Formal: Assignment #8

Due on November 11, 2022 at 4:00 PM

Professor Matthew Patitz 4:10 PM

Piam Chittisane

Problem 1

Create a turing machine T such that for an input A where A is a DFA:

1. If every state in A is an accept state, accept. Otherwise, reject.

Problem 2

Proof by contradiction
Suppose a bijection f exists between B and N.
We can create a sequence $x \in B$ such that x isn't paired with anything in N.
If we take the nth digit of the nth sequence of B and change it to anything else (arbitrarily), we can create a new x on the diagonal that is in B, but doesn't correspond to any of the natural numbers. This is a contradiction, meaning that B must be uncountable.

Problem 3

We can map the three tuple (i, j, k) to the infinite subset of N where $f(n) = 2^i 3^j 5^k$. Since the prime factorizations of a number is unique to itself, we can guarantee the relationship is one-to-one. The relationship is also onto since we can generate any prime factorization of the form $2^i 3^j 5^k$ with any arbitrary i, j, or k. This means a bijection exists, meaning the set T is countable.

Problem 4

Construct a turing machine T that takes in the encoding of a DFA M and string w:

1. Construct a string w^R
2. Run string w through DFA M
3. If w is accepted by M, run string w^R through DFA M, otherwise T rejects
4. If w^R is accepted by M, T accepts, otherwise T rejects

S is decidable.

Problem 5

Proof by contradiction
Assume T is a decidable language and that R decides it. Construct a turing machine J such that on input $\langle M, w \rangle$ where M is a turing machine and w is the input string:

1. Create a turing machine S such that on an input x
   (a) if x has the form 01, accept
   (b) else, run the input w on turing machine M and accept if M accepts w, otherwise reject
2. run R on $\langle S \rangle$
3. If R accepts, J accepts, reject otherwise.