gradeup

Divisibility

Divisibility Theorem:
Divisibility rules:
1. Divisibility by 1: All the integers are divisible by 1.
2. Divisibility by 2: A number is said to be divisible by 2 when the last digit of the given number is even i.e. 0, 2, 4, 6, 8.
Example 1: 68, 484, 89232, 5820, 1446 all numbers are divisible by 2.
3. Divisibility by 3: A number is divisible by 3 when the sum of all the digits of the given number is either 3 or a multiple of 3.
Example 2: Check whether 78342 is divisible by 3 or not.
Solution:
Sum of the digits of 78342 = 7 + 8 + 3 + 4 + 2 = 24. Here, 24 is clearly a multiple of 3. Thus, the given number 78342 will be exactly divisible by 3.
Example 3: Check whether 27353 is divisible by 3 or not.
Solution:
Sum of the digits of 27353 = 2 + 7 + 3 + 5 + 3 = 20. Here, 20 is clearly not a multiple of 3. Thus, the given number 27353 will not be divisible by 3.
4. Divisibility by 4: A number is divisible by 4 when the last two digits of the given number is divisible by 4 or any multiple of 4.
Example 4: Check whether 482 is divisible by 4 or not.
Solution:
Consider last two digits of 482, which is 82. Here, 82 is not divisible by 4 or multiple of 4. Thus, 482 is not divisible by 4.
Example 5: Check whether 378244 is divisible by 4 or not.
Solution:
Consider last two digits of 378244, which is 44. Here, 44 is divisible by 4 or in other words 44 is a multiple of 4. Thus, 378244 is divisible by 4.
5. Divisibility by 5: : A number is divisible by 5 when the last digit of the given number is either 0 or 5.
Example 6: Check whether 5795 is divisible by 5 or not.
Solution:
Here, the last digit of the given number is 5. Thus, 5795 is divisible by 5.
Example 7: Check whether 8732 is divisible by 5 or not.
Solution:
Here, the last digit of the given number is 2. Thus, 8732 is not divisible by 5.
We can also verify this by dividing 8732 by 5 which will leave 2 as remainder. Thus, 8732 is not divisible by 5.
6. Divisibility by 6: : A number is divisible by 6 when the given number is both 2 and 3 as 6 = 2 x 3.
Example 8: Check whether 27498 is divisible by 6 or not.
Solution:
Here, 27498 is exactly divisible by 2 as the last digit of the given number is even.
Also, Sum of the digits of 27498 = 2+7+4+9+8 = 30 which is divisible by 3. Thus, 27498 is also divisible by 3.
Since 27498 is divisible by both 2 and 3, the given number 27498 will be divisible by 6.
7. Divisibility by 7:
There are various methods to find divisibility by 7 for any given number. These are as following:
a. Step 1: First form pairs of three-three digits from the right end of the given number.
Step 2: Now add all the alternating pairs at odd places and even places simultaneously and find the difference between them. If the number obtained is exactly divided by 7 then the given number is said to be divisible by 7.
Example 9: Check whether 57498 is divisible by 7 or not.
Solution:
Step 1: Form pairs of three-three digits from the right end.
Thus, 57498 is written as 057 498 (Add 0's in the beginning of the number if necessary)
Step 2: Since only two such are available here. Simply go for the difference between them.
Difference = 498 – 057 = 441. Now check if 441 is divided by 7 or not.

Here, 441 is exactly divisible by 7. So, the given number 57498 will also be divisible by 7.
Example 10: Check whether 92384623 is divisible by 7 or not.
Solution:
Step 1: Form pairs of three-three digits from the right end.
Thus, 92384623 is written as 092 384 623 (Add 0's in the beginning of the number if necessary)
Step 2: Now add all the alternating pairs at odd places and even places simultaneously and find the difference between them.
Sum of pairs at odd places = 092 + 623 = 715
Sum of pairs at even places = 384
Now, Difference = 715 – 384 = 331. Now check if 331 is divided by 7 or not.
Here, 331 is not exactly divisible by 7. So, the given number 92384623 will not be divisible by 7.
b. Subtract 2 times of digit at the right end from the rest of the number and repeat the process. Then check if the obtained number is divisible by 7 or not. If yes then the given number will be divisible by 7 otherwise not.
Example 11: Check whether 57498 is divisible by 7 or not.
Solution:
2 times the last digit from the right end = 2 x 8 = 16
Subtract it from the rest of the number which is 5749.
Thus, 5749 – 16 = 5733
Again, 574 – (2x9) = 556
Again, 55 – (2x6)= 43
Now check 43 is divisible by 7 or not.
Since, 43 is not exactly divisible by 7 then the given number 57498 will not be divisible by 7.
8. Divisibility by 8: A number is divisible by 8 when the last three digits of the given number is divisible by 8 or any multiple of 8.
Example 12: Check whether 274432 is divisible by 8 or not.
Solution:
Here, we consider last three digits of the given number.
So, when 432 is divided by 8, it gives zero as remainder which means that 432 is completely divisible by 8. Hence 274432 will also be completely divisible by 8.
9. Divisibility by 9: A number is divisible by 9 when the sum of all the digits of the given number is divisible by 9 or a multiple of 9.
Example 13: Check whether 873477 is divisible by 9 or not.
Solution:
Sum of the digits = 8+7+3+4+7+7 = 36
Since 36 is a multiple of 9 or is completely divisible by 9 then the given number 873477 will be exactly divisible by 9.
10. Divisibility by 10: Since 10 can be broken down into 2 multiplied by 5. So, any number that is divisible by 2 and 5 simultaneously will also be divisible by 10. Or if the last digit is 0 then the given number will be exactly divisible by 10.
Example 14:
18720 is divisible by 10 (As it is divisible by both 2 and 5).
But 39235 is not divisible by 10 because it is completely divisible by 5 but not by 2.
11. Divisibility by 11: The divisibility rule of 11 is as following:
Step 1: Find the sum of the digits at odd places and even places.
Step 2: Find the difference between the sums. Check whether the result is 0 or 11 or a multiple of 11. If yes then the given number will be exactly divisible by 11.
Example 15: Check whether 873477 is divisible by 11 or not.
Solution:
Step 1: Sum of the digits at odd places = 8+3+7 = 18
Sum of the digits at even places = 7+4+7 = 18
Step 2: Difference between the sums = 18 – 18 = 0
Since the result is 0. The given number 873477 will be exactly divisible by 11.

12. Divisibility by 12: We can see that 12 is obtained by 3 multiplied by 4. So, for a number to be divisible by 12, it has to be exactly divisible by 3 and 4 simultaneously.
13. Divisibility by 13: The divisibility of a number by 13 is determined as following:
Step 1: First form pairs of three-three digits from the right end of the given number.
Step 2: Now add all the alternating pairs at odd places and even places simultaneously and find the difference between them.
Step 3: Now multiply the last digit of the obtained difference with 4 and add it to rest of the number.
Step 4: If the result is 0 or 13 or a multiple of 13, then the given number will be divisible by 13 otherwise not.

Example 16: Check whether 27364672 is divisible by 13 or not.

Solution:
Step 1: First form pairs of three-three digits from the right end of the given number.
Thus, 27364672 can be written as 027 364 672
Step 2: Now add all the alternating pairs at odd places and even places simultaneously and find the difference between them.
Difference = (027 + 672) - (364) = 335
Step 3: Now multiply the last digit of the obtained difference with 4 and add it to rest of the number.
In 335, last digit is 5, when it is multiplied by 4 = 5x4 = 20
Subtracting it from the rest of the number = 33 – 20 = 13
Step 4: Clearly the result is 13 which is multiple of 13. Thus, the given number 27364672 is completely divisible by 13.

Divisibility rule of powers of 2:

When the divisor is in the form of 2^n, then check the last n digits of the given number if they are completely divisible by 2^n or not. If yes then the given number will be exactly divisible by 2^n.

Example:
A number will be divisible by $2^1 = 2$ when the last 1 digit of the given number is completely divisible by 2^1.
A number will be divisible by $2^2 = 4$ when the last 2 digits of the given number is completely divisible by 4.
A number will be divisible by $2^3 = 8$ when the last 3 digits of the given number is completely divisible by 8.
A number will be divisible by $2^4 = 16$ when the last 4 digits of the given number is completely divisible by 16.
A number will be divisible by $2^5 = 32$ when the last 5 digits of the given number is completely divisible by 32.

Other Divisibility Rules:

1. If a number in the form of XXXXXX (6 times repitition), then the number will be completely divisible by 3, 7, 11, 13, 21, 37, 101.
2. If a number in the form of abcabc, then the number will be completely divisible by 7, 11, 13, 1001.
3. $(a^n + b^n)$ is completely divisible by (a + b) when n is an odd number.

Example 17: Which of the following will completely divide $(13^9 + 3^{18})$:
a. 17
b. 19
c. 7
d. 11

Solution:
$(13^9 + 3^{18})$ can be written as $(13^9 + 9^9)$ which is in form of $(a^n + b^n)$ and n = 9 is an odd number.
Thus, $(13^9 + 9^9)$ is completely divisible by (13 + 9) = 22.
Thus, 22 or 11 and 2 will divide the given number.
4. $(a^n - b^n)$ is completely divisible by (a + b) & (a - b) when n is an even number.

Example 18: $(5^{10} - 1024)$ will be completely divisible by:

a. 3
b. 7
c. 3 and 7
d. None of the above
Solution:
Given expression (5^{10} - 1024) can be written as ($5^{10} - 2^{10}$).
Thus, ($5^{10} - 2^{10}$) has even power of 10 and is in the form of ($a^n - b^n$).
So, ($5^{10} - 2^{10}$) is divisible by (5 + 2) and (5 - 2) or 7 and 3.
5. ($a^n + b^n + c^n + d^n$ +………..) is exactly divisible by (a + b + c + d +…………) when n is an odd number.
Example 19: The expression ($3^{37} + 5^{37} + 6^{37}$) is exactly divisible by:
a. 11
b. 14
c. 15
d. 19
Solution:
Here power n = 37 which is an odd number.
Thus, ($3^{37} + 5^{37} + 6^{37}$) will be exactly divisible by (3 + 5 + 6) = 14.
Example 20: $(49)^{15} - 1$ is completely divisible by:
a. 50
b. 51
c. 29
d. 8
Solution:
Here, n= 15 which is an odd number.
And 1 can be written as 1^{15}.
So, $(49)^{15} - 1^{15}$ will be divisible by 49 – 1 = 48.
Since, 48 is not the option, then the given number will be all the factors of 48. Thus, 8 is the answer.
Successive Division:
It is a division in which the quotient of the dividend is taken and is used as dividend of the next division.
Example 21: When 325 is successively divided by 3, 5, 11 then the remainders are:
Solution:

Divisor	Dividend	Quotient	Remainder
3	325	108	1
5	108	21	3
11	21	1	10

Thus, remainders are 1, 3 and 10 respectively.