Digital Signal Processing
Tutorial 3

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Task 1: Designing a DTMF System

Dual-tone multi-frequency (DTMF) signaling is used for telephone signaling over the line in the voice-frequency band to the call switching center. Basically, the sound you hear when dialing a number on the pad has two frequency components characterizing it which can be described by

$$x(n) = \sin\left(2\pi \frac{f_i}{f_s} n\right) + \sin\left(2\pi \frac{f_j}{f_s} n\right)$$

with $i = 1, \ldots, 4$ and $j = 5, \ldots, 7$.
The audio signal you send is decoded at the receiver side to get the information which numbers have been dialed. The DTMF keypad frequencies are given in the following scheme.

	$f_5 = 1209$ Hz	$f_6 = 1336$ Hz	$f_7 = 1477$ Hz
$f_1 = 697$ Hz	1	2	3
$f_2 = 770$ Hz	4	5	6
$f_3 = 852$ Hz	7	8	9
$f_4 = 951$ Hz	$*$	0	$\#$

a) One possibility to decode the signal is to use a bandpass FIR filter bank at the receiver side. Sketch a system diagram of the filter bank needed for this task.

b) The following specifications are given for the filter design:

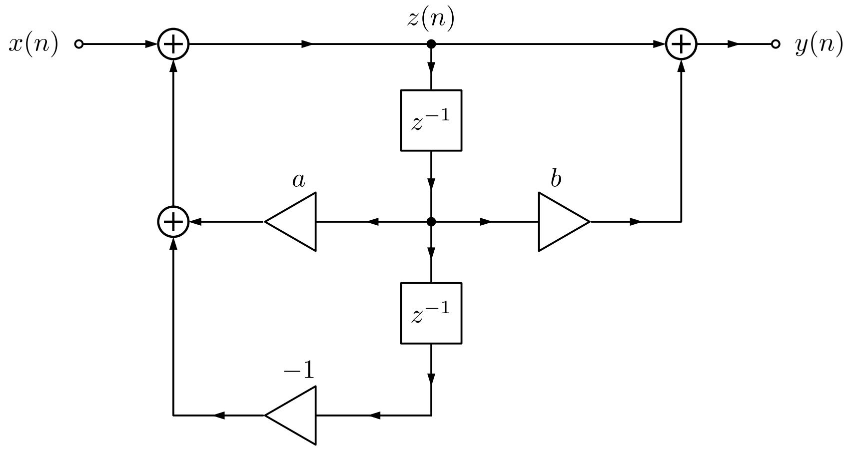
- center frequency: f_m, $m = 1, ..., 7$
- sampling frequency: 8 kHz
- passband bandwidth: 30 Hz
- transition bandwidth: 20 Hz
- passband ripple: ≤ 1 dB
- stopband attenuation: ≥ 20 dB

Draw the magnitude frequency response of an ideal filter and give an expression for $|H_d(e^{j\omega})|$ - used to detect the presence of frequency f_m.

c) Design all bandpass FIR filters in MATLAB using an FIR filter with Kaiser's window.

d) The maximal signal duration of a DTMF signal is specified to be $T_{max} = 70\text{ms}$. State the main drawbacks in terms of the filter order for using the designed FIR filter bank.

Consider the filter structure given below which implements the Goertzel algorithm. It is used in practice to decode DTMF signals. The filter coefficients are $a = 2\cos(2\pi k/N)$ and $b = -\exp(j2\pi k/N)$ where $k = f_m/f_s \cdot N$ is the DFT frequency index, and N is the signal length. Note that the depicted filter effectively calculates the DFT value of frequency k for the input signal $x(n)$, so that $X_N(e^{j\frac{2\pi}{N}k}) = y(N)$.



e) Determine the transfer function $H(z) = \frac{Y(z)}{X(z)}$.

Download the DTMF signal *dtmf_signal.mat* from the web page.

f) Show all filter outputs for this signal using the designed FIR filters. **Hint:** Export your designed filters to the workspace and use the function *filter*.

g) Implement the Goertzel algorithm for all frequencies. **Hint:** An individual filter structure has to be designed for each DTMF keypad frequency. Use your result from part (f) and the function *filter*.

h) Which key is encoded in the signal.

Task 2: Stochastic Process and Stationarity

Given the stochastic process

$$X(n) = \cos(\omega_0 n + \phi)$$

where ω_0 is a fixed frequency and ϕ is a uniformly distributed phase on $[-\pi, \pi)$, find

a) the expected value of $X(n)$ and

b) the covariance function $c_{XX}(n_1, n_2)$. Is $X(n)$ a wide-sense stationary process?

Now consider the stochastic process

$$Z(n) = X(n) + V(n)$$

where $X(n)$ is the same process as given above and $V(n)$ is white noise with variance σ^2, and independent of $X(n)$.

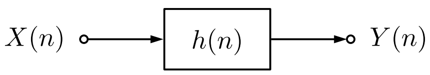
c) Calculate the expected value of $Z(n)$.

d) Calculate the covariance function $c_{ZZ}(n_1, n_2)$. Is $Z(n)$ a wide-sense stationary process?

e) Calculate the spectrum $C_{ZZ}(e^{j\omega})$.

Task 3: Spectrum of a Stochastic Process

Consider the linear time-invariant (LTI) system of as shown below, where $X(n)$ is a white process satisfying $X(n) \sim \mathcal{N}(0, 1),\ \forall n \in \mathbb{Z}$, and $Y(n)$ is given by $Y(n) = X(n) + X(n-1), \quad n \in \mathbb{Z}$.

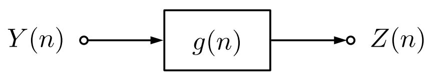


a) Find the transfer function of the LTI system, $H(e^{j\omega})$.

b) Find the covariance function $c_{YY}(\kappa),\ \kappa \in \mathbb{Z}$.

c) Show that $C_{YY}(e^{j\omega}) = \left(1 + e^{j\omega}\right)\left(1 + e^{-j\omega}\right)$.

The process $Y(n)$ is now passed through a LTI system with transfer function $G(e^{j\omega})$ as shown in the second figure where $Z(n)$ is given by $Z(n) = Y(n) - \frac{1}{2}Y(n-1), \quad n \in \mathbb{Z}$.



d) Give a closed-form expression for $C_{ZZ}(e^{j\omega})$ using the result of part (c).