ECON 457 - A01
Computational Economics*
UVIC - Department of Economics
Spring Term 2023/24

Assignment 3
Due on Brightspace before ***11.59pm March 19th*** *2024*
Please create and submit a pdf file, making sure that it's readable and unlocked.
The file name has to follow this template: 457_PS3_Surname_Name_StudentNumber.pdf

You can cooperate with other students, but no group submissions will be accepted
If you do cooperate, please list the other students' names in the cover page
No "photocopy answers" will be accepted
No late submissions will be accepted

NOTE: YOU MUST INCLUDE THE ASSIGNMENT COVER PAGE
(Failure to do so will entail a 5-point deduction from the grade received)

Remarks: Your answers have to be submitted in a "report" format. Relying on Jupyter is the easiest option. The codes you developed have to be included as well in the pdf file. Devote some time to give the graphs, plots and tables a format easy to understand. Also the way you present your answers matters for the final grade. Even if a question is mainly technical, ***briefly*** *explain what you are doing, stressing the economic meaning of the various steps whenever possible. Being able to convey your thoughts effectively is an asset also in real life.*

Question 1: An AD-AS Model (50 Marks)

We work with the AD-AS model for a closed economy that we discussed in class. You may want to modify the Python notebook I provided (4_AD-AS.ipynb).

(a) We rely on the same formulas that we used in class (for the definition of output, the consumption function, the investment function, the demand for liquidity, the production function, and proportional income taxes).

For the parameters, use the following values:

$$c = 0.6, b = 1500, k = 4.0, h = 1000, t = 0.2, \alpha = 0.4$$

For the autonomous components, use the following values:

$$\overline{C} = 160, \overline{I} = 100, \overline{L} = 225$$

For the exogenous variables, use the following values:

$$\overline{G} = 200, \overline{W} = 10, \overline{K} = 5000$$

For the value of money supply $\overline{M}$, use the average value of the monetary aggregate M2 for the year 1983. You can find the M2 data for the U.S. economy at the following website:

`https://fred.stlouisfed.org/series/WM2NS`

Find numerically the value of productivity $\overline{A}$ that delivers an endogenous price level P equal to 1. Explain your implementation and, in a Table, report the values of GDP, the Price level, Consumption, Investment and the interest rate. Comment.

(b) Now we use the model to decompose the increase in the price level observed in the U.S. economy between the early 1980s and the early 2000s. You can find the CPI data at the following website (notice that the price index is normalized, such that between 1982-1984 its average value is 100):

`https://fred.stlouisfed.org/series/CPIAUCSL`

For the value of productivity $\overline{A}$, use the value you obtained in part (a). Since nominal wages over those two decades almost doubled, set $\overline{W} = 20$. For the money supply $\overline{M}$, use the average value of the monetary aggregate M2 for the year 2003. For the remaining parameters/autonomous components/exogenous variables, use the same values as in part (a). Does the model account for the observed increase in the price level between 1983 and 2003? How much of the observed increase in the price level is accounted for by the higher nominal wages? How much of the observed increase in the price level is accounted for by the higher money supply? Comment.

Question 2: Utility Maximization (50 Marks)

A consumer has preferences over two commodities x_1 and x_2. These preferences are represented by the following utility function:

$$U(x_1, x_2) = [\alpha x_1^\gamma + (1-\alpha)x_2^\gamma]^{\frac{1}{\gamma}}$$

α and γ are parameters, and are such that $0 < \alpha < 1$ and $-\infty < \gamma < 1$. The consumer wants to maximize utility by choosing the optimal values x_1^* and x_2^* , subject to the budget constraint:

$$p_1 x_1 + p_2 x_2 = I$$

where $p_i > 0$ denotes the price of good x_i and I denotes income.

(a) To start with, set the parameters to their benchmark values: $\alpha = 0.5, \gamma = -1, p_1 = 1, p_2 = 1, I = 10$.
Note: you may want to substitute the budget constraint into the objective function.
Write a code in Python that maximizes the consumer's utility, by using the golden search algorithm. Report your results and comment on your findings. Plot the objective function in a two-dimensional graph.

(b) Now consider the effect of changing some parameter values.
In one case they are:

$$\alpha = 0.5, \gamma = -5, p_1 = 1, p_2 = 1, I = 10.$$

In another case they are:

$$\alpha = 0.25, \gamma = -1, p_1 = 1, p_2 = 1, I = 10.$$

In a final case they are:

$$\alpha = 0.5, \gamma = -1, p_1 = 1, p_2 = 2, I = 20.$$

Report your results in a table and comment on your findings.

(c) Now the consumer's preferences are over three commodities x_1, x_2, and x_3. The utility function is:

$$U(x_1, x_2, x_3) = [\alpha x_1^\gamma + \beta x_2^\gamma + (1-\alpha-\beta)x_3^\gamma]^{\frac{1}{\gamma}}$$

α, β and γ are parameters, and are such that $0 < \alpha < 1$, $0 < \beta < 1$ and $-\infty < \gamma < 1$. The consumer wants to maximize utility by choosing the optimal values x_1^*, x_2^* and x_3^* , subject to the budget constraint:

$$p_1 x_1 + p_2 x_2 + p_3 x_3 = I$$

The parameters have these values: $\alpha = 0.45, \beta = 0.35, \gamma = -1, p_1 = 2, p_2 = 3, p_3 = 4, I = 10$.
Note: you may want to substitute the budget constraint into the objective function.
Write a code in Python that maximizes the consumer's utility, using a multivariate algorithm of your choice. Report your results and comment on your findings. Plot the objective function both in a three-dimensional graph and with a contour plot.