Digital Signal Processing

Tutorial 1

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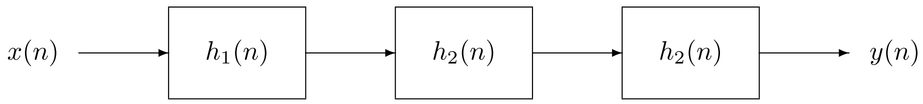
Task 1: Signals and Systems

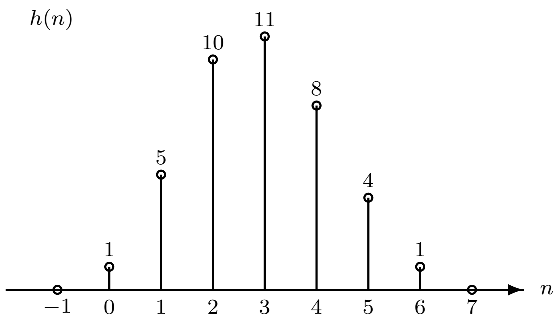
Consider the cascade of three causal linear time-invariant systems illustrated in the following figure:



The unit sample response $h_2(n)$ is given by:

$$h_2(n) = u(n) - u(n-2)$$

and the overall unit sample response is as shown in the following figure:



a) Find the unit sample response $h_1(n)$. **Hint:** calculation in the z-domain.

b) Find the response of the overall system to the input given by:

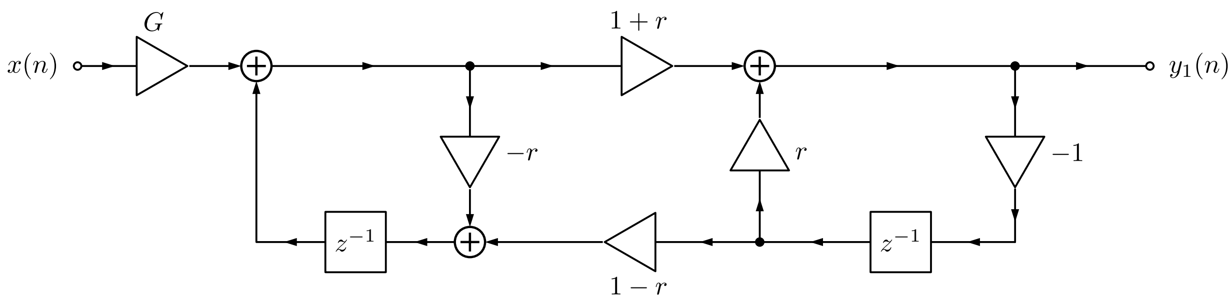
$$x(n) = \delta(n) - \delta(n-1).$$

(i.) in the z-domain and

(ii.) in the time domain using convolution.

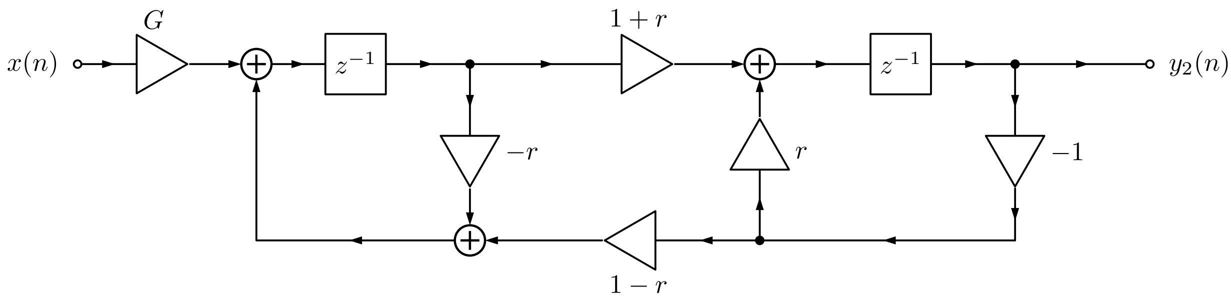
Task 2: System Flow Graphs

Consider the discrete-time system as depicted below:



a) Write the set of difference equations corresponding to the flow graph given above.
   Hint: Use appropriate auxiliary signal(s) if necessary.

b) Determine the system function $H_1(z) = Y_1(z)/X(z)$ of the system and determine the magnitudes and angles of the poles of $H_1(z)$ as a function of r for $-1 < r < 1$.



c) How is the system function $H_2(z) = Y_2(z)/X(z)$ related to $H_1(z) = Y_1(z)/X(z)$.

Task 3: Filter Application

We consider a simple example where we want to enhance the quality of a recorded speech signal that was disturbed by high-frequency ambient noise. This noise is band-limited and only appears at high frequencies.

a) Determine the ideal filter and the corresponding unit sample response. Why is it not realizable?

b) The ideal filter you have determined is to be approximated by an FIR filter. What design methods are available?

c) Using the windowing method, find an appropriate window to design a filter that fulfills the following specifications:

Stop-band attenuation:	≥ 25dB
Pass-band ripple:	≤ 3dB
Pass-band edge:	5000 Hz
Stop-band edge:	7000 Hz
Sampling rate:	22050 Hz

Proceed as follows:

(i) Determine the length, the order and the delay of the filter.

(ii) Sketch the frequency response of the low-pass filter and explain how it is related to the desired unit sample response and the used window function.

We now apply the designed filter to the recorded signal. Download the sound file “speechCrickets.wav” from the course web page and store it as a variable in MATLAB.

(iii) Analyze the signal using the spectrogram, which is a standard tool in speech processing. Try to distinguish speech from disturbances. Is the transition band defined above reasonable? (**Hint:** You might find the command `specgramdemo` useful.)

(iv) Design the previous filter using `fdatool` with the parameters you have determined and apply it to the signal.

(v) Now design two other filters with cut-off frequencies $f_{c2} = 4000\text{Hz}$, $f_{c3} = 2000\text{Hz}$ and compare the results. What effects do you recognize in terms of noise suppression and speech quality?

(vi) What changes do you expect if the power of the noise is increased further?

(vii) Now, load the ”speechAWGN.wav” which consists of the same speech file and Additive White Gaussian Noise (AWGN). Again apply the designed filters. Explain any differences you hear in the output signals. Which filter gives a higher Signal-to-Noise-Ratio (SNR)? Why?

Task 4: Multi-Band Filter Design

Consider the following ideal frequency response for a multi-band filter:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\frac{N-1}{2}}, & 0 \le |\omega| < 0.3\pi \\ 0, & 0.3\pi < |\omega| < 0.6\pi \\ 0.5e^{-j\omega\frac{N-1}{2}}, & 0.6\pi < |\omega| \le \pi \end{cases}$$

The unit sample response $h_d\left(n\right)$ is multiplied by a Kaiser window with $N = 49$ and $\beta = 3.68$, resulting in a linear phase FIR system with unit sample response $h\left(n\right)$.

a) What is the delay of the filter?

b) Determine the ideal unit sample response $h_d\left(n\right)$.

c) Determine the set of approximation error specifications which are satisfied by the FIR filter; i.e., determine the parameters $\alpha_1, \alpha_2, \alpha_3, B, C, \omega_{p1}, \omega_{p2}, \omega_{s1}$ and ω_{s2} (assume $A = 42.4256$) in

$$\begin{aligned} B - \alpha_1 \le \quad & \left|H(e^{j\omega})\right| \le B + \alpha_1, && 0 \le \omega \le \omega_{p1}, \\ & \left|H(e^{j\omega})\right| \le \alpha_2, && \omega_{s1} \le \omega \le \omega_{s2}, \\ C - \alpha_3 \le \quad & \left|H(e^{j\omega})\right| \le C + \alpha_3, && \omega_{p2} \le \omega \le \pi. \end{aligned}$$

Hint: Assume that the ripples at discontinuities do not overlap.

d) Discuss the requirements for the above made assumption to be legitimate. Additionally, state what the consequences are for the specifications of the filter, if the assumption cannot be made (i.e. the ripples overlap).

e) Design this band-stop filter using the MATLAB FDA Tool. The band-stop filters may be considered as the superposition of one low-pass and one high-pass filter. Use the same values of β and N in order to design these filters.

f) What should be the cut-off frequencies of low-pass and high-pass filters to construct the multi-band filter specified above?

g) Construct two filters using MATLAB FDA Tool and superpose them to obtain the multi-band filter.

h) Observe the frequency response of this multi-band filter.

i) Discuss which problems can arise from the superposition of the two filters. Which advantages and disadvantages does this approach have?

j) How would you design the multi-band filter without MATLABs FDA tool? Is there a way to avoid the use of two filters? If so, describe the approach and any differences you expect in the resulting filter.