

Question 2

```
In [2]: #importing required libraries
from sympy import *
import matplotlib.pyplot as plt
%config InlineBackend.figure_format='retina'
%matplotlib inline
init_printing(use_latex = True)
from matplotlib import rcParams
import pandas as pd
import pandas_datareader as pdr
import datetime
import numpy as np
plt.style.use('fivethirtyeight')
#Setting font style and size
rcParams['font.family'] = 'serif'
rcParams['font.size'] = 16
import math
```

```
In [3]: #Question 1
#importing data with pandas
filename = 'CA_data.txt'
df = pd.read_csv(filename,delim_whitespace = True,parse_dates = True,index_col
```

```
In [4]: #checking that the import was done correctly

df.head()
```

```
Out[4]:
```

	C	G	I
Quarter			
1981-01-01	109859750000	56521500000	43427750000
1981-04-01	109707500000	56411000000	44640750000
1981-07-01	109094500000	57100000000	43572500000
1981-10-01	109304250000	57255750000	43120750000
1982-01-01	107629750000	56774000000	41716750000

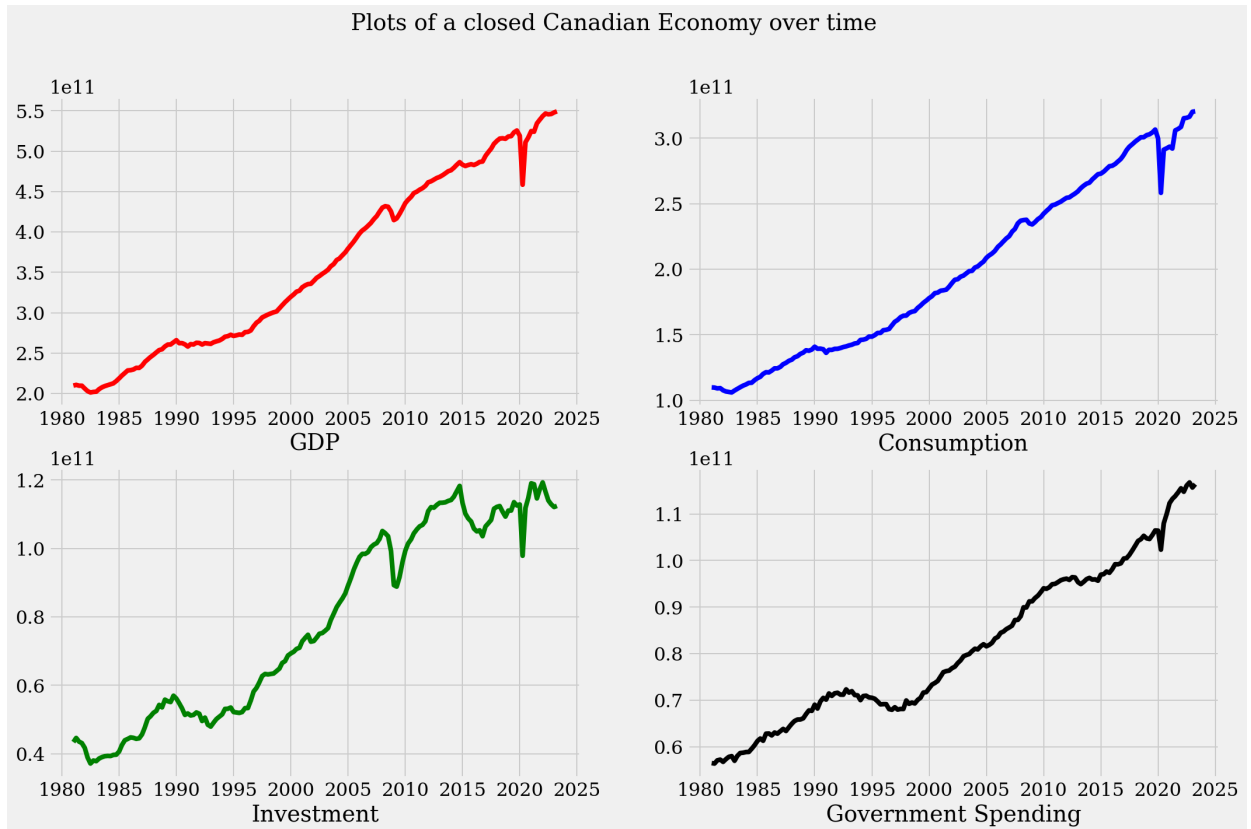
Part a.

```
In [5]: #Question 2 part a
x = df.index
C = df['C']
I = df['I']
G = df['G']
Y = C + I + G

fig,ax = plt.subplots(2,2,sharey = False,sharex = False, figsize = (16,10))
fig.suptitle("Plots of a closed Canadian Economy over time")
ax[0,0].plot(x,Y,color = 'red',linestyle = '-',linewidth = 4)
ax[0,1].plot(x,C,color = 'blue',linestyle = '-',linewidth = 4)
```

```
ax[1,0].plot(x,I,color = 'green',linestyle = '-',linewidth = 4)
ax[1,1].plot(x,G,color = 'black',linestyle = '-',linewidth = 4)
ax[0,0].set_xlabel('GDP')
ax[0,1].set_xlabel('Consumption')
ax[1,0].set_xlabel('Investment')
ax[1,1].set_xlabel('Government Spending')
```

Out[5]: Text(0.5, 0, 'Government Spending')

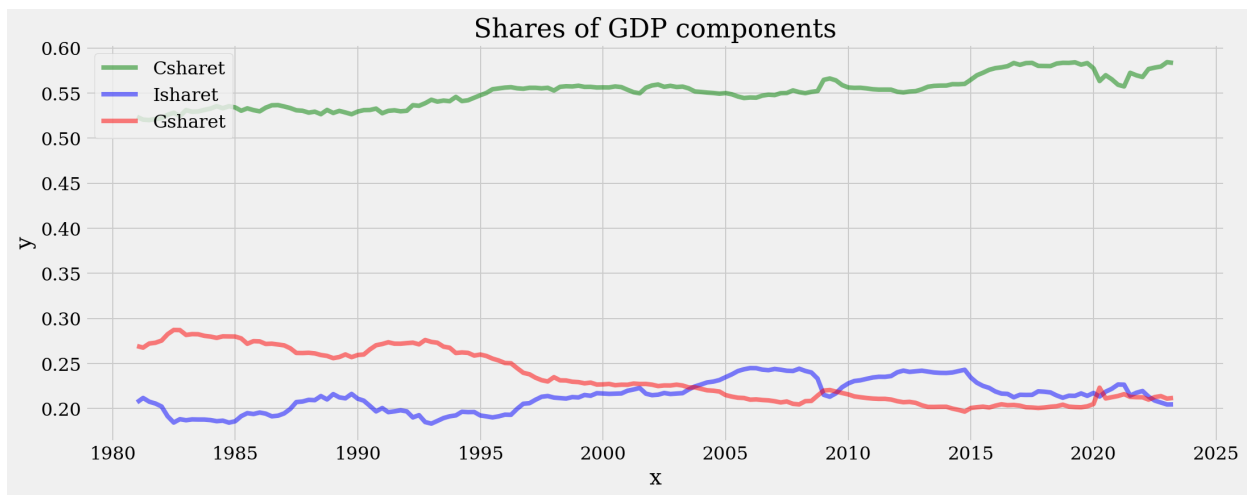


Part b.

```
In [89]: #Question 2 part b
x = df.index
Share_C = C/Y
Share_I = I/Y
Share_G = G/Y

fig = plt.figure(figsize=(16,6))
plt.plot(x, Share_C, color='g', linestyle='-', linewidth=4, alpha=0.5, label =
plt.plot(x, Share_I, color='b', linestyle='-', linewidth=4, alpha=0.5, label =
plt.plot(x, Share_G, color='r', linestyle='-', linewidth=4, alpha=0.5, label =
plt.legend(loc='upper left')
plt.title('Shares of GDP components')
plt.xlabel('x')
plt.ylabel('y')
```

Out[89]: Text(0, 0.5, 'y')



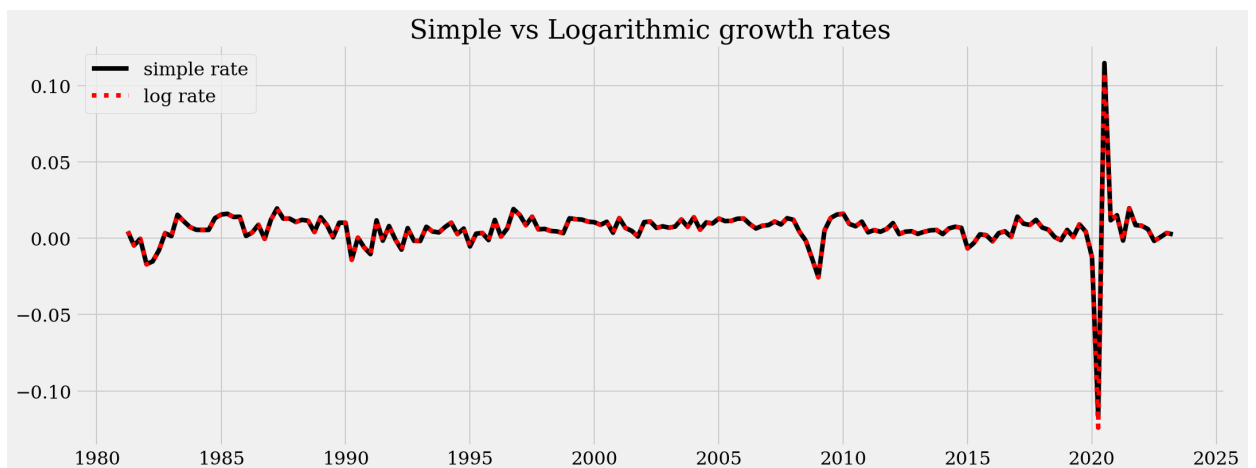
-Consumption is consistently the largest share of GDP components -Government spending has seemed to trend downwards, rising briefly in 2020 presumably due to COVID-19. -Investment makes the smallest component, with an upward trend until 2020

Part c

```
In [28]: simple_gdp_growth_rate = Y.pct_change()
log_growth_rate = np.log(Y/Y.shift(1))
#log_growth_rate = [np.log(Y[i-2]) - np.log(Y[i-1])] for i in range(Y.size)]
```

```
In [33]: x = df.index
fig = plt.figure(figsize=(16,6))
plt.plot(x, simple_gdp_growth_rate, color = 'black', linestyle = 'solid', linewidth=1)
plt.plot(x, log_growth_rate, color = 'red', linestyle = 'dotted', linewidth = 4, label = 'log rate')
plt.title("Simple vs Logarithmic growth rates")
plt.legend()
```

```
Out[33]: <matplotlib.legend.Legend at 0x7f8103ab7e20>
```



The simple and log growth rates are approximately equal with a noticeably larger difference in log rates during 2020.

```
In [9]: s = simple_gdp_growth_rate
l = log_growth_rate
```

```
percentage_deviation = ((s - l) / ((s + l) / 2)) * 100
percentage_deviation.head()
```

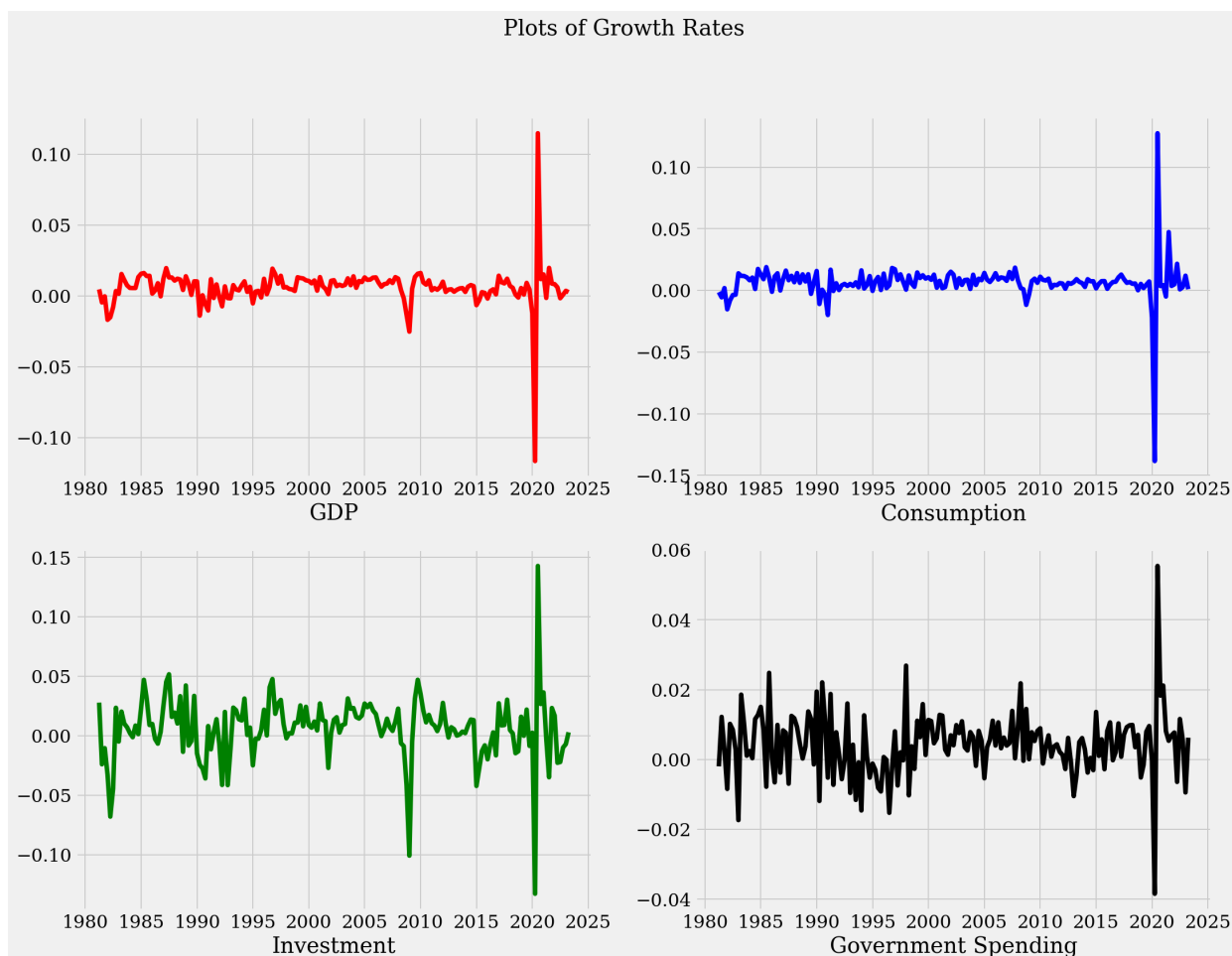
```
Out[9]: Quarter
1981-01-01      NaN
1981-04-01      0.226030
1981-07-01     -0.235862
1981-10-01     -0.020562
1982-01-01     -0.855032
dtype: float64
```

Part d

```
In [10]: gdp_growth_rate = Y.pct_change()
consumption_growth_rate = C.pct_change()
investment_growth_rate = I.pct_change()
fiscal_spending_growth_rate = G.pct_change()
x = df.index

fig,ax = plt.subplots(2,2,sharey = False,sharex = False, figsize = (16,12))
fig.suptitle("Plots of Growth Rates")
ax[0,0].plot(x,gdp_growth_rate,color = 'red',linestyle = '-',linewidth = 4)
ax[0,1].plot(x,consumption_growth_rate,color = 'blue',linestyle = '-',linewidth = 4)
ax[1,0].plot(x,investment_growth_rate,color = 'green',linestyle = '-',linewidth = 4)
ax[1,1].plot(x,fiscal_spending_growth_rate,color = 'black',linestyle = '-',linewidth = 4)
ax[0,0].set_xlabel('GDP')
ax[0,1].set_xlabel('Consumption')
ax[1,0].set_xlabel('Investment')
ax[1,1].set_xlabel('Government Spending')
```

```
Out[10]: Text(0.5, 0, 'Government Spending')
```



Part e

```
In [11]: growth_rates = pd.DataFrame({'GDP':gdp_growth_rate,'Consumption':consumption_g
                                     'Investment':investment_growth_rate,'Government Spei
growth_rates.describe()
```

```
Out[11]:
```

	GDP	Consumption	Investment	Government Spending
count	169.000000	169.000000	169.000000	169.000000
mean	0.005817	0.006491	0.005989	0.004328
std	0.014488	0.016309	0.026209	0.009413
min	-0.116689	-0.138411	-0.132777	-0.038433
25%	0.002754	0.002707	-0.002651	-0.000358
50%	0.006416	0.006864	0.008631	0.004368
75%	0.010967	0.010640	0.020386	0.009594
max	0.114709	0.127537	0.142823	0.055440

```
In [12]: growth_rates.corr()
```

Out[12]:

	GDP	Consumption	Investment	Government Spending
GDP	1.000000	0.942299	0.811308	0.576212
Consumption	0.942299	1.000000	0.605532	0.487783
Investment	0.811308	0.605532	1.000000	0.281520
Government Spending	0.576212	0.487783	0.281520	1.000000

Consumption has the highest mean growth rate, and the highest correlation to GDP, followed by investment, and government spending having the least of the three.

Question 1

Part b.

```
In [60]: #alpha = 0.5, epsilon = -2.0
#demand function = 0.5p**-2
#deltap = -1*p**-3
p = 0.5
for i in range(100):

    f = 0.5 * p**-2 - 2 * p**0.5
    d = -1 * p**-3 - p**-0.5
    deltap = -f/d

    p = p + deltap
    print(p)

    if abs(deltap) < 1.e-8:
        break
print(f"Computed in {i} iterations")
```

```
0.5622236189720814
0.5740285524734964
0.5743489537536681
0.5743491774984085
0.5743491774985175
Computed in 4 iterations
```

part c.

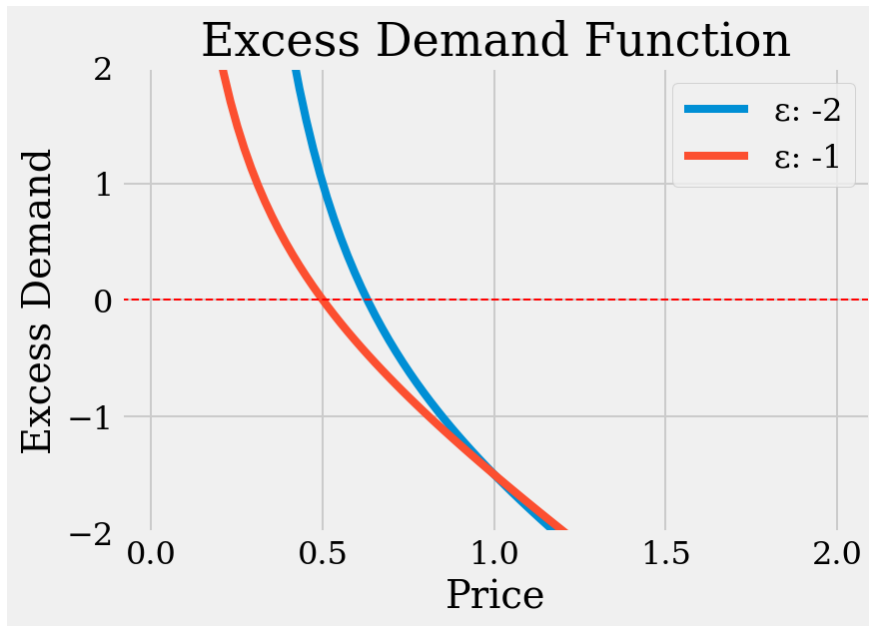
```
In [77]: p = np.linspace(0.01,2,100)
elasticities = [-2,-1]

def Excess_Demand(p,elas):
    return 0.5*p**elas - 2*p
for elasticity in elasticities:
    plt.plot(p,Excess_Demand(p,elasticity),label = f"ε: {elasticity}")
```

```
plt.xlabel('Price')
plt.ylabel('Excess Demand')
plt.title('Excess Demand Function')
plt.axhline(y=0, color='r', linestyle='--', linewidth=1)

plt.legend()
plt.ylim(-2,2)
```

Out[77]: $\left(-2.0, 2.0 \right)$



At an excess demand of 0, which would be the equilibrium level of demand and supply, the equilibrium price is exactly 0.5 at an ϵ of -1 and approximately 0.6 at an ϵ of -2, which is consistent with the analytical answers

part d.

```
In [79]: def excess_demand(p):
          return np.exp(-2*p) - 0.01*p - p**2

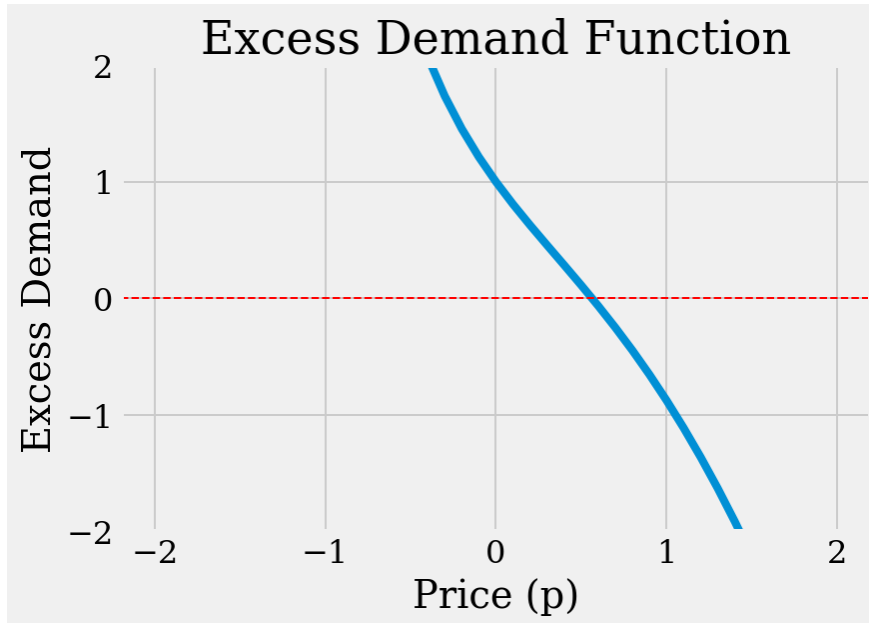
start = -2
step = 0.1
end = 2

p_values = np.arange(start, end+step, step)
excess_demand_list = excess_demand(p_values)
plt.plot(p_values, excess_demand_list, label='Excess Demand')
plt.xlabel('Price (p)')
plt.ylabel('Excess Demand')
plt.title('Excess Demand Function')
plt.axhline(y=0, color='r', linestyle='--', linewidth=1)

plt.ylim(-2,2)

plt.grid(True)
```

```
plt.show()
```



Because of the power of the exponent, excess demand will decrease asymptotically for higher prices p . The equilibrium at Excess Demand = 0 corresponds to a price of approximately 0.5.

```
In [87]: p = 0.0
for i in range(100):

    f=np.exp(-2*p) - 0.01*p- p**2
    d=-2*np.exp(-2*p) - 0.01 - p*2
    deltap = -f/d

    p = p + deltap
    print(p)

    if abs(deltap) < 1.e-8:
        break
print(f"Price = {p},computed in {i} iterations")

0.49751243781094534
0.564708557449029
0.5639687247565134
0.5639686164601598
0.5639686164601576
Price = 0.5639686164601576,computed in 4 iterations
```

part e.

```
In [81]: def excess_demand(p):
    return np.exp(-p) - 0.01*p- p**4
```



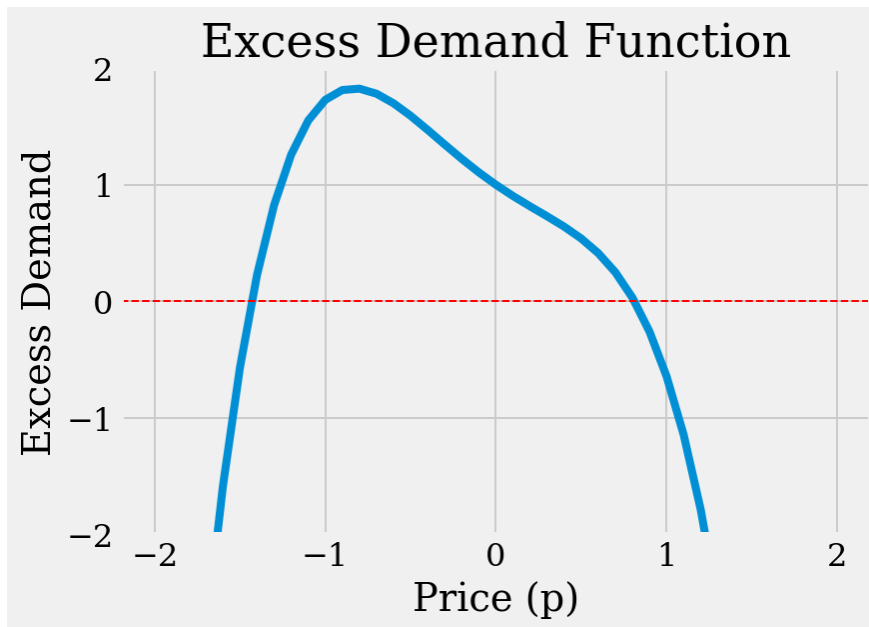
```

start = -2
step = 0.1
end = 2

p_values = np.arange(start, end+step, step)
excess_demand_list = excess_demand(p_values)
plt.plot(p_values, excess_demand_list, label='Excess Demand')
plt.xlabel('Price (p)')
plt.ylabel('Excess Demand')
plt.title('Excess Demand Function')
plt.axhline(y=0, color='r', linestyle='--', linewidth=1)
plt.ylim(-2,2)

plt.grid(True)
plt.show()

```



Due to the power and symbol of the exponents for e^p and p^4 , function will curve downwards. Since prices cannot be negative, the equilibrium price will be approximately 0.8

```

In [88]: p = 0
         for i in range(100):

             f=np.exp(-p) - 0.01*p- p**4
             d= (-np.exp(-p)) - 0.01- 4*p**3
             deltap = f/d

             p = p - deltap
             print(p)

             if abs(deltap) < 1.e-8:
                 break
         print(f"Price = {p},computed in {i} iterations")

```

```
0.9900990099009901
0.849537232663613
0.814329837165257
0.8124344026543568
0.8124292170425147
0.8124292170038313
Price = 0.8124292170038313, computed in 5 iterations
```

In []:

```
In [78]: p = 0.3
for i in range(100):

    f=0.5*p**2 - 2*p**1
    d=-1 * p **3 - 2
    deltap = -f/d

    p = p + deltap
    print(p)

    if abs(deltap) < 1.e-8:
        break
print(f"Computed in {i} iterations")
```

```
0.42694497153700195
0.5541627107526501
0.6201635563767954
0.6298065982821767
0.6299604873303536
0.6299605249474344
0.6299605249474366
Computed in 6 iterations
```

In []: