Formal: Assignment #7

Due on November 2, 2022 at 4:00 PM

Professor Matthew Patitz 4:10 PM

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Problem 1

Let this machine be represented as a k-tape Turing Machine.

1. Let tape 1 be the input string w.
2. Write a 1 to the first cell of tape 2 and move the head right and write a cell in the second cell of tape 2, making 2 in unary. Then reset the head to the start of tape 2.
3. Move the head right and read each 1 in tape 2. For each 1 read in tape 2, mark a 1 in tape 1 and mark a 1 in tape 2 until a blank is read in tape 1.
4. If there are unmarked 1's when a blank is read in tape 1, this means the number isn't divisible by the number in tape 2. Add a 1 to the end of tape 2 and unmark both tapes and reset the heads to the start for both. Repeat from step 3.
5. If there are no unmarked 1's left when a blank is read in tape 1, this means the number is divisible by the number in tape 2. Since the number is a composite number, we accept.

Problem 2

Show that the collection of decidable languages is closed under the operations of

Part A

complementation

Solution

Decidable languages are closed under complementation. For a given decidable Turing machine, its complement can be made by simply swapping its accept and reject states. In other words, if the machine usually accepts, reject. If the machine usually rejects, accept.

Part B

intersection

Solution

Decidable languages are closed under intersection. For two given decidable Turing machines, their intersection can be created by making a 2 tape Turing machine. The input string is copied to both tapes. If you take the accept states from one of the machines and attach it to the start state of the other machine, the accept state of the entire machine would be the intersection. While the machine is in its first half, it uses tape 1. While the machine is in its second half, it uses tape 2.

Part C

concatenation

Solution

Nondeterministically divide the string w into two sections a and b, where a goes from empty to w, and b is the rest of the string. Run the string a in the the first turing machine in the concatenation. If it accepts, run the string b on the second Turing machine in the concatenation. If that accepts, the whole machine accepts. If either reject, the whole thing rejects.

Problem 3

Solution

I is encoded as rows of c's represented as numbers in binary separated by semicolons. At the end of each row is a single symbol to act as a sort of endline character (c;c;c;c;). X, y and c would all be represented in binary. Generally speaking, to read a given pixel, the head of the tape would move over y 's and read past x ;'s to read the specific color number. This would be compared against the c before the .