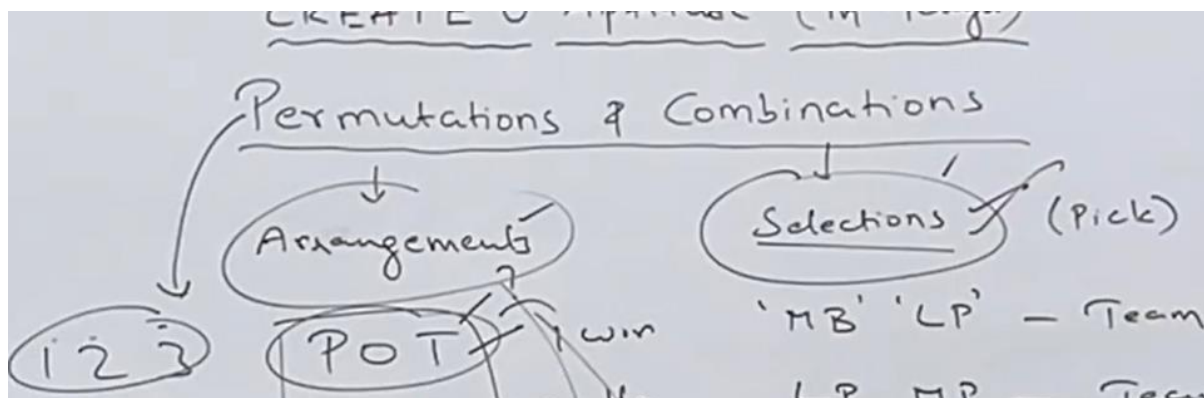


PERMUTATIONS & COMBINATIONS



PERMUTATIONS & COMBINATIONS

$${}_5P_3 \rightarrow 5 \times 4 \times 3$$

$${}_7P_2 \rightarrow 7 \times 6$$

$${}_8P_4 \rightarrow 8 \times 7 \times 6 \times 5$$

$${}_nP_n = n!$$

$${}_5P_5 = 5 \times 4 \times 3 \times 2 \times 1 = 5!$$

$${}_5P_4 = 5 \times 4 \times 3 \times 2 = 5!$$

$${}_nP_{n-1} = n!$$

$${}_nP_1 = n$$

$${}_5P_0 = 1$$

$${}_7P_6 = 7!$$

$${}_7P_7 = 7!$$

$${}_5P_1 = 5$$

$${}_5P_0 = 1$$

$$5C_2 \Rightarrow \frac{5 \times 2}{2 \times 1}$$

$$8C_3 = \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$$

$$10C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$$

$$10C_6 \Rightarrow \frac{10 \times 9 \times 8 \times 7 \times \boxed{6 \times 5}}{\boxed{6 \times 5} \times 4 \times 3 \times 2 \times 1}$$

$$10C_6 = 10C_4$$

$$6P_3 = 6 \times 5 \times 4$$

$$6C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1}$$

$$7P_5 \neq 7P_2$$

$$7C_5 = 7C_2$$

In how many ways, the letters of the word YUVRAJ be arranged?

Sol : ${}^6P_6 \Rightarrow 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
 $= 720 \text{ ways}$

$$\begin{array}{r} 40320 \\ \times 9 \\ \hline 362880 \end{array}$$

$$\begin{array}{l} 1! = 1 \\ 2! = 2 \\ 3! = 6 \\ 4! = 24 \\ 5! = 120 \\ 6! = 720 \\ 7! = 5040 \\ 8! = 40320 \\ 9! = 362880 \\ 10! = 3628800 \end{array}$$

TENDULKAR

$$9! = 362880$$

~~TENDULKAR~~

$$25200 \times 11 = \boxed{277200}$$

$$\Rightarrow \frac{11!}{3!3!2!2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1}$$

$$= 277200$$

In how many ways YUVRAJ can be arranged such that 'Vowels' are always together.

YUVRAJ

$$\begin{array}{cccccc} Y & V & R & J & (U & A) \\ 1 & 2 & 3 & 4 & 5 & 6 \end{array} = 5!2!$$

$$= 120 \times 2 = \underline{\underline{240}}$$

In how many ways 'TENDULKAR' can be arranged such that 'Vowels' are always together.

T E N D U L K A R

$$\begin{array}{ccccccccc} \text{T} & \text{N} & \text{D} & \text{L} & \text{K} & \text{R} & \text{E} & \text{U} & \text{A} \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & & \end{array} \quad = 7! 3! = 5040 \times 6 = 30240$$

In how many ways 'ZAHEERABAD' can be arranged such that 'Vowels' are always together.

Z A H E E R A B A D

$$\begin{array}{ccccccccc} \text{Z} & \text{H} & \text{R} & \text{B} & \text{D} & \text{A} & \text{A} & \text{A} & \text{E} \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \quad = \frac{9! 5!}{3! 2!} = \frac{362880 \times 120}{6 \times 2} = 3628800$$

In how many ways 'DRAAVID' can be arranged such that 'Vowels' are always together.

D R A A V I D

$$\begin{array}{ccccccccc} \text{D} & \text{R} & \text{A} & \text{A} & \text{V} & \text{I} & \text{D} \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \quad = \frac{7! 3!}{2! 2!} = \frac{5040 \times 6}{2 \times 2} = 1512$$

Permutations & Combinations

In how many ways, you can arrange the letters of the word TENDULKAR, such that Vowels are never together

Sol

$\overset{1}{T} \overset{2}{\textcircled{E}} \overset{3}{N} \overset{4}{\textcircled{U}} \overset{5}{L} \overset{6}{\textcircled{A}} \overset{7}{R}$

$\sqrt{\text{C}} \quad \sqrt{\text{C}} \quad \sqrt{\text{C}} \quad \sqrt{\text{C}} \quad \sqrt{\text{C}} \quad \sqrt{\text{C}} \quad \sqrt{\text{C}}$
 $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$

$$\Rightarrow 6! \cdot {}^7P_3 \quad V \text{ C V C V C V C V C V}$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 7 \times 6 \times 5 = 720 \times 210 = 151200$$

Permutations & Combinations

In how many ways, you can arrange the letters of the word YUVRAT, such that Vowels are never together

Sol

$\overset{1}{Y} \overset{2}{\textcircled{U}} \overset{3}{V} \overset{4}{\textcircled{A}} \overset{5}{T}$

$V - V - V - V - V$

$$4! \cdot {}^5P_2 = 24 \times 5 \times 4 = 480$$

$$\boxed{{}^{C+1}P_V}$$

Permutations & Combinations

In how many ways, you can arrange the letters of the word YUVRAT, such that Vowels are never together

Sol

Y U V R A T
1 2 3 4 5

V - V - V - V - V

$$4! \cdot {}^5P_2 = 24 \times 5 \times 4$$

$$= \underline{\underline{480}}$$

$$\frac{{}^{C+1}P_V}{C!}$$

Permutations & Combinations

In a Joint Family, there are 7 Girls & 4 Boys.

In how many ways they can stand for a photograph such that no two boys are together.

B G B G B G B G B G B G B

$${}^8P_4 \Rightarrow 5040 \times 8 \times 7 \times 6 \times 5$$

$$= 5040 \times 1680 = \underline{\underline{8467200}}$$

$$\begin{array}{r} 5040 \times 1680 \\ \hline 8467200 \end{array}$$

6 Girls and 4 Boys joined a maths Tuition. In how many ways, they can sit in a straight line such that all girls are together.

4B 'GG'

B₁ B₂ B₃ B₄ (G₁ G₂ G₃ G₄ G₅ G₆)

$$= (5! 6!)$$

$$= 120 \times 720$$

$$= \underline{\underline{86400}}$$

Permutations & Combinations

In how many different ways can letters of the word PROBLEM be arranged such that Vowels occupy only 'odd' places.

$$\begin{array}{ccccccccc} P & R & \textcircled{O} & B & L & \textcircled{E} & M & & \\ 1 & 2 & 3 & 4 & 5 & & & & \end{array} \Rightarrow 5!^4 P_2$$
$$\begin{array}{ccccccccc} \overset{1}{P} & \overset{2}{R} & \overset{3}{O} & \overset{4}{B} & \overset{5}{L} & \overset{6}{E} & \overset{7}{M} & & \\ \hline & & & & & & & & \end{array} \Rightarrow 120 \times 4 \times 3$$
$$= 120 \times 120$$
$$= \underline{14400}$$

Permutations & Combinations

How many 4 digit numbers can be formed from $\{2, 4, \overset{\checkmark}{7}, 3, 0, 6, 8\}$

① Without Repetition $\underline{6 \times 6 \times 5 \times 4} = 720$

② Repetition allowed $\underline{6 \times 7 \times 7 \times 7} = 343 \times 6$
 $= \underline{2058}$

Permutations & Combinations

How many 3 digit numbers can be formed from $\{2, 3, 4, 5, 6, 7, 8, 9\}$

① Without Repetition $\underline{8 \times 7 \times 6} \Rightarrow {}^8P_3 = 336$

② Repetition allowed $\underline{8 \times 8 \times 8} = 8^3 = 512$

Permutations & Combinations

How many 4 digit numbers can be formed which are divisible by 5 from $\{2, 3, 4, 5, 6, 7\}$

$$\underline{6 \times 6 \times 6 \times 1}$$
$$\Rightarrow \underline{\underline{216}}$$

Permutations & Combinations

How many 4 digit numbers can be formed which are divisible by 5 from $\{2, 3, 4, 5, 0, 7\}$

$$\underline{5 \times 6 \times 6 \times 2}$$
$$\Rightarrow \underline{\underline{360}}$$

Permutations & Combinations

How many 4 digit ATM PINs are possible?

$(0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$

$$\underline{10 \times 10 \times 10 \times 10}$$
$$10^4 = \underline{\underline{10000}}$$

Permutations & Combinations

From a group of 12 members, in how many ways, a committee of 9 members be formed.

$${}^{12}C_9 \Rightarrow {}^{12}C_3 = \frac{12 \times 11 \times 10}{3 \times 2 \times 1}$$

$$\frac{12 \times 11 \times 10 \times \cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4}}{\cancel{9 \times 8 \times 7 \times 6 \times 5 \times 4} \times 3 \times 2 \times 1}$$

$$= \underline{\underline{220 \text{ ways}}}$$

In how many ways can 4 boys & 3 girls be selected from 5 boys & 6 girls.

$${}^5C_4 \times {}^6C_3$$

$${}^5C_1 \times {}^6C_3 = 5 \times \frac{\cancel{6} \times 5 \times 4}{3 \times 2 \times 1}$$

$$= \underline{\underline{100 \text{ ways}}}$$

And \times
or $+$

A committee of 5 members is going to be formed from 3 Trainees, ^x4 Professors & 6 Research Scholars.

In how many ways, they can be selected for the committee, so that there are 2 Trainees and 3 Research Scholars?

$${}^3C_2 \times {}^6C_3 = {}^3C_1 \times {}^6C_3 = 3 \times \frac{\cancel{6} \times 5 \times 4}{3 \times 2 \times 1}$$

$$= \underline{\underline{60 \text{ ways}}}$$

Permutations & Combinations

Out of 5 Men and 4 Women, a committee of 5 members has to be formed. Find the no. of ways in which this can be done, such that there has to be at least 2 women.

(2W and 3M) (or) (3W and 2M) (or) (4W and 1M)

$${}^4C_2 {}^5C_3 + {}^4C_3 {}^5C_2 + {}^4C_4 {}^5C_1$$

$${}^4C_2 {}^5C_3 + {}^4C_1 {}^5C_2 + {}^4C_0 {}^5C_1$$

$$\left(\frac{4 \times 3}{2 \times 1} \times \frac{5 \times 4}{2 \times 1} \right) + \left(4 \times \frac{5 \times 4}{2 \times 1} \right) + (1 \times 5)$$

$$= 60 + 40 + 5 = \underline{105 \text{ ways}}$$

Permutations & Combinations

Out of 5 Men and 4 Women, a committee of 5 members has to be formed. Find the no. of ways in which this can be done, such that there has to be At most 2 women.

(2W & 3M) (or) (1W and 4M) (or) (0W & 5M)

$${}^4C_2 {}^5C_3 + {}^4C_1 {}^5C_4 + {}^5C_5$$

$${}^4C_2 {}^5C_3 + {}^4C_1 {}^5C_4 + {}^5C_0 \Rightarrow \underline{81}$$

$$\Rightarrow \left(\frac{4 \times 3}{2 \times 1} \times \frac{5 \times 4}{2 \times 1} \right) + (4 \times 5) + 1$$

$$= 60 + 20 + 1 = \underline{81}$$

A box contains 2 white balls, 3 Black Balls, 4 Red Balls. In how many ways 3 Balls can be drawn from the Box, such that at least one black ball is included in the draw?

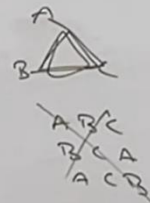
$(2W), 3B, (4R)$
 (One B and 2 others) or (2 Black & one other) or 3 Black
 $\underline{3 \text{ Balls}} \rightarrow {}^3C_1 {}^6C_2 + {}^3C_2 {}^4C_1 + {}^3C_3$
 $\rightarrow \left(\frac{3 \times 6 \times 5}{2} \right) + (3 \times 4) + 1$
 $= 45 + 12 + 1 = 58$

10 Students are participating in a Race. In how many ways, can the first 3 Prizes be won?

~~${}^{10}C_3$~~ $({}^{10}P_3)$ A B C D E F G H I J
 $10 \times 9 \times 8$
 $= 720 \text{ ways}$

How many Triangles can be formed by joining the Vertices of Octagon.

8C_3 ~~8P_3~~
 $\Rightarrow \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$



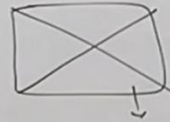
Permutations & Combinations

How many Diagonals can be formed by joining the Vertices of Hexagon?

No. of Diagonal

$$\frac{n(n-3)}{2}$$

$$= \frac{6 \times (6-3)}{2} = \frac{6 \times 3}{2} = 9 \text{ Diagonal}$$



$$\frac{4 \times (4-3)}{2} = \frac{4 \times 1}{2} = 2$$

In how many ways can one or more of six friends be invited for a dinner?

$$2^6 - 1 = 64 - 1 = 63$$

In how many ways 3 consonants and 2 vowels are selected from the word "RAINBOW"?

R A I N B O W
1 2 3 4

4 Consonants & 3 vowels

3 consonants & 2 vowels

$${}^4C_3 \times {}^3C_2 = 4 \times 3 = 12 \text{ ways}$$

Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels are formed?

$${}^7C_3 \times {}^4C_2$$

$$\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}$$

210 × 6 = 1260

Selections Arrange

Combination Permutation

1 1

210 × 5! = 210 × 120 = 25200

5!

R G L A E

20 members attended the party. If each person in the party shakes hand with every other person once, find the total no. of hand shakes?

$${}^{20}C_2 = \frac{20 \times 19}{2 \times 1} = 190$$

$$A \leftrightarrow B$$

$$AB = BA$$

Combination

$$\overline{AB} = \overline{BA}$$

Permutation

$$AB \neq BA$$

20 members attended the party. If each person in the party gives one gift to every other person in the party, find the total no. of gifts exchanged?

$${}^{20}P_2 = 20 \times 19 = 380$$

Permutation

$$A \rightarrow B \neq B \rightarrow A$$

$$\overline{AB} \neq \overline{BA}$$

Arran

Permutations & Combinations

- (20) Teams participated in a Tournament. If each team plays one match with every other team in the tournament find the total no. of matches played.

$${}^{20}C_2 = \frac{20 \times 19}{2} = 190 \text{ matches}$$

$$\text{Ind v/s Pak} = \text{Pak v/s Ind}$$

$$AB = BA$$

Combinations

Hand Shakes
Matches \rightarrow Combinations $\rightarrow {}^nC_2$

Gifts
Tickets \rightarrow Permutations $\rightarrow {}^nP_2$

There are 20 Railway stations b/w Chennai & Bangalore. How many different kinds of tickets must be printed so as to enable a passenger to travel from one place to another?

