Formal: Assignment #3

Due on September 22, 2022 at 11:59pm

Professor Matthew Patitz 4:10 PM

Piam Chittisane

Problem 1

Prove Theorem 1 (i.e. if L is a regular language, then $\overline{L} = \Sigma^*/L$ is also regular).

Solution

If L is a regular language, that must mean there is a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes the language. Since $\overline{L}$ accepts any expression that isn't accepted by L, all that needs to be done is complement the accepting states. DFA $\overline{M} = (Q, \Sigma, \delta, q_0, Q/F)$ should exist such that $L(\overline{M}) = \overline{L}$.

Problem 2

Give regular expressions generating the following languages. In all parts, the alphabet is {0, 1}.

Part A

$\{w \mid \text{every odd position of } w \text{ is a } 1\}$

Solution

$R = (1\Sigma)^*$

Part B

$\{w \mid w \text{ contains at least two 0s and at most one } 1\}$

Solution

$R = 000^* + ((01 + 10)0 + 000^*1)0^*$

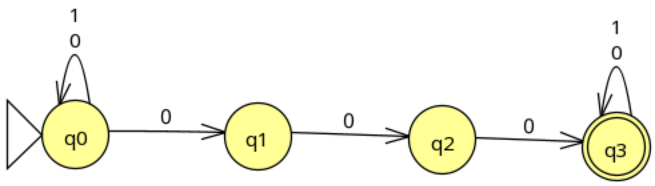
Problem 3

Use the procedure described in Lemma 1.55 to covert the following regular expressions to nondeterministic finite automata.

Part A

$(0 \cup 1)^*000(0 \cup 1)^*$

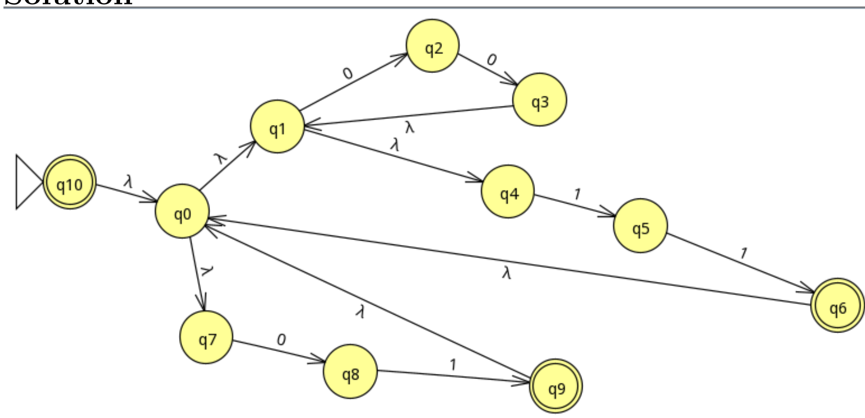
Solution



Part B

$(((00)^*(11)) \cup 01)^*$

Solution



Problem 4

Use the procedure described in Lemma 1.60 to covert the following finite automata to regular expressions.

Part A

Solution

$(a^*ba^*b)^*a^*ba^*$

Part B

Solution

$((a \cup b)a^*b(ba^*b)^*a)^*(\epsilon \cup (a \cup b)a^*b(ba^*b)^*)$

Problem 5

Let $\Sigma = \{0, 1, +, =\}$ and $ADD = \{x = y + z \mid x, y, z$ are binary integers, and x is the sum of y and $z\}$
Prove that ADD is not regular.

Solution

Proof. Let string s be $1^p = 1^p + 0^p$. Since 1^p is as long as the pumping length p, the substring y must consist of only 1s. For any arbitrary y, if y is pumped down (if $y = 0$), the equality of $x = y + z$ does not hold since there will be less 1s in x than y. □

Problem 6

Prove that the following language is not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement. $\{w \mid w \in \{0, 1\}^*$ is not a palindrome$\}$

Solution

Proof. Assume the language L is regular.
The language $\overline{L} = \{w \mid w \in \{0, 1\}^*$ is a palindrome$\}$ must be regular due to closure under complement.
Consider the string 0^p10^p that is accepted by $\overline{L}$.
Since 0^p is as long as the pumping length p, the substring y must only consist of 0s.
If substring y is pumped up or pumped down, the number of 0s before 1 will not equal the number of 0s after, resulting in a whole string that isn't a palindrome.
Therefore, $\overline{L}$ is not a regular language. If $\overline{L}$ is not a regular language, L is not a regular language through closure, resulting in a proof by contradiction.

□