Digital Signal Processing

Tutorial 5

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Task 1: Non-Parametric Spectrum Estimation

We want to compare the following non-parametric spectrum estimators: (i) the periodogram, (ii) the modified periodogram with a Hamming window, (iii) the Bartlett method, and (iv) the Welch method with a Hamming window and 50% overlap. For performance assessment of spectrum estimators $\hat{C}_{XX}(e^{j\omega})$, we want to use the so-called variability $\mathcal{V}$ which is defined as

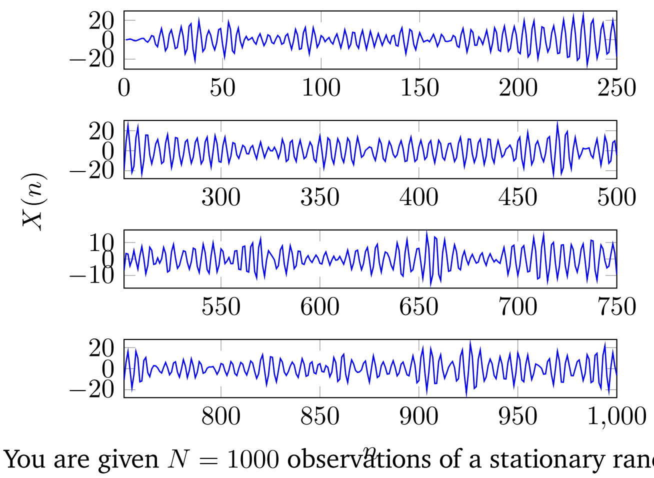
$$\mathcal{V} = \frac{\mathrm{Var}[\hat{C}_{XX}(e^{j\omega})]}{\mathrm{E}[\hat{C}_{XX}(e^{j\omega})]^2}$$

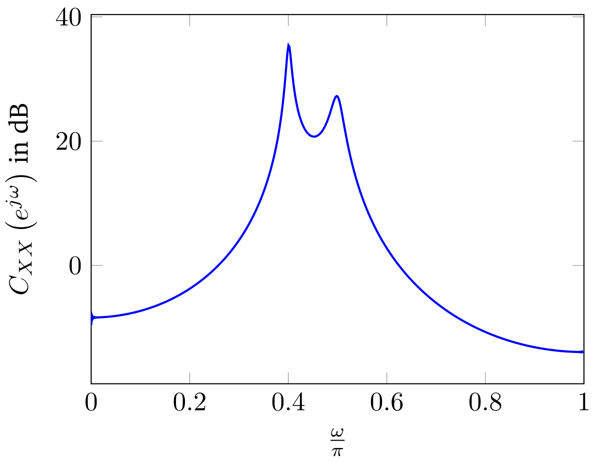
a) Briefly summarize methods (i)-(iv) and state their asymptotic statistical properties.

b) Determine $\mathcal{V}$ for the described methods. For periodogram averaging, take L segments of length M. Assume N, M to be large so that the asymptotic results from the manuscript can be applied. Only consider $\omega \neq 0$.

Another figure of merit for spectral estimators is the product $Q = \mathcal{V}\Delta\omega$, where $\Delta\omega$ denotes the resolution bandwidth of the spectrum estimator. More specifically, $\Delta\omega$ is the 3-dB main-lobe width of the Fourier tranform of the window function. Use $\Delta\omega = 0.89 \cdot 2\pi/N$ for the rectangular window and $\Delta\omega = 1.30 \cdot 2\pi/N$ for the Hamming window of length N.

c) Calculate Q for methods (i)-(iv). You may have to express the results from b) as a ratio between M and N. Comment on your results. Is Q a reasonable figure of merit for a spectral estimator?

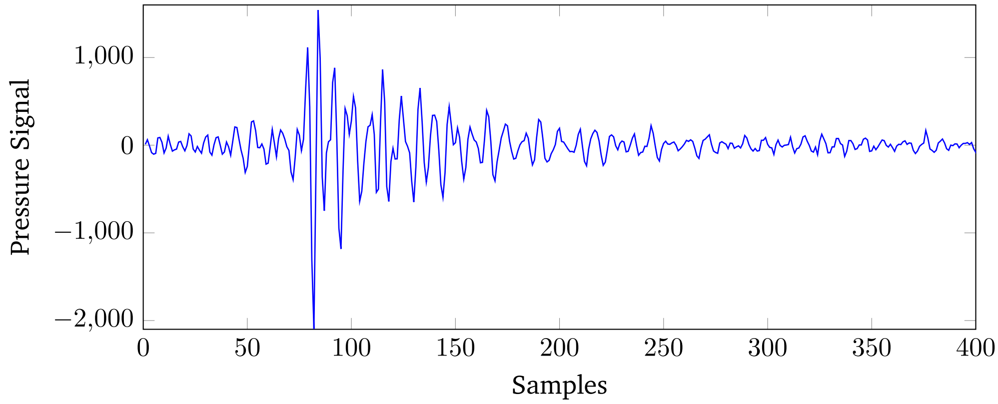




You are given $N = 1000$ observations of a stationary random process $X(n)$, which are plotted in the figure above. The true spectrum is also provided for later comparison. Assume for the time being that there is no available additional information on the process and we thus have to estimate the spectrum using the non-parametric methods described above.

d) Download the file `ar3data.mat` from the moodle course. Calculate and plot the different estimates (i)-(iv) using MATLAB and compare them to the true spectrum. For periodogram averaging, use a segment length of $M = 100$. Which method provides the best estimation in terms of bias and variance?

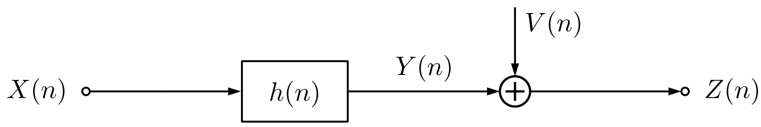
We will now estimate the spectrum of a real-world pressure signal, depicted in the figure below. More specifically, this pressure signal is a high-pass filtered recording of a pressure sensor in a combustion chamber of a conventional spark-ignition engine. Diagnosis of these signals can be used for knock detection and motor control. Note that the signal is not strictly stationary and shows transient characteristics. Regardless, the aforementioned methods can be applied for *mode detection*.



e) Download the file `pressure.mat` from the moodle course. Use MATLAB to calculate and plot a suitable non-parametric spectrum estimate. Can you detect the four different modes in the signal?

Task 2: System Identification

Consider a stable linear time-invariant system as shown below. The input $X(n)$ is a zero-mean real-valued white noise



process with variance σ_X^2. The disturbance signal $V(n)$ is a zero-mean real-valued random process, which is independent of $X(n)$. The aim is to estimate the system transfer function $H(e^{j\omega}) = \mathcal{F}\{h(n)\}$ assuming that observations of $X(n)$ for $n = 0, \ldots, N-1$ and corresponding observations of $Z(n)$ are available. This problem is known as *system identification*.

a) Derive expressions for the cross-covariance function $c_{ZX}(n)$ and the cross-spectrum $C_{ZX}(e^{j\omega})$.

b) How can the system transfer function $H(e^{j\omega})$ be estimated given an estimate of the cross-spectrum $\hat{C}_{ZX}(e^{j\omega})$?

c) Construct an estimator of the cross-spectrum $C_{ZX}(e^{j\omega})$ using the averaged cross-periodogram. Describe the effect of the number of segments L on the bias and variance of the estimator.

d) As an alternative to b), the transfer function may also be estimated using a time domain approach. Show that $H(e^{j\omega})$ can also be estimated given an estimate of the cross-covariance function $\hat{c}_{ZX}(n)$ by

$$\hat{H}(e^{j\omega}) = \frac{1}{\hat{\sigma}_X^2} \sum_{n=-p}^{p} \hat{c}_{ZX}(n) e^{-j\omega n}.$$

What is the influence of p on the statistical properties of $\hat{H}(e^{j\omega})$?