Digital Signal Processing
Tutorial 2

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Task 1: FIR Filter Design

An ideal differentiator has the frequency response

$$H(e^{j\omega}) = j\omega, \qquad -\pi \le \omega < \pi.$$

a) Sketch the real and imaginary parts of $H(e^{j\omega})$ for $0 \le \omega < 2\pi$.

The inverse Fourier transform of $H(e^{j\omega})$ is given by

$$h(n) = \begin{cases} \frac{(-1)^n}{n} & n = \pm 1, \pm 2, \pm 3, \dots \\ 0 & n = 0. \end{cases}$$

b) Is the FIR filter with unit sample response $h(n)$ realizable? If not, how can it be made realizable?

Multiplying $h(n)$ with the window function

$$b(n) = \begin{cases} \frac{((\gamma N)!)^2}{(\gamma N - n)!(\gamma N + n)!} & |n| \le N \\ 0 & \text{elsewhere.} \end{cases}$$

yields an LTI system with unit sample response

$$h_1(n) = b(n)h(n),$$

c) Is the filter with unit sample response $h_1(n)$ realizable? If not, how can it be made realizable?

d) Show that the filter described by $h_1(n)$ has a (generalized) linear phase. Depending on N, what is the type of the realizable filter?

e) What is the relationship between the two systems with unit sample responses $h(n)$ and $h_1(n)$ in the frequency domain?

f) Which values may γ take? How does its choice influence the window shape? What happens for $\gamma \to \infty$?
Hint:

$$\frac{((\gamma N)!)^2}{(\gamma N - n)!(\gamma N + n)!} = \prod_{i=1}^{|n|} \frac{\gamma N - (i-1)}{\gamma N + i} \quad \text{for} \quad n \in \{\pm 1, \pm 2, \dots, \pm N\}$$

g) Plot the magnitude of the frequency response $H_1(e^{j\omega})$ (e.g. using python or Matlab) for $N = 20$ and $\gamma \in \{1, 5, \infty\}$. How do γ and N influence the frequency response?

Task 2: IIR Filter Design - Bilinear Transform

A digital lowpass filter is required to meet the following specifications:

Pass-band edge	0.24π rad/cycle
Stop-band edge	0.5π rad/cycle
Pass-band attenuation	≤ 1 dB
Stop-band attenuation	≥ 4 dB

a) The filter is to be designed using the bilinear transform with $T_d = 2$. A Butterworth filter will be used to approximate the analog lowpass characteristic.

   (i) Determine the minimum order as well as the 3dB cut-off frequency of the Butterworth filter such that the passband specification is exactly met.

   (ii) Determine the poles of the magnitude squared transfer function $|H_c(s)|^2$. Afterwards, find the transfer function $H_c(s)$ so that the lowpass filter is causal and stable.

b) Find the system transfer function of the digital filter, $H(z)$, corresponding to the analog filter designed in part (a).

c) Can the digital filter obtained in part (b) be implemented using direct convolution in practice? Justify your answer. How could $H(z)$ be implemented using the same number of delay elements as the filter order. Draw a signal flow graph.

d) How does N change if we increase/decrease the transition width or the passband and stopband tolerance bands?

Task 3: IIR Filter Design - Impulse Invariance Method

Design a digital lowpass filter with the impulse invariance method. The specifications should be the same as in Problem 2.

a) Determine the minimum order as well as the 3dB cut-off frequency of the analog Butterworth filter such that the passband specification is exactly met. Also use $T_d = 2$.

b) Why do we not obtain the same order as with the bilinear transform?

c) Implement the *analog* Butterworth filter in MATLAB and convert it into a digital filter using the impulse invariance method.

d) Implement the filter designed in Problem 2 using the function for the bilinear transform.

e) Compare the two filters, particularly with regard to their magnitude response. Which one would you prefer? (**Hint:** You can use the Filter Visualization Tool (*fvtool*) to display the filters.)