes)
 $1' = 60''$ (seconds)

Eg. Convert 12.875° into degree, minutes and seconds.

$$12.875^\circ \rightarrow 12^\circ 52' 30''$$

$$.875^\circ \Rightarrow .875 \times 60$$

$$\underline{\underline{52.5}}$$

$$.5 \times 60$$

CENTESIMAL SYSTEM

Another important system of measurement of angles is the centesimal system. The principal unit of this system is grade denoted by 'g'. As in the Centesimal system, one right angle is divided into 100 equal parts called grades and each grade is further subdivided into 100 minutes and each minute into 100 seconds. In mathematical form, this can be represented as:

$$\begin{aligned}\text{One Right angle} &= 100g \\ 1g &= 100 \text{ minutes} \\ 1 \text{ minute} &= 100 \text{ seconds}\end{aligned}$$

CIRCULAR SYSTEM

This is the most commonly used system of measurement. The unit of measurement used in circular system is radian.

$$1 \text{ Right angle} = \frac{\pi^c}{2}$$

Eg. Convert 7° into degree, minutes and seconds.

$$1 \text{ Right} = \frac{\pi}{2}^{\circ}$$

$$\pi^{\circ} = 180^{\circ}$$

$$1^{\circ} = \frac{180 \cdot 7}{2211} = \frac{630}{11} \rightarrow 57 \frac{3}{11}$$

$$57^{\circ} 16' 22''$$

$$\frac{3^{\circ}}{11} \rightarrow \frac{3}{11} \times 60$$

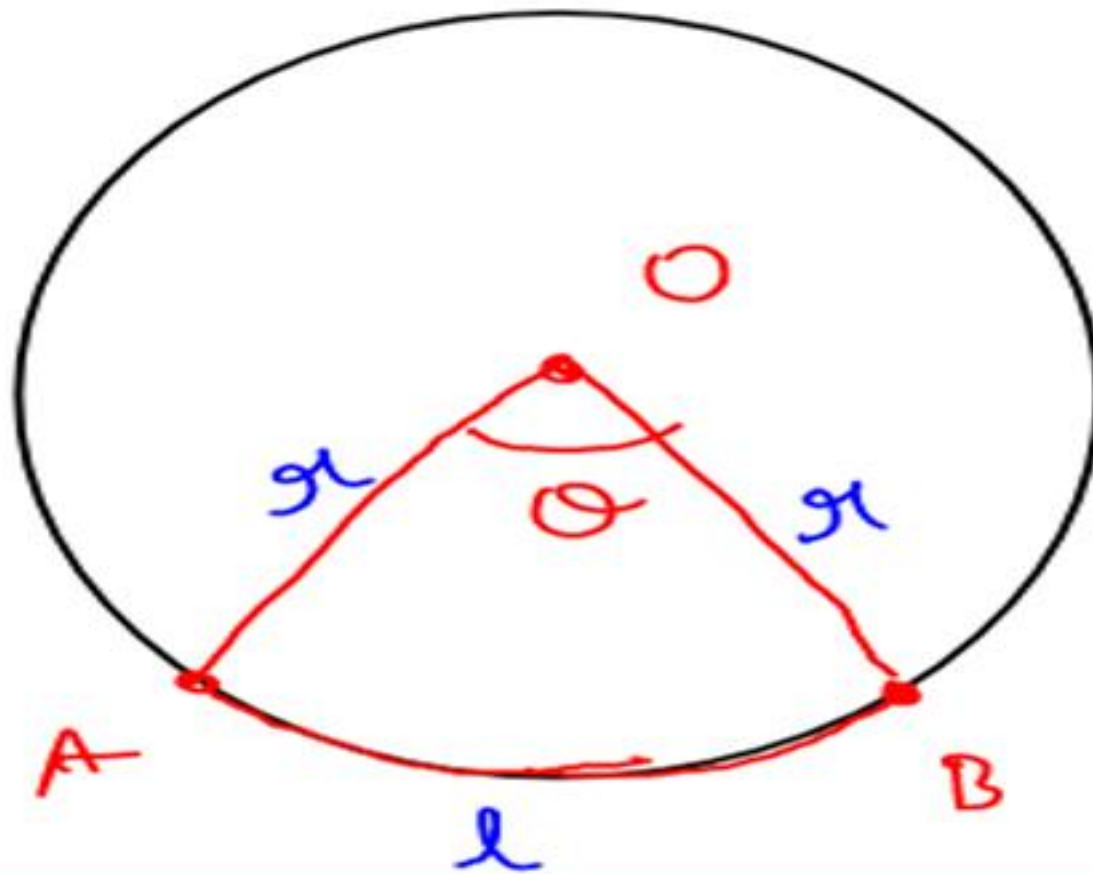
$$\frac{180}{11} = 16 \frac{4}{11}$$

$$\frac{4'}{11} \rightarrow \frac{4}{11} \times 60$$

$$\frac{240}{11} = 21 \frac{9}{11}$$

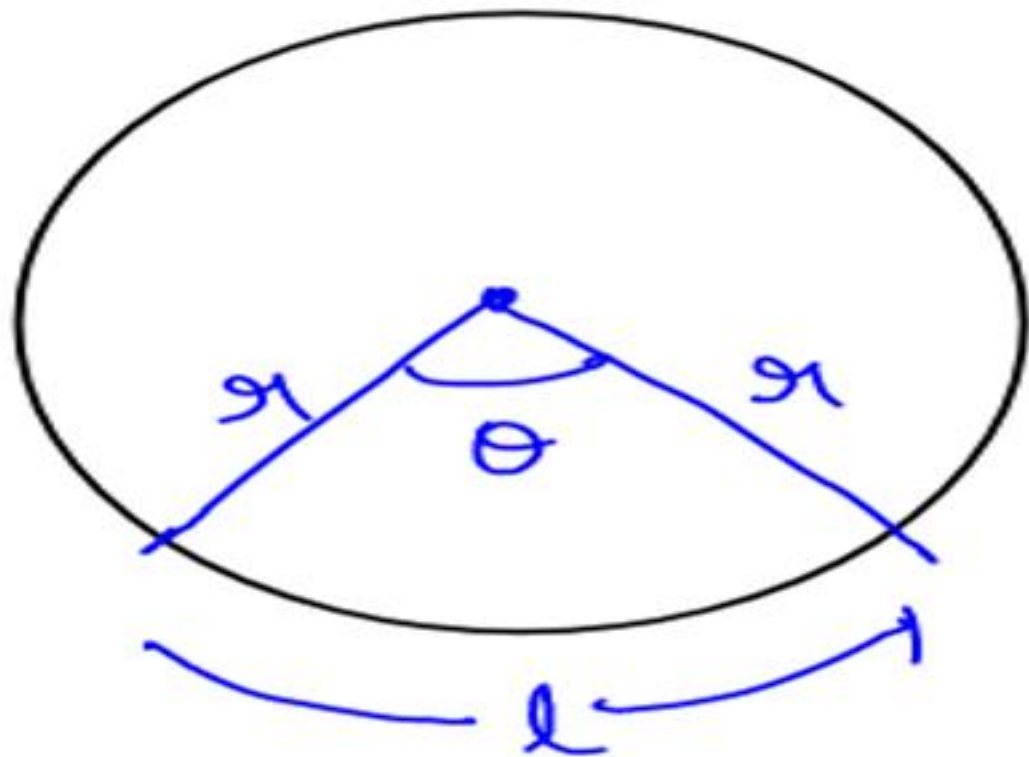
DEFINITION OF RADIAN

One radian, written as 1^c is the measure of angle subtended at the centre of the circle by an arc of length equal to the radius of the circle.



$$\theta = 1^c$$

when $\boxed{l = r}$

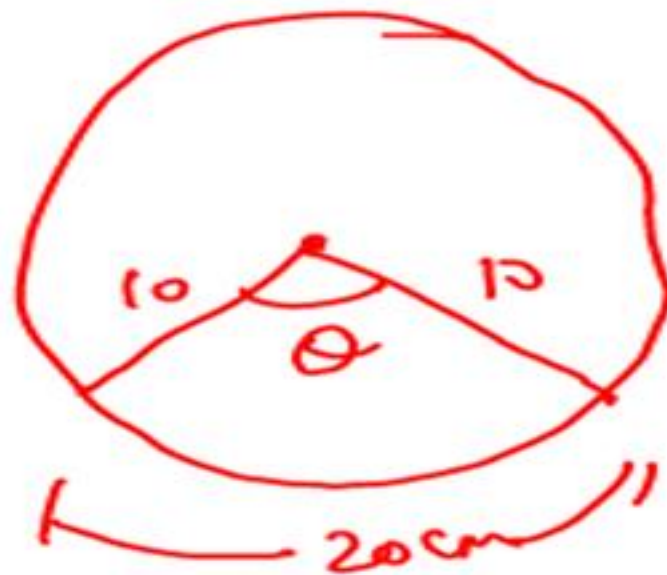


$$\theta = \frac{l}{r}$$

↓

(Radians)


eg



$$\theta = \frac{20}{10}^2$$

$$\theta \rightarrow \underline{\underline{2\text{radian}}}$$

1 RADIAN IN TERMS OF DEGREE


$$1^c = 57^\circ 16' 22''$$

RELATION BETWEEN THE VARIOUS SYSTEMS OF MEASUREMENT

$$\begin{aligned}1 \text{ Right angle} &= 90^\circ \\1 \text{ Right angle} &= 100 G \\1 \text{ Right angle} &= \frac{\pi}{2}^c\end{aligned}$$

$$\frac{D}{90} = \frac{G}{100} = \frac{2^c}{\pi}$$

$D \rightarrow$ No. of degrees
in an angle

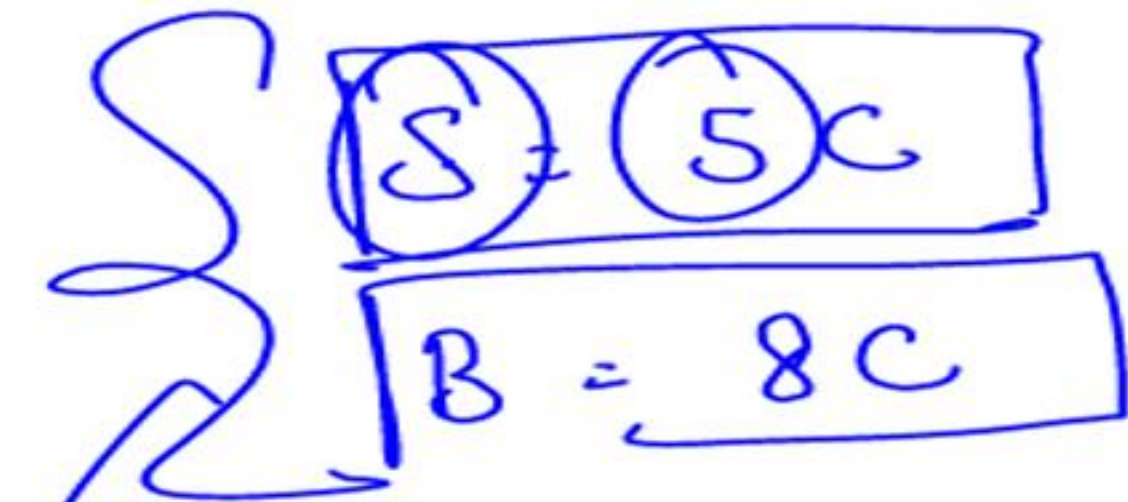
$G \rightarrow$ No. of grades
in an angle

$c \rightarrow$ No. of radians
in an angle

* Real life

(S) Small Packet \rightarrow 5 chocolates

(B) Big Packet \rightarrow 8 chocolates



$$1C = \frac{S}{5}$$

$$1C = \frac{B}{8}$$

$$\frac{S}{5} = \frac{B}{8}$$

Ques

Radians

$$\times \frac{180}{\pi} \rightarrow$$

Degrees

Degrees

$$\times \frac{\pi}{180} \rightarrow$$

Radians

+

$$\pi^c = 180^\circ$$

$$1^c = \frac{180}{\pi}$$

Q1. Convert the following into radians.

(i) $30^\circ \rightarrow 30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$

(v) $22\frac{1}{2}^\circ \rightarrow \frac{\pi}{8}$

(ii) $45^\circ \rightarrow 45 \cdot \frac{\pi}{180} = \frac{\pi}{4}$

(vi) $135^\circ \rightarrow \frac{3\pi}{4}$

(iii) $15^\circ \rightarrow \frac{\pi}{12}$

(vii) $112.5^\circ \rightarrow \frac{5\pi}{8}$

(iv) $75^\circ \rightarrow \frac{5\pi}{12}$

(viii) $15.625^\circ \rightarrow \frac{25\pi}{288}$

Handwritten calculation for (viii):

$$15.625 \times \frac{\pi}{180} = \frac{25\pi}{288}$$

Handwritten steps for conversion of 15.625 to fraction:

$$15.625 = \frac{15625}{1000} = \frac{125 \times 125}{8 \times 40} = \frac{125}{8} \times \frac{125}{40} = \frac{125}{8} \times \frac{25}{8} = \frac{3125}{64}$$

Handwritten steps for conversion of 180 to fraction:

$$180 = \frac{180}{1} = \frac{180 \times 16}{16} = \frac{2880}{16}$$

Q2. Convert the following into degrees.

$$(i) \frac{\pi}{6} \rightarrow \frac{180}{6} = 30^\circ$$

$$(v) \frac{\pi}{12} \xrightarrow{\text{Radian} \times \frac{180}{\pi} \rightarrow \text{Degree}} 15^\circ$$

OR

$$(ii) \frac{\pi}{4} \rightarrow 45^\circ$$

$$(vi) \frac{\pi}{3} \rightarrow 60^\circ \quad \pi \rightarrow 180^\circ$$

$$(iii) \frac{\pi}{2} \rightarrow 90^\circ$$

$$(vii) \frac{2\pi}{3} \rightarrow 120^\circ \quad \frac{2 \times 60}{1} \rightarrow 120^\circ$$


$$(iv) \frac{\pi}{8} \rightarrow 22\frac{1}{2}^\circ$$

$$(viii) \frac{3\pi}{4} \rightarrow 135^\circ$$

Q3. If \underline{D} , \underline{G} and \underline{R} denote respectively the number of degrees, grades and radians in an angle, then

(a) $\frac{D}{100} = \frac{G}{90} = \frac{2R}{\pi}$

(b) $\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi}$

 (c) $\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$

(d) $\frac{D}{90} = \frac{G}{100} = \frac{R}{2\pi}$

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Ans. (c)

Q4. Two angles of a triangle are $\frac{1}{2}$ and $\frac{1}{3}$ radian, find the degree measure of 3rd angle.

(a) $132\frac{1^\circ}{11}$

(b) $132\frac{2^\circ}{11}$

☒ (c) $132\frac{3^\circ}{11}$

(d) 132°

Sum of 3 Angles of $\Delta = 180^\circ$

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6} \text{ radian}$$

$$= \frac{5}{6} \cdot \frac{180}{22} \cdot 7$$

$$180 - \frac{5}{6} \cdot \frac{180}{22} \cdot 7$$

$$180 \left[1 - \frac{5 \cdot 7}{6 \cdot 22} \right]$$

$$= \frac{15}{22} \cdot \frac{180}{11} \left[\frac{97}{5 \cdot 22} \right] = \frac{1455}{11}$$

Ans. (c)

Q5. In $\triangle ABC$, $\angle ABC = 75^\circ$ & $\angle ACB = \frac{\pi}{12}$ then find circular measure of $\angle BAC$.

(a) $\frac{5\pi}{12}$ radian (b) $\frac{\pi}{3}$ radian

(c) $\frac{\pi}{6}$ radian ~~(d) $\frac{\pi}{2}$ radian~~

$$75^\circ + 15^\circ + \angle A = 180^\circ$$

$$\angle A = 90^\circ$$

$$90 \cdot \frac{\pi}{180 \cdot 2}$$

Ans. (b)

PYQ of SSC

Q6. Find the radian measure of $63^\circ 14' 51''$

Ist

Detailed Method

✓ (a) $\left(\frac{2811\pi}{8000}\right)^\circ$

(b) $\left(\frac{3811\pi}{8000}\right)^\circ$

(c) $\left(\frac{4811\pi}{8000}\right)^\circ$

(d) $\left(\frac{5811\pi}{8000}\right)^\circ$

$$63 + \frac{14^\circ}{60} + \frac{51''}{60 \cdot 60}$$

$$63 + \frac{7}{30} + \frac{17}{1200}$$

$$\frac{75600 + 280 + 17}{1200} = \left(\frac{75897}{1200}\right)^\circ$$

$$\frac{28118433}{75897}$$

$$\frac{75897}{1200}$$

$$\frac{1200}{4}$$

$$\times \frac{\pi}{180}$$

$$= \left(\frac{2811\pi}{8000}\right)^\circ$$

Ans. (a)

$$63^{\circ} 14' 51'' \rightarrow ?? \text{ radians}$$

$$\approx 60^{\circ} \approx \frac{1}{3} \pi$$

$$\checkmark \frac{2811\pi}{8000}$$

~~$$\frac{4811\pi}{8000}$$~~

~~$$\frac{3811\pi}{8000}$$~~

~~$$\frac{5811\pi}{8000}$$~~

$$\frac{4000\pi}{8000} \rightarrow \frac{1}{2} \pi$$

$$\underline{\underline{5.99997}} \times \underline{\underline{3.00013}}$$

$$6 \times 3$$

$$\approx 18$$

Find the app

A 18.002

B 18.001

C 17.997

D 17.995

$$\theta = \frac{l}{r}$$

where,

θ is the angle made by an arc (in radians) at the centre of the circle.

l is the length of the arc of the circle.

r is the radius of the circle.

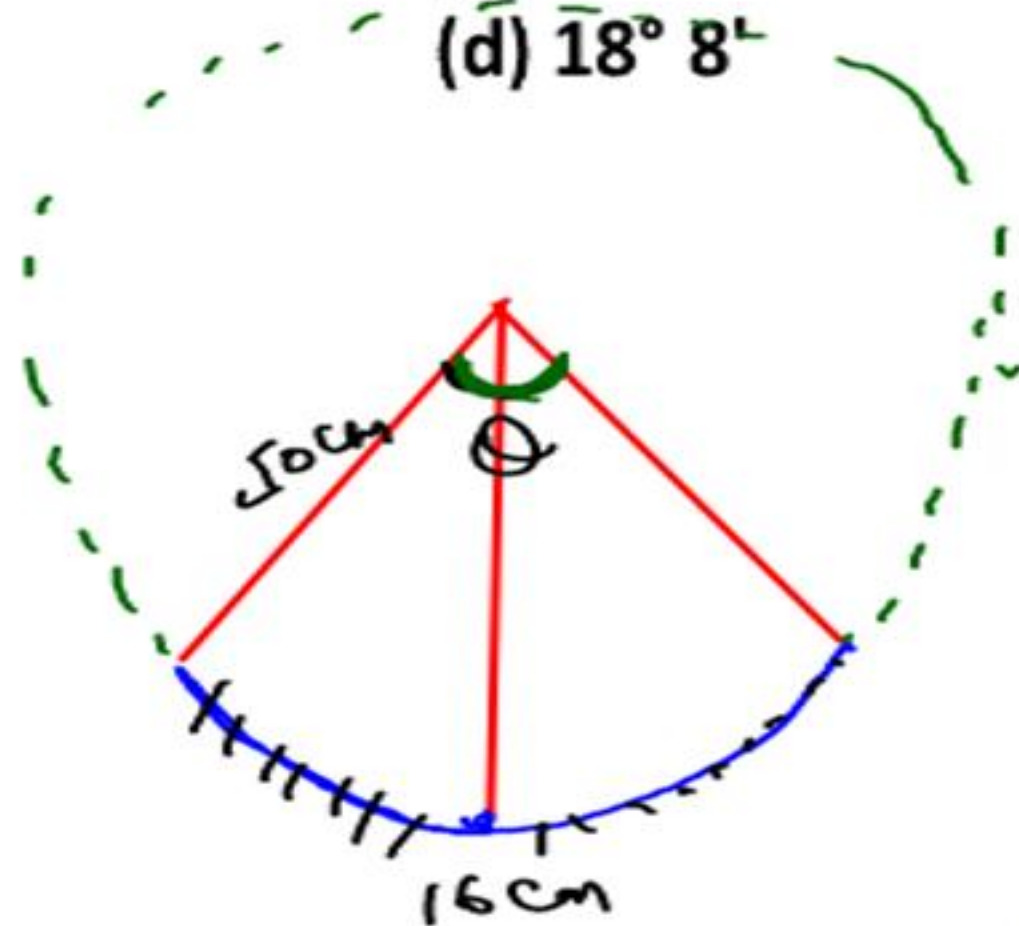
Q7. Find the angle (in degree) through which a pendulum swings, if its length is 50 cm and the tip describes an arc of length 16 cm.

(a) $18^\circ 25'$

(b) $18^\circ 35'$

(c) $18^\circ 20'$

(d) $18^\circ 8'$



$$\theta = \frac{l}{r}$$

$$= \frac{168}{5025}$$

$$\frac{8}{525} \times \frac{180}{2211} \times 7 = \frac{1008}{55}$$

$$18 \left(\frac{18}{55} \right) \Rightarrow \underline{\underline{18^\circ 20'}}$$

$$\frac{18}{55}$$

$$\approx \frac{1}{3}^\circ \approx 20'$$

Ans. (c)

Q8. If the arcs of same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of their radii.

(a) 5 : 6

(b) 5 : 4

(c) 4 : 5

(d) 6 : 5

$$\overset{4}{\cancel{60}} : \overset{5}{\cancel{75}}$$

radii

$$\underline{\underline{5 : 4}}$$



$$\theta = \frac{l}{r}$$

$$l \rightarrow \underline{\text{constant}}$$

$$\theta \propto \frac{1}{r}$$

Ans. (b)

Q9. Length of minute hand of a clock is 35 cm. Find the arc length made by hand in 18 seconds. (take $\pi = \frac{22}{7}$)

(a) 11 cm

☒ (b) 1.1 cm

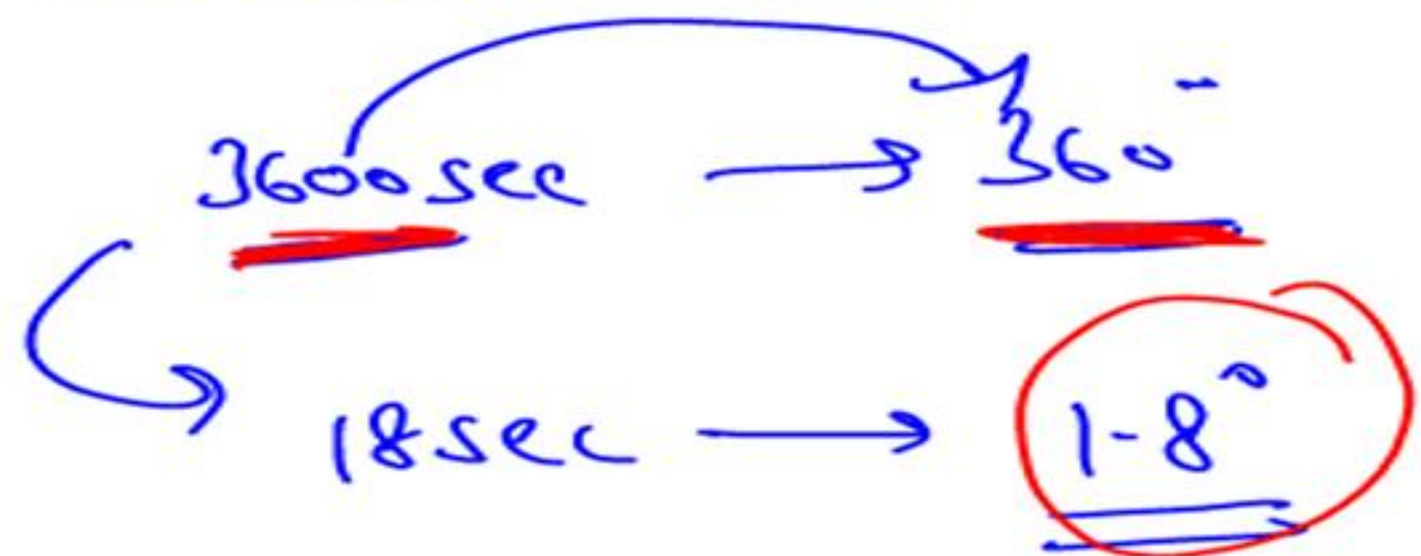
(c) 6.6 cm

(d) 6 cm

MIN HAND

1 hr $\rightarrow 360^\circ$

60-60 sec $\rightarrow 360^\circ$



$$\theta = \frac{l}{r}$$

$$1.8 \times \frac{\pi}{180} = \frac{l}{35}$$

$$\frac{1.8 \times \frac{22}{7}}{180} \times 35 = l$$

$$l = 1.1 \text{ cm}$$

Ans. (b)

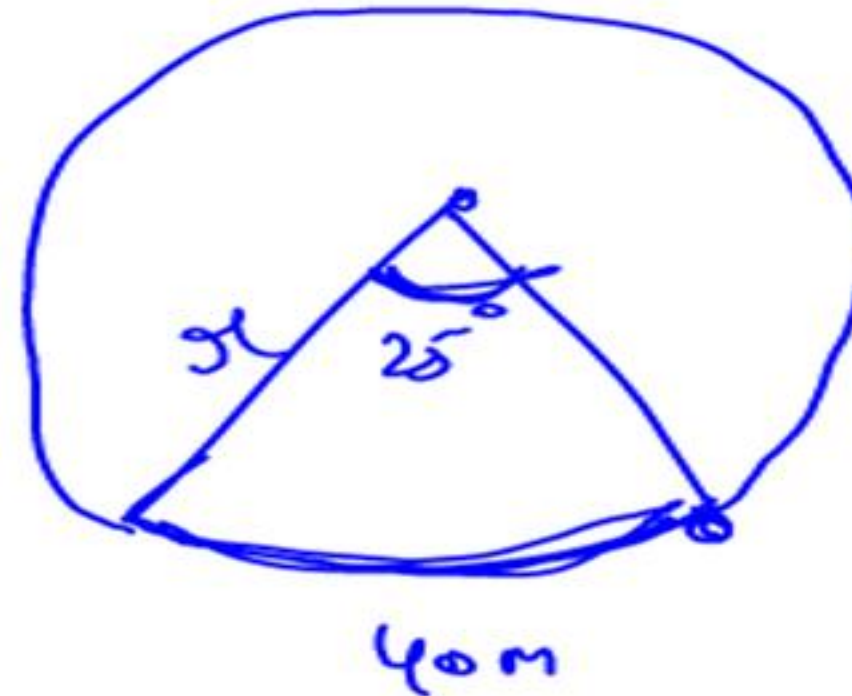
Q10. A railway train is travelling on a circular curve. What radius should be kept to turn it at 25° angle, at the distance, at the distance of 40m in the direction of path?

☒ (a) 91.64 m

(b) 90.46 m

(c) 89.64 m

(d) 93.64 m



$$\theta = \frac{l}{r}$$

$$\frac{25 \cdot \pi}{180} = \frac{40}{r}$$

$$r = \frac{36 \cancel{20}^4 \times 40}{\cancel{180} \times \cancel{25} \times \cancel{22} \times 7} = \frac{1008}{11} \Rightarrow 91 \frac{7}{11}$$

$$= \frac{1008}{11} \Rightarrow 91 \frac{7}{11}$$



Ans. (a)

Q11. A railway train is travelling on a circular curve of 1500 metres radius at the rate of 66 km/hr. Through what angle has it turned in 10 seconds?

(a) $\left(\frac{11}{90}\right)^\circ$

(b) $\left(\frac{11}{80}\right)^\circ$

(c) $\left(\frac{11}{95}\right)^\circ$

(d) $\left(\frac{11}{60}\right)^\circ$

Homework

Ans. (a)

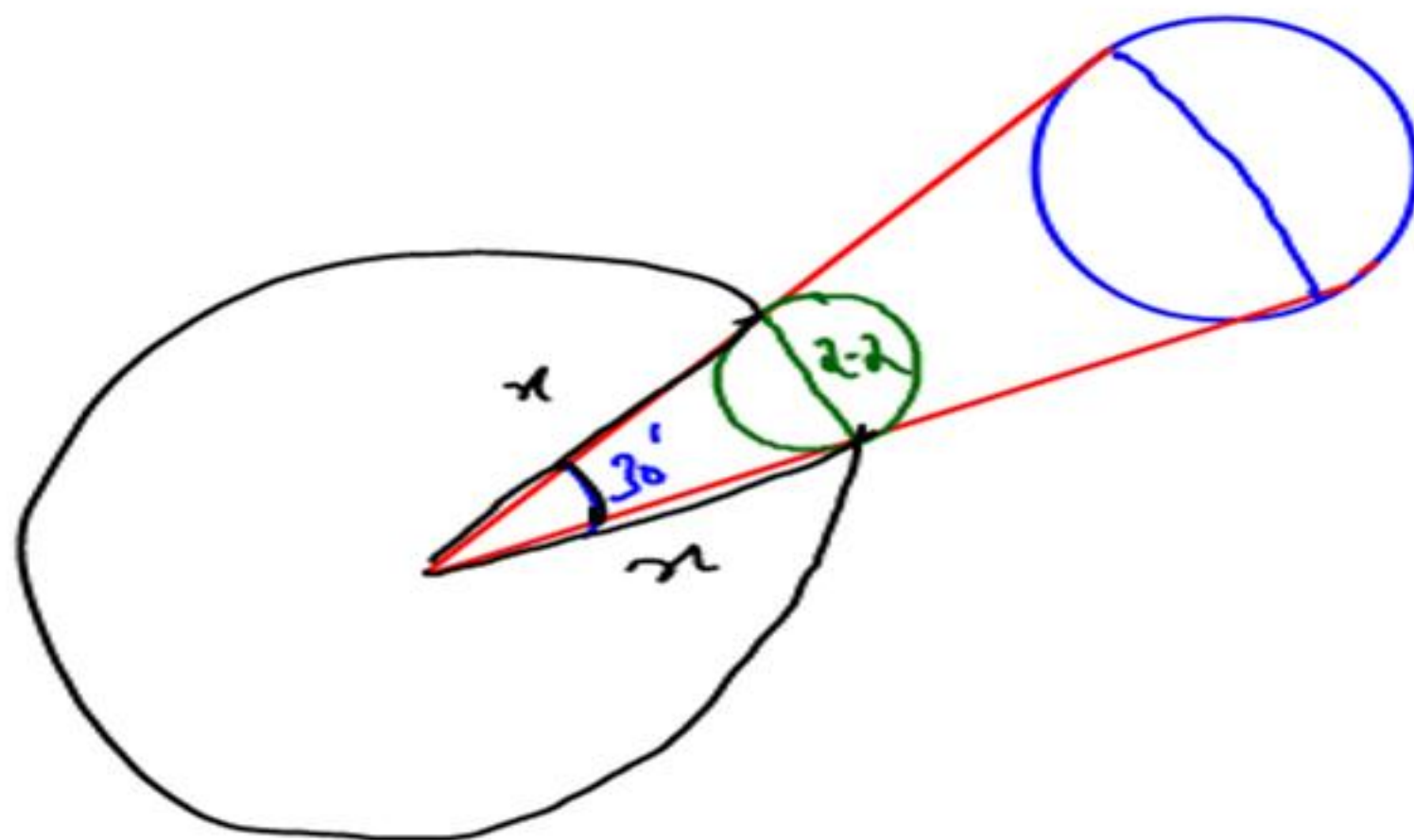
Q12. If the angular diameter of the moon be $30'$, how far from the eye a coin of diameter 2.2 cm be kept to hide the moon

(a) 252 cm

(b) 200 cm

(c) 150 cm

(d) 180 cm



$$\theta = \frac{l}{r}$$

$$\frac{1}{2} \cdot \frac{\pi}{180} = \frac{2.2}{x}$$

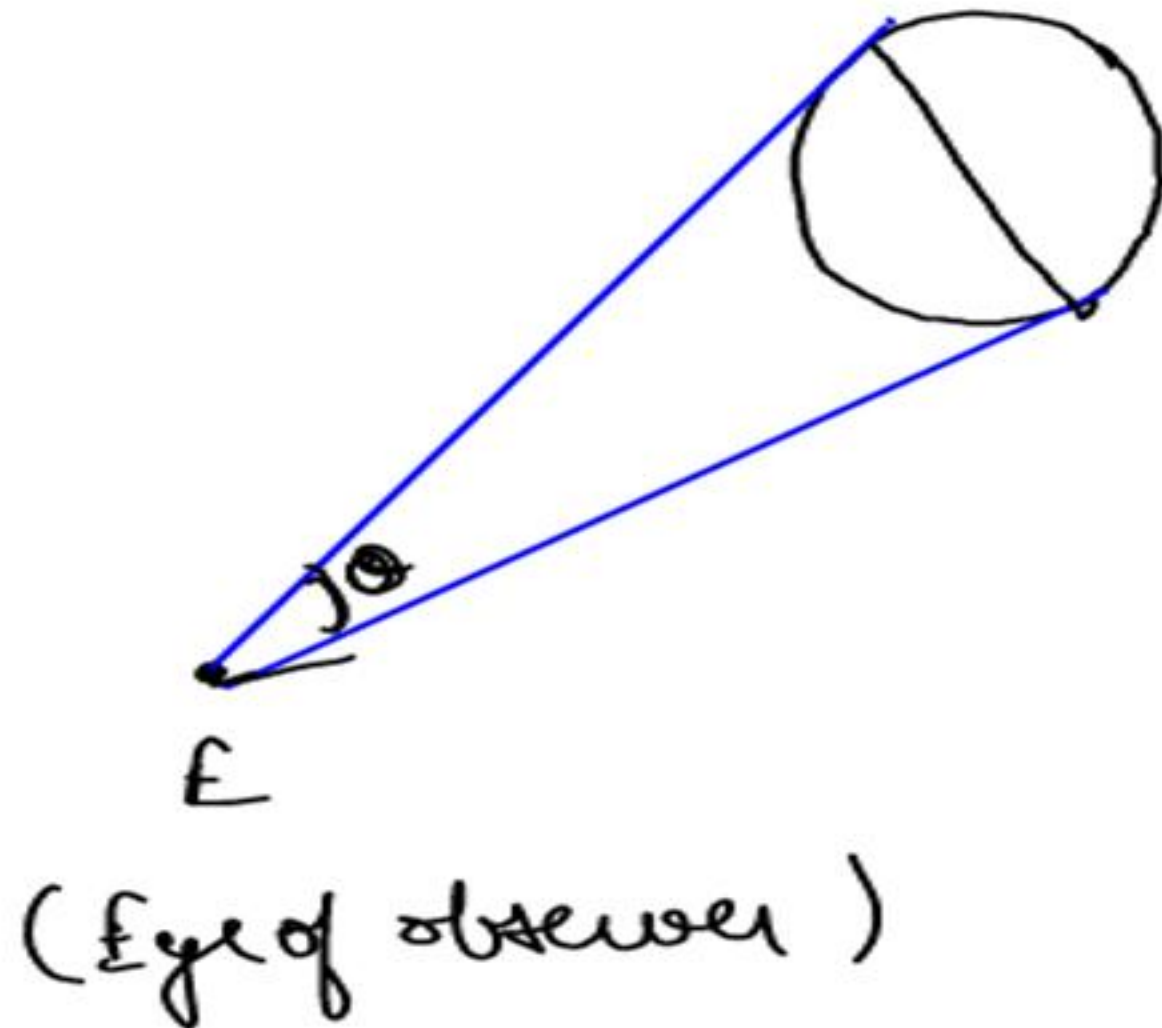
$$x = \frac{2.2 \times 2 \times 180}{\pi}$$

$$= \underline{\underline{252 \text{ cm}}}$$

Ans. (a)

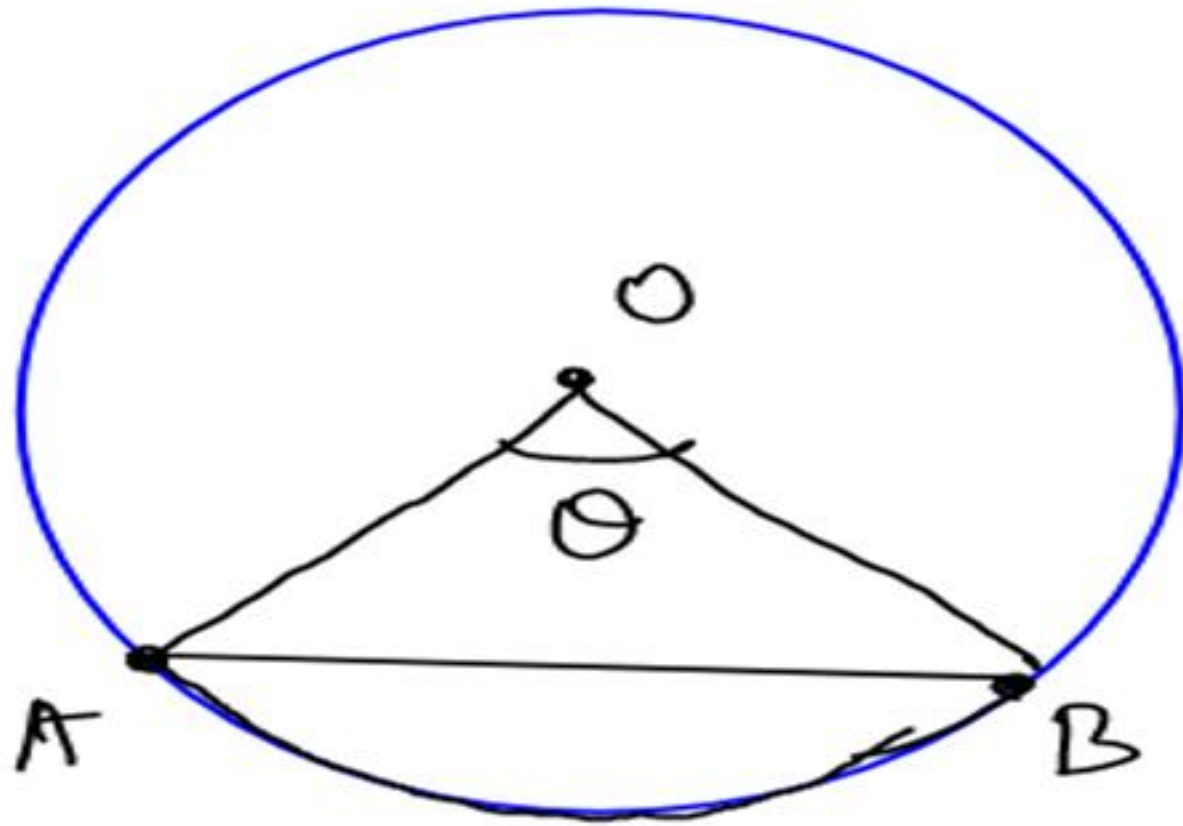
ANGULAR DIAMETER OF MOON

Angle made by the moon at the eye of observer.



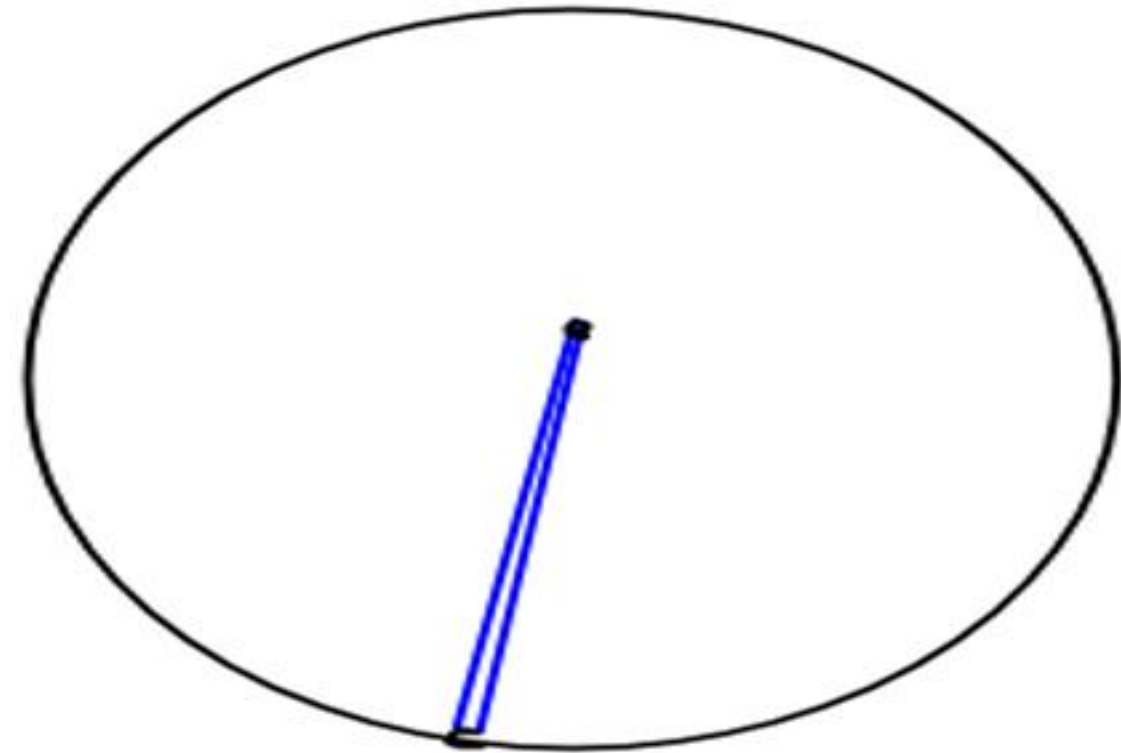
$\theta \rightarrow$ Angular Diameter

Note: If angle made at the centre of an arc is very small $< 1^\circ$, then there is almost no difference between length of arc and chord.



$AB \rightarrow$ chord

$\widehat{AB} \rightarrow$ arc



Q13. The moon's distance from the earth is 3,60,000 km and its diameter subtends an angle of $30'$ at the eye of the observer. Find the diameter of the moon.

(a) 100π km

(b) 1000π km

(c) 1500π km

(d) 2000π km

Homework

Ans. (b)