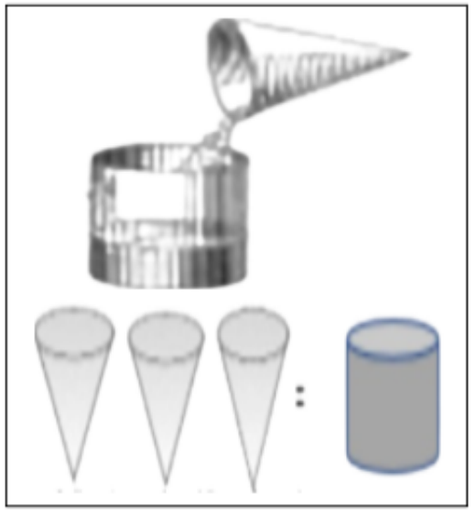
Finds the Volume of Cylinders, Pyramids, Cones, and Spheres, and Solves Routine and Non-Routine Problems Related to it.

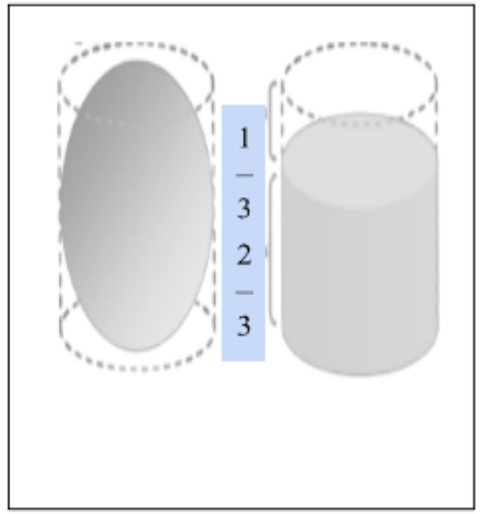
Let us try to build a connection and derive a formula between the volume of these matching solids **(same radius and same height).**
Suppose that we will try to fill these **solid figures with water or sand.**

Figure 1.



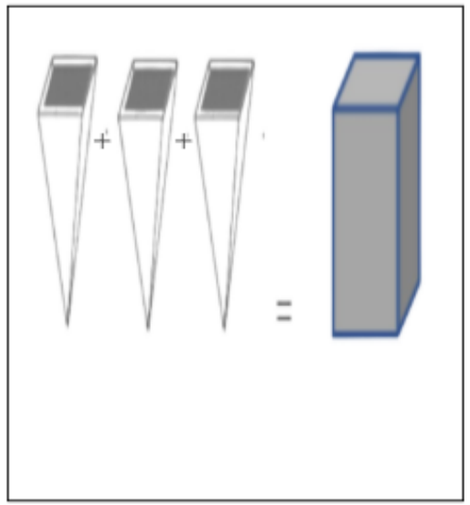
Trivia: Do you know that it takes three cones to fill the cylinder with sand or water?

Figure 2.



Trivia: The space a sphere takes is 2/3 of the volume of the cylinder.

Figure 3.



Trivia: It takes exactly 3 full pyramids to fill the prism.

What mathematical formula can you derive for the **volume of a cone?**

Note: The volume of a cylinder is **three times** the volume of the cone, or the volume of a cone is $\frac{1}{3}$ that of the cylinder.

What is the formula to be used to find the volume of a cylinder?

V = B x h , Where: B = Area of the base

$$B = \pi r \qquad \pi = 3.14$$

$$V = \pi r h$$

How do we write the formula for the volume of a cone?

V = 1/3 Bh , Where B = area of the base

$$B = \pi r \qquad \pi = 3.14$$

h = height of the cone

$$V = \frac{Bh}{3} \quad \text{or} \quad \frac{\pi r h}{3} \quad \text{or} \quad \frac{1}{3}\pi r$$

VOLUME OF A CONE

Example: A cone hat has a diameter of 8 cm and a height of 10 cm; what is its volume?

What is being **asked?**

- The volume of the cone.

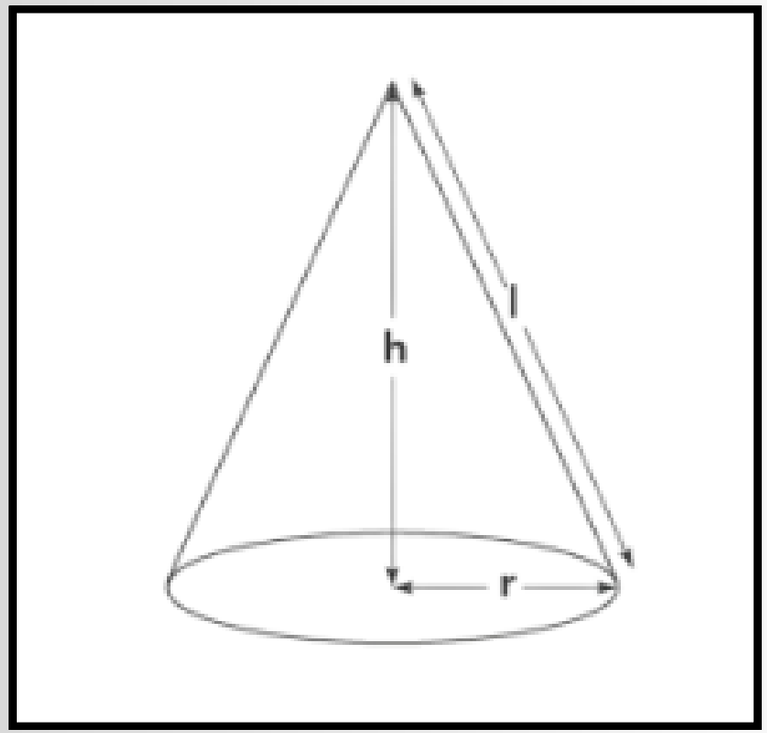
What are the **given facts?**

- d = 8 cm, h = 10 cm

What is the **operation to be used?**

- Formula for the volume of a cone.

- $V = \frac{1}{3}Bh$ or $\frac{1}{3}\pi r^2$



What is the **number sentence?** $V = \frac{1}{3}$ Bh or $\frac{1}{3}\ \pi r^2 = N$

Write the **solution** with the correct label.

Bh = 4 cm

$V = \frac{1}{3}$ Bh or $\frac{1}{3}\ \pi r^2$

$V = \frac{1}{3}$ (3.14) × 2 cm × 2 cm × 10 cm

$V = \frac{1}{3}$ (3.14 × 4 cm ×10 cm)

$V = \frac{1}{3}$ (3.14 × 40 cm)

$V = \frac{1}{3}$ (125.6 cm)

V = 41.87 cm^3

Given:

h = 10 cm

d = 8 cm

The volume of the cone is **41.87 cm^3**.

VOLUME OF A CYLINDER

Example: Lilly is molding a cylindrical candle with a diameter of 16 cm and a height of 22 cm. How much wax does she need to mold the candle? What is the shape of the candle? Cylinder

How are you going to solve the problem? Find the volume of the cylindrical candle.

What is the formula for the volume of a cylinder? $\mathbf{V = \pi r^2 h}$

Write your solution and answer.

$V = \pi r^2 h$

$V = (3.14)\ (8)^2 (22\ cm)$

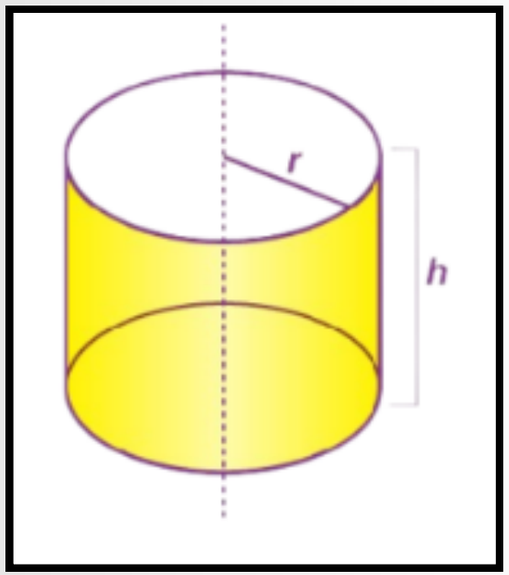
$V = 3.14 \times 64 \times 22\ cm$

$V = 200.96 \times 22$

$\mathbf{V = 4421.12\ cm^3}$

d= 16 cm

h= 22 cm



Lilly needs **4421.12 cm^3** of wax to mold the candle.

VOLUME OF A PYRAMID

Example: The volume of a pyramid is given by the formula:

$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{area of base} \times \text{height}$$

In a bazaar, Terry bought a square pyramid glass keychain for his bag. The base of the keychain is 5 cm by 6.5 cm, and its height is 8 cm. Find the volume of the glass used to make the glass pyramid.

What is being **asked**?

- The volume of the glass pyramid keychain.

What is the **shape of the base**?

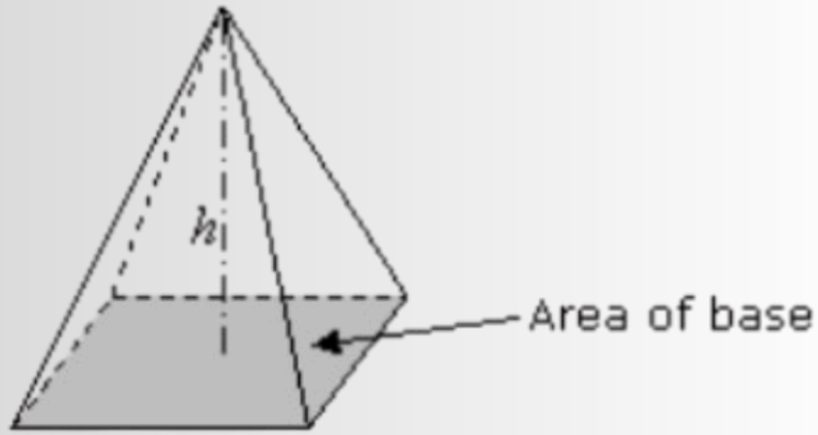
- Rectangle

What are the **given facts**?

- 5 in by 6.5 in h = 8 in

What is the **operation to be used**?

- Formula for the volume of pyramid



VOLUME OF A PYRAMID

In a bazaar, Terry bought a square pyramid glass keychain for his bag. The base of the keychain is 5 cm by 6.5 cm, and its height is 8 cm. Find the volume of the glass used to make the glass pyramid.

Solution: Volume of Pyramid = $\frac{1}{3}$ × area of base × height

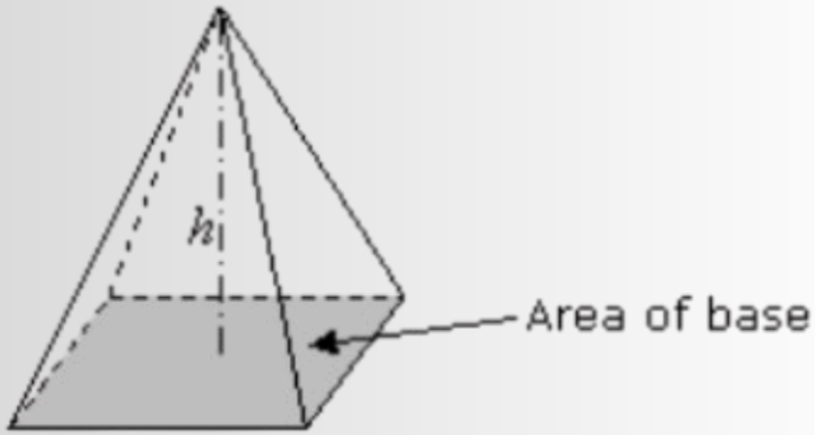
$$V = \frac{1}{3}Bh \; or V = \frac{1}{3}(l.w.h)$$

$$V = \frac{1}{3}(5 \times 6.5)(8)$$

$$V = \frac{1}{3}(32.5)(8)$$

$$V = \frac{1}{3}(260)$$

$$V = 86.67 \ cm^3$$



The volume of the glass pyramid keychain is $86.67 \ cm^3$

VOLUME OF A SPHERE

Example 4: Lacy wants to know how much water a sphere can hold with a radius of 12 cm. Find the volume. Use 3.14 for pi.

What is **being asked**?

- The volume of the water the sphere can hold.

What are the **given facts**?

- r = 12 cm

What is the **operation to be used**?

- Use the formula for the volume of a sphere.

$$V = \frac{4}{3} \times \pi r^3$$

VOLUME OF A SPHERE

Example 4: Lacy wants to know how much water a sphere can hold with a radius of 12 cm. Find the volume. Use 3.14 for pi.

Write the number sentence and your final answer with its correct label.

Given: **r = 12**

Solution:

$$V = \frac{4}{3} \times \pi r^3 = N$$

$$V = \frac{4}{3} \times 3.14 \times 12^3$$

$$V = \frac{4}{3} \times 3.14 \times 1728$$

$$V = \frac{4}{3} \times 5425.92$$

$$V = 7234.56 cm^3$$

The volume of the water the sphere can hold is **7,234.56** cm^3

Remember: The **volume** of a solid figure is the amount of **space inside it**. Volume is measured in **cubic units** (**m^3**,**cm^3**,**dm^3**, etc.), which means the total number of cubes it takes to **fill a solid figure**.