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MENSURATION-3D

Part-7

Agenda

Basic concept of Pyramid

→ All formulas

& Relationship → (38-40) min

Tetrahedron

Questions

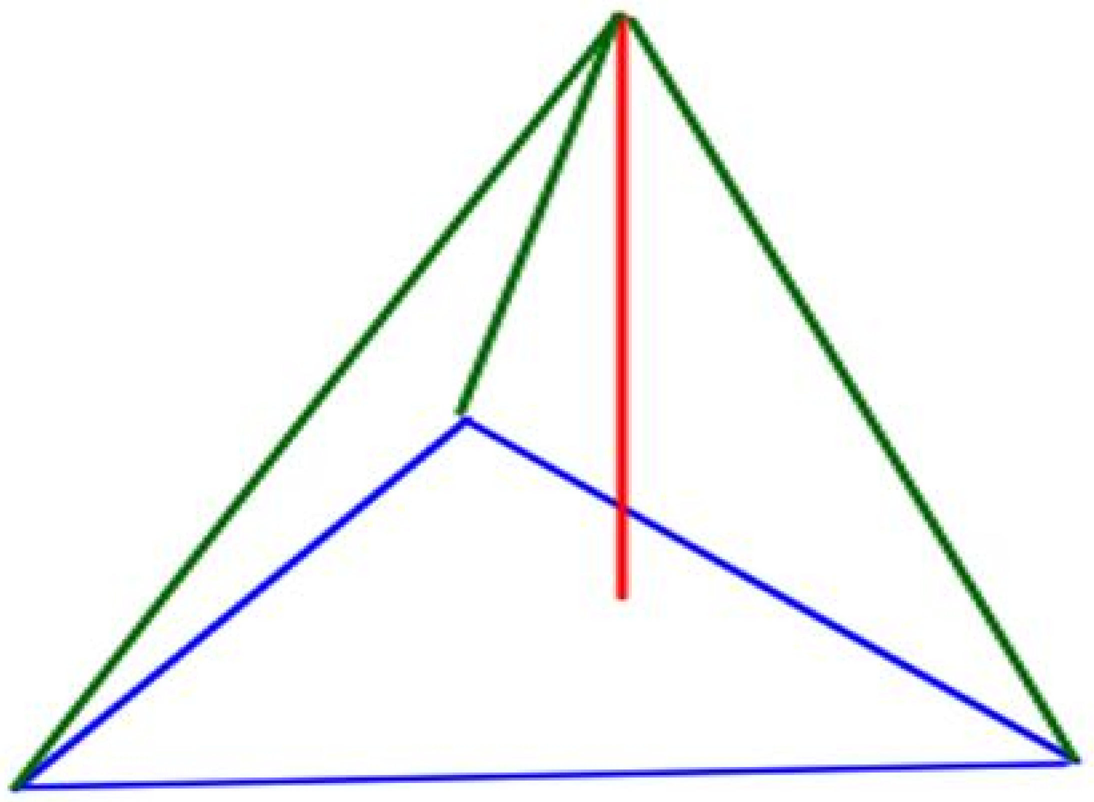
12Q → (36-38) min

$\boxed{74-78}$

Pyramid

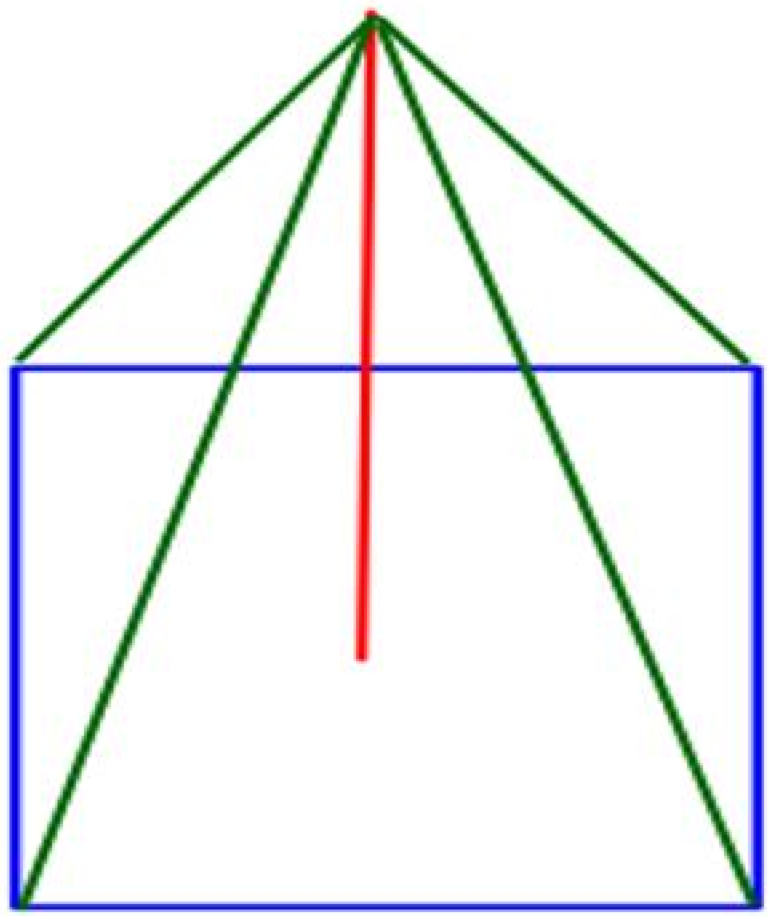
What is Pyramid ?

we have only Base (Polygon)

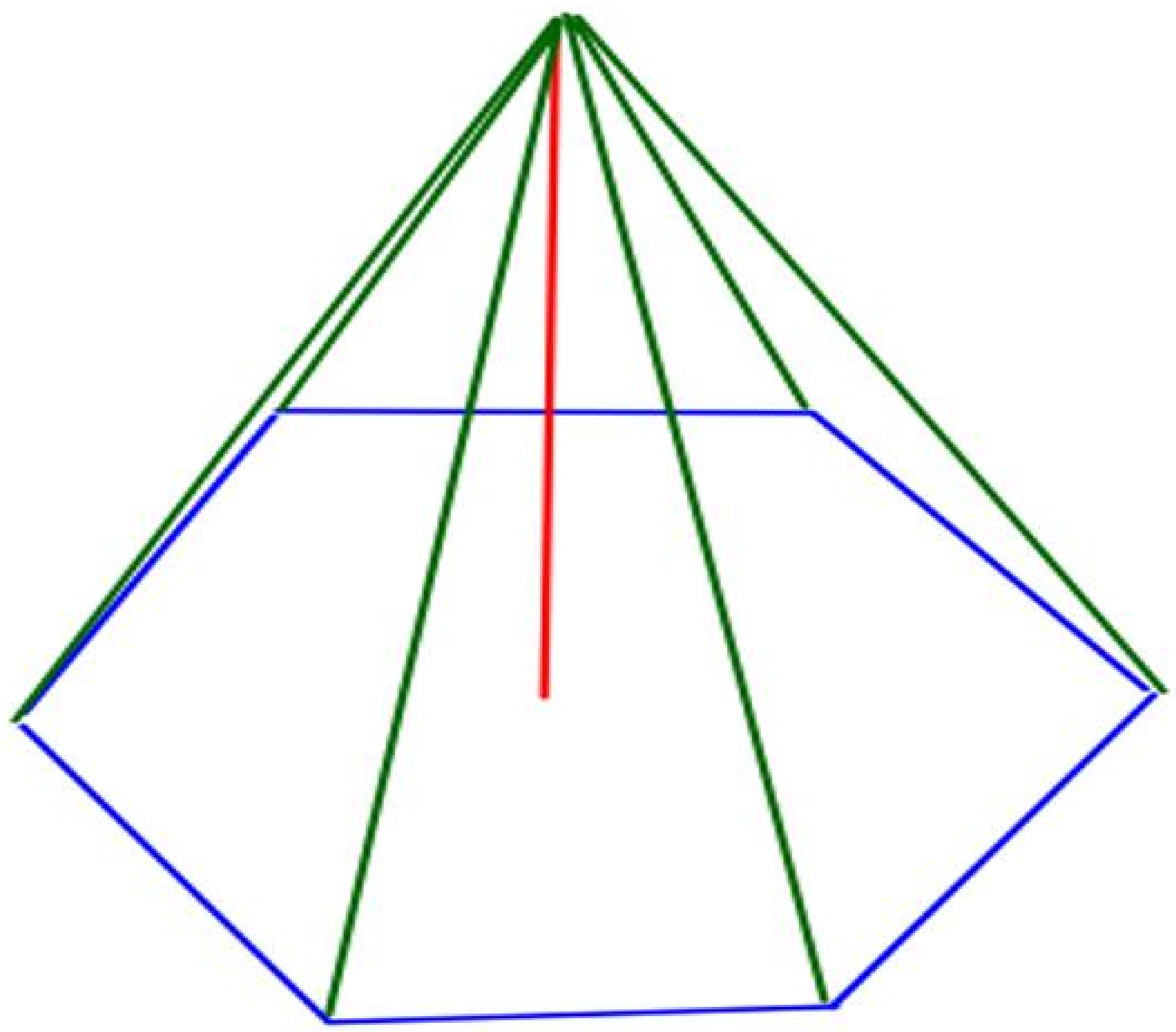


lateral surfaces → Triangles

Name → Triangular Pyramid



→ Square Pyramid

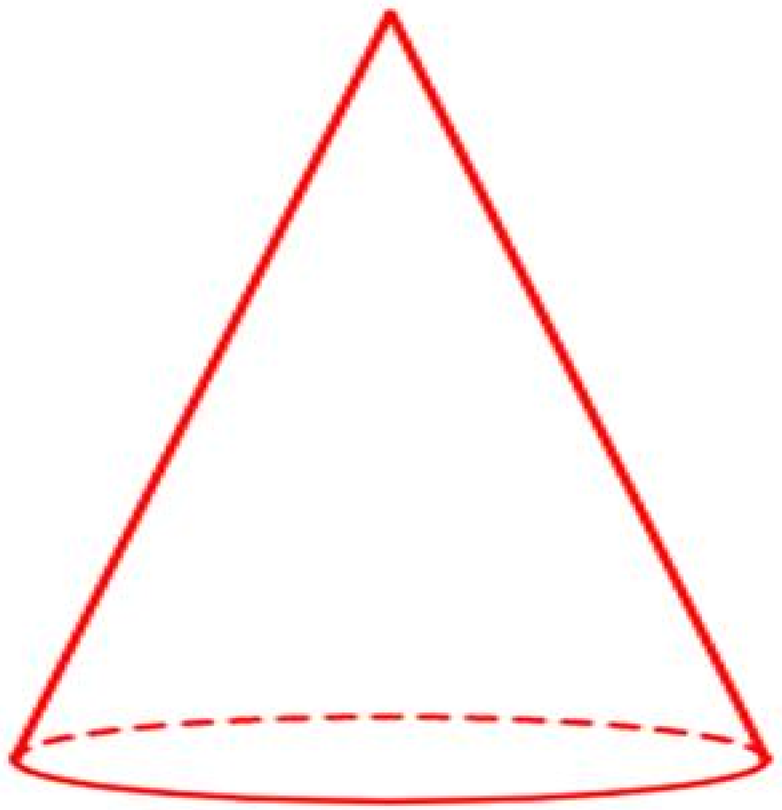


Hexagonal

Pyramid

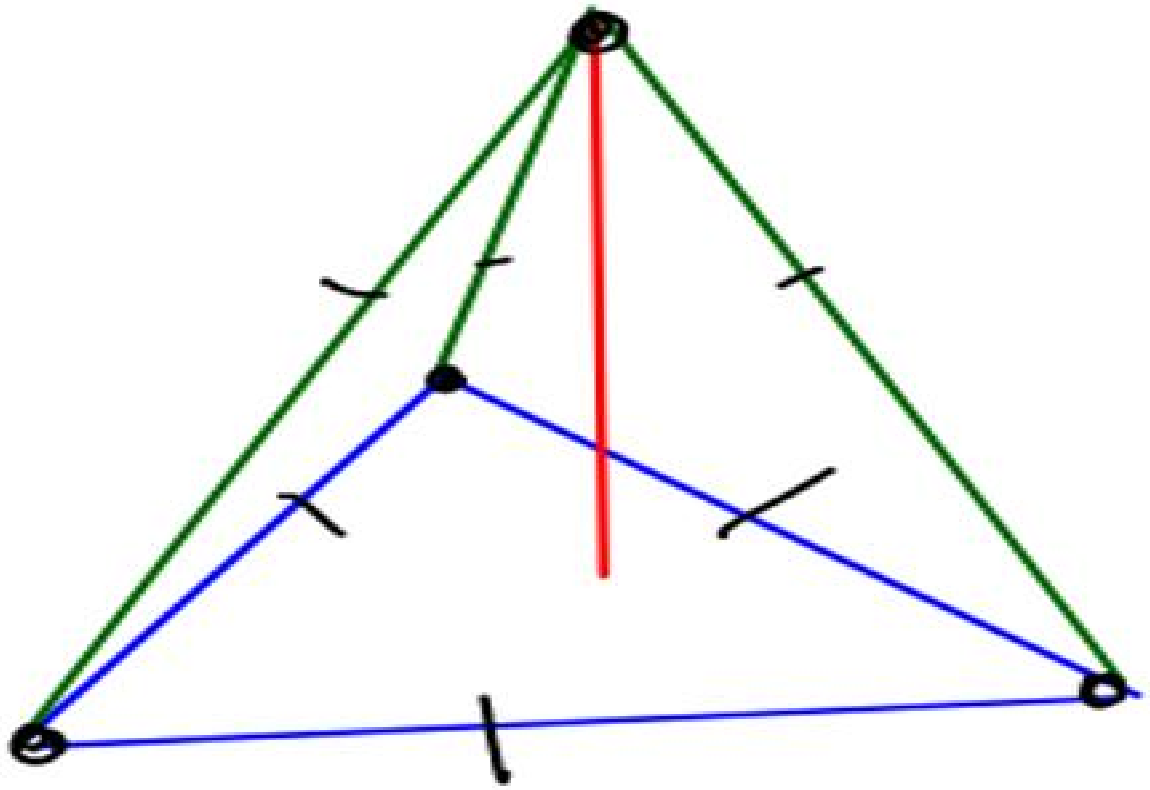
In SSC 90% Question of Pyramid will have Equilateral Δ, Square or Regular Hexagon as their base

Is Right Circular Cone a Pyramid?



No Because Base is not a polygon

Faces, Vertices and Edges of Pyramid



face → n+1

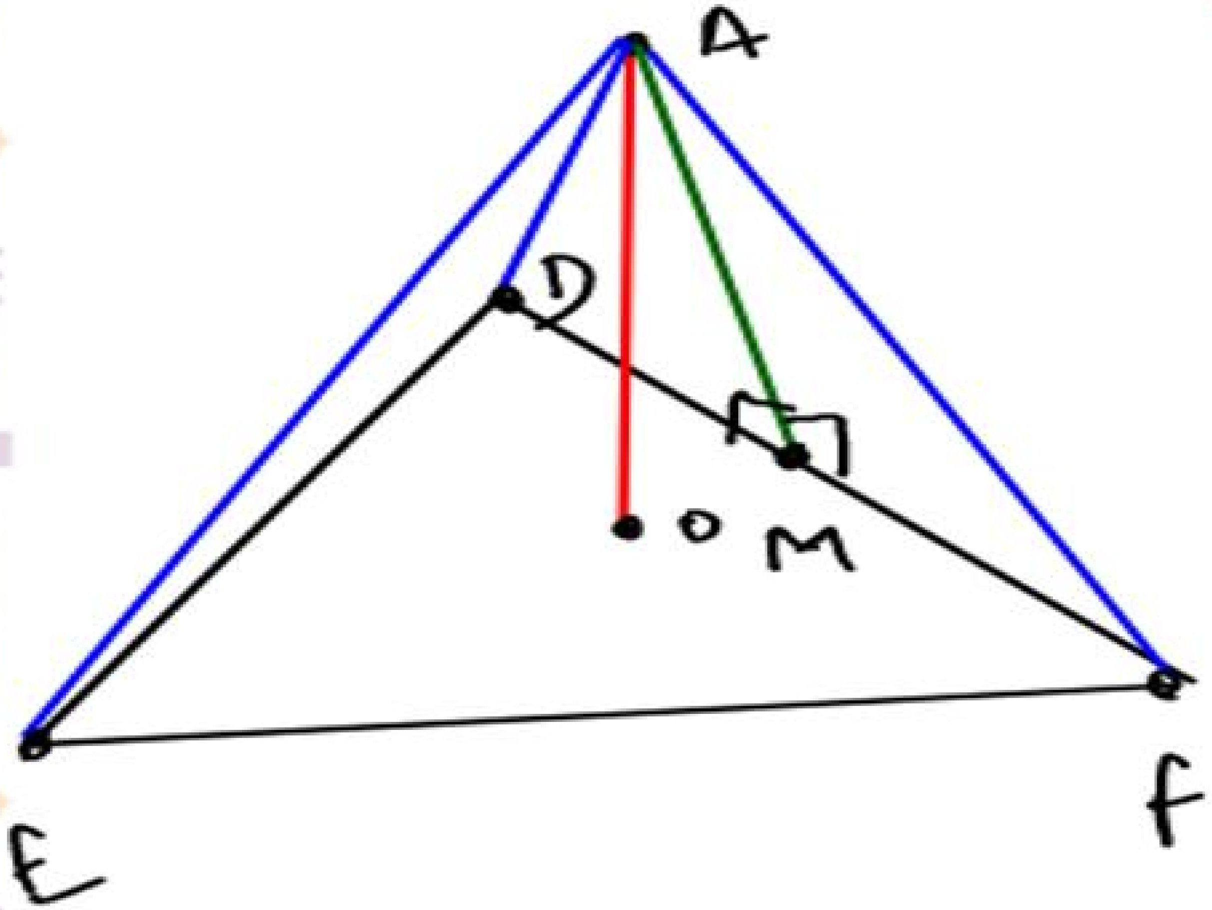
vertices → n+1

Edges → 2n

If Base → n sides

$\boxed{F+V-E=2}$

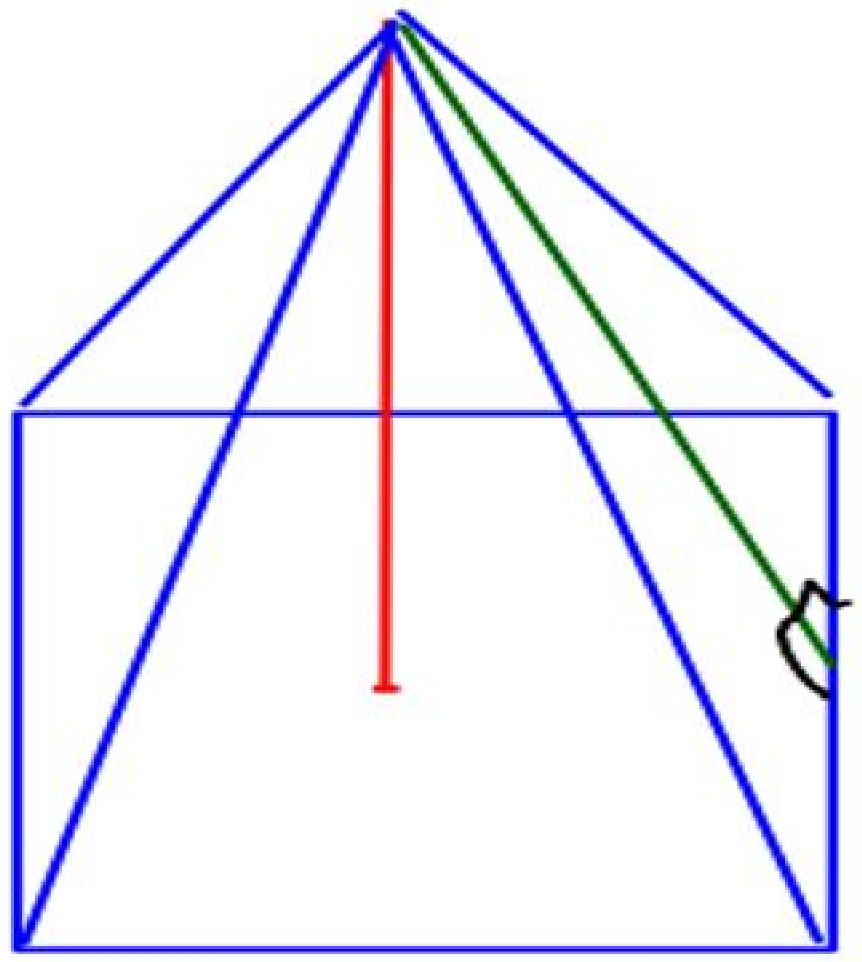
Height, Slant Height & Slant Edge of Pyramid



AO → Height of Pyramid (Red)

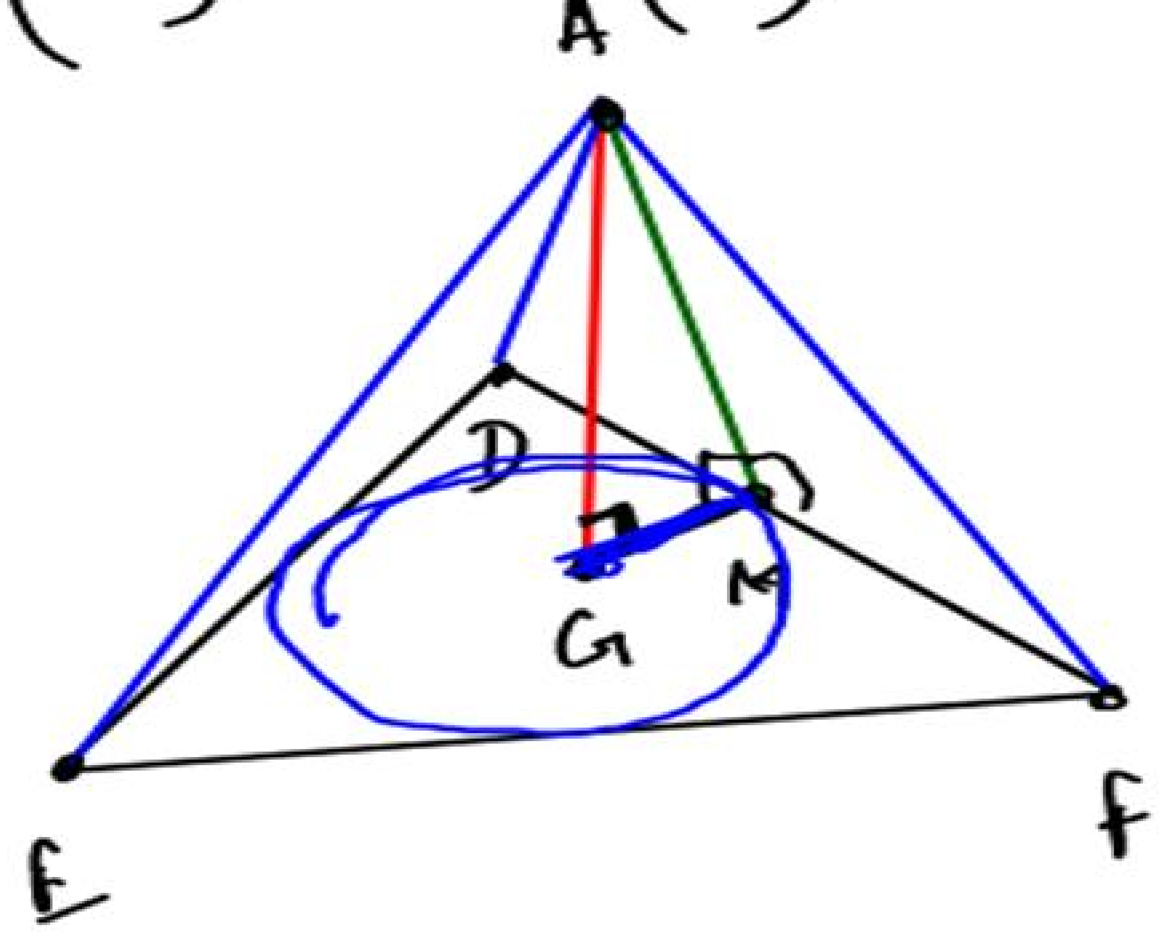
(Blue) AF | AE | AD → Slant edges

(Green) AM → Slant height



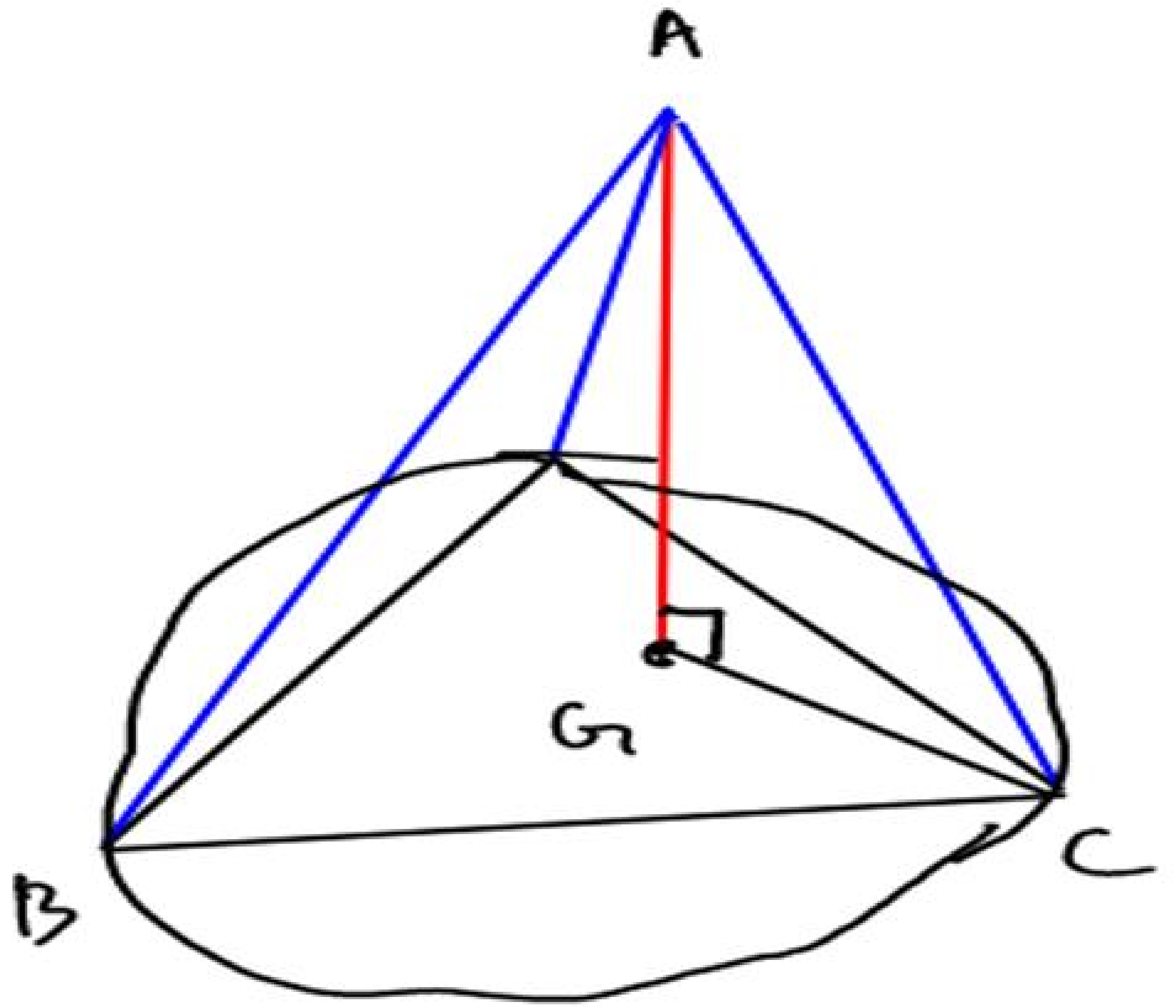
Relationship between Height, Slant Height & Slant Edge of Pyramid

(h) (l) (E)



$$\boxed{l^2 = h^2 + r^2}$$

$r \rightarrow$ inradius of Δ



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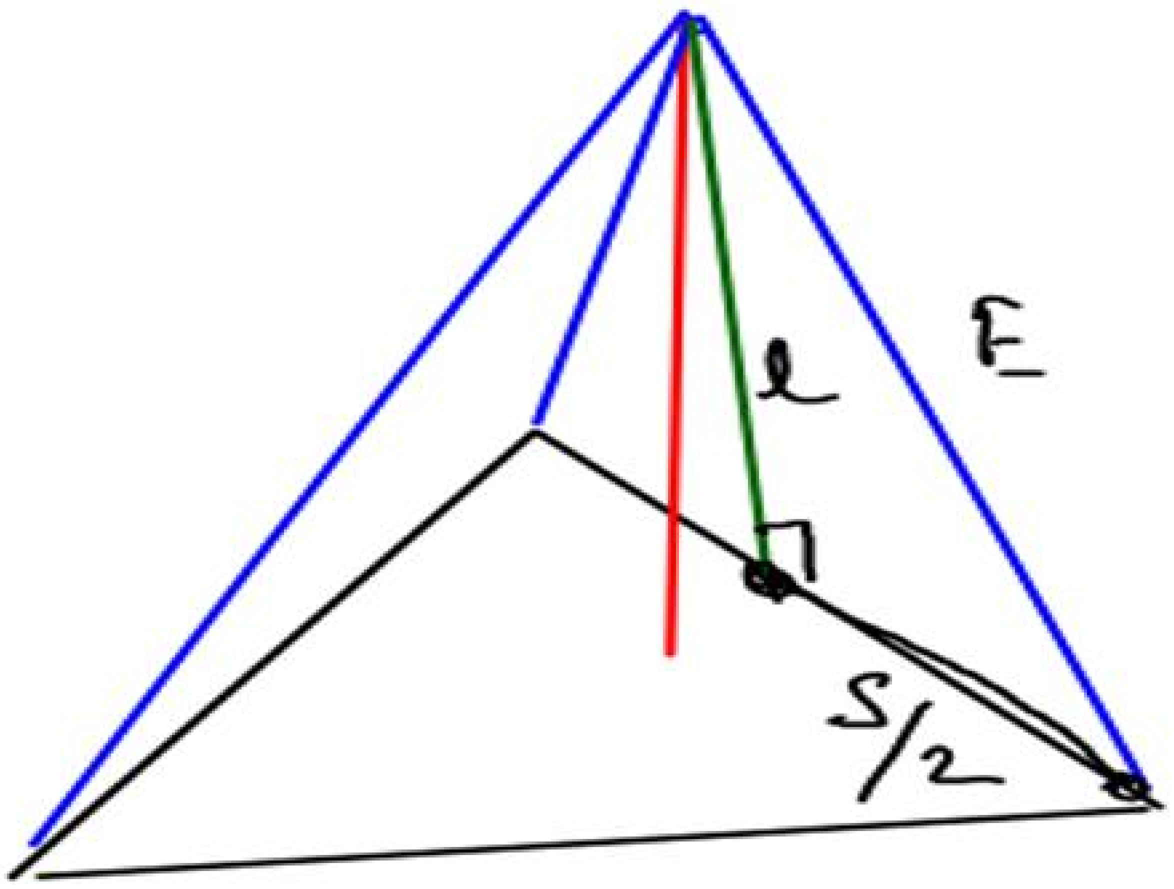
ΔACG

$$AC^2 = AG^2 + GC^2$$

$$\boxed{E^2 = h^2 + R^2}$$

R → Circumradius



$$\boxed{E^2 = l^2 + \left(\frac{s}{2}\right)^2}$$

$$r = \frac{\text{Area}}{\text{Semi} - \text{perimeter}}$$

$$R = \frac{abc}{4 \cdot \text{Area}}$$

	r	**R**
Equilateral Δ	$\frac{S}{2\sqrt{3}}$	$\frac{S}{\sqrt{3}}$
Square	$\frac{S}{2}$	$\frac{S}{\sqrt{2}}$
Regular Hexagon	$\frac{\sqrt{3}}{2}S$	S

Inradius (r) of a Regular Polygon of n-sides

where length of each side = S

$$r = \frac{S}{2}\cot\frac{180}{n}$$

$$R = \frac{S}{2}\operatorname{cosec}\frac{180}{n}$$

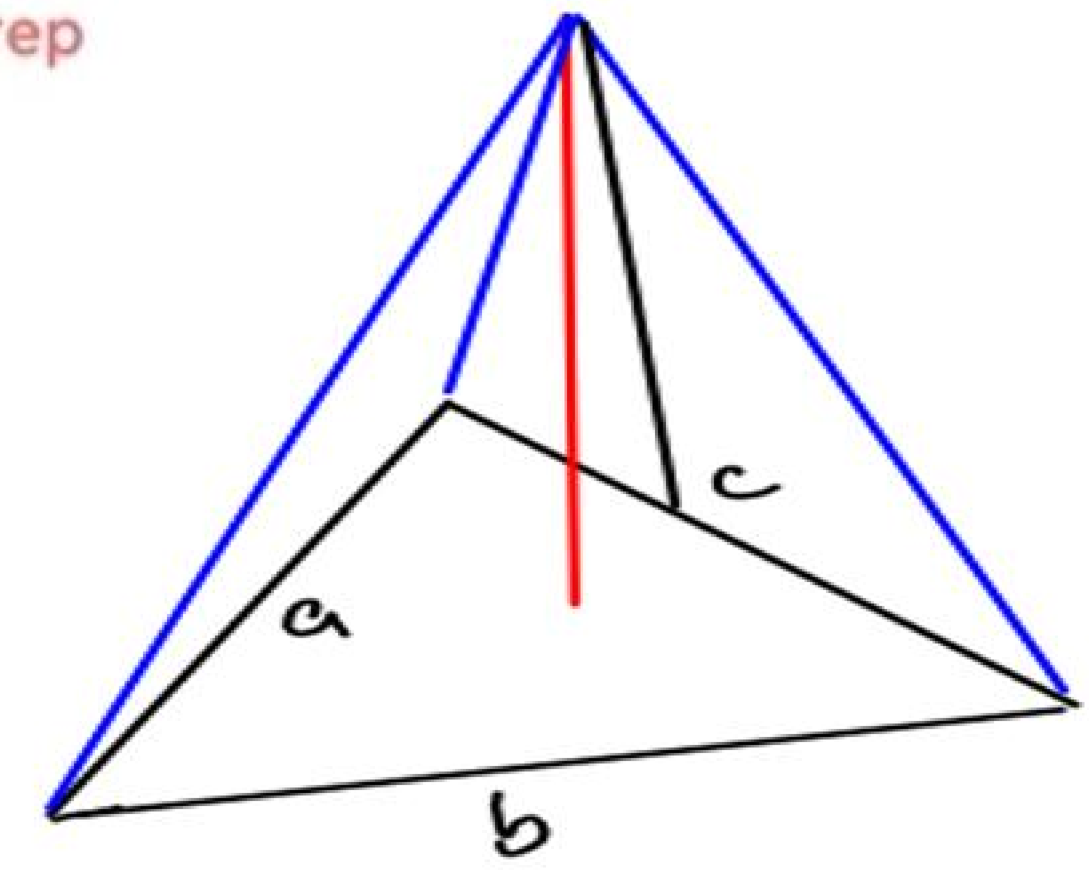
$*\quad l^2 = h^2 + r^2$

$E^2 = h^2 + R^2$

$E^2 = l^2 + \left(\frac{s}{2}\right)^2$

$*\quad r = \frac{s}{2}\cot\left(\frac{180}{n}\right)$ $\qquad R = \frac{s}{2}\operatorname{cosec}\frac{180}{n}$

	r	R
Eq Δ	$\frac{s}{2\sqrt{3}}$	$\frac{s}{\sqrt{3}}$
Square	$\frac{s}{2}$	$\frac{s}{\sqrt{2}}$
Regular Hexagon	$\frac{\sqrt{3}s}{2}$	s



LSA

$$\frac{1}{2}a\cdot l + \frac{1}{2}b\cdot l + \frac{1}{2}c\cdot l$$

$$\cdot\left(\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c\right)$$

$$LSA = \frac{1}{2}p\cdot l$$

$$= \frac{1}{2}\times \text{Perimeter of Base}$$

Formulae used in Pyramid

(i) Lateral surface area $= \frac{1}{2}$ Perimeter of the base × Slant Height

(ii) Total surface area = Lateral surface area + Area of the base

(iii) Volume $= \frac{1}{3}$ Area of the base × Height

Base (Eq Δ)

$$LSA = \frac{1}{2} \times 3s \times l$$

$$TSA = \frac{1}{2} \times 3s \times l + \frac{\sqrt{3}}{4} s^2$$

$$\text{Volume} = \frac{1}{3} \times \frac{\sqrt{3}}{4} s^2 \times h$$

height of Pyramid

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Base (Square)

$$LSA = \frac{1}{2} \times 4S \times l$$

$$TSA = \frac{1}{2} \times 4S \times l + S^2$$

$$Volume = \frac{1}{3} \times S^2 \times h$$

height of Pyramid

$$(i)\ l^2 = h^2 + r^2$$

$$(ii)\ E^2 = h^2 + R^2$$

$$(iii)\ E^2 = l^2 + \left(\frac{S}{2}\right)^2$$

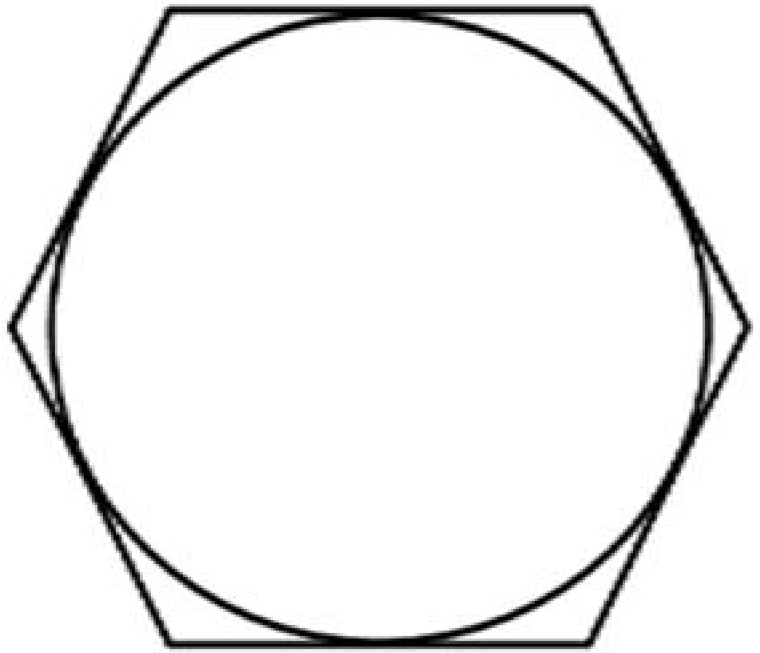
Where,

r **= inradius of Polygon**

R **= Circumradius of Polygon**

l **= Slant height of pyramid**

INRADIUS OF A REGULAR POLYGON



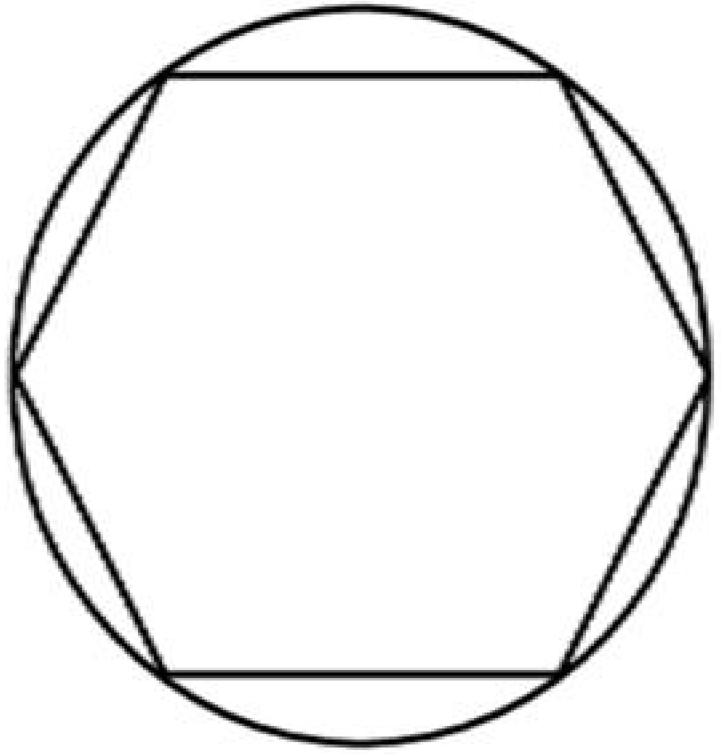
$$r = \frac{S}{2}\cot\left(\frac{180}{n}\right)$$

$n = 6$

$r = \frac{S}{2}\cot 30$

$= \frac{S}{2} \cdot \sqrt{3}$

CIRCUMRADIUS OF A REGULAR POLYGON



$$R = \frac{S}{2}\operatorname{cosec}\left(\frac{180}{n}\right)$$

$n = 6$

$R = \frac{S}{2} \cdot \operatorname{cosec} 30$

$= S$

1. The base of a right pyramid is an equilateral triangle of side 4 cm each. Each slant edge is 5 cm long. The volume of the pyramid is:

(a) $\frac{4\sqrt{8}}{3}$ cm³ (b) $\frac{4\sqrt{60}}{3}$ cm³

(c) $\frac{4\sqrt{59}}{3}$ cm³ (d) $\frac{4\sqrt{61}}{3}$ cm³

Volume $= \frac{1}{3}$ (Area of Base) (height)

Eq Δ

$S = 4$ cm

$E = 5$ cm

Volume

Volume $= \frac{1}{3} \cdot \frac{\cancel{\sqrt{3}}}{\cancel{4}} \cdot \cancel{16}^{4} \cdot \frac{\sqrt{59}}{\cancel{\sqrt{3}}}$

$E^2 = h^2 + R^2$

$25 = h^2 + \frac{16}{3}$

$h^2 = \frac{59}{3}$

$h = \frac{\sqrt{59}}{\sqrt{3}}$

Ans. (c)

2. The base of a right pyramid is an equilateral triangle of side $10\sqrt{3}$ cm. If the total surface area of the pyramid is $270\sqrt{3}$ sq. cm. its height is :

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(a) $12\sqrt{3}$ cm (b) $10\sqrt{3}$ cm

(c) 10 cm (d) 12 cm

$TSA = \frac{1}{2} p \cdot l + \text{Area}$

Eq Δ

$S = 10\sqrt{3}$

$TSA = 270\sqrt{3}$

$h = ??$

$\frac{1}{2} \cdot 30\sqrt{3} \cdot l + \frac{\sqrt{3}}{4} \cdot (10\sqrt{3})^2 = 270\sqrt{3}$

$15\cancel{\sqrt{3}} l + 75\cancel{\sqrt{3}} = 270\cancel{\sqrt{3}}$

$15 l = 195$ $\boxed{l = 13 \text{ cm}}$

$l^2 = h^2 + r^2$

$169 = h^2 + 5^2$

$\boxed{h = 12}$

Ans. (d)

3. A right pyramid of 6 m height has a square base in which the diagonal is $\sqrt{1152}\ m$. Volume of the pyramid is:

(a) 144 m^3 (b) 576 m^3 (c) 228 m^3 (d) 1152 m^3

Square

$D = \sqrt{1152}$

Volume = ??

$h = 6$

$\text{Volume} = \frac{1}{3}(A)\ h$

$= \frac{1}{3} \cdot \frac{1152}{2}\ 6$

$= 1152\ m^3$

Ans. (d)

4. The base of a right pyramid is a square of side 10 cm. If the height of the pyramid is 12 cm, then its total surface area is:

(a) 400 cm^2
(b) 460 cm^2
(c) 260 cm^2
(d) 360 cm^2

Square

$S = 10cm$

$h = 12cm$

$TSA = ??$

$TSA = \frac{1}{2} p \cdot l + Area$

$= \frac{1}{2} \cdot 40 \cdot l + 10^2$

$l^2 = h^2 + r^2$

$l^2 = 144 + 25$

$l = 13$

$\Rightarrow \frac{1}{2} \cdot 40 \cdot 13 + 100$

$= 360cm^2$

Ans. (d)

5. If the slant height of a right pyramid with square base is 4 metre and the total slant surface of the pyramid is 12 sq. metres, then the ratio of total slant surface and area of the base is:

(a) 16 : 3 (b) 24 : 5

(c) 32 : 9 (d) 12 : 3

$l = 4m$

$\frac{1}{2} p \cdot l = 12$

$\frac{LSA}{\text{Area of Base}} = ??$

$\frac{1}{2} p \cdot 4 = 12$

$p = 6$

$4s = 6$ $s = \frac{3}{2}$

$\frac{12}{\frac{9}{4}} = \frac{\cancel{48}\ 16}{\cancel{9}\ 3}$

Ans. (a)

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6. A square pyramid has side of its base 20 cm and height 45 cm is melted and recast into triangular pyramids of equilateral base of side 10 cm and height $10\sqrt{3}$ cm. What are the total numbers of triangular pyramids.

(a) 24 (b) 20 (c) 27 (d) 28

Volumes would be same

$$\frac{1}{\cancel{3}}\cdot(\cancel{20}^{\,4})^2\cdot\cancel{45}^{\,\cancel{9}\,3} = n\cdot\frac{1}{\cancel{3}}\cdot\frac{\cancel{\sqrt{3}}}{\cancel{4}_{\,2}}(\cancel{10})^2\cdot\cancel{10\sqrt{3}}$$

$$\boxed{n = 24}$$

Ans. (a)

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7. There is a pyramid on a base which is a regular hexagon of side 2a cm. If every slant edge of this pyramid is of length 5a/2 cm, then the volume of this pyramid is

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PYQ of SSC

(a) $3a^3$ cm^3
(b) $3\sqrt{2}\,a^3$ cm^3
(c) $3\sqrt{3}\,a^3$ cm^3
(d) $6a^3$ cm^3

Regular Hexagon

$S = 2a$

$E = \frac{5a}{2}$

Volume = ??

$$\text{Volume} = \frac{1}{3}\,6\,\frac{\sqrt{3}}{4}\cdot(2a)^2\cdot h$$

$$\longrightarrow \frac{1}{\cancel{3}}\cdot\cancel{6}^{3}\cdot\frac{\sqrt{3}\cdot\cancel{4}a^2}{\cancel{4}}\cdot\frac{\cancel{3}a}{\cancel{2}}$$

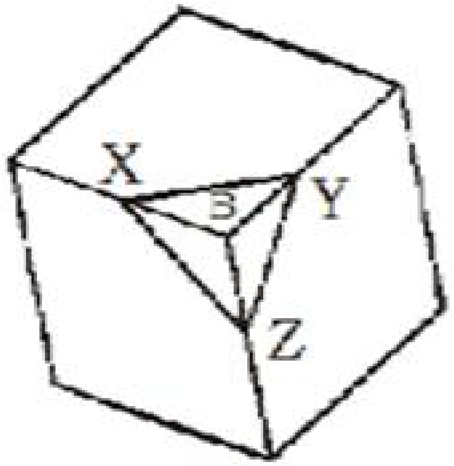
$$E^2 = h^2 + R^2$$

$$\frac{25a^2}{4} = h^2 + 4a^2$$

$$h = \frac{3a}{2}$$

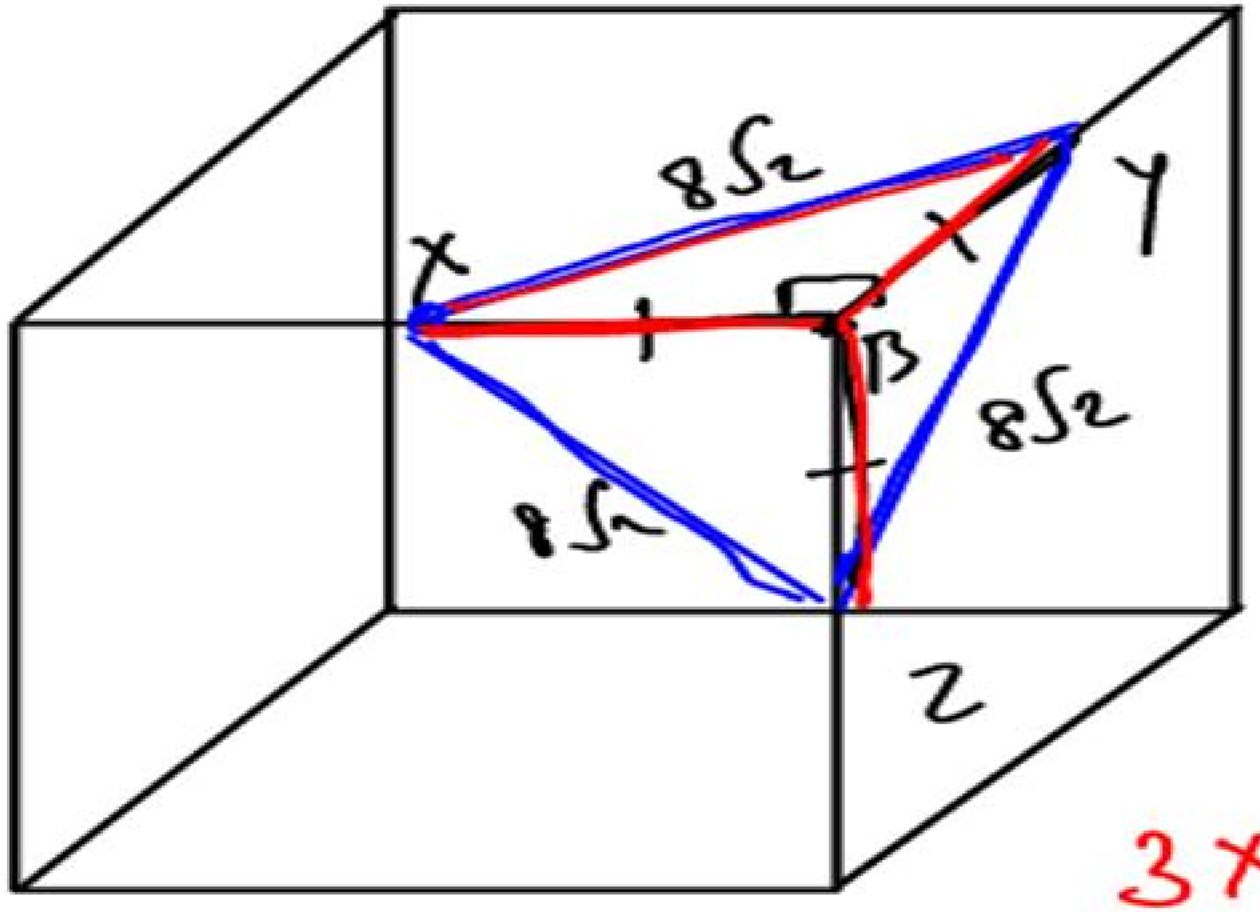
Ans. (c)

8. A right triangular pyramid XYZB is cut from cube as shown in figure. The side of cube is 16 cm. X, Y and Z are mid points of the edges of the cube. What is the total surface area (in cm^2) of the pyramid?



(a) $48\,[(\sqrt{3})+1]$ (b) $24[4+(\sqrt{3})]$

(c) $28[6+(\sqrt{3})]$ (d) $32[3+(\sqrt{3})]$



XYZB $\longrightarrow$ Pyramid

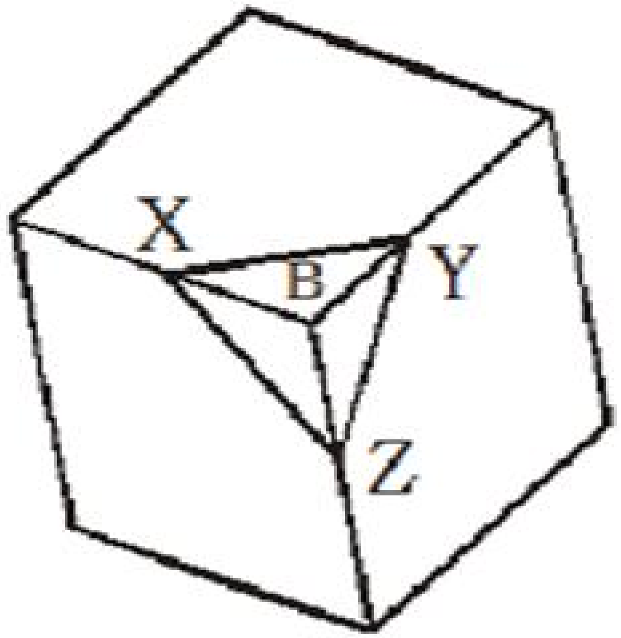
TSA

$$3\times\frac{1}{\cancel{2}}\times\cancel{8}^{4}\times 8+\frac{\sqrt{3}}{4}\times(8\sqrt{2})^2$$

$$96+32\sqrt{2}$$

$$(96+32\sqrt{2})\ cm^2$$

Ans. (d)

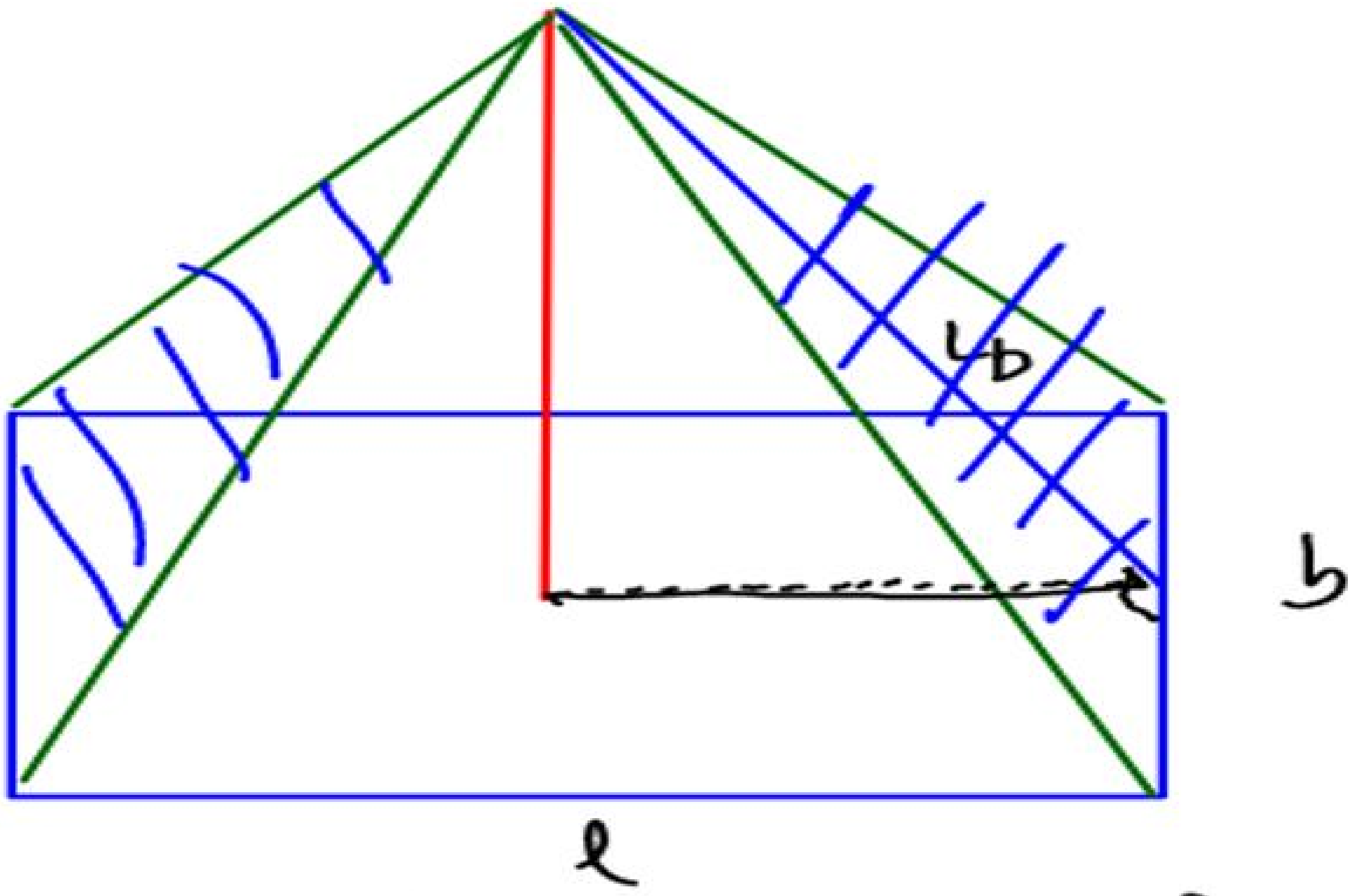


If Base of a Pyramid $\rightarrow$ RECTANGLE

There will be 2 different slant heights :

$L_L \rightarrow$ Slant height on length

$L_B \rightarrow$ Slant height on breadth



$$(L_b)^2 = h^2 + \left(\frac{l}{2}\right)^2$$

$$(L_l)^2 = h^2 + \left(\frac{b}{2}\right)^2$$

$$(L_B)^2 = (H)^2 + \left(\frac{Length}{2}\right)^2$$

$$(L_L)^2 = (H)^2 + \left(\frac{Breadth}{2}\right)^2$$

$$LSA = 2\left(\frac{1}{2} \cdot b \cdot L_B\right) + 2\left(\frac{1}{2} \cdot l \cdot L_L\right)$$

$$\boxed{LSA = b \cdot L_B + l \cdot L_L}$$

Here, $b \rightarrow$ breadth **$l \rightarrow$ length**

$L_B \rightarrow$ Slant height on breadth

$L_L \rightarrow$ Slant height on length

9. The height of a right pyramid is 12 cm. If the base of the pyramid is a rectangle whose length and breadth are 18 cm and 10 cm respectively. What is the total surface area of the pyramid?

(a) 384 cm^2
(b) 564 cm^2
(c) 580 cm^2
(d) 600 cm^2

Imp

PYQ of SSC

$h = 12$ cm

Base Rectangle

$l = 18 \quad b = 10$

TSA = ??

TSA = LSA + Area of Base

$TSA = (l \cdot L_L + b \cdot L_B) + l \cdot b$

$L_L = \sqrt{h^2 + \left(\frac{b}{2}\right)^2} \Rightarrow 13$

$L_B = \sqrt{h^2 + \left(\frac{l}{2}\right)^2} \Rightarrow 15$

$TSA \rightarrow 18 \cdot 13 + 10 \cdot 15 + 18 \cdot 10$

$234 + 150 + 180$

Ans. (b)

REGULAR TETRAHEDRON

Is a Pyramid whose base is an equilateral Δ & lateral surfaces are also equilateral Δ

→ 4 equilateral Δ

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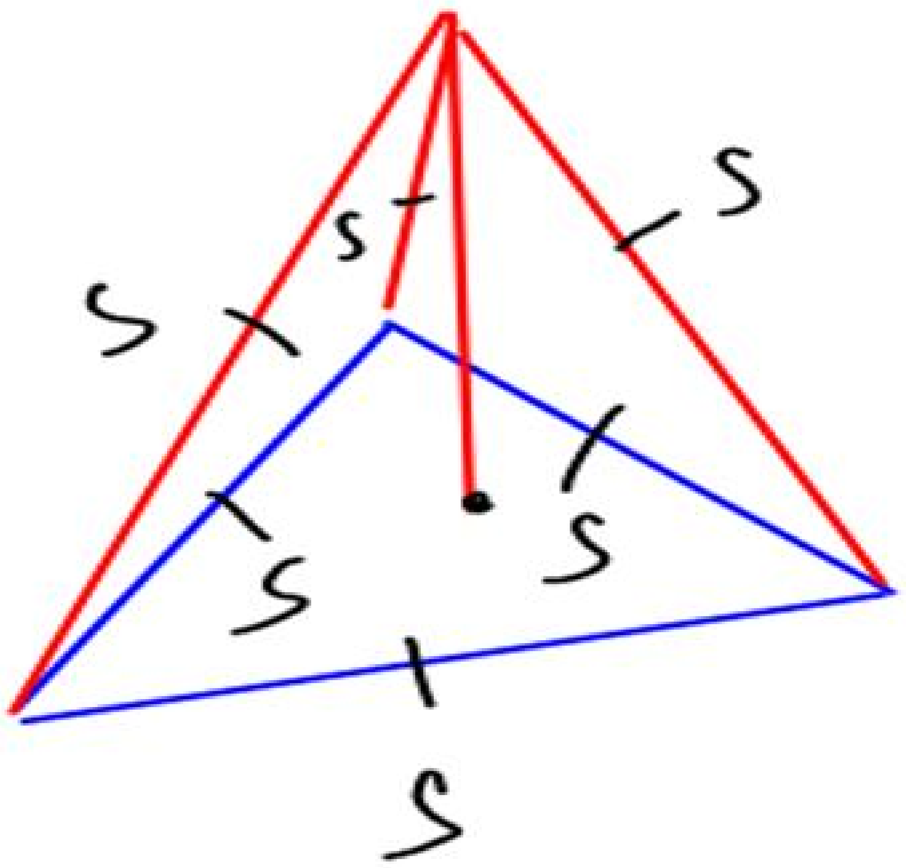
$$LSA = 3 \cdot \frac{\sqrt{3}}{4} s^2$$

$$TSA = \sqrt{3} \cdot s^2$$

$$\text{Volume} = \frac{1}{3} \text{Area of Base} \times \text{Height}$$

$$\frac{1}{3} \times \frac{\cancel{\sqrt{3}}\, s^2}{\cancel{4}_{\,2\sqrt{2}}} \times \frac{\cancel{\sqrt{2}}\, s}{\cancel{\sqrt{3}}}$$

$$\rightarrow \frac{s^3}{6\sqrt{2}} \checkmark\checkmark$$



$$H = \frac{\sqrt{2}S}{\sqrt{3}}$$

Height of Regular Tetrahedron

$$E^2 = H^2 + R^2$$

$$S^2 = H^2 + \frac{S^2}{3}$$

$$\boxed{H = \frac{\sqrt{2}S}{\sqrt{3}}}$$

Regular Tetrahedron

$$LSA = 3 \cdot \frac{\sqrt{3}\,S^2}{4}$$

$$TSA = \sqrt{3}\,S^2$$

$$Volume = \frac{S^3}{6\sqrt{2}}$$

10. If each edge of a regular tetrahedron is 4 cm. Its volume (in cubic cm) is:

(a) $\frac{16\sqrt{3}}{3}$ (b) $16\sqrt{3}$

(c) $\frac{16\sqrt{2}}{3}$ (d) $16\sqrt{2}$

Volume $= \frac{s^3}{6\sqrt{2}}$

$\frac{4 \cdot \cancel{4}^{2} \cdot \cancel{4}^{2}}{3\,\cancel{6}\,\cancel{\sqrt{2}}} \cdot \frac{\sqrt{2}}{\cancel{\sqrt{2}}}$

$\frac{16\sqrt{2}}{3}$

Ans. (c)

11. The length of each edge of a regular tetrahedron is 12 cm. The area (in sq. cm) of the total surface of the tetrahedron is :

(a) $288\sqrt{3}$ (b) $144\sqrt{2}$

(c) $108\sqrt{3}$ (d) $144\sqrt{3}$

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$S = 12 cm$

$TSA = \sqrt{3} \cdot S^2$

$\Rightarrow 144\sqrt{3}\ cm^2$

Ans. (d)

12. If the ratio of the height and the volume of a regular tetrahedron is $1:\sqrt{3}$, then the difference of the total lateral surface area and the area of the base is:

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(a) $2\sqrt{3}\ \text{cm}^2$ (b) $4\sqrt{3}\ \text{cm}^2$

(c) $6\sqrt{3}\ \text{cm}^2$ (d) $9\sqrt{3}\ \text{cm}^2$

90sec

$$\frac{h}{V} = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{\sqrt{2}}{\sqrt{3}}\cancel{s}\cdot 6\sqrt{2}}{\cancel{\sqrt{3}}\,\cancel{s^3}\,s^2} = \frac{1}{\cancel{\sqrt{3}}}$$

$$\boxed{s^2 = 12}$$

LSA − Area of Base

$$\cancel{2}\cdot\frac{\sqrt{3}}{\cancel{4}\,2}s^2 \longrightarrow \frac{\sqrt{3}}{2}\times 12 = 6\sqrt{3}$$

Ans. (c)