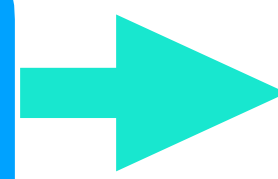
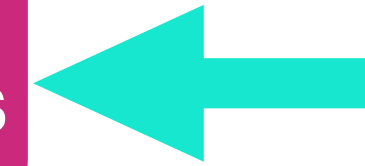


We have Items, every Item has weight and profit.



Unbounded
Knapsack Problems



Rod Cutting :

Given a rod of length 'n', we are asked to cut the rod and sell the pieces in a way that will maximize the profit. We are also given the price of every piece of length 'i' where $1 \leq i \leq n$.

Example:

Lengths: [1, 2, 3, 4, 5]

Prices:[2, 6, 7, 10, 13]

Rod Length: 5

Output : 14

{1,2,2} max profit can be possible.

Lets see all possible positive options :::

Lengths: [1, 2, 3, 4, 5]

Prices: [2, 6, 7, 10, 13]

Rod Length: 5

$$\Rightarrow \{1,1,1,1,1\} \Rightarrow 5 * 2 = 10 \text{ RS}$$

$$\Rightarrow \{1,1,1,2\} \Rightarrow 3 * 2 + 6 = 12 \text{ RS}$$

$$\Rightarrow \{1,1,3\} \Rightarrow 2 * 2 + 7 = 11$$

$$\Rightarrow \{1,4\} \Rightarrow 2 + 10 = 12$$

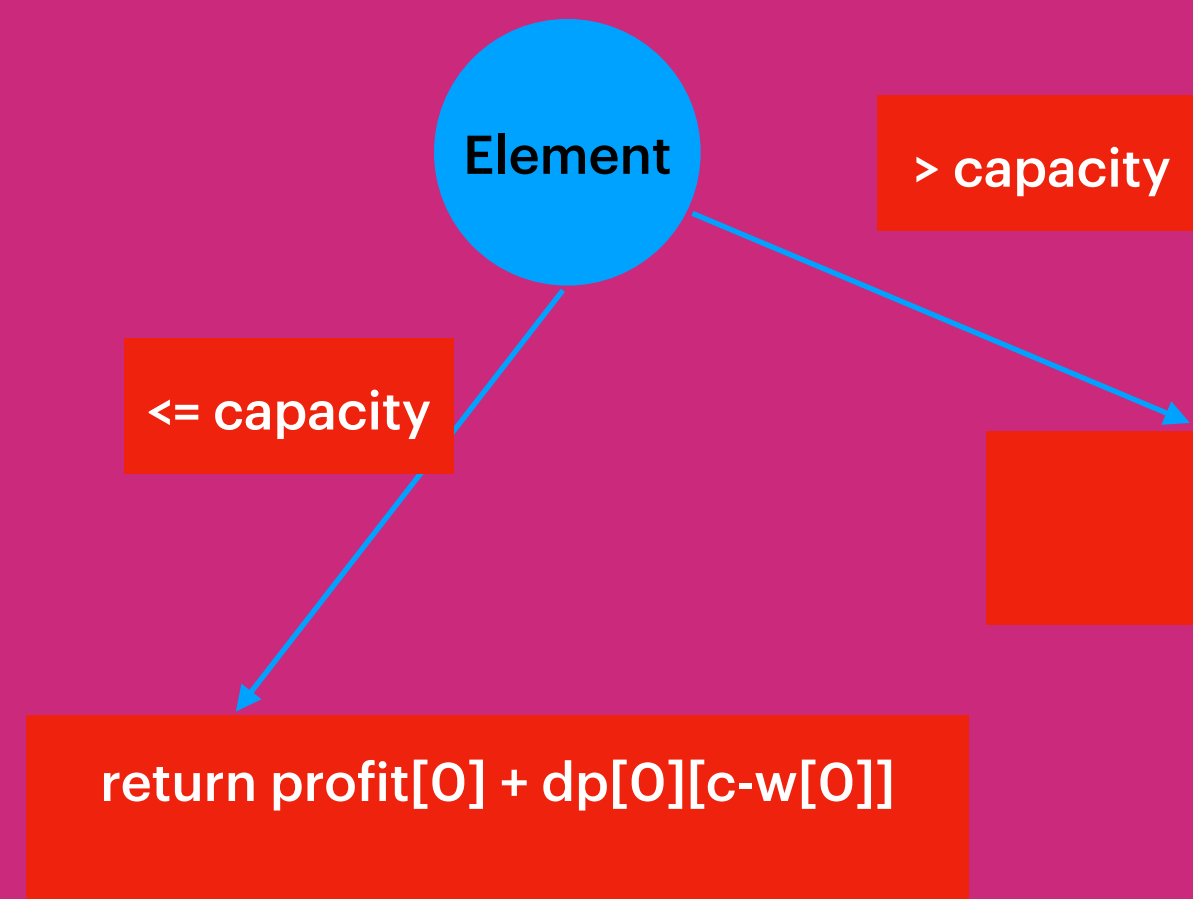
$$\Rightarrow \{1,2,2\} \Rightarrow 1 * 2 + 2 * 6 = 14 \text{ RS}$$

$$\Rightarrow \{2,3\} \Rightarrow 1 * 6 + 1 * 7 = 13 \text{ RS}$$

$$\Rightarrow \{5\} \Rightarrow 1 * 13 = 13 \text{ RS}$$

So Max Profit we got is $\{1,2,2\} \Rightarrow 14$

Lengths: [1, 2, 3, 4, 5]
Prices: [2, 6, 7, 10, 13]
Rod Length: 5



Calculating subproblem with element 1 :
Index = 0, w{1} p{2} , Rod Length: 5

$$c(5) = dp[0][5] = \{1,1,1,1,1\} \Rightarrow 5 * 2 = 10$$

Lets see how it works !!!!

$$c(0) = dp[0][0] = w(0) > c(0) = 0$$

$$\begin{aligned} c(1) &= dp[0][1] = p[0] + dp[0][1-1] = p[0] + dp[0][0] \\ &= 2 + c(0) = 2 + 0 = 2 \end{aligned}$$

$$\begin{aligned} c(2) &= dp[0][2] = p[0] + dp[0][2-1] = p[0] + dp[0][1] \\ &= p[0] + c(1) = 2 + 2 = 4 \end{aligned}$$

$$\begin{aligned} c(3) &= dp[0][3] = p[0] + dp[0][3-1] = p[0] + dp[0][2] = p[0] + c[2] \\ &= 2 + 4 = 6 \end{aligned}$$

$$c(4) = p[0] + c(3) = 2 + 6 = 8$$

$$c(5) = p[0] + c(4) = 2 + 8 = 10$$

Calculating subproblem with element 1 :

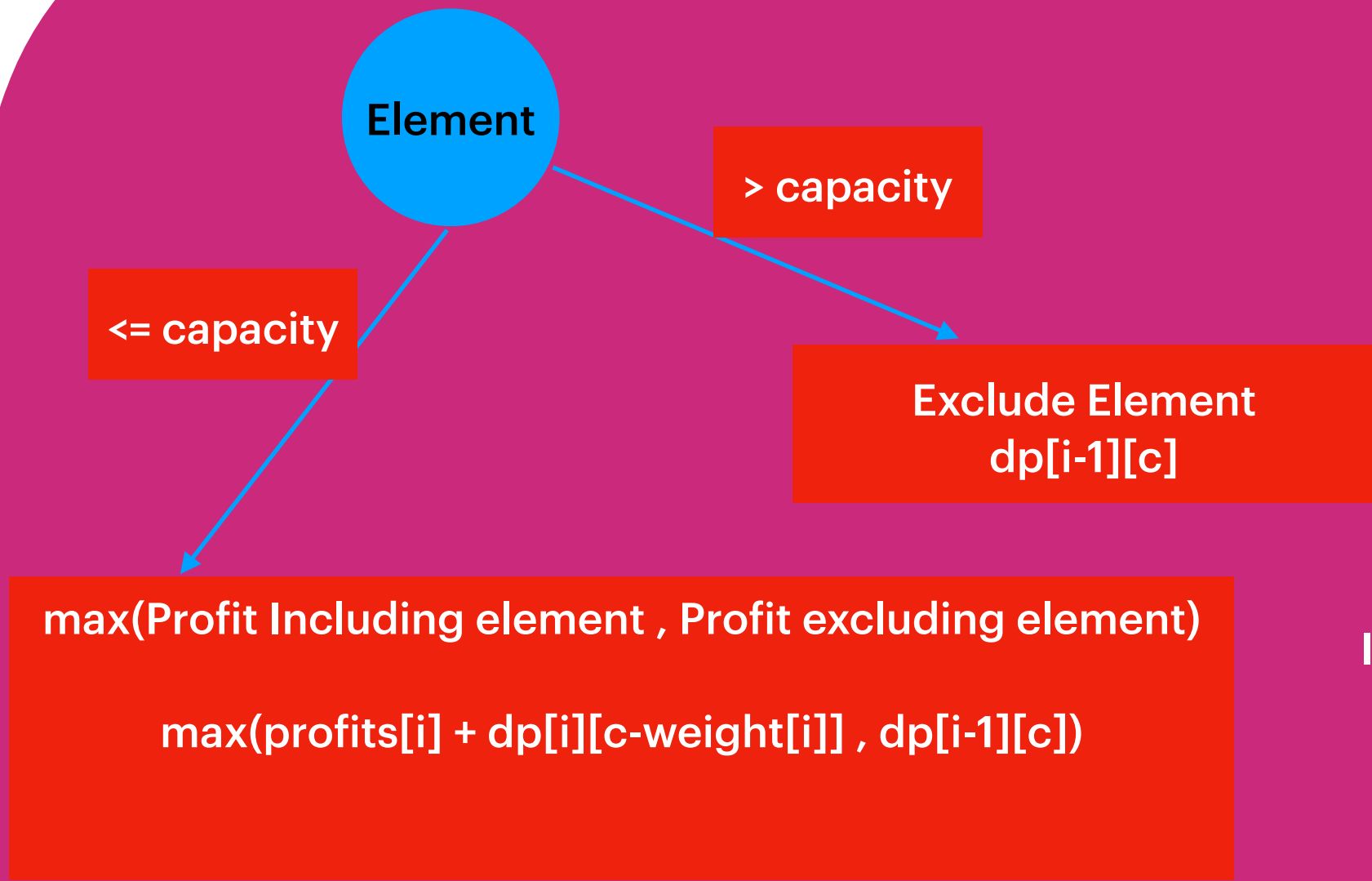
When we consider only one element, we should see either we can include or exclude the element.

If weight of element > capacity
then profit = 0

Otherwise

$$\begin{aligned} \text{profit} &= \text{currentElementProfit} + \\ &\quad \text{profitOfCapacityLeft} \\ &= \text{profit}[0] + dp[0][\text{capacity} - w[0]] \end{aligned}$$

Calculating subproblem with elements 2
Index = 1, w{1,2} p{2,6} , Rod Length i.e capacity : 5



Calculating subproblem with elements 2
Index = 1, w{1,2} p{2,6} , Rod Length i.e capacity : 5

$c(0) = dp[1][0] = w(2) > c(0) = 0$

$c(1) = dp[1][1] = w(2) > c(1) = dp[0][1] = 2$

$c(2) = dp[1][2] = w(2) \leq c(2) = 6$

Exclude Element = $dp[0][2] = 4$

Include Element = $p[1] + dp[1][2-2] = p[1] + dp[1][0] = 6 + 0 = 6$

Max(IncludingElement, Excluding Element)

Max(6,4) = 6

$c(3) = dp[1][3] = w(2) \leq c(3) =$

Exclude Element = $dp[0][3] = 6$

Include Element = $p[1] + dp[1][3-2] = p[1] + dp[1][1] = 6 + 2 = 8$

Max(IncludingElement, Excluding Element)

Max(8,6) = 8

$c(4) = dp[1][4] = w(2) \leq c(4) =$

Exclude Element = $dp[0][4] = 8$

Include Element = $p[1] + dp[1][4-2] = p[1] + dp[1][2] = 6 + 6 = 12$

Max(IncludingElement, Excluding Element)

Max(12,8) = 12

$c(5) = dp[1][5] = w(2) \leq c(5) =$

Exclude Element = $dp[0][5] = 10$

Include Element = $p[1] + dp[1][5-2] = p[1] + dp[1][3] = 6 + 8 = 14$

Max(IncludingElement, Excluding Element)

Max(14,10) = 14

Lengths: [1, 2, 3, 4, 5]
Prices: [2, 6, 7, 10, 13]
Rod Length: 5

For zero capacity all the items gives 0 profit.

When we are solving for the 1st element.
i.e at index 0.

We don't have option of comparison so

If weight of the 1st element is \leq capacity then value will be, $\text{profits}[0] + \text{dp}[0][\text{c} - \text{weight}[0]]$.

If weight of the 1st element is $>$ capacity then value is 0.

Prices	Length	Index
2	1	0
6	2	1
7	3	2
10	4	3
13	5	4

Target Sum					
0	1	2	3	4	5
0	2	4	6	8	10
0	2	6	8	12	14
0	2	6	8	12	14
0	2	6	8	12	14
0	2	6	8	12	14



Element weight

\leq capacity

$>$ capacity

Max(Including element , excluding element)
 $\max(\text{profits}[i] + \text{dp}[i][\text{c} - \text{weight}[i]] , \text{dp}[i-1][\text{c}])$

Exclude Elem
 $\text{dp}[i-1][\text{c}]$

