2.1] A discrete time signal $x(n)$ is defined as

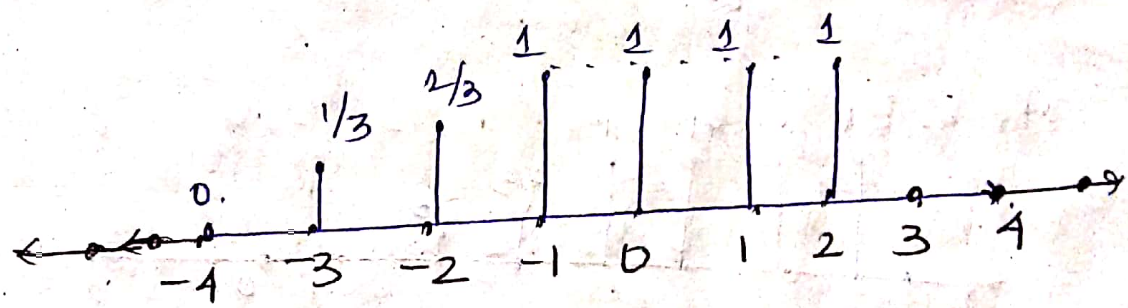
$$x(n) = \begin{cases} 1+\frac{n}{3}; & -3 \le n \le -1 \\ 1; & 0 \le n \le 3 \\ 0; & \text{elsewhere} \end{cases}$$

a] Determine its values and sketch signal $x(n)$

$$x(n) = \{\ldots\ldots 0, 0, \tfrac{1}{3}, \tfrac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1; 0 \ldots\}$$

$n =$	-3	-2	-1	0	1	2	3
$x(n) =$	$1-\frac{3}{3}$	$1-\frac{2}{3}$	$1-\frac{1}{3}$	1	1	1	1
$\Rightarrow$	0	1/3	2/3	1	1	1	1

Sketch of $x[n]$



b]. Sketch the signals that result if we:

1. First fold $x(n)$ and then delay the resulting signal by four samples.

$x(-n) = \{\ldots 0, 1, 1, 1, 1, 2/3, 1/3, 0, 0, \ldots\}$
$\uparrow$

delay by 1

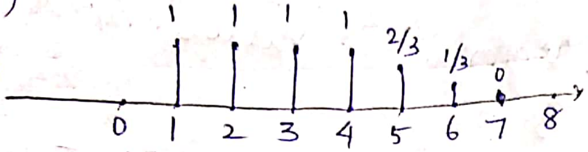
$x(-n+4) = \{\ldots 0, 0, 0, 0, \underset{\uparrow}{0}, 1, 1, 1, 1, 2/3, 1/3, 0, 0\}$

$\downarrow$

ie

$x(-(n-4))$

$\Rightarrow x(-n+4)$



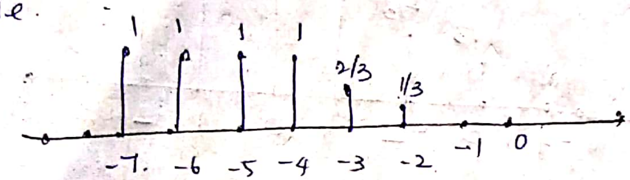
2]. First delay $x(n)$ by four samples and then fold the resulting signal.

$x(n) \xrightarrow{\text{delay by 4}} x(n-4) \xrightarrow{\text{fold}} x(-n+4)$.

$x(n-4) \Rightarrow \{\ldots \underset{\uparrow}{0}, 0, 1/3, 2/3, 1, 1, 1, 1, 2/3, 0, 0 \ldots\}$

$x(-n-4) \Rightarrow \{\ldots 1, 1, 1, 1, 2/3, 1/3, \underset{\uparrow}{0}, 0 \ldots\}$

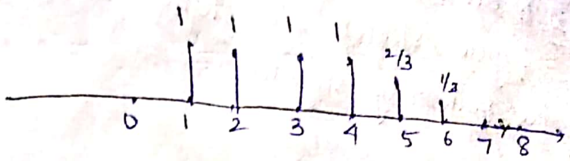
ie.



c] Sketch the signal $x(-n+4)$

$x(-n+4) = \{\ldots, \underset{\uparrow}{0}, 1, 1, 1, 1, 2/3, 1/3, 0, \ldots\}$

$x(-n+4)$.



d] Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$

Method 1

$x(n) \to x(n+k) \to x(-n+k)$

shift [left] by k [if +ve]
[right] if k [-ve]
Then flip the resultant

Method 2.

$x(n) \to x(-n) \to x(-(n-k))$
$\to x(-n+k)$

First flip the signal
perform the reverse
operation of shifting
ie - k

We mostly prefer Method 1 many cases.

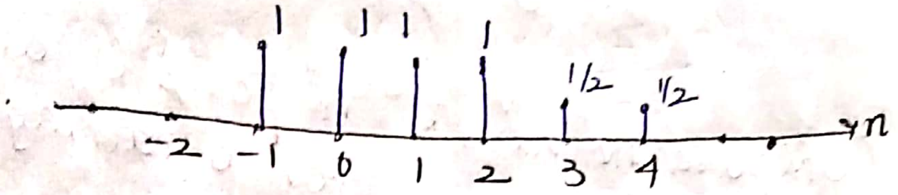
doubt

e] Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$.

Yes, we can express $x(n)$ in terms of $\delta(n)$ and $u(n)$.

$x[n] = \frac{1}{3}\delta(n+2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$.

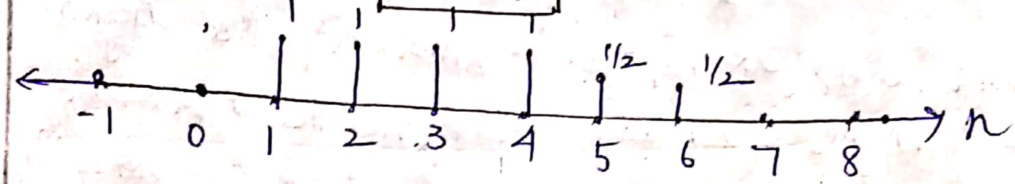
2.2] A discrete-time-signal $x(n)$ is shown in the figure. Sketch and label carefully each of the following signals.



(a) $x(n-2)$. $\Rightarrow$ $x(n) = \{0, 1, \underset{\uparrow}{1}, 1, 1, 1/2, 1/2, 0 \ldots\}$

$x(n-2) = \{\ldots \underset{\uparrow}{0}, 1, 1, 1, 1, 1/2, 1/2, 0 \ldots\}$

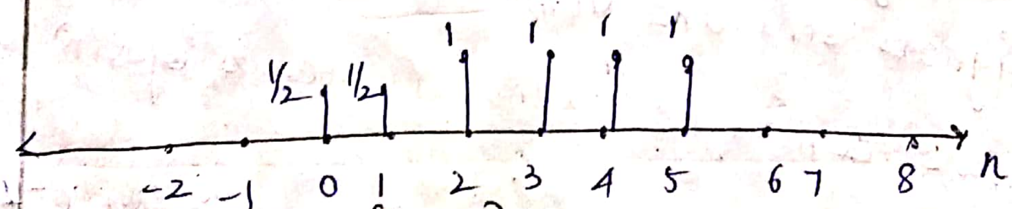
$\boxed{x(n-2)}$



(b) $x(4-n) = x(-n+4) \Rightarrow x(n+4) \xrightarrow{\text{flip}} x(-n+4)$

$x(n+4) = \{\ldots 0, 1, 1, 1, 1, 1/2, \underset{\uparrow}{1/2}, 0 \ldots\}$

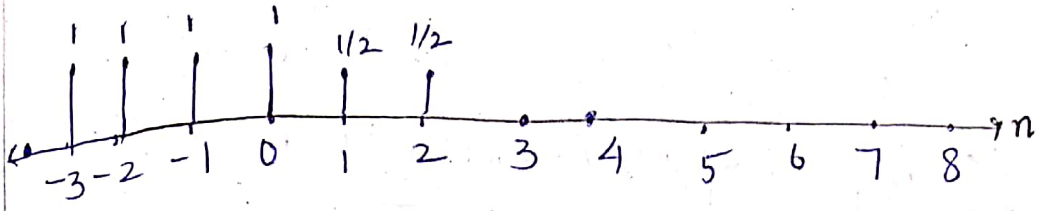
$x(-n+4) = \{\ldots 0, \underset{\uparrow}{1/2}, 1/2, 1, 1, 1, 1, 0, \ldots\}$



$x(-n+4)$

(c) $x(n+2) = \{\ldots\ldots 0, 1, 1, 1, \underset{\uparrow}{1}, 1/2, 1/2, 0, \ldots\}$

$\boxed{x(n+2)}$



(d) $x(n)\,u(2-n)$.

$u(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, \ldots\}$

$u(n+2) = \{\ldots 0 \ldots 1, \ldots 1, \underset{\uparrow}{1}, 1, 1, \ldots\}$

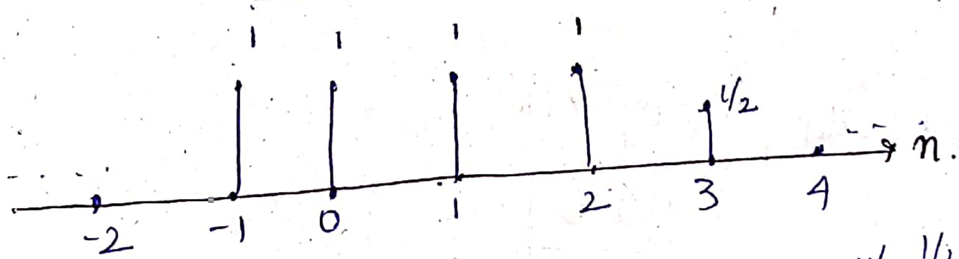
$u(-n+2) = \{\ldots 1\ 1, 1, \underset{\uparrow}{1}, 1, 1, 0 \ldots\}$

$x(n) = \{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, 1/2, 1/2, 0 \ldots\}$

$x(n-1)\delta(n-3)$

$x(n)u(2-n) = \{\ldots 0, 1, \underset{\uparrow}{1}, 1, 1, 1/2, 0 \ldots\}$

$\boxed{x(n)u(2-n)}$



(e). $x(n-1)\,\delta(n-3)$. $\Rightarrow$ $x(n) = \{\ldots 0, 0, 1, \underset{\uparrow}{1}, 1, 1, 1/2, 1/2, 0 \ldots\}$

$x(n-1) \Rightarrow \{\ldots 0, \underset{\uparrow}{1}, 1, \underset{\uparrow}{1}, 1, 1/2, 1/2, 0, \ldots\} = y(n)$

$\therefore\ \cancel{x}y(n).\,\delta(n-3) = y(3)\,\delta(n-3)$

$= 1\,\cancel{x_2}\,\delta(n-3)$

ie. . . . $\{\underset{\uparrow}{0}\ 0., 0, 1, 0, \ldots\}$

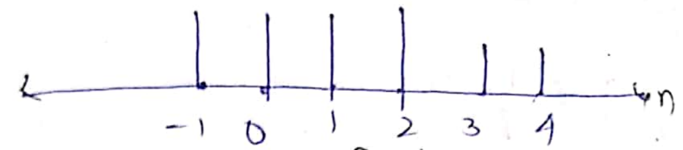
(f) $x(n^2) \Rightarrow \{\ldots\ldots 0, x(4), x(1), x(0), x(1), x(4), x(9), x(16)\}$

$= \{\ldots 0, 1/2, 1, \underset{\uparrow}{1}, 1, 1/2, 0 \ldots\}$

(g) even part of $x(n) = \dfrac{x(n)+x(-n)}{2}$

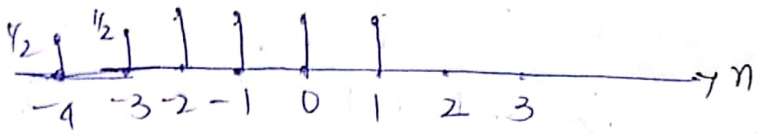
$x(n)$

(a)

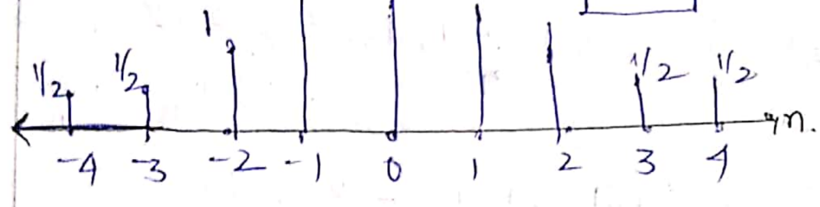


$x(-n)$ +

(b)



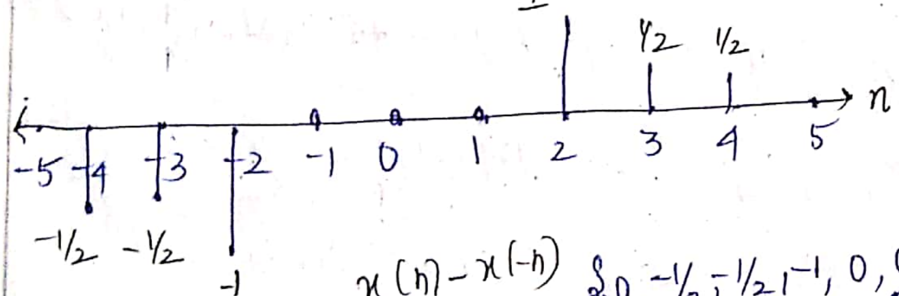
$\boxed{a+b}$



$\dfrac{x(n)+x(-n)}{2} = \{\ldots 0, 1/4, 1/4, 1/2, 1, \underset{\uparrow}{1}, 1, 1/2, 1/4, 1/4, 0 \ldots\}$

(h) Odd part of $x(n)$.

$\boxed{a-b}$



$x_o(n)$ $\dfrac{x(n)-x(-n)}{2} = \{0, -1/2, -1/2, -1, 0, \underset{\uparrow}{0}, 0, 1, 1/2, 1/2, 0 \ldots\}$

2.3] Show that (a) $\delta(n) = u(n) - u(n-1)$

Sol:- $u(n) - u(n-1) = \{\ldots 0, \underset{\uparrow}{1}, 0, \ldots\}$

$= \delta(n).$

(b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k).$

$\sum_{k=-\infty}^{n} \delta(k) = \delta(-k) + \ldots + \delta(n)$

① $\rightarrow$ $u(n) \begin{cases} = 1 & \text{if } n \geq 0 \\ = 0 & \text{if } n < 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) \Rightarrow \delta(n) + \delta(n-1) + \ldots + \delta(n-\infty)$

② $\rightarrow$ $u(n) = \begin{cases} = 1 & \text{if } n \geq 0 \\ = 0 & \text{if } n \leq 0 \end{cases}$

Reason:- $0, \infty \Rightarrow$ all positive number..

If $n - k = 0 \Rightarrow \delta(0) = 1;$

hence 'n' must be a positive integer.

Hence from ① and ②;

$u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k).$

2.4] Show that any signal can be decomposed into an even and odd component. Is the decomposition unique? Illustrate your arguments using an example.

$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$

$$x_e(n) = \frac{x(n)+x(-n)}{2} \Rightarrow$$

$$x(-n) = \{6, 5, \underset{\uparrow}{4}, 3, 2\} \; ; \; x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

$$* \; x_e(n) = \frac{x(n)+x(-n)}{2} = \frac{\{8, 8, 8, 8, 8\}}{2}$$

$$= \{4, 4, \underset{\uparrow}{4}, 4, 4\}$$

$$* \; x_o(n) = \frac{x(n)-x(-n)}{2} = \frac{\{6, 5, \underset{\uparrow}{4}, 3, 2\} - \{2, 3, \underset{\uparrow}{4}, 5, 6\}}{2}$$

$$= \frac{\{4, 2, 0, 2, 4\}}{2}$$

$$= \{2, 1, \underset{\uparrow}{0}, 1, 2\}$$

Hence the decomposition is always unique

2.5 Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energy (powers) of its even and odd components.

$x_1(n)^2 + x(-n)^2$

Ans Method 1:- $x_e(n) = \frac{x(n)+x(-n)}{2}$; $\frac{x(n)+x(-n)}{2}$

$$x_o(n) = \frac{x(n)-x(-n)}{2} \quad x_o(n) = \frac{x(n)-x(-n)}{2}$$

$$\sum_{n=-\infty}^{\infty} x_e(n)^2 = \sum_{n=-\infty}^{\infty} \left(\frac{x(n)+x(-n)}{2}\right)^2 \quad \frac{x^2(n) + x^2(-n)}{4} + x$$

$$= \frac{1}{4}\left[\sum_{n=-\infty}^{\infty} x(n)^2 + \sum_{n=-\infty}^{\infty} x(-n)^2 + \sum_{n=-\infty}^{\infty} 2x(n)x(-n)\right] \rightarrow (1)$$

Similarly $\sum_{n=-\alpha}^{\alpha} x_o^2(n) = \sum_{n=-\alpha}^{\alpha} \frac{1}{4}\left[x^2(n)+x^2(-n) - 2x(n)x(-n)\right]$ → ②

Adding ① and ②

$$\sum_{n=-\alpha}^{\alpha}\left(x_e^2(n)+x_o^2(n)\right) = \frac{1}{4}\sum_{n=-\alpha}^{\alpha} 2x^2(n)+2x^2(-n)$$

as $\boxed{x^2(n) = x^2(-n)}$

$$= \sum_{n=-\alpha}^{\alpha} x(n)^2$$

$$\therefore \boxed{\sum_{n=-\alpha}^{\alpha} x^2(n) = \sum_{n=-\alpha}^{\alpha} x_e^2(n) + \sum_{n=-\alpha}^{\alpha} x_o^2(n)}$$

Hence it is proved that energy of signal is equal to the sum of energies of its even and odd components.

Method 2:- We first prove that

$$\sum_{n=-\alpha}^{\alpha} x_e(n)\,x_o(n) = 0;$$

$$\sum_{n=-\alpha}^{\alpha} x_e(n)\,x_o(n) = \sum_{m=-\alpha}^{\alpha} x_e(-m)\,x_o(-m).$$

Note Eg:- ~~Let $x_e(n) = \{1, 0, 1\}$; $x_o(n) = \{-1, 0, 1\}$~~ ⇒ Sum = 0

~~$x_e(n)\,x_o(n) = \{-1, 0, 1\}$~~

$\{1, 0, 1\}$
$\{1, 0, -1\}$

Similarly

$x_e(-n)\,x_o(-n) = \{1, 0, -1\}$

sum = 1 − 1 = 0. Hence equal //

$$= - \sum_{m=-\infty}^{\infty} x_e(m) x_o(m)$$

$$\sum_{n=-\infty}^{\infty} x_e(n) x_o(n) = - \sum_{n=-\infty}^{\infty} x_e(n) x_o(n)$$

which is violating condition; hence

$$\sum_{n=-\infty}^{\infty} x_e(n) x_o(n) = 0 \text{ W.}$$

Then $\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$

$$= \sum_{n=-\infty}^{\infty} [x_e^2(n) + x_o^2(n) + 2 x_e[n] x_o[n]]$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2 \sum_{n=-\infty}^{\infty} x_e(n) x_o(n)$$

$$\boxed{\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n)}$$

2.6] Consider the system $y(n) = T[x(n)] = x(n^2)$.

(a) Determine if the system is time invariant.

Sol:- System is time variant

$$x(n) \longrightarrow y(n) = x(n^2)$$

$$x(n-k) \rightarrow x((n-k)^2) \neq y(n-k) = x(n^2-k)$$

Hence the system is time variant.

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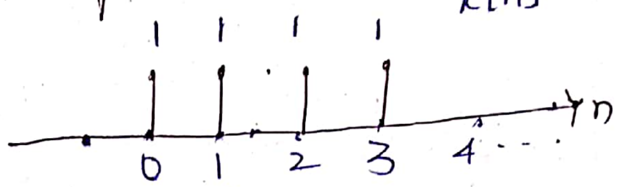
(b) To clarify the result in part (a) assume that the signal $x(n) = \begin{cases} 1; & 0 \le n \le 3 \\ 0; & \text{elsewhere.} \end{cases}$

is applied to the system.

1] Sketch the spectrum of signal $x(n)$.

Sol:-



$x[n]\ \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$

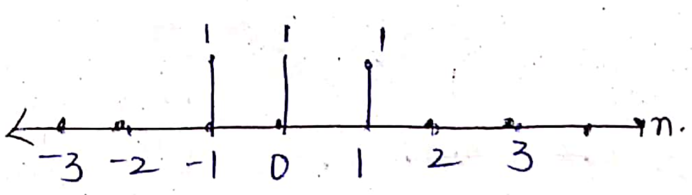
2] Determine and sketch the signal $y(n) = T[x(n)]$

$x[n^2] \Rightarrow y(n) = \{\ldots\ y(0), y(1), y(2) \ldots\}$

$= \{\ldots, x(1), x(0), x(1), x(4) \ldots\}$

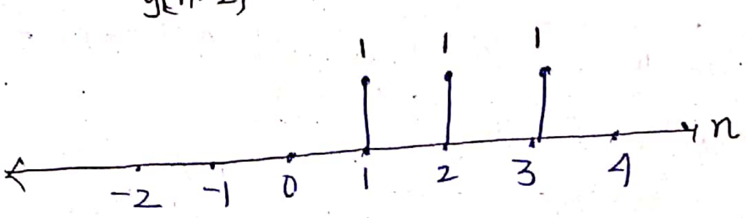
$X = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 0, 0, \ldots\}$

$y(n) = x[n^2] \quad = \quad \{\ldots 0, 1, \underset{\uparrow}{1}, 1, 0 \ldots\}$



3] Sketch the signal $y_2(n) = y(n-2)$

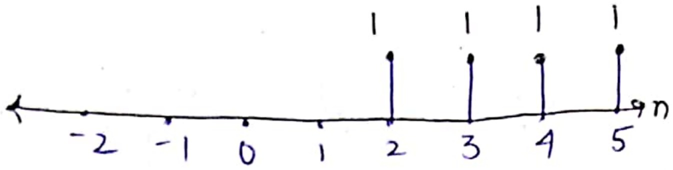
$y(n-2) = \{\ldots \underset{\uparrow}{0}, 1, 1, 1, 0 \ldots\}$



4] Sketch the signal $x_2(n) = x[n-2]$

$$x(n-2) = \{\ldots 0, 0, 1, 1, 1, 1, 0 \ldots\}$$
$$\uparrow$$

$x[n-2]$



5] Determine and sketch the signal

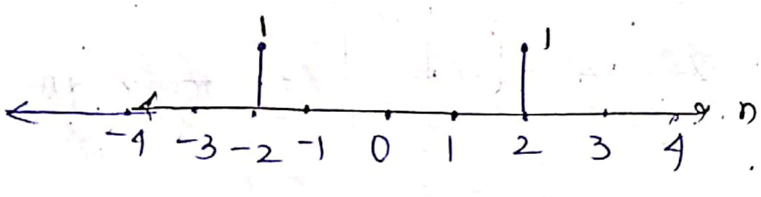
$$y_2(n) = T[x_2(n)]$$

$$y_2(n) = x_2[n^2] = \{\ldots\ldots y_2(-2), y_2(-1), \ldots y_2(0) \ldots\}$$

$$= \{\ldots x((-5)^2), x(16), x(9), x(4), x(1), x(0), x(1), x(4), x(9), x(16), x(25) \ldots\}$$

$$= \{\ldots 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0 \ldots\}$$
$$\uparrow$$

$$y_2(n) = \{\ldots 0, 1, 0, 0, 0, 1, 0 \ldots\}$$
$$\uparrow$$



$y_2[n]$.

6) Compare the signals $y_2(n)$ and $y(n-2)$. What is your conclusion.

$$y_2(n) = T[x_2(n)] = T[x(n-2)] \to (1)$$

$$y(n-2) \to (2) \quad \text{Here } 1 \neq 2$$

Hence the given system is time variant system

Q. Repeat part (b) for the system.

$y(n) = T[x(n)] = x(n) - x(n-1)$.

Can you use this result to make any statement about the time invariance of this system? why?

sol:- $x(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots \}$

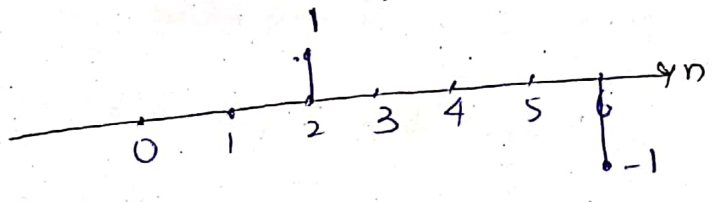
1. $y(n) = x(n) - x(n-1)$

2. $y(n) = \{ \ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots \} - \{ \ldots \underset{\uparrow}{0}, 1, 1, 1, 1 \ldots \}$

$= \{ \ldots \underset{\uparrow}{1}, 0, 0, 0, -1, \ldots \}$

3. $y(n-2) = \{ \ldots \underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1, 0 \ldots \}$

4.



4. $x(n-2) = \{ \ldots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \ldots \} = x_2(n)$

5. $y_2(n) = x_2(n) - x_2(n-1)$

$= \{ \ldots \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0 \ldots \} - \{ \ldots \underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1 \}$

$= \{ \ldots \underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1, 0 \ldots \}$

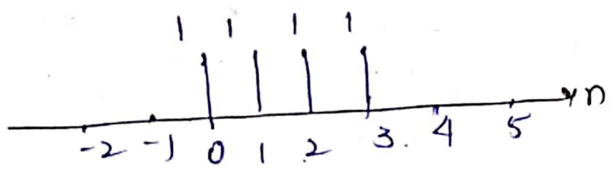
c] $y_2(n) = y(n-2)$. Hence the given system is time-invariant.

By the proof that if
$y(n,k) = y(n-k)$; then it is time-invariant

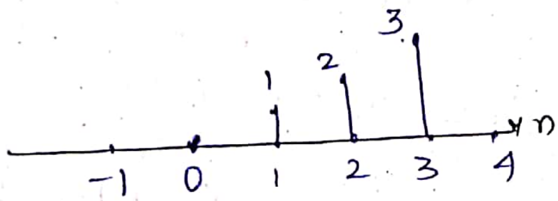
but this example alone does not constitute a proof.

d] Repeat the parts (b) and (c) for the system
$y(n) = T[x(n)] = n\,x(n)$.

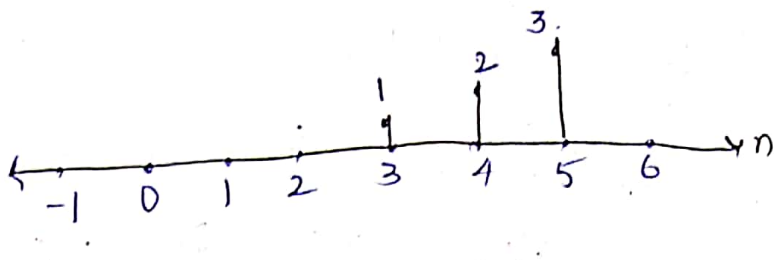
1. $x(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots\}$



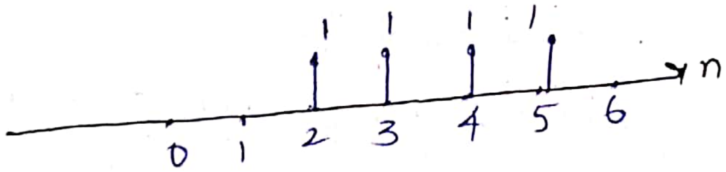
2. $y(n) = n\,x(n) = \{0 \cdot 0;\ 0 \times 1,\ 1 \times 1,\ 2 \times 1,\ 3 \times 1,\ 0 \ldots\}$
$y(n) = \{\ldots 0, \underset{\uparrow}{1}, 2, 3, 0 \ldots\}$



3] $y_2'(n) = y(n-2) = \{\ldots 0, 0, 0, 1, 2, 3, 0, \ldots\}$ with the arrow (↑) under the first 0 after the dots.



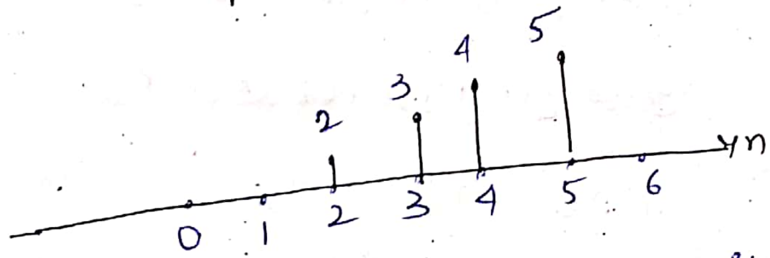
4] $x(n-2) = \{\ldots 0, 0, 1, 1, 1, 1, 0, \ldots\}$ with the arrow (↑) under the first 0.



5] $y_2(n) = n x_2(n)$

$= \{\ldots 0, 2\times1, 3\times1, 4\times1, 5\times1, 0 \ldots\}$

$= \{\ldots 0, 0, 2, 3, 4, 5, 0 \ldots\}$ with the arrow (↑) under the first 0.



6] $y_2(n) \neq y(n-2)$. Hence the given system is time ~~in~~ variant.

Q.7] A discrete time system can be

1. Static or dynamic
2. Linear or Non-linear
3. Time variant or time invariant
4. Casual or Non-Causal
5. Stable or Unstable

Examine the following systems with respect to the above properties.

(a) $y[n] = \cos[x(n)]$.

(i). Static as $y[1] = \cos[x(1)]$

$y(n) = f(x(n))$ only static.

(ii) Non-linear as

$y_1(n) = \cos(x_1(n))$

$y_2(n) = \cos(x_2(n))$

$y_1(n) + y_2(n) = \cos(x_1(n)) + \cos(x_2(n))$

$\neq y(n) \neq \cos(x_1(n) + x_2(n))$

Hence Non-linear.

(iii) Time invariant

① ← $y(n,) = \cos[x(n)] \Rightarrow y(n,k) = \cos[x(n-k)]$

② ← $y(n-k] = \cos[x(n-k)]$

from ① and ② system is time-invariant

(iv) Casual ⇒ as static it is casual.

$f = \{$ present.

(v). Stable as $|x(n)| < \alpha;$

$y(n) = \cos(|x(n)|)$ $\left\{\begin{array}{l}\text{as cos is} \\ \text{always } -1 \text{ to } 1 \\ \text{bounded}\end{array}\right\}$

∴ System is stable

(b) $y(n) = \sum_{k=-\alpha}^{n+1} x(k)$. d; L, IV, NC, U

$= \sum x(-\alpha) + \dots + x(n) + x(n+1)$

ie $y(1) = y(-\alpha) \dots + x(1) + x(2)$

(i) dynamic as $y(1)$ depends on $x(2)$.

(ii) Linear as $y_1(n) = \sum x_1(n)$;
$y_2(n) = \sum x_2(n)$

$y(n) = y_1(n) + y_2(n) = \sum (x_1(n) + x_2(n))$

Similary scaling also.

(iii) Time invariant

$y(n) = \sum_{k=-\alpha}^{n+1} x(k)$; $y(n,k) = \sum_{k=-\alpha}^{n+1-k} x(k)$

Similarly $y(n-k) = \sum_{k=-\alpha}^{n+1-k} x(k)$.

Hence Time Invariant

(iv) System is Non-Casual.

(v) System is Unstable. Because if $x(k) = u(k)$; the output becomes

$y(n) = \sum_{k=-\alpha}^{n+1} u(k) = \begin{cases} 0; & n < 0 \\ n+2; & n \geq 0 \end{cases}$

Hence it is stable.

(c) $y(n) = x(n) \cos(w_0 n)$.

Sol:- (i) Static $\quad y(0) = x(0) \cos(w_0(0))$

$y(1) = x(1) \cos(w_0(1))$

(ii) Linear

$y_1(n) = x_1(n) \cos(w_0 n)$

$y_2(n) = x_2(n) \cos(w_0 n)$

$y(n) = y_1(n) + y_2(n) = [x_1(n) + x_2(n)] \cos(w_0 n)$.

(iii) Time Variant

$y(n) = x(n) \cos(w_0 n)$

$y(n,k) = x(n-k) \cos(w_0 n) \rightarrow ①$

$y(n-k) = x(n-k) \cos(w_0(n-k)) \rightarrow ②$

$1 \neq 2$; Hence time Variant.

(iv). Casual as it is static.

(v) Stable as $\quad |x(n)| < N = N$;

$|\cos(w_0 n)| = 1$;

then $y(n)$ is also stable.

(d) $y(n) = x(-n+2)$.

[i] Dynamic ; $\quad y(0) = x(2)$; hence dynamic

[ii] Linear $\quad y_1(n) = x_1(-n+2)$, $y_2(n) = x_2(-n+2)$

$\Rightarrow y(n) = y_1(n) + y_2(n)$.

[iii] Time Variant

$y[n] = x[-n+2]$; $y[n,k] = x[-n-k+2]$

$y[n-k] = x[-n+k+2]$

[iv] Non casual [depends on future inputs]

$y[0] = x[2]$

[v] Stable ; as $|x[n]| < \alpha$, stable.

[c] $y(n) = \text{Trun}[x(n)]$; where $\text{Trun}[x(n)]$ denotes the integer part of $x(n)$; obtained after truncation

Sol:- [i] Static ; $y[0] = \text{Trunc}[x(0)]$

[ii] Non linear.

$x_1(n) \Rightarrow y_1[n] = \text{Trunc}[x_1(n)]$

$y_2(n) = \text{Trunc}[x_2(n)]$

$y[n] \neq y_1(n) + y_2(n)$.

Xi Eg:- 1.6 ; 1.3

$\downarrow$ $\downarrow$ $1+1 = 2$

O/P $\to$ 1 1 $\neq$

$1.6 + 1.3 = 3.1 \to$ O/P $= 3$

Hence Non linear.

[iii] Time invariant;

$y(n,k) = y(n-k)$; hence time invariant

[iv] Casual as it is static

(v) stable as bounded i/p gives bounded o/p.

(f) $y(n) = \text{Round}[x(n)]$; where $\text{Round}[x(n)]$ denotes the integer part of $x(n)$ obtained by rounding.

sol:- Similar to above system is static, Non-Linear, Time invariant, Casual and stable.

Non-linearity: Let i/p $\Rightarrow$ 1.6 $\downarrow$ 2 + 1.3 $\downarrow$ 2 = 4

$\neq$

i/p $(1.6 + 1.3) = 2.9 \rightarrow 3$

Hence Non-linear.

Remark:- The system in parts (e) and (f) are quantizers that perform truncation and rounding respectively.

(g) $y(n) = |x(n)|$. System is

static; $y(0) = |x(0)|$

Time invariant; $y(n,k) = y(n-k)$.

Casual: As it satisfies the condition of static.

Stable :- As bounded i/p gives bounded o/p.

$x[n] = 1$; $y[n] = 1$;

Non linear $x_1[n] = 1 \rightarrow y_1[n] = 1$

$x_2[n] = 1 \rightarrow y_2[n] = 1$

$y[n] = |x_1[n] + x_2[n]| = 0 \neq y_1[n] + y_2[n]$

System is Non-linear.

[h] $y(n) = x(n).u(n)$.

sol:- Static:- as $y[0] = x[0]u[0]$

doubt * Time Variant :-

as $y(n) = x(n)u(n) \xrightarrow{x(n-k)} x(n-k)u(n)$

$y(n-k) = x(n-k)u(n-k)$

Eg:- $x(n-2)u(n) \neq x(n-2)u(n-2)$

at $k=2$

* Linear:- $y_1(n) = x_1(n)u(n)$;

$y_2(n) = x_2(n)u(n)$

$y(n) = y_1(n) + y_2(n) = [x_1(n) + x_2(n)]u(n)$.

* Casual as it is static

* Stable. [as multiplication of stable with unstable; gives only the stable quantity

[i] $y(n) = x(n) + n x(n+1)$.

Dynamic :- as $y[1] = x[1] + x[2]$

Hence it is dynamic

Linear :- $y_1(n) = x_1(n) + n x_1(n+1)$

$y_2(n) = x_2(n) + n x_2(n+1)$.

$y = y_1(n) + y_2(n) = [x_1(n) + x_2(n)] + n[x_1(n+1) + x_2(n+1)]$

[iii] Time Variant as

$y[n] = x(n) + n\,x(n+1);$

$y(n,k) = x(n-k) + n\,x(n+1-k) \rightarrow ①$

$y(n-k) = x(n-k) + (n-k)\,x(n+1-k) \rightarrow ②$

As $1 \neq 2$; hence the system is time variant

[iv] Non Casual → As it depends on future

[v] Stable.

Note that the bounded input $x(n) = u(n)$; produces unbounded output

[5] $y(n) = x(2n)$.

Sol:- * Dynamic as $y[2] = x[4]$.

* Linear :- as $y_1[n] \rightarrow x_1[2n]$

$y_2[n] \rightarrow x_2[2n]$

$\rightarrow y[n] = y_1(n) + y_2(n) = x_1(2n) + x_2(2n)$

* Time Variant as

$y(n,k) = x(2n-k);$ $y(n-k) = x(2n-2k)$

Here $y(n,k) \neq y(n-k)$

* Non Casual [depends on future]

$y(2) = x(4)$

* Stable; $|x(2n)| < M < \infty;$

then $|y(n)| < \infty;$

bounded.

k] $y(n) = \begin{cases} x(n) & ; \text{ If } x(n) \geq 0 \\ 0 & ; \text{ If } x(n) < 0 \end{cases}$

* Static : $y(0) = x(0)$; as it depends on present input only

* Non-linear; Lets $x_1[n] = 0.3 \Rightarrow y_1[n] = 0.3$
$x_2[n] = -0.7 \rightarrow y_2[n] = 0$

$y[n] = x_1[n] + x_2[n] = -0.4 \Rightarrow y[n] = 0$

Here $y[n] \neq y_1[n] + y_2[n]$

* It is time-Invariant

$y(n, k) = y(n-k)$

* Casual as it is static.

* Stable $\rightarrow$ as bounded input produces the ~~pro~~ bounded output.

* doubt

l] $y(n) = x[-n]$.

* Dynamic as $y[-2] = x[2]$.

* Linear $\rightarrow$ $y_1[n] = x_1[-n]$; $y_2[n] = x_2[-n]$

$\Rightarrow y[n] = y_1[n] + y_2[n] = x_1[-n] + x_2[-n]$.

* Time variant;

* Non casual [depends on future values]

* stable $\rightarrow$ as bounded input produces the bounded ~~input~~. output

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m] $y(n) = \text{sign}[x(n)]$.

1 Static :- $y[0] = \text{sign}[x(0)]$

$y[1] = \text{sign}[x(1)]$

2 Nonlinear :- Let $x_1[n] = 0.3 \rightarrow 1$

$x_2[n] = -0.7 \rightarrow 0$

3 $\text{sign}[(0.4)] = -1$.

Time invariant :- $y(n) = \text{sign}[x(n)]$

$y(n, k) = \text{sign}[x(n-k)]$

4

$y(n-k) = \text{sign}[x(n-k)]$. &

Stable system

n] The ideal sampling system with input $x_a(t)$ and output $x(n) = x_a(nT)$, $-\alpha < n < \alpha$.

Sol:- Static $\rightarrow$ $x(0) = x_a(0)$

Linear $\rightarrow$ $x_1 a(nT) + x_2 a(nT) = y(n)$

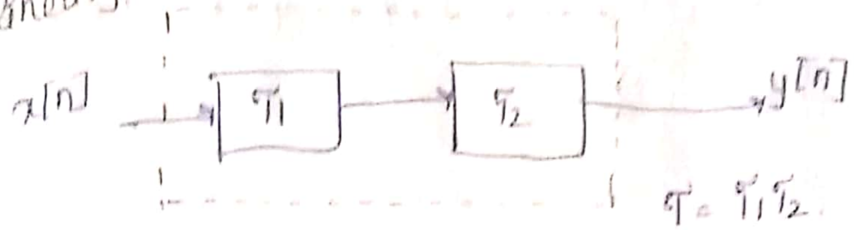
Time Invariant $\rightarrow$

Casual and $\rightarrow$ As it is static

Stable $\rightarrow$ bounded input gives the bounded output.

2.8 Two discrete-time systems T_1 and T_2 are connected in cascade to form a new system T as shown in fig. P2.8. Prove or dispove the following statements.

a] If T_1 and T_2 are linear, then T is linear [i.e the cascade connection of two linear systems is linear]



$T = T_1 T_2$

sol:- True if

$v_1(n) = T_1[x_1(n)]$ and $v_2(n) = T_1[x_2(n)]$

then $\alpha_1 x_1(n) + \alpha_2 x_2(n)$ yields

$\alpha_1 v_1(n) + \alpha_2 v_2(n)$

by the linearity property of T_1. Similarly if

$y_1(n) = T_2[v_1(n)]$ and

$y_2(n) = T_2[v_2(n)]$; then

$\beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$

by the linearity property of T_2. Since

$v_1(n) = T_1[x_1(n)]$ and

$v_2(n) = T_2[x_2(n)]$; it follows that

$A_1 x_1(n) + A_2 x_2(n)$.. yields the output

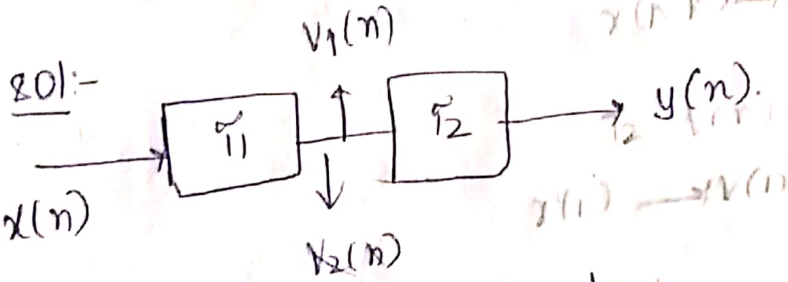
$A_1 T[x_1(n)] + A_2 T[x_2(n)]$

b] If T_1 and T_2 are casual; then T is casual.

Sol:- True. T_1 is casual $\Rightarrow v(n)$ depends only on $x(k)$ for $k \le n$. T_2 is casual $\Rightarrow$ $y(n)$ depends only on $v(k)$ for $k \le n$. Therefore; $y(n)$ depends only on $x(k)$ for $k \le n$. Hence, T is casual.

(k) If T_1 and T_2 are time invariant; then T is time invariant.

Sol:-



$x(n) \xrightarrow{T_1} v(n)$ and

TIV $x(n-k) \xrightarrow{T_1} v(n-k)$

for T_2 $\quad v(n) \rightarrow y(n)$

and $\quad v(n-k) \rightarrow y(n-k)$

Hence for $T_1 T_2$; if $\quad x(n) \rightarrow y(n)$ and

$x(n-k) \rightarrow y(n-k)$

Therefore $T = T_1 T_2$ is time invariant.

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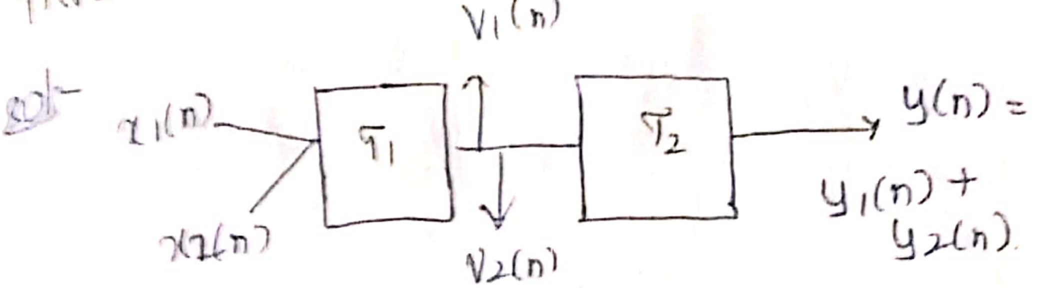
d) If T_1 and T_2 are linear and time invariant, the same holds for T.

sol–



$$\alpha_1 x_1(n) \xrightarrow{T_1} \alpha_1 v_1(n)$$

$$\alpha_2 x_2(n) \xrightarrow{T_1} \alpha_2 v_2(n)$$

$$\alpha_1 x_1(n) + \alpha_2 x_2(n) \xrightarrow[\text{Linear}]{T_1} \alpha_1 v_1(n) + \alpha_2 v_2(n)$$

$$\alpha_1 x_1(n-k) + \alpha_2 x_2(n-k) \xrightarrow[T_1]{LTI} \alpha_1 v_1(n-k) + \alpha_2 v_2(n-k)$$

Actual

$$\alpha_1 v_1(n-k) + \alpha_2 v_2(n-k) \xrightarrow[\text{as LTI}]{T_2} \alpha_1 y_1(n-k) + \alpha_2 y_2(n-k).$$

for T_2

$$v_1(n) \xrightarrow{T_2} y_1(n)$$

$$v_2(n) \xrightarrow{T_2} y_2(n)$$

$$\beta_1 v_1(n) + \beta_2 v_2(n) \xrightarrow{T_2} \beta_1 y_1(n) + \beta_2 y_2(n)$$

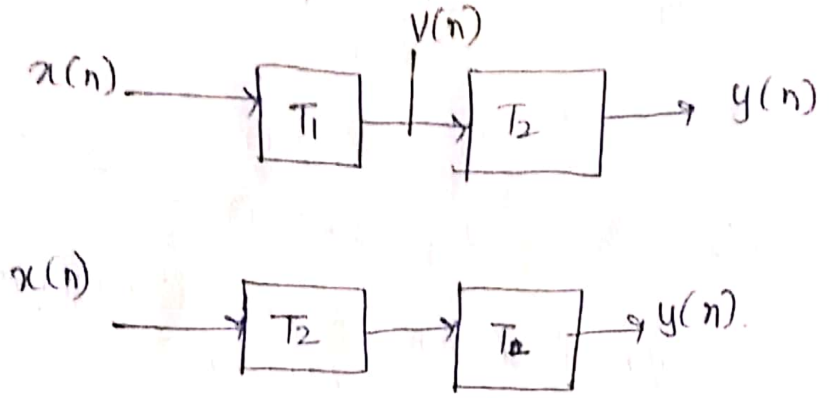
$$\beta_1 v_1(n-k) + \beta_2 v_2(n-k) \xrightarrow{T_2} \beta_1 y_1(n-k) + \beta_2 y_2(n-k).$$

Hence for $T_1 T_2$ if $x(n) \longrightarrow y(n)$ and $x(n-k) \longrightarrow y(n-k)$.

Therefore $T = T_1 T_2$ is time invariant

(e) If T_1 and T_2 are LTI; & then interchanging their order doesnot change the system T.

Sol:-



① case

① $v(n) \rightarrow T_1[x(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \xrightarrow{L} \alpha_1 v_1(n) + \alpha_2 v_2(n)$

$\alpha_1 x_1(n-k) + \alpha_2 x_2(n-k) \xrightarrow{LTI} \alpha_1 v_1(n-k) + \alpha_2 v_2(n-k)$.

② $y(n) \rightarrow T_2[v(n)]$

$\beta_1 v_1(n) + \beta_2 v_2(n) \xrightarrow{L} \beta_1 y_1(n) + \beta_2 y_2(n)$

$\beta_1 v_1(n-k) + \beta_2 v_2(n-k) \xrightarrow{LTI} \beta_1 y_1(n-k) + \beta_2 y_2(n-k)$.

$A_1 x_1(n) + A_2 x_2(n) \xrightarrow{O/P} A_1 T[x_1(n)] + A_2 T[x_2(n)]$

$T = T_1 T_2$; T is linear.

2nd block.

$V(n) \longrightarrow T_2[x(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \longrightarrow \alpha_1 V_1(n) + \alpha_2 V_2(n)$

$\Rightarrow$

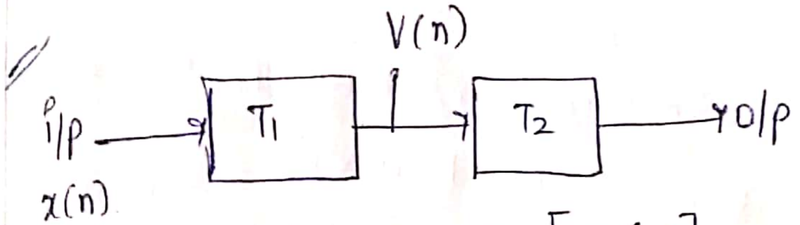
$y(n) \longrightarrow T_1[v(n)] = T_1[T_2[x(n)]]$

$= T_1[\alpha_1 V_1(n) + \alpha_2 V_2(n)]$

$= y_1(n) + y_2(n) = T_1 T_2[\alpha_1 x_1(n) + \alpha_2 x_1(n)]$

yields same output.

(f) As in part (e) except that T_1, T_2 are now time varying (Hint: Use an example).



$v(n) \longrightarrow T_1[x(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \xrightarrow{L} \alpha_1 V_1(n) + \alpha_2 V_2(n)$

$\alpha_1 x_1(n-k) + \alpha_2 x_2(n-k) \longrightarrow \alpha_1 V_1(n-g) + \alpha_2 V_2(n-g)$

$k \neq g$

$y(n) \longrightarrow T_2[V(n)]$

$\alpha_1 V_1(n-g) + \alpha_2 V_2(n-g) \longrightarrow \alpha_1 y_1(n-h) + \alpha_2 y_2(n-h)$

$g \neq h$

Similarly if we reverse the block the overall output changes accordingly

eg:- $y_1(n)$ ∠ let us consider;

doubt

$T_1 : y(n) = n x(n)$ and

$T_2 : y(n) = n x(n+1)$

Then;

$$T_2[T_1[\delta(n)]] = T_2(0) \quad 0.\delta(n) = 0$$
$$= 0$$

$$T_1[T_2[\delta(n)]] = T_1[\delta(n+1)]$$
$$= -\delta(n+1)$$
$$\neq 0.$$

9] If T_1 and T_2 are nonlinear; then T is nonlinear.

Sol:- $T_1 : y(n) = x(n) + b$ and

$T_2 : y(n) = x(n) - b$ where $b \neq 0$

$$T[x(n)] = T_2[T_1[x(n)]]$$
$$= T_2[x(n) + b]$$
$$= x[n] + \cancel{b} - \cancel{b}$$
$$= x[n];$$ Linear

Hence false.

h] If T_1 and T_2 are stable; then T is stable

$$v(n) = \sum x[n]\, T_1[n]$$

as $|v(n)| \leq \sum |x[n]|\, |T_1[n]|$

as $|T_1[n]|$ is stable;

If $x[n]$ is bounded then $v(n)$ is bounded.

$$y(n) = \sum v(n)\, T_2[n].$$

then $|y(n)| = \sum |v(n)|\, |T_2(n)|$ as [$T_2[n]$ is stable]

If $v[n]$ is bounded then $y[n]$ is also

Then $y[n]$ is bounded if $x[n]$ is bounded.

Hence "T is stable"

i] Show by an example that the inverse of parts (c) and (h) do not hold in general.

Inverse of (c), T_1 and T_2 are noncasual

$\Rightarrow T$ is noncasual. Example:-

$T_1 : y(n) = x(n+1)$ and

$T_2 : y(n) = x(n-2)$

$\Rightarrow T : y(n) = x(n-1);$

2.9 Let T be an LTI, relaxed and BIBO stable s/m with input $x(n)$ and output $y(n)$. Show that:

(a) If $x(n)$ is periodic with period N [i.e. $x(n) = x(n+N)$ for all $n \geq 0$], the output $y(n)$ tends to a periodic signal with the same period.

Sol. $y(n) = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ as $x(n)=0; k<0$

$= \sum_{k=0}^{\infty} x[k]h[n-k]$ Let $\sum_{m=0}^{\infty} x[m]h[n-m]$

$n-m=k$ | $m=\infty$

at $m=0$ | $n-\infty=k$

$k=n$ | $k=-\infty$

$$y(n) = \sum_{k=n}^{\infty} x(n-m)h(m)$$

$$y(n) = \sum_{k=-\infty}^{n} h(k)x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)x(n+N-k)$$

$$= \sum_{k=-\infty}^{n+N} h(k)x(n-k)$$ [as $x(n)=x(n+N)$ periodic with period N]

$$= \sum_{k=-\infty}^{n} h(k)x(n-k) + \sum_{k=n+1}^{n+N} h(k)x(n-k)$$

$$= y(n) + \sum_{k=n+1}^{n+N} h(k)x(n-k)$$

For a BIBO system, $\lim_{n\to\infty} |h(n)| = 0$. Therefore

$$\lim_{n \to \alpha} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0 \quad \text{and}$$

$$\lim_{N \to \alpha} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0 \quad \text{and}$$

$$\lim_{N \to \alpha} y(n+N) = y(N).$$

(b) If $x(n)$ is bounded and tends to a constant, the output will also tend to a constant;

(c)

Q.10 The following input-output pairs have been observed during the operation of a time-invariant system

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

Can you draw the conclusions regarding the linearity of the system. What is the impulse response of the system?

Sol:- The given system is non linear.

Reason:-

Coming to $x_2(n)$ and $x_3(n)$:

$$x_2(n) = \{0, \underset{\uparrow}{0}, 3\} \xrightarrow{[T]} \{0, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n-1) = \{0, \underset{\uparrow}{0}, 0, 1\} \xleftrightarrow{[T]} \{1, 2, \underset{\uparrow}{1}\}$$

$\Downarrow$

$$x_3(n+1) = \{0, \underset{\uparrow}{0}, 1\} \xrightarrow{(T.I)} \{\underset{\uparrow}{1}, 2, 1\}$$

Now if the system is linear then

$$3 \times x_3(n+1) \longleftrightarrow \{\underset{\uparrow}{3}, 6, 3\}$$

$$\{\underset{\uparrow}{0}, 0, 3\}$$

But $\{3, 6, 3\} \neq \{0, \underset{\uparrow}{1}, 0, 2\}$

Hence, the system is Non-linear.

11]. The following input-output pairs have been observed during the operation of a linear system:

$$x_1(n) = \{-1, \underset{\uparrow}{2}, 1\} \xleftrightarrow{T} y_1(n) = \{1, \underset{\uparrow}{2}, -1, 0, 1\}$$

$$x_2(n) = \{1, -1, -1\} \xleftrightarrow{T} y_2(n) = \{-1, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{0, \underset{\uparrow}{1}, 1\} \xleftrightarrow{T} y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$$

Sol:- Since $x_1(n) + x_2(n) = \delta(n)$ and the system is linear,

The impulse response of the system is given by $y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$.

If the system is further time-invariant, then it would be an LTI System

Then $x_3(n) * h(n) = \{0, 1, 1\} * \{0, 3, -1, 2, 1\}$
$\qquad\qquad\qquad\qquad\quad \uparrow \qquad\qquad \uparrow$

$y(n) = \{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

But the the given o/p $y_3(n) = \{\underset{\uparrow}{1}, 2, 1\}$

Hence system is time variant

Q.12] The only available information about a system consists of N input-output pairs of signals $y_i(n) = T[x_i(n)]$; $i = 1, 2, --, N$.

(a) What is the class of input signals for which we can determine the output, using the information above; if the system is known to be a linear?

sol:- Any weighted linear combination of the signals $x_i(n)$; $i = 1, 2, \ldots N$.

(b) The same as above; if the system is known to be time invariant

sol:- Any $x_i(n-k)$; where k is any integer and $i = 1, 2, \ldots, N$

Q13 Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is

$$\sum_{n=-\alpha}^{\alpha} |h(n)| \leq M_h < \alpha$$ for some

constant M_h.

Sol:- A system is BIBO stable if and only if a bounded input produces a bounded output

$$y(n) = \sum_{k=-\alpha}^{\alpha} x[k]\, h[n-k]$$ where let $|x(n-k)| \leq M_x$

$$|y(n)| \leq \sum_{k} |h(k)|\, |x(n-k)|$$

$$\leq M_x \sum_{k} |h(k)|$$

Therefore $|y(n)| < \alpha$ for all n; if and only if

$$\sum_{k} |h(k)| < \alpha.$$

14. Show that: (a) A relaxed linear system is casual if and only if for any input $x(n)$ such that

$x(n) = 0$ for $n < n_0 \Rightarrow y(n) = 0$ for $n < n_0$.

Sol:- A system is casual $\Leftrightarrow$ the output becomes nonzero after the input becomes non-zero. Hence;

$x(n) = 0$ for $n < n_0 \Rightarrow y(n) = 0$ for $n < n_0$

(ii) A relaxed LTI system is casual iff only if

$h(n)=0$ for $n<0$.

$y(n)=\sum_{-\alpha}^{n} h(k)x(n-k)$; where $[x(n)=0$ for $n<0$

if $h(k)=0$ for $k<0$ then

$y(n)=\sum_{0}^{n} h(k)x(n-k)$; and hence

$y(n)=0$ for $n<0$.

On the other hand; if $y(n)=0$ for $n<0$; then

$\sum_{-\alpha}^{n} h(k)x(n-k) \Rightarrow h(k)=0$; $k<0$.

Q1.5 (a) Show that for any real or complex constant a, and any finite integer numbers M and N we have.

$$\sum_{n=M}^{N} a^n = \begin{cases} \frac{a^M - a^{N+1}}{1-a} & \text{if } a\neq 1 \\ N-M+1 & \text{if } a=1. \end{cases}$$

sol:- if $a=1 \Rightarrow \sum_{n=M}^{N} 1 = N-M+1$

if $a\neq 1 \Rightarrow \sum_{n=M}^{N} a^n = a^M + \dots + a^N$

$\Rightarrow a^M[1+\dots+a^{N-M}]$

$\Rightarrow a^M\left[\frac{1-a^{N-M+1}}{1-a}\right] = \frac{a^M - a^{N+1}}{1-\alpha}$

(b) Show that if $|a| < 1$; then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}.$$

sol:- For $M = 0$; $|a| < 1$; and $N \to \infty$;

$$\sum_{n \geq 0}^{\infty} a^n = 1 + a + a^2 + \cdots = \frac{1}{1-a}; \text{[I.P]}$$

2.16 (a) If $y(n) = x(n) * h(n)$; show that

$$\Sigma_y = \Sigma_x \Sigma_h \text{ ; where } \Sigma_x = \sum_{n=-\infty}^{\infty} x(n).$$

sol:- Method 1:-

$$y(-\infty) + \cdots + y(1) + \cdots y(\infty) = \sum_{k=-\infty}^{\infty} x(k) h(-\infty - k) +$$

$$\cdots + \sum_{k=-\infty}^{\infty} x(k) h(\infty - k).$$

$$= \sum_{k=-\infty}^{\infty} x(k) \left[h(-\infty - k) + \cdots + h(\infty - k) \right]$$

$$\sum_n y(n) = \sum_{k=-\infty}^{\infty} x(k) \sum_{n=-\infty}^{\infty} h(k)$$

$$= \left(\sum_k h(k) \right) \left(\sum_n x(n) \right)$$

Method 2:- $y(n) = \sum_k h(k) x(n-k)$

$$\sum_n y(n) = \sum_n \sum_k h(k) x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$= \left(\sum_k h(k) \right) \left(\sum_n x(n) \right).$$

(b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the above.

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

<u>Sol:-</u> $y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum_n y(n) = 35$.

$\sum_k h(k) = 5$; $\sum_k x(k) = 7$

$\therefore 35 = 7 \times 5$ ✓✓

(2) $x(n) = \{1, 2, -1\}$; $h(n) = x(n)$.

$\Rightarrow y(n) = \{1, 4, 2, -4, 1\}$

	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$\sum_n y(n) = 4$.

$\sum_n x(n) = 2$;

$\sum_k h(k) = 2$; $4 = 2 \times 2$ ✓✓

(3) $x(n) = \{0, 1, -2, 3, -4\}$; $h(n) = \{1/2, 1/2, 1, 1/2\}$

$h(-n) = \{1/2, 1, 1/2, 1/2\}$

$y(n) = \{0, 1/2, \frac{-1}{2}, 3/2,$

0 1 -2 3

1/2 -1 +3/2 -2

-3 +2

	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

$\{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$

$\sum_n y(n) = -5$

$\sum_n x(n) = -2;$

$\sum_n h(n) = 5/2$

$-5 = -2 \times 5/2$

(4) $x(n) = \{1, 2, 3, 4, 5\}; \quad h(n) = \{1\}$

$y(n) = \{1, 2, 3, 4, 5\}; \quad \sum y(n) = 15$

[5] $x(n) = \{\underset{\uparrow}{1}, -2, 3\}; \quad h(n) = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$

$h(-n) = \{1, 1, 1, 1, 0, 0\}$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n (y(n)) = 8; \quad \sum_n h(n) = 4; \quad \sum_n x(n) = 2.$

[6] $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}; \quad h(n) = \{1, \underset{\uparrow}{-2}, 3\}$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8; \quad \sum_n h(n) = 4; \quad \sum_n x(n) = 2$

[7] $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}; \quad h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$\{-1, -1, 0, \underset{\uparrow}{1}\}$

$y(n) = \{0, 1, 4, -4, -5, -1, +3\}$

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8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$; $h(n) = u(n)$

$x(n) = \delta(n) + \delta(n-1) + 2\delta(n-2)$

$y(n) = u(n) + u(n-1) + 2u(n-2)$.

$\sum_n y(n) = \infty$ $\quad \sum_n x(n) = 4$; $\quad \sum_n h(n) = \infty$.

9] $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$; $h(n) = \{1, -2, -3, 4\}$

	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4

$y(n) = \{1, -1, -5, +2, 3, \underset{\uparrow}{-5}, 1, 4\}$

$\sum_n y(n) = 0$

$\sum_n h(n) = 0$; $\quad \sum_n x(n) = 4$.

10] $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$; $h(n) = x(n)$.

$y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y(n) = \{36\}$

$\sum_n h(n) = \sum_n x(n) = 6$.

	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

11] $x(n) = (1/2)^n u(n)$; $\quad h(n) = (1/4)^n u(n)$.

$$y(n) = \sum_{k=-\alpha}^{\alpha} (1/2)^k u(k)\,(1/4)^{n-k}\, u(n-k).$$

$$= \sum_{k=0}^{n} (1/2)^k (1/4)^{n-k} \;=\; \sum_{k=0}^{n} (1/2)^{2n-k}$$

$$y(n) = (1/2)^{2n} \sum_{k=0}^{n} 2^k$$

$$= (1/2)^{2n} \cdot \frac{1-2^{n+1}}{1-2}$$

$$= (1/2)^{2n} (2^{n+1} - 1)$$

$$\Rightarrow 2^{-n+1} - 2^{-2n} \Rightarrow \left[2(1/2)^n - (1/4)^n\right] u(n)$$

$$\sum_n h(n) \Rightarrow \frac{1}{1-1/4} = \frac{4}{3}$$

$$\sum_n x(n) = 2 \Rightarrow \frac{8}{3}$$

$$2(2) - \frac{1}{1-1/4}$$

-3×4

$4 + 1$

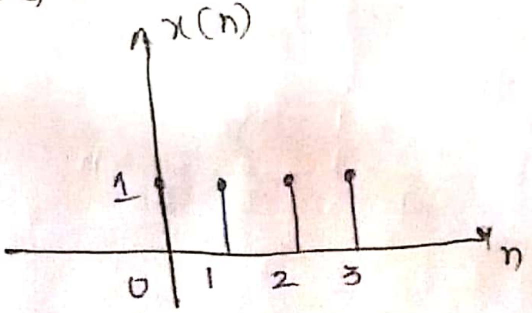
$$\Rightarrow 2 \cdot \sum_{n \geq 0}^{\alpha} (1/2)^{n} - \sum_{n \geq 0}^{\alpha} (1/4)^n$$

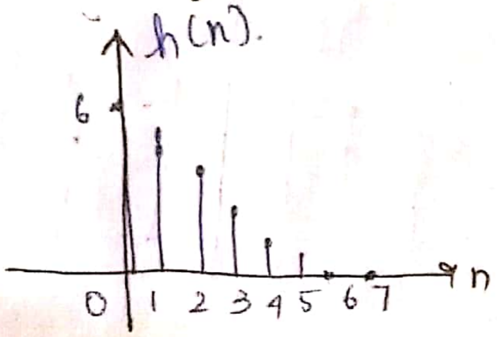
$4 - \frac{4}{3}$

$$\Rightarrow 2 \cdot \frac{1}{1-1/2} - \frac{1}{1-1/4}$$

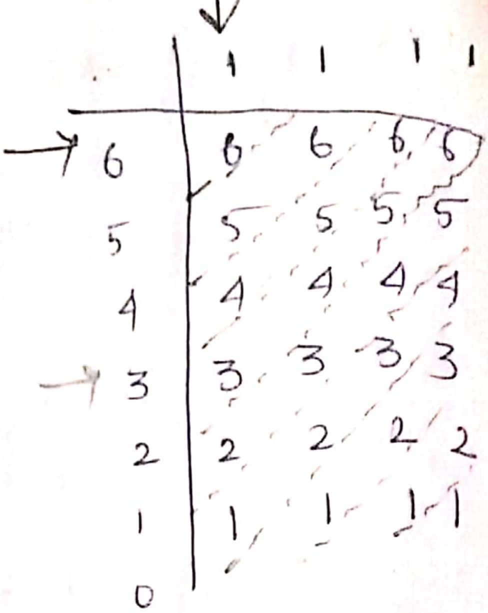
$$\Rightarrow 2(2) - 4/3 \Rightarrow 8/3$$

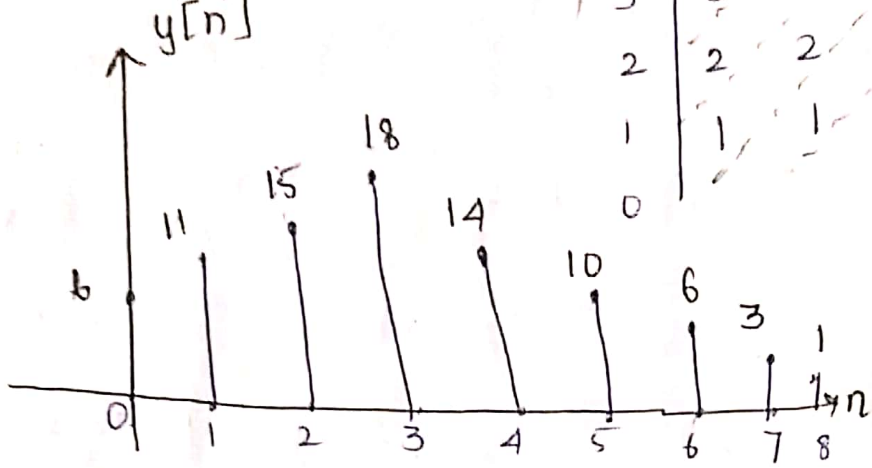
2.17 Compare and plot the convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of signals shown.



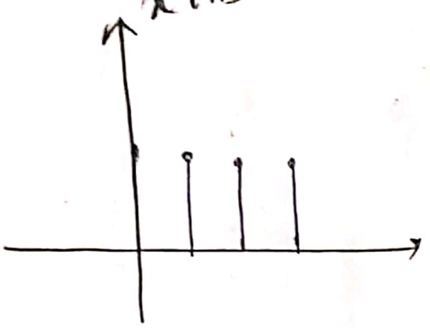


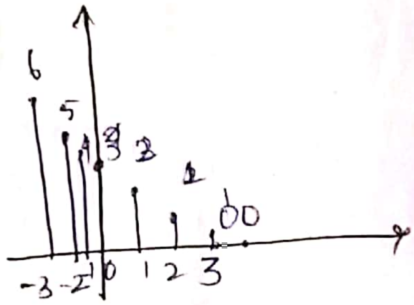
$y(n) = \{ \underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1 \}$





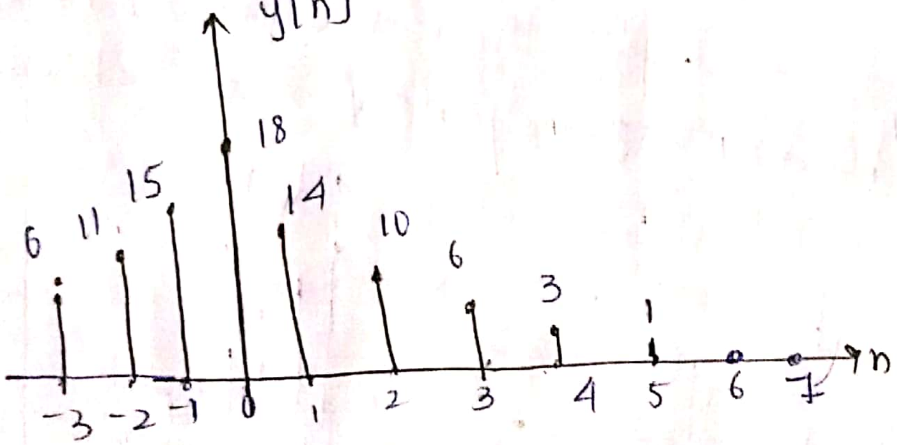
(b)



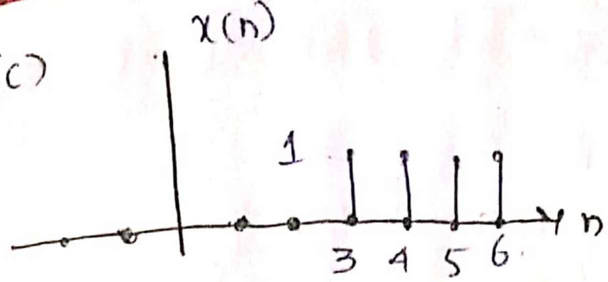


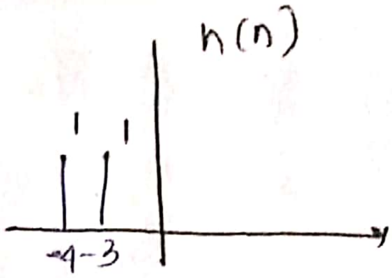
Same as above

$y(n) = \{ 6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1 \}$



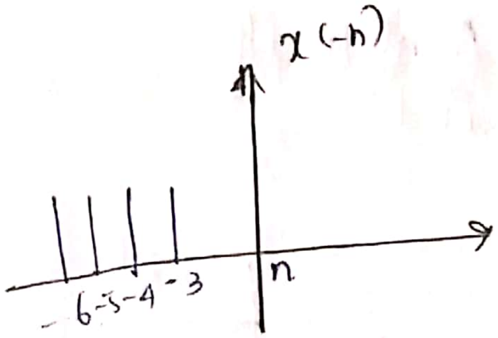
(c)





$\{1, 2, 2, 2, 1\}$
$\uparrow$

0	0	3	4	5	6
	1	3	4	5	6
	1	3	4	5	6

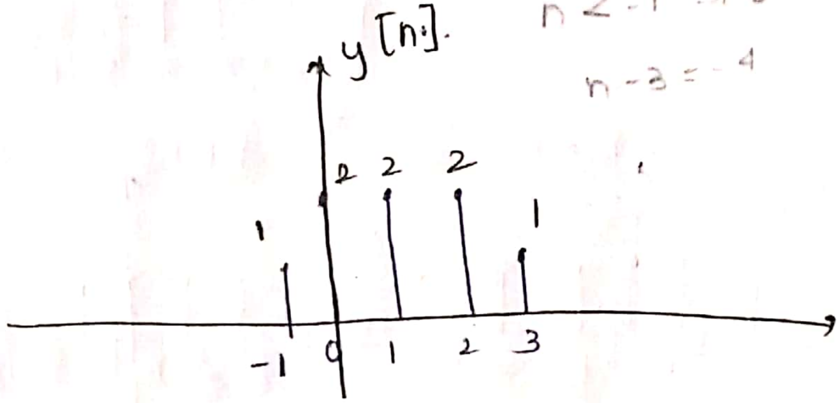


0	0	0	0	1	1	1	1
0	0	0	0	0	0	0	
0	0	0	0	0	0	0	
1	0	0	0	1	1	1	
1	0	0	0	1	1	1	1

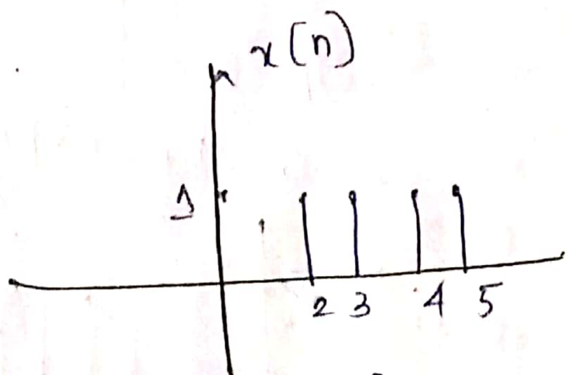
$n-3 < -4$

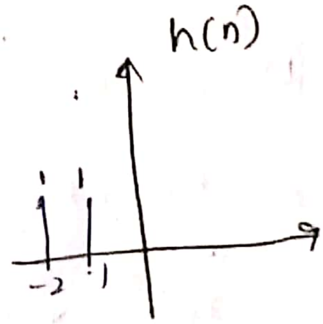
$n < -1 \Rightarrow 0$

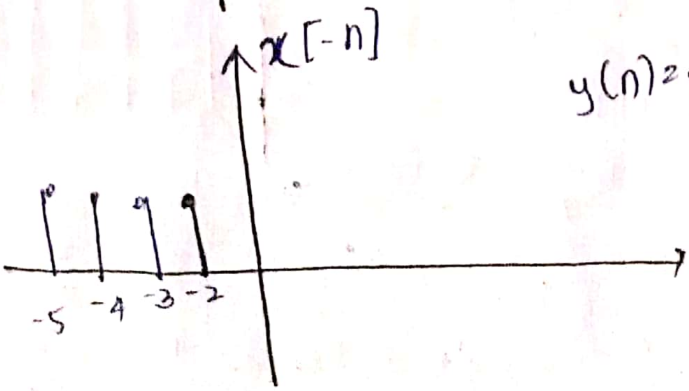
$n-3 = -4$



(d)







$y(n) = \{1, 2, 2, 2, 1\}$
$\uparrow$

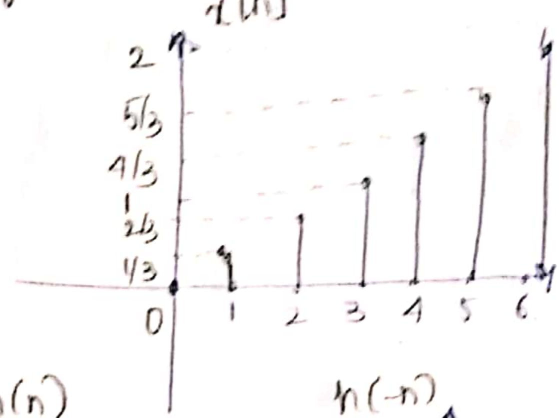
2.18 Determine and sketch the convolution $y(n)$ of the signals

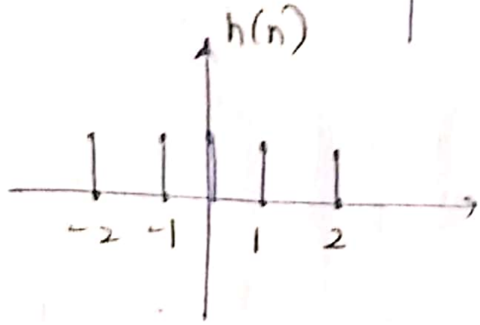
$$x(n) = \begin{cases} \frac{1}{3}n ; & 0 \le n \le 6 \\ 0 ; & \text{elsewhere} \end{cases}$$

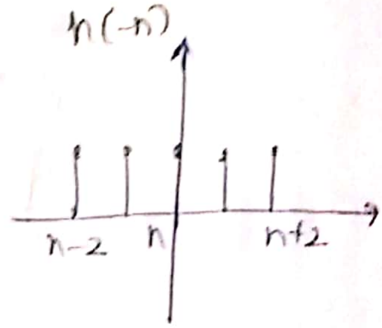
$$h(n) = \begin{cases} 1 ; & -2 \le n \le 2 \\ 0 ; & \text{elsewhere} \end{cases}$$

(a) Graphically (b) Analytically

(a) Graphically





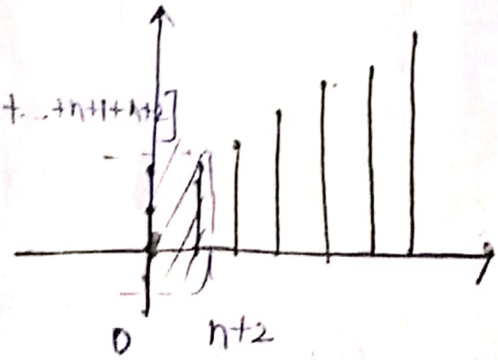


1) $n+2 < 0$, $n < -2$ $\Rightarrow$ O/P = 0

2). $n+2 > 0$ and $n-2 < 0$ $\Rightarrow$ $n > -2$ and $n < 2$

$$\sum_{0}^{n+2} (1/3) n = 1/3 . [0 + 1 + . . + n+1 + n+2]$$

$= 1/3 [0+1+$



Eg -2 ; 0

-1 ; $\sum_0^1 (1/3) \Rightarrow 1/3$

0 ; $\sum_0^2 (1/3) n \Rightarrow 1/3 (2+1) = 1$

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$1 \Rightarrow \sum_{0}^{3} (1/3)^{n} \Rightarrow 1/3\,(1+2+3) = 2.$

$2 \Rightarrow \sum_{0}^{4} (1/3)^{n} \Rightarrow 1/3\,(1+2+3+4) = \frac{10}{3}.$

$$③ \quad \sum_{n=2}^{n+2} (1/2)^{n} \Rightarrow 1/3 \left[n-2 + n-1 + n + n+1 + n+2 \right]$$

$$\Rightarrow 1/3 \left[5n \right] = \frac{5n}{3}$$

ie $n-2 > 0 \Rightarrow n > 2; \quad n+2 < 6 \Rightarrow n < 4.$

$n \Rightarrow 2;\ 10/3 \quad n=3;\ 5; \quad n=4;\ \frac{20}{3};$

$5 \to 6; \quad 6 \to$

$n-2 < 6 \Rightarrow n < 8$ and

$n+2 > 6 \Rightarrow n > 4.$

$$④ \quad \sum_{n-2}^{6} (1/3)^{n} = (1/3)\left[n-2 + n-1 + \cdots + 6 \right]$$

$$n=5 \Rightarrow \sum_{3}^{6} (1/3)^{n} \Rightarrow [1/3]\,[6+5+4+3]$$

$$\Rightarrow \frac{18}{3} = 6.$$

$y(n) = \{ 1/3, 1, 2, 10/3, 5, 20/3, 6, 5, 11/3, 2 \}$
$\quad\quad\quad\quad\ \ \uparrow$

<u>Analytically:-</u>

$\{ 1/3, 1, 2, 10/3, 5,$
$\quad\ \ \uparrow$
$20/3, 6, 5,$
$1/3, 2 \}.$

	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

$y(n)$

19 Compute the convolution $y(n)$ of the signals

$$x(n) = \begin{cases} \alpha^n; & -3 \le n \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Sol:- $y(n) = \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k)$

$= \sum_{k=-\alpha}^{\alpha} \alpha^k . [u(n) - u(n-k)]$

Wrong answer

$y(n) = \sum_{-3}^{5} \alpha^k u(n) - \sum_{-3}^{5} \alpha^k u(n-k)$

$y(n) = [\alpha^{-3} \ldots + \alpha^5]\, u(n) - [\alpha^{-3} \ldots + \alpha^5]\, u(n-k)$

$\alpha^{-3}[1 + \ldots + \alpha^8]$

$y(n) = \sum_{k=0}^{4} h(k)\, x(n-k)$

$= \sum_{k=0}^{4} 1 . \alpha^{n-k}$

$$\left\{ \begin{array}{l} x(n) = \{\alpha^{-3}, \ldots \alpha^5\} \\ h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\} \end{array} \right.$$

$y(n) = \sum_{k=0}^{4} \alpha^{n-k}$

$\therefore y(n) = \sum_{k=0}^{4} x(n-k); \quad -3 \le n \le 9.$

$= 0$ otherwise.

$y(-3) \Rightarrow \sum_{k=0}^{4} x(-3-k)$

$= x(-3) + x(-4)^{0} + x(-5) + x($

$= \alpha^{-3}$

$y(-2) \Rightarrow \sum_{k=0}^{4} x(-2-k) = x(-2) + x(-3) + x(-4) + x(-5) + x(-6) = \alpha^{-3} + \alpha^{-2}$

$y(-1) \Rightarrow \sum_{k=0}^{4} x(-1-k) = x(-1) + x(-2) + x(-3) + 0$

$= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y(0) = \sum_{k=0}^{4} x(-k) \Rightarrow x(0) + x(-1) + x(-2) + x(-3)$

$= 1 + \alpha^{-1} + \alpha^{-2} + \alpha^{-3}$

$y(1) = \sum_{k=0}^{4} x(1-k) \Rightarrow x(1) + x(0) + x(-1) + x(-2) + x(-3) + x($

$= 1 + \alpha^{-1} + \alpha^{-2} + \alpha^{-3} + \alpha$

$y(2) \Rightarrow \sum_{k=0}^{4} x(2-k) \Rightarrow x(2) + x(1) + x(0) + x(2-3) + x(2-4)$

$\Rightarrow \alpha^2 + \alpha + 1 + \alpha^{-1} + \alpha^{-2}$

$y(3) \Rightarrow \sum_{k=0}^{4} x(3-k) \Rightarrow x(3) + x(2) + x(1) + x(0) + x(-1)$

$\Rightarrow \alpha^{-1} + 1 + \alpha + \alpha^2 + \alpha^3$

$y(4) \Rightarrow \sum_{k=0}^{4} x(4-k) \Rightarrow x(4) + x(3) + x(2) + x(1) + x(0)$

$= 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4$

$y(5) \Rightarrow \sum_{k=0}^{4} x(5-k) = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y(6) \Rightarrow \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$y(7) \Rightarrow \alpha^3 + \alpha^4 + \alpha^5$

$y(8) = \alpha^4 + \alpha^5$

$y(9) = \alpha^5$

20 Consider the following three operations

(a) Multiply the integer numbers; 131 & 222

(b) Compute the convolution of signals

$\{1, 3, 1\} * \{1, 2, 2\}$

(c) Multiply the polynomials:

$1+3x+x^2$ and $1+2x+2x^2$

(d) Repeat part (a) for the numbers 1.31 & 12.2

(e) Comment on your results.

Sol:- (a) $131 \times 122 = 15982$

(b) $\{1, 5, 9, 8, 2\}$

	1	3	1
1	1	3	1
2	2	6	2
2	2	6	2

(c) $1+5x+9x^2+8x^3+2x^4$

(d) $1.31 \times 12.2 = 15.982$

(e) These are different ways to perform the convolution operation.

21. Compute the convolution $y(n) = x(n) * h(n)$ of the following pair of signals

(a) $x(n) = a^n u(n)$; $h(n) = b^n u(n)$ where $a \neq b$ and when $a = b$.

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$x(n) = a^n u(n); \quad h(n) = b^n u(n)$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} a^k u(k)\, b^{n-k} u(n-k)$$

$$= \sum_{0}^{n} a^k b^{n-k} = b^n \sum_{0}^{n} (a/b)^k$$

$$= b^n \left[\frac{1-(a/b)^{n+1}}{1-a/b} \right]$$

$$= \frac{b^{n+1}\left[1-(a/b)^{n+1}\right]}{b-a} = \frac{b^{n+1}-a^{n+1}}{b-a} u(n) \quad a \neq b$$

$$y(n) = \begin{cases} \dfrac{b^{n+1}-a^{n+1}}{b-a} u(n) & a \neq b. \\ b^n (n+1) u(n); & a = b \end{cases}$$

(b) $x(n) = \begin{cases} 1 ; & n = -2, 0, 1 \\ 2 ; & n = -1 \\ 0 ; & \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

Sol:- $x(n) = \{1, 2, \underset{\uparrow}{1}, 1\};$

$h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$

	1	-1	0	0	1	1
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
1	1	-1	0	0	1	1
1	1	-1	0	0	1	1

(c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y(n) = \{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, -1\}$.

	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

(d) $x(n) = u(n) - u(n-5)$;

$h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$

<u>Sol:</u> $x(n) = \{0, \underset{\uparrow}{1}, 1, 1, 1\}$

$h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1\}$

$h'(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$h(n) = h'(n) + h'(n-9)$

$y(n) = y'(n) + y'(n-9)$

$y'(n) = x(n) * h'(n)$

$y'(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 4, 4, 3, 2, 1\}$

	1	1	1	1
0	0	0	0	0
0	0	0	0	0
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$y(n) = x * h$

	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1

$y(n) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 4, 4, 3, 2, 1, 1, 2, 3, 4, 4, 4, 3, 2, 1\}$

22. Let $x(n)$ be the input signal to a discrete-time filter with impulse response $h_i(n)$ and let $y_i(n)$ be the corresponding output.

(a) Compute and sketch $x(n)$ and $y_i(n)$ in the following cases using the same scale in all the figures.

$x(n) = \{\underset{\uparrow}{1}, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1(n) = \{1, 1\}$; $h_2(n) = \{1, 2, 1\}$;

$h_3(n) = \{\underset{\uparrow}{1/2}, 1/2\}$; $h_4(n) = \{1/4, 1/2, 1/4\}$

$h_5(n) = \{\underset{\uparrow}{1/4}, -1/2, 1/4\}$

Sketch $x(n), y_1(n), y_2(n)$ on one graph and $x(n), y_3(n), y_4(n), y_5(n)$ on another graph.

Sol:-

	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}$

$y_1(n) = x(n) + x(n-1)$

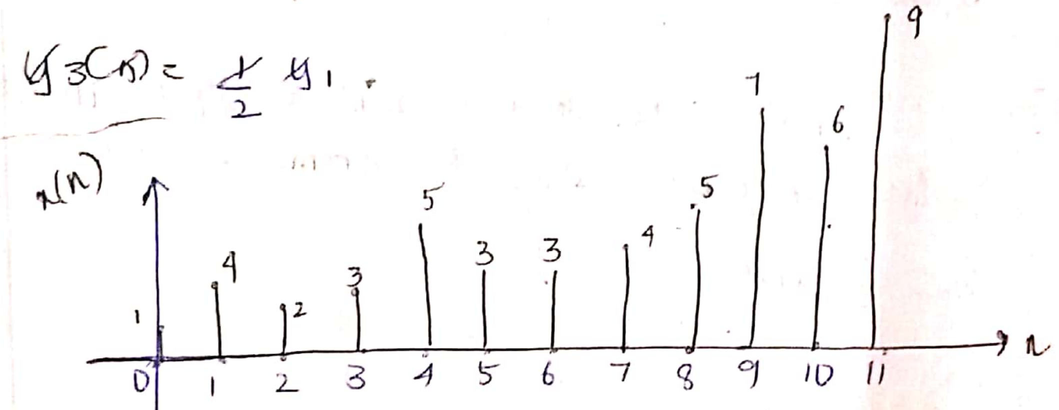
$= \{1, 5, 6, 5, 8, 8, 6, 9, 9, 12, 13, 15, 9\}$

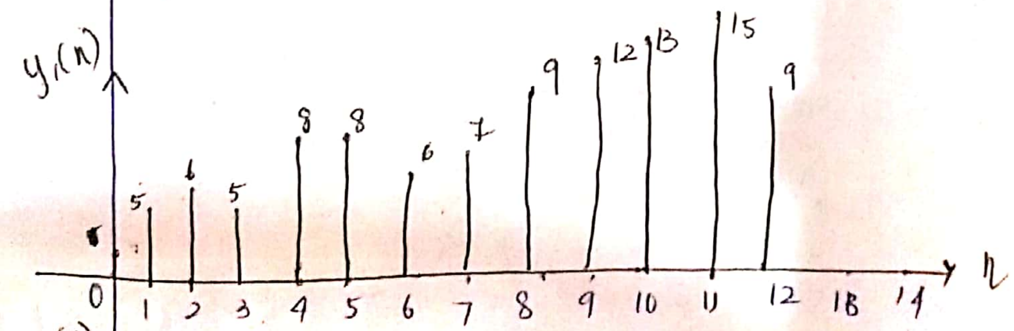
$y_2(n) =$

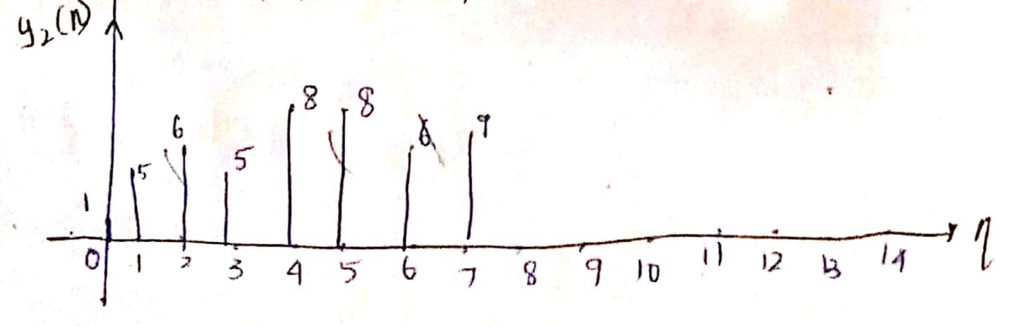
$\{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

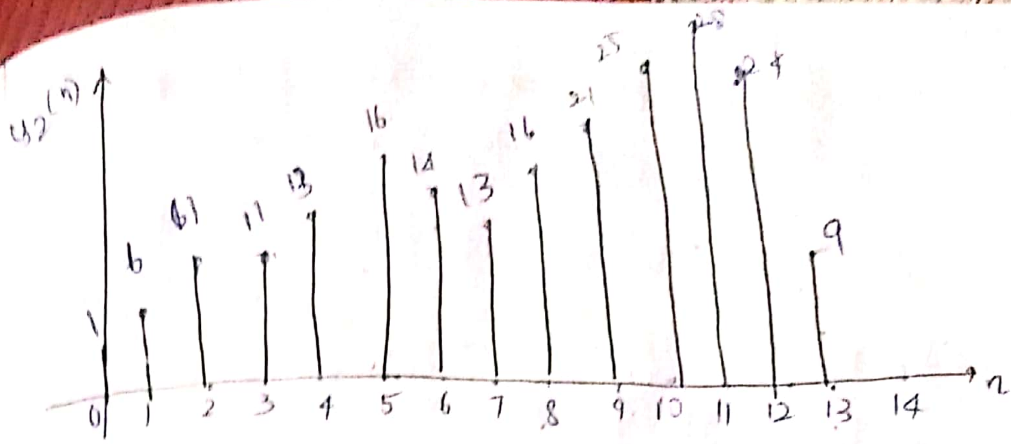
	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_3(n) = \frac{1}{2} y_1$

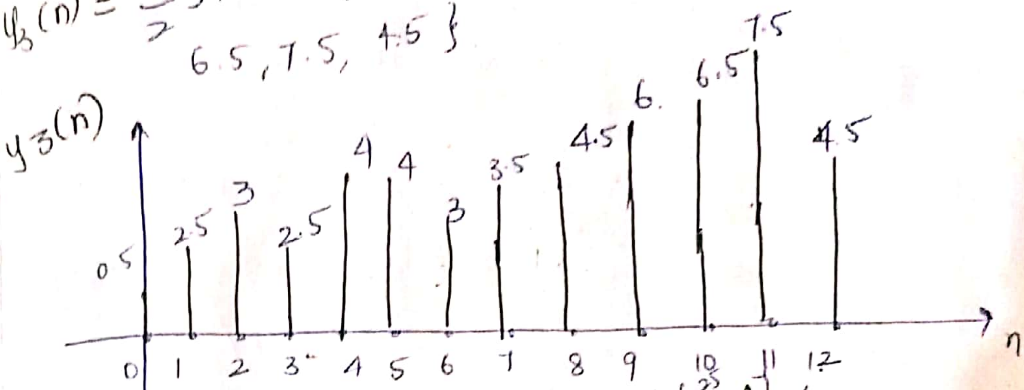


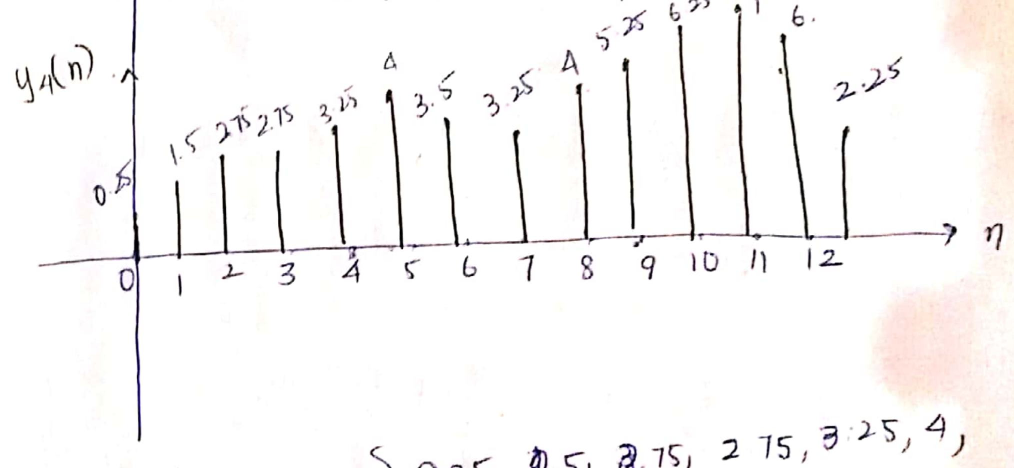






$y_3(n) = \frac{1}{2} y_1(n) = \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6.5, 7.5, 4.5\}$





$y_4(n) = \frac{1}{4} y_2(n) = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 4, 5.25, 6.25, 7, 6, 2.25\}$

(b) What is the difference between $y_1(n)$ and $y_2(n)$, and between $y_3(n)$ and $y_4(n)$?

Sol:- $y_3(n) = \frac{1}{2} y_1(n)$ because; $h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$ because, $h_4(n) = \frac{1}{4} h_2(n)$

(c) Comment on the smoothness of $y_2(n)$ and $y_4(n)$. Which factors affect the smoothness?

sol: $y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$; but $y_4(n)$ will appear even smoother because of the smaller scale factor.

(d) Compare on the $y_4(n)$ with $y_5(n)$. What is the difference? Can you explain it?

$\times \frac{1}{4}$

	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
-2	-2	-8	-4	-6	-10	-6	-6	-8	-10	-14	-12	-18
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_5(n) = \frac{1}{4}\{1, 2, -5, 3, 1, -4, 2, 1, 0, 1, -3, 4, -12, 9\}$

$= \{0.25, 0.5, -1.25, -0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, 2.25\}$

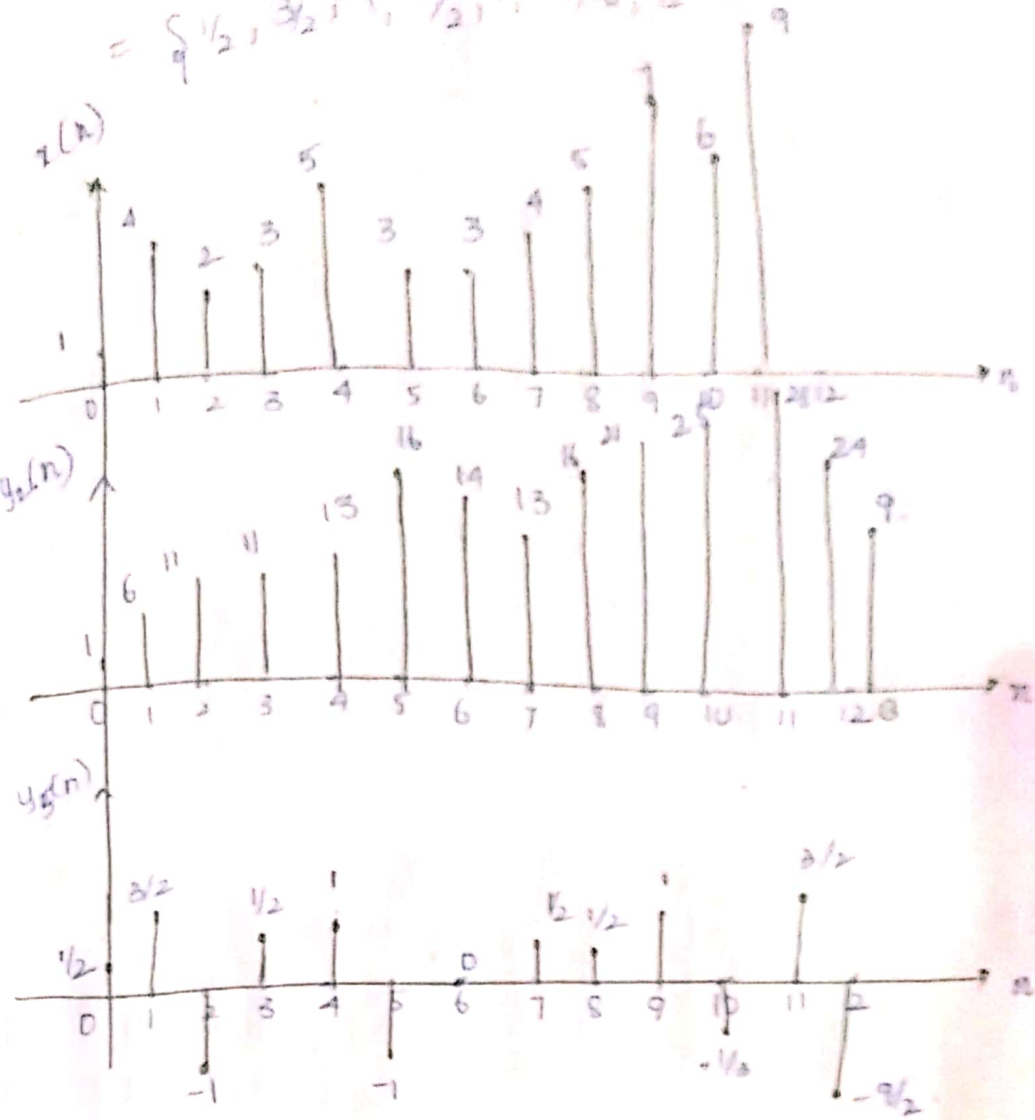
System '4' is more smoother than '5'. The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$.

(e) Let $h_6(n) = \{1/2, -1/2\}$. Compute $y_6(n)$. Sketch $x(n)$, $y_2(n)$ and $y_6(n)$ on the same figure and comment on the results.

	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
-1	-1	-4	-2	-3	-5	-3	-3	-4	-5	-7	-6	-9

$y_5(n) = \frac{1}{2}\{1, 3, -2, 1, 2, -2, 0, 1, 1, 2, -1, 3, -9\}$

$= \{1/2, 3/2, -1, 1/2, 1, -1, 0, 1/2, 1/2, 1, -1/2, 3/2, -9/2\}$



$y_2(n)$ is more smoother than $y_5(n)$ because of negative value in $h_6(n)$

23. Express the output $y(n)$ of a linear time-invariant system with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$

24. The discrete time system

$$y(n) = ny(n-1) + x(n), \quad n \geq 0$$

is at rest [i.e $y(-1) = 0$]. Check if the system is Linear Time Invariant and BIBO stable

Sol:- Let $x_1(n) \rightarrow$

$$y_1(n) = ny_1(n-1) + x_1(n)$$

$$x_2(n) \rightarrow y_2(n) = ny_2(n-1) + x_2(n)$$

Let $x(n) = a x_1(n) + b x_2(n)$

produces the output

$$y(n) = ny(n-1) + x(n), \text{ where}$$

$$y(n) = a y_1(n) + b y_2(n)$$

$y(n) =$

Hence the system is linear. If the input is we have

$$y(n-1) = (n-1)\,y(n-2) + x(n-1) \text{ But}$$

$$y(n-1) = n\,y(n-2) + x(n-1)$$

Hence, the system is time variant. If $x(n) = u(n)$; then $|x(n)| \le 1$. But for this bounded input, the output is

$y(0) = 1,\quad y(1) = 1+1 = 2, \ldots$ which is unbounded. Hence, the system is unstable.

Q5 Consider the signal $r(n) = a^n u(n)$; $0 < a < 1$

(a) Show that any sequence $x(n)$ can be decomposed as $x(n) = \sum_{n=-\infty}^{\infty} c_k\, r(n-k)$

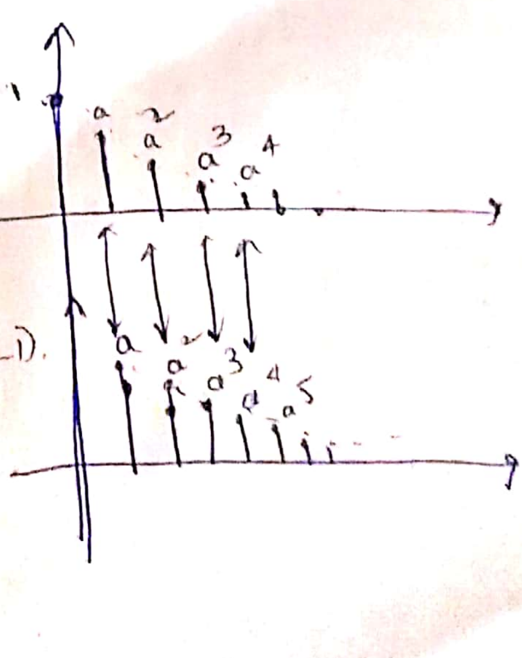
and express c_k in terms of $x(n)$.

Sol:-

$r(n) = a^n u(n)$

$\delta(n) = r(n) - a\,r(n-1)$ and,

$\delta(n-k) = r(n-k) - a\,r(n-k-1)$.



Then;

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\left[\gamma(n-k) - a\gamma(n-k-1)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-(k+1-1))$$

$$= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left[x(k) - ax(k-1)\right]\gamma(n-k)$$

Thus $c_k = x(k) - ax(k-1)$.

(b) $$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} c_k\gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k T\left[\gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k g(n-k)$$

(c) $$h(n) = T[\delta(n)] = T[\gamma(n) - a\gamma(n-1)]$$

$$= g(n) - ag(n-1)$$

Q. Determine the zero-input response of the system described by the second-order difference equation

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Sol:

$$x(n) = 0 \Rightarrow 3y(n-1) + 4y(n-2) = 0$$

$$\Rightarrow y(n-1) + \frac{4}{3}y(n-2) = 0$$

$$y(-1) = -\frac{4}{3}y(-2)$$

$$y(0) = -\frac{4}{3}y(-1) = \left(\frac{-4}{3}\right)^2 y(-2)$$

$$y(1) = \frac{-4}{3}y(0) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2) \rightarrow \text{zero-input response}$$

Q. Determine the particular solution of the difference equation

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$

when the forcing function is $x(n) = 2^n u(n)$.

Sol:- To determine particular solution; let us first obtain homogeneous solution i.e.

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = 0$$

for input $x(n)$ characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0 \Rightarrow \lambda = 1/2, 1/3$$

Hence, $y_h(n) = C_1(1/2)^n + C_2(1/3)^n$

The particular solution to

$$x(n) = 2^n u(n) \text{ is.}$$

$$y_p(n) = K(2^n)u(n)$$

Substitute this solution into the difference equation. Then, we obtain

$$K(2^n)u(n) - K(5/6)2^{n-1}u(n-1) + K(1/6)(2^{n-2})u(n-2) = 2^n u(n)$$

$$\Rightarrow K(2^2)u(n) - K(5/6) \times 2 \times u(n-1) + K(1/6)u(n-2) = 4u(n)$$

for $n = 2 \Rightarrow 4K - \frac{K5}{3} + \frac{K}{6} = 4$

$$\Rightarrow \frac{24K - 10K + K}{6} = 4$$

$$\Rightarrow 24K - 9K = 24$$

$$\Rightarrow 15K = 24 \Rightarrow \boxed{K = 8/5}$$

Therefore, the total solution is

$$y(n) = y_p(n) + y_h(n) = 8/5(2^n)u(n) + C_1(1/2)^n u(n) + C_2(1/3)^n u(n).$$

To determine C_1 & C_2, assume that

$$y(-1) = y(-2) = 0$$

$$\Rightarrow y(0) = 1, \quad y(1) = \frac{5}{6} + 2 = \frac{17}{6}$$

$\Rightarrow$ Thus $\frac{8}{5} + c_1 + c_2 = 1 \Rightarrow c_1 + c_2 = -3/5$

$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = -\frac{11}{5}$

and therefore, $c_1 = -1$; $c_2 = 2/5$

Hence; the total solution is

$$y(n) = \left[\frac{8}{5}(2)^n - (1/2)^n + 2/5(1/3)^n\right]u(n).$$

2. In example 2.4.8; equation (2.4.30) separate the output sequence $y(n)$ into the transient response and the steady state response. Plot these two responses for $a_1 = -0.9$.

Sol $y(n) = (-a_1)^{n+1} y(-1) + \frac{1-(-a_1)^{n+1}}{1+a_1}$; $n \geq 0$.

As $y(\infty) = 0 + \frac{1}{1+a_1} \rightarrow$ steady state response

Hence $\frac{1-(-a_1)^{n+1}}{1+a_1}$ steady state response.

at $n=0 \Rightarrow \frac{1+a_1}{1+a_1} = 1$

$n=1 \Rightarrow \frac{1-a_1^2}{1+a_1} = 1-a_1 = 1+0.9 = 1.9$

0.5129
0.729

$n=2 \Rightarrow \frac{1-(-a_1)^3}{1+a_1} \Rightarrow \frac{1+0.729}{0.1} = 17.29$

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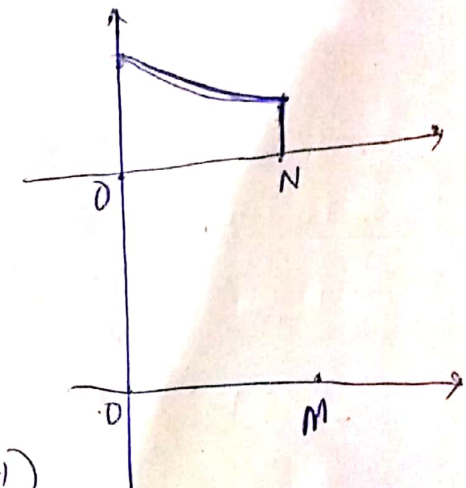
29 Determine the impulse response for the cascade of two linear time-invariant systems having impulse responses.

$h_1(n) = a^n[u(n) - u(n-N)]$ and

$h_2(n) = [u(n) - u(n-M)]$



$h_1(n) * h_2(n)$

$\Rightarrow a^n[u(n) - u(n-N)] *$

$[u(n) - u(n-M)]$

$= a^n[r(n-1) - r(n-N-1)$

$- r(n-M-1) + r(n-N+M-1))$

Determine the response $y(n), n \geq 0$ of the system described by the second-order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

to the input $x(n) = 4^n u(n)$.

The input to the system be λ^n

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = \lambda^n + 2\lambda^{n-1} \, 0$$

$\Rightarrow$

$\Rightarrow$ characteristic equation is

$$\lambda^2 - 3\lambda - 4 = 0 \Rightarrow \lambda = 4, -1 \text{ and}$$

$$y_h(n) = C_1(4)^n + C_2(-1)^n$$

Since '4' is the characteristic root and the excitation is

$x(n) = 4^n u(n)$; we assume a particular solution of the form

$$y_p(n) = kn4^n u(n)$$

Then

$$k[n4^n u(n) - 3(n-1)4^{n-1} u(n-1) - 4(n-2) \cdot 4^{n-2} u(n-2)]$$
$$= 4^n u(n) + 2(n-1)4^{n-1} u(n-1)$$

for $n = 2$

$$k[n4^2 u(n) - 3(n-1) \times 4 u(n-1) - 4(n-2) u(n-2)]$$
$$= 4^2 u(n) + 2(n-1) \times 4 u(n-1)$$

for $n = 2$

$$\Rightarrow k[32 - 12] = 16 + 8 \Rightarrow k = 6/5$$

The total solution is

$$y(n) = y_p(n) + y_h(n)$$
$$= \left[6/5\, n 4^n + c_1 4^n + c_2(-1)^n \right] u(n)$$

To solve for c_1 & c_2 let us assume that $y(-1) = y(-2) = 0$; Then;

$y(0) = 1$ and $c_1 + c_2 = 1$

$$y(1) = 3y(0) + 4 + 2$$

$y(1) = 9$ ∴ $\frac{6}{5} \times 4 + 4c_1 - c_2 = 9$

∴ $c_1 = \frac{26}{25}$ and $c_2 = \frac{-1}{25}$

Hence the total solution is

$$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n \right] u(n).$$

31. Determine the impulse response of the following casual system:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

Note:- When the input is impulse, particular solution doesnot exist; only homogeneous solution exists.

Hence the characteristic values are $\lambda = 4, -1$.

Hence $y_h(n) = c_1 4^n + c_2(-1)^n$.

When $x(n) = \delta(n)$; we find that

$y(0) = 1$; $y(1) = 3y(0) = 2$

$\Rightarrow y(1) = 5$ W

Hence
$$c_1 + c_2 = 1$$
$$4c_1 - c_2 = 5$$
$$5c_1 = 6 \Rightarrow c_1 = \frac{6}{5};\ c_2 = \frac{-1}{5}$$

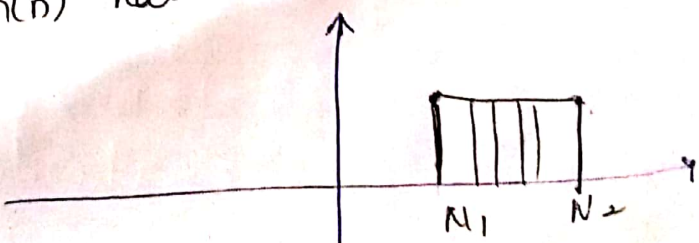
Hence $h(n) = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right]u(n)$.

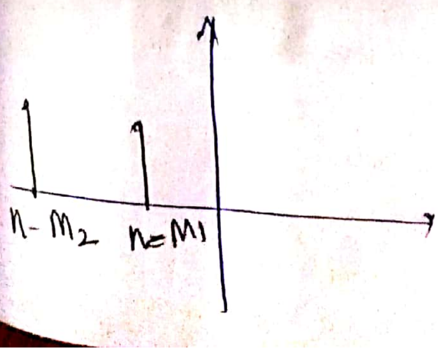
32. Let $x(n)$; $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite-duration signals.

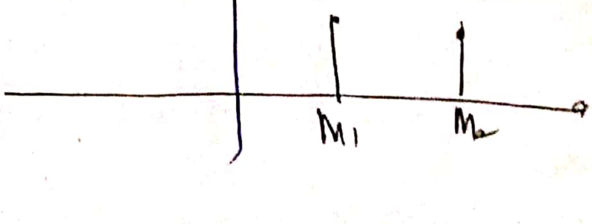
(a) Determine the range $L_1 \le n \le L_2$ of their convolution; in terms of N_1, N_2, M_1 and M_2.

Sol:- $L_1 = N_1 + M_1$; $L_2 = N_2 + M_2$

(b) Determine the limits of the cases of partial overlap from the left, full overlap and partial overlap from the right. For convenience assume that $h(n)$ has shorter duration than $x(n)$.







Complete overlap.

$n - M_2 < N_1 \Rightarrow n < N_1 + M_2$

$n - M_1 \not< N_2 \Rightarrow n < M_1 + N_2$

Low limit $= N_1 + M_2$

Higher Limit $= N_2 + M_1$

Partial overlap from left

$n - m_2 < N_1 \Rightarrow N_1 + M_2$

Low limit $= N_1 + M_1$

High limit $= N_1 + M_2 - 1$ [as included in complete]

Partial overlap from right.

low $= N_2 + M_1 + 1$ [complete overlap]

Higher $= N_2 + M_2$.

(c) Illustrate the validity of your results by computing the convolution of the signals

$$x(n) = \begin{cases} 1 ; & -2 \le n \le 4 \\ 0 ; & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2 ; & -1 \le n \le 2 \\ 0 ; & \text{elsewhere} \end{cases}$$

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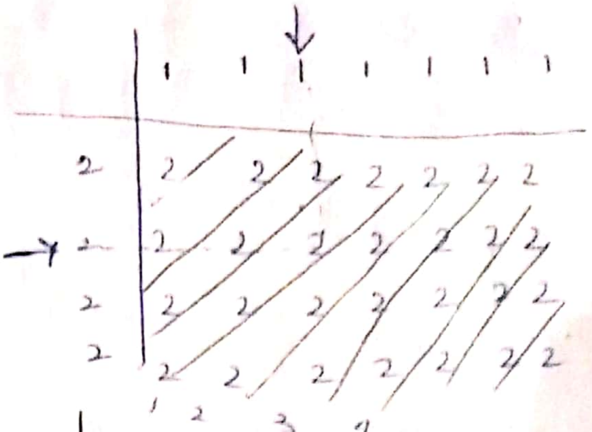


$\Rightarrow y(n) =$

$\{2, 4, 6, 8, 8, 8, 8, 6, 4, 2\}$

$2 \quad 8$

Here complete overlap $= 0 : 3 = 0 : 3$

partial " from left $= -3 : -1 = -3 : 1$

partial " from right $= 4 : 6 = 4 : 6$

$N_1 = -2$

$N_2 = 4$

$M_1 = -1$

$M_2 = 2$

complete $\Rightarrow N_1 + M_2 : M_1 + N_2 \Rightarrow 0 : 3$

partial from left $\Rightarrow N_1 + M_1 = -3$

$N_1 + M_2 - 1 = 1$

partial from right $\Rightarrow N_2 + M_1 + 1 = 4$

$N_2 + M_2 = 6.$

Hence verified.

33 Determine the impulse response and the unit step response of the systems described by the difference equation

(a) $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

Sol: Impulse response $x(n) = \delta(n)$.

For the impulse response; only the homogeneous solution exists; but not particular solution

Let input be λ^n & $y(n)$.

$\Rightarrow \quad \lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

Hence the characteristic equation is

$\lambda^2 - 0.6\lambda + 0.08 = 0. \Rightarrow \lambda = 0.2;\ 0.4$ Hence

$y_h(n) = C_1(0.2)^n + C_2(0.4)^n.$

With $x(n) = \delta(n)$; the initial conditions are

$y(0) = 1 = C_1 + C_2$

$y(1) = 0.6\,y(0) + 0 = C_1(0.2) + C_2(0.4)$

$\Rightarrow 0.2C_1 + 0.4C_2 = 0.6.$

$\Rightarrow C_1 + 2C_2 = 3.$

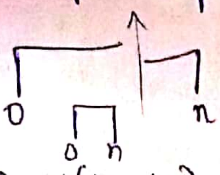
$C_1 + C_2 = 1$

$C_2 = 2 \ ; \ C_1 = -1$

Therefore $h(n) = \left[-1(0.2)^n + 2(0.4)^n\right]u(n).$

The step response is $u(n) * h(n)$

$\Rightarrow u(n) * \left[-(0.2)^n + 2(0.4)^n\right]u(n)$



as $u(k) \cdot u(n-k)$

$\downarrow$

defined only for 0 to n.

$\Rightarrow \sum_{k=0}^{n}\left[2(2/5)^{n-k} - (1/5)^{n-k}\right]$

$\Rightarrow$

$$\Rightarrow 2\left(2/5\right)^{n} \sum_{k=0}^{n} \left(2/5\right)^{-k} - \left(1/5\right)^{n} \sum_{k=0}^{n} \left(1/5\right)^{-k}$$

$$\Rightarrow 2\left(2/5\right)^{n} \left[\frac{1-\left(2/5\right)^{-(n+1)}}{1-\left(2/5\right)^{-1}}\right] - \left(\frac{1}{5}\right)^{n} \left[\frac{1-\left(1/5\right)^{-(n+1)}}{1-\left(1/5\right)^{-1}}\right]$$

$$\Rightarrow 2\left(2/5\right)^{n} \frac{\left[1-\left(2/5\right)^{-n-1}\right]}{-3/2} + \left(1/5\right)^{n} \times \frac{1}{4}\left[1-\left(1/5\right)^{-n-1}\right]$$

$$\Rightarrow \frac{-4}{3}\left[\left(2/5\right)^{n} - 5/2\right] + \frac{1}{4}\left[\left(1/5\right)^{n} - 5\right].$$

$$\Rightarrow \frac{-4}{3} \times \frac{2}{5}\left[\left(2/5\right)\right.$$

or

$$2\left(2/5\right)^{n} \sum_{k=0}^{n} \left(5/2\right)^{k} - \left(1/5\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$\Rightarrow 2\left(2/5\right)^{n} \left[\frac{1-\left(5/2\right)^{n+1}}{1-5/2}\right] - \left(\frac{1}{5}\right)^{n} \left[\frac{1-5^{n+1}}{1-5}\right].$$

$$\Rightarrow 2\left(2/5\right)^{n} \left[\frac{1-\left(\cancel{2}/\cancel{5}\right)^{-n-1}}{-3/2}\right] - \left(1/5\right)^{n} \left[\frac{1-\left(\cancel{1}/5\right)^{-n-1}}{1-5}\right]$$

$$\Rightarrow \frac{-\cancel{2}4}{3}\left[\left(2/5\right)^{n} - \right.$$

(b) $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$

Sol:- Only homogeneous solution exists

Hence the characteristic equation is

$\Rightarrow \quad \lambda^2 - 0.7\lambda + 0.1 = 0.$

$\lambda_1 = 1/2; \quad \lambda_2 = 1/5.$

$y_h(n) = C_1(1/2)^n + C_2(1/5)^n$

$y(0) = 2; \quad y(1) = 0.7(2) = 1.4.$

$C_1 + C_2 = 2; \quad \Rightarrow \quad C_1 + C_2 = 2$

$\frac{C_1}{2} + \frac{C_2}{5} = 1.4 \qquad C_1 + 0.4C_2 = 2.8$

$0.6C_2 = -0.8$

$C_1 = 2 + \frac{4}{3} = \frac{10}{3} \qquad C_2 = -\frac{4}{3}$

$\therefore h(n) = \left[\frac{10}{3}(1/2)^n - \frac{4}{3}(1/5)^n\right] u(n)$

$s(n) = \sum_{k=0}^{n} h(n-k)$

$= \frac{10}{3}(1/2)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}(1/5)^n \sum_{k=0}^{n} 5^k$

$= \frac{10}{3}\left(\frac{1}{2}\right)^n \left(\frac{1-2^{n+1}}{1-2}\right) - \frac{4}{3}(1/5)^n \left[\frac{1-5^{n+1}}{1-5}\right]$

$= -\frac{10}{3}\left[1/2 \right.$

$\Rightarrow \frac{10}{3} \times \left(\frac{1}{2}\right)^n \left[2^{n+1} - 1\right] u(n) - \frac{1}{3}(1/5)^n (5^{n+1} - 1) u(n)$

A Consider a system with impulse response

$$h(n) = \begin{cases} (1/2)^n ; & 0 \le n \le 4 \\ 0 ; & \text{elsewhere} \end{cases}$$

Determine the input $x(n)$ for $0 \le n \le 8$ that will generate the output sequence

$$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \dots\}$$

Sol:-

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1/2	$\frac{x_0}{2}$	$\frac{x_1}{2}$	$\frac{x_2}{2}$	$\frac{x_3}{2}$	$\frac{x_4}{2}$	$\frac{x_5}{2}$	$\frac{x_6}{2}$	$\frac{x_7}{2}$	$\frac{x_8}{2}$
1/4	$\frac{x_0}{4}$	$\frac{x_1}{4}$	$\frac{x_2}{4}$	$\frac{x_3}{4}$	$\frac{x_4}{4}$	$\frac{x_5}{4}$	$\frac{x_6}{4}$	$\frac{x_7}{4}$	$\frac{x_8}{4}$
1/8	$\frac{x_0}{8}$	$\frac{x_1}{8}$	$\frac{x_2}{8}$	$\frac{x_3}{8}$	$\frac{x_4}{8}$	$\frac{x_5}{8}$	$\frac{x_6}{8}$	$\frac{x_7}{8}$	$\frac{x_8}{8}$
1/16	$\frac{x_0}{16}$	$\frac{x_1}{16}$	$\frac{x_2}{16}$	$\frac{x_3}{16}$	$\frac{x_4}{16}$	$\frac{x_5}{16}$	$\frac{x_6}{16}$	$\frac{x_7}{16}$	$\frac{x_8}{16}$

Hence $x_0 = 1$; $x_1 + \frac{x_0}{2} = 2$

$\Rightarrow x_1 = 3/2$

$y_2 = x_2 + \frac{x_1}{2} + \frac{x_0}{4} = 2.5$

$\Rightarrow x_2 + \frac{3}{4} + \frac{1}{4} = 2.5 \Rightarrow x_2 = 1.5$

$y_3 \Rightarrow 3 = x_3 + \frac{x_2}{2} + \frac{x_1}{4} + \frac{x_0}{8} \Rightarrow 3 = x_3 + \frac{3}{4} + \frac{3}{8} + \frac{1}{8}$

$\Rightarrow x_3 = 7/4$

$y_4 \Rightarrow x_4 + \frac{x_3}{2} + \frac{x_2}{4} + \frac{x_1}{8} + \frac{x_0}{16} = 3$

$\Rightarrow x_4 \Rightarrow 3 - \frac{7}{8} - \frac{3}{8} - \frac{3}{16} - \frac{1}{16}$

$\Rightarrow x_4 = 3/2$

$y_5 \Rightarrow x_5 + \frac{x_4}{2} + \frac{x_3}{4} + \frac{x_2}{8} + \frac{x_1}{16} = 3$

$$\Rightarrow x_5 = 3 - \frac{3}{4} - \frac{7}{16} - \frac{3}{16} - \frac{3}{32} = \frac{49}{32}$$

$y_6 \Rightarrow x_6 + \frac{x_5}{2} + \frac{x_4}{4} + \frac{x_3}{8} + \frac{x_2}{16} = 2$

$$\Rightarrow x_6 = 2 - \frac{49}{64} - \frac{3}{8} - \frac{7}{32} - \frac{3}{64} = \frac{19}{32}$$

$y_7 \Rightarrow x_7 + \frac{x_6}{2} + \frac{x_5}{4} + \frac{x_4}{8} + \frac{x_3}{16} = 1$

$$\Rightarrow x_7 = 1 - \frac{7}{32} - \frac{3}{16} - \frac{49}{32 \times 4} - \frac{19}{32 \times 2}$$

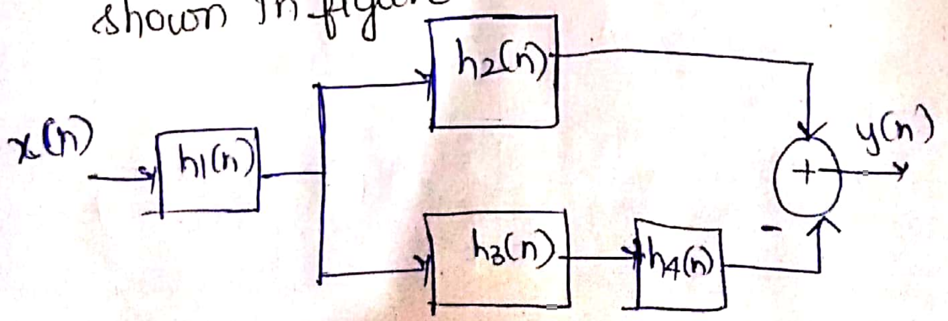
$$= \frac{-11}{128}$$

$x_8 \Rightarrow x_8 + \frac{x_7}{2} + \frac{x_6}{4} + \frac{x_5}{8} + \frac{x_4}{16} = 0$

$$\Rightarrow x_8 = -\frac{3}{2 \times 16} - \frac{49}{32 \times 8} - \frac{19}{32 \times 4} + \frac{11}{128 \times 2}$$

$$x_8 = \frac{-25}{64}$$

35 Consider the interconnection of LTI systems as shown in figure



(a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$.

$$h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$$

(b) Determine $h(n)$ when

$$h_1(n) = \{\underset{\uparrow}{1/2}, 1/4, 1/2\}$$

$$h_2(n) = h_3(n) = (n+1)u(n);$$

$$h_4(n) = \delta(n-2)$$

Sol -

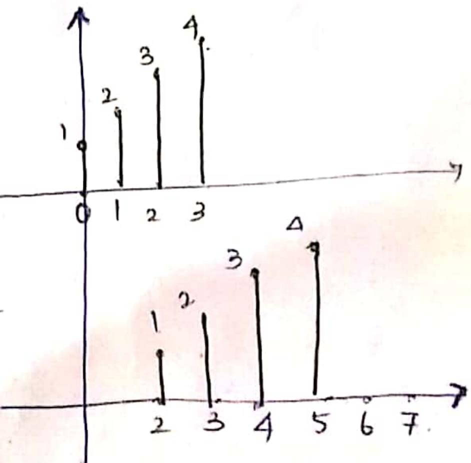
$$h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2).$$

$$= (n-1)u(n-2). = x(n)$$

$h_1(n) * h_2(n) \Rightarrow$

$$h_2(n) - x(n) = (n+1)u(n) - (n-1)u(n-2).$$

$$\Rightarrow 2u(n) - \delta(n)$$



$$h(n) = \left[\tfrac{1}{2}\delta(n) + \tfrac{1}{4}\delta(n-1) + \tfrac{1}{2}\delta(n-2)\right] * [2u(n) - \delta(n)]$$

$$= u(n) - \tfrac{1}{2}\delta(n) + \tfrac{1}{2}u(n-1) - \tfrac{1}{4}\delta(n-1)$$

$$+ u(n-2) - \tfrac{1}{2}\delta(n-2)$$

$$= \left[\delta(n) - \tfrac{1}{2}\delta(n)\right] + \left[\delta(n-1) + \tfrac{1}{2}\delta(n-1) - \tfrac{1}{4}\delta(n-1)\right]$$

$$+ u(n-2) + \tfrac{1}{2}u(n-2) + u(n-2)$$

$$= \tfrac{1}{2}\delta(n) + \tfrac{5}{4}\delta(n-1) + \left[-\tfrac{1}{2}\delta(n-2) + \tfrac{5}{2}\delta(n-2)\right] + \tfrac{5}{2}u(n-3)$$

$= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

(c) Determine the response of the system in part (b) if $x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

Sol:- $y(n) = x(n) * h(n)$

$y(n) = h(n+2) + 3h(n-1) - 4h(n-3)$

$\Rightarrow \frac{1}{5}\delta(n+2) + \frac{15}{4}\delta(n-2) + 2$

$\Rightarrow \frac{1}{2}\delta(n+2) + \frac{5}{4}\delta(n+1) + 2\delta(n) + \frac{5}{2}u(n-1)$

$+ \frac{3}{2}\delta(n-1) + \frac{15}{4}\delta(n-2) + 6\delta(n-3) + \frac{15}{2}u(n-4)$

$- 2\delta(n-3) - 5\delta(n-4) - 8\delta(n-5) - 10u(n-6)$

$n=-2 \Rightarrow 1/2$; $n=-1 \Rightarrow 5/4$; $n=0 \Rightarrow 2$

$n=1 \Rightarrow 5/2 + 3/2 = 4$

$n=2 \Rightarrow \frac{15}{4} + \frac{5}{2} = \frac{25}{4}$

$n=3 \Rightarrow 6 - 2 + 5/2 = 4 + 5/2 = \frac{13}{2}$

$n=4 \Rightarrow 5/2 + \frac{15}{2} - 5 = \frac{15}{2} - 5/2 = 5$

$n=5 \Rightarrow 5/2 + 15/2 - 8 = 2$

$n=6 \Rightarrow 5/2 + 15/2 - 10 = 0$; $n \geq 7 = 0$ ---

$y(n) = \{ 1/2, 5/4, \underset{\uparrow}{2}, 4, \frac{25}{4}, 13/2, 5, 2, 0 \ldots \}$

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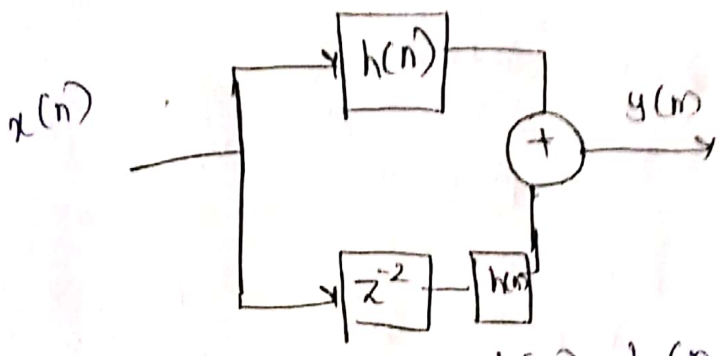
2.36] Consider the system in figure 36 with $h(n) = a^n u(n)$; $-1 < a < 1$. Determine the response $y(n)$ of the system to the excitation

$$x(n) = u(n+5) - u(n-10)$$

Sol:-



$\Rightarrow$ $g(n) = h'(n) = h(n) - h(n-2)$.

$$= a^n u(n) - a^{n-2} u(n-2).$$

$$y(n) = x(n) * h'(n)$$

$$= a^n u(n) * u(n+5) - a^{n-2} u(n-2) * u(n+5)$$

$$- a^n u(n) * u(n-10) + a^{n-2} u(n-2) * u(n-10)$$

(1) $= \sum_{k=-\alpha}^{\alpha} u(k+5) \cdot a^{n-k} u(n-k)$

$$= \sum_{-5}^{n} a^{n-k} = a^n \left[\sum_{-5}^{n} a^{-k} \right]$$

$\sum_{-n}^{5} a^k$ $\Rightarrow \frac{1-a^{n+6}}{1-a}$ $\Rightarrow$ $a^n \left[a^5 + a^4 + \cdots + a^{-n} \right]$ $\Rightarrow a^n \left[\frac{1-a^{n+5}}{1-} \right]$

$\Rightarrow$ $a^n \left[\frac{1 - (1/a)^{n+5}}{1 - 1/a} \right]$ $\Rightarrow 1 + a + \cdots + a^{n+5}$

$\Rightarrow 1 - a$

$\sum_{-n}^{+5} a^k$ $\Rightarrow$ $\frac{a^{n+1}}{a^{n+5}} \left[\frac{a^{n+5} - 1}{a-1} \right] = \frac{a^{n+1}}{a-1}$ $\Rightarrow \frac{1 - a^{n+6}}{1-a} u(n+5)$

$1 + a^{n+6}$

$\Rightarrow$

$\Rightarrow$

$$\text{ie}\ \sum_{k=-5}^{n} a^{n-k} = a^{n}\left[a^{5} + a^{4} + \dots + a^{-n}\right]$$

$$= 1 + a + \dots + a^{n+5}$$

$$= \frac{1-a^{n+6}}{1-a} = \frac{a^{n+6}-1}{a-1}\, u(n+5)$$

$$- a^{n-2}u(n-2) * u(n+5)$$

$$= \sum_{-\infty}^{\infty} u(k+5)\, a^{n-2-k}\, u(n-2-k)$$

$$= \sum_{-5}^{n-2} a^{n-2-k} = \sum_{-3}^{n} a^{n-\cancel{2}-k+\cancel{2}} = \sum_{k=-3}^{n} a^{n-k}$$

$$= 1 + a + \dots + a^{n-2+5}$$

$$= 1 + a + \dots + a^{n+3} = \frac{a^{n+4}-1}{a-1}\, u(n+3)$$

$$+ a^{n}u(n) * u(n-10) = \sum_{-\infty}^{\infty} u(k-10)\, a^{n-k}\, u(n-k)$$

$$= \sum_{10}^{n} a^{n-k}$$

$$= 1 + \dots + a^{n-10}$$

$$= \frac{a^{n-9}-1}{a-1}\, u(n-10)$$

$$- a^{n-2}u(n-2) * u(n-10) = \sum_{k=-\infty}^{\infty} u(k-10)\, a^{n-2-k}\, u(n-2-k)$$

$$= \sum_{10}^{n-2} a^{n-2-k} = \sum_{12}^{n} a^{n-k} = 1 + \dots + a^{n-12}$$

$$= \frac{a^{n-11}-1}{a-1}\, u(n-12)$$

Hence $y(n) = \frac{a^{n+16}-1}{a-1} u(n+15) - \frac{a^{n-9}-1}{a-1} u(n-10)$

$- \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-12)$

3.7] Compute and sketch the step response of the system

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

sol:- $h(n) = \frac{1}{M} [u(n) - u(n-M)]$

$s(n) = u(n) * \frac{1}{M} [u(n) - u(n-M)]$

$= \frac{1}{M} \left[\sum_{k=0}^{n} 1 - \sum_{M}^{n} 1 \right]$

if $n \geq M$ $= \frac{1}{M} [n+1 - [n-M+1]] = 1$

if $n < M \Rightarrow s(n) = \frac{1}{M} \left[\sum_{k=0}^{n} 1 - 0 \right]$

$= \frac{1}{M} (n+1) = \frac{n+1}{M}$

Hence $s(n) = \begin{cases} \frac{n+1}{M} ; & n < M \\ 1 ; & n \geq M. \end{cases}$

3] Determine the range of value of the parameter "a" for which the linear time-invariant system with impulse response

$$h(n) = \begin{cases} a^n; & n \geq 0;\ n\ \text{even} \\ 0; & \text{otherwise.} \end{cases}$$

is stable.

Sol:- $\sum_{n=-\alpha}^{\alpha} |h(n)| = \sum_{n=0;\ n\ \text{even}}^{\alpha} |a|^n$

$$= \sum_{0}^{\alpha} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

Stable if $|a| < 1$.

29] Determine the response of the system with impulse response $h(n) = a^n u(n)$ to the input signal $x(n) = u(n) - u(n-10)$

Sol:- $h(n) = a^n u(n)$. The response to $h(n)$ is

$$y_1(n) = \sum_{k=0}^{\alpha} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k} = a^n \left[1 + a + \dots + a^n\right]$$

$$= \frac{1-a^{n+1}}{1-a} u(n).$$

Then $y(n) = y_1(n) - y_1(n-10)$

$$= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right].$$

40] Determine the response of the (relaxed) system characterized by the impulse response $h(n) = (\frac{1}{2})^n u(n)$ to the input signal

$$x(n) = \begin{cases} 1; & 0 \leq n < 10 \\ 0; & \text{otherwise.} \end{cases}$$

sol- Similar to above problem; with $a = \frac{1}{2} = 0.5$

$$\therefore y(n) = \frac{1}{1-0.5}\left[\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\tfrac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

$$= 2\left[\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)u(n) - \left(1-\left(\tfrac{1}{2}\right)^{n-9}\right)u(n-10)\right]$$

41] Determine the response of the (relaxed) system characterized by the impulse response $h(n) = (\frac{1}{2})^n u(n)$ to the input signals

(a) $x(n) = 2^n u(n)$

(b) $x(n) = u(-n)$.

Sol:- (a) $y(n) = x(n) * h(n)$

$$= \sum_{k=-\alpha}^{\alpha} (1/2)^{k} u(k) \cdot 2^{n-k} u(n-k)$$

$$= 2^{n} \sum_{0}^{n} (1/2)^{2k} = 2^{n} \sum_{0}^{n} (1/4)^{k}$$

$$= 2^{n} \frac{1-(1/4)^{n+1}}{3/4}$$

$$= \frac{2}{3}\left[2^{n+1} - 2^{-2n-2+n}\right]$$

$$= \frac{2}{3}\left[2^{n+1} - (1/2)^{n+1}\right] u(n)$$

(b) $y(n) = x(n) * h(n)$

$$= \sum_{k=-\alpha}^{\alpha} (1/2)^{k} u(k) \cdot u(-$$

$$= \sum_{k=-\alpha}^{\alpha} u(-k) (1/2)^{n-k} u(n-k)$$

$k-n$

$$= \sum_{k=0}^{-\alpha} (1/2)^{n-k} =$$

$$= 1 + 1/2 + \ldots + 1/2$$

$$= 1 + 2 + \ldots$$

$$= \sum_{k=2}^{\alpha} (1/2)^{k} u(k) \, u(n+k)$$

Case 1 ⇒ $n < 0$ ⇒ $\sum_{k=0}^{\infty} (1/2)^k = 2$ → $\underline{\underline{n=0}}$

Case 2 ⇒ $n \geq 0$ ⇒ $\sum_{k=0}^{-\infty} (1/2)^{n-k}$ ie

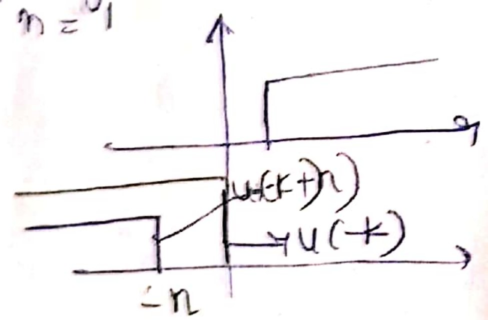
$= (1/2)^n + (1/2)^{n+1} + \ldots$

$= (1/2)^n [1 + 1/2 + 1/2^2 + \ldots]$

$= (1/2)^n [2] = 2(1/2)^n; \quad n \geq 0.$

Reason- $u(n-k)$ ⇒ shift by $u(k) \to u(n+k)$

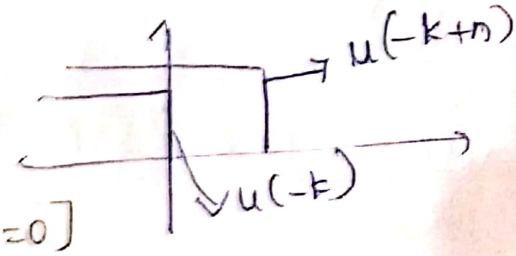
$n = 1$



Case 2 ⇒

$u(-k+n)$

limits consider ⇒ n



for $n = -1$

Case 1 limits: $-\infty$ to 0. [n=0]

↓

Last. Hence

42 Three systems with impulse responses $h_1(n) = \delta(n) - \delta(n-1)$; $h_2(n) = h(n)$ and $h_3(n) = u(n)$ are connected in cascade.

(a) What is the impulse response; $h_c(n)$ of the overall system?

Sol:- $h_c(n) = h_1(n) * h_2(n) * h_3(n)$

$= [h_1(n) * h_3(n)] * h_2(n)$ [commutative property]

$= [\delta(n) - \delta(n-1)] * u(n) * h_2(n)$

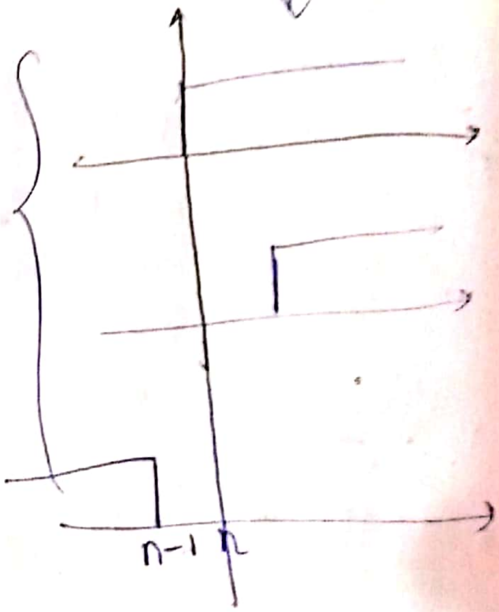
$= [u(n) - u(n-1)] * h_2(n)$

Not needed $\Big\{ = \sum_{k=-\alpha}^{\alpha} h(n)\, u(n-k) - \sum_{k=-\alpha}^{\alpha} h(n)\, u(n-k+1)$

Method 1

$= \sum_{k=0}^{n} h(k) - \sum_{k=0}^{n-1} h(k)$ explanation

$= h(n)$



Method 2:-

$u(n) - u(n-1) = \delta(n)$

$\Rightarrow \delta(n) * h(n)$

$= h(n)$

(4.) Does the order of the interconnection affect the overall system?

Sol:- No, the order of the interconnection doesnot affect the overall system

43] (a) Prove and explain graphically the difference between the relations

$$x(n)\cdot\delta(n-n_0) = x(n_0)\,\delta(n-n_0) \quad \text{and}$$

$$x(n) * \delta(n-n_0) = x(n-n_0)$$

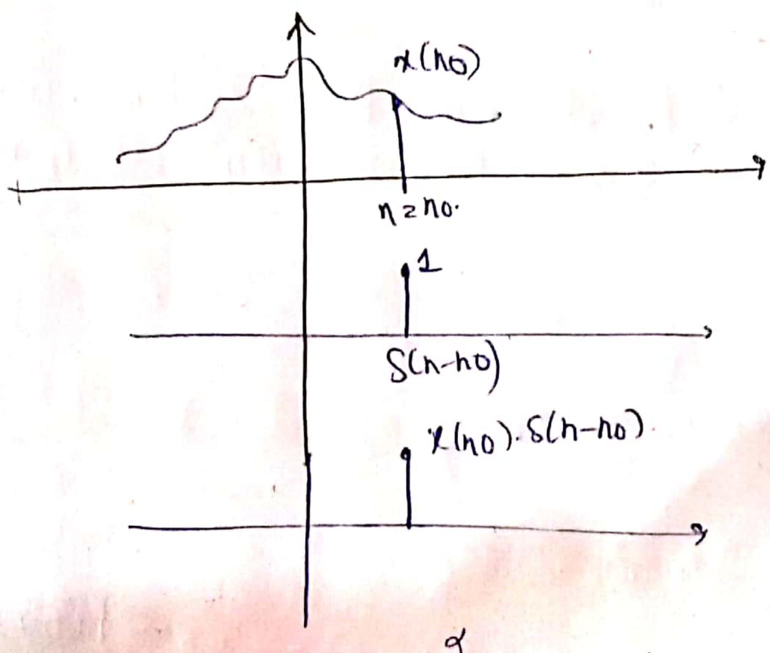
As $\quad x(n) = \sum_{k=-\alpha}^{\alpha} x(k)\,\delta(n-k)$

$$x(n)\cdot\delta(n-n_0) = \sum_{k=-\alpha}^{\alpha} x(k)\,\delta(n-k) \cdot \delta(n-n_0)$$

$$\text{at } n = n_0 \Rightarrow \sum_{k=-\alpha}^{\alpha} x(k)$$

$$\Rightarrow \left[x(-\alpha)\,\delta(n+\alpha) + \dots + x(n_0)\,\delta(n-n_0) + \dots\ x(\alpha)\,\delta(n-\alpha)\right] \cdot \delta(n-n_0)$$

at $n = n_0$ $= $ Then $\quad x(n_0)\cdot\delta(n-n_0)$



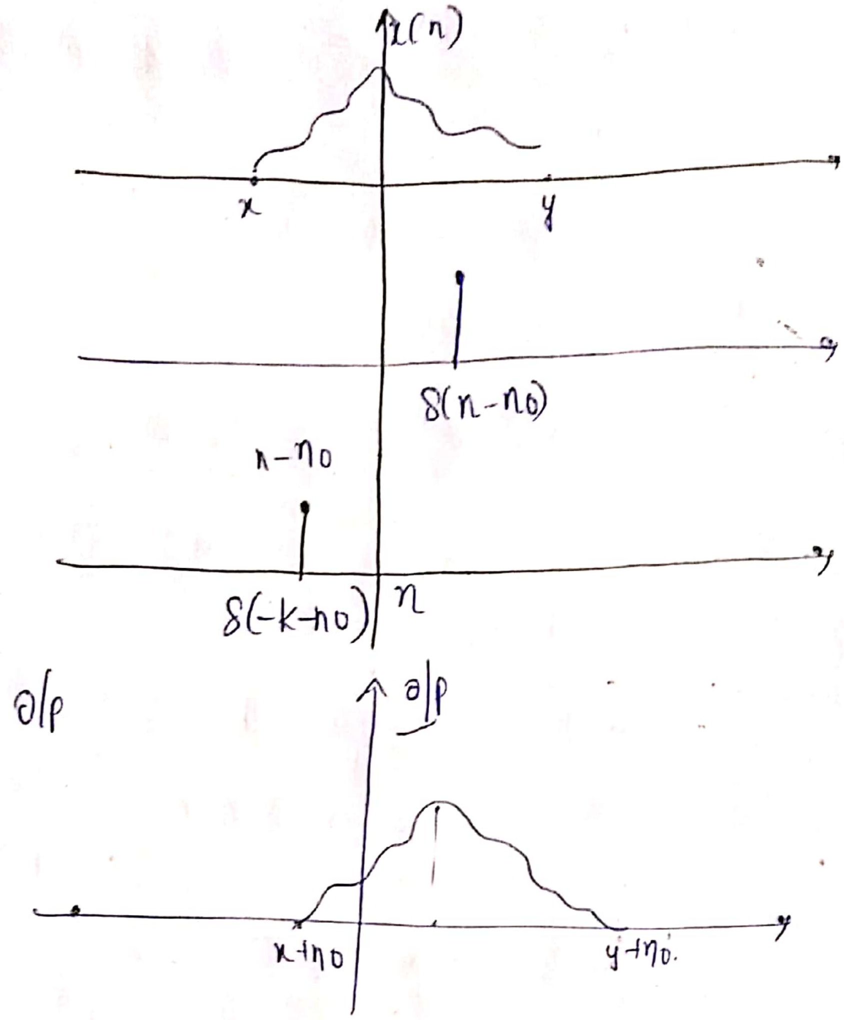
(b) $x(n) * \delta(n-n_0) = \sum_{k=-\alpha}^{\alpha} x(k)\,\delta(n-k-n_0)$

$$= \sum_{k=-\alpha}^{\alpha} x(k)\cdot\delta(n-n_0-k)$$

as $\delta(n)$ even function

$$= \sum_{k=-\alpha}^{\alpha} x(n-n_0)\,\delta(k-n+n_0)$$

$$= x(n-n_0)$$



(b) Show that the discrete time system ; which is described by a convolution summation, is LTI and relaxed.

Sol:- $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) = h(n) * x(k)$

Linearity $x_1(n) \to y_1(n) = h(n) * x_1(n)$

$x_2(n) \to y_2(n) = h(n) * x_2(n)$

Then $x(n) = \alpha x_1(n) + \beta x_2(n) \to y(n) = h(n) * x(n)$

$y(n) = h(n) * [\alpha x_1(n) + \beta x_2(n)]$

$= \alpha y_1(n) + \beta y_2(n)$

Hence the given system is linear.

Time invariance $x(n) \rightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$$= \sum_k h(k)\, x(n-n_0-k)$$

$$= y(n-n_0)$$

Hence it is time Invariant; hence it is an LTI system

Relaxed?

(c) What is the impulse response of the system described by $y(n) = x(n-n_0)$?

sol:- $h(n) = \delta(n-n_0)$ is the impulse response of the system.

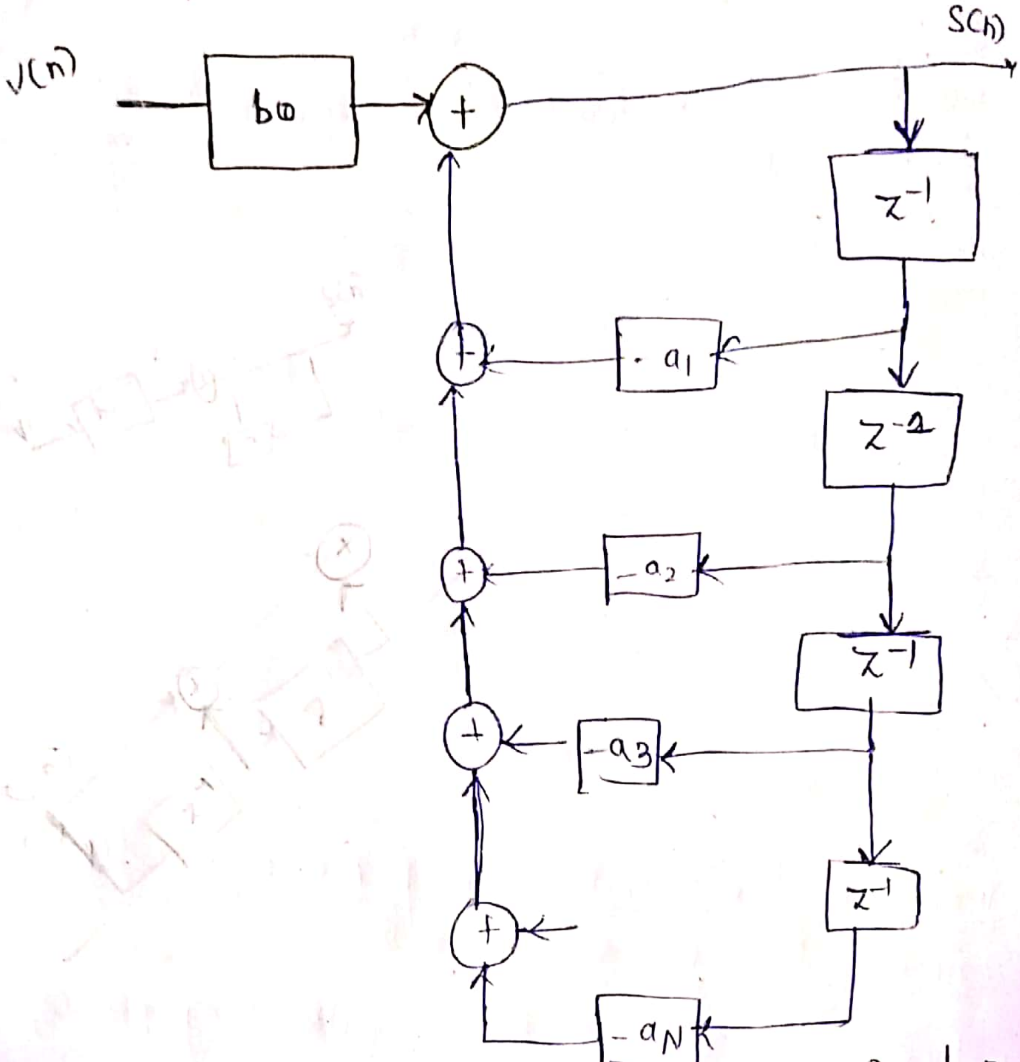
4] Two signals $s(n)$ and $v(n)$ are related through the following difference equations.

$$s(n) + a_1 s(n-1) + \cdots + a_N s(n-N) = b_0 v(n).$$

Design the block diagram realization of

(a) The system that generates $s(n)$ when excited by $v(n)$

Sol:- $s(n) = -a_1 s(n-1) - a_2 s(n-2) - \dots - a_N s(n-N)$
$+ b_0 v(n)$.



(b) The system that generates $v(n)$ when excited by $s(n)$

$$v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + a_2 s(n-2) + \dots + a_N s(n-N)\right].$$

(c) What is the impulse response of the cascade interconnection of system in parts a and b?

Sol:- It has only homogeneous solution

$y_h(n) =$

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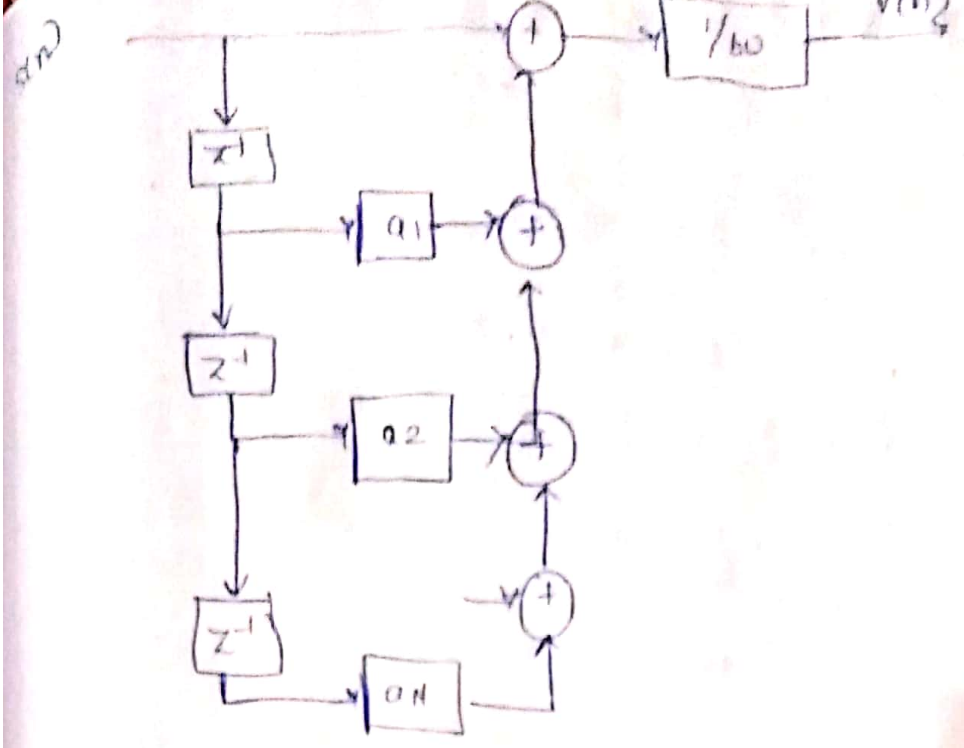


$$\lambda^n + a_1\lambda^{n-1} + \dots + a_N\lambda^{n-N} = 0$$

$$\lambda^N + a_1\lambda^{N-1} + \dots + a_N = 0 \rightarrow \text{characteristic equation}$$

Let roots be $\lambda_0, \lambda_1, \dots, \lambda^{N-1}$

solution is $c_1\lambda_0^n + c_2\lambda_1^n + \dots + c_N\lambda^{N-1}$

out ⁄⁄

(b) $v(n) = \frac{1}{b_0}\left[s(n) + a_1 s(n-1) + \dots + a_N s(n-1)\right]$

both have same → homogeneous solution is

$v(n) = 0 \Rightarrow$ no solution at all

45 Compute the zero state response of the system described by the difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$$

to the input $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$

by solving the difference equation recursively.

Sol:- $y(n) = -\frac{1}{2} y(n-1) + x(n) + 2x(n-2)$

As x starts at -2

$y(-2) = -\frac{1}{2} y(-3) + x(-2) + 2x(-4)$

$= 0 + 1 + 2(0) = 1$

$y(-1) = -\frac{1}{2} y(-2) + x(-1) + 2x(-3)$

$= -\frac{1}{2}(1) + 2 = 3/2$

$y(0) = -\frac{1}{2} y(-1) + x(0) + 2x(-2)$

$= -\frac{1}{2}(3/2) + 3 + 2(1)$

$= 5 - 3/4 = 17/4$

$y(1) = -\frac{1}{2} y(0) + x(1) + 2x(-1)$

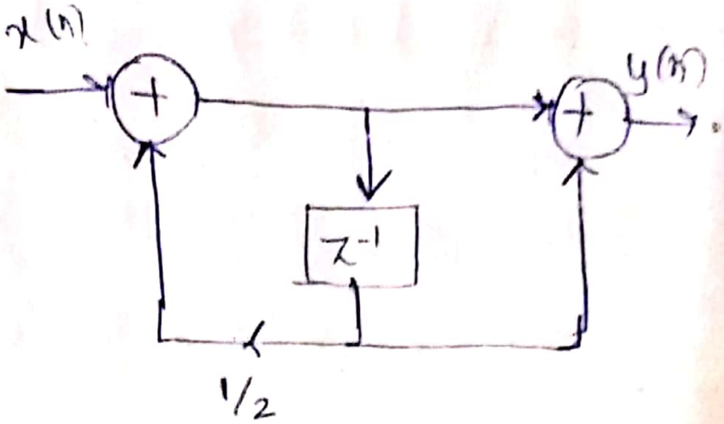
$= -\frac{1}{2}(17/4) + 4 + 2(2)$

$= 8 - \frac{17}{8} = \frac{47}{8}$ etc..

Note:- We have considered from $y(-2) \Rightarrow$ as $y(-2)$ depends on $x(-2)$ where x starts $\Rightarrow$ $y(-3)$ depends on $x(-3)$ which is zero; hence we start from $y(-2)$ //

47. Consider the discrete time system shown in figure below.

Sol:-



48] Consider the system described by the difference equation.

$$y(n) = a y(n-1) + b x(n).$$

(a) Determine b in terms of a so that

$$\sum_{n=-\alpha}^{\alpha} h(n) = 1.$$

Homogeneous solution ⇒ [Impulse response]

⇒ $y^{(n)} - a\lambda^{n-1} = 0$ ⇒ a

⇒ $h(n) = c_1 a^n u(n)$

$y(0) = b$ ⇒ c_1 ⇒ $h(n) = b a^n u(n)$

$$\sum_{n=-\alpha}^{\alpha} h(n) = \frac{b}{1-a} = 1 \Rightarrow \boxed{b = 1-a}$$

(b) Compute the zero-state step response $s(n)$ of the system and choose b so that $s(\alpha) = 1$.

Sol:-

$$s(n) = \sum_{k=0}^{n} h(n-k) = b.\sum_{k=0}^{n} a^{n-k}.$$

$$= b.\left[1 + a + - + a^n\right]$$

$$= b\left[\frac{1 + a^{n+1}}{1-a}\right]u(n)$$

$$s(\alpha) = \frac{b}{1-a} = 1 \Rightarrow b = 1-a.$$

(c) [Compute the zero-state res]. Compare the values of b obtained in parts (a) and (b) What did you notice?

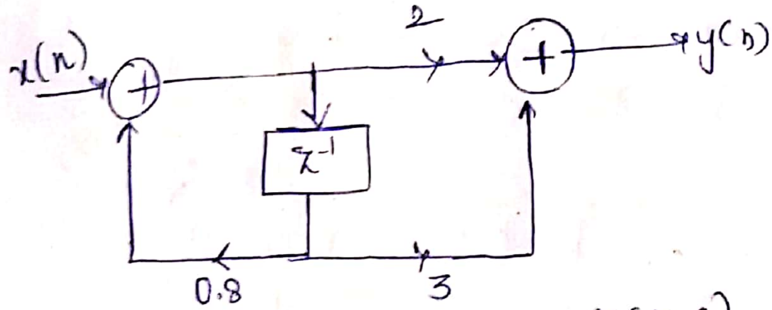
Sol:- $b = 1-a$; In both cases they are equal.

49] A discrete-time system is realized by the structure shown in figure.

(a) Determine the impulse response.

(b) Determine a realization for its inverse system that is; the system which produces $x(n)$ as an output when $y(n)$ is used as an input

Sol:



$$y(n) = 0.8\,y(n-1) + 2x(n) + 3x(n-1).$$

The characteristic equation is.

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8.$$

$$y_h(n) = c\,(0.8)^n$$

Let us first consider the response of the system.

$y(n) - 0.8\,y(n-1) = x(n)$; to $x(n) = \delta(n)$

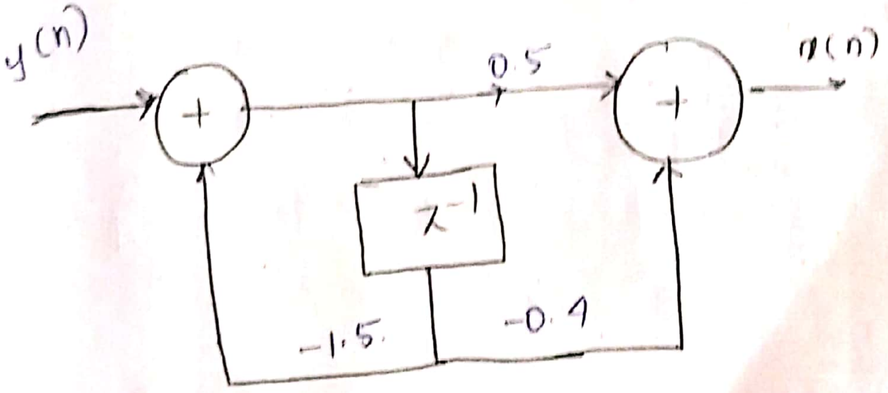
Since $y(0) = 1$; it follows that $c = 1$; Then the impulse response of the original system is

$$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$$

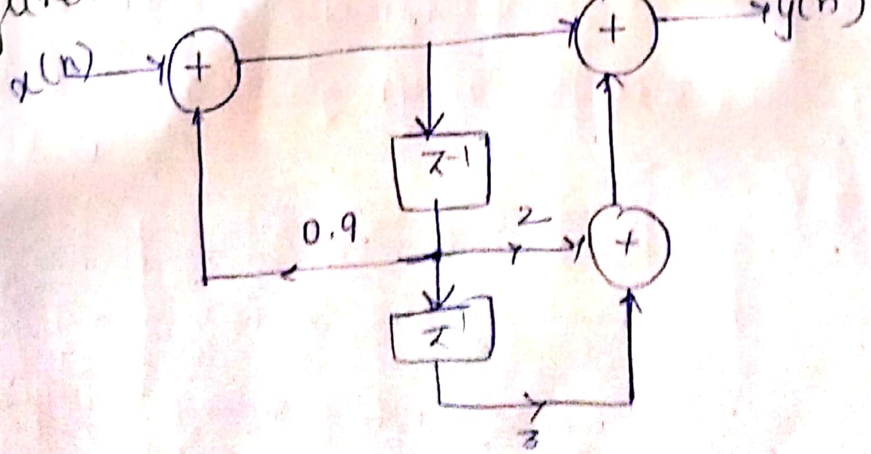
$$= 2\,\delta(n) + 4.6(0.8)^n u(n-1)$$

(b) The Inverse system is characterized by the difference equation.

$$x(n) = -1.5\,x(n-1) + \frac{1}{2}\,y(n) - 0.4\,y(n-1)$$



50. Consider the discrete-time system shown in figure.



(a) Compute the first six values of the impulse response of the system.

(b) Compute the first six values of the zero-state step response of the system.

(c) Determine an analytical expression for the impulse response of the system.

Sol:- (a) $y(n) = 0.9y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

For $x(n) = \delta(n)$, we have

$y(0) = 1$

$y(1) = 0.9(1) + 2 = 2.9$

$y(2) = 0.9(2.9) + 3 = 5.61$

$y(3) = 5.61 \times 0.9 = 5.049$

$y(4) = 5.049 \times 0.9 = 4.544$

$y(5) = 4.544 \times 0.9 = 4.090$

(b) Step response is nothing but an accumulator

Hence $s(0) = y(0) = 1;$

$s(1) = y(0) + y(1) = 3.90$

$s(2) = y(0) + y(1) + y(2) = 3.9 + 5.61 = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 9.51 + 5.049$

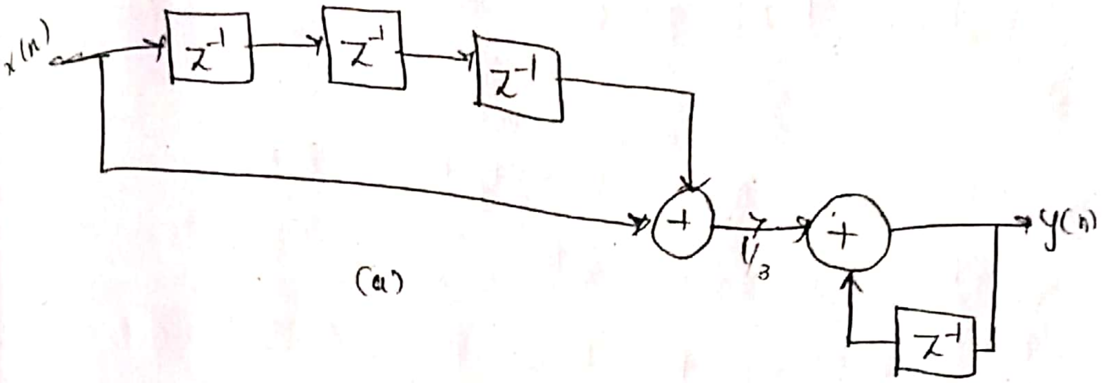
$= 14.56$

$s(4) = \sum_{0}^{4} y(n) = 19.10$

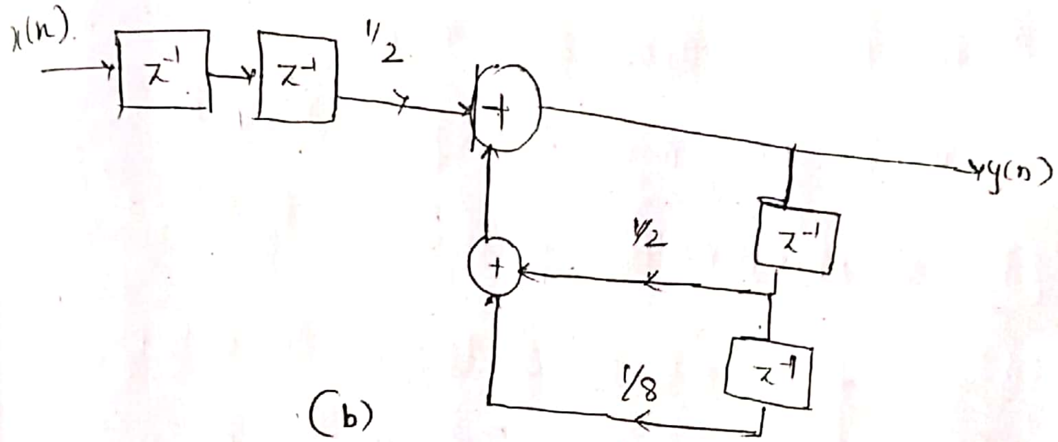
$s(5) = \sum_{0}^{5} y(n) = 23.19$

(c) $h(n) = \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

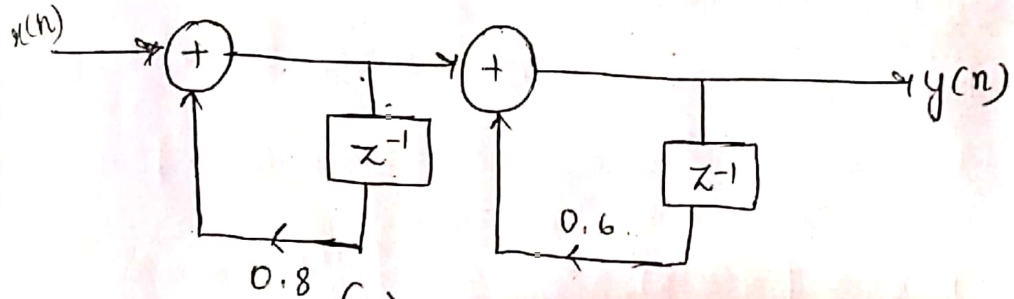
5.1] Determine and sketch impulse response of the following systems for $n=0,1,\dots,9$.



(a)



(b)



(c)

(d) Classify the systems above as FIR or IIR.

(e) Find an explicit expression for the impulse response of system in part 'c'.

Sol:- (a) $\frac{1}{3}x(n) + \frac{1}{3}x(n-3) + y(n-1)$

for $x(n) = \delta(n)$; we have

$h(n) = \{1/3, 1/3, 1/3, 2/3, 2/3 \dots\}$

(b) $y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + \frac{1}{2}x(n-2)$.
with $x(n) = \delta(n)$ and $y(-1) = y(-2) = 0$; we
obtain $h(n) = \{0, 0, 1/2, 1/4, 3/16, 1/8, 11/128 \cdots\}$

doubt

(c) $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$
with $x(n) = \delta(n)$ and $y(-1) = y(-2) = 0$; we
obtain $h(n) = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, \cdots\}$

(d) All the three systems are IIR

(e) $y(n) = 1.4y(n-1) - 0.48y(n-2) + x(n)$

The characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$ Hence $\lambda = 0.8, 0.6$ and

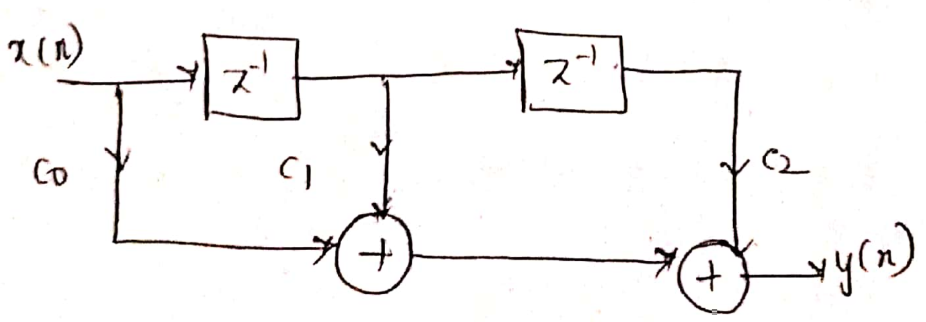
$y_h(n) = c_1(0.8)^n + c_2(0.6)^n$ for $x(n) = \delta(n)$.

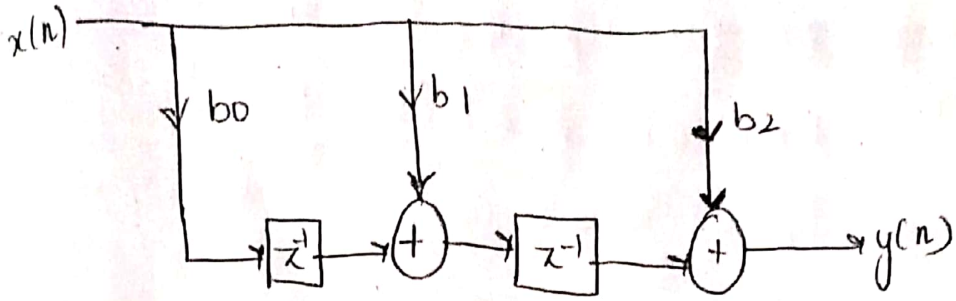
We have; $c_1 + c_2 = 1$ and $0.8c_1 + 0.6c_2 = 1.4$

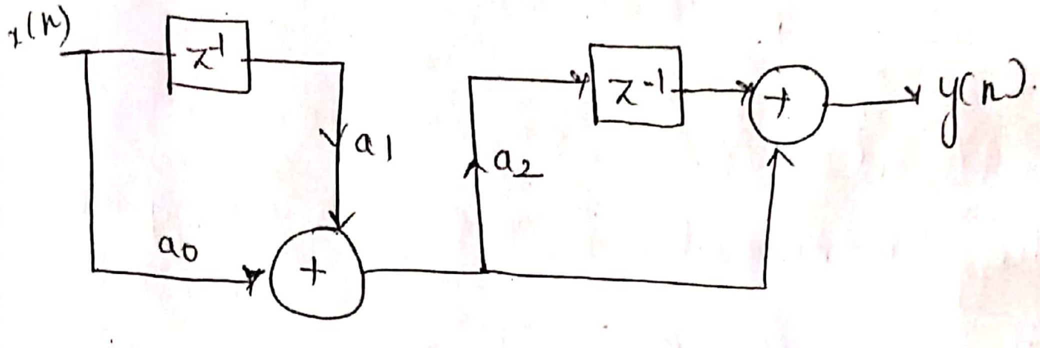
$\Rightarrow c_1 = 4$; $c_2 = -3$. Therefore

$h(n) = [4(0.8)^n - 3(0.6)^n]\,u(n)$.

52] Consider the system shown in figure







(a) Determine the and sketch their impulse responses $h_1(n)$, $h_2(n)$ and $h_3(n)$.

(b) Is it possible to choose the coefficients of these systems in such a way that

$$h_1(n) = h_2(n) = h_3(n)$$

sol:- (a) $y_1(n) = C_0 x(n) + C_1 x(n-1) + C_2 x(n-2)$

$$h_1(n) = C_0 \delta(n) + C_1 \delta(n-1) + C_2 \delta(n-2).$$

$$y_2(n) = b_2 x(n) + b_1 x(n-1) + b_0 x(n-2).$$

$$h_2(n) = b_2 \delta(n) + b_1 \delta(n-1) + b_0 \delta(n-2).$$

$$y_3(n) = a_0 x(n) + (a_1 + a_0 a_2) x(n-1) + a_1 a_2 x(n-2).$$

$$h_3(n) = a_0 \delta(n) + (a_1 + a_0 a_2) \delta(n-1) + a_1 a_2 \delta(n-2).$$

(b) The only question is whether

$h_3(n) - h_2(n) = h_1(n)$; let

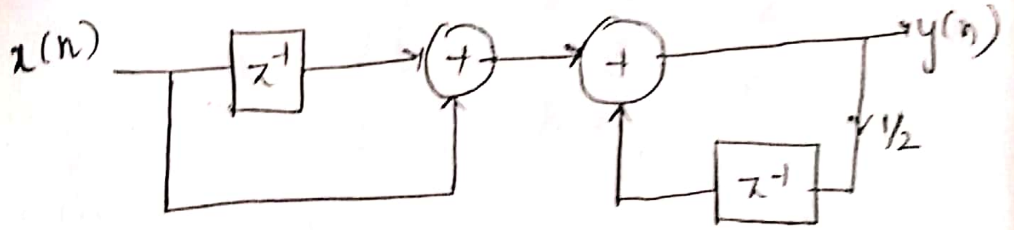
$a_0 = c_0$; $a_1 + a_2 c_0 = c_1$; $a_2 a_1 = c_2$. Here

$$\frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$$

$\Rightarrow$ $c_0 a_2^2 - c_1 a_2 + c_2 = 0$, for $c_0 \neq 0$;

The quadratic has a real solution if and only if $c_1^2 - 4c_0 c_2 \geq 0$.

53] Consider the system shown in figure.



(a) Determine the impulse response $h(n)$

(b) Show that $h(n)$ is equal to the convolution of the following signals:

$$h_1(n) = \delta(n) + \delta(n-1)$$

$$h_2(n) = (1/2)^n u(n).$$

Sol:- $y(n) = 1/2\, y(n-1) + x(n) + x(n-1)$

For $y(n) - 1/2\, y(n-1) = \delta(n)$; the solution is

$$h(n) = (1/2)^n u(n) + (1/2)^{n-1} u(n-1).$$

(b) $h_1(n) * h_2(n) = h_2(n) * [\delta(n) + \delta(n-1)]$

$= h_2(n) + h_2(n-1)$

$= (1/2)^n u(n) + (1/2)^{n-1} u(n-1)$

5] Compute and sketch the convolution $y_i(n)$ and correlation $r_i(n)$ sequences for the following pair of signals and comment on the results obtained

(a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$ $h_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

Correlation $= \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

(b) $x_1(-n) * h(n)$

$\Rightarrow \{4, 2, \underset{\uparrow}{1}\}; \{1, 1, 1, 1, 1\}$

$\{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

(b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$; $h_2(n) = \{1/2, 1, \overset{\downarrow}{2}, 1, 1/2\}$

$\{1/2, 0, \underset{\uparrow}{3/2}, -2, 1/2, -6, -5/2, 1/2, -2\}$

	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
→ 2	0	2	-4	6	-8
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

Correlation $r_1(n) = \{1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

Note that convolution = correction, because $h_2(-n) = h(n)$.

(c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$; $h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$

	1	2	3	4
→ 4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

convolution $= y_3(n)$

$\Rightarrow \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Correlation $= r_3(n)$

$= \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
→ 4	4	8	12	16

(d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$; $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Convolution $\Rightarrow y_3(n+3)$

$= \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

Correlation $\Rightarrow r_3(n+3)$

$= \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

55] The zero-state response of a casual LTI system to the input $x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y(n) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$. Determine its impulse response.

sol:- The length of $h(n) = 2$

as $4 + x - 1 = 5$.

$\Rightarrow x + 3 = 5 \Rightarrow x = 2 = $ length of $h(n)$

	1	3	3	1
h_0	h_0	$3h_0$	$3h_0$	h_0
h_1	h_1	$3h_1$		

Here $h_0 = 1$;

$h_1 \Rightarrow ?$ $h_1 + 3h_0 = 4$

$\Rightarrow h_1 = 1$

$\therefore h(n) = \{1, 1\}$

doubt

56] Prove by direct substitution the equivalence of equations (2.5.9) and (2.5.10); which describe the direct form II structure; to the relation (2.5.6); which describes direct form I structure.

solve: $$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \rightarrow (2.5.6)$$

$$w(n) = -\sum_{k=1}^{N} a_k w(n-k) + x(n) \rightarrow (2.5.9)$$

$$y(n) = \sum_{k=0}^{M} b_k w(n-k) \rightarrow (2.5.10)$$

from (2.5.9) we obtain

$$x(n) = w(n) + \sum_{k=1}^{N} a_k w(n-k). \quad (A)$$

By substituting (2.5.10) for $y(n)$ and (A) into (2.5.6);

$$\sum_{k=0}^{M} b_k w(n-k)$$

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57] Determine the response $y(n), n \geq 0$ of the system described by the second-order difference equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

and the input is $x(n) = (-1)^n u(n)$

and the initial conditions are $y(-1) = y(-2) = 0$

Sol: $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$; $\lambda = 2, 2$. Hence

$y_h(n) = C_1 2^n + C_2 n 2^n$

The particular solution is $y_p(n) = k(-1)^n u(n)$

Substitution this solution into the difference equation; we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

$\Rightarrow$ $\left[k(-1)^2 - 4k(-1) + 4k\right] = 1 + 1$

$\Rightarrow$ $k + 4k + 4k = 2 \Rightarrow k = 2/9$.

Hence the total solution is

$$y(n) = \left[C_1 2^n + C_2 2^n + 2/9 (-1)^n\right] u(n)$$

from the initial conditions, we obtain;

$y(0) = 1$; $y(1) = 2$ $\Rightarrow$ $y(1) - 4 = -1 - 1$

$\Rightarrow$ $y(1) = 2$

$C_1 + 2/9 = 1 \Rightarrow C_1 = 7/9$

$2c_1 + 2c_2 - \frac{2}{9} = 2 \Rightarrow c_2 = 1/3.$

$$y(n) = \left[7/9 (2)^n + 1/3 n . 2^n + 2/9 (-1)^n \right] u(n)$$

58] Determine the impulse response $h(n)$ for the system described by the second-order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

Sol:- From the above problem.

$$h(n) = [c_1 2^n + c_2 (n) 2^n] u(n);$$

with $y(0) = 1$; $y(1) = 3$ we have

$$c_1 = 1, \quad 2c_1 + 2c_2 = 3 \Rightarrow c_2 = 1/2$$

Thus $h(n) = \left[2^n + \frac{1}{2} n 2^n \right] u(n)$

59] Show that any discrete-time signal $x(n)$ can be expressed as.

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k).$$

where $u(n-k)$ is a unit step delayed by k units in time; that is

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise} \end{cases}$$

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Sol:- Method1:-

$$x(n) = x(n) * \delta(n)$$
$$= x(n) * \frac{d}{dn} u(n)$$
$$= \frac{d}{dn} x(n) * u(n)$$
$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

Method2:-

$$x(n) = x(n) * \delta(n) = x(n) * [u(n) - u(n-1)]$$
$$= [x(n) - x(n-1)] * u(n)$$
$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

60. Show that the output of an LTI system can be expressed in terms of its unit step response $s(n)$ as follows

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$
$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] s(n-k)$$

Sol:- Let $h(n)$ be the impulse response of the system

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$\Rightarrow h(k) = s(k) - s(k-1).$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$
$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k).$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, s(n-k).$$

Method 2:

$$s(n) = h(n) * u(n)$$

$$s(n-1) = h(n) * u(n-1)$$

$$\Rightarrow s(n) - s(n-1) = h(n) * \delta(n) = h(n). \rightarrow (1)$$

(or) $s(n) = h(n) * u(n)$; $s(n-1) = h(n-1) * u(n)$

$$s(n) - s(n-1) = [h(n) - h(n-1)] * u(n).$$

$$y(n) = x(n) * h(n) = x(n) * [s(n) - s(n-1)]$$

$$\Rightarrow y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k).$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, s(n-k).$$

6] compute the corelation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences.

$$x(n) = \begin{cases} 1; & n_0 - N \le n \le n_0 + N \\ 0; & \text{otherwise.} \end{cases}$$

$$y(n) = \begin{cases} 1; & -N \le n \le N \\ 0; & \text{otherwise.} \end{cases}$$

$$r_{xx}(l) \Rightarrow x(n) * x(-n) = \sum_{k=-\infty}^{\infty} x(k)\, x(k+l)$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} x(-k)\, x(l-k).$$

$$x(n) = \begin{cases} 1 ; & n_0 - N \le n \le n_0 + N \\ 0 ; & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1 ; & -N \le n \le N \\ 0 ; & \text{otherwise} \end{cases}$$

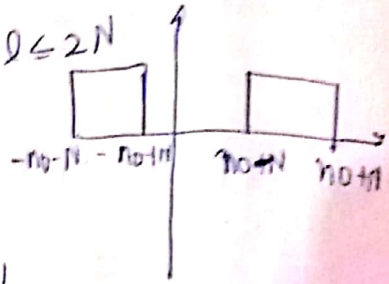
$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by $n_0 - N \le n \le n_0 + N$.

$n_0 - N \le n - l \le n_0 + N$.

which implies

$-2N \le l \le 2N$



ie $-n_0 - N + \cancel{n_0} - N = -2N$

$\cancel{n_0} + N + - n_0 + N = 2N$

range from $-2N \le l \le 2N$.

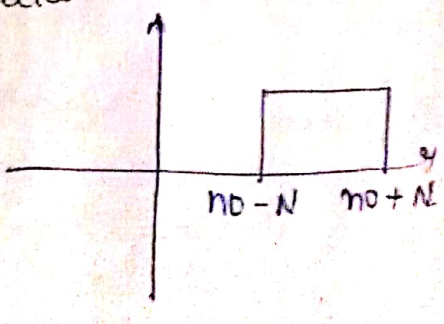
For a given shift l; the no. of terms in the summation for which both $x(n)$ and $x(n-l)$ are non-zero is $2N+1-|l|$ and the value of each term is 1. Hence,

$$r_{xx}(l) = \begin{cases} 2N+1-|l| ; & -2N \le l \le 2N \\ 0 ; & \text{otherwise} \end{cases}$$

For $r_{xy}(l)$ we have

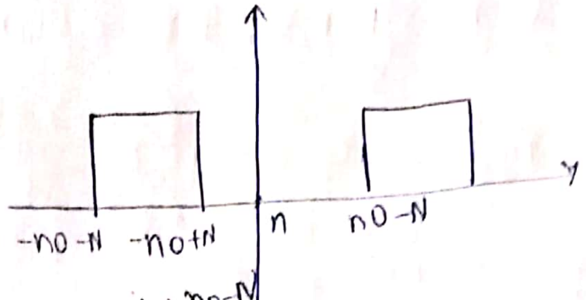
$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0| ; & n_0-2N \le l \le n_0+2N \\ 0 ; & \text{otherwise} \end{cases}$$

Another method:- Correlation



$n + n0 - N$

$\sum_{n0-N}^{n+n0-N} 1.$



$n + n0 - N$

$-n0 + N <$

$= n + n0 - N - n0 + N + 1$

$l + n0 \cancel{+N} < n0 + N$

$=$

2] Determine the autocorrelation sequences of the following signals.

(a) $x(n) = \{1, 2, 1, 1\}$ with the arrow under the first 1

(b) $y(n) = \{1, 1, 2, 1\}$ with the arrow under the first 1

Sol:- $r_{xx}(l) = x(n) * x(-n)$

$x(-n) \{1, 1, 2, 1\}$ with the arrow under the last 1

	1	2	1	1
1	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1

$r_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$ with the arrow under the 7

* $r_{xx}(l) = \sum_{n=-\alpha} x(n) x(n-l)$

$r_{xx}(-3) = x(0) x(3) = 1.$

(b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$ $\quad y(-n) = \{1, 2, 1, \underset{\uparrow}{1}\}$

$r_{yy}(l) = y(n) * y(-n)$

	1	1	2	1
1	1	1	2	1
2	2	2	4	2
1	1	1	2	1
1	1	1	2	1

$\{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$

We observe that

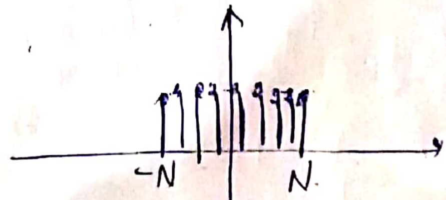
$y(n) = x(-n+3)$; which is equivalent to reversing the sequence $x(n)$. This has not changed the autocorrelation sequence..

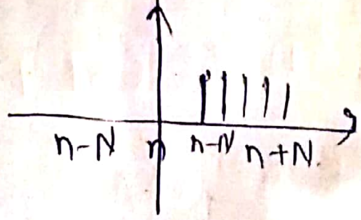
63] What is the normalized autocorrelation sequence of the signal $x(n)$ given by

$$x(n) = \begin{cases} 1; & -N \le n \le N \\ 0; & \text{otherwise} \end{cases}$$

Sol:-

$$\sum_{-N}^{n+N} 1.1 = 2N+1+n$$





$$\sum_{n-N}^{N} 1.1 = 2N+1-n.$$

$$r_{xx}(n) = \begin{cases} 2N+1+n & -2N \le n < 0 \\ 2N+1-n & 0 \le n \le 2N \end{cases}$$

Hence $r_{xx}(l) = \begin{cases} 2N+1-|l| & ; -2N \le l \le 2N \\ 0 & ; \text{otherwise} \end{cases}$

$r_{xx}(0) = 2N+1$.

Therefore the normalized autocorrelation is

$$\rho_{xx}(l) = \frac{1}{2N+1}(2N+1-|l|) \; ; \; -2N \leq l \leq 2N$$

$$= 0 \qquad ; \text{ otherwise.}$$

64] An Audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling; is

$$x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$$

where k_1 and k_2 are the delays of the two echoes

(a) Determine the autocorrelation $r_{xx}(l)$ of the signal $x(n)$

(b) Can we obtain r_1, r_2, k_1 and k_2 by observing $r_{xx}(l)$?

(c) What happens if $r_2 = 0$?

Sol:- (a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$

$$= \sum_{n=-\infty}^{\infty} [s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)] *$$

$$[s(n-l) + r_1 s(n-l-k_1) + r_2 s(n-l-k_2)]$$

$$= (1+r_1^2+r_2^2) \cdot r_{ss}(l) + r_1[r_{ss}(l+k_1)+r_{ss}(l-k_1)] +$$
$$+ r_2 [r_{ss}(l+k_2) + r_{ss}(l-k_2)] +$$
$$r_1 r_2 [r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)]$$

(b) $r_{xx}(l)$ has peaks at $l=0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. Suppose that $k_1<k_2$. Then, we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

(c) If $r_2=0$; the peaks occur at $l=0$ and $l=\pm k_1$. Then; it is easy to obtain r_1 and k_1.

6.5] Time-delay estimation in radar. Let $x_a(t)$ be the transmitted signal and $y_a(t)$ be the received signal in a radar system where

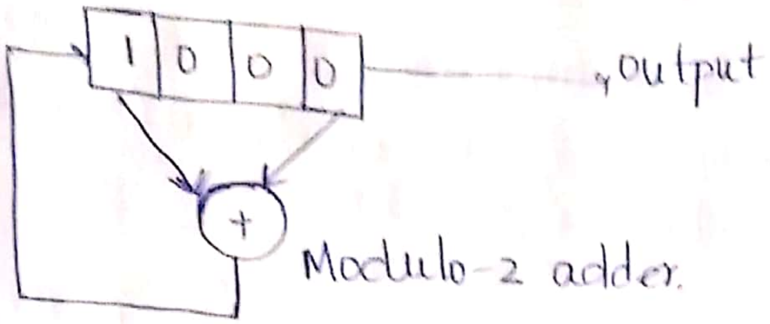
$$y_a(t) = a x_a(t-t_d) + v_a(t).$$

and $v_a(t)$ is additive random noise. The signals $x_a(t)$ and $y_a(t)$, are sampled in the receiver; according to the sampling theorem; and are possessed digitaly to determine the time delay and hence the distance of the object The resulting discrete time signals are

$$x(n) = x_a(nT)$$

$$y(n) = y_a(nT) = a x_a(nT - DT) + v_a(nT)$$

$$\triangleq a x(n-D) + v(n)$$



(a) Explain how we can measure the delay D by computing the crosscorrelation $r_{xy}(l)$

sol -

Chapter 1

Problems:

1 Classify the following signals according to whether they are (1) one or multidimensional (2) single or multichannel (3) continuous or discrete time and (4) Analog or digital (in amplitude). Give a brief explanation

(a) Closing prices of utility stocks on the New York Stock Exchange.

sol:- one dimensional, multichannel, discrete time and digital.

(b) A color movie.

sol:- Multi dimensional, single channel, continuous-time, analog.

(c) Position of the steering wheel of a car in motion relative to car's reference frame.

sol:- one dimensional, single channel, continuous-time, analog.

(d) Position of the