Q4 1.)

The perceptron rule is

$$q(x) = \begin{cases} 1 & \text{if } \sum w_i x_i + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

If we multiply all the weights & biases by a constant $c > 0$ we get:

$$q(x) = \begin{cases} 1 & \text{if } \sum c w_i x_i + c b > 0 \\ 0 & \text{if } \sum c w_i x_i + c b \leq 0 \end{cases}$$

$$q(x) = \begin{cases} 1 & \text{if } c(\sum w_i x_i + b) > 0 \\ 0 & \text{if } c(\sum w_i x_i + b) \leq 0 \end{cases}$$

Now $\because c > 0$ the sign of $c(\sum w_i x_i + b)$ depends on the sign of $\sum w_i x_i + b$

Thus the rule holds & multiplying by a positive constant doesn't affect the behavior of the network

2.) Sigmoidal activation func^n

$$a = \frac{e^z}{1+e^z}$$

where $z = w^T x + u$

If all weights and biases are multiplied by a constant c we have

$$cz = \sum c w_i x_i + cu$$
$$= c(\sum w_i x_i + u)$$

Inputting cz in the activation func^n a

$$a = \frac{e^{cz}}{1+e^{cz}} = \frac{1}{1+e^{-cz}}$$

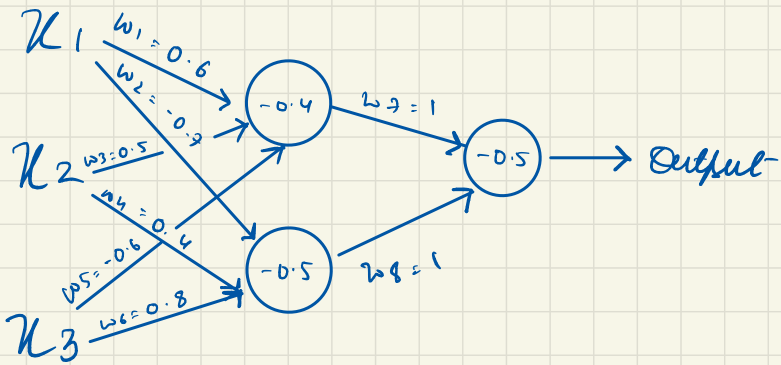
Applying the limit

$$\lim_{c \to \infty} a = \begin{cases} 1 & \text{if } z > 0 \\ 0 & \text{if } z < 0 \end{cases}$$

Again we see that the sign of $\sum w_i x_i + b$ determines whether the neuron outputs 1 or 0

This is akin to the perceptron rule, thus if $\Sigma w_i x_i + u > 0$, $a = 1$ else $a = 0$

3.)



			Layer 1		Layer 2	
x_1	x_2	x_3	n_1	n_2	n_3	O/P
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	0	1	1
0	1	1	0	1	1	1
1	0	0	1	0	1	1
1	0	1	0	0	0	0
1	1	0	1	0	1	1
1	1	1	1	1	1	1

$x = [0\ 0\ 1]$

$n1 \rightarrow \Sigma wx + u = -0.6 - 0.4 = -1$

$a(n1) = 0$

$n2 \rightarrow 0.8 - 0.5 = 0.3$

$a(n2) = 1$

$n3 = 1 + 0 - 0.5, \quad a(n3) = 1$

$x = [0\ 1\ 0]$

$n1 = 0.5 - 0.4 = 0.1, \quad a(n1) = 1$

$n2 = 0.4 - 0.5 = -0.1, \quad a(n2) = 0$

$n3 = 1 + 0 - 0.5 = 0.5, \quad a(n3) = 1$

$x = [0\ 1\ 1]$

$n1 = 0.5 - 0.6 - 0.4, \quad a(n1) = 0$

$n2 = 0.4 + 0.8 - 0.5, \quad a(n2) = 1$

$n3 = 1 + 0 - 0.5, \quad a(n3) = 1$

$x = [1\ 0\ 0]$

$n1 = 0.6 - 0.4 = 0.2, \quad a(n1) = 1$

$n2 = -0.7 - 0.5 = -1.2, \quad a(n2) = 0$

$n3 = 1 + 0 - 0.5 = 0.5$
$a(n3) = 1$

$x = [1\ 0\ 1]$

$n1 = 0.6 - 0.6 - 0.4$, $a(n1) = 0$

$n2 = -0.7 + 0.8 - 0.5$, $a(n2) = 0$

$n3 = 0 - 0.5$, $a(n3) = 0$

$x = [1\ 1\ 0]$

$n1 = 0.6 + 0.5 - 0.4 = 0.7$, $a(n1) = 1$

$n2 = -0.7 + 0.4 - 0.5 = -0.8$, $a(n2) = 0$

$n3 = 1 + 0 - 0.5 = 0.5$, $a(n3) = 1$

$x = [1\ 1\ 1]$

$n1 = 0.6 + 0.5 - 0.6 - 0.4 = 0.1$, $a(n1) = 1$

$n2 = -0.7 + 0.4 + 0.8 = 0.5$, $a(n2) = 1$

$n3 = 1 + 1 - 0.5 = 1.5$, $a(n3) = 1$

4.)

			Layer 1		Layer 2	
x_1	x_2	x_3	n_1	n_2	n_3	O/P
0	0	0	0.4	0.37	0.56	1
0	0	1	0.26	0.57	0.33	0
0	1	0	0.52	0.47	0.62	1
0	1	1	0.18	0.66	0.58	1
1	0	0	0.54	0.23	0.56	1
1	0	1	0.4	0.4	0.57	1
1	1	0	0.66	0.31	0.61	1
1	1	1	0.52	0.62	0.65	1

$x = [0 \ 0 \ 0]$

$$n_1 = -0.4, \quad \sigma(n_1) = \frac{e^{-0.4}}{1+e^{-0.4}} \approx 0.4$$

$$n_2 = -0.5, \quad \sigma(n_2) = \frac{e^{-0.5}}{1+e^{-0.5}} \approx 0.37$$

$$n_3 = 0.4 + 0.37 - 0.5, \quad \sigma(n_3) = \frac{e^{0.27}}{1+e^{0.27}} \approx 0.56$$

$O/P = 1$

$x = [0\ 0\ 1]$

$n_1 = -0.6 - 0.4 = -1$, $\sigma(n_1) = 0.26$

$n_2 = 0.8 - 0.5 = 0.3$, $\sigma(n_2) = 0.57$

$n_3 = 0.26 + 0.57 - 0.5$, $\sigma(n_3) = 0.33$

$O/P = 0$

$x = [0\ 1\ 0]$

$n_1 = 0.5 - 0.4 = 0.1$, $\sigma(n_1) = 0.52$

$n_2 = 0.4 - 0.5 = -0.1$, $\sigma(n_2) = 0.47$

$n_3 = 0.52 + 0.47 - 0.5 = 0.49$,
$\sigma(n_3) = 0.62$

$O/P = 1$

$x = [0\ 1\ 1]$

$n_1 = -1.5$, $\sigma(n_1) = 0.18$

$n_2 = 0.7$, $\sigma(n_2) = 0.66$

$n_3 = 0.66 + 0.18 - 0.5 = 0.34$
$\sigma(n_3) = 0.58$

$o/p = 1$

$x = [1\ 0\ 0]$

$x_1 = 0.2$, $a(x_1) = 0.54$

$x_2 = -1.2$, $a(x_2) = 0.23$

$x_3 = 0.77 - 0.5 = 0.27$, $a(x_3) = 0.56$

$o/p = 1$

$x = [1\ 0\ 1]$

$x_1 = -0.4$, $a(x_1) = 0.4$

$x_2 = -0.4$, $a(x_2) = 0.4$

$x_3 = 0.3$, $a(x_3) = 0.57$

$o/p = 1$

$x = [1\ 1\ 0]$

$x_1 = 0.7$, $a(x_1) = 0.66$

$x_2 = -0.8$, $a(x_2) = 0.31$

$x_3 = 0.97 - 0.5 = 0.47$

$a(x_3) = 0.61$, $o/p = 1$

$x = [1\ 1\ 1\ 1]$

$n_1 = 0.1$, $a(n_1) = 0.52$

$n_2 = 0.5$, $a(n_2) = 0.62$

$n_3 = 0.64$, $a(n_3) = 0.65$

$o/p = 1$

5.)