MTH 180 , exam 3

Name : Mani Subramani

I certify that all work on this exam is my own: S.Manikandan

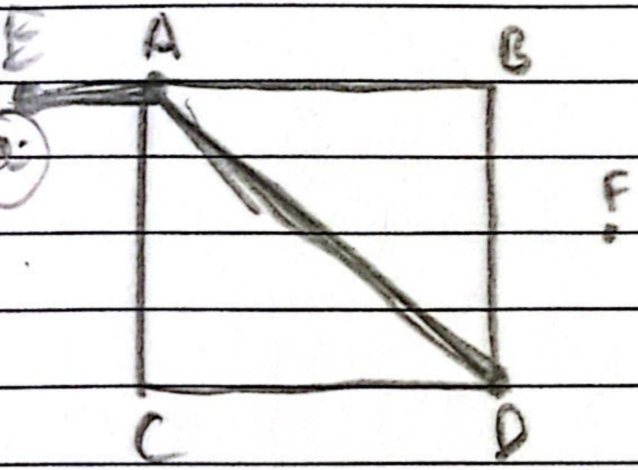
1.) a. $deg(v_1) = 3$, $deg(v_2) = 5$, $deg(v_3) = 5$, $deg(v_4) = 4$, $deg(v_5) = 3$

total deg $= 3+5+5+4+3 = \boxed{20}$

b. Yes, the graph is connected as there is a connecting path between each pair of vertices.

c. No, there is no Euler circuit as $d(v_1)$ is an odd vertex. For there to be an Euler circuit, each vertex must be even.

2.) (a)



deg(A) = 4
deg(B) = 2
deg(C) = 2
deg(D) = 3
deg(E) = 1
deg(F) = 0

(b) It would be impossible as a simple graph cannot have parallel edges or loops, and each vertex can only be joined by 4 other vertices and not 5 in this case, as the total available vertices here are just 5.

3.) $A = $ "aba"
$B = $ "ccd"

$$n(A) + n(B) - n(A \cap B) = n(A \cup B)$$

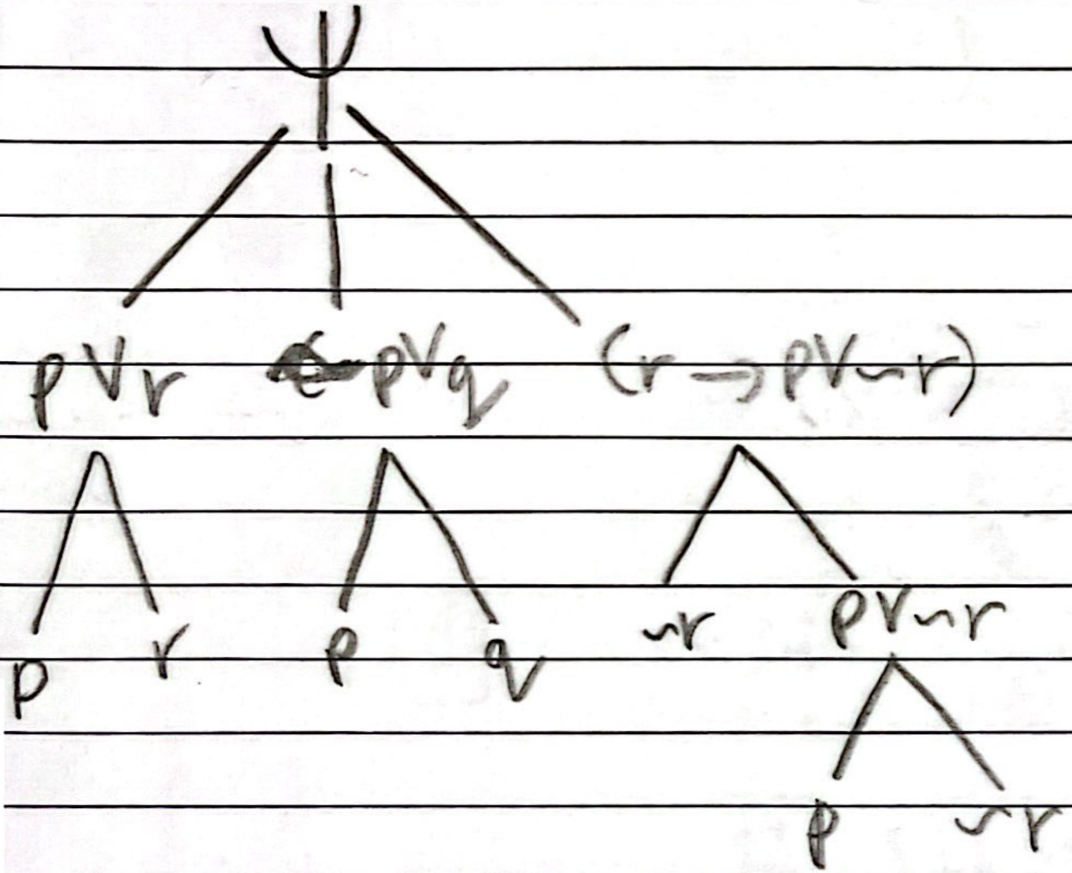
$n(A) = n(B) = 4^6$, $n(A \cap B) = 4^3$

$n(A \cup B) = 4^6 + 4^6 - 4^3 = 4096 + 4096 - 64 = 8192 - 64 = \boxed{8128}$

$\therefore$ There are 8128 such item strings

4.) truth table :

p	q	r	$(p \vee r)$	$(\sim p \vee q)$	$(r \rightarrow p \vee \sim r)$	Ψ
T	T	T	T	T	T	T
T	T	F	T	T	F	F
T	F	T	T	F	T	F
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	T	F	F	T	T	F
F	F	T	T	T	F	F
F	F	F	F	T	T	T



this tree shows Ψ is satisfiable

5.) (a) No, as one of the elements 'b' is repeated in this case. there needs to be 4 different elements in the set for it to be considered a "4-permutation".

(b) number of elements 'n' in S = 6

$$6P4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2} = \frac{\cancel{720}^{360}}{\cancel{2}} = \boxed{360}$$

$\boxed{\therefore \text{ there are 360 different 4 permutations in S}}$

(c) $$6C4 = \frac{6!}{4!(6-4)!} = \frac{720}{4!(2!)} = \frac{720}{4\cdot3\cdot2\cdot1} = \frac{\cancel{720}^{15}}{\cancel{24}} = 15$$

$\boxed{\therefore \text{ there are 15 different 4 combinations in S}}$

Scanned with CamScanner

6) (a) $C^8_5 \cdot C^7_3 = \frac{8(7)(6)}{3(2)(1)} \cdot \frac{7(6)(5)}{3(2)(1)} = 56 \cdot 35 = \boxed{1960}$

(b) 6 women:

$$\left.\begin{array}{l} C^8_6 = \frac{8(7)}{2(1)} = \frac{56}{2} = 28 \\ C^7_2 = \frac{7(6)}{2(1)} = \frac{42}{2} = 21 \end{array}\right\} 588$$

$+$

$$\left.\begin{array}{l} C^8_7 = \frac{8}{1} = 8 \\ C^7_1 = \frac{7(1)}{1} = 7 \end{array}\right\} 56$$

$+$

$$\left.\begin{array}{l} C^8_8 = 1 \\ C^7_0 = 1 \end{array}\right\} 1$$

$588+56+1 = 645$

6 men:

$$\left.\begin{array}{l} C^8_2 = 28 \\ C^7_6 = 7 \end{array}\right\} 196$$

$$\left.\begin{array}{l} C^8_1 = 8 \\ C^7_7 = 1 \end{array}\right\} 8$$

$196+8 = 204$

$= 645+204 = \boxed{849}$

Scanned with CamScanner

a) position 1 = 0 $\therefore 2^{7-2} = 2^5 =$ $\boxed{\text{32 possible strings}}$

position 7 = 0

2 choices for remaining 5 positions

b) 2 choices for remaining 3 positions = $\boxed{2^3 = 8 \text{ possible strings}}$

8-) a. $\frac{7!}{4!(7-4!)} = \frac{7(6)(5)}{3(2)(1)} = \boxed{35}$

b. $\binom{7}{0} = 1$ $\binom{7}{3} = \frac{7(6)(5)}{3(2)(1)} = 35$

$\binom{7}{1} = \frac{7}{1} = 7$ $\binom{7}{4} = 35$

$\binom{7}{2} = \frac{7(6)}{2(1)}$ $1+7+21+35+35 = \boxed{99}$

c. Bit strings with no 1's $= \binom{7}{0} = 1$

Bit strings with one 1 $= \binom{7}{1} = 7$

complement $= 1+7 = 8$

$\therefore 2^7 - 8 =$ $\boxed{\text{120 bit strings}}$