
Linkable Ring Signature Algorithm

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I. SIGNATURE GENERATION

Let $G = \langle g \rangle$ be a group of large prime order (preferably *safeprime*) q . Let $H_1 : \{0, 1\}^* \rightarrow \mathbb{Z}_q$ and $H_2 : \{0, 1\}^* \rightarrow G$ be independent hash functions. Each user $i \in \{1, \dots, n\}$ has a distinct public key y_i and a private key x_i such that $y_i = g^{x_i}$. Let $L = \{y_1, \dots, y_n\}$ be the list of n public keys.

Input: $m \in \{0, 1\}^*$, list of public keys $L = \{y_1, \dots, y_n\}$, private key x_π corresponding to y_π $1 \leq \pi \leq n$.

- 1: Compute $h = H_2(L)$ and $\tilde{y} = h^{x_\pi}$.
 - 2: Pick $u \in_R \mathbb{Z}_q$ and compute $c_{i+1} = H_1(L, \tilde{y}, m, g^u, h^u)$.
 - 3: For $i = \pi + 1, \dots, n, 1, \dots, \pi - 1$ pick $s_i \in_R \mathbb{Z}_q$ and compute $c_{i+1} = H_1(L, \tilde{y}, m, g^{s_i} y_i^{c_i}, h^{s_i} \tilde{y}^{c_i})$
 - 4: Compute $s_\pi = u - x_\pi c_\pi \bmod q$
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The signature is $\sigma_L(m) = (c_1, s_1, \dots, s_n, \tilde{y})$.

II. SIGNATURE VERIFICATION

Input: $\sigma_L(m) = (c_1, s_1, \dots, s_n, \tilde{y})$ and list of public keys $L = \{y_1, \dots, y_n\}$.

- 1: Compute $h = H_2(L)$ and for $i = 1, \dots, n$, compute $z'_i = g^{s_i} y_i^{c_i}$ and $z''_i = h^{s_i} \tilde{y}^{c_i}$ and then $c_{i+1} = H_1(L, \tilde{y}, m, z'_i, z''_i)$ if $i \neq n$.
 - 2: Check $c_1 \stackrel{?}{=} H_1(L, \tilde{y}, m, z'_n, z''_n)$. If yes, accept signature else reject.
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III. LINKABILITY TESTING

Given: Two signatures $\sigma'_L(m') = (c'_1, s'_1, \dots, s'_n, \tilde{y}')$ and $\sigma''_L(m'') = (c''_1, s''_1, \dots, s''_n, \tilde{y}'')$ corresponding to messages m' and m'' .

- 1: Verify signatures : $\sigma'_L(m')$ and $\sigma''_L(m'')$
 - 2: Check if $\tilde{y}' = \tilde{y}''$. If so, signatures are created by the same signer. Otherwise, signatures are generated by different signers.
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