Unit Digit

Unit Digit and Last two digits

Unit Digit : Unit digit represents the last digit of any given number or it is obtained by getting the remainder when the given mathematical expression is divided by 10.

Example 1: Find the unit digit of N = 9382496,

Here N is a 7-digit number where 6 at the end represents the Unit digit. …Ans

Unit Digit of Powers of number:

Unit digit can also be found when the given expression is in terms of power to a number.

Base No.→ Power ↓	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	1	4	9	6	5	6	9	4	1
3	1	8	7	4	5	6	3	2	9
4	1	6	1	6	5	6	1	6	1

Note:

From the above table it can be inferred:

1. Any number raised to a power repeats itself after a power of 4. Thus every number has a cyclicity of 4 or less different digits as their unit digit. $2^1 = \underline{2}$, $2^2 = \underline{4}$, $2^3 = \underline{8}$ & $2^4 = 1\underline{6}$ and after that it starts repeating.

So, the cyclicity of 2 has 4 different numbers 2, 4, 8, 6.

Same is true for 3,7 and 8 as given in the table

2. $1^n = 1$ for any value of n. Same is true for 5 and 6.

3. 4 and 9 has a cyclicity of 2. i.e. $4^1 = 4$, $4^2 = 6$, $4^3 = 4$ and so on.

Every number could be generally taken as having a cyclicity of 4 as far as unit digits are concerned for simplicity.

Example 2: For the given expression $(1023)^{127}$, find the value of unit digit.

Unit can be found in following method:

Take unit digit of the base number which is 3. Now, 3^{127}. 3 has cyclicity of power 4 after that the powers start repeating itself and yield the same no. Here, power 127 = 4 × 31 + 3. Thus, 3^{127} will be reduced to 3^3. Thus, $3^3 = 27$ which means the last digit of given expression $(1023)^{127}$ will be 7.

Ex.: $(43287)^{845} = (7)^{4\times211+1} = 7^1 = 7$ will be the unit digit. … Ans

Unit Digit for multiplication of numbers:

$N = 33 \times 19 \times 71 \times 94 \times 58 \times 23$, For the following expression the unit digit will be obtained by dividing it by 10 and finding the remainder:

$$\frac{33 \times 19 \times 71 \times 94 \times 58 \times 23}{10} \rightarrow \frac{(+3) \times (-1) \times (+1) \times (+4) \times (-2) \times (+3)}{10}$$

$$\rightarrow \frac{(-3) \times (+4) \times (-6)}{10} \rightarrow +\frac{72}{10} \rightarrow R = +2$$

Thus, 2 will be the unit digit of given expression.

Alternatively, Unit digit can also be found by multiplying the last digits of the given expression

$N = 33 \times 19 \times 71 \times 94 \times 58 \times 23.$

Here, $3 \times 9 \times 1 \times 4 \times 8 \times 3 = 27 \times 4 \times 24 = 28 \times 24 = 8 \times 4 = 32 \rightarrow \text{Unit Digit} = 2$

Unit Digit of a factorial of a number:

Example 3: Find the unit digit of 724!

We know that the value of

1! = 1

2! = 2

3! = 6
4! = 24
5! = 120
6! = 720

Since unit digit of 5! = 0, thus factorial of any number >= 5 will also have it's unit digit as zero as 5! is always multiplied in any higher number factorial

Thus, unit digit of 724! = 0. …Ans

Unit Digit of powers of a power:

Example 4: Find the unit digit of $23^{24^{25}}$

Since 23 is the base number so we can treat 3 as the base

Since 3 has a cyclicity of 4, so we have to divide the power by 4 and check its nature.

Now Power is 24^{25}

Since 24 is completely divisible by 4, then 24^{25} will also by divisible by 4.

Thus the power is of the form 4k and we know $3^{4k} = 3^4 = 1$ (unit digit)…Ans

Also, when powers are given in factorial, then the unit digit of the expression is calculated as following:

Example 5: Find the unit digit of $(3648)^{283!}$,

It can be clearly seen that power 283! is in the form of 4k (As 4 is one of the multiplicative quantity in any number's factorial that is greater than 4). Thus, 283! Will be equivalent to the power of 4.

Unit digit $= 8^4 = 4096 = 6$ …Ans

Last Two Digits:

The last two digits represent the last two digits at the end of any given number or it is obtained by getting the remainder when the given mathematical expression is divided by 100.

Ex.: N = 9382496, Here N is a 7-digit number where 96 at the end represents the last two digits.

Similarly, $N = 28 \times 19 \times 71 \times 96 \times 58 \times 23$, For the following expression the last two digits will be obtained by dividing it by 100 and finding the remainder:

$$\frac{28 \times 19 \times 71 \times 96 \times 58 \times 23}{100} \text{ can be written as } \frac{28 \times 19 \times 71 \times 24 \times 58 \times 23}{25}$$

$$\frac{(+3) \times (-6) \times (-4) \times (-1) \times (+8) \times (-2)}{25} \rightarrow \frac{(72) \times (16)}{25} \rightarrow \frac{(-3) \times (-9)}{25}$$

$$\rightarrow \frac{+27}{25} \rightarrow R = +2$$

But we simplified the given expression by dividing the numerator and the denominator by 4. Thus, we have to multiply it by 4. So, R = +2 × 4 = +8 or 08 which represents the last two digits of the given expression.

Last two digits of number ending with 1,3,7 and 9:

For such number we have to make the unit digit = 1 and then the trick is explained in the example below

Example 6: Find last two digits of $(238641)^{386}$

Clearly the last digit here is 1

For ten's digit we need to multiply the 2nd last digit of the base with the last digit of power.

Here $(2386\mathbf{4}1)^{38\mathbf{6}}$ that would be 4x6 = 2**4**

And the last digit of that product is the ten's digit of the expression = 4

So the last two digits for the given expression is 41 …. Ans

Now for 3, 7 and 9 :

We have to successively take squares until the last digit becomes 1 and then find the last two digits
$3^1 = 3$, $3^2 = 9$, $3^3 = 27$, **$3^4 = 81$**
$7^1 = 7$, $7^2 = 49$, $7^2 = 343$, **$7^4 = 2401$**
$9^1 = 9$, **$9^2 = 81$**
Example 7: Find the last two digits of $(17)^{263}$
As mentioned above 7^4 has unit digit 1. So
So we first need to find the last two digits of 17^4
$17^2 = 2$**89**
Last two digits of $17^4 = 89^2 = 79$**21**
Now $17^{263} = 17^{4x65 + 3} = 17^{4x65} \times 17^3$
Considering only last 2 digits 17
$17^{4x65} \times 17^3 = 21^{65} \times 17^3$
$= 01 \times 13 = 13$

Last two digits of number ending with 2, 4, 6 and 8:

We know
$2 = 2$
$4 = 2^2$
$6=2x3$
And $8 = 2^3$
So once we establish the method to find last two digits of the number ending with two, we can follow the rest of numbers using the above relation by breaking the number in simpler prime factors.
$2^1=02$, $2^2 = 04$, $2^3 = 08$, $2^4 = 16$ ………… $2^{10} = 10$**24**
$2^{20} = (2^{10})^2 = 76$ as last two digits
Again $2^{30} = (2^{10})^3 = 24$ as last two digits
Thus for 2 after every power of 10 the last two digits become 24 and 76 alternatively
i.e.
Last two digits of following expression
$(2^{10})^{odd} = 24$
$(2^{10})^{even} = 76$
Example 8: Find the last two digits of $(54)^{283}$
$54 = 2 \times 3^3$
So $(54)^{283} = (2 \times 3^3)^{283} = 2^{283} \times 3^{849} = (2^{10})^{84} \times 2^3 \times 81^{70} + 3^3$
Considering only last two digits, we get $(2^{10})^{84} \times 2^3 \times 81^{70} + 3^3 = 76x08x01x27 =$ 164**16**
Hence last two digits are 16.

For numbers ending with 5:

Tens digit of number or base	Units digit of power	Last two digits
Even	Even	25
Even	Odd	25
Odd	Even	25
Odd	Odd	75

Example 8: Find last two digit of 326735^{31}

As ten's digit of base is 3 (odd) and unit digit of power is 1(odd)

So last two digits are 75