Digital Signal Processing

Tutorial 6

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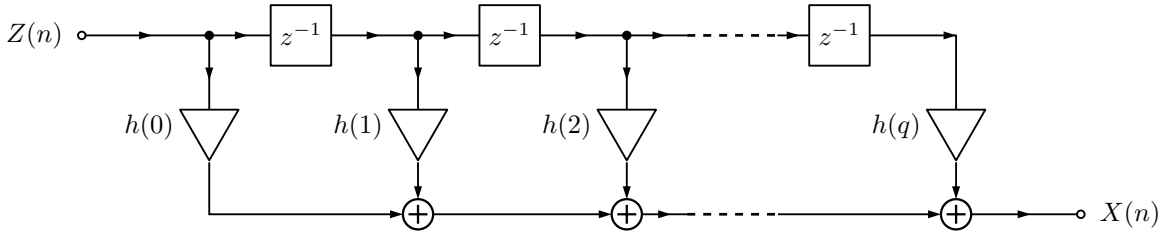
Task 1: Process identification

Consider the model given below where $Z(n)$ is a real-valued white noise process with variance σ_Z^2.



a) Find the difference equation for the system.

b) What type of process is $X(n)$?

Assume the covariance function $\hat{c}_{XX}(n)$ given by

$$\hat{c}_{XX}(n) = \begin{cases} \frac{1}{6}\sigma_Z^2, & n = -2 \\ \frac{-35}{36}\sigma_Z^2, & n = -1 \\ \frac{62}{36}\sigma_Z^2, & n = 0 \\ \frac{-35}{36}\sigma_Z^2, & n = 1 \\ \frac{1}{6}\sigma_Z^2, & n = 2 \\ 0, & \text{otherwise.} \end{cases}$$

(c) Which model order(s) is most appropriate for process $X(n)$ based on the given $\hat{c}_{XX}(n)$? Justify your answer.

Task 2: Auto-regressive process

Given the AR(p) process of observations $x(0), \ldots, x(N-1)$ as

$$X(n) + \sum_{k=1}^{p} a_k X(n-k) = Z(n)\,,$$

where $Z(n)$ is white noise with variance σ_Z^2.

a) Derive the Yule-Walker equations.

b) Assume that the filter, which relates $X(n)$ with $Z(n)$, is causal. Develop a parametric estimator for $a_1, \ldots, a_p$.

c) Derive a parametric estimator for σ_Z^2.

d) Construct an estimator $\hat{C}_{XX}(e^{j\omega})$ of the spectrum $C_{XX}(e^{j\omega})$ as a function of the parameters $a_1, \ldots, a_p$ and σ_Z^2. Is the estimator consistent?

Task 3: Moving average process

Given the MA(q) process of observations $x(0), \ldots, x(N-1)$ as

$$X(n) = Z(n) + \sum_{l=1}^{q} b_l Z(n-l)\,,$$

where $Z(n)$ is white noise with variance σ_Z^2.

a) Derive the Yule-Walker equations.

b) Develop a parametric estimator for $b_1, \ldots, b_q$ and σ_Z^2.

c) Construct an estimator $\hat{C}_{XX}(e^{j\omega})$ of the spectrum $C_{XX}(e^{j\omega})$ as a function of the parameters $b_1, \ldots, b_q$ and σ_Z^2.

Task 4: Parametric and non-parametric comparison

Download the provided MATLAB files *sample1.mat* and *Cxx1.mat*. *sample1.mat* contains observations $x(n)$, $n = 0, \ldots, 1023$ for the process $X(n)$. The file *Cxx1.mat* comprises of two vectors `Cxx` and `freq`. which represent the true spectrum $C_{XX}(e^{j\omega})$ and the 2π normalized frequency axis for `Cxx`.

a) Use the periodogram and Bartlett's method to estimate the spectrum $C_{XX}(e^{j\omega})$. What are the differences? (**Hint:** Use the commands `periodogram` and `pwelch` from the signal processing toolbox.)

b) Estimate the spectrum $C_{XX}(e^{j\omega})$ in a parametric way. Use an AR(2), AR(6) and AR(10) process for the estimation. Which one is best? What are the differences to the non-parametric methods?
(**Hint:** Use the command `aryule` from the signal processing toolbox.)

For the following use the files *sample2.mat* and *Cxx2.mat*

(c) Use the periodogram and Bartlett's method to estimate the spectrum $C_{XX}(e^{j\omega})$.

(d) Estimate the spectrum with an AR(2), AR(6) and AR(10) process. Which one is best? Compare your results with the results of the non-parametric estimation methods. Explain.

(e) Estimate the spectrum with an MA(2), MA(6) and MA(8) process. Which one is best? Compare your results with the results of the AR estimations. (**Hint:** Use the provided file *maparam.m*.)

Task 5: Model and order selection

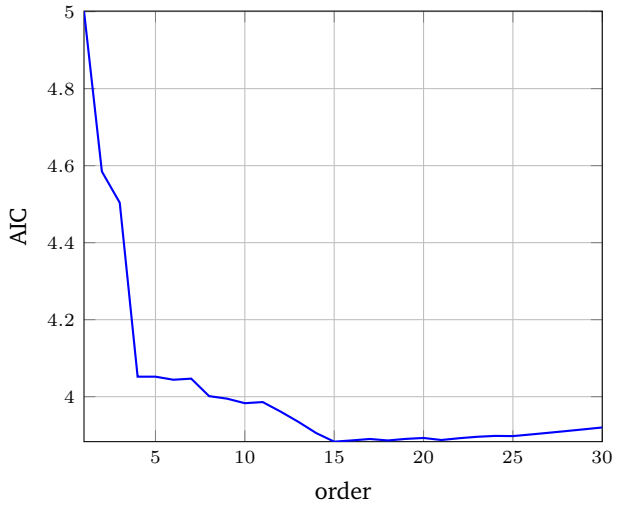
The provided MATLAB file *pressure.mat* contains real pressure data measured in the cylinder of an engine. The spectrum of this process is unknown. In order to estimate it, we need to know which parametric model suits best and which order is appropriate.

a) Explain the following statement: The AR-model is good for spectra with significant peaks, while the MA models a spectrum with valleys in it.

b) First use the Welch estimator to estimate the spectrum of the provided pressure data and decide if an AR or a MA model is more appropriate in this case.

c) Which order do you need at least to see the right number of peaks/valleys shown by your welch estimate? Try various values between 5 and 20.

d) Now we want to choose the order which suits best. A good approach is Akaike's information criterion (AIC) described in the manuscript. What is the meaning of the AIC? The AIC for this process is already computed and depicted below. Which order is the best according to AIC?



e) Now estimate the spectrum using the model chosen in (c) and the order given by the AIC.