Lecture

Adaptive Filters

Lecture 1: Introduction

Content of this lecture

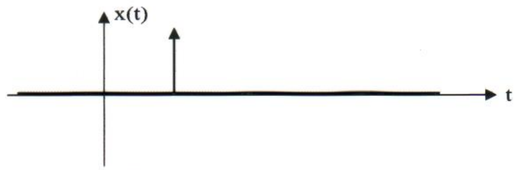
- Review of random processes and their properties
- Notations
- Review of static filters in time & frequency domain
- Different generalized applications of adaptive filters
- Applications of adaptive filters in audio processing systems
- Overview about adaptive filters introduced in this lecture series

Examples of Signal Models

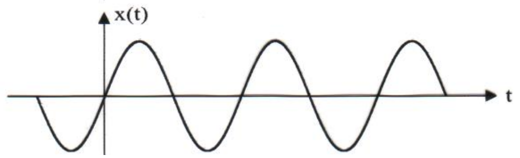
Examples:

A single signal:
(deterministic signal,
test signal)

impulse



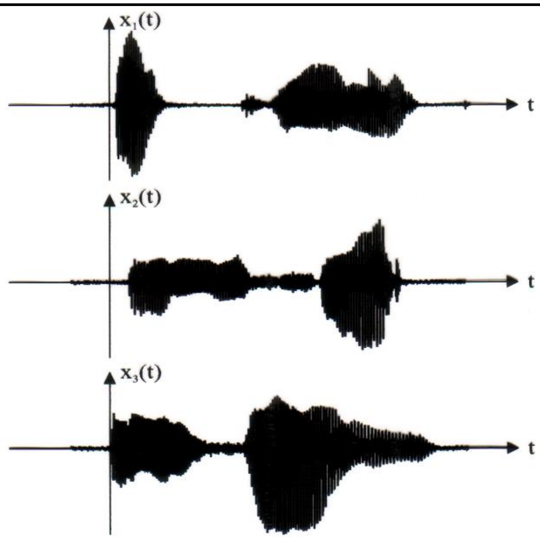
periodic signal



A random process
(random signal, statistical signal, random process):

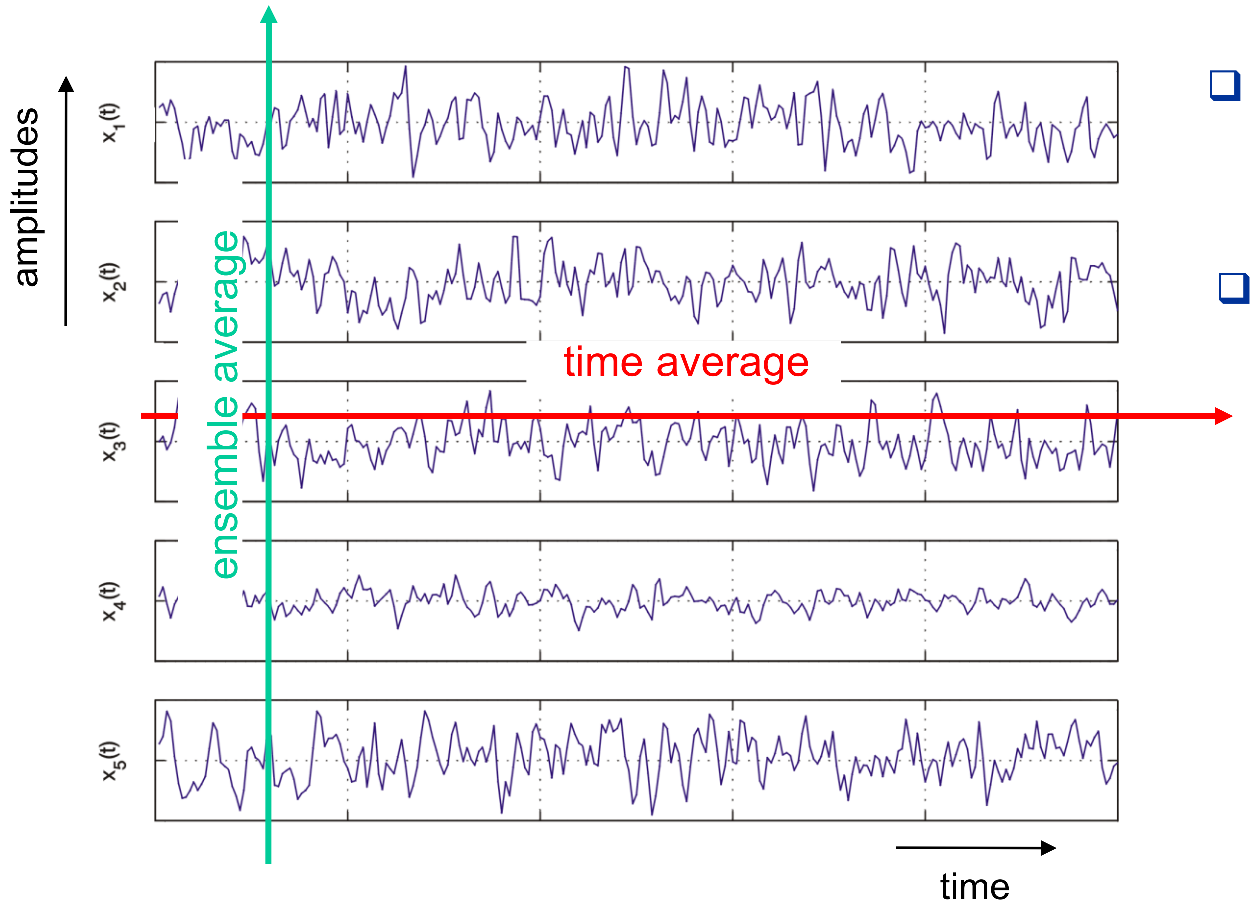
Ensemble of signals with common properties

Example: speech



Random Processes

- Examples of sample functions of one random process:



- Possible averages over the ensembles or time

- Random processes characterized by probability density functions (pdf):

$$f_x(x, n)$$

Notation: Part 1

- Scalar notation:
  - Signals: $x(n)$ — *Discrete time index*
  - Impulse responses (time varying): $h_i(n)$ — *Coefficient index*
  - Example for a (real-value) convolution: $$y(n) \;=\; \sum_{i=0}^{N-1} x(n-i)\, h_i(n)$$

- Vector notation:
  - Signal vectors: $\boldsymbol{x}(n) \;=\; \Big[x(n),\, x(n-1),\, \ldots,\, x(n-N+1)\Big]^{\mathrm{T}}$ — *Bold, lower case*
  - Vectors of impulse responses (time varying): $\boldsymbol{h}(n) \;=\; \Big[h_0(n),\, h_1(n),\, \ldots,\, h_{N-1}(n)\Big]^{\mathrm{T}}$
  - Example for a (real-value) convolution : $y(n) \;=\; \boldsymbol{x}^{\mathrm{T}}(n)\, \boldsymbol{h}(n) \;=\; \boldsymbol{h}^{\mathrm{T}}(n)\, \boldsymbol{x}(n)$

- Matrices:

$$\boldsymbol{A}(n) \;=\; \begin{bmatrix} a_{00}(n) & a_{01}(n) & \ldots & a_{0N}(n) \\ a_{10}(n) & a_{11}(n) & \ldots & a_{1N}(n) \\ \vdots & \vdots & & \vdots \\ a_{M0}(n) & a_{M1}(n) & \ldots & a_{MN}(n) \end{bmatrix}$$

Bold, upper case

Notation: Part 2

- Random processes („Ensemble of signals“):
  - **Notation:** $x(n),\; x_1(n),\; x_2(n)$ — ***No differentiation in notation of deterministic signals and random processes – other notations:*** $\boldsymbol{x}(\eta, n),\; x(\omega, n),\; X(n)$
  - **Probability density function:** $f_x(x,n),\; f_{x_1 x_2}(x_1, x_2, n_1, n_2)$
  - **Stationary random processes:**

$$f_x(x,\, n) \;=\; f_x(x,\, n+n_0) \;=\; f_x(x)$$
$$f_{x_1 x_2}(x_1,\, x_2,\, n_1,\, n_2) \;=\; f_{x_1 x_2}(x_2,\, x_2,\, n_1+n_0,\, n_2+n_0) \;=\; f_{x_1 x_2}(x_1,\, x_2,\, n_2-n_1)$$

  - **Expectation values for random processes**:

Non-stationary random processes:

Linear mean value: $$m_x^{(1)}(n) = \mathrm{E}\Big\{x(n)\Big\} \;=\; \int\limits_{x=-\infty}^{\infty} x\, f_x(x,n)\, dx$$

Quadratic mean value: $$m_x^{(2)}(n) = \mathrm{E}\Big\{x^2(n)\Big\} \;=\; \int\limits_{x=-\infty}^{\infty} x^2\, f_x(x,n)\, dx$$

Variance: $$\sigma_x^2(n) = \mathrm{E}\{[x(n) - m_x^{(1)}(n)]^2\} \;=\; \int\limits_{x=-\infty}^{\infty} (x - m_x^{(1)}(n))^2\, f_x(x,n)\, dx$$

Linear transform mean value: $$\mathrm{E}\Big\{g\big(x(n)\big)\Big\} \;=\; \int\limits_{x=-\infty}^{\infty} g(x)\, f_x(x,n)\, dx$$

Stationary random processes:

$$m_x^{(1)} \;=\; \int\limits_{x=-\infty}^{\infty} x\, f_x(x)\, dx$$

$$m_x^{(2)} \;=\; \int\limits_{x=-\infty}^{\infty} x^2\, f_x(x)\, dx$$

$$\sigma_x^2 \;=\; \int\limits_{x=-\infty}^{\infty} (x - m_x^{(1)})^2\, f_x(x)\, dx$$

$$\mathrm{E}\Big\{g\big(x(n)\big)\Big\} = \int\limits_{x=-\infty}^{\infty} g(x)\, f_x(x)\, dx$$

Notation: Part 3

- Auto- und cross-correlation for real-value, stationary random processes:
  - Auto-correlation function:

$$\mathrm{E}\Big\{x(n)\,x(n+l)\Big\} \;=\; r_{xx}(l)$$

  - Cross-correlation function:

$$\mathrm{E}\Big\{x(n)\,y(n+l)\Big\} \;=\; r_{xy}(l)$$

  - Auto power spectral density:

$$S_{xx}(\Omega) \;=\; \sum_{l=-\infty}^{\infty} \mathrm{E}\Big\{x(n)\,x(n+l)\Big\}\,e^{-j\Omega\,l} \;=\; \sum_{l=-\infty}^{\infty} r_{xx}(l)\,e^{-j\Omega\,l}$$

  - Cross power spectral density:

$$S_{xy}(\Omega) \;=\; \sum_{l=-\infty}^{\infty} \mathrm{E}\Big\{x(n)\,y(n+l)\Big\}\,e^{-j\Omega\,l} \;=\; \sum_{l=-\infty}^{\infty} r_{xy}(l)\,e^{-j\Omega\,l}$$

Notation: Part 4

- Stationary, white noise:
  - Auto-correlation function:

$$r_{xx}(l)\Big|_{\text{white noise}} = \sigma_x^2\,\delta_K(l) = \begin{cases} \sigma_x^2, & \text{if } l = 0, \\ 0, & \text{else} \end{cases}$$

  - Auto power spectral density:

$$S_{xx}(\Omega)\Big|_{\text{white noise}} = \sigma_x^2$$

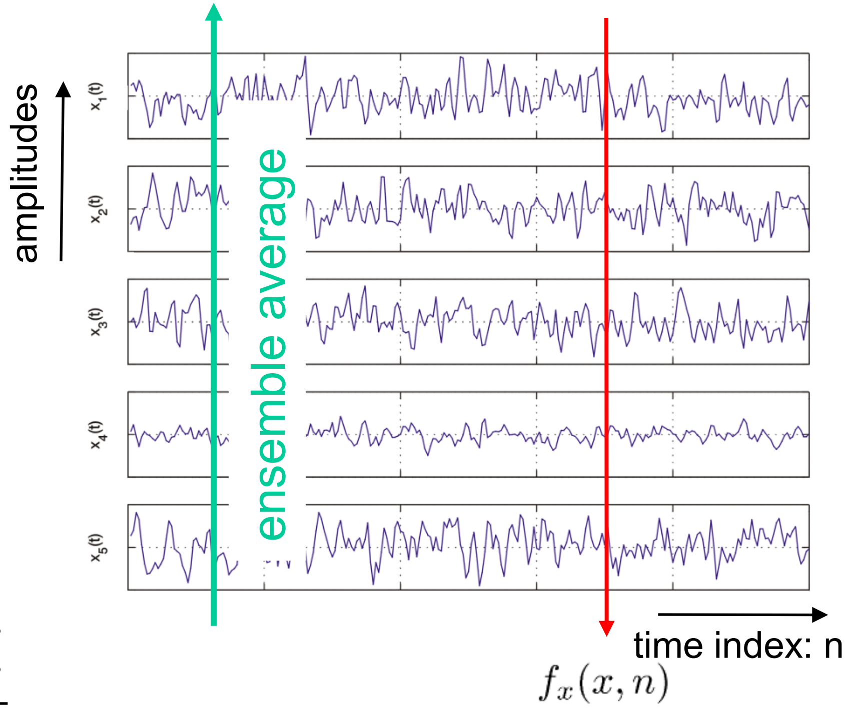
- Literature:
  - E. Hänsler: *Statistische Signale: Grundlagen und Anwendungen, Kapitel 3 – Zufallsprozesse,* Springer, 2001
  - A. Zoubir: *Digital Signal Processing, Chapter 7 – Random Variables and Stochastic Processes*, Vorlesungsskript, Darmstadt, 2005

Random Processes: Ensemble Average

- Definition of the **time dependent** "Probability density function" (pdf) of a random process: $f_x(x,n)$

- Analyzing a random process at one time instance, one generates a random variable.

- As for the random variable, one may also define statistical characteristic values, such as the "expectation" for a random process:



Random variable:

$$m_x^{(1)} = \mathrm{E}\{\, x \,\} = \begin{cases} \int\limits_{-\infty}^{+\infty} x\, f_x(x)dx & \text{continuous amplitudes} \\ \sum\limits_{\forall x_i} x_i\, p_x(x_i) & \text{discrete amplitudes} \end{cases}$$

Random process:

$$m_x^{(1)}(n) = \mathrm{E}\{\, x(n) \,\} = \begin{cases} \int\limits_{-\infty}^{+\infty} x\, f_x(x,n)dx & \text{continuous amplitudes} \\ \sum\limits_{\forall x_i} x_i\, p_x(x_i,n) & \text{discrete amplitudes} \end{cases}$$

Random Processes: Example for the expectation

- **Example of a process with a time dependent mean:**

  Random process with sinusoidal sample function and random phase

  - Process definition: Sinus function with random phase:

$$x(t) = \sin(\omega_0 t + \varphi)$$

  - Density of the random phase:

$$f_\varphi(\varphi) = \begin{cases} 2/\pi & \text{for } 0 \le \varphi \le \pi/2 \\ 0 & \text{elsewhere} \end{cases}$$

  - Mean calculation:

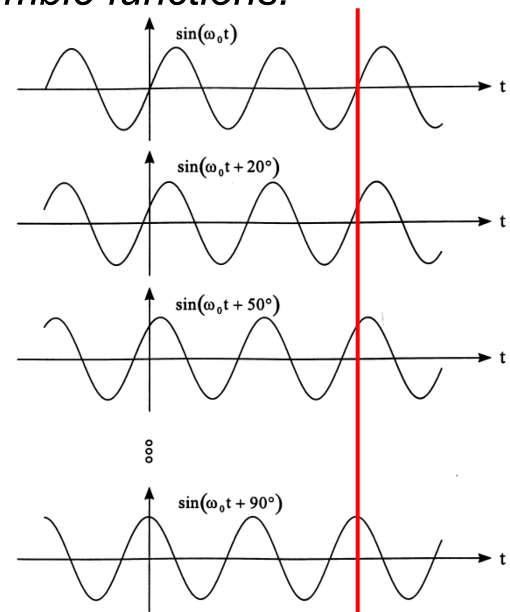
$$\begin{aligned} m_x^{(1)}(t) &= \mathrm{E}\{\, x(t) \,\} \\ &= \mathrm{E}\{\, \sin(\omega_0 t + \varphi) \,\} \\ &= \int_{\varphi=0}^{\pi/2} \sin(\omega_0 t + \varphi)\, \frac{2}{\pi}\, d\varphi \end{aligned}$$

density

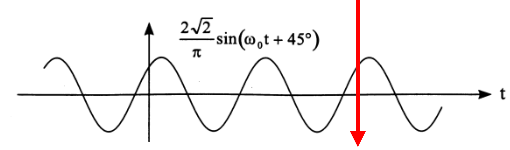
limits of the density

$$\boxed{m_x^{(1)}(t) = 2\sqrt{2}/\pi \, \sin(\omega_0 t + \pi/4)}$$

Ensemble functions:



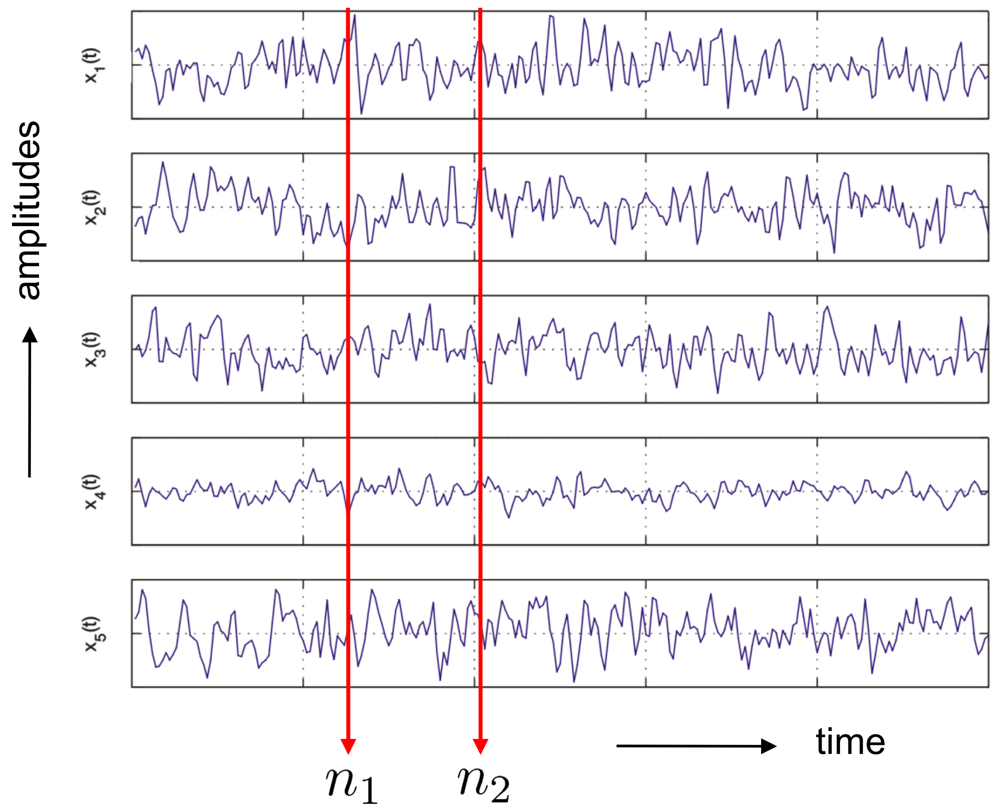
Mean value:



Random Processes: Stationarity (I)

1) Probability density functions are invariant against translations

$$
\begin{aligned}
m_x^{(1)} &= \mathrm{E}\left\{ x(n) \right\} \\
m_x^{(2)} &= \mathrm{E}\left\{ |x(n)|^2 \right\} \\
\sigma_x^2 &= \mathrm{E}\left\{ |x(n) - m_x^{(1)}|^2 \right\}
\end{aligned}
$$



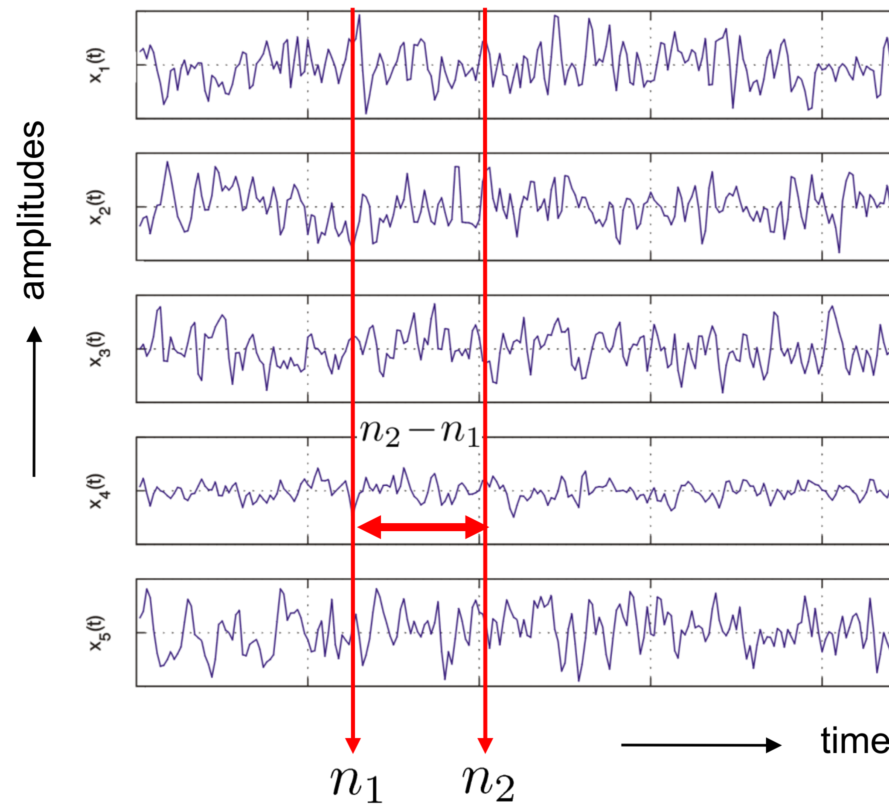
$$f_x(x, n_1) = f_x(x, n_2) = f_x(x)$$

Random Processes: Stationarity (II)

2) Joint probability density functions only depend on the time difference

$$r_{xx}(l) = \mathrm{E}\{\, x(n)^* x(n+l) \,\}$$

$$S_{xx}(\Omega) = \sum_{l=-\infty}^{+\infty} r_{xx}(l)\, e^{-j\,\Omega\, l}$$



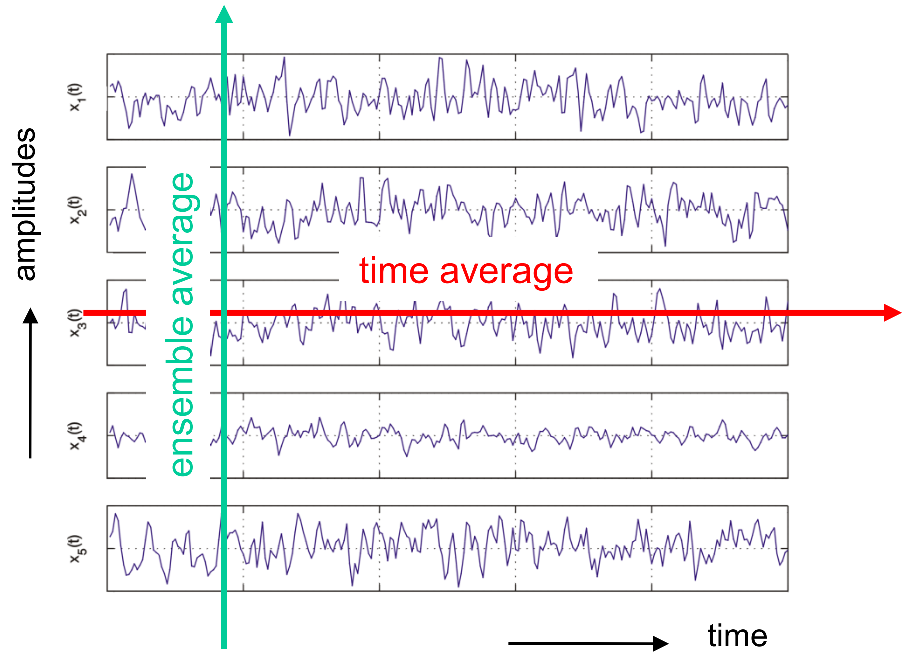
$$f_{x_1 x_2}(x_1, x_2, n_1, n_2) = f_{x_1 x_2}(x_1, x_2, n_2 - n_1)$$

Random Processes: Ergodic Processes

- **Definition of ergodic processes:**

Each sample function represents with probability one the entire random process

=> Ensemble mean values may be replaced by time mean values



Random Processes (RP) : Statistical Values

- In dependence of the properties of the random processes the statistical values may be calculated differently:

Signal models

conj.-complex for complex processes

	mean value	autocorrelation function
non-stationary RP	$m_x^{(1)}(n) = \mathrm{E}\{\, x(n) \,\}$	$r_{xx}(n_1, n_2) = \mathrm{E}\{\, x^*(n_1)\, x(n_2) \,\}$
stationary RP	$m_x^{(1)} = \mathrm{E}\{\, x(n) \,\}$	$r_{xx}(l) = \mathrm{E}\{\, x^*(n)\, x(n+l) \,\}$
ergodic RP	$m_x^{(1)} = \lim\limits_{n_0 \to \infty} \frac{1}{2n_0 + 1} \sum\limits_{n=-n_0}^{n_0} x_\nu(n)$	$r_{xx}(l) = \lim\limits_{n_0 \to \infty} \frac{1}{2n_0 + 1} \sum\limits_{n=-n_0}^{+n_0} x_\nu^*(n) x_\nu(n+l)$

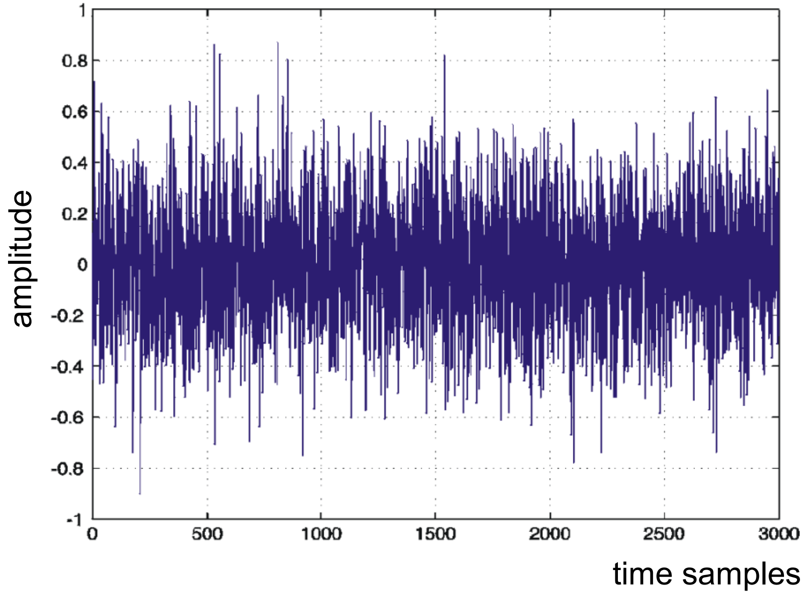
increased complexity

closer to reality

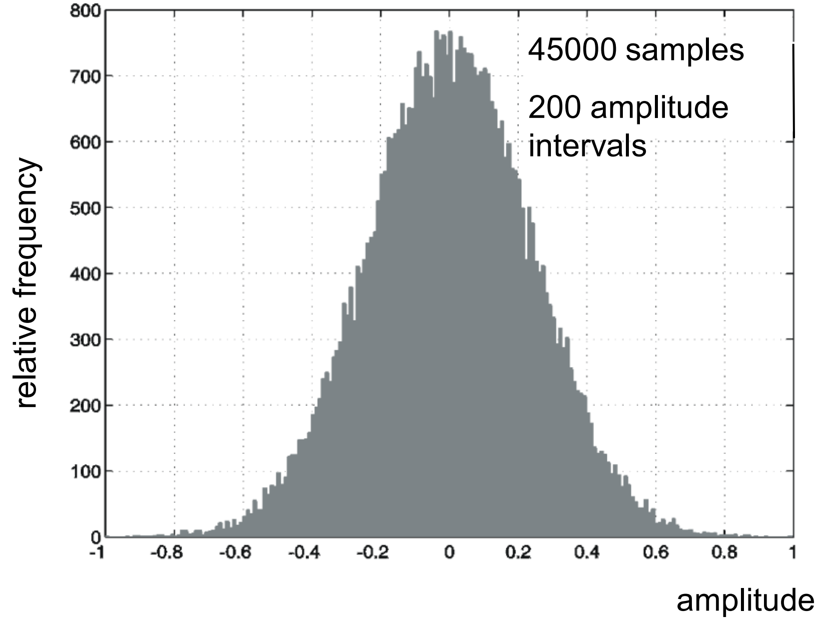
one sample function of the random process

Signal distributions: Gaussian Noise

- One ensemble function:



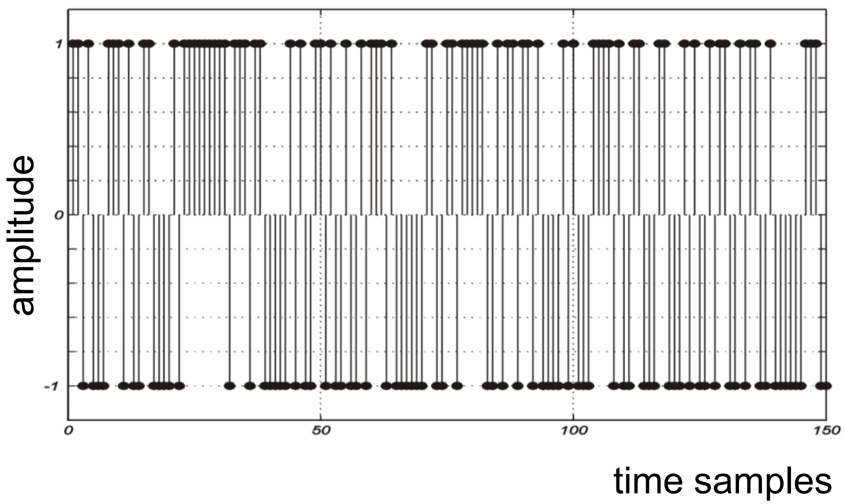
- Histogram of a Gaussian distributed signal:



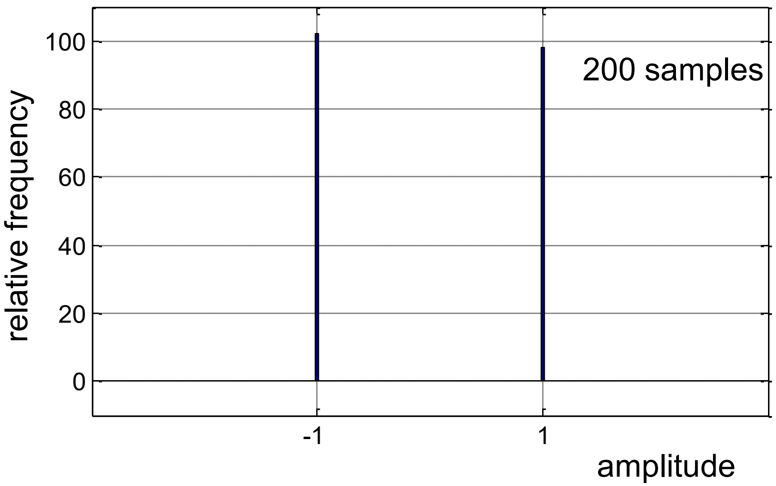
- **Caution**:
  Amplitude distribution of a signal should NOT be confused with correlation properties!
  I.e., Gaussian noise can be white or colored (example: car noise!)

Signal distributions: Binary Noise

- One ensemble function:



- Histogram of a binary noise:



- **Caution**:
  Amplitude distribution of a signal should NOT be confused with correlation properties!
  I.e., Binary noise is typically white but not necessarily!

Random Processes: The autocorrelation matrix

- For many calculations with random signals, it is helpful to define a “random vector” :

$$\boldsymbol{x}(n) \;=\; \Big[x(n),\;\; x(n-1),\;\; x(n-2),\;\; \ldots\;\; x(n-N+1)\Big]^{\mathrm{T}}$$

- For this definition, $x(n)$ is typically assumed to be a stationary random process.

- Definition of the autocorrelation matrix $\boldsymbol{R}_{xx}$, which exhibits Toeplitz structure, due to the property: $r^*_{xx}(l) = \mathrm{E}\{\; x(n)\, x^*(n+l)\;\} = \mathrm{E}\{\; x(n-l)\, x^*(n)\;\} = r_{xx}(-l)$

$$\begin{aligned}
\boldsymbol{R}_{xx}^{(N)} \;&=\; \mathrm{E}\left\{\boldsymbol{x}(n)\,\boldsymbol{x}(n)^{\mathrm{H}}\right\} \\
&=\; \begin{bmatrix} r_{xx}(0) & r_{xx}(1) & \ldots & r_{xx}(N-1) \\ r^*_{xx}(1) & r_{xx}(0) & \ldots & r_{xx}(N-2) \\ \vdots & \vdots & \ddots & \vdots \\ r^*_{xx}(N-1) & r^*_{xx}(N-2) & \ldots & r_{xx}(0) \end{bmatrix}_{(N\times N)}
\end{aligned}$$

Random Processes: The autocorrelation matrix

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Numerical properties of Toeplitz matices:

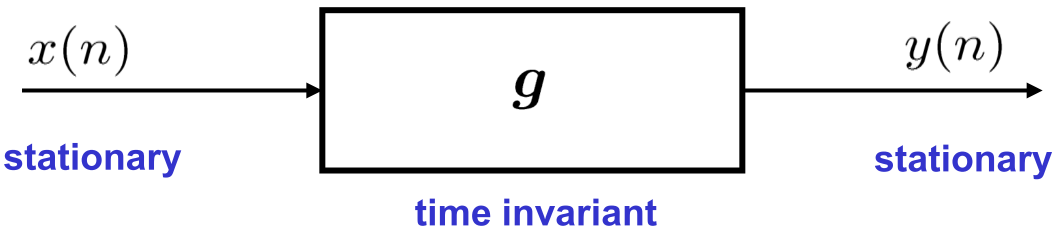
- For many calculations with random signals it is helpful to define a "random vector" : $\boldsymbol{x}(n) \;=\; \Big[x(n),\;\; x(n-1),\;\; x(n-2),\;\; \ldots\;\; x(n-N+1)\Big]^{\mathrm{T}}$
- The summation of two Toeplitz matrices requires *O(N)* floating point operations
- The multiplication of two Toeplitz matrices requires *O(N log N)* floating point operations
- Equation systems with Toeplitz matrices can be solved by *O(N²)* floating point operations
- The inversion of a positive definite Toeplitz matrix requires *O(N²)* floating point operations

Random Processes: Property of the autocorrelation matrix

- An important property of autocorrelation matrices:

$\boxed{\boldsymbol{R}_{xx} \text{ is non-negative definite}}$ being equivalent to non-negative eigenvalues.

- Proof via a linear system transformation:



$$
\begin{aligned}
\boldsymbol{x}(n) &= \left[\, x(n),\, x(n-1),\, x(n-2),\, \ldots\, ,\, x(n-N+1) \,\right]^{\mathrm{T}} \\
\boldsymbol{g} &= \left[\, g_0,\, g_1,\, g_2,\, \ldots\, ,\, g_{N-1} \,\right]^{\mathrm{T}} \\
y(n) &= \boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{x}(n) = \boldsymbol{x}^{\mathrm{T}}(n)\, \boldsymbol{g}^{*} \\
\mathrm{E}\left\{ |y(n)|^2 \right\} &= \mathrm{E}\left\{ \boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{x}(n)\, \boldsymbol{x}^{\mathrm{H}}(n)\, \boldsymbol{g} \right\} = \boldsymbol{g}^{\mathrm{H}}\, \mathrm{E}\left\{ \, \boldsymbol{x}(n)\, \boldsymbol{x}^{\mathrm{H}}(n) \, \right\} \, \boldsymbol{g} \\
&= \boxed{\boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{R}_{xx}\, \boldsymbol{g} \geq 0} \quad \text{independent of the choice of } \boldsymbol{g}
\end{aligned}
$$

Random Processes: Property of the autocorrelation matrix

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- We use the property to show that when this relation is fulfilled

  $\boldsymbol{g}^{\mathrm{H}} \boldsymbol{R}_{xx} \boldsymbol{g} \geq 0$ independent of the choice of $\boldsymbol{g}$

  => 1) all eigenvalues are real & non-negative => 2) the matrix is non-negative definite

- Definition of eigenvalues:

$$\begin{aligned} \boldsymbol{R}_{xx} \boldsymbol{h}_i &= \lambda_i \boldsymbol{h}_i \\ \boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{R}_{xx} \boldsymbol{h}_i &= \lambda_i \boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{h}_i \end{aligned}$$

- Known properties: $\boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{R}_{xx} \boldsymbol{h}_i \geq 0$ (reason: previous slide)

  both are real values → $\boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{h}_i \geq 0$ (reason: quadratic value)

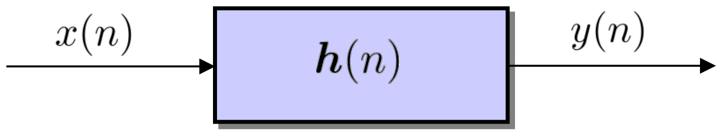
- => All eigenvalues are non-negative and real values:

$$\lambda_i = \frac{\boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{R}_{xx} \boldsymbol{h}_i}{\boldsymbol{h}_i^{\mathrm{H}} \boldsymbol{h}_i} \geq 0$$

  being equivalent to the property that the auto-correlation matrix is not-negative definite is equal to semi-positive definite

Digital Filters – Filtering in the time domain

- Time variant FIR filter (real values):



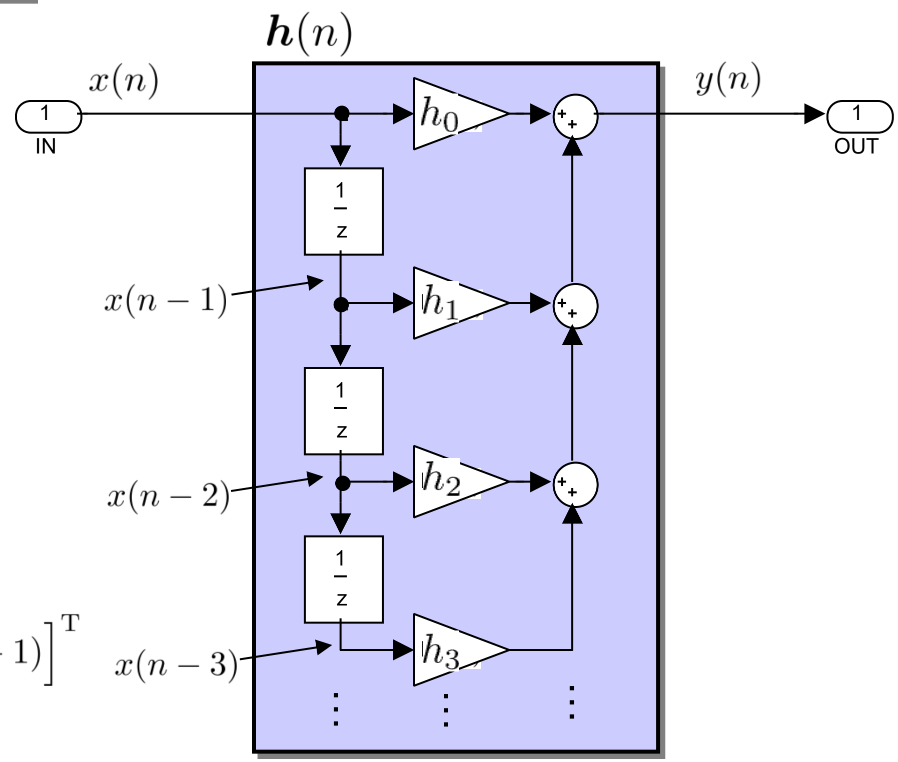
- Notations:

Scalar:

Discrete time index

$$y(n) = \sum_{i=0}^{N-1} x(n-i)\, h_i(n)$$

Coefficient index



Vectors:

$$y(n) = \boldsymbol{x}^{\mathrm{T}}(n)\, \boldsymbol{h}(n) = \boldsymbol{h}^{\mathrm{T}}(n)\, \boldsymbol{x}(n)$$

$$\boldsymbol{x}(n) = \Big[x(n),\, x(n-1),\, \ldots,\, x(n-N+1)\Big]^{\mathrm{T}}$$

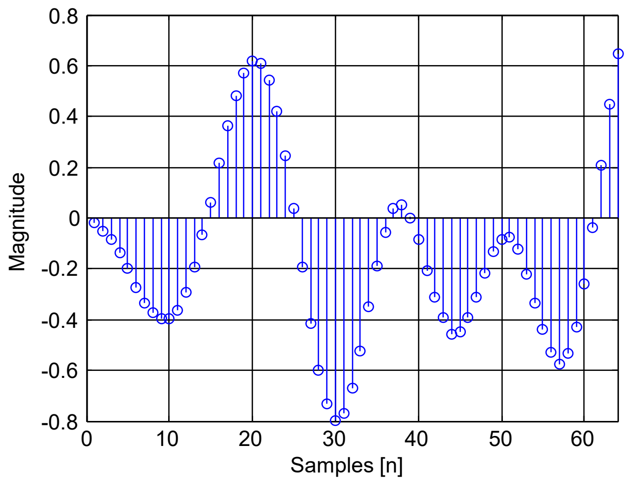
$$\boldsymbol{h}(n) = \Big[h_0(n),\, h_1(n),\, \ldots,\, h_{N-1}(n)\Big]^{\mathrm{T}}$$

Digital Filters – Filtering in the frequency domain

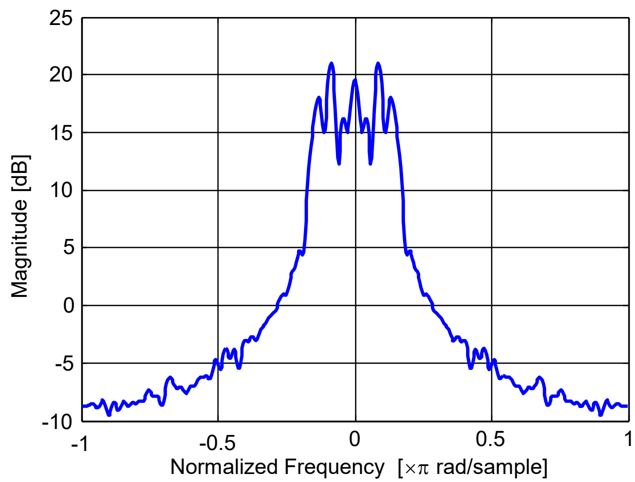
- Use of DTFT (Discrete Time Fourier Transform):

$$x(n) \circ\!\!-\!\!\bullet\ X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\,\Omega n}$$

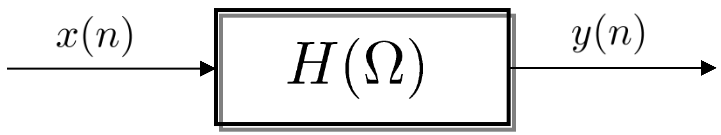
Time discrete signal:



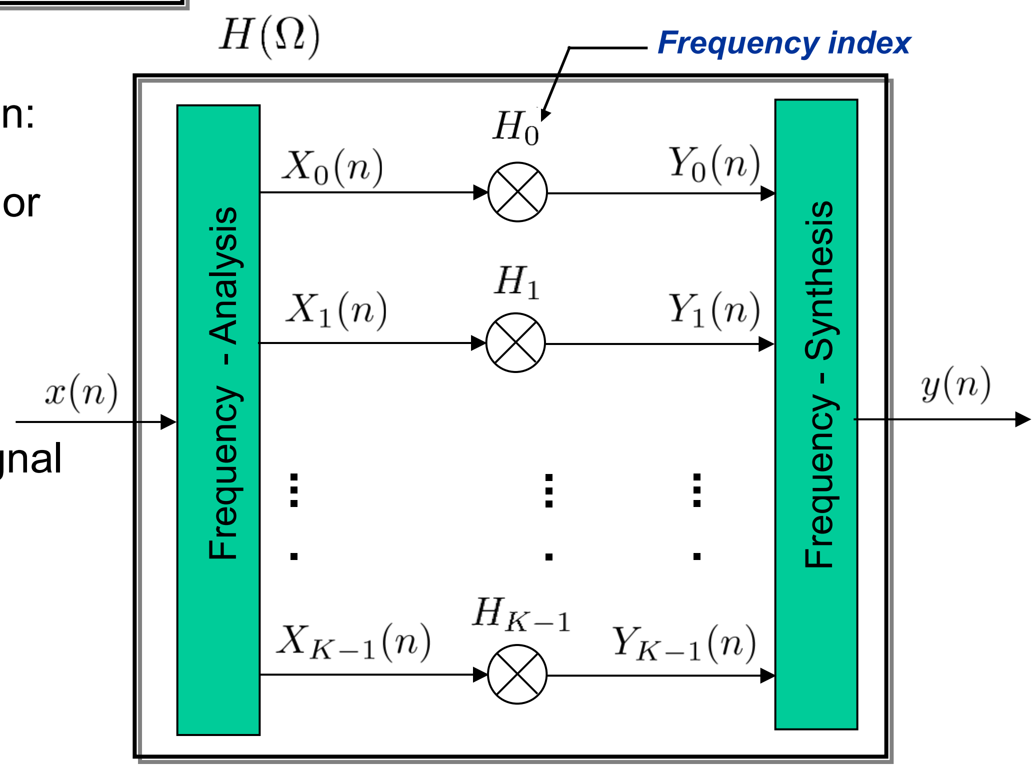
Frequency continuous signal (periodic with 2π, Symmetric for real-value signal samples):



Digital Filters – Filtering in the frequency domain



- Implementation in the frequency domain:
  - Use of fast convolution (“overlap-add” or “overlap-save” approaches)
  - Or use of filterbank concepts
  - Each decompose time-consecutive signal blocks with an FFT (length K) into the frequency domain
  - Filtering is performed by multiplication of the frequency domain filter blocks



IIR-filter: typically not used for adaptive filtering

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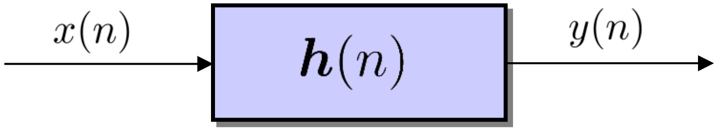
- Time variant IIR filter (real values):



No adaptation of IIR filters considered in this lecture
=> Stability hard to guaranty!

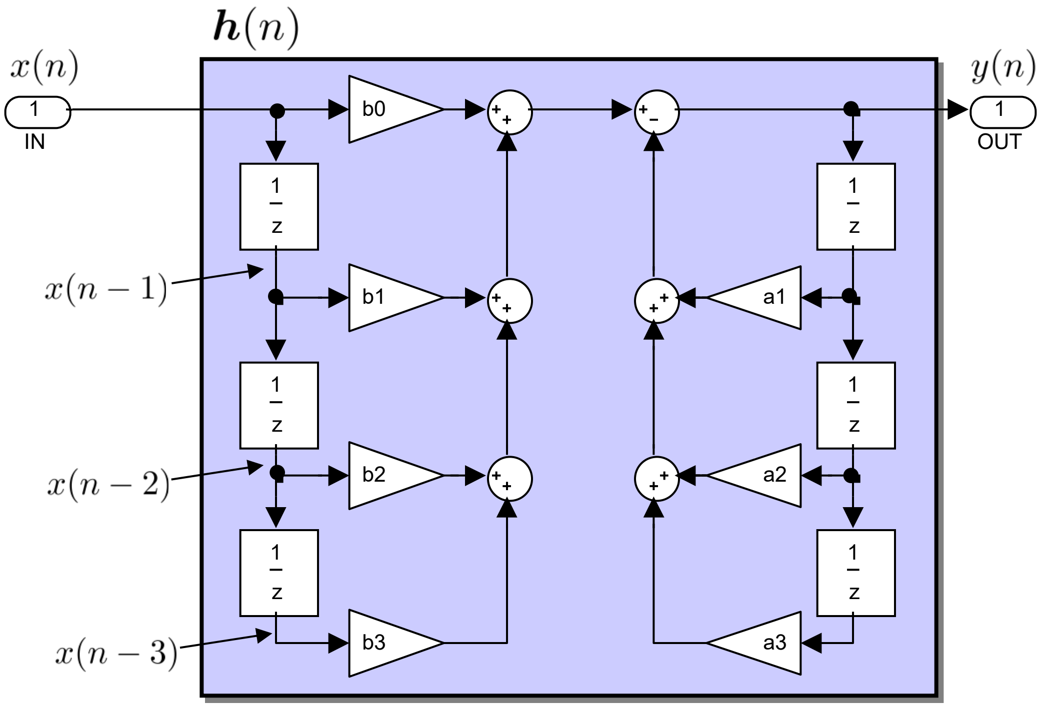
- Notations:

Scalar:

$$y(n) = \sum_{i=0}^{N-1} x(n-i)\, b_i(n) - \sum_{i=1}^{M-1} y(n-i)\, a_i(n)$$

$$h_i \circ\!\!-\!\!\bullet\; H(\Omega)$$

$$H(\Omega) = \frac{\sum_{i=0}^{N-1} b_i\, e^{-j\, i\Omega}}{1 + \sum_{i=1}^{M-1} a_i\, e^{-j\, i\Omega}}$$

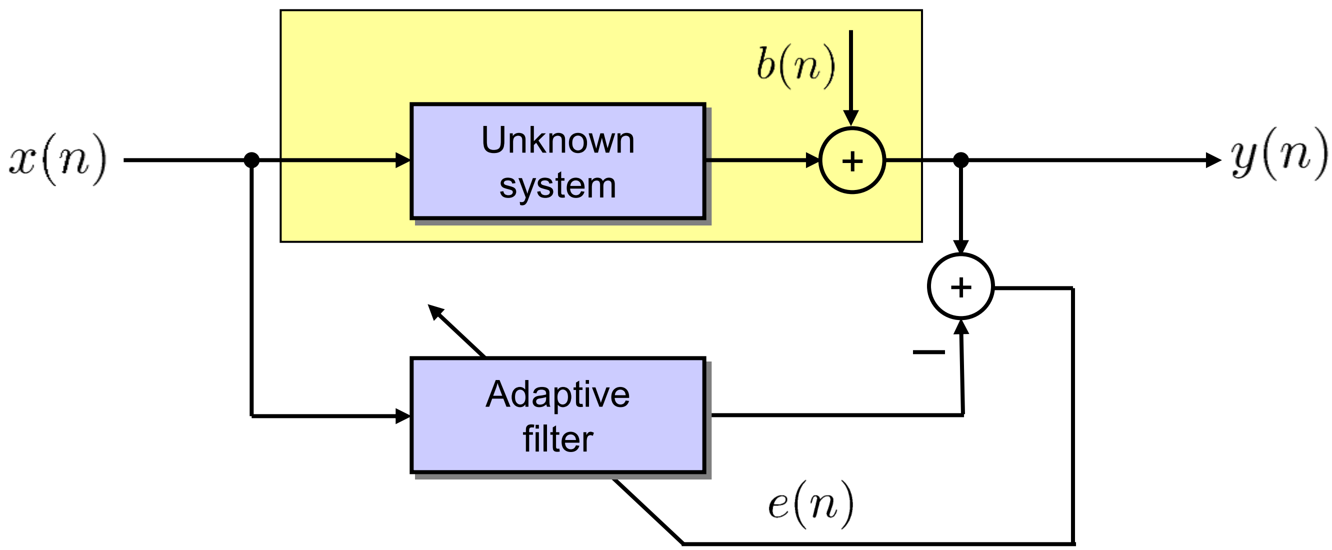


Applications of adaptive filter - Overview

- System identification
- Inverse modeling
- Prediction
- Noise compensation with noise reference
- Noise reduction without noise reference

System identification

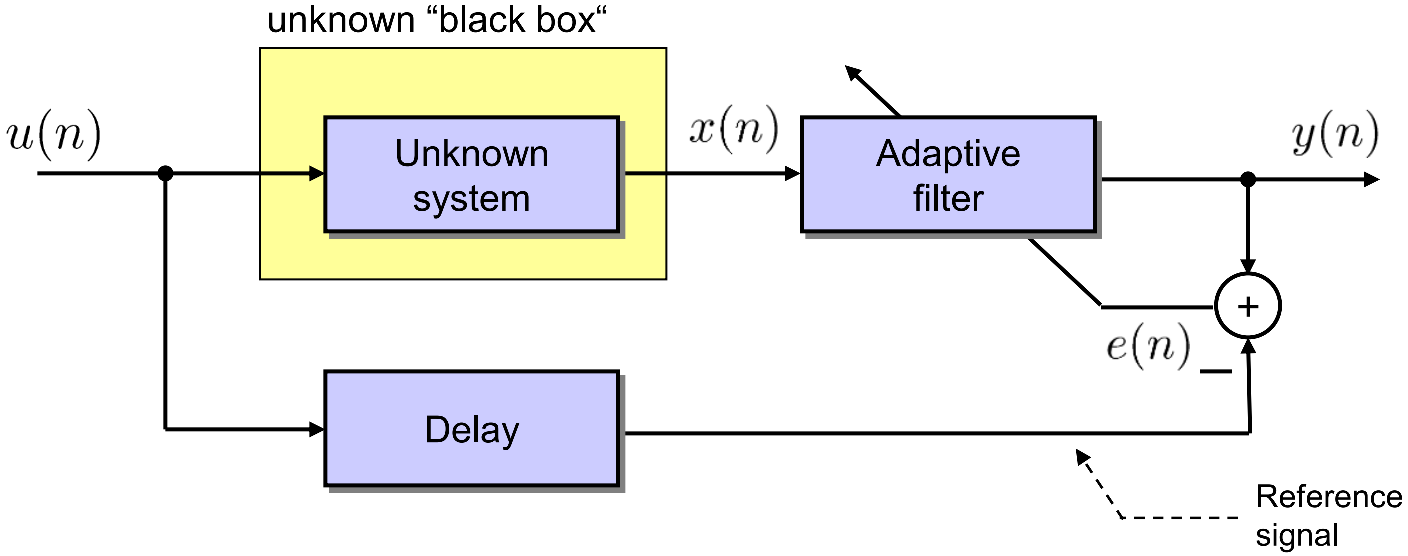
- **Target:**
  - Model an unknown linear system with an adaptive filter
  - Error signal should only contain signal $b(n)$ and no excitation components



- **Applications:**
  - Echo cancellation: Hands-free telephones
  - Feedback cancellation: Hearing aids, speaker plants, in-car communication

Inverse modeling

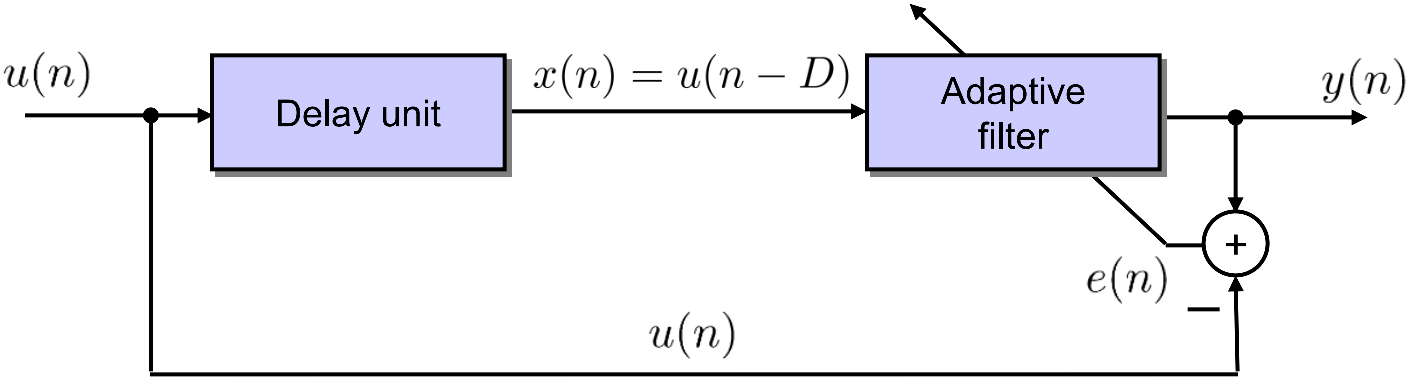
- **Target:**
  - Invert / equalize the „unknown“ system
  - Usually excitation signal unknown; typical approach: send a training sequence



- **Applications:**
  - Equalization of transmission channels, e.g. mobile transmission
  - Equalization of loudspeakers or other acoustic transmission paths

Prediction

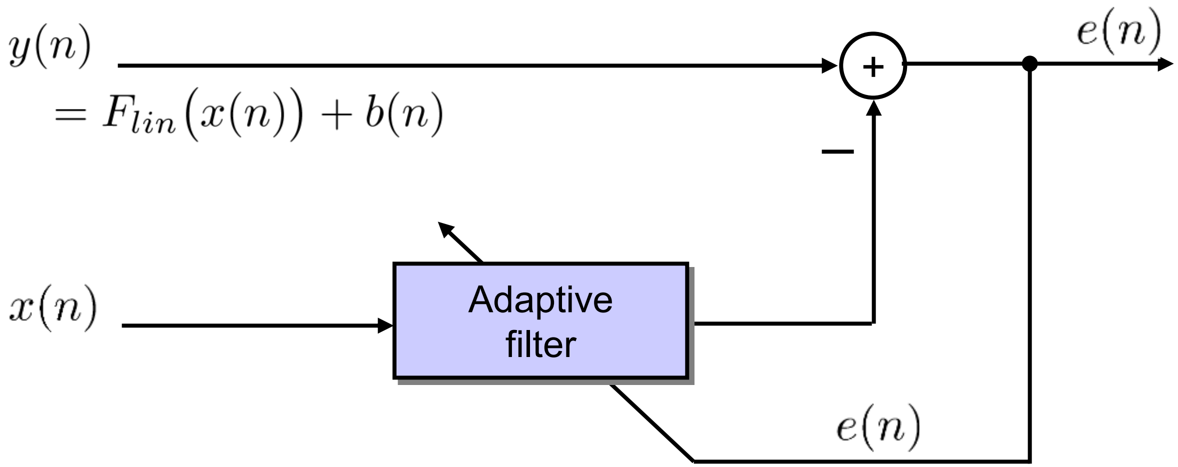
- **Target:**
  - Predict future signal samples based on the sequence of the previous signal samples



- **Applications:**
  - Source coding: Removal of signal redundancy for efficient signal coding and transmission; also, for pictures (MPEG Coding)
  - Parametric spectral estimation

Noise compensation with reference signal

- **Target:**
  - One virtual noise source $x(n)$
  - Unknown propagation path to noisy source signal detection point



- **Applications:**
  - Removal of reference noise
  - Active noise control
  - Array processing / Beamforming, e.g., Griffith-Jim beamformer or GSC (Generalized Sidelobe Canceller)
  - Medical applications: Enhancement of ECG and EEG signals

Noise reduction without reference

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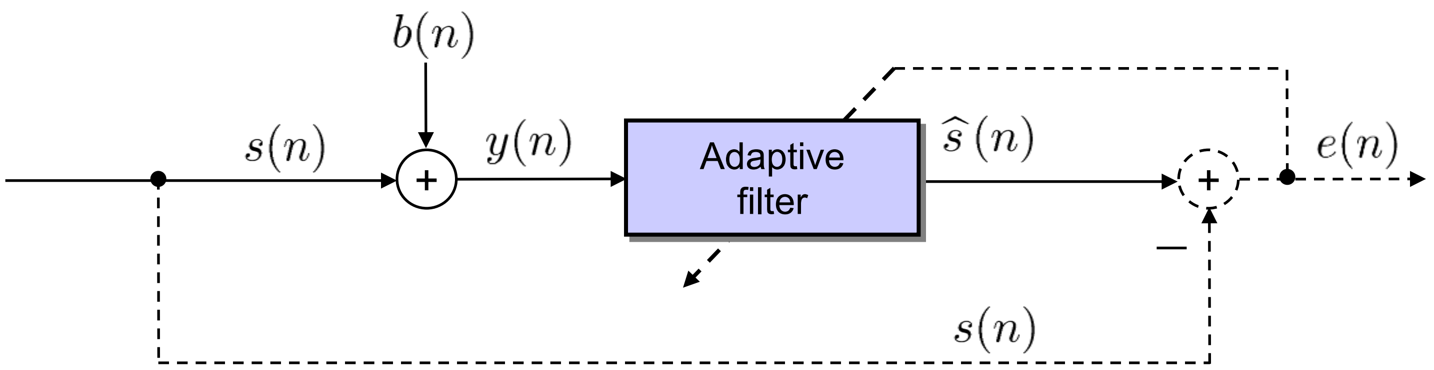
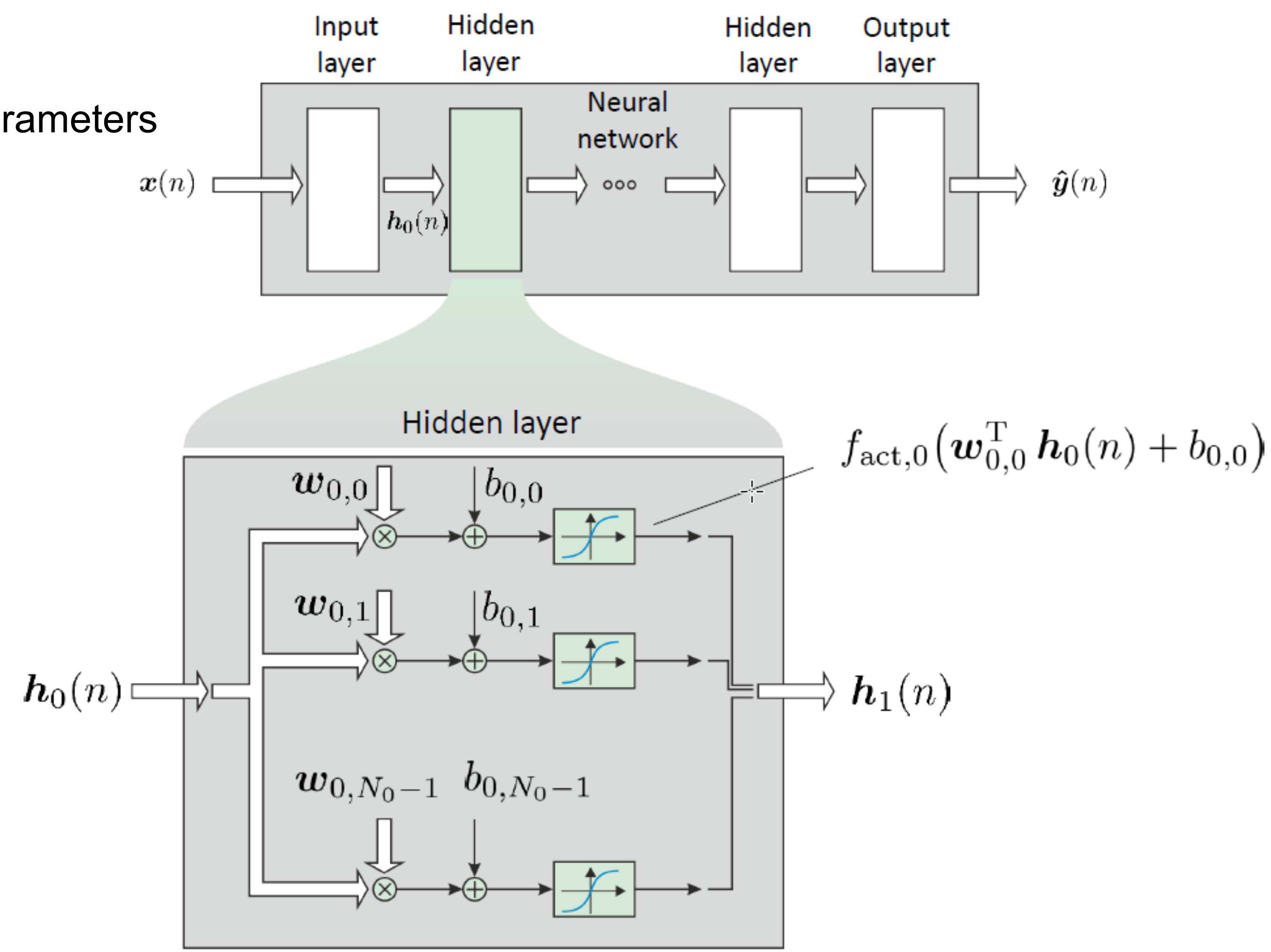
- **Target:**
  - Cancel noise based on different properties of target signal and noise, e.g. different properties concerning “stationarity” of these signals



- **Applications:**
  - Cancellation of noise where only a noisy signal is present
  - Noise reduction in cell phones and hearing aids

Adaptation in Neural Networks

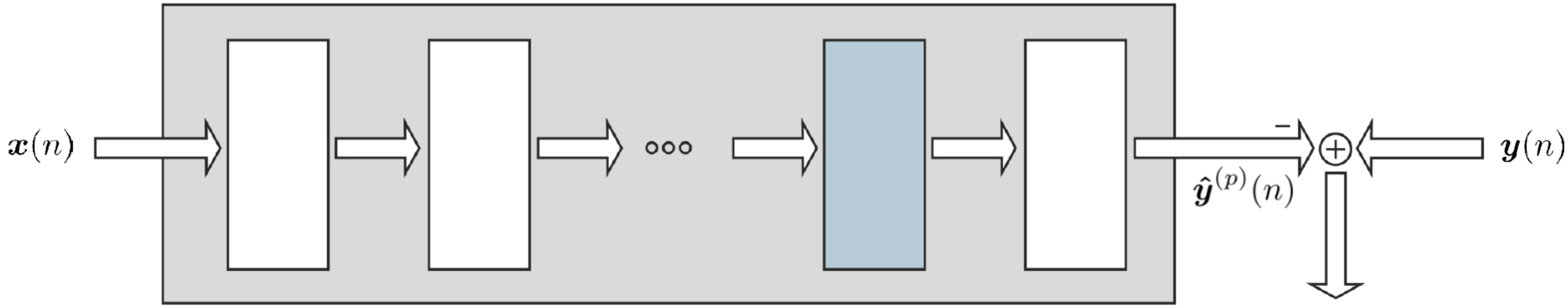
- **Target:**
  - Train the network parameters
  - Weight vectors **w** and bias term b



Adaptation in Neural Networks

- **Procedure:**
  - Based on an error signal against reference training data calculate a loss.
  - Gradient calculation of the loss wrt. the training parameters.
  - Update the parameters.

Compute in forward direction $\tilde{h}^{(p)}_{M-1,i}(n)$ and $x^{(p)}_{M-1,i}(n)$!



Update helping variables in backward direction

$$\delta^{(p)}_{M-1,i}(n) \quad = \quad f'_{\text{act},M-1}\big(x^{(p)}_{M-1,i}(n)\big) \sum_k \delta^{(p)}_{M,k}(n)\, \tilde{w}^{(p)}_{M,k,i}$$

and update the parameter of the second last layer

$$\tilde{w}^{(p+1)}_{M-1,i,j} \quad = \quad \tilde{w}^{(p)}_{M-1,i,j} + \alpha \sum_{n=0}^{N-1} \delta^{(p)}_{M-1,i}(n)\, \tilde{h}^{(p)}_{M-1,j}(n).$$

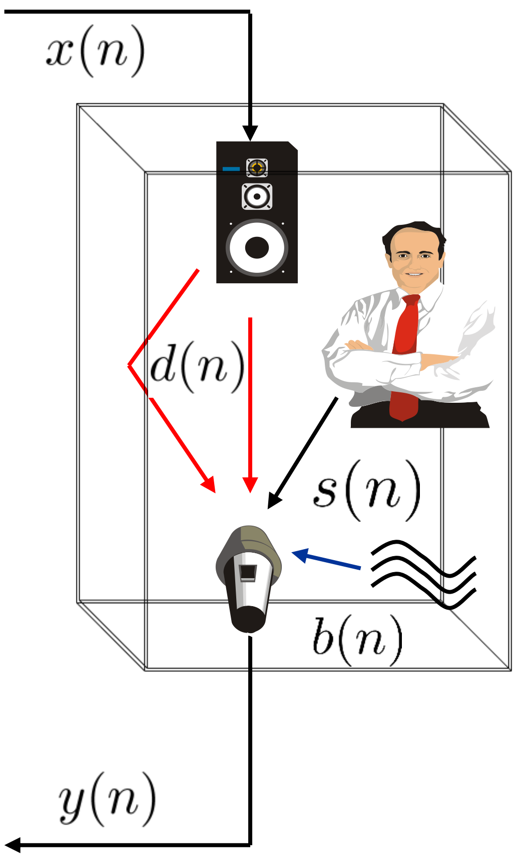
Example for the

System Identification

setup in the audio processing domain

System identification example:
Loudspeaker-room-microphone (LRM) systems

Signal of the far-end speaker



Microphone signal

$x(n)$: excitation signal

$d(n)$: echo (*desired signal for the adaptation*)

$s(n)$: local speech signal

$b(n)$: background noise

$y(n)$: microphone signal

$$y(n) = s(n) + d(n) + b(n)$$

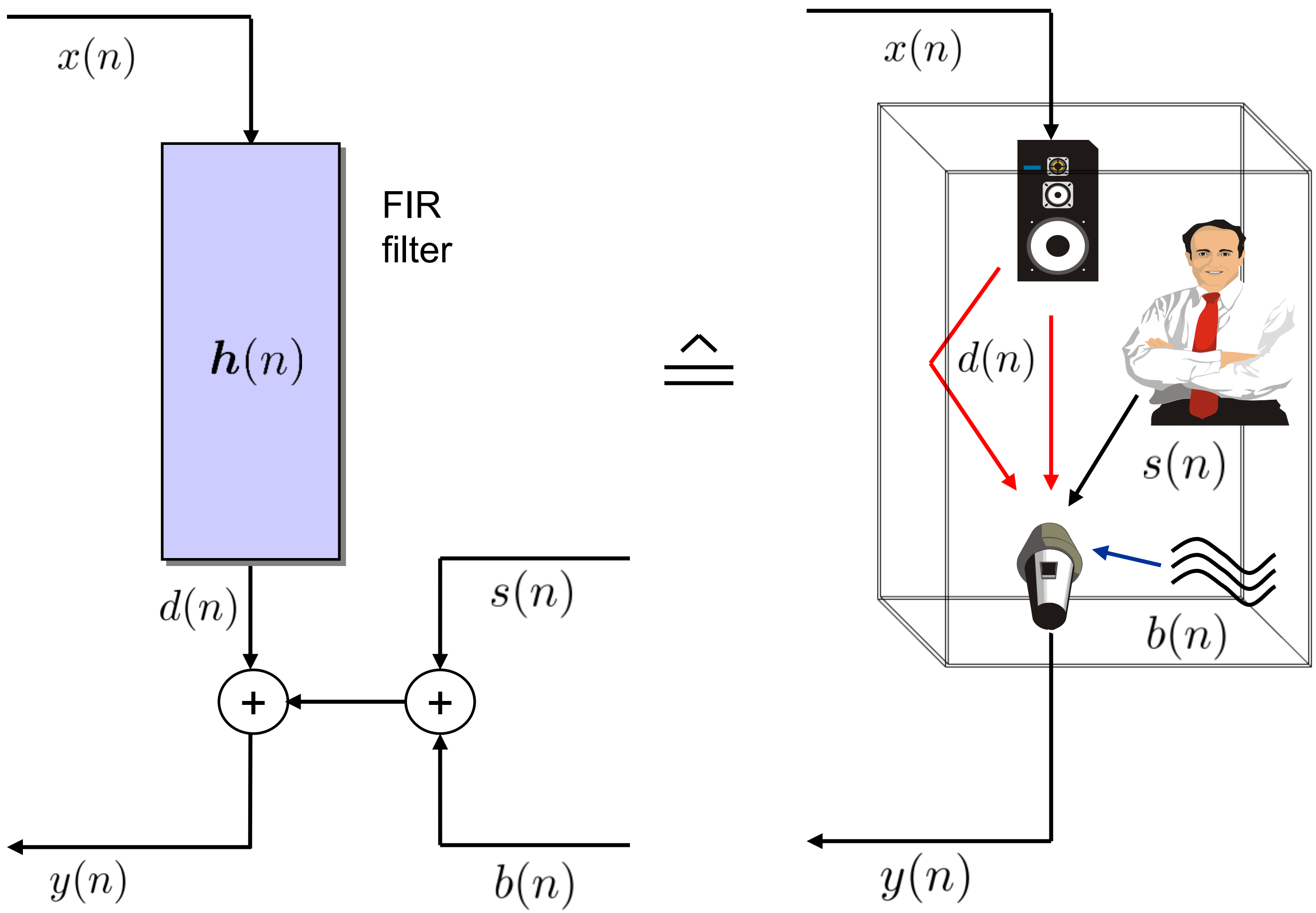
Loudspeaker-room-microphone (LRM) systems

Assumption:

The loudspeaker-room-microphone system (LRM-System) can be modeled as a linear system: i.e.

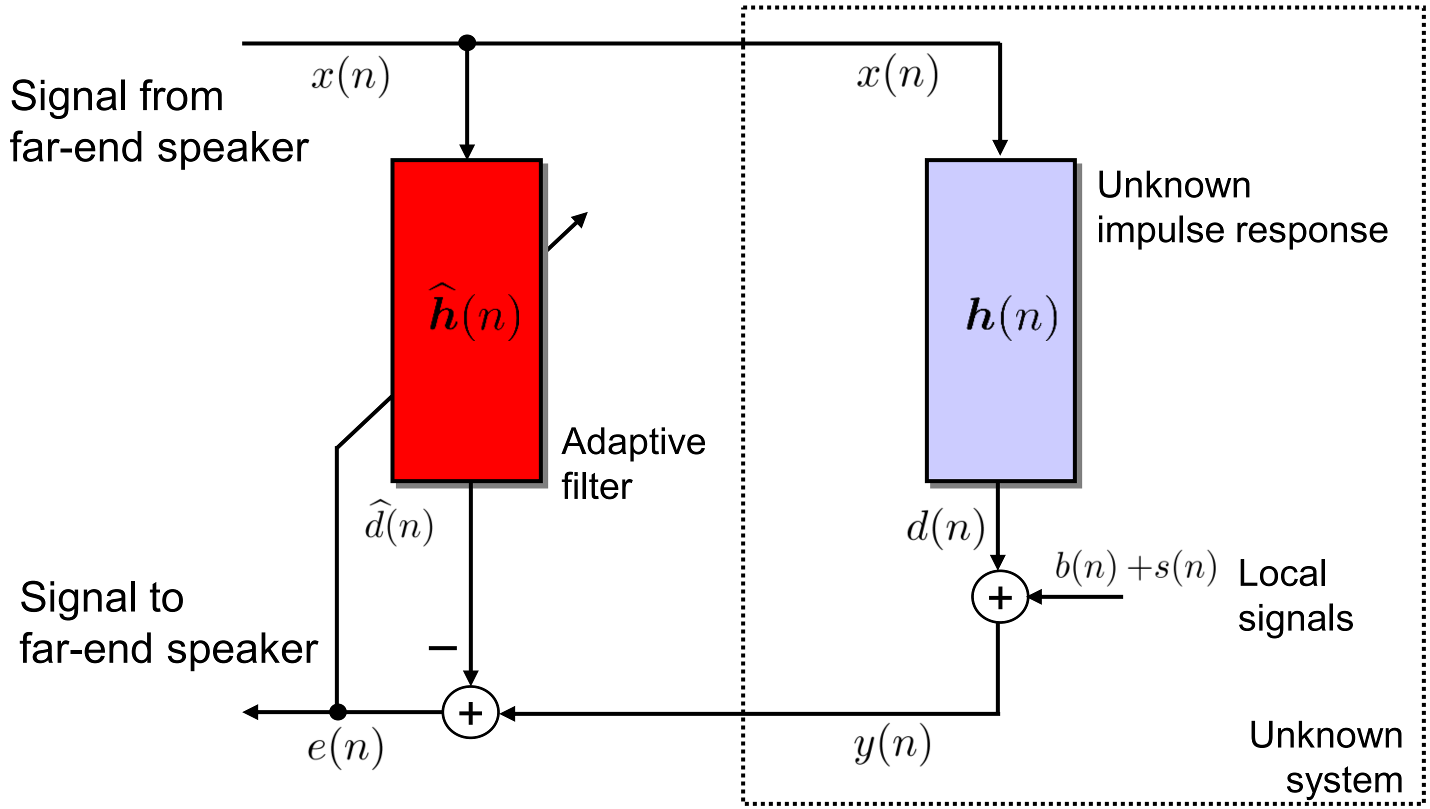
-linear loudspeakers

- no clipping at the microphone

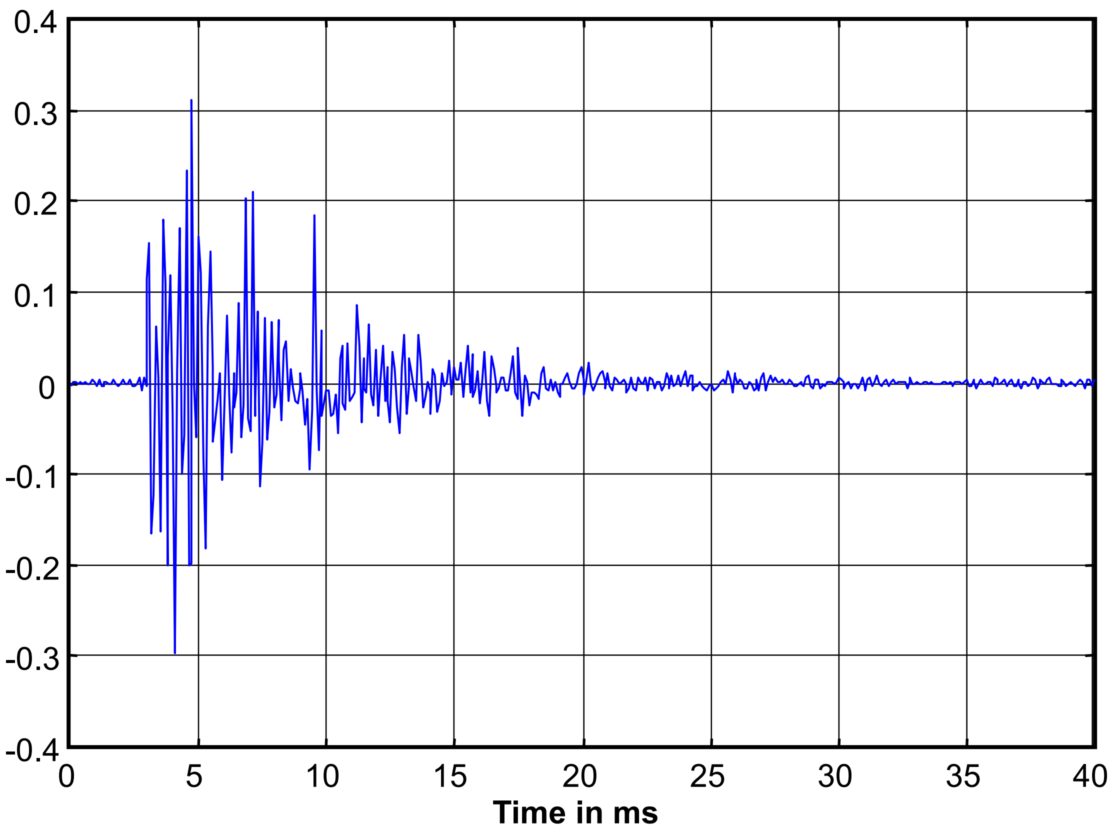


System identification

- Echo cancellation for hands-free telephones: Internal replica of external system:



Loudspeaker-room-microphone (LRM) systems



Constraints:

- Volume of
  a car cabin: 5 ... 15 m³

Properties:

- Short latency
- Direct path after 3 - 4 ms
- Specific reflections and …
- Diffuse echoes

Background noise

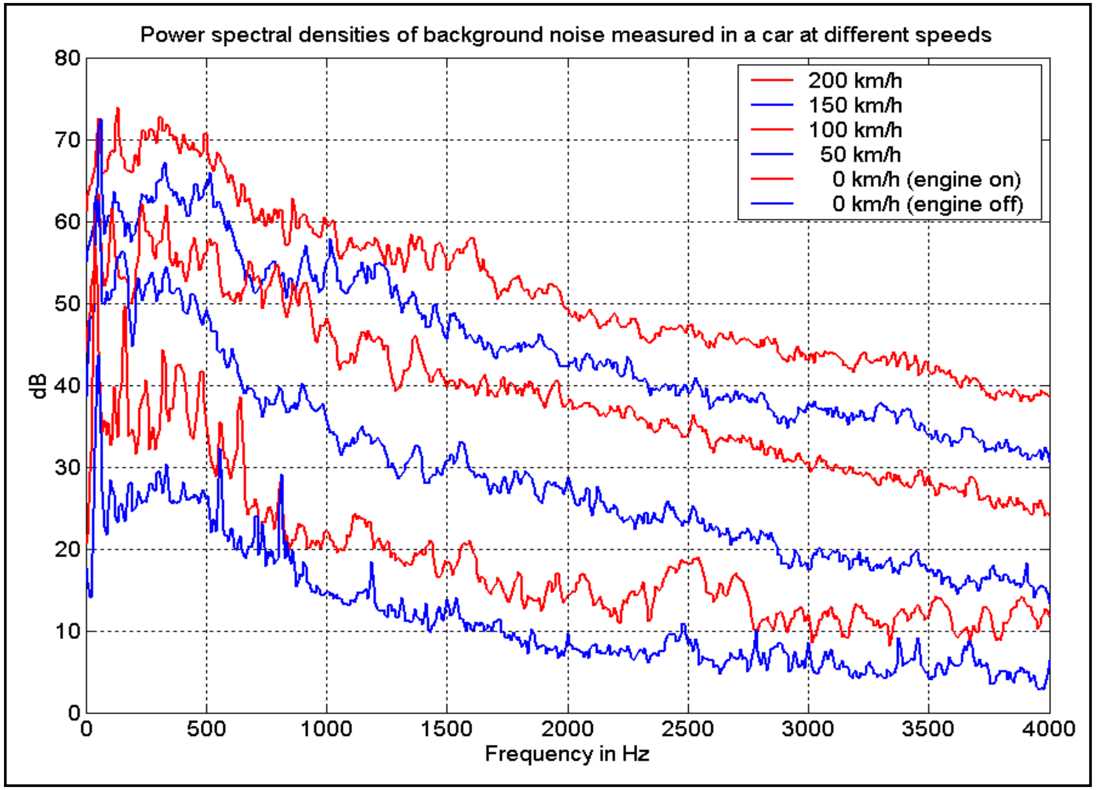
- **Components of car noise:**

  External:

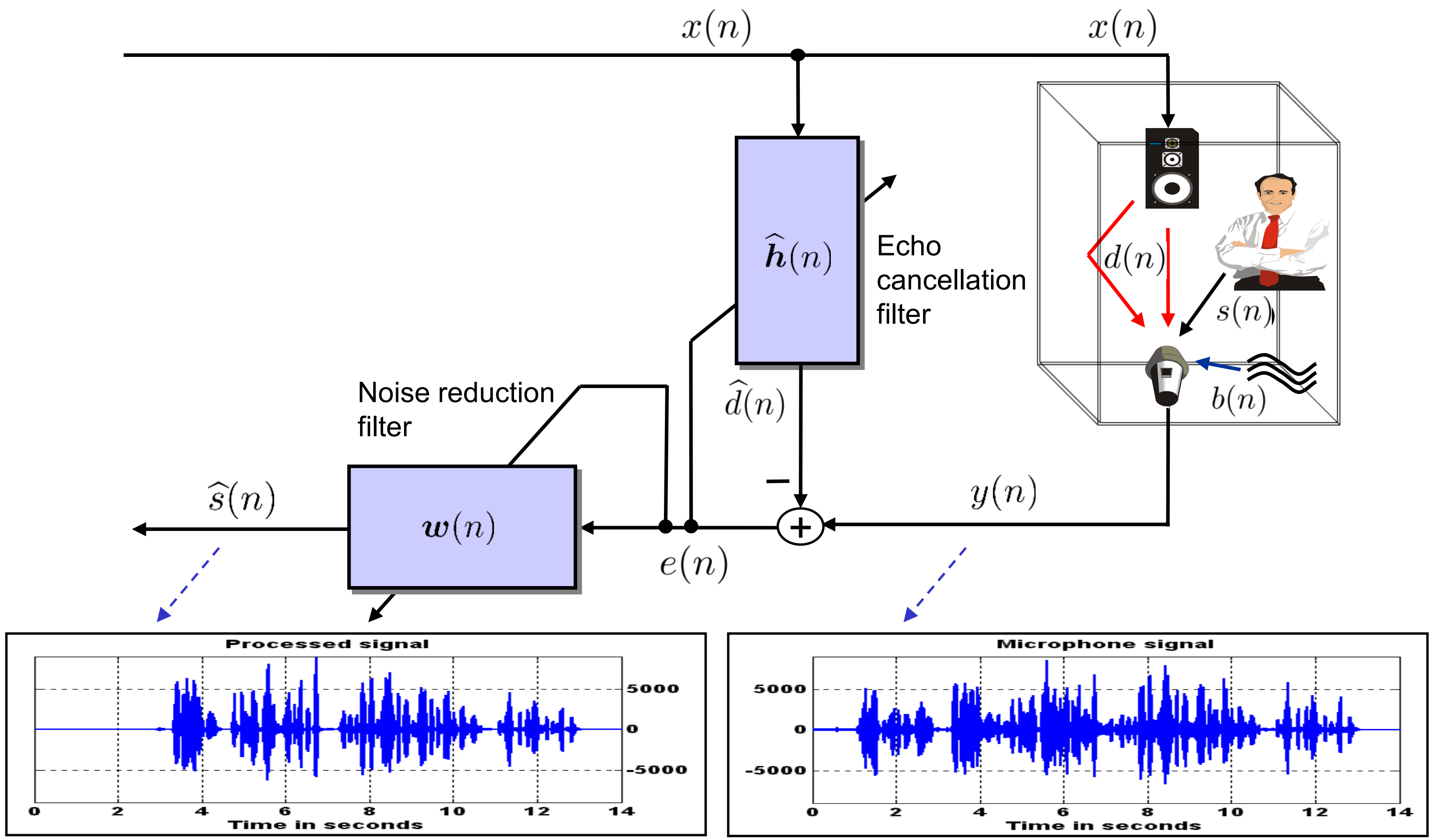
  - Engine noise
  - Wind noise
  - Tire noise

  Internal:

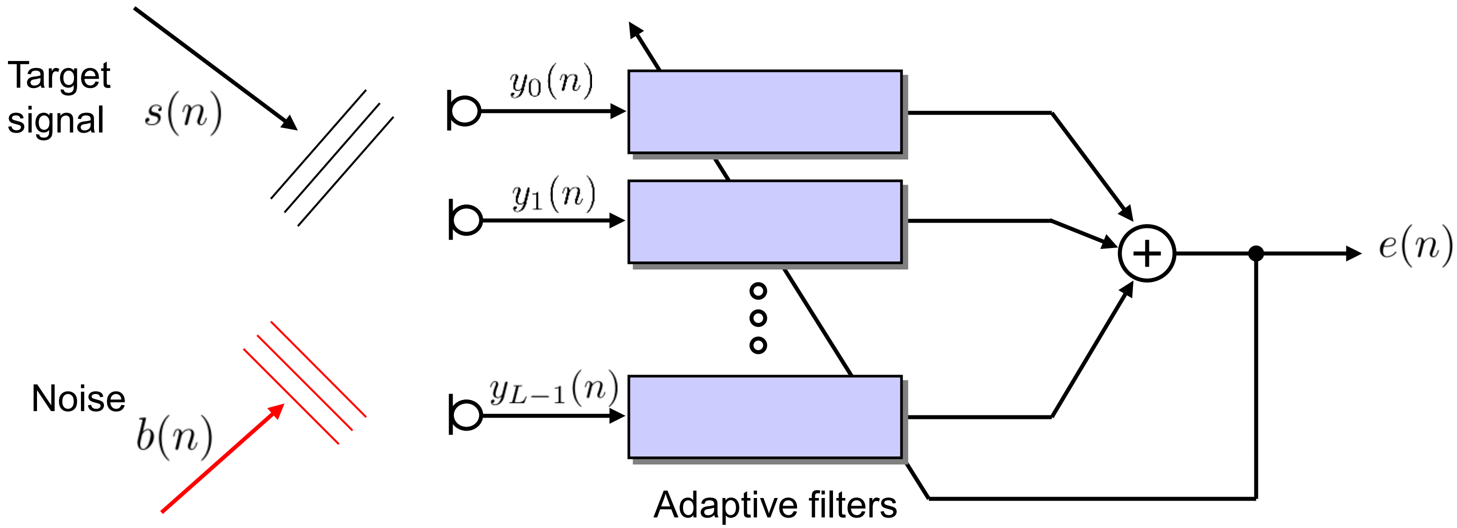
  - Air condition



A simple hands-free telephone with two adaptive filters



Beamforming – general setup



Beamformer:

- Minimization of the output power under the constraint not to attenuate the target signal
- Target signal direction has to be known
- Real-world setup limits the performance: e.g., signal reflections in the cabin and not ideally calibrated microphones

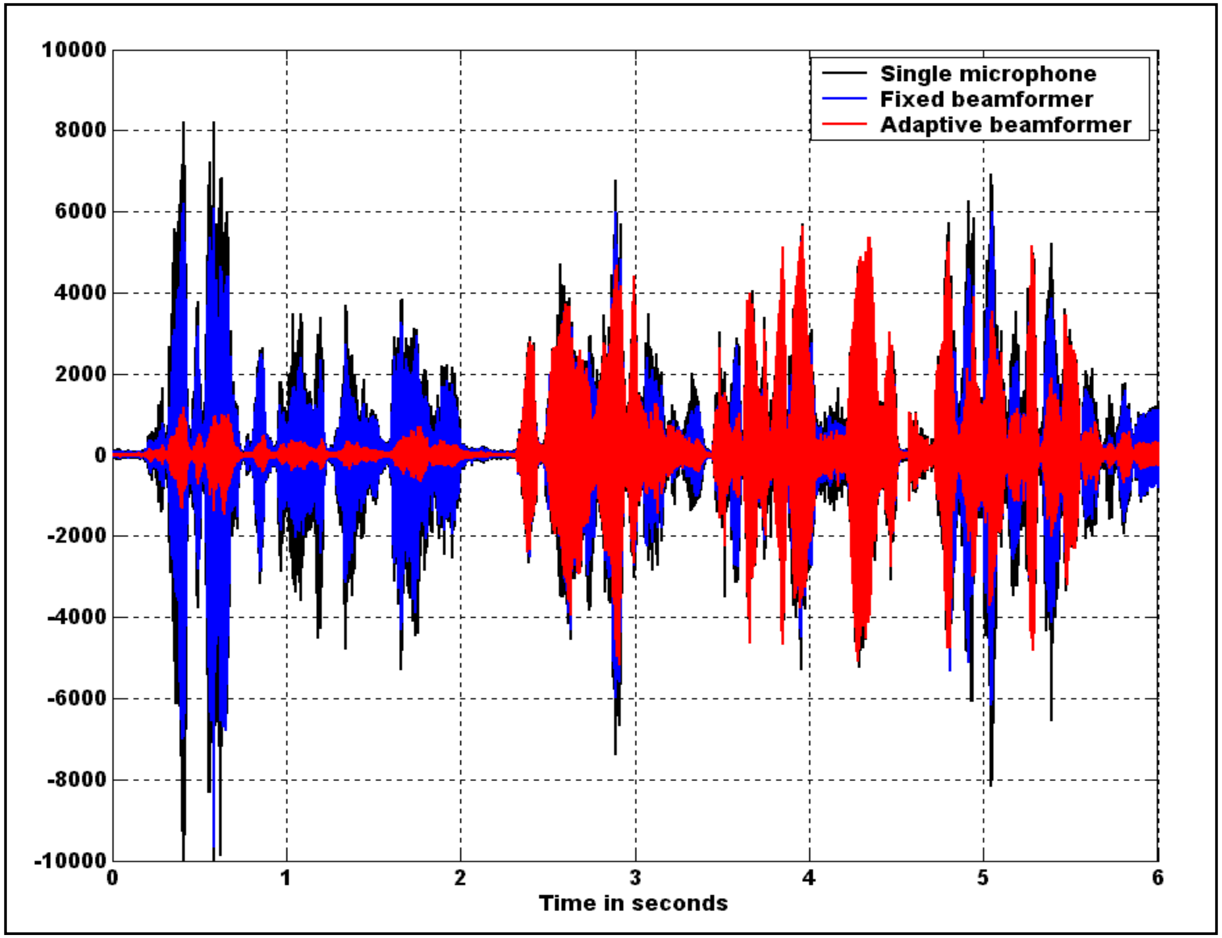
Beamforming – audio examples

- 4-channel beamformer (5 cm distance)
- Directional noise source (Loudspeaker on co-driver‘s side)
- Possible noise reduction > 15 dB by adaptive filtering of the microphone signals

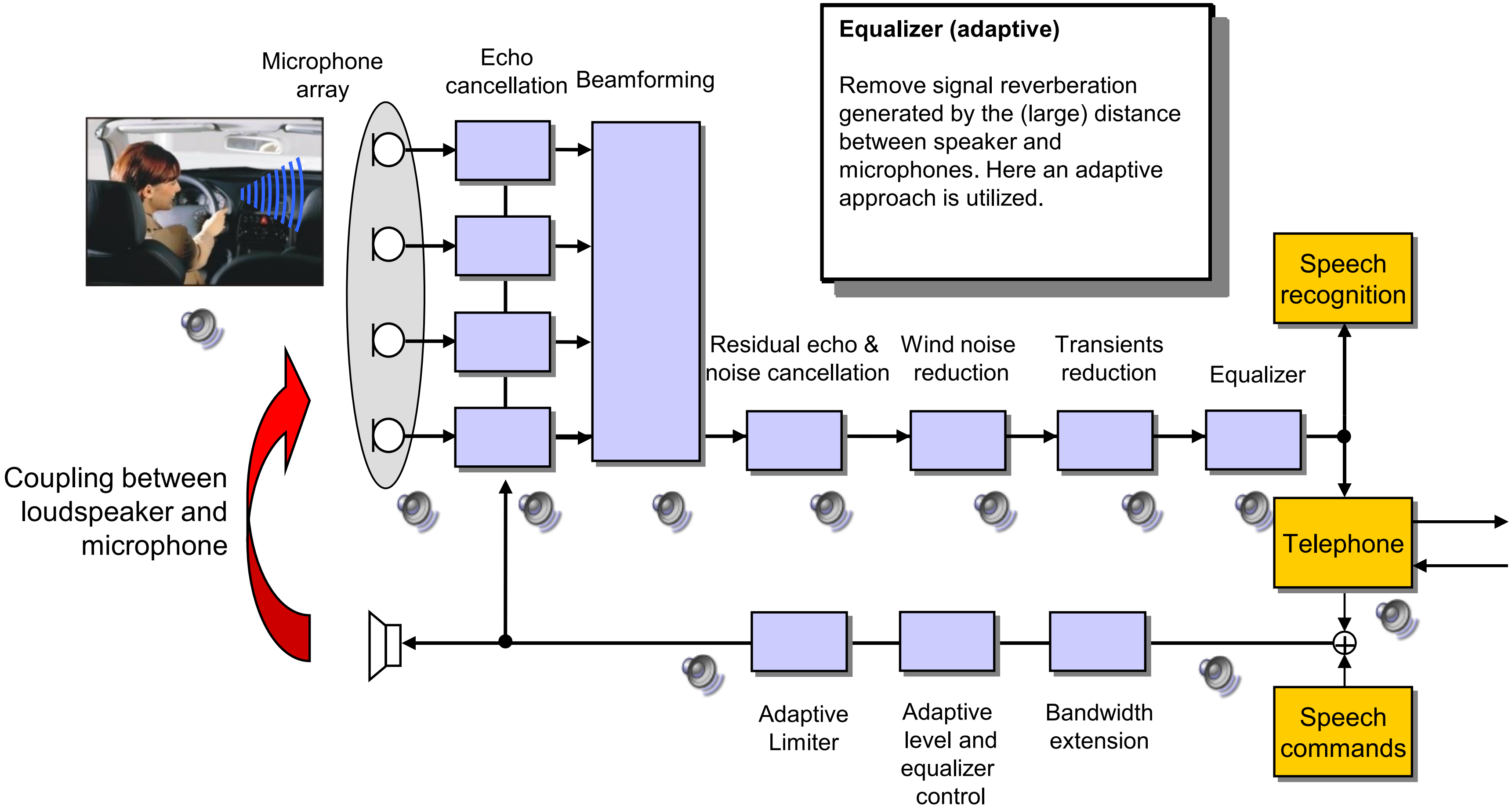
Input signal at one microphone

Fixed beamformer

Adaptive beamformer



Overview of a hands-free telephone in cars



Overview of all measures

Bandwidth extension

Missing frequency components are estimated and synthesized.

Target: Increase speech signal quality.

Adaptive level and equalizer control

Automatic volume adjustment in dependence of the background noise level. Additionally the input signal spectrum is modified in order to be as less as possible masked by the background noise.

Adaptive limiter

Adaptive limitation of the loudspeaker level in order to avoid non-linear saturation effects of the loudspeakers and microphone (constraint for the application of linear filters!).

Echo cancellation

Loudspeaker signals are reflected in the car, coupled into the micro-phone signals and fed back as echoes to the far-end speaker.

Target: Echo estimation and subtraction.

Beamforming

Two or more microphone signals are filtered and combined to amplify signals from a desired direction and attenuate other signals.
Effect: Only method to increase speech intelligibility in noise.

Residual echo and noise cancellation

Remaining echo and noise components (after echo cancelation and beamforming) are additionally attenuated.
Target: Cancel residual echoes and stationary noise.

Wind noise reduction

Open windows and air condition may generate wind noise at the microphones.

Target: Detect wind noise activity and activate a suppression routine.

Reduction of transients

Transient signal components, such as turn indicator generate problems esp. for speech recognition systems.

Target: Remove short impulsive noise signals.

Equalizer (adaptive)

Remove signal reverberation generated by the (large) distance between speaker and microphones. Here an adaptive approach is utilized.

General aspects to consider when applying adaptive filters

Systemdesign

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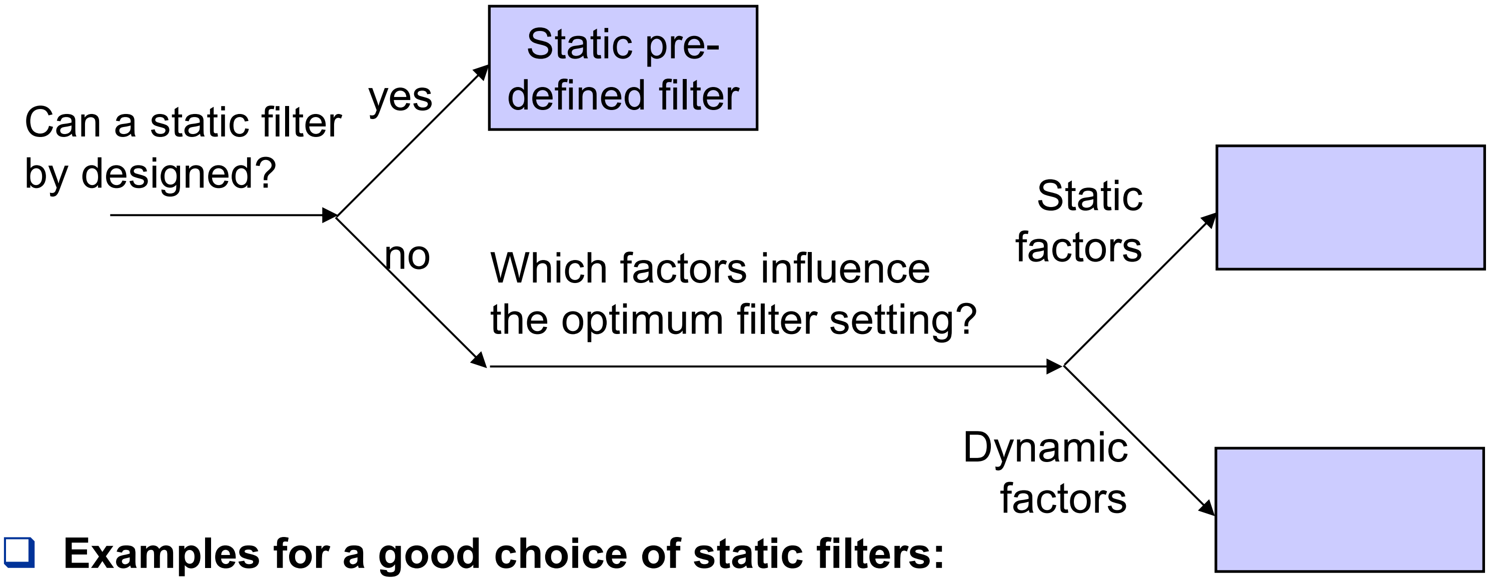
- **Choice of adaptive filters vs. static filters:**
  - Is an adaptive filter necessary?

- **Choice of the selectivity of the adaptive filter:**
  - Time domain: Filter length
  - Frequency domain: Frequency resolution

- **Choice of the adaptation rule:**
  - What is the excitation signal? More sophisticated rules are only interesting in case of non-white excitation
  - Which is the performance criterion for the application: E.g., adaptation speed, robustness of adaptation, etc.?

- **Choice and design of an adaptation control:**
  - Based on an analysis of the application scenario: Which interferers are present? Which properties do these interferers have?

Choice of adaptive filters vs. static filters (I)

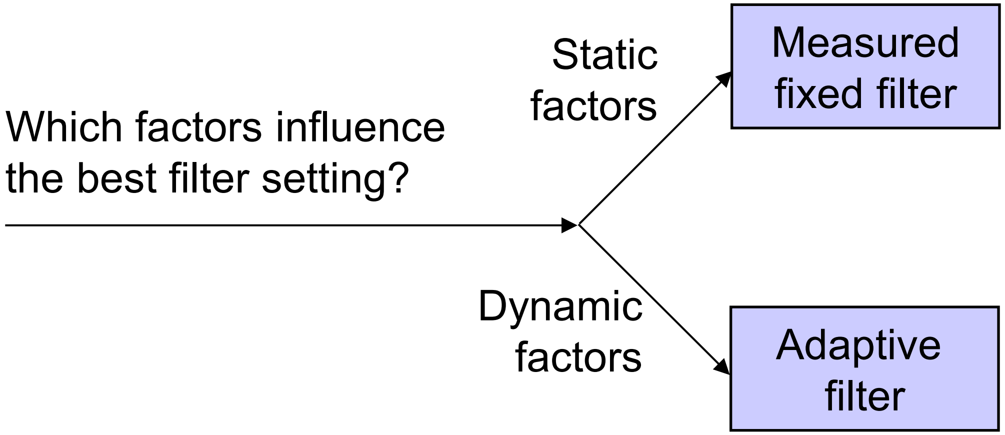


- **Examples for a good choice of static filters:**

  1) Narrow frequency interferer
     => design a notch filter at specific frequency

  2) Short regular bursts:
     => short & regular signal attenuation

Choice of adaptive filters vs. static filters (II)



- **Static system:**
  - T-coil feedback in hearing aids

  - Magnetic coupling of receiver signal into T-coil
  - Depends on the static position of receiver and T-coil

  => One path measurement; then coefficient freeze

- **Dynamic system:**
  - Acoustic feedback in hearing aids

  - Acoustic coupling between receiver and hearing aid microphones strongly depend on the „wearing situation“:
  - Are there obstacles present: hat, telephone, hand, etc.

  => Dynamic adaptation required

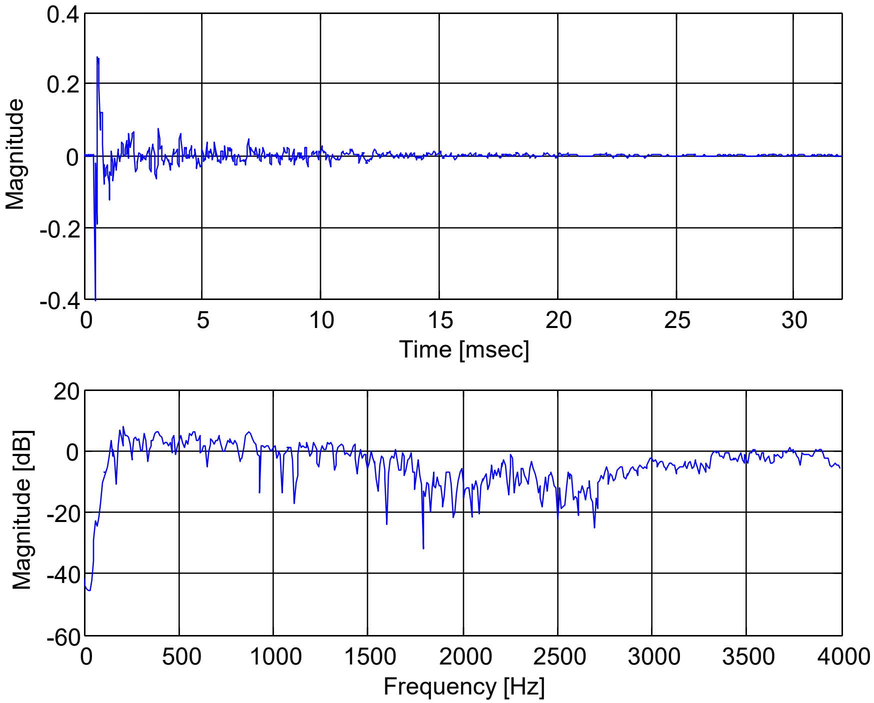
Choice of the selectivity of adaptive filters (I)

- Time domain: Choice of the length of the adaptive filter
- Frequency domain: Frequency resolution: FFT size

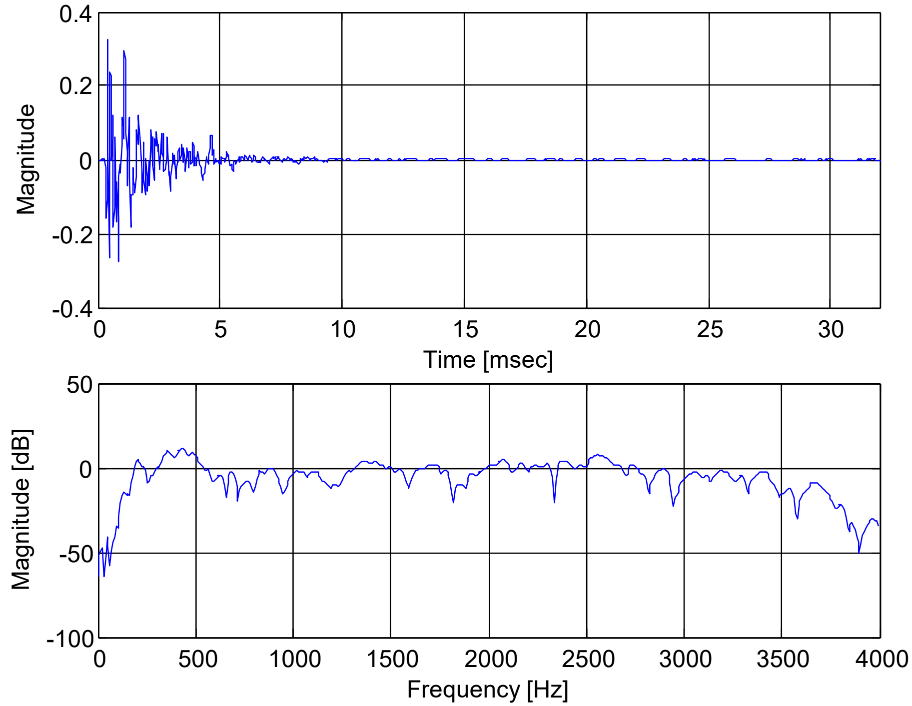
- Measurement of typical settings of the systems to identify:

LRM:
Loudspeaker
Microphone
Room

LRM system of an office room:



LRM system of a car:

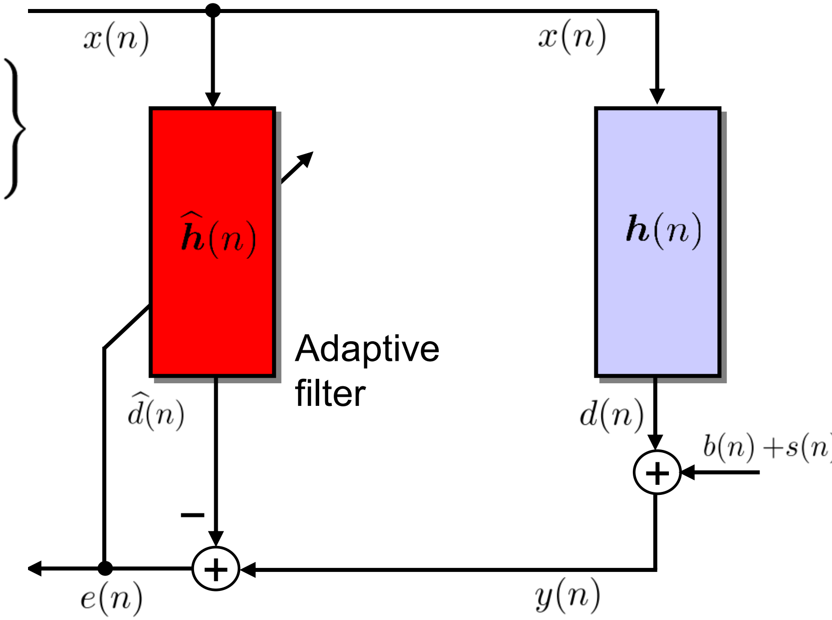


Choice of the selectivity of adaptive filters (II)

- Calculation of max. echo attenuation in dependence of length *N* of the adaptive filter:

  For time-invariant system, white excitation and in case of $b(n) + s(n) = 0$:

$$
\begin{aligned}
\sigma_e^2(N) &= \mathrm{E}\left\{e^2(n,N)\right\} \\
&= \mathrm{E}\left\{\left(\sum_{v=0}^{\infty} h_v\, x(n-v) - \sum_{v=0}^{N-1} \hat{h}_v\, x(n-v)\right)^2\right\} \\
&= \mathrm{E}\left\{\left(\sum_{v=N}^{\infty} h_v\, x(n-v)\right)^2\right\} \\
&= \mathrm{E}\left\{\sum_{v=N}^{\infty}\sum_{u=N}^{\infty} h_v\, h_u\, x(n-v)\, x(n-u)\right\} \\
&= \sum_{v=N}^{\infty}\sum_{u=N}^{\infty} h_v\, h_u\, r_{xx}(v-u)
\end{aligned}
$$



$$\boxed{\sigma_e^2(N) = \sigma_x^2 \sum_{v=N}^{\infty} h_v^2}$$

⟵ for white excitation ⟶

$$\boxed{\sigma_y^2 = \sigma_x^2 \sum_{v=0}^{\infty} h_v^2}$$

Choice of the selectivity of adaptive filters (III)

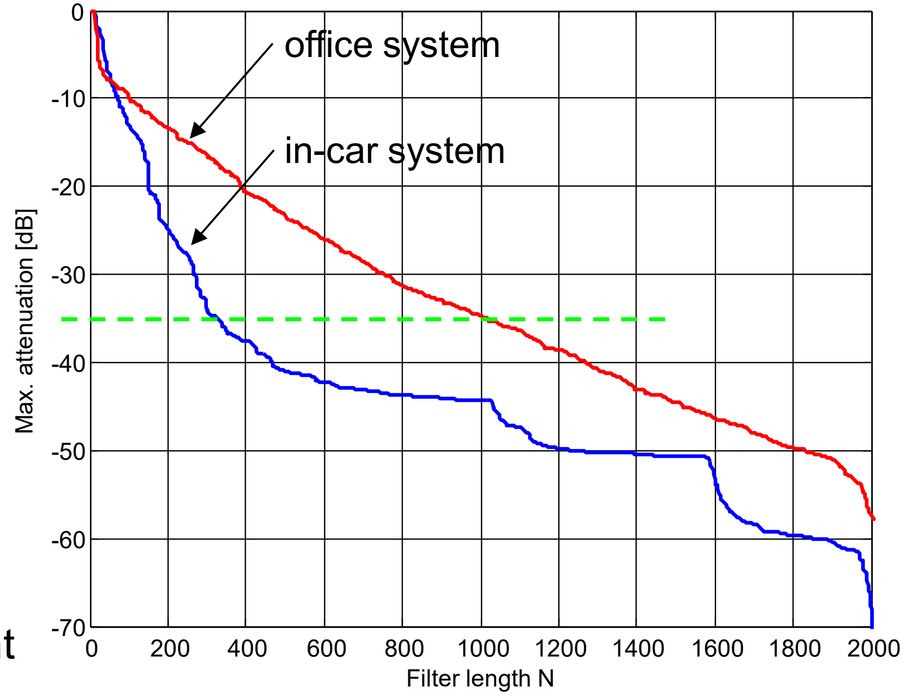
- Estimate max. required attenuation by the adaptive filter:

  Example for 35 dB:
  => ~ 300 filter taps for in-car system
  ~1000 filter taps for office system
  necessary

- Local signals mask (in dependence of frequency!) the residual echo:

  => Echo attenuation ~5 dB below the
  local signal power is typically sufficient

$$att_{max} = \frac{\sigma_e^2(N)}{\sigma_y^2} = \frac{\sum_{v=N}^{\infty} h_v^2}{\sum_{v=0}^{\infty} h_v^2}$$



Choice of the adaptation rule

- **What is the excitation signal?**

  - We will learn about different adaptation rules: (LMS, AP, RLS)
  - They are computationally more or less demanding
  - Advantage of more complex rules only in case of non-white, strongly correlated signal excitation

- **What is the performance criterion of the specific application?**

  - Adaptation speed:
    - Select more sophisticated adaptation rule
    - Avoid too long filters, since adaptation speed is proportional to filter length

  - Robustness of the adaptation vs. noise:
    - Select a basic and robust adaptation rule
    - Invest computational effort into adaptation control

Choice and design of the adaptation control

- **What is to consider?**

  - Analysis of excitation signal and local signal (disturbance) presence:

  - **Analysis of temporal and spectral activity**

    => In case of stationary signals: fixed adaptation speed possible
    => In case of instationary signals: adaptive control necessary

  - **Analysis of correlation properties of excitation signals**

    Such correlations are present for feedback cancellation and intercom systems

    => Specific control or decorrelation measures are necessary

Adaptive systems of this lecture (I)

- **Wiener filter:**

  - Filter designed for stochastic signals
  - Design assumes stationary signals
    => Design of a fixed filter
    Calculation of the optimum filter in one step

  - **Design in the time-domain:**
    Based on auto- and cross-correlation functions

  - **Design in the frequency domain:**
    Based on auto- and cross-power spectral densities

  - **Design of time-varying / adaptive filters:**
    Block processing; modeling of input signals as short-term stationary

Adaptive systems of this lecture (II)

- **Prediction:**

  - Prediction of future signal samples based on current and previous ones

  - Specific but very powerful application of adaptive filters

  - Can be derived from the Wiener filter solution

  - An effective order-based recursion procedure will be explained

  - Applications in
    - source coding and
    - parametric spectral estimation

Adaptive systems of this lecture (III)

- **Adaptive filtering:**

  - **Step-by-step calculation** of the optimum Wiener solution:

  - **RLS (recursive least squares):** Recursive calculation of the least squares solution; optimum solution for the current input signal frame

  - **LMS / NLMS (normalize least mean squares):** Stochastic approaches replacing the "expectation" by the current input signal values

  - **AP (affine projection):** LMS based procedures of higher order in order to enhance adaptation speed in case of non-white input signals

Adaptive systems of this lecture (IV)

- **More on adaptive systems:**

  - **Kalman filters:** Alternative to the Wiener filter; does not need the assumption of stationary signals

  - **Particle filters:** Applied for models for which Kalman filters are also used, however, here, no assumptions about linear systems and Gaussian inputs required

  - **Neural Network training:** Apply optimal gradient-based procedures with appropriate loss functions to train the network parameters

Summary

- General review of random processes.

- Introduction to adaptive filter applications.
- General application of adaptive procedures.

- Special application to echo cancellation, noise reduction, beamforming.
- Parameters of adaptive filters:
  - Selectivity (length or frequency resolution)
  - Adaptation rule
  - Control methods for adaptation control

- Next lecture:
  - Optimal filter for stationary random signal: Wiener filter.
  - Wiener filter: Basis for the design of adaptive filters..