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Sensor signal processing

Two-Dimensional Direction Finding With Parallel Nested Arrays Using DOA Matrix Method

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Abstract—This article develops a new 2-D direction estimation algorithm using parallel arrays that consist of two parallel nested subarrays. The key idea behind the presented algorithm is to use the difference coarray properties embedded in the auto- and cross correlation matrices of the parallel nested arrays to form a direction-of-arrival (DOA) matrix with enhanced degree of freedoms for 2-D angle estimation. The presented algorithm does not require 2-D nonlinear searching or parameter paring processing. Simulation results demonstrate the superiority of the new algorithm.

Index Terms—Sensor signal processing, direction finding, DOA matrix method (DMM), nested arrays.

I. INTRODUCTION

Estimation of 2-D arrival angles using sensor array technique is a key problem in many engineering applications such as wireless communications and radar. Many efficient 2-D direction finding algorithms using regularly planar array structures such as rectangular [1], [2], L-shaped [3]–[6], and parallel shaped [7]–[11] have been proposed over the past few decades. A regularly planar array usually contains several uniformly spaced linear subarrays, and consequently, has limited degree of freedom (DOF), since an L-element uniformly linear array (ULA) can resolve $L-1$ signals at the most.

Nested arrays, first proposed in [12], can offer enhanced DOF for array processing. Specially, for 1-D direction finding, an L-element linear nested array can offer $\mathcal{O}(L^2)$ DOFs to identify $K > L$ source signals. In recent years, several works on the application of nested arrays for 2-D direction finding are presented [13]–[17]. Pal and Vaidyanathan [13], [14] design the so-called 2-D nested array for 2-D array processing. A 2-D nested array contains two uniformly spaced linear subarrays with different element spacing. Lan *et al.* [15] propose a rank reduce (RARE) algorithm based on the 2-D nested array. Niu *et al.* [16] and Dong *et al.* [17] develop algorithms for 2-D direction finding using L-shaped nested array that consists of two identical but orthogonal linear nested subarrays. However, the above-mentioned algorithms require either parameter searching [13]–[16] or paring [16]. In addition, the algorithm in [17] presumes a special space-time structure to define spatial–temporal correction matrices for joint diagonalization.

In this article, we consider the using of nested arrays for 2-D direction finding in a simple way. We propose computationally efficient algorithm using parallel nested arrays using the direction-of-arrival (DOA) matrix method (DMM). The parallel nested array considered here is composed of two identical but parallel linear nested subarrays. The basic idea of the new algorithm is to use the difference coarray properties embedded in the auto- and cross correlation matrices of the parallel nested arrays to form a DOA matrix, whose eigenvalues and eigenvectors together give the 2-D direction estimates with correctly paring. The presented algorithm thus does not require any nonlinear searching or parameter paring processing. The proposed method has a similarity with the original DMM presented in [10] and [11] because both methods obtain the 2-D DOAs from the eigenvalues and eigenvectors of the defined DOA matrices. However, since the array geometry of the proposed method is different from those in [10] and [11], the proposed method can be considered as the extension of the methods in [10] and [11] by exploiting the nested array properties embedded in the array geometry.

II. SIGNAL MODEL

We consider that there are K narrow-band source signals impinging upon an irregularly parallel shape array. The kth source signal is parameterized by azimuth angle ϕ_k and elevation angle θ_k, with $\phi_k \in [0, \pi]$ and $\theta_k \in [0, \pi/2]$. The irregularly parallel shape array is composed of two subarrays, where the first subarray is deployed along y-axis and the second is deployed by moving the first subarray along x-axis with distance d_x. The source-array geometry configuration is shown in Fig. 1.

Each subarray contains L sensors, which are placed in a two-level linear nested structure, as shown in Fig. 2. This linear nested subarray consists of two cascaded ULAs, where the first ULA contains L_1 sensors with spacing d_y, and the second ULA has $L_2 = L - L_1$ sensors with spacing $D_y = (L_1 + 1)d_y$. Letting $\boldsymbol{p}$ be an $L \times 1$ vector, which contains the y-axis coordinates of the sensors, we have $\boldsymbol{p} = [0, 1, \ldots, L_1 - 1, L_1, 2L_1, \ldots, L_2 L_1]d_y$.

The output data of the ℓth sensor in the first subarray is expressed as

$$z_{1,\ell}(t) = \sum_{k=1}^{K} e^{j2\pi/\lambda p_\ell v_k} s_k(t) + n_{1,\ell}(t) \tag{1}$$

where λ denotes the wavelength of the signals; $v_k = \sin\theta_k \sin\phi_k$ and $s_k(t)$, respectively, signify the kth signal's y-axis direction cosine and the waveform; and $n_{1,\ell}(t)$ is the additive noise measured by the ℓth sensor of the first subarray. The data measured by the first subarray can be expressed as the following $L \times 1$ vector:

$$\boldsymbol{z}_1(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}_1(t) \tag{2}$$

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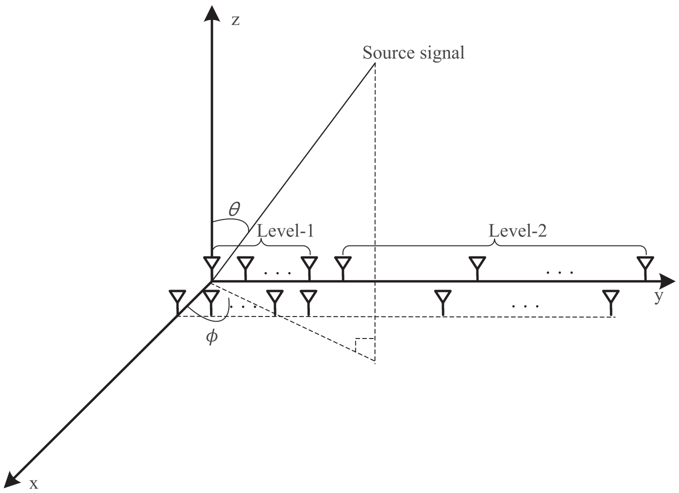

Fig. 1. Source-array geometry configuration for the proposed algorithm. The irregularly parallel shape array consists of two linear nested subarrays.

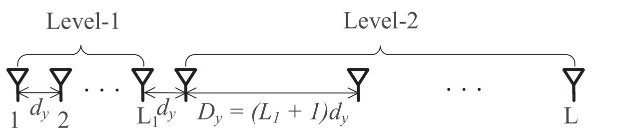

Fig. 2. Illustration of two-level linear nested array that contains two concatenated ULAs. The inner ULA contains L_1 sensors with spacing d_y, and the outer ULA has $L_2 = L - L_1$ sensors with spacing $D_y = (L_1 + 1)d_y$.

where $\boldsymbol{A} = [\boldsymbol{a}(v_1), \boldsymbol{a}(v_2), \ldots, \boldsymbol{a}(v_K)]$ is the $L \times K$ subarray manifold, with $\boldsymbol{a}(v_k) = [e^{j2\pi/\lambda p_1 v_k}, e^{j2\pi/\lambda p_2 v_k}, \ldots, e^{j2\pi/\lambda p_L v_k}]^T$ being the $L \times 1$ subarray response vector for the kth signal, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T$ is the $K \times 1$ signal vector, and $\boldsymbol{n}_1(t) = [n_{1,1}(t), n_{1,2}(t), \ldots, n_{1,L}(t)]^T$ is the $L \times 1$ noise vector. Similarly, the $L \times 1$ data vector measured by the second subarray is of the form

$$\boldsymbol{z}_2(t) = \boldsymbol{A}\Phi_u \boldsymbol{s}(t) + \boldsymbol{n}_2(t) \tag{3}$$

where Φ_u is a $K \times K$ diagonal matrix, with the kth diagonal entry being $e^{j2\pi/\lambda d_x u_k}$, $u_k = \sin\theta_k \cos\phi_k$ is the x-axis direction cosine of the kth signal, and $\boldsymbol{n}_2(t) = [n_{2,1}(t), n_{2,2}(t), \ldots, n_{2,L}(t)]^T$ is the $L \times 1$ additive noise vector of the second subarray.

The problem of the letter is to estimate the directions $\{\theta_k, \phi_k k = 1, \ldots, K\}$ of the impinging signals from a total of N snapshots taken at the distinct instants $\{t_n : n = 1, \ldots, N\}$. We will develop a new algorithm using the DMM in Section III. For this purpose, the following assumptions are made.

1) The direction cosines u and v are pairwise distinct, i.e., $u_1 \neq u_2 \neq \cdots \neq u_K$ and $v_1 \neq v_2 \neq \cdots \neq v_K$. This implies that the proposed method would fail when some of the signals have the same direction cosines.
2) The number of source signals is known in advance or correctly estimated.
3) The source signals are all statistically independent of each other.
4) The noise is assumed to be spatial–temporal white Gaussian distributed with power σ_n^2, and is uncorrelated with the source signals.

III. ALGORITHM DEVELOPMENT

A. DOA Matrix Formulation

The first step of the proposed algorithm is to formulate a DOA matrix with enhanced DOFs. Based on the assumptions made above, the autocorrelation matrix of data measured by the first subarray is of the form

$$\boldsymbol{R}_a = E\{\boldsymbol{z}_1(t)\boldsymbol{z}_1^H(t)\} = \boldsymbol{A}\boldsymbol{R}_s\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I} \tag{4}$$

where the superscript H represents the conjugate transpose and $\boldsymbol{I}$ denotes the identity matrix. The $K \times K$ signal correlation, matrix $\boldsymbol{R}_s = \mathrm{diag}[\rho_1, \rho_2, \ldots, \rho_K]$ is diagonal, where ρ_k is the power of the kth source signal. Since $\boldsymbol{R}_s$ is diagonal, the vectorization version of $\boldsymbol{R}_a$ can be written compactly as

$$\boldsymbol{v}_a^{'} = (\boldsymbol{A}^* \odot \boldsymbol{A})\boldsymbol{\rho} + \sigma_n^2\boldsymbol{1}_e \tag{5}$$

where the superscript $*$ stands for the complex conjugate, $\odot$ represents the Khatri–Rao product, $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_K]^T$, $\boldsymbol{1}_e = [\boldsymbol{e}_1^T, \boldsymbol{e}_2^T, \ldots, \boldsymbol{e}_L^T]^T$, with $\boldsymbol{e}_\ell$ being a vector of all zeros except for 1 at the ℓth position and the superscript T being the transpose operator. Since the noise power in (5) can be easily estimated by using the eigenvalue estimation method, we can remove the noise term in (5) and obtain

$$\boldsymbol{v}_a = \boldsymbol{v}_a^{'} - \sigma_n^2\boldsymbol{1}_e = (\boldsymbol{A}^* \odot \boldsymbol{A})\boldsymbol{\rho}. \tag{6}$$

Next, utilizing the difference coarray property of the nested array, it is easy to verify that the columns of $(\boldsymbol{A}^* \odot \boldsymbol{A})$ have only $\bar{L} = 2L_2(L_1 + 1) - 1$ different items. Remove the repeated rows from $(\boldsymbol{A}^* \odot \boldsymbol{A})$ and sort the remaining rows such that the ℓth row is associated with the y-axis coordinate $(\bar{\ell} - L_2(L_1 + 1))$. Then, we can form a new vector by picking out the nonrepeated items of $\boldsymbol{v}_a$ as

$$\bar{\boldsymbol{v}}_a = \boldsymbol{B}\boldsymbol{\rho} \tag{7}$$

where $\boldsymbol{B} = [\boldsymbol{b}(v_1), \ldots, \boldsymbol{b}(v_K)]$ is an $\bar{L} \times K$ matrix, with the kth column being $\boldsymbol{b}(v_k) = [e^{j2\pi/\lambda d_y(1-L_2(L_1+1))v_k}, e^{j2\pi/\lambda d_y(2-L_2(L_1+1))v_k}, \ldots, e^{j2\pi/\lambda d_y(L_2(L_1+1)-1)v_k}]^T$ being an $\bar{L} \times 1$ vector.

Obviously, (8) manifests the single snapshot direction finding problem with manifold matrix $\boldsymbol{B}$ and signal vector $\boldsymbol{\rho}$. In order to estimate the directions of the signals, the vector $\bar{\boldsymbol{v}}_a$ is divided into $L_2(L_1 + 1)$ overlapping subvectors, each of which contains $L_2(L_1 + 1)$ elements. Then, the mth subvector can be expressed as

$$\bar{\boldsymbol{v}}_{a,m} = \boldsymbol{B}_m\boldsymbol{\rho} \tag{8}$$

where $\boldsymbol{B}_m = [e^{j2\pi/\lambda d_y(m-L_2(L_1+1))v_k}, \ldots, e^{j2\pi/\lambda d_y(m-1)v_k}]^T$ is the mth to $(m + L_2(L_1 + 1) - 1)$th rows of $\boldsymbol{B}$. For all $m = 1, \ldots, L_2(L_1 + 1))$, we can form an $L_2(L_1 + 1)) \times L_2(L_1 + 1))$ matrix

$$\bar{\boldsymbol{V}}_a = [\bar{\boldsymbol{v}}_{a,1}, \bar{\boldsymbol{v}}_{a,2}, \ldots, \bar{\boldsymbol{v}}_{a,L_2(L_1+1))}]. \tag{9}$$

Since $\boldsymbol{B}_m$ is related to $\boldsymbol{B}_1$ as

$$\boldsymbol{B}_m = \boldsymbol{B}_1\Phi_v^m \tag{10}$$

where Φ_v is a $K \times K$ diagonal matrix, with the kth diagonal entry being $e^{j2\pi/\lambda d_y v_k}$, we can express $\bar{\boldsymbol{V}}_a$ as

$$\bar{\boldsymbol{V}}_a = \boldsymbol{B}_1\mathcal{D}(\boldsymbol{\rho})\boldsymbol{B}_1^H \tag{11}$$

where $\mathcal{D}(\cdot)$ denotes the operator that generates a diagonal matrix by putting the vector in parentheses on the main diagonal.

Analogously, the cross correlation matrix of data measured by the first and the second subarrays is

$$\boldsymbol{R}_b = E\{\boldsymbol{z}_2(t)\boldsymbol{z}_1^H(t)\} = \boldsymbol{A}\Phi_u\boldsymbol{R}_s\boldsymbol{A}^H. \tag{12}$$

The vectorization version of $\boldsymbol{R}_b$ is

$$\boldsymbol{v}_b = (\boldsymbol{A}^* \odot \boldsymbol{A}\Phi_{\mathbf{u}})\boldsymbol{\rho} = (\boldsymbol{A}^* \odot \boldsymbol{A})\Phi_{\mathbf{u}}\boldsymbol{\rho}. \tag{13}$$

After removing the repeating items from $\boldsymbol{v}_b$, we have

$$\bar{\boldsymbol{v}}_b = \boldsymbol{B}\Phi_{\mathbf{u}}\boldsymbol{\rho}. \tag{14}$$

Since the vector $\bar{\boldsymbol{v}}_b$ is of the same size as $\bar{\boldsymbol{v}}_a$, we can divide the vector $\bar{\boldsymbol{v}}_b$ into $L_2(L_1+1)$ overlapping is of.....” Please check."?>subvectors to construct an $L_2(L_1+1) \times L_2(L_1+1)$ matrix

$$\bar{\boldsymbol{V}}_b = [\bar{\boldsymbol{v}}_{b,1}, \bar{\boldsymbol{v}}_{b,2}, \ldots, \bar{\boldsymbol{v}}_{b,L_2(L_1+1))}] \tag{15}$$

where $\bar{\boldsymbol{v}}_{b,m}$ is the mth to $(m + L_2(L_1+1) - 1)$th rows of $\bar{\boldsymbol{v}}_b$. The matrix $\bar{\boldsymbol{V}}_b$ can then be expressed as

$$\bar{\boldsymbol{V}}_b = \boldsymbol{B}_1\Phi_u\mathcal{D}(\boldsymbol{\rho})\boldsymbol{B}_1^H. \tag{16}$$

Using $\boldsymbol{B}$ as full column rank due to its Vandermonde structure and assumption 1), we have

$$\bar{\boldsymbol{V}}_a^\dagger = \left(\mathcal{D}(\boldsymbol{\rho})\boldsymbol{B}_1^H\right)^\dagger \boldsymbol{B}_1{}^\dagger \tag{17}$$

where the superscript † represents the pseudo-inverse operator. From (16) and (17), the so-called DOA matrix is constructed as

$$\boldsymbol{D} = \bar{\boldsymbol{V}}_b\bar{\boldsymbol{V}}_a^\dagger = \boldsymbol{B}_1\Phi_v\boldsymbol{B}_1^\dagger. \tag{18}$$

B. Azimuth–Elevation Angle Estimation

In (18), we can see that the DOA matrix is related to the phase factors $\{e^{j2\pi/\lambda d_y v_k}, k = 1, 2, \ldots, K\}$ through the diagonal elements of Φ_v, and to phase factors $\{e^{j2\pi/\lambda d_x u_k}, k = 1, 2, \ldots, K\}$ through the kth columns of the matrix $\boldsymbol{B}_1$. Using this relationship, the direction cosines of the incoming signals can be estimated.

Write the eigendecomposition of $\boldsymbol{D}$ as

$$\boldsymbol{D} = \boldsymbol{F}\Psi_v\boldsymbol{F}^{-1}. \tag{19}$$

It can be easily verified from (18) and (19) that the kth diagonal element of Φ_v equal to the kth nonzero diagonal element of Ψ_v, and the kth column of $\boldsymbol{B}_k$, is a scalar multiple of the eigenvector associated with the kth nonzero (largest) diagonal element of Ψ_v. Let $\{[\Psi_v]_{k,k} = e^{j2\pi/\lambda d_y \hat{v}_k}, k = 1, 2, \ldots, K\}$ be the nonzero eigenvalues of $\boldsymbol{D}$, and let $\{\boldsymbol{f}_k, k = 1, 2, \ldots, K\}$ be the eigenvectors associated with these eigenvalues. Then, the x-axis direction cosine estimates $\hat{u}_k$ can be estimated from $\boldsymbol{f}_k$ as

$$\hat{u}_k = \frac{1}{L_2(L_1+1)-1}\sum_{m=1}^{L_2(L_1+1)-1}\frac{\arg(f_{k,m+1}/f_{k,m})}{2\pi d_x/\lambda} \tag{20}$$

where $f_{k,m}$ denotes the mth element of $\boldsymbol{f}_k$. The y-axis direction cosine estimates $\hat{v}_k$ can be computed as

$$\hat{v}_k = \frac{\arg\{[\Psi_v]_{k,k}\}}{2\pi d_y/\lambda}, \quad k = 1, 2, \ldots, K. \tag{21}$$

With the estimation of direction cosines $\hat{u}_k$ and $\hat{v}_k$, the azimuth–elevation angles of the kth signal can be estimated as

$$\hat{\theta}_k = \arcsin\left(\sqrt{\hat{u}_k^2 + \hat{v}_k^2}\right) \tag{22}$$

$$\hat{\phi}_k = \angle(\hat{u}_k + j\hat{v}_k). \tag{23}$$

C. Implementation of the Algorithm

In this section, we summarize the implementation of the presented algorithm as follows.

S1) Estimate the auto- and cross correlation matrices $\hat{\boldsymbol{R}}_a = \frac{1}{N}\sum_{n=1}^N \boldsymbol{z}_1(t_n)\boldsymbol{z}_1^H(t_n)$ and $\hat{\boldsymbol{R}}_b = \frac{1}{N}\sum_{n=1}^N \boldsymbol{z}_2(t_n)\boldsymbol{z}_1^H(t_n)$.

S2) Obtain the vectors $\hat{\boldsymbol{v}}_a^{'}$ and $\hat{\boldsymbol{v}}_b$ by vectorizing the matrices $\hat{\boldsymbol{R}}_a$ and $\hat{\boldsymbol{R}}_b$.

S3) Estimate the noise variance $\hat{\sigma}_n^2$ by averaging the $L-K$ smallest eigenvalues of $\hat{\boldsymbol{R}}_a$ and obtain the vector $\hat{\boldsymbol{v}}_a$.

S4) Obtain the vectors $\hat{\bar{\boldsymbol{v}}}_a$ and $\hat{\bar{\boldsymbol{v}}}_b$ by removing the repeated elements from $\hat{\boldsymbol{v}}_a$ and $\hat{\boldsymbol{v}}_b$.

S5) Formulate the matrices $\hat{\bar{\boldsymbol{V}}}_a$ and $\hat{\bar{\boldsymbol{V}}}_b$ using (9) and (15).

S6) Estimate the DOA matrix $\hat{\boldsymbol{D}} = \hat{\bar{\boldsymbol{V}}}_b\hat{\bar{\boldsymbol{V}}}_a^\dagger$.

S7) Perform eigenvalue decomposition to $\hat{\boldsymbol{D}}$ to obtain the estimates $\hat{\boldsymbol{F}}$ and $\hat{\Psi}_v$.

S8) Compute direction cosine estimates $\hat{u}_k$ and $\hat{v}_k$ using (20) and (21).

S9) Compute the azimuth–elevation angle estimates $\hat{\theta}_k$ and $\hat{\phi}_k$ using (22) and (23).

D. Remarks

As discussed in Section III, the proposed algorithm estimates the y-axis direction cosines $\hat{v}_k$ from the eigenvalues of the matrix $\hat{\boldsymbol{D}}$, and estimates the x-axis direction cosines $\hat{u}_k$ from the eigenvectors of $\hat{\boldsymbol{D}}$. Since the eigenvalues and their eigenvectors are paired, the estimated direction cosines $\hat{u}_k$ and $\hat{v}_k$ are therefore automatically matched without any additional pairing match processing.

The maximum number of source signals that can be identified is determined by the signal subspace dimension of $\hat{\boldsymbol{D}}$. Since the dimension of $\hat{\boldsymbol{D}}$ is $L_2(L_1+1) \times L_2(L_1+1)$, the proposed algorithm can resolve $K \leq L_2(L_1+1) - 1$ source signals with an array of $2L$ physical sensors.

IV. SIMULATIONS

Several simulation results are provided to demonstrate the performance of the presented algorithm in this section. Parallel nested array in Fig. 1 with $L_1 = 3$ and $L_2 = 2$ is used. The result in each of the figures given in the following is obtained from 500 independent Monte Carlo trials, unless otherwise stated.

First, we examine the identifiability performance of the proposed algorithm for a parallel nested array with $L_1 = 3$ and $L_2 = 2$. Seven monochromatic signals with the following direction cosines $(u_1, \ldots, u_7) = (0.1, 0.2, 0.3, 0.6, 0.7, 0.4, 0.5)$ and $(v_1, \ldots, v_7) = (0.2, 0.4, 0.7, 0.1, 0.3, 0.8, 0.5)$ are assumed to impinge upon the array. The signal-to-noise ratio (SNR) and the number of snapshots are set as 20 dB and 1000, respectively. Fig. 3 shows the direction cosines estimates for ten independent runs. We can see from the figure that the presented algorithm can resolve these seven signals, as claimed in Section III-D: $K \leq L_2(L_1+1) - 1$.

Next, the performance of the presented algorithm is compared with the following algorithms: parallel-shaped array of uniform spacing with $L = 6$, parallel-shaped array of uniform spacing with $L = 12$, and the interlaced double-precision (IDP) algorithm using L-shaped nested array [16]. For the proposed algorithm, $L_1 = L_2 = 3$ is considered. Thus, the total aperture of the above four algorithms is $5d_y$, $11d_y$, $9d_y$, and $9d_y$, respectively. The signals are assumed to come from the direction cosines $u_1 = 0.1$, $v_1 = 0.2$ and $u_2 = 0.2$, $v_2 = 0.3$. The number of snapshots used is $N = 200$. Fig. 4 shows the root-mean-squared errors (RMSEs) of u_1 estimates versus the SNR

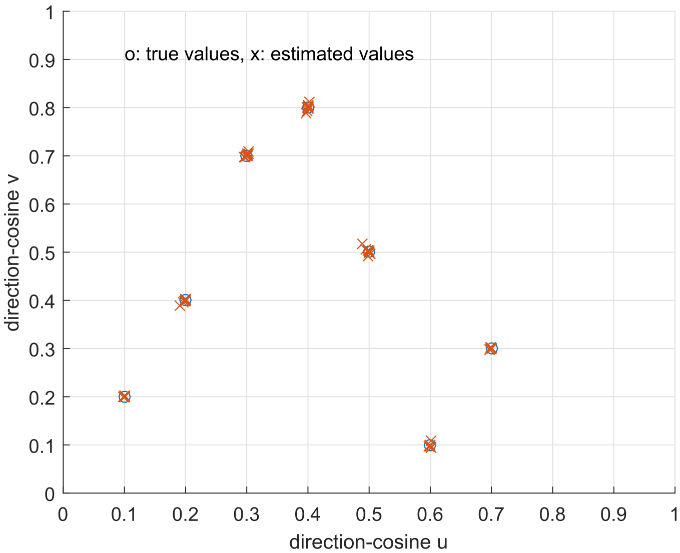

Fig. 3. Direction cosine estimates for ten independent runs. $(u_1, \ldots, u_7) = (0.1, 0.2, 0.3, 0.6, 0.7, 0.4, 0.5)$ and $(v_1, \ldots, v_7) = (0.2, 0.4, 0.7, 0.1, 0.3, 0.8, 0.5)$.

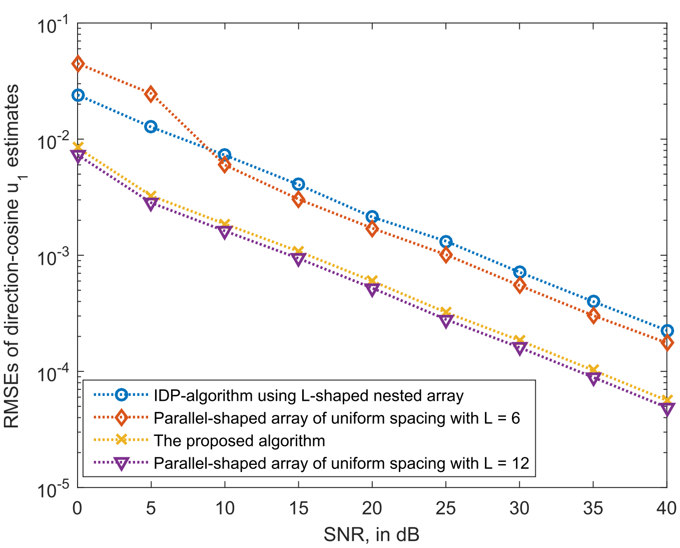

Fig. 4. RMSEs of direction cosine u_1 estimates. Two uncorrelated signals with direction cosines $u_1 = 0.1$, $v_1 = 0.2$ and $u_2 = 0.2$, $v_2 = 0.3$ impinge upon the array.

varying from 0 to 40 dB. As seen from Fig. 4, the proposed algorithm has performance better than the IDP algorithm and the parallel-shaped array of uniform spacing with $L = 6$ in terms of lower RMSEs. Also, note that the performance of the proposed algorithm is an approach to the parallel-shaped array of uniform spacing with $L = 12$, where the physical sensors used are twice as much as used in the proposed algorithm.

V. CONCLUSION

We have proposed a new algorithm to estimate 2-D directions using parallel arrays that consist of two parallel nested subarrays. The key idea of the proposed algorithm is to exploit the difference coarray properties embedded in the auto- and cross correlation matrices of the parallel nested arrays to form a DOA matrix with enhanced DOFs for automatically paired 2-D angle estimation. The proposed algorithm is computationally efficient in that no 2-D nonlinear searching is involved.

REFERENCES

[1] X. Xu and Z. Ye, "Two-dimensional direction of arrival estimation by exploiting the symmetric configuration of uniform rectangular array," *IET Radar, Sonar Navig.*, vol. 6, no. 5, pp. 307–313, Jun. 2012.

[2] P. Heidenreich, A. M. Zoubir, and M. Rubsamen, "Joint 2-D DOA estimation and phase calibration for uniform rectangular arrays," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4683–4693, Sep. 2012.

[3] J. E. F. del Rio and M. F. Catedra-Peraz, "The matrix pencil method for two-dimensional direction of arrival estimation employing L-shaped array," *IEEE Trans. Antennas Propag.*, vol. 45, no. 11, pp. 1693–1694, Nov. 1997.

[4] N. Tayem and H. M. Kwon, "L-shape 2-dimensional arrival angle estimation with propagator method," *IEEE Trans. Antennas Propag.*, vol. 53, no. 5, pp. 1622–1630, May 2005.

[5] S. Kikuchi, H. Tsuji, and A. Sano, "Pair-matching method for estimating 2-D angle of arrival with a cross-correlation matrix," *IEEE Antennas Wireless Propag. Lett.*, vol. 5, pp. 35–40, 2006.

[6] J. F. Gu and P. Wei, "Joint SVD of two cross-correlation matrices to achieve automatic pairing in 2-D angle estimation problems," *IEEE Antennas Wireless Propag. Lett.*, vol. 6, pp. 553–556, 2007.

[7] Y. Wu, G. Liao, and H. C. So, "A fast algorithm for 2-D direction-of-arrival estimation," *Signal Process.*, vol. 83, pp. 1827–1831, 2003.

[8] V. S. Kedia and B. Chandna, "A new algorithm for 2-D DOA estimation," *Signal Process.*, vol. 60, pp. 325–332, 1997.

[9] J. He and Z. Liu, "Extended aperture 2-D direction finding with a two-parallel-shape-array using propagator method," *IEEE Antennas Wireless Propag. Lett.*, vol. 8, pp. 323–327, 2009.

[10] Q. Y. Yin, R. W. Newcomb, and L. H. Zou, "Estimating 2-D angles of arrival via two parallel linear arrays," in *Proc. Int. Conf. Acoust., Speech, Signal Process.*, Glasgow, U.K, 1989, vol. 4, pp. 2803–2806.

[11] L. Jin and Q. Y. Yin, "Space-time DOA matrix method," *Acta Electron. Sin.*, vol. 28, no. 7, pp. 8–12 2000.

[12] P. Pal and P. P. Vaidyanathan, "Nested array: A novel approach to array processing with enhanced degrees of freedom," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4167–4181, Aug. 2010.

[13] P. Pal and P. P. Vaidyanathan, "Nested arrays in two dimensions, Part I: Geometrical considerations," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4694–4705, Sep. 2012.

[14] P. Pal and P. P. Vaidyanathan, "Nested arrays in two dimensions, Part II: Application in two dimensional array processing," *IEEE Trans. Signal Process.*, vol. 60, no. 9, pp. 4706–4718, Sep. 2012.

[15] X. Lan, Y. Li, and E. Wang, "A RARE algorithm for 2D DOA estimation based on nested array in massive MIMO system," *IEEE Access*, vol. 4, pp. 3806–3814, 2016.

[16] C. Niu, Y. Zhang, and J. Guo, "Interlaced double-precision 2-D angle estimation algorithm using L-shaped nested arrays," *IEEE Signal Process. Lett.*, vol. 23, no. 4, pp. 522–526, Apr. 2016.

[17] Y. Dong, C. Dong, Y. Zhu, G. Zhao, and S. Liu, "Two-dimensional DOA estimation for L-shaped array with nested subarrays without pair matching," *IET Signal Process.*, vol. 10, no. 9, pp. 1112–1117, Dec. 2016.