Lecture

Speech and Audio Signal Processing

Lecture 7: Beamforming, Part II

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Part I:

- Introduction
- Characteristics of multi-microphone systems
- *Differential* beamformer
- *Delay-and-sum* beamformer

Part II:

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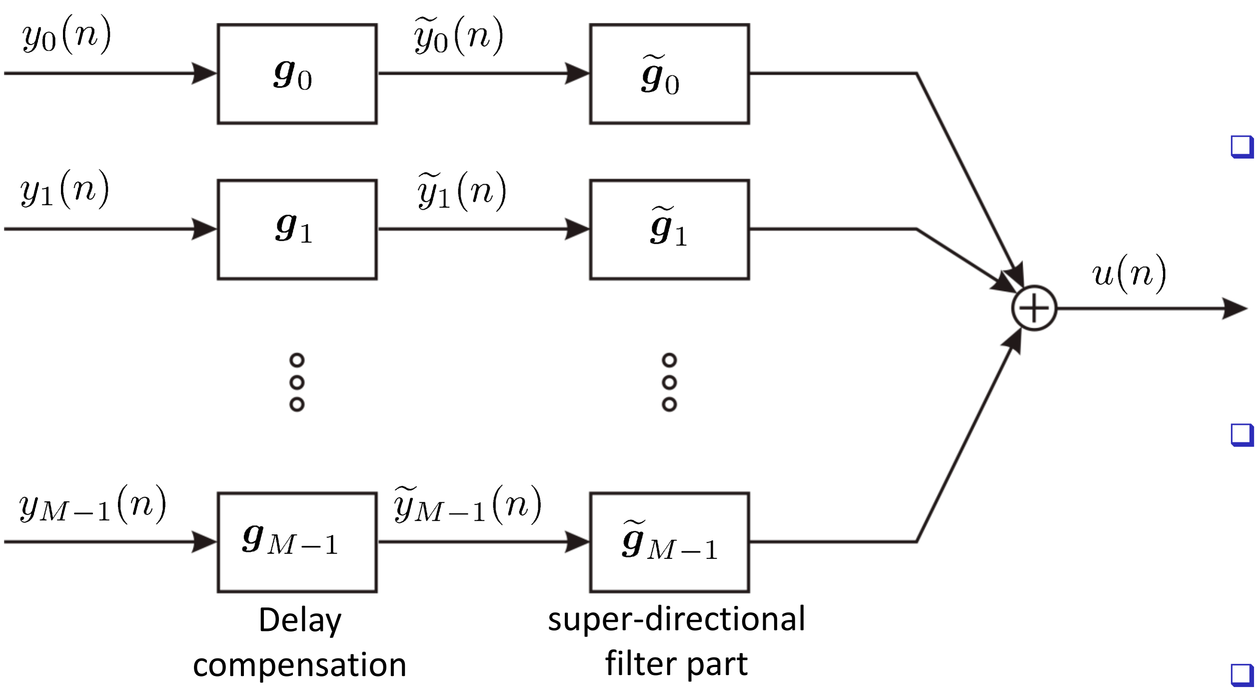
  - Minimum Variance Distortionless Response (MVDR) beamformer
  - Multi-channel Wiener Filter
  - Linear Constrained Minimum Variance Beamformer
- Interference compensation
- Audio examples and results

Filter-and-sum beamformer

Basic structure:



- **Target:**
  Improve the beamformer performance additionally by filtering
- **Definition of „Super-directivity“:**
  Noise suppression is higher than with a *delay-and-sum* setup.
- The super-directional filters are designed such as to optimize the broadside beampattern.
- Other target signal directions are realized by the delay compensation.

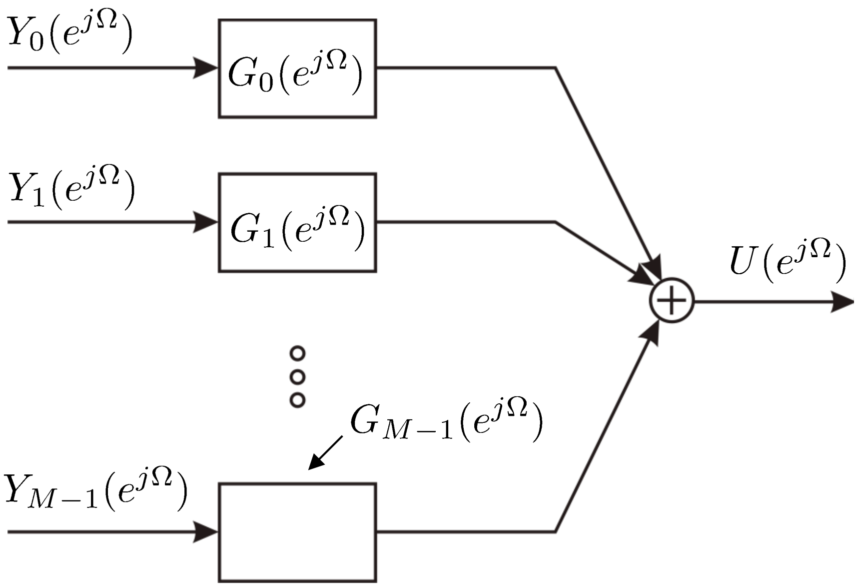
A constrained beamformer:

Minimum Variance Distortionlees Response: MVDR beamformer

MVDR Beamformer: Frequency domain solution

Optimization setup in the *frequency domain* [*].

Design of a fixed filter, Minimization of the interference power by keeping the desired signal unmodified



- **Input:**

Target point source

$$\boldsymbol{y}(e^{j\Omega}) = S(e^{j\Omega})\,\boldsymbol{d} + \boldsymbol{v}(e^{j\Omega})$$

Interference

with: $\boldsymbol{y}(e^{j\Omega}) = \left[Y_0(e^{j\Omega}), Y_1(e^{j\Omega}), \ldots, Y_{M-1}(e^{j\Omega})\right]^{\mathrm{T}}$

$\boldsymbol{v}(e^{j\Omega}) = \left[V_0(e^{j\Omega}), V_1(e^{j\Omega}), \ldots, V_{M-1}(e^{j\Omega})\right]^{T}$

- **Output:**

$$U(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})$$

with:

$$\boldsymbol{g}(e^{j\Omega}) = \left[G_0(e^{j\Omega}), G_1(e^{j\Omega}), \ldots, G_{M-1}(e^{j\Omega})\right]^{\mathrm{T}}$$

Target source propagation delay: $\boldsymbol{d} = \left[e^{-j\Omega\, f_s \tau_0}, \ldots, e^{-j\Omega\, f_s \tau_{M-1}}\right]^{\mathrm{T}}$

Broad side target source: $\boldsymbol{d} = [1, \ldots, 1]^{\mathrm{T}}$

[*] E.A.P. Habets, et. al.: New Insights Into the MVDR Beamformer in Room Acoustics, IEEE Trans. On Speech and Audio processing, vol. 18, no. 1, Jan. 2010

MVDR Beamformer: Frequency domain solution

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Design criteria:

- Error signal:

$$\begin{aligned} E(e^{j\Omega}) &= U(e^{j\Omega}) - S(e^{j\Omega}) \\ &= \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega}) - S(e^{j\Omega}) \\ &= \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\left[S(e^{j\Omega})\,\boldsymbol{d} + \boldsymbol{v}(e^{j\Omega})\right] - S(e^{j\Omega}) \\ &= \left[\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} - 1\right] S(e^{j\Omega}) \; + \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{v}(e^{j\Omega}) \end{aligned}$$

- Minimization criterion:

$$\begin{aligned} J((\boldsymbol{g}(e^{j\Omega})) &= \mathrm{E}\left\{|E(e^{j\Omega})|^2\right\} \\ &= \left|\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} - 1\right|^2 S_{SS}(\Omega) \\ &\quad + \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{VV}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) \end{aligned}$$

- Minimization of interference signal power at the output…

… while keeping the target signal components unchanged:

$$\boxed{S_{U_vU_v}(\Omega) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{VV}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min}$$

$$U_v(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{v}(e^{j\Omega})$$

$$\boxed{\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} \stackrel{!}{=} 1}$$

$$U_s(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,S(e^{j\Omega})\,\boldsymbol{d} \stackrel{!}{=} S(e^{j\Omega})$$

- **PSD definition:**

$$S_{V_iV_j}(\Omega) = \mathrm{E}\left\{V_i(e^{j\Omega})V_j^*(e^{j\Omega})\right\}$$

- **PSD matrix of the noise components:**

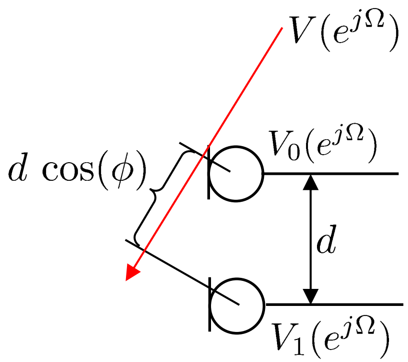
$$\boldsymbol{S}_{VV}(\Omega) = \begin{bmatrix} S_{V_0V_0}(\Omega) & S_{V_0V_1}(\Omega) & \dots & S_{V_0V_{M-1}}(\Omega) \\ S_{V_1V_0}(\Omega) & S_{V_1V_1}(\Omega) & \dots & S_{V_1V_{M-1}}(\Omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{V_{M-1}V_0}(\Omega) & S_{V_{M-1}V_1}(\Omega) & \dots & S_{V_{M-1}V_{M-1}}(\Omega) \end{bmatrix}$$

MVDR Beamformer: Examples for the noise PSD matrix

- **Supposing two microphones:**

$$\boldsymbol{S}_{VV}(\Omega) = \begin{bmatrix} S_{V_0V_0}(\Omega) & S_{V_0V_1}(\Omega) \\ S_{V_1V_0}(\Omega) & S_{V_1V_1}(\Omega) \end{bmatrix}$$

- **First assumption: Noise arriving from one specific direction (directional noise)**



with: $V_1(e^{j\Omega}) = V_0(e^{j\Omega})\, e^{-j\Omega\, \frac{d\ \cos(\phi)}{c}\, f_s}$

one obtains:

$$\begin{aligned} S_{V_0V_1}(\Omega) &= \mathrm{E}\left\{V_0(e^{j\Omega})\, V_1^*(e^{j\Omega})\right\} \\ &= \mathrm{E}\left\{V_0(e^{j\Omega})\, V_0^*(e^{j\Omega})\, e^{+j\Omega\, \frac{d\ \cos(\phi)}{c}\, f_s}\right\} \\ &= S_{V_0V_0}(\Omega) e^{+j\Omega\, \frac{d\ \cos(\phi)}{c}\, f_s} \\ S_{V_1V_0}(\Omega) &= S_{V_0V_0}(\Omega) e^{-j\Omega\, \frac{d\ \cos(\phi)}{c}\, f_s} \end{aligned}$$

resulting in:

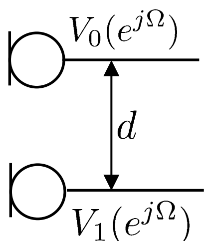
$$\boxed{\boldsymbol{S}_{VV}(\Omega) = S_{V_0V_0}(\Omega) \begin{bmatrix} 1 & e^{j\Omega\, \frac{d\ \cos(\phi)}{c}\, f_s} \\ e^{-j\Omega\, \frac{d\ \cos(\phi)}{c}\, f_s} & 1 \end{bmatrix}}$$

MVDR Beamformer: Examples for the noise PSD matrix

- **Supposing two microphones:**

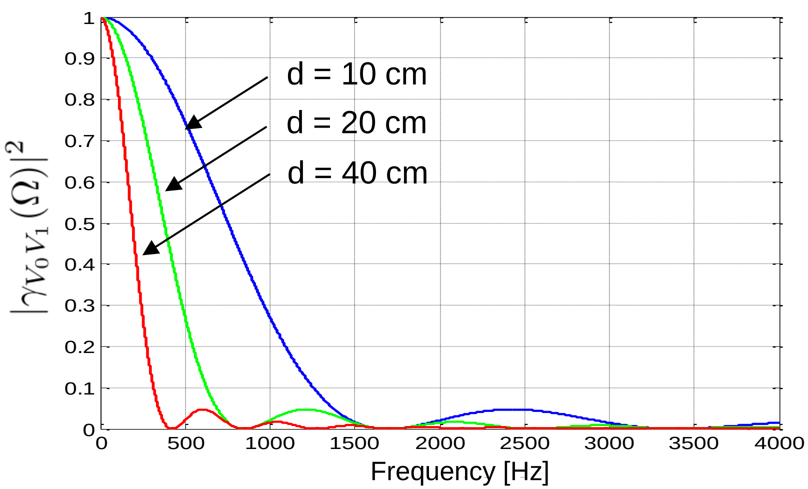
$$\boldsymbol{S}_{VV}(\Omega) = \begin{bmatrix} S_{V_0V_0}(\Omega) & S_{V_0V_1}(\Omega) \\ S_{V_1V_0}(\Omega) & S_{V_1V_1}(\Omega) \end{bmatrix}$$

  - **Second assumption: Diffuse noise**



Kohärenzfunktion:

with:
$$\gamma_{V_0V_1}(\Omega) = \frac{S_{V_0V_1}(\Omega)}{\sqrt{S_{V_0V_0}(\Omega)\,S_{V_1V_1}(\Omega)}}$$
$$S_{V_0V_0}(\Omega) = S_{V_1V_1}(\Omega)$$



one obtains:

$$S_{V_0V_1}(\Omega) = \gamma_{V_0V_1}(\Omega)\,S_{V_0V_0}(\Omega)$$
$$S_{V_1V_0}(\Omega) = \gamma^*_{V_0V_1}(\Omega)\,S_{V_0V_0}(\Omega)$$

resulting in:

$$\boxed{\boldsymbol{S}_{VV}(\Omega) = S_{V_0V_0}(\Omega)\begin{bmatrix} 1 & \gamma_{V_0V_1}(\Omega) \\ \gamma^*_{V_0V_1}(\Omega) & 1 \end{bmatrix}}$$

MVDR Beamformer: Frequency domain solution

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- Minimization of the interference signal by preserving the target signal component:

$$\boxed{S_{U_vU_v}(\Omega) \;=\; \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{VV}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min} \qquad \boxed{\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} \;\stackrel{!}{=}\; 1}$$

- Lagrange approach:

$$J\left(\boldsymbol{g}(e^{j\Omega})\right) = \frac{1}{2}\,\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{VV}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) + \lambda\left(\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} - 1\right) \stackrel{!}{=} \min$$

$$\nabla_{\boldsymbol{g}}\,J\left(\boldsymbol{g}(e^{j\Omega})\right) = \frac{\partial}{\partial \boldsymbol{g}^*}J\left(\boldsymbol{g}(e^{j\Omega})\right) \stackrel{!}{=} 0$$

- Results in:

=> $\boldsymbol{S}_{VV}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) + \lambda\,\boldsymbol{d} \stackrel{!}{=} 0$

$$\left.\begin{aligned} \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega}) &= -\lambda\,\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(\Omega) \\ \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} &\stackrel{!}{=} 1 \end{aligned}\right\} \;\Rightarrow\; -\lambda\,\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(\Omega)\,\boldsymbol{d} \stackrel{!}{=} 1$$

$$\left.\begin{aligned} \lambda &= \frac{-1}{\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(\Omega)\,\boldsymbol{d}} \\ \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega}) &= -\lambda\,\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(\Omega) \end{aligned}\right\} \;\Rightarrow\; \boxed{\boldsymbol{g}_{\mathrm{MVDR}}(e^{j\Omega}) = \frac{\boldsymbol{S}_{VV}^{-1}(\Omega)\,\boldsymbol{d}}{\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(\Omega)\,\boldsymbol{d}}}$$

MVDR Beamformer: Frequency domain solution

Se eu vou manter o sinal desejado intacto, então reduzir o ruido é também reduzir o sinal de saida (erro + sinal)

Alternative design criterion based on the minimization of the output signal power:

- Minimization of the **output** signal power…

$$U(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})$$

$$\boxed{S_{UU}(\Omega) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{YY}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min}$$

… while keeping the target signal components unchanged:

$$U_s(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,S(e^{j\Omega})\,\boldsymbol{d} \stackrel{!}{=} S(e^{j\Omega})$$

$$\boxed{\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} \stackrel{!}{=} 1}$$

- Minimization of the output power with respect to $\boldsymbol{G}(e^{j\Omega})$ …

$$S_{UU}(\Omega) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{YY}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min$$

$$\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,[\boldsymbol{S}_{SS}(\Omega) + \boldsymbol{S}_{VV}(\Omega)]\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min$$

$$\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,[\boldsymbol{d}\,S_{SS}(\Omega)\,\boldsymbol{d}^{\mathrm{H}} + \boldsymbol{S}_{VV}(\Omega)]\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min$$

with: $\boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{d} = 1$ => $S_{SS}(\Omega) + \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,[\boldsymbol{S}_{VV}(\Omega)]\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min$

… equivalent to the minimization of the interference signal power :

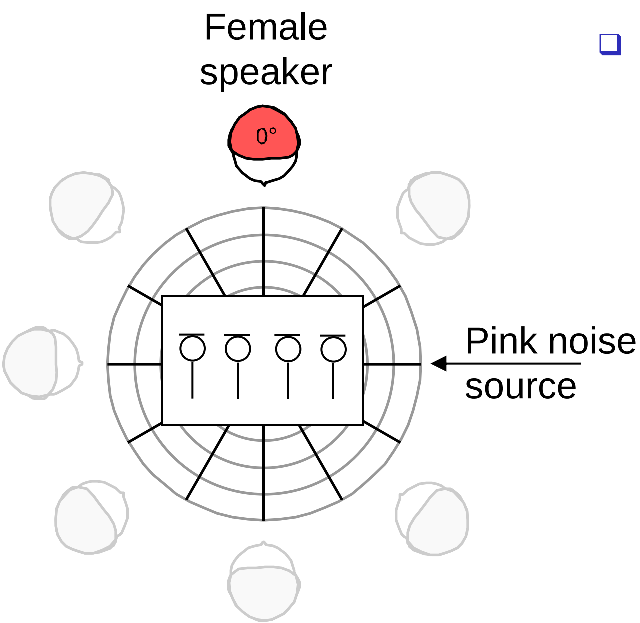
$$\boxed{S_{U_vU_v}(\Omega) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{VV}(\Omega)\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min}$$

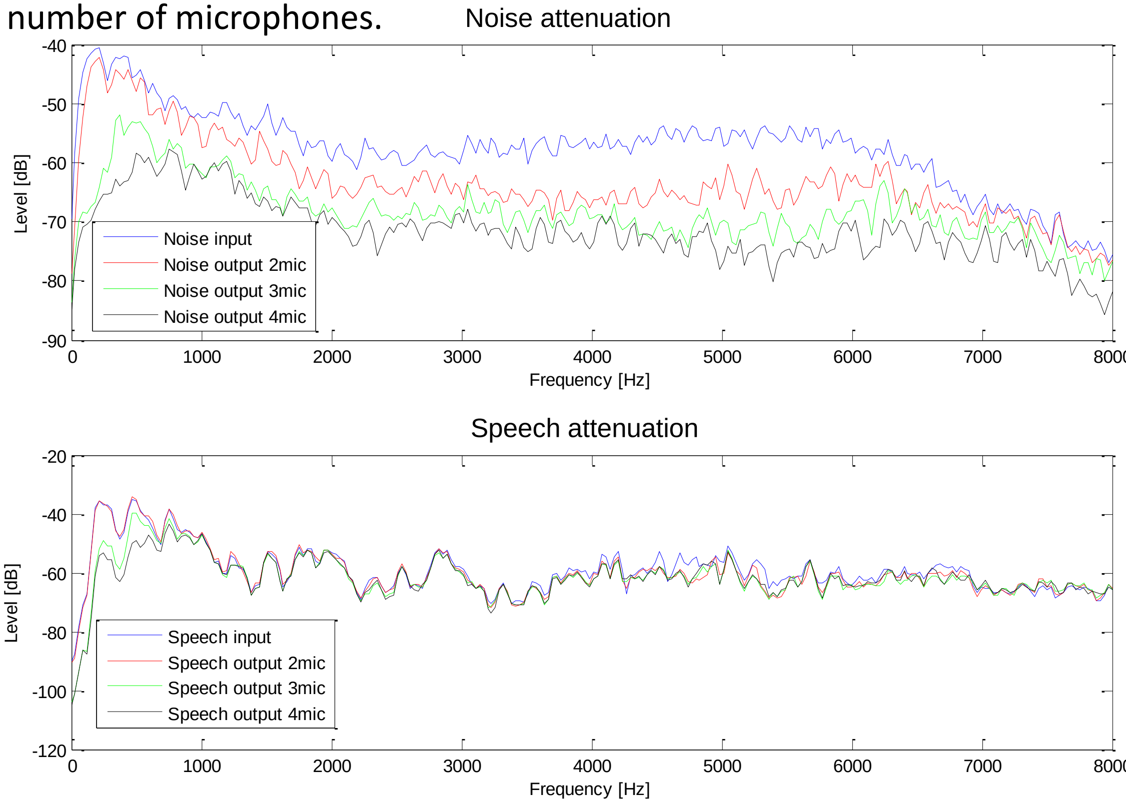
Aqui pelo menos temos acesso a Syy

=> $$\boxed{\boldsymbol{g}_{\mathrm{MVDR}}(e^{j\Omega}) = \frac{\boldsymbol{S}_{YY}^{-1}(\Omega)\,\boldsymbol{d}}{\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{YY}^{-1}(\Omega)\,\boldsymbol{d}}}$$

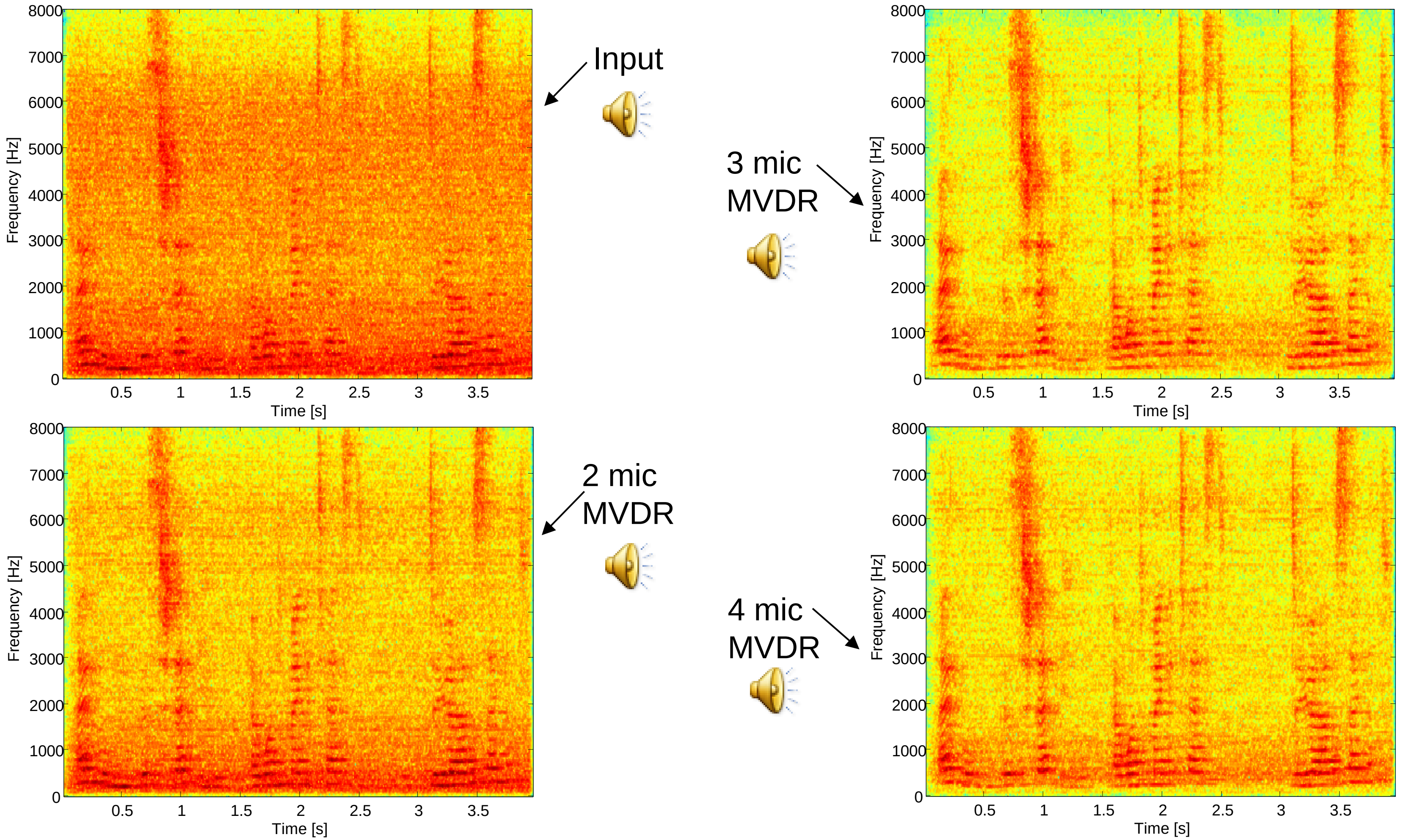
Example results

- Simulation results:
  - Stronger noise attenuation with more microphones (distance between each microphone: 3 cm)
  - Speech attenuation for lower frequencies with increasing number of microphones.



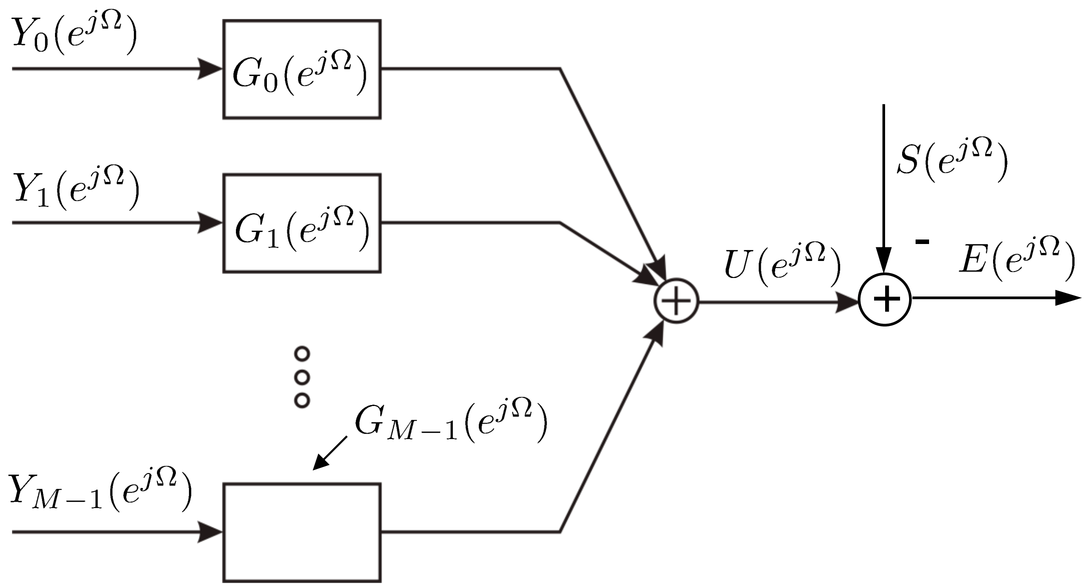


Example results



Multi-channel Wiener filter

Multi-channel Wiener filter



$$U(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})$$

$$E(e^{j\Omega}) = S(e^{j\Omega}) - \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})$$

- **Filter design criterion:**
  Min. the mean square error

$$\mathrm{E}\left\{|E(e^{j\Omega})|^2\right\} \stackrel{!}{=} \min$$

$$\mathrm{E}\left\{\left[S(e^{j\Omega}) - \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})\right]\left[S^*(e^{j\Omega}) - \boldsymbol{y}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{g}(e^{j\Omega})\right]\right\} \stackrel{!}{=} \min$$

$$S_{SS}(e^{j\Omega}) - \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{YS}(e^{j\Omega}) - \boldsymbol{S}_{SY}^{\mathrm{T}}(e^{j\Omega})\,\boldsymbol{g}(e^{j\Omega}) + \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{YY}(e^{j\Omega})\,\boldsymbol{g}(e^{j\Omega}) \stackrel{!}{=} \min$$

$$\frac{\partial}{\partial \boldsymbol{g}^*}\left[S_{SS}(e^{j\Omega}) - \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{YS}(e^{j\Omega}) - \boldsymbol{S}_{SY}^{\mathrm{T}}(e^{j\Omega})\,\boldsymbol{g}(e^{j\Omega}) + \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{S}_{YY}(e^{j\Omega})\,\boldsymbol{g}(e^{j\Omega})\right] \stackrel{!}{=} 0$$

=> $\boldsymbol{S}_{YS}(e^{j\Omega}) = \boldsymbol{S}_{YY}(e^{j\Omega})\,\boldsymbol{g}(e^{j\Omega})$

Wiener solution:

$$\boldsymbol{g}_{\mathrm{Wiener}}(e^{j\Omega}) = \boldsymbol{S}_{YY}^{-1}(e^{j\Omega})\,\boldsymbol{S}_{YS}(e^{j\Omega})$$

Multi-channel Wiener filter

$$\boxed{\boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \boldsymbol{S}_{YY}^{-1}(e^{j\Omega})\,\boldsymbol{S}_{YS}(e^{j\Omega})}$$

With uncorrelated target and noise components:

$$\boldsymbol{S}_{YS}(e^{j\Omega}) = S_{SS}(e^{j\Omega})\boldsymbol{d} \longleftarrow \text{Vector}$$

$$\left.\begin{aligned} \boldsymbol{S}_{YY}(e^{j\Omega}) &= S_{SS}(e^{j\Omega})\boldsymbol{d}\boldsymbol{d}^{\mathrm{H}} + \boldsymbol{S}_{VV}(e^{j\Omega}) \\ S_{SS}(e^{j\Omega})\boldsymbol{d}\boldsymbol{d}^{\mathrm{H}} &= \boldsymbol{S}_{YY}(e^{j\Omega}) - \boldsymbol{S}_{VV}(e^{j\Omega}) \end{aligned}\right\} \longleftarrow \text{Matrices}$$

$$\boldsymbol{S}_{YS}(\Omega) = \begin{bmatrix} S_{Y_0 S}(\Omega) \\ S_{Y_1 S}(\Omega) \\ \vdots \\ S_{Y_{M-1} S}(\Omega) \end{bmatrix}$$

- **Different notations:**

$$\boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \left[S_{SS}(e^{j\Omega})\boldsymbol{d}\boldsymbol{d}^{\mathrm{H}} + \boldsymbol{S}_{VV}(e^{j\Omega})\right]^{-1} S_{SS}(e^{j\Omega})\boldsymbol{d}$$

$$\boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \boldsymbol{S}_{YY}^{-1}(e^{j\Omega})\,\hat{\boldsymbol{S}}_{YS}(\Omega)$$

$$\hat{\boldsymbol{S}}_{YS}(\Omega) = \begin{bmatrix} S_{Y_0 Y_0}(\Omega) \\ S_{Y_1 Y_0}(\Omega) \\ \vdots \\ S_{Y_{M-1} Y_0}(\Omega) \end{bmatrix} - \begin{bmatrix} S_{V_0 V_0}(\Omega) \\ S_{V_1 V_0}(\Omega) \\ \vdots \\ S_{V_{M-1} V_0}(\Omega) \end{bmatrix}$$

- **Applications of multi-channel Wiener filters** [Doc 05, Doc 07]:
  Estimation of the noise PSD matrix in speech pauses
  => Calculation of the target signal PSD matrix by
  subtracting the input PSD matrix and the estimated noise PSD matrix
  using the relation: $S_{SS}(e^{j\Omega})\boldsymbol{d} = \hat{\boldsymbol{S}}_{YS}(e^{j\Omega})$

[Doc 05] S. Doclo et. al.: "**Extension of the Multi-Channel Wiener Filter with Localization Cues for Noise Reduction in Binaural Hearing Aids**", in *Proc. IEEE IWAENC, pp. 221-224, Sept. 2005*
[Doc 07] T. Bogaert, S. Doclo, M. Moonen:
"**Binaural Cue Preservation for Hearing Aids using an Interaural Transfer Function Multichannel Wiener Filter**", in *Proc. IEEE ICASSP*, vol. 4, pp. 565 - 568, Apr. 2007

Multi-channel Wiener filter

$$\boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \boldsymbol{S}_{YY}^{-1}(e^{j\Omega}) \left[\begin{bmatrix} S_{Y_0Y_0}(\Omega) \\ S_{Y_1Y_0}(\Omega) \\ \vdots \\ S_{Y_{M-1}Y_0}(\Omega) \end{bmatrix} - \begin{bmatrix} S_{V_0V_0}(\Omega) \\ S_{V_1V_0}(\Omega) \\ \vdots \\ S_{V_{M-1}V_0}(\Omega) \end{bmatrix} \right]$$

- **Problems:**
  - Detection of speech pauses essential for the estimation of the noise correlation matrix in speech pauses only.
  - Noise correlation matrix depends on the spatial and spectral characteristics of the noise.
  - No interference cancellation of non-stationary signals (speech) possible since they typically change their characteristic during target speech signal activity.

Relation between MVDR BF and Multi-channel Wiener filter

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$$\boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \left[S_{SS}(e^{j\Omega})\boldsymbol{d}\boldsymbol{d}^{\text{H}} + \boldsymbol{S}_{VV}(e^{j\Omega})\right]^{-1} S_{SS}(e^{j\Omega})\boldsymbol{d}$$

- **Matrix inversion lemma:**

$$\left[\boldsymbol{A}^{-1} + \boldsymbol{B}\boldsymbol{C}^{-1}\boldsymbol{B}^{\text{H}}\right]^{-1} = \boldsymbol{A} - \boldsymbol{A}\boldsymbol{B}\left[\boldsymbol{C} + \boldsymbol{B}^{\text{H}}\boldsymbol{A}\boldsymbol{B}\right]^{-1}\boldsymbol{B}^{\text{H}}\boldsymbol{A}$$

$$\left.\begin{aligned} \boldsymbol{A} &= \boldsymbol{S}_{VV}^{-1}(e^{j\Omega}) \\ \boldsymbol{B} &= \sqrt{S_{SS}(e^{j\Omega})}\boldsymbol{d} \\ \boldsymbol{C} &= 1 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \boldsymbol{C} + \boldsymbol{B}^{\text{H}}\boldsymbol{A}\boldsymbol{B} &= 1 + S_{SS}(e^{j\Omega})\,\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d} \quad \text{: scalar} \\ \boldsymbol{A}\boldsymbol{B}\,\boldsymbol{B}^{\text{H}}\boldsymbol{A} &= S_{SS}(e^{j\Omega})\,\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega}) \end{aligned}$$

Multiplication gives the scalar element known from denom.

$$\Rightarrow \quad \boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \left[\boldsymbol{S}_{VV}^{-1}(e^{j\Omega}) - \frac{S_{SS}(e^{j\Omega})\,\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})}{1 + S_{SS}(e^{j\Omega})\,\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}}\right] S_{SS}(e^{j\Omega})\boldsymbol{d}$$

$$= \left[\frac{S_{SS}(e^{j\Omega})}{1 + S_{SS}(e^{j\Omega})\,\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}}\right] \boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}$$

$$\boxed{\boldsymbol{g}_{\text{Wiener}}(e^{j\Omega}) = \left[\frac{S_{SS}(e^{j\Omega})}{S_{SS}(e^{j\Omega}) + \left(\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}\right)^{-1}}\right] \frac{\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}}{\boldsymbol{d}^{\text{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}}}$$

Relation between MVDR BF and Multi-channel Wiener filter

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$$\boldsymbol{g}_{\mathrm{Wiener}}(e^{j\Omega}) = \left[\frac{S_{SS}(e^{j\Omega})}{S_{SS}(e^{j\Omega}) + \left(\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}\right)^{-1}}\right] \frac{\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}}{\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(e^{j\Omega})\boldsymbol{d}}$$

$$\boldsymbol{g}_{\mathrm{MVDR}}(e^{j\Omega}) = \frac{\boldsymbol{S}_{VV}^{-1}(\Omega)\,\boldsymbol{d}}{\boldsymbol{d}^{\mathrm{H}}\boldsymbol{S}_{VV}^{-1}(\Omega)\,\boldsymbol{d}}$$

Single channel Wiener filter

Noise PSD after summation according to target signal direction of arrival

- **Concluding:**
  The Multi-channel Wiener filter combines an MVDR beamformer with a single channel Wiener filter.

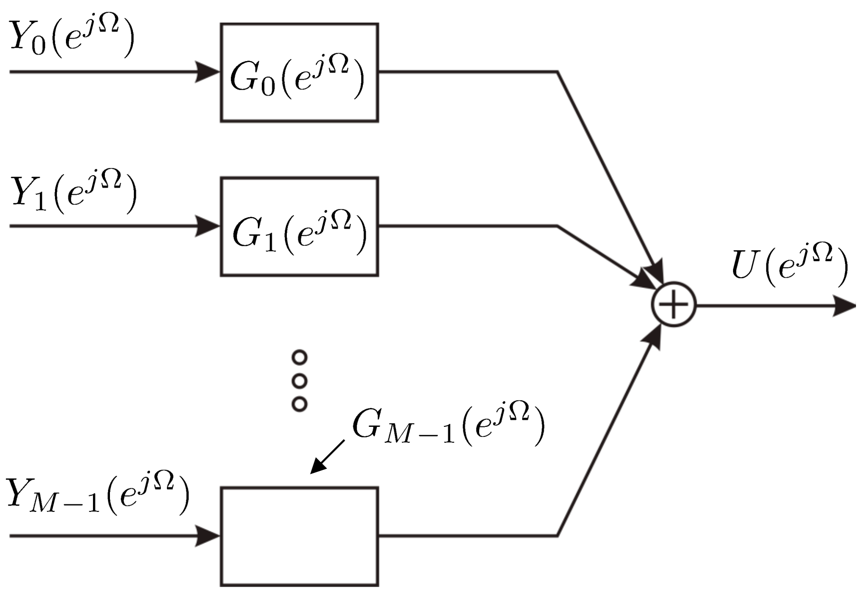
  Target signal distortion introduced is according to the single channel Wiener filter.

A beamformer allowing more than one constraint:

Linear Constrained Minimum Variance (LCMV) beamformer

LCMV Beamformer

- **Optimization setup in the *frequency domain*.**



- **Input:**

Target point source

Directional interferer source

$$\boldsymbol{y}(e^{j\Omega}) = S(e^{j\Omega})\,\boldsymbol{d} + \Psi(e^{j\Omega})\,\boldsymbol{b} + \boldsymbol{v}(e^{j\Omega})$$

Other interference: unknown spatial characteristic

with: $\boldsymbol{y}(e^{j\Omega}) = \left[Y_0(e^{j\Omega}), Y_1(e^{j\Omega}), \ldots, Y_{M-1}(e^{j\Omega})\right]^{\mathrm{T}}$

$\boldsymbol{v}(e^{j\Omega}) = [V_0(e^{j\Omega}), V_1(e^{j\Omega}), \ldots, V_{M-1}(e^{j\Omega})]^{T}$

- **Output:**

$$U(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})$$

with: $\boldsymbol{g}(e^{j\Omega}) = \left[G_0(e^{j\Omega}), G_1(e^{j\Omega}), \ldots, G_{M-1}(e^{j\Omega})\right]^{\mathrm{T}}$

	Target source propagation model:	Directional interferer propagation model:
Free field:	$\boldsymbol{d} = \left[e^{-j\Omega\, f_s \tau_0}, \ldots, e^{-j\Omega\, f_s \tau_{M-1}}\right]^{\mathrm{T}}$	$\boldsymbol{b} = \left[e^{-j\Omega\, f_s \eta_0}, \ldots, e^{-j\Omega\, f_s \eta_{M-1}}\right]^{\mathrm{T}}$
General model (frequency resp. required):	$\boldsymbol{d} = \left[D_0(e^{j\Omega}), \ldots, D_{M-1}(e^{j\Omega})\right]^{\mathrm{T}}$	$\boldsymbol{b} = \left[B_0(e^{j\Omega}), \ldots, B_{M-1}(e^{j\Omega})\right]^{\mathrm{T}}$

LCMV Beamformer

- Input signal model:

$$\boldsymbol{y}(e^{j\Omega}) = S(e^{j\Omega})\,\boldsymbol{d} + \Psi(e^{j\Omega})\,\boldsymbol{b} + \boldsymbol{v}(e^{j\Omega})$$

$$\boldsymbol{y}(e^{j\Omega}) = \boldsymbol{d}\,S(e^{j\Omega}) + \boldsymbol{b}\,\Psi(e^{j\Omega}) + \boldsymbol{v}(e^{j\Omega})$$

- Spatial filter:

$$U(e^{j\Omega}) = \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\,\boldsymbol{y}(e^{j\Omega})$$

$\boldsymbol{y} = \boldsymbol{d}\,S + \boldsymbol{b}\,\Psi + \boldsymbol{v}$ $\longleftarrow$ without freq.index $\longrightarrow$ $U = \boldsymbol{g}^{\mathrm{H}}\,\boldsymbol{y}$

- Filter design:

$U = \boldsymbol{g}^{\mathrm{H}}\,[\boldsymbol{d}\,S + \boldsymbol{b}\,\Psi + \boldsymbol{v}]$ $\longrightarrow$ Minimization of the corresponding power

$S_{UU}(e^{j\Omega}) \longrightarrow$ min

under the constraints: $\boldsymbol{g}^{\mathrm{H}}\,\boldsymbol{d} \overset{!}{=} D_0$ 1) Output signal should be equiv. to the target signal component as present at the 1. microphone

$\boldsymbol{g}^{\mathrm{H}}\,\boldsymbol{b} \overset{!}{=} 0$ 2) Directional interferer cancellation

combined constraints: $\boldsymbol{g}^{\mathrm{H}}\,\boldsymbol{C} \overset{!}{=} \boldsymbol{w}^{\mathrm{H}}$ with: $\boldsymbol{C} = [\boldsymbol{d}, \boldsymbol{b}]$ Constraint matrix

$\boldsymbol{w} = [D_0^*, 0]^{\mathrm{T}}$ Constraint vector

Combined: $\boxed{S_{UU}(e^{j\Omega}) \longrightarrow \text{min} \quad \text{with: } \boldsymbol{g}^{\mathrm{H}}\,\boldsymbol{C} \overset{!}{=} \boldsymbol{w}^{\mathrm{H}}}$ Allows up tp M-1 constraints

LCMV Beamformer

- Minimization of the Lagrange function:

$$J(\boldsymbol{g}) = \frac{1}{2} S_{UU} + \left(\boldsymbol{g}^{\mathrm{H}} \boldsymbol{C} - \boldsymbol{w}^{\mathrm{H}}\right) \boldsymbol{\lambda} \longrightarrow \text{min} \qquad \text{with: } \boldsymbol{\lambda} = [\lambda_1, \lambda_2]^{\mathrm{T}}$$

$$= \frac{1}{2} \boldsymbol{g}^{\mathrm{H}} S_{\boldsymbol{yy}} \boldsymbol{g} + \left(\boldsymbol{g}^{\mathrm{H}} \boldsymbol{C} - \boldsymbol{w}^{\mathrm{H}}\right) \boldsymbol{\lambda}$$

$$\nabla_{\boldsymbol{g}} J(\boldsymbol{g}) = \frac{\partial}{\partial \boldsymbol{g}^*} J(\boldsymbol{g}) \stackrel{!}{=} 0$$

$$S_{\boldsymbol{yy}} \boldsymbol{g} + \boldsymbol{C} \boldsymbol{\lambda} \stackrel{!}{=} 0$$

$$\boldsymbol{g} = -S_{\boldsymbol{yy}}^{-1} \boldsymbol{C} \boldsymbol{\lambda}$$

$$\boldsymbol{g}^{\mathrm{H}} = -\boldsymbol{\lambda}^{\mathrm{H}} \boldsymbol{C}^{\mathrm{H}} S_{\boldsymbol{yy}}^{-1}$$

$$\boldsymbol{g}^{\mathrm{H}} \boldsymbol{C} = -\boldsymbol{\lambda}^{\mathrm{H}} \boldsymbol{C}^{\mathrm{H}} S_{\boldsymbol{yy}}^{-1} \boldsymbol{C}$$

$$\boldsymbol{w}^{\mathrm{H}} = -\boldsymbol{\lambda}^{\mathrm{H}} \boldsymbol{C}^{\mathrm{H}} S_{\boldsymbol{yy}}^{-1} \boldsymbol{C}$$

$$\boldsymbol{\lambda}^{\mathrm{H}} = -\boldsymbol{w}^{\mathrm{H}} \left(\boldsymbol{C}^{\mathrm{H}} S_{\boldsymbol{yy}}^{-1} \boldsymbol{C}\right)^{-1}$$

$$\boldsymbol{\lambda} = -\left(\boldsymbol{C}^{\mathrm{H}} S_{\boldsymbol{yy}}^{-1} \boldsymbol{C}\right)^{-1} \boldsymbol{w}$$

$$\boxed{\boldsymbol{g}_{\mathrm{LCMV}} = S_{\boldsymbol{yy}}^{-1} \boldsymbol{C} \left(\boldsymbol{C}^{\mathrm{H}} S_{\boldsymbol{yy}}^{-1} \boldsymbol{C}\right)^{-1} \boldsymbol{w}}$$

- Comparison to the MVDR beamformer:

$$\boldsymbol{g}_{\mathrm{MVDR}}(e^{j\Omega}) = \frac{\boldsymbol{S}_{YY}^{-1}(\Omega)\, \boldsymbol{d}}{\boldsymbol{d}^{\mathrm{H}} \boldsymbol{S}_{YY}^{-1}(\Omega)\, \boldsymbol{d}} \qquad \text{with one constraint:} \quad \boldsymbol{g}^{\mathrm{H}}(e^{j\Omega})\, \boldsymbol{d} \stackrel{!}{=} 1$$

LCMV Beamformer

- Alternative procedure:

$$S_{UU} = \boldsymbol{g}^{\mathrm{H}}\, S_{\boldsymbol{yy}}\, \boldsymbol{g} \qquad\qquad S_{\boldsymbol{yy}} = \boldsymbol{d}^{\mathrm{H}}\, S_{ss}\, \boldsymbol{d} + \boldsymbol{b}^{\mathrm{H}}\, S_{\Psi\Psi}\, \boldsymbol{b} + S_{\boldsymbol{vv}}$$

$$= \boldsymbol{d}\, S_{ss}\, \boldsymbol{d}^{\mathrm{H}} + \boldsymbol{b}\, S_{\Psi\Psi}\, \boldsymbol{b}^{\mathrm{H}} + S_{\boldsymbol{vv}}$$

with: $\boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{d} \overset{!}{=} D_0$

and $\boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{b} \overset{!}{=} 0 \longrightarrow S_{UU} = D_0\, S_{ss}\, D_0{}^{*} + \boldsymbol{g}^{\mathrm{H}}\, S_{\boldsymbol{vv}}\, \boldsymbol{g}$

$$\begin{aligned} J(\boldsymbol{g}) &= \frac{1}{2} S_{UU} + \left(\boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{C} - \boldsymbol{w}^{\mathrm{H}}\right) \boldsymbol{\lambda} \longrightarrow \text{min} \\ &= \frac{1}{2}\left[D_0\, S_{ss}\, D_0{}^{*} + \boldsymbol{g}^{\mathrm{H}}\, S_{\boldsymbol{vv}}\, \boldsymbol{g}\right] + \left(\boldsymbol{g}^{\mathrm{H}}\, \boldsymbol{C} - \boldsymbol{w}^{\mathrm{H}}\right) \boldsymbol{\lambda} \end{aligned}$$

with: $\boldsymbol{\lambda} = [\lambda_1, \lambda_2]^{\mathrm{T}}$

$$\nabla_{\boldsymbol{g}} J(\boldsymbol{g}) = \frac{\partial}{\partial \boldsymbol{g}^{*}} J(\boldsymbol{g}) \overset{!}{=} 0$$

$$S_{\boldsymbol{vv}}\, \boldsymbol{g} + \boldsymbol{C}\, \boldsymbol{\lambda} \overset{!}{=} 0 \xrightarrow{\text{same procedure as previous slide}} \boxed{\boldsymbol{g}_{\mathrm{LCMV}} = S_{\boldsymbol{vv}}^{-1}\, \boldsymbol{C} \left(\boldsymbol{C}^{\mathrm{H}}\, S_{\boldsymbol{vv}}^{-1}\, \boldsymbol{C}\right)^{-1} \boldsymbol{w}}$$

Spatially uncorrelated white noise: $S_{\boldsymbol{vv}} = \sigma^2\, \boldsymbol{I} \longrightarrow \boldsymbol{g}_{\mathrm{LCMV}} = \boldsymbol{C}\left(\boldsymbol{C}^{\mathrm{H}}\, \boldsymbol{C}\right)^{-1} \boldsymbol{w}$

LCMV Beamformer

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❑ Adaptive LCMV beamformer

Most critical topic: How to design a constrained adaptation?

Concept: Separation of $\boldsymbol{g}$ into $\boldsymbol{g}_c$ and $\tilde{\boldsymbol{g}}_c$

Achieved by using a projection matrix $\boldsymbol{P}_c = \boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{C}^{\mathrm{H}}$

$$\left.\begin{aligned} \boldsymbol{g}_c &= \boldsymbol{P}_c\,\boldsymbol{g} \\ \tilde{\boldsymbol{g}}_c &= (\boldsymbol{I} - \boldsymbol{P}_c)\,\boldsymbol{g} \end{aligned}\right\} \quad \boldsymbol{g}_c + \tilde{\boldsymbol{g}}_c = \boldsymbol{g}$$

Now: 1) $\boldsymbol{g}_c$ fulfills the constraints

2) $\tilde{\boldsymbol{g}}_c$ orthogonal to the constraints => any solution possible, i.e., a free adaptation!

Proof: 1)

$$\boldsymbol{g}_c = \boldsymbol{P}_c\,\boldsymbol{g}$$

$$\boldsymbol{g}_c = \boldsymbol{P}_c\,(S_{\boldsymbol{yy}}^{-1}\,\boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,S_{\boldsymbol{yy}}^{-1}\,\boldsymbol{C})^{-1}\,\boldsymbol{w})$$

$$\boldsymbol{g}_c = \boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{C}^{\mathrm{H}}\,S_{\boldsymbol{yy}}^{-1}\,\boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,S_{\boldsymbol{yy}}^{-1}\,\boldsymbol{C})^{-1}\,\boldsymbol{w}$$

$$\boldsymbol{g}_c = \boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{w} \longrightarrow \boxed{\boldsymbol{C}^{\mathrm{H}}\boldsymbol{g}_c = \boldsymbol{w} \longrightarrow \boldsymbol{g}_c \text{ fulfills the constraints}}$$

LCMV Beamformer

Proof: 2) $\tilde{\boldsymbol{g}}_c = (\boldsymbol{I} - \boldsymbol{P}_c)\,\boldsymbol{g}$

$\tilde{\boldsymbol{g}}_c = \boldsymbol{g} - \boldsymbol{g}_c$ with: $\boldsymbol{g}_c = \boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{w}$ (s. prev. equation on last slide):

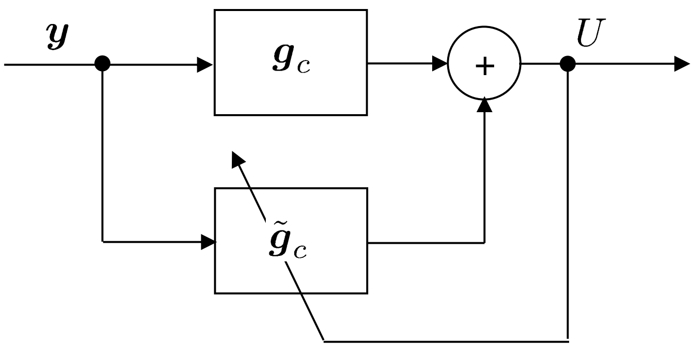
$$\tilde{\boldsymbol{g}}_c = \boldsymbol{g} - \boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{w}$$

$$\boldsymbol{C}^{\mathrm{H}}\,\tilde{\boldsymbol{g}}_c = \boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{g} - \boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{w}$$

$$\boldsymbol{C}^{\mathrm{H}}\,\tilde{\boldsymbol{g}}_c = \boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{g} - \boldsymbol{w} = 0$$

$\boxed{\boldsymbol{C}^{\mathrm{H}}\,\tilde{\boldsymbol{g}}_c = 0 \longrightarrow \tilde{\boldsymbol{g}}_c \text{ orthogonal to the constraints}}$

- Adaptive setup: $U = \boldsymbol{g}^{\mathrm{H}}\,\boldsymbol{y}$



Upper filter fixed (optimal for spatially uncorrelated white noise, s. page 23):

$$\boldsymbol{g}_c = \boldsymbol{C}\,(\boldsymbol{C}^{\mathrm{H}}\,\boldsymbol{C})^{-1}\,\boldsymbol{w}$$

Lower filter adaptive, multiplication with $\tilde{\boldsymbol{P}}_c = (\boldsymbol{I} - \boldsymbol{P}_c)$ ensures projection on null-space

$$\tilde{\boldsymbol{g}}_c = \tilde{\boldsymbol{P}}_c\,\boldsymbol{g}$$

LCMV Beamformer

- Adaptive procedure setup:

Null-space constraint

$$\tilde{\boldsymbol{g}}_c(n+1) = \tilde{\boldsymbol{g}}_c(n) - \frac{\mu(n)}{\mathrm{NORM}} \tilde{\boldsymbol{P}}_c \boldsymbol{y}(n)\, U^*(n)$$

- Motivation for the adaptive procedure, i.e., adaptation into direction of neg. gradient:

with: $S_{UU} = \boldsymbol{g}^{\mathrm{H}}\, S_{\boldsymbol{yy}}\, \boldsymbol{g}$

the gradient is: $\nabla_{\boldsymbol{g}} S_{UU} = \dfrac{\partial}{\partial \boldsymbol{g}^*} [\boldsymbol{g}^{\mathrm{H}}\, S_{\boldsymbol{yy}}\, \boldsymbol{g}] \;\; = S_{\boldsymbol{yy}}\, \boldsymbol{g}$ with: $S_{\boldsymbol{yy}} = \mathrm{E}\{\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}\}$

Instantaneous value: $\approx \boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}\, \boldsymbol{g}$

$\approx \boldsymbol{y}\, U^*$

Norm: $\mathrm{NORM} = \|\tilde{\boldsymbol{P}}_c \boldsymbol{y}(n)\|^2$

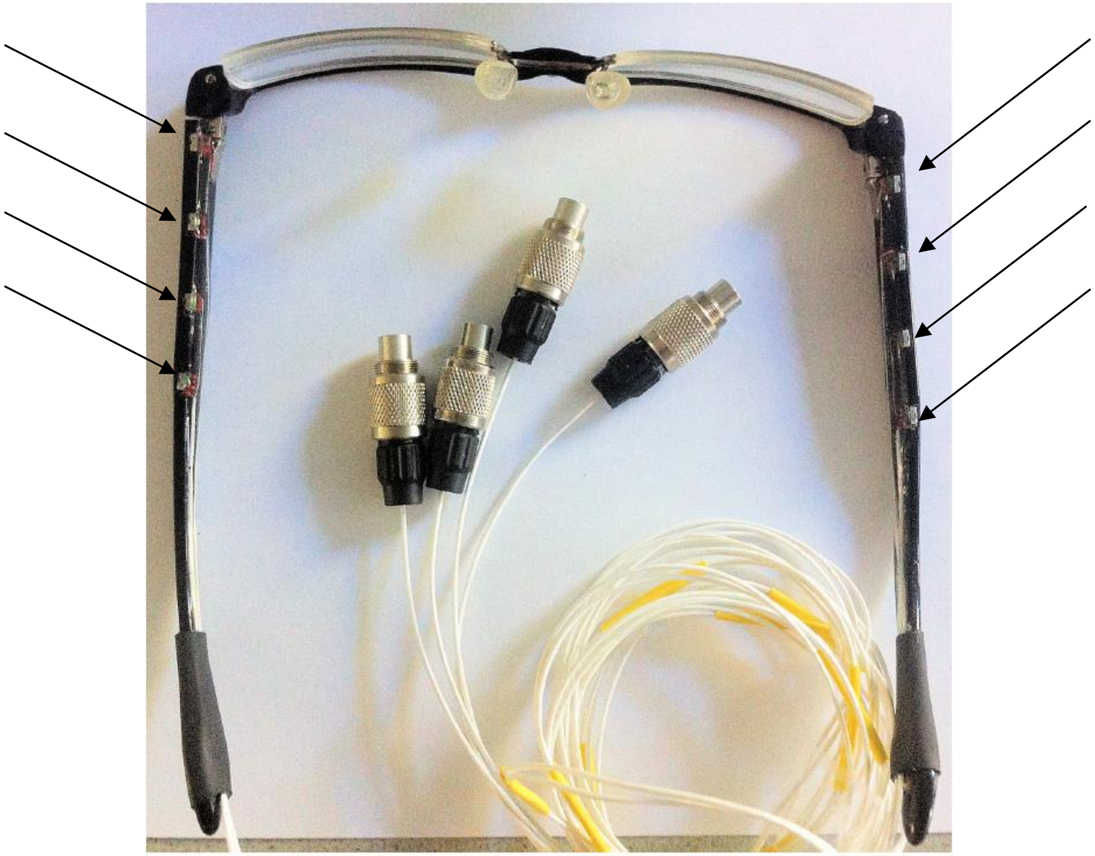
- The adaptive procedure:

$$\tilde{\boldsymbol{g}}_c(n+1) = \tilde{\boldsymbol{g}}_c(n) - \mu(n) \frac{\tilde{\boldsymbol{P}}_c \boldsymbol{y}(n)\, U^*(n)}{\|\tilde{\boldsymbol{P}}_c \boldsymbol{y}(n)\|^2}$$

$$\boxed{\tilde{\boldsymbol{g}}_c(n+1) = \tilde{\boldsymbol{P}}_c \left[\tilde{\boldsymbol{g}}_c(n) - \mu(n) \frac{\boldsymbol{y}(n)\, U^*(n)}{\|\tilde{\boldsymbol{P}}_c \boldsymbol{y}(n)\|^2} \right]}$$

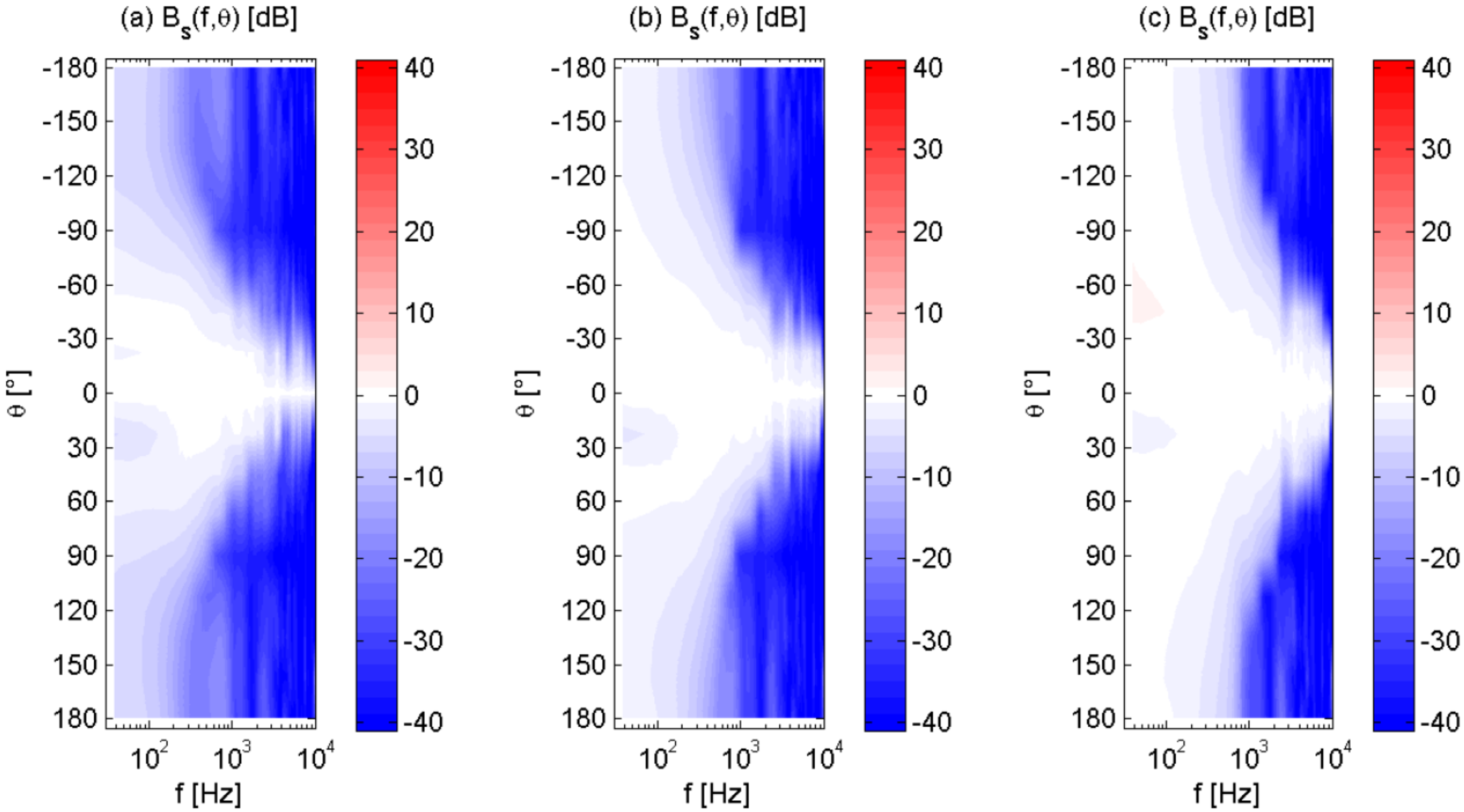
Avoids numerical problems to fulfill constraint

8-mic setup in glasses



LCMV Beamformer with 4 mics on one side of glasses

- Beampattern for 10, 5, and 0 dB white noise gain limitation (left to right)

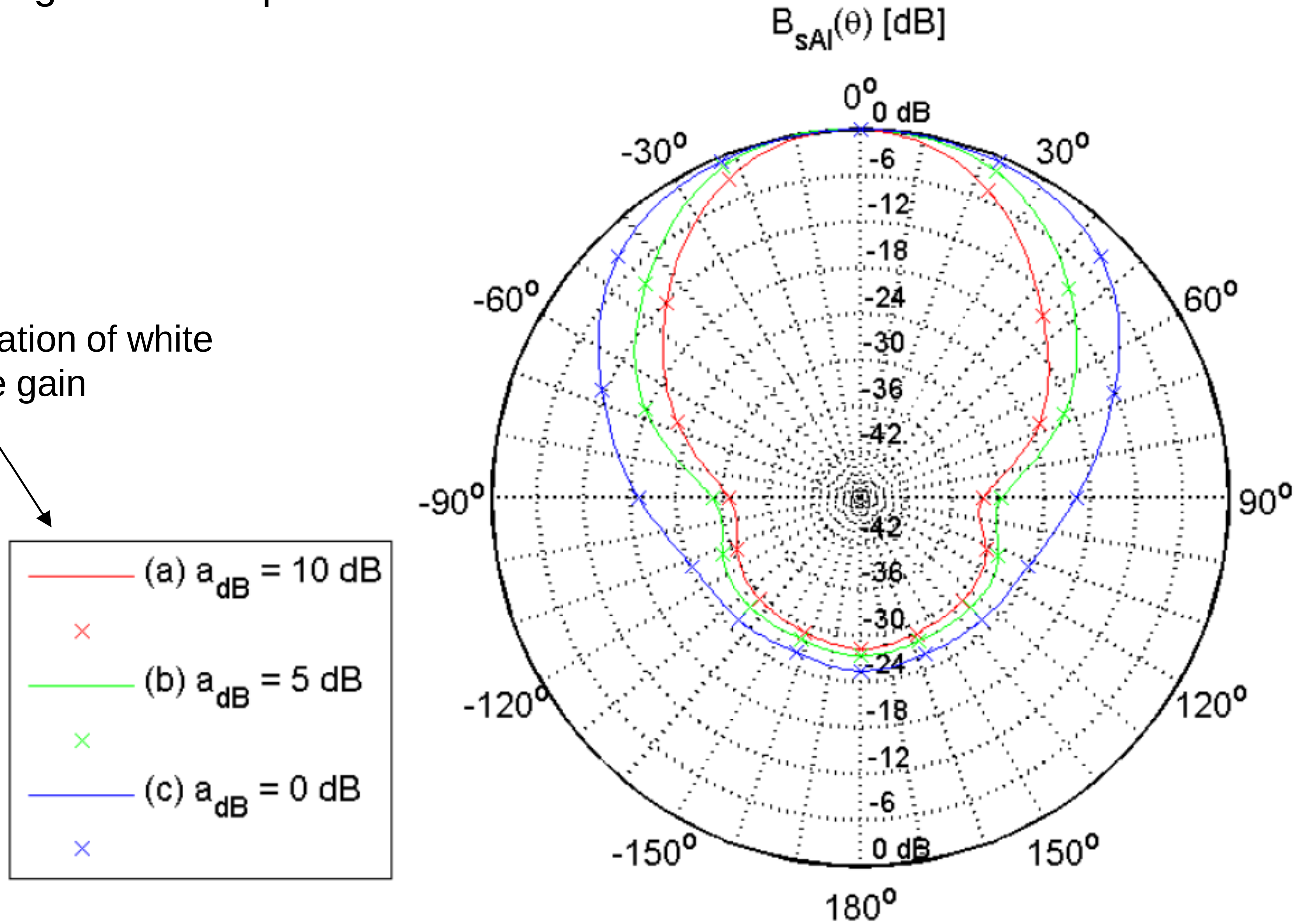


sAIDI = 13.1 dB | sAIDI = 9.94 dB | sAIDI = 7.04 dB

LCMV Beamformer with 4 mics on one side of glasses

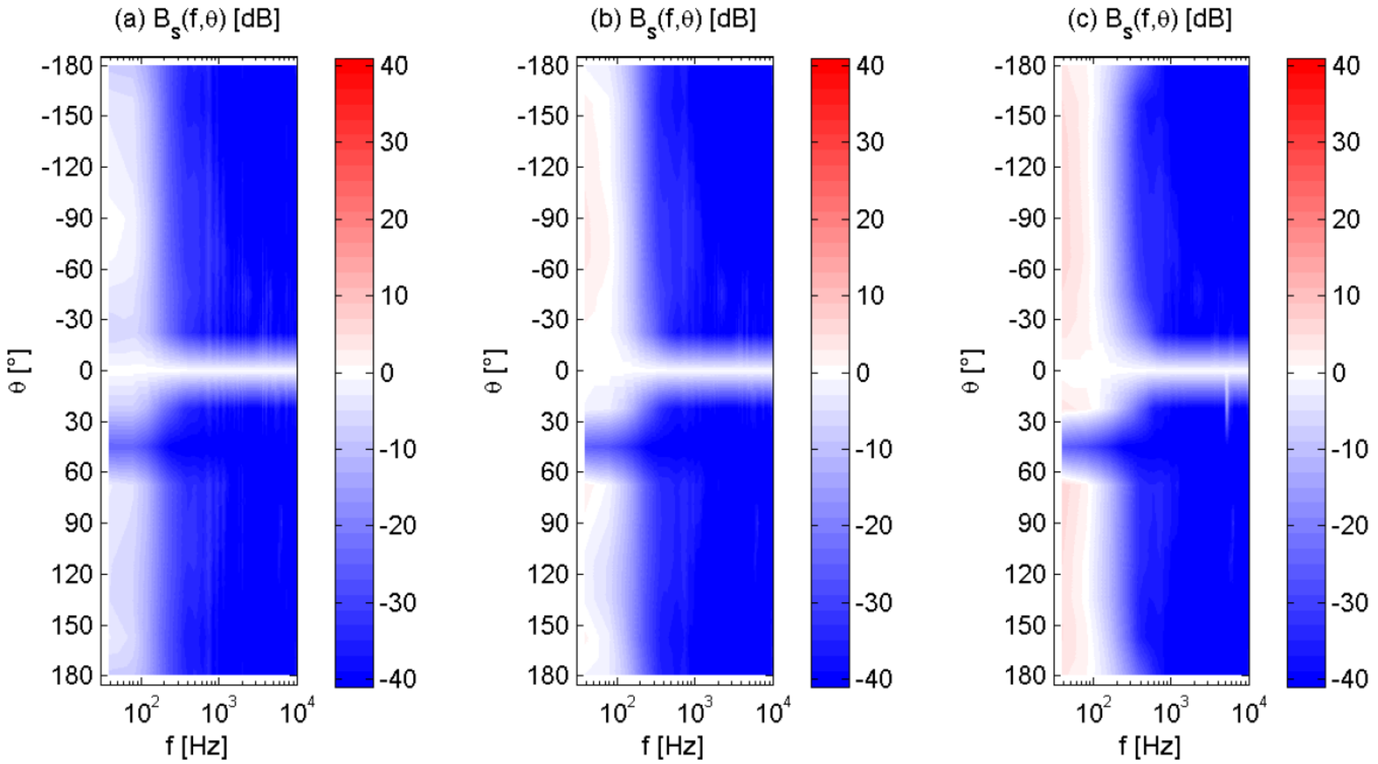
- AI-weighted Beampattern

Limitation of white noise gain



LCMV Beamformer with 8 mics in glasses

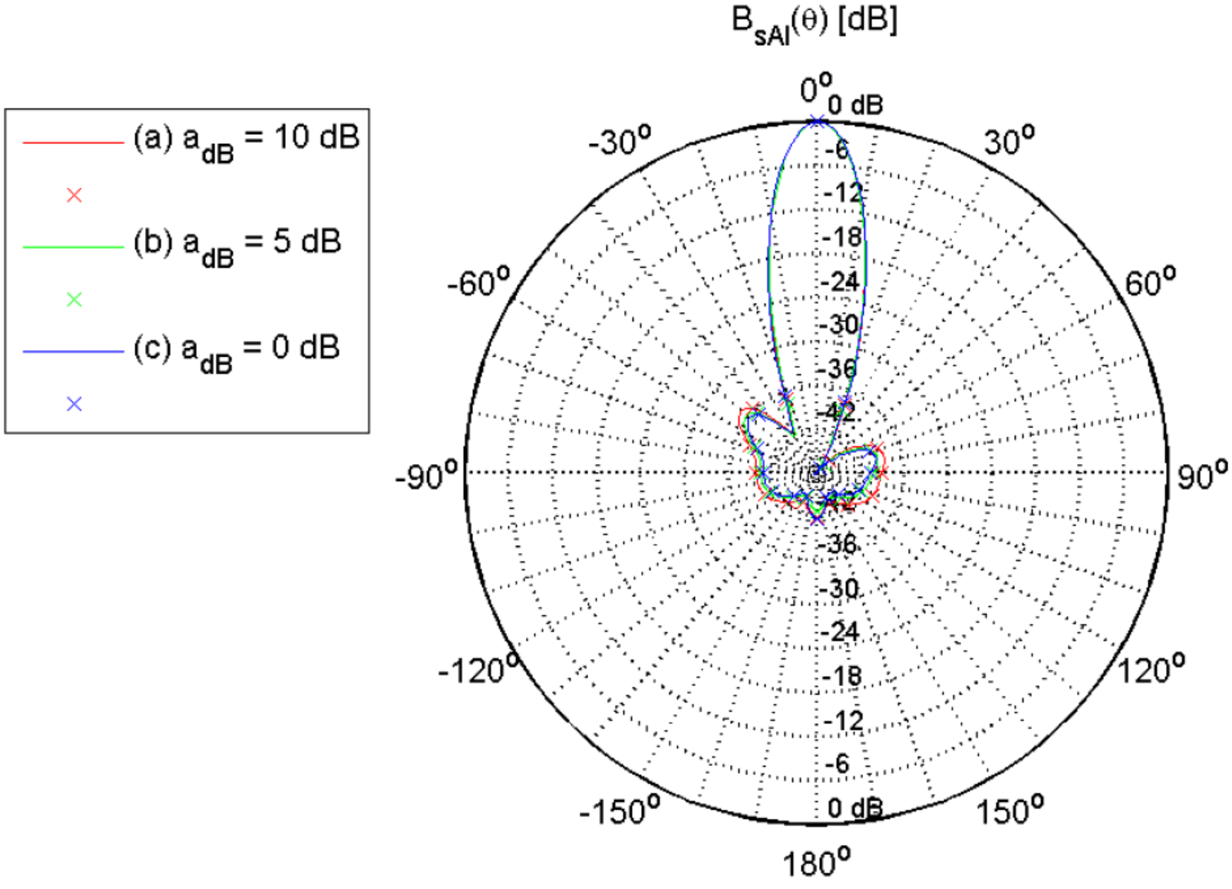
- Beampattern for 10, 5, and 0 dB white noise gain limitation (left to right)



sAIDI = 39.2 dB　　sAIDI = 40.1 dB　　sAIDI = 39.8 dB

LCMV Beamformer with 8 mics in glasses

- AI-weighted Beampattern



Interference cancellation:

The Generalized Sidelobe Canceller:
GSC beamformer
An alternative for an constrained adaptive beamformer

Interference compensation

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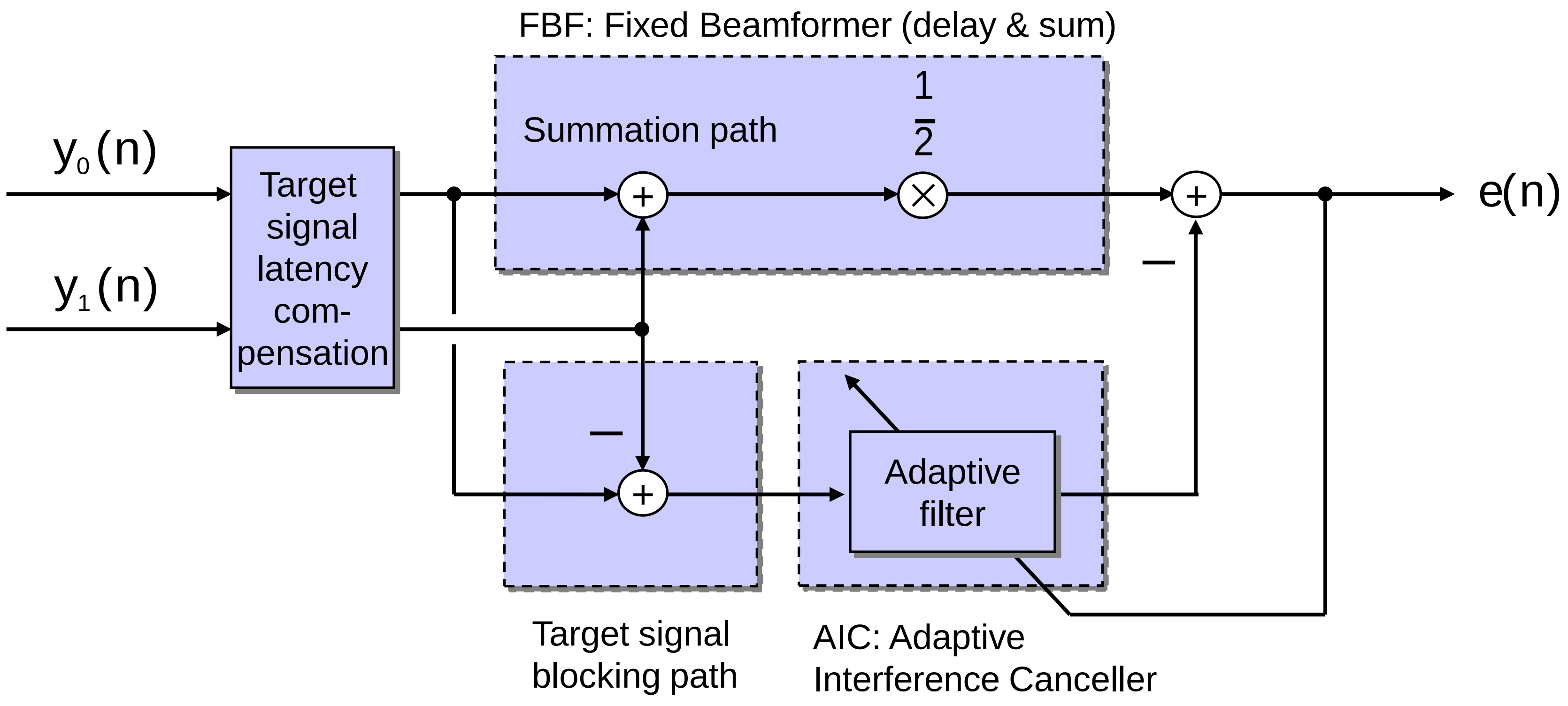
Basic principle:

- For the design of filter-and-sum beamformer or MVDR beamformer an assumption of the noise field is necessary or the noise field characteristic needs to be known. For the multi-channel Wiener filter, the noise auto-correlation matrix must be estimated.
- Target: Use another kind of adaptive procedure than the LCMV approach to minimize the interference signal power.
- A direct minimization of the beamformer output signal power would lead to the „null“ solution.
- Therefore, a constrained adaptation comparable to the MVDR approach has to be applied also for the adaptive solution.
- The constraint should be considered in the filter structure by target signal „blocking“.
- The task of the target signal blocking is to remove the target signal components, but all interfering components. Based on the „noise only“ signal, a signal minimization approach is possible without constraints.

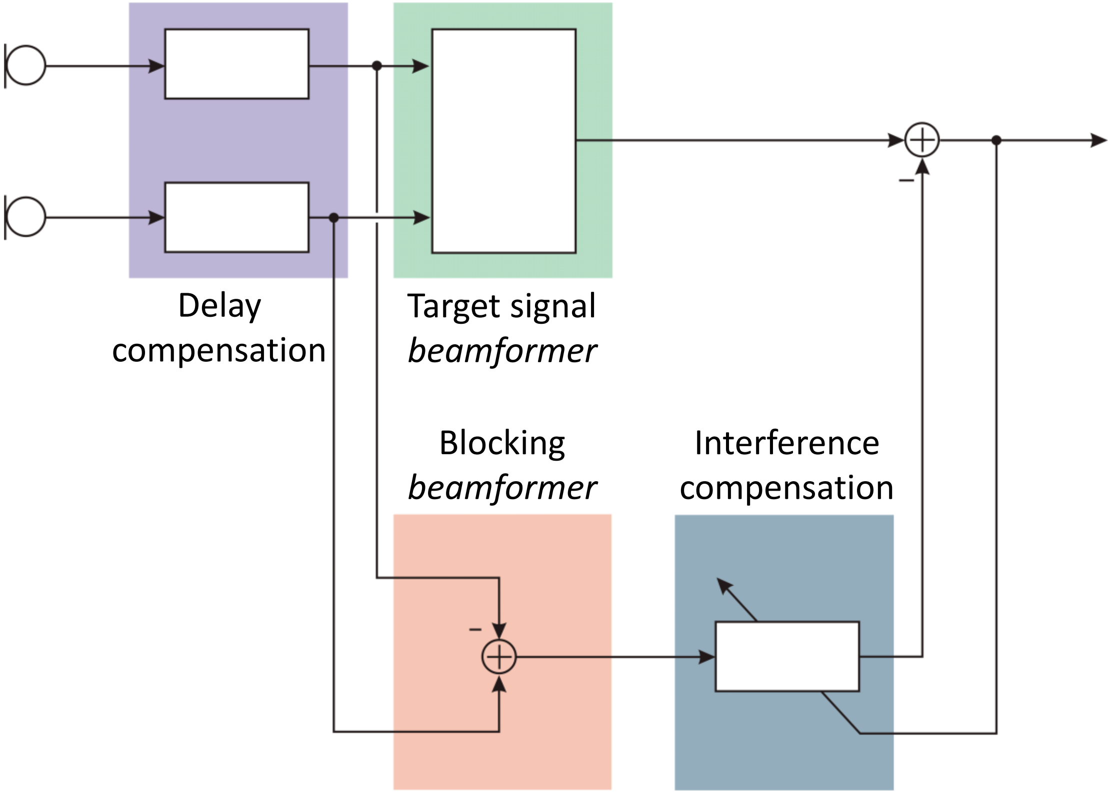
Interference compensation

- **Griffith-Jim Beamformer:**
  Very simple approach with a delay-and-sum beamformer in the signal path:



- **The generalized sidelobe canceller (GSC):**

  More general structure with an arbitrary target signal beamformer and a fixed blocking beamformer:



Interference compensation

Fixed blocking beamformer:

Advantages:

- Very simple and computationally efficient structure.
- Also, a fixed filtering (instead of a subtraction only) can be applied to have a broader angular range for the target signal suppression.

Disadvantages:

- In case of a non-optimum target signal blocking (non-frontal target signal, non-optimal delay compensation, etc.) some target signal components remain in the noise reference path and may lead to target signal cancellation and distortion.

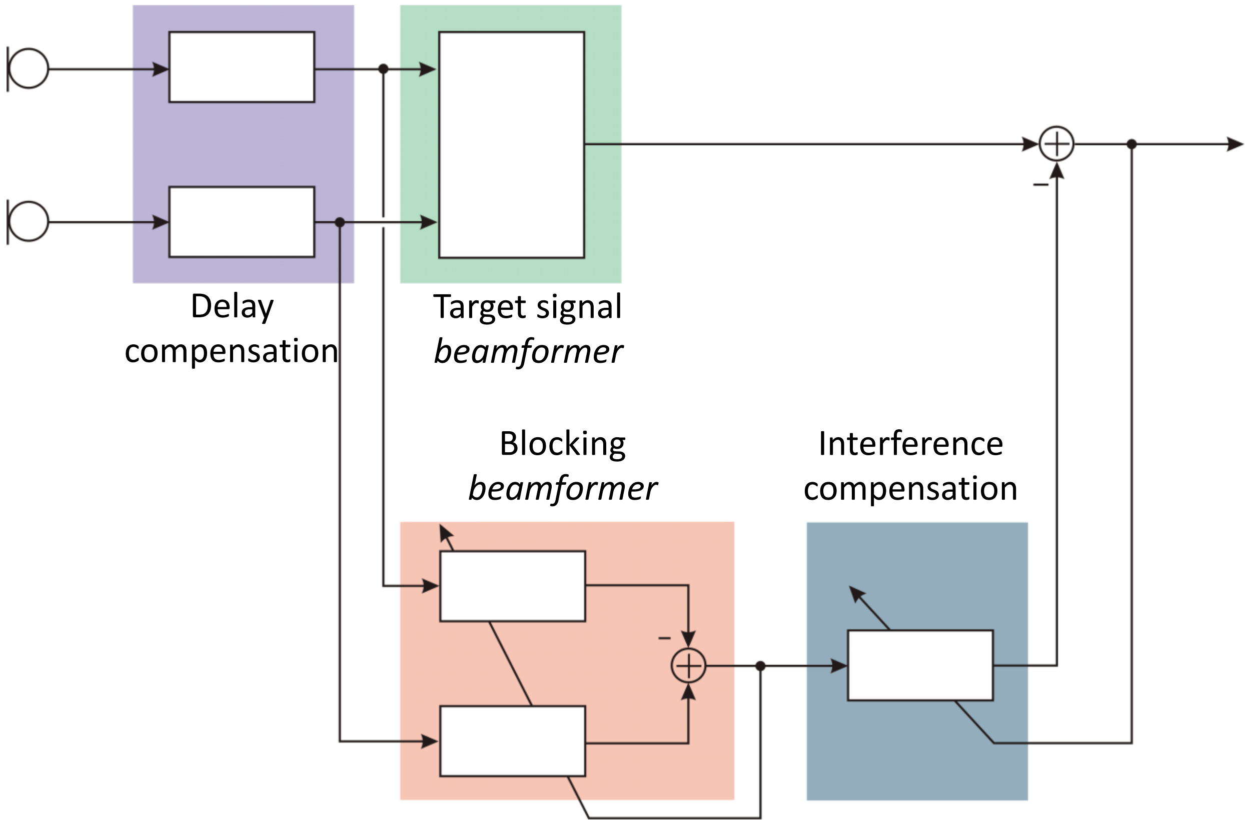
- Reverberation components are neither cancelled by the blocking structure. Therefore, they are removed by the interference canceller. This may be desired for late reverberation but not for early reverberation components.

Conclusion:

- The fixed blocking structure is mostly only utilized for a „pre-classification“ of the current acoustic situation. Following, adaptive more sophisticated structures are used.

Interference compensation

More general structure with an arbitrary target signal beamformer and an adaptive blocking beamformer:



Interference compensation

Adaptive blocking beamformer:

Advantages:

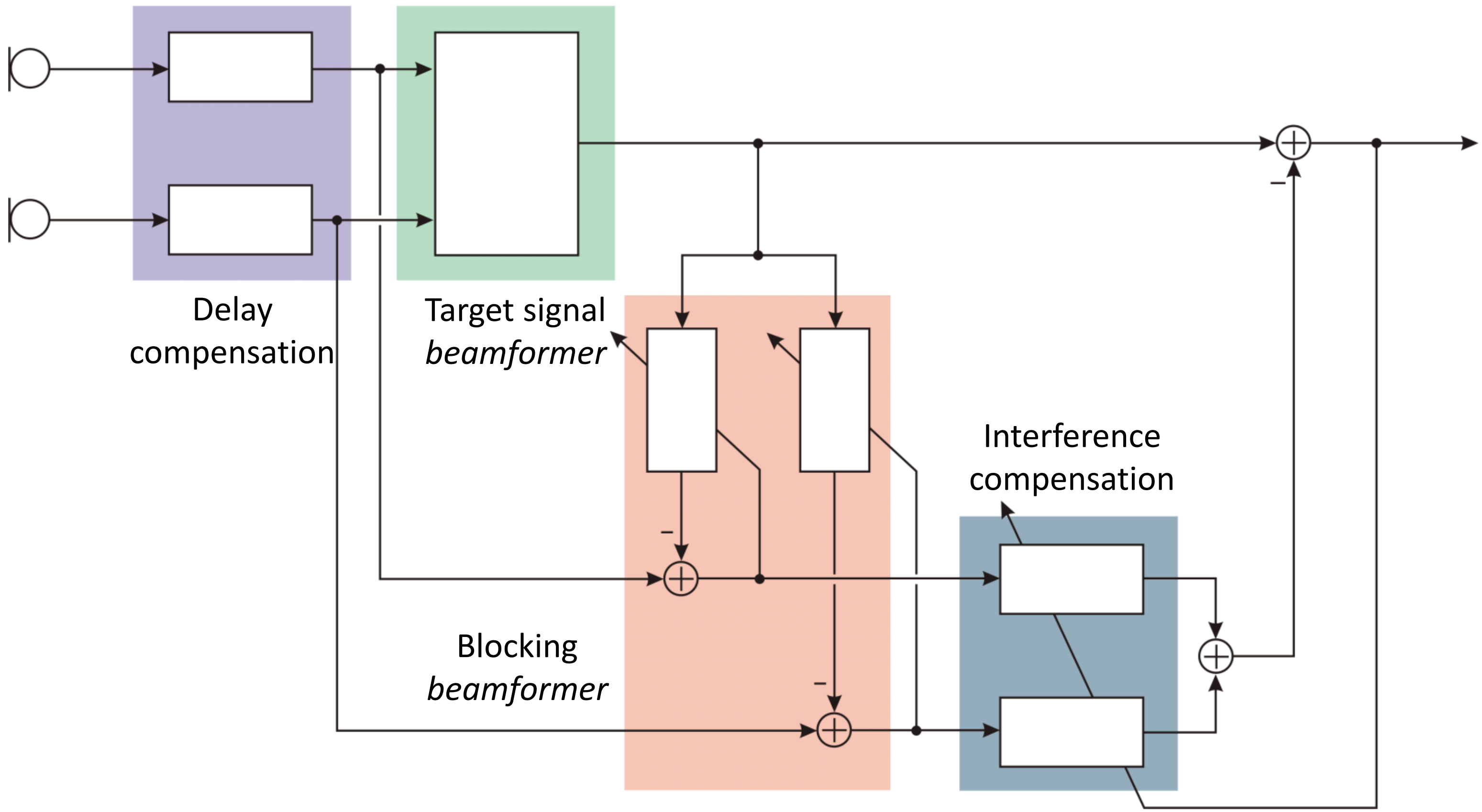
- Errors of the compensation of the target signal propagation delay can be reduced (assuming a proper situation classification, e.g., detection of target signal activity).
- Reverberation components can be removed from the noise reference which reduces the target signal distortion at the output.
- The structure can be utilized for target signal localization.

Disadvantages:

- A **constrained adaptation** is necessary for the blocking beamformer filter (e.g., the sum of the filter norm must be kept constant in order to avoid the „zero“ solution).
- A robust control of the adaptive filters is necessary.

Interference compensation

Blocking beamformer as adaptive difference system between the microphone signals and the beamformer output:



Interference compensation

Blocking beamformer as adaptive difference system between the microphone signals and the beamformer output:

Advantages:

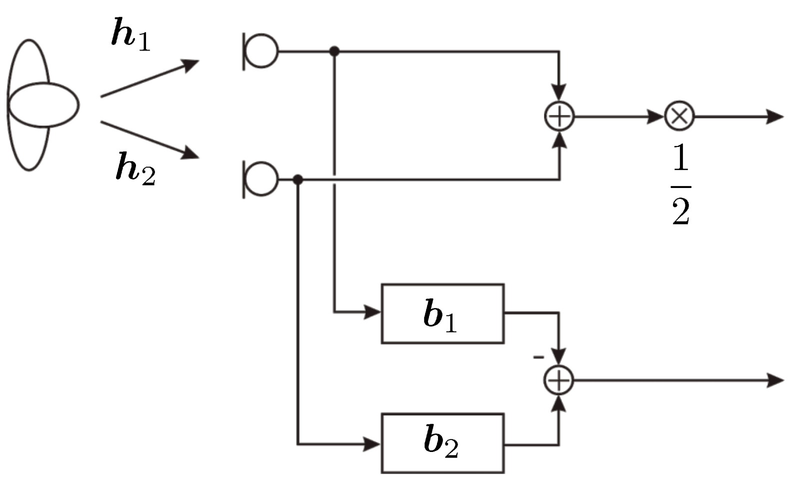
- Reverberation components can be removed from the noise reference which reduces the target signal distortion at the output.
- The reference signal of the target speaker (beamformer output) has a better signal-to-noise ratio than the microphone signals.
- Only one input signal of the adaptive blocking filter (reduced memory requirements).

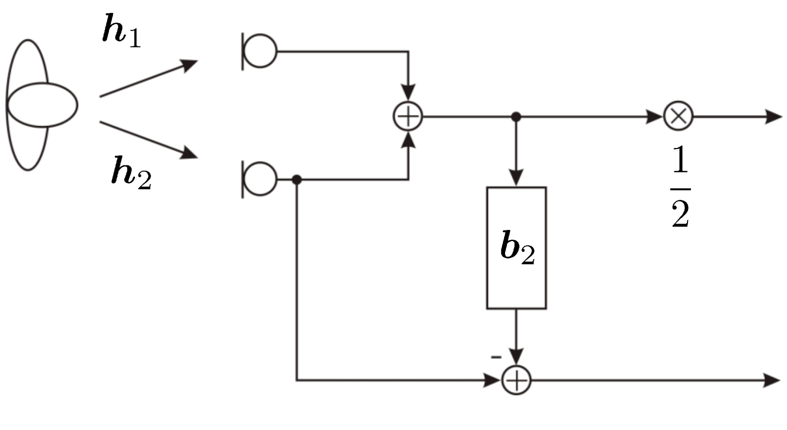
Disadvantages:

- Typically, more filter coefficients are necessary for the inversion of the room impulse response => see next slide.
- Even though the computational complexity is higher (more coefficients) this solution is typically the preferred one, since no constrained adaptation is necessary.
- A robust control is also necessary to ensure that the target signal components are well removed by the blocking matrix.

Interference compensation

Differences of the blocking structures:





$$H_1(e^{j\Omega})\,B_1(e^{j\Omega}) = H_2(e^{j\Omega})\,B_2(e^{j\Omega})$$

$$\Longrightarrow \quad \begin{aligned} B_1(e^{j\Omega}) &= H_2(e^{j\Omega})\,C(e^{j\Omega}) \\ B_2(e^{j\Omega}) &= H_1(e^{j\Omega})\,C(e^{j\Omega}) \end{aligned}$$

$$H_2(e^{j\Omega}) = \left[H_1(e^{j\Omega}) + H_2(e^{j\Omega})\right] B_2(e^{j\Omega})$$

$$\Longrightarrow \quad B_2(e^{j\Omega}) = \frac{H_2(e^{j\Omega})}{H_1(e^{j\Omega}) + H_2(e^{j\Omega})}$$

An inversion of the impulse response has to be modelled by an adaptive FIR filter (long FIR impulse response necessary)

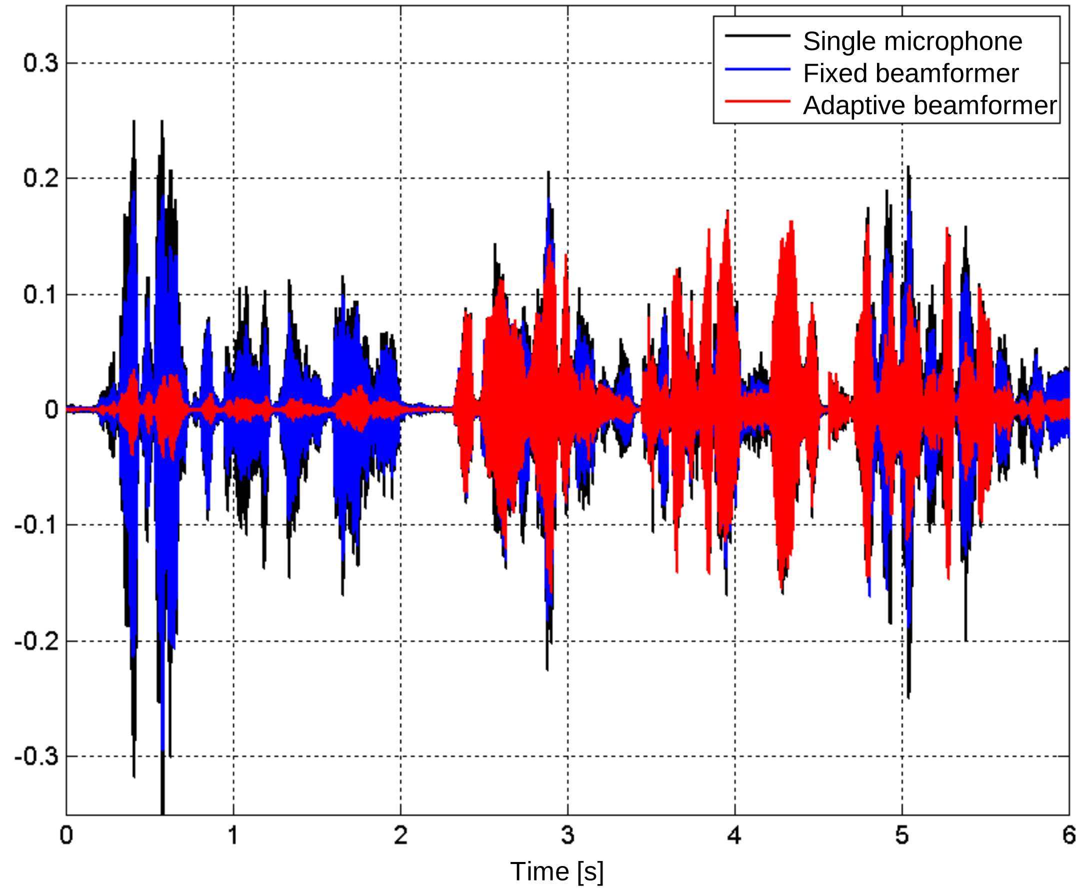
Audio examples

- 4 channel *array*
- Point noise source (one car loudspeaker)
- Noise suppression > 15 dB by adaptive filtering of the mirco-phone signals

Single microphone

Fixed *beamformer*

Adaptive *beamformer*



Comparing different beamformer approaches

Name	Properties	Critical issues
Differential Beamformer	- Endfire setup - Limited microphone distance - Adaptively steers direction of noise cancellation	- Microphone noise amplification - Microphone matching
Delay-and-sum beamformer	- Broadside setup - Fixed setting beamformer - Simple structure	- Limited SNR enhancement especially for low frequencies
MVDR beamformer	- Broadside setup - Fixed setting - No target signal distortion	- No inherent adaptation - Spatial noise field has to be known
Multi-channel Wiener Filter	- Adaptive procedure - Combined MVDR beam-former and 1-ch Wiener filter	- Voice activity detection necessary - Only stationary noise interference cancellation possible
LCMV beamformer	- Fixed and adaptive design - Several constraints possible	- Additional WNG (white noise gain) control necessary
GSC	- Adaptive procedure - Inherent target signal preservation constraint	- Computationally demanding - Sophisticated adaptive filter control necessary

References on MVDR, LCMV and GSC beamformers

[1] M. Souden, J. Benesty, S. Affes,
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IEEE Trans. Signal Process., vol. 58, no. 9, pp. 4925-4935, Sep. 2010

[2] H. Cox, R. M. Zeskind, and M. M. Owen. Robust adaptive beamforming. *IEEE Transactions on Acoustics, Speech, and Signal Processing*, 35(10): 1365–1376, October 1987

[3] Benesty J, Souden M, Chen J: A perspective on multichannel noise reduction in the time domain. *Appl. Acoustics* 2013, 74(3):343-355. 10.1016

[4] E. A. P. Habets, J. Benesty, I. Cohen, S. Gannot and J. Dmochowski, "New Insights Into the MVDR Beamformer in Room Acoustics," in *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 18, no. 1, pp. 158-170, Jan. 2010

[5] O. Hoshuyama, A. Sugiyama, and A. Hirano, "A robust adaptive beamformer for microphone arrays with a blocking matrix using constrained adaptive filters," in IEEE Transactions on Signal Processing, vol. 47, no. 10, October 1999, pp. 2677–2684.

[6] W. Herbordt and W. Kellermann, "Efficient frequency-domain realization of robust generalized sidelobe cancellers," in IEEE fourth Workshop on Multimedia Signal Processing, October 2001, pp. 377–382.

Summary

Summary

- Filter & sum beamformer: MVDR beamformer
- Multi-channel Wiener filter
- Relation between MVDR beamformer and Multi-channel Wiener filter
- LCMV Beamformer including adaptation
- Generalized sidelobe canceller

Next week:

- Fundamental frequency and cepstral based processing