

Lab Report - 5

Author: Venugopal R - 211EE160

Email: venugopalr.211ee160@nitk.edu.in

Note:

The value of α used in this document is given by: $\alpha = 1 + \text{mod}(160; 3) = 2$

Problem 1. (Butterworth filter design)

Design a low pass digital Butterworth filter which has a maximum passband ripple of α dB, and an edge frequency of 10 Hz. The filter also should have a minimum stopband attenuation of 40 dB from a stopband edge frequency of 20 Hz. Assume a sampling frequency of 720 samples/sec.

(1. Find the transfer function of the filter.)

(Solution) The transfer function coefficients are as follows,

$b = [2.0372\text{e-}11, 1.6297\text{e-}10, 5.7042\text{e-}10, 1.1408\text{e-}09, 1.4260\text{e-}09, 1.1408\text{e-}09, 5.7042\text{e-}10, 1.6297\text{e-}10, 2.0372\text{e-}11]$

$a = [1. , -7.5132, 24.7104, -46.4662, 54.6396, -41.1423, 19.3719, -5.2148, 0.6144]$

(2.. Plot its pole-zero plot. Comment on the system stability from the plot.)

(Solution) Plotting the Pole-zero plot of the Butterworth filter,

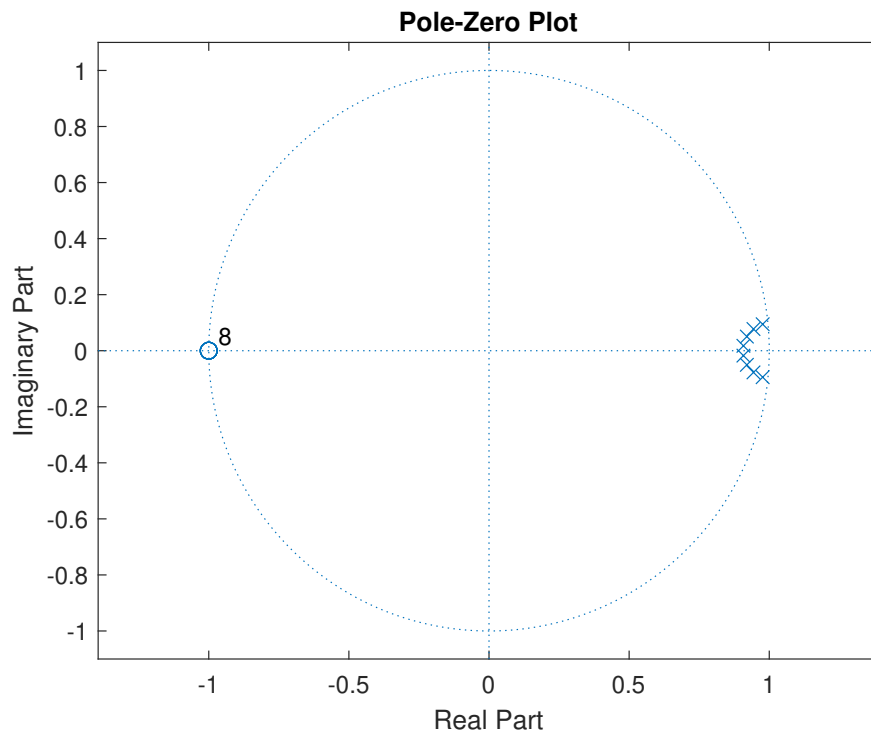


Fig 1: Pole-Zero plot of Butterworth filter

The system is stable because the poles reside inside the unit circle in the pole-zero plot. This tells us that this system will produce an output that will reach a steady state value after some time.

(3. Plot also the bode plot (with respect to frequency in Hz). Compare (plot on the same graph with legends) the impulse response and step response of the filter for a duration of 1 sec. Write down your observations.)

(Solution)

The Bode plot of the Butterworth filter is as follows,

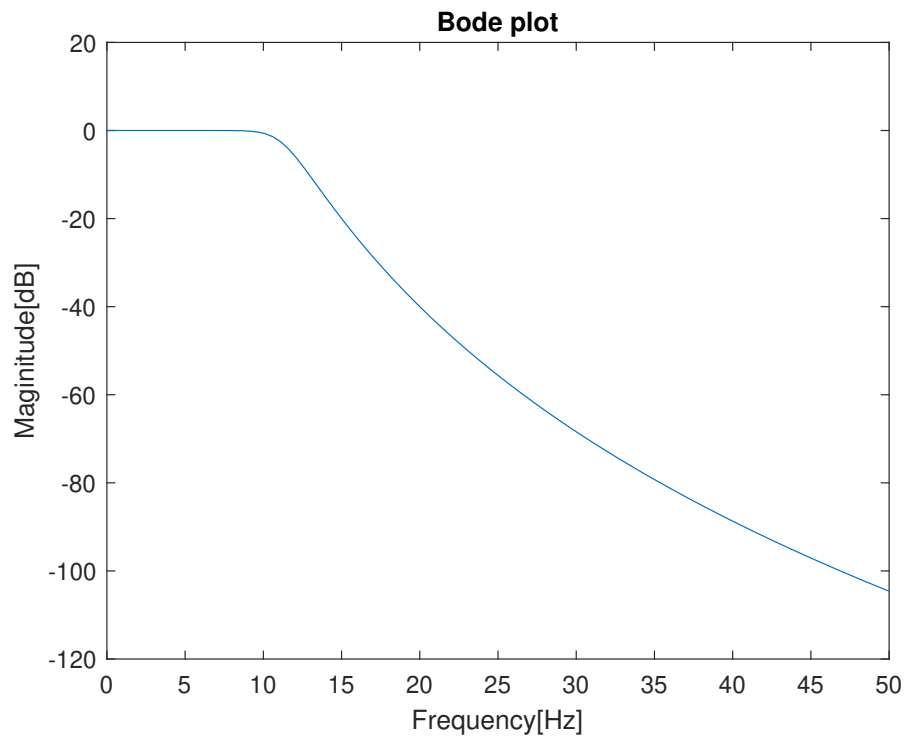


Fig 2: Bode plot of Butterworth filter

The step response and impulse response of this system is as follows,

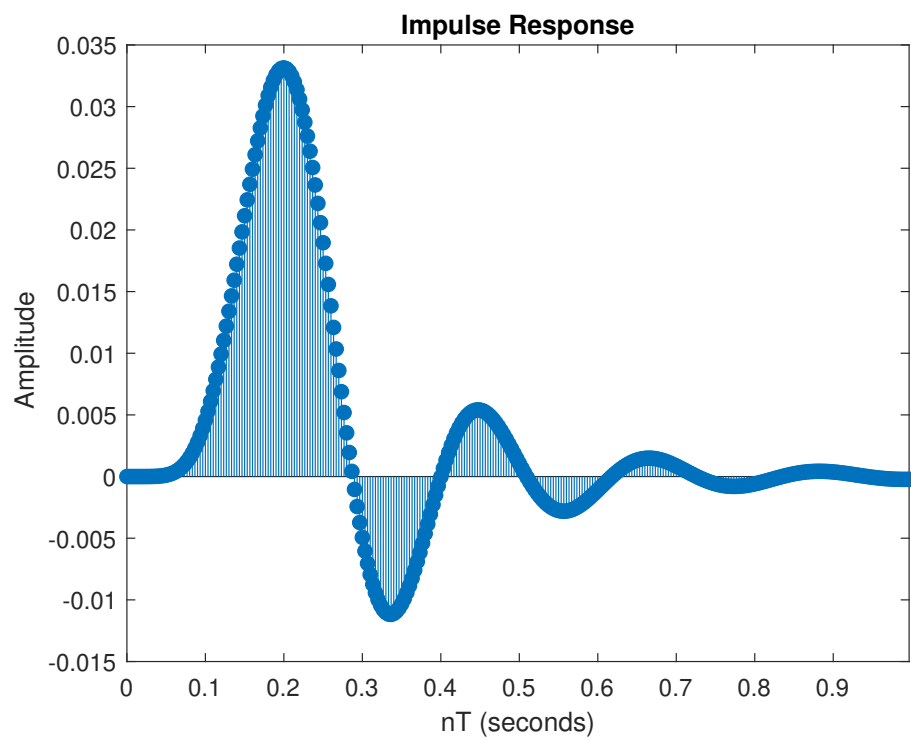


Fig 3: Impulse response of Butterworth filter

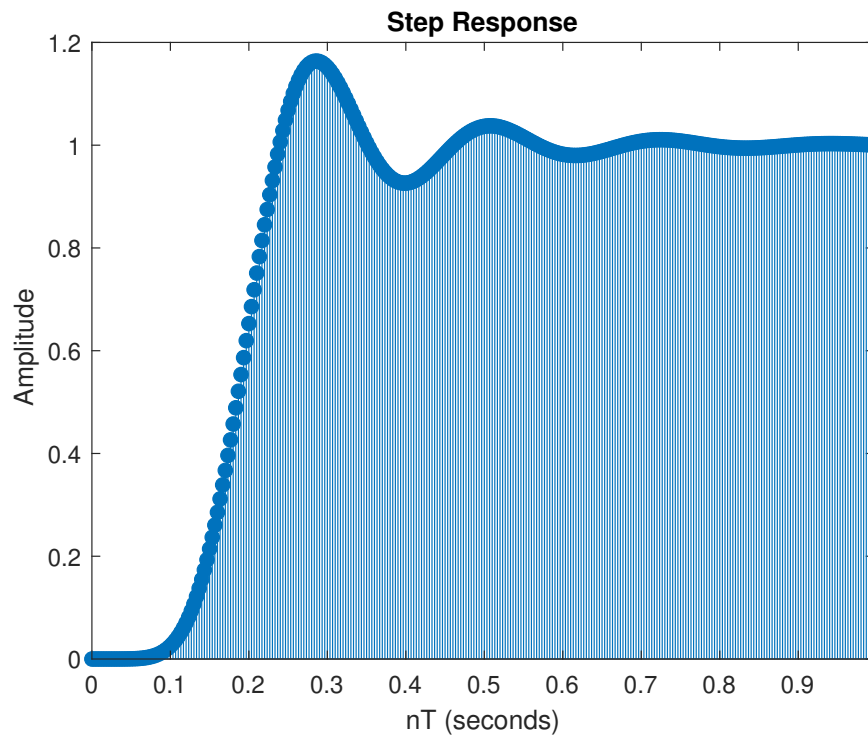


Fig 4: Step reponse of Butterworth filter

As inferred from the Pole-Zero plot, we can see that the system is stable and attains a steady state value.

Problem 2. (Filtering) Use the Butterworth filter to filter the ECG data ($FS = 720$ Hz) stored in the text file. Plot the filtered output and compare it with the original signal in the same figure.

(Solution) The input ECG data is as follows,

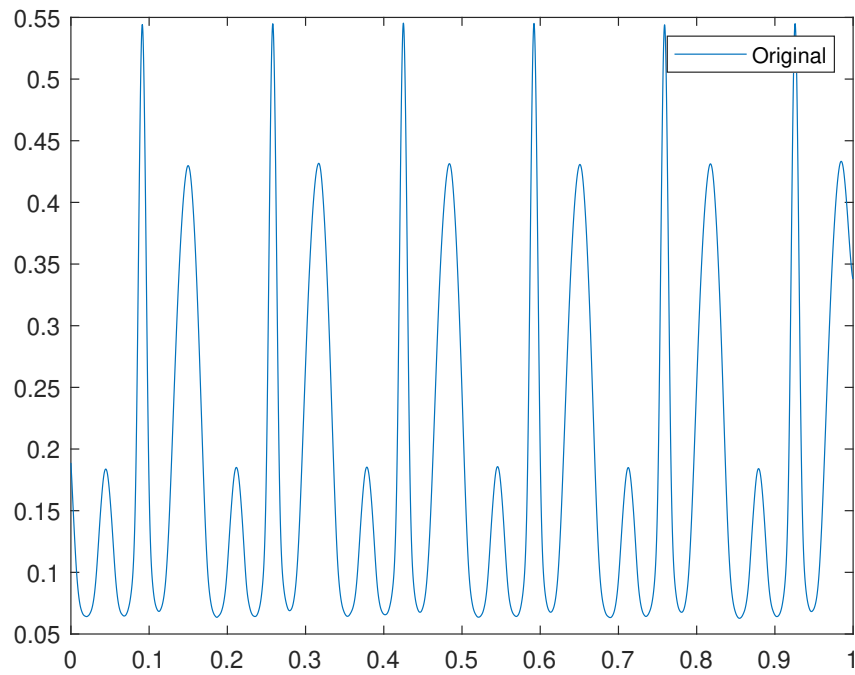


Fig 6: Original ECG Signal plot

Applying Butterworth filter to the ECG signal with the sampling frequency of 720 Hz and the pass band as 0-30 Hz from 10dB to -30dB. Comparing the original and the filtered signal, we get the following plot.

After applying the Butterworth filter, we get the following Bode Plot.

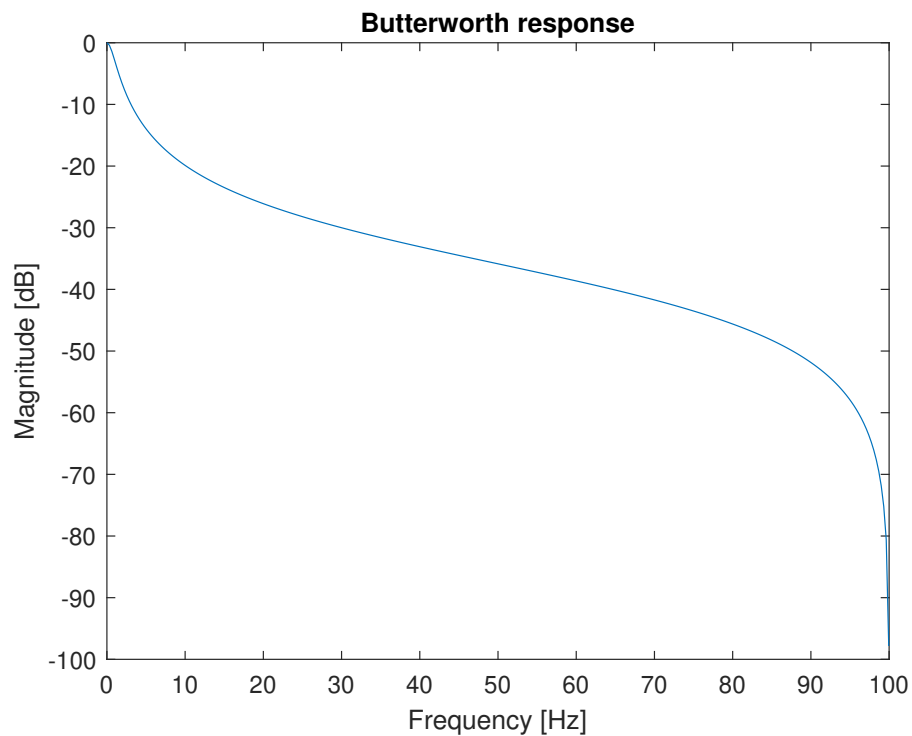


Fig 7: Bode plot of Butterworth filter applied to ECG signal

The filtered ECG signal is as follows,

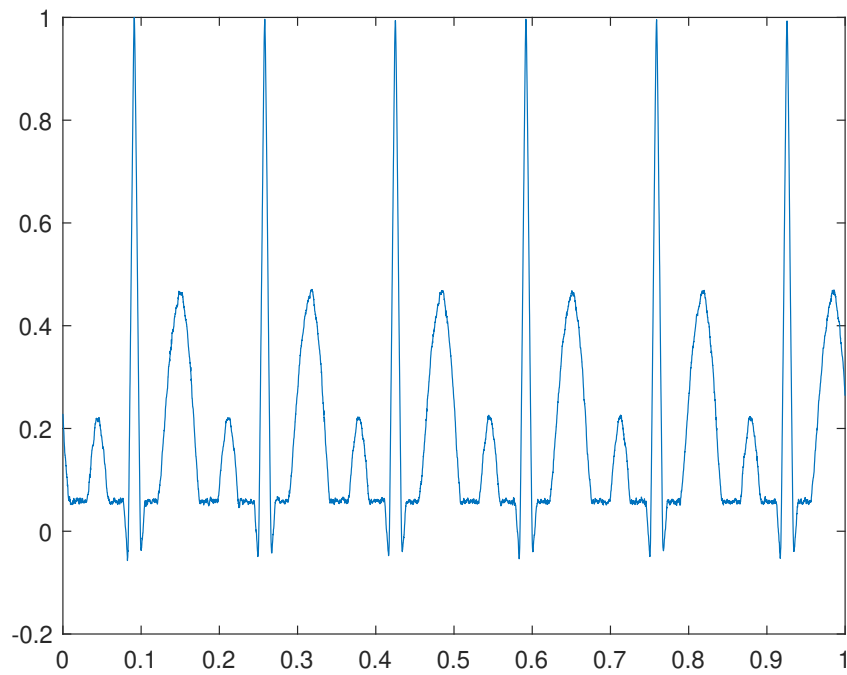


Fig 8: Filtered ECG signal

Problem 3. (Filtering — Time-Frequency Analysis) Plot the spectrogram of the `instruα.wav` (Same as Exp-5). You may use any window of your choice and sample duration for the window. Design a digital Butterworth band pass filter to only extract the fundamental (The first major peak after DC) and remove the rest including the DC. Write it into a wav file and listen. Also plot the spectrogram to ensure that you only have the fundamentals.

(Solution) Reading the data and plotting the spectrogram of the signal from `instru2.wav` file after windowing with a hanning window,

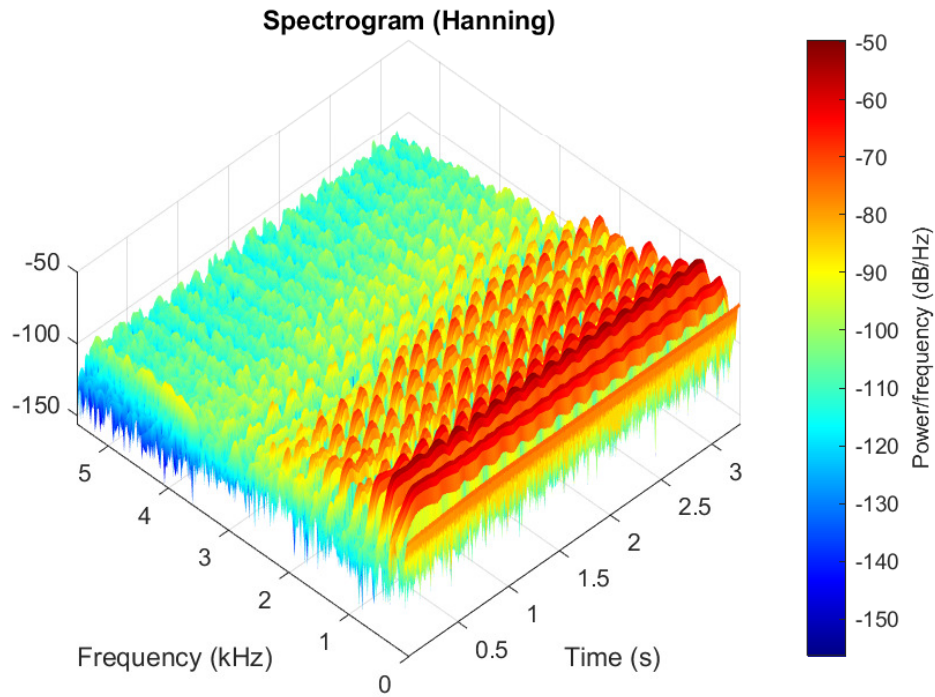


Fig 9: Instru2.wav signal spectrogram

Now, applying Butterworth filter by setting the passband frequency from 450 Hz-600Hz, from -60dB to 30 dB, we get the following Butterworth response,

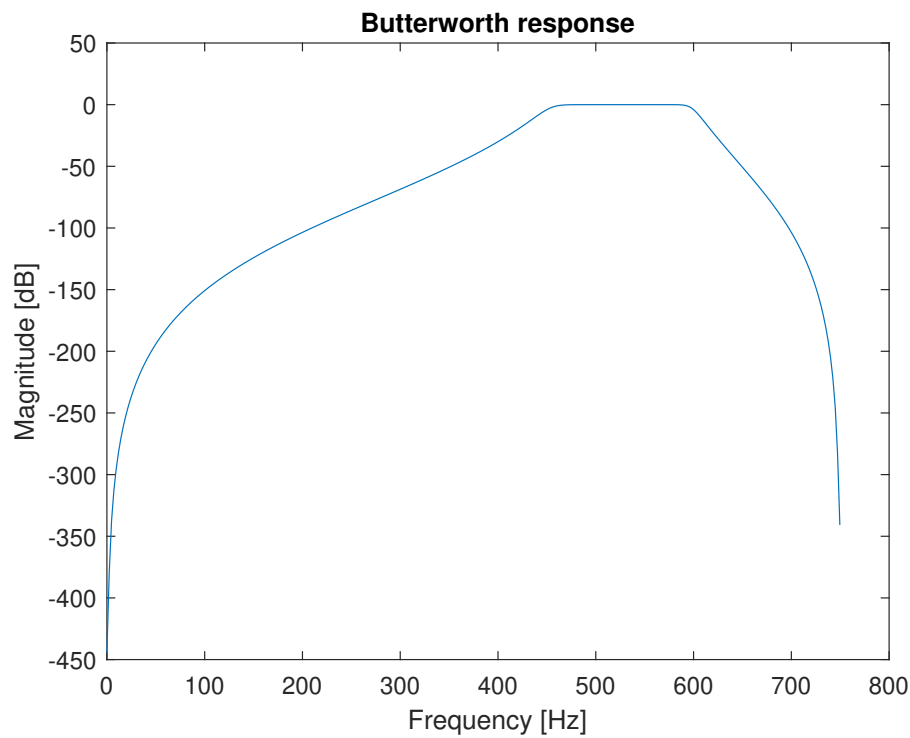


Fig 10: Butterworth response of Instru2.wav file

The new spectrogram of the signal clearly shows us the pitch which has been extracted by applying the filter.

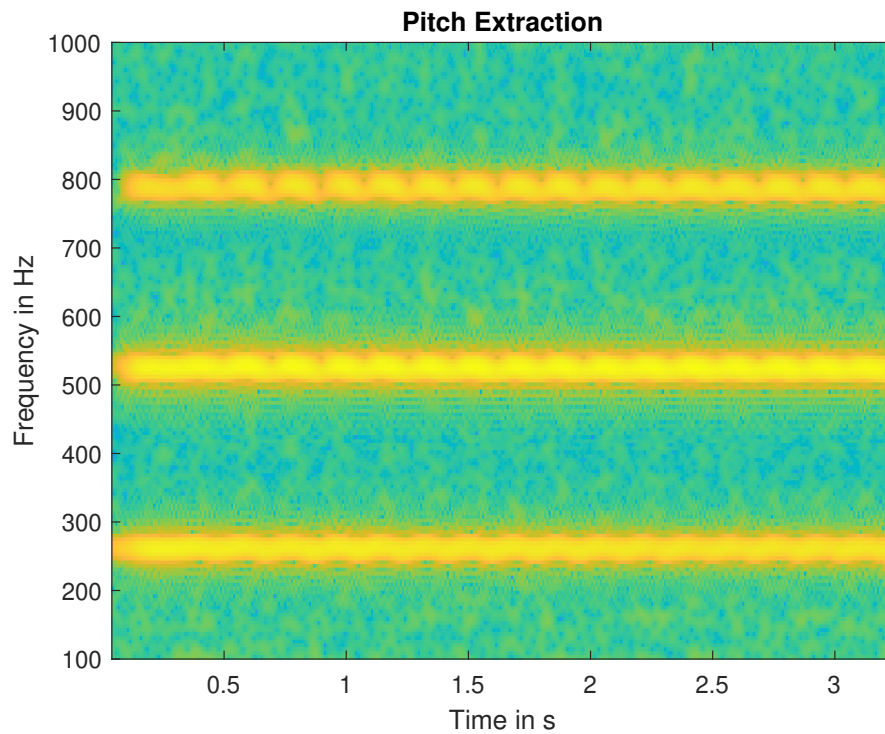


Fig 11: Filtered instru2.wav file signal

Problem 4. (Chebyshev filter design)

Try the lowpass filter specifications used in Problem 1 with Type I Chebyshev's filter

(1. Compare the system order w.r.t Butterworth)

(Solution)

In case of the Chebyshev filter the order of the system is 5, whereas, for the Butterworth filter, the order of the system was 8. This implies that you would need fewer components to physically implement the Chebyshev filter compared to the Butterworth filter.

(2. Plot also the bode plot (with respect to frequency in Hz). Plot the bode of the Butterworth filter from Problem 1 in the same picture and compare them)

Plotting the Bode plot for the Chebyshev filter with same parameters as problem 1,

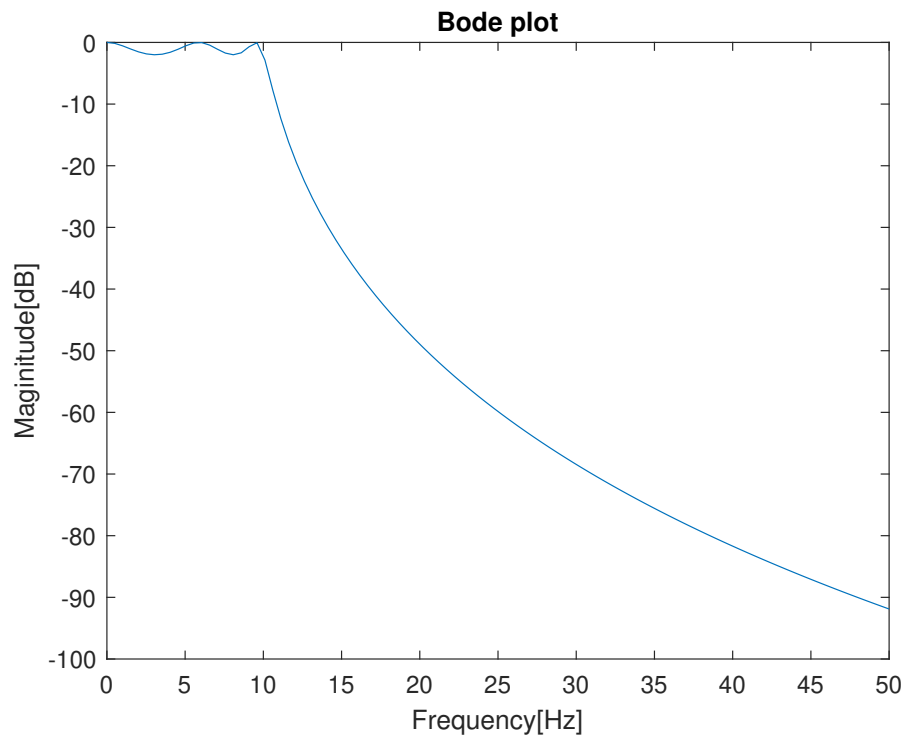


Fig 12: Bode plot of Chebyshev filter

Plotting the Bode plot for the Butterworth filter,

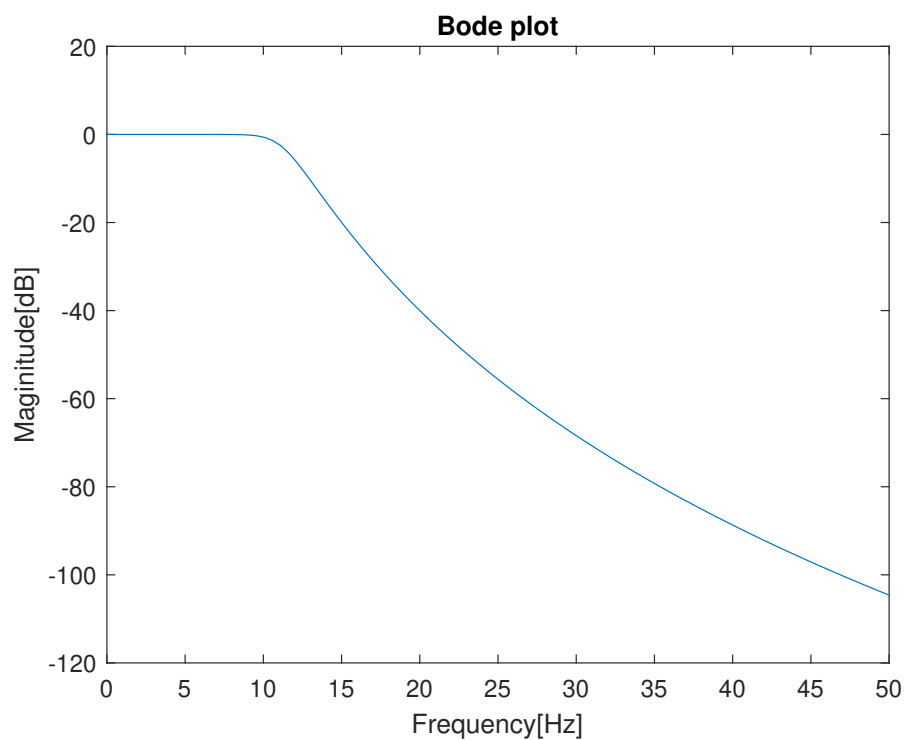


Fig 13: Bode plot of Butterworth filter

Comparing both the filters, we can see that the Chebyshev filter can suppress the necessary frequency components better than the butterworth filter.

(3. In a single graph compare the impulse response and step response of the

Butterworth and Chebyshev filter. Write your observations.)

(Solution) Comparing the step and impulse response of both the filters in a single graph

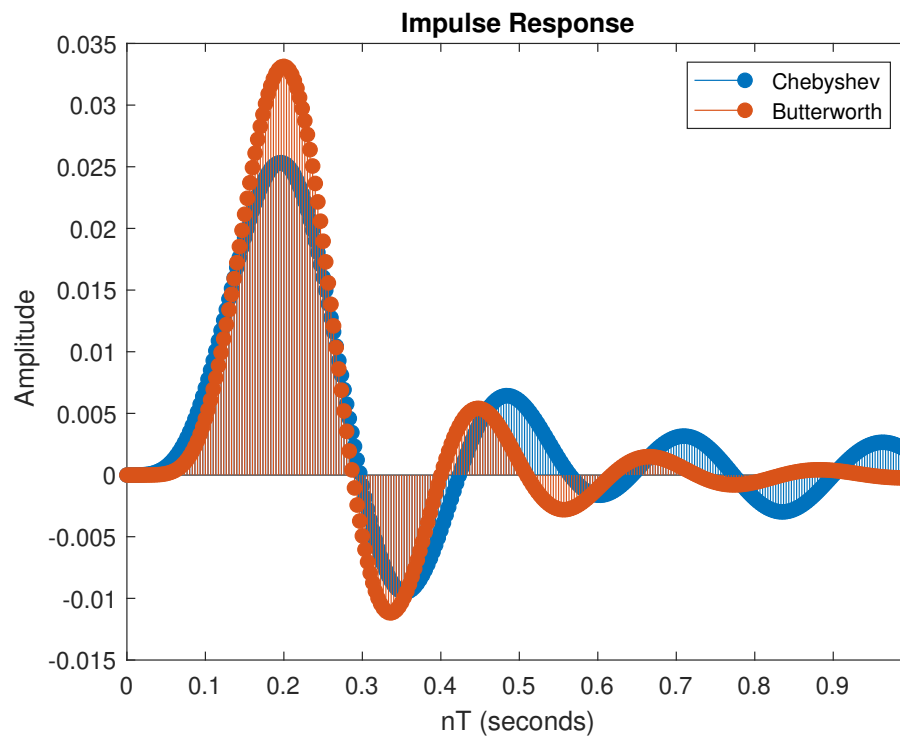


Fig 14: Impulse response of both filters

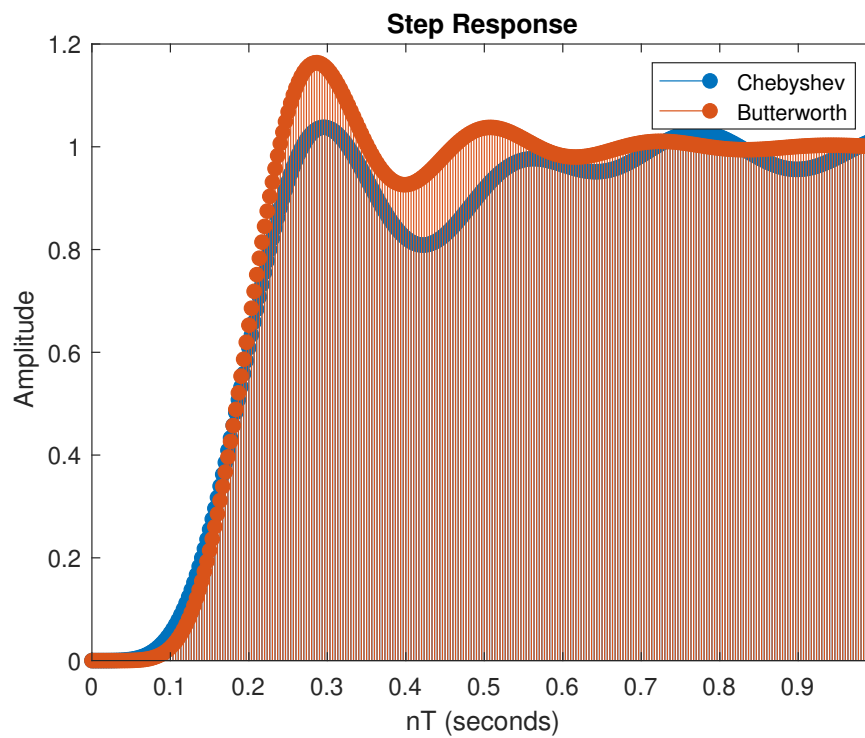


Fig 15: Step response of both filters

From these graphs we can observe that the Butterworth filter reaches the steady state value faster, whereas the Chebyshev filter is able to reduce the value of maximum peak overshoot

better than the Butterworth filter.

A Code Repositories

<https://github.com/VenugopalRadhakrishnan/DSP-Lab>.