Formative Assessment 5

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Problem 8.18

List all samples of size *n*=2 that are possible (with replacement) from the population in Problem 8.17.

The population consists of the credit hours X={9,12,15}, and each has a probability p(x)=1/3.

Since we are sampling with replacement, the possible pairs of X are:

{(9,9),(9,12),(9,15),(12,9),(12,12),(12,15),(15,9),(15,12),(15,15)}

Use R to plot the sampling distribution of the mean to show that $\mu_{\bar{x}} = \mu$,and show that $\sigma^2_{\bar{x}} = \sigma^2/2$.

```
X <- c(9, 12, 15)
p <- rep(1/3, 3)

samples <- expand.grid(X1=X, X2=X)

samplemeans <- rowMeans(samples)

sampling_table <- data.frame(Sample_1 = samples$X1, Sample_2 = samples$X2, Sample_Mean = samplemeans)

print(sampling_table)
```

```
##   Sample_1 Sample_2 Sample_Mean
## 1        9        9         9.0
## 2       12        9        10.5
## 3       15        9        12.0
## 4        9       12        10.5
## 5       12       12        12.0
## 6       15       12        13.5
## 7        9       15        12.0
## 8       12       15        13.5
## 9       15       15        15.0
```

```
meansamplemean <- mean(samplemeans)

populationmean <- sum(X * p)

varsamplemean <- var(samplemeans)

populationvariance <- sum((X^2 * p)) - populationmean^2

expectedvarsamplemean <- populationvariance / 2

list(
  samples = samples,
  samplemeans = samplemeans,
  meansamplemean = meansamplemean,
  populationmean = populationmean,
  varsamplemean = varsamplemean,
  expectedvarsamplemean = expectedvarsamplemean
)
```

```
## $samples
##   X1 X2
## 1  9  9
## 2 12  9
## 3 15  9
## 4  9 12
## 5 12 12
## 6 15 12
## 7  9 15
## 8 12 15
## 9 15 15
##
## $samplemeans
## [1]  9.0 10.5 12.0 10.5 12.0 13.5 12.0 13.5 15.0
##
## $meansamplemean
## [1] 12
##
## $populationmean
## [1] 12
##
## $varsamplemean
## [1] 3.375
##
## $expectedvarsamplemean
## [1] 3
```

Problem 8.21

A population consists of the four numbers 3, 7, 11, and 15. Consider all possible samples of size 2 that can be drawn with replacement from this population. Find:

a. the population mean

b. the population standard deviation

c. the mean of the sampling distribution of means

```
samples <- expand.grid(population, population)
samplemeans <- rowMeans(samples)
samplingdistmean <- mean(samplemeans)
samplingdistmean
```

```
## [1] 9
```

d. the standard deviation of the sampling distribution of means.

```
samplingdistsd <- sd(samplemeans)
samplingdistsd
```

```
## [1] 3.265986
```

Verify parts (c) and (d) directly from (a) and (b) by using suitable formulas. Using the formulas:

- Mean of the sampling distribution = Population mean
- Standard deviation of the sampling distribution = Population standard deviation divided by the square root of the sample size

```
verifysd <- popsd / sqrt(2)
verifysd
```

```
## [1] 3.651484
```

Problem 8.34

Find the probability that of the next 200 children born. Assume equal probabilities for the births of boys and girls.

Given: * The probability of a boy being born, p = 0.5 * The number of trials, n = 200

a. less than 40% will be boys

- 40% of 200 children = 0.40×200=80 boys.
- We want to find P(X<80).

```
phata <- 0.40
za <- (phata - meanprop) / seprop
za
```

```
## [1] -2.828427
```

```
proba <- pnorm(za)
proba
```

```
## [1] 0.002338867
```

b. between 43% and 57% will be girls

- 43% and 57% of 200 children correspond to: 0.43×200=86 girls, or 114 boys.
- 57% of 200 = 0.57×200=114 girls, or 86 boys.
- We want to find P(86≤X≤114), where X is the number of girls.

```
phatb1 <- 1 - 0.57
phatb2 <- 1 - 0.43

zb1 <- (phatb1 - meanprop) / seprop
zb2 <- (phatb2 - meanprop) / seprop


probb <- pnorm(zb2) - pnorm(zb1)
probb
```

```
## [1] 0.9522851
```

c. more than 54% will be boys

- 54% of 200 children = 0.54×200=108 boys.
- We want to find P(X>108).

```
phatc <- 0.54
zc <- (phatc - meanprop) / seprop
zc
```

```
## [1] 1.131371
```

```
probc <- 1 - pnorm(zc)
probc
```

```
## [1] 0.1289495
```

Problem 8.49

The credit hour distribution at Metropolitan Technological College is as follows:

Find μ and σ^2. Give the 25 (with replacement) possible samples of size 2, their means, and their probabilities.

Mean:

```
x <- c(6, 9, 12, 15, 18)
px <- c(0.1, 0.2, 0.4, 0.2, 0.1)

mu <- sum(x * px)
mu
```

```
## [1] 12
```

σ^2:

```
sigma2 <- sum((x - mu)^2 * px)
sigma2
```

```
## [1] 10.8
```

```
samples <- expand.grid(x, x)
samplemeans <- rowMeans(samples)

sampleprobs <- outer(px, px, "*")
sampleprobs <- as.vector(sampleprobs)

results <- data.frame(Sample1 = samples[,1], Sample2 = samples[,2],
                      Mean = samplemeans, Probability = sampleprobs)


results
```

```
##    Sample1 Sample2 Mean Probability
## 1        6       6  6.0        0.01
## 2        9       6  7.5        0.02
## 3       12       6  9.0        0.04
## 4       15       6 10.5        0.02
## 5       18       6 12.0        0.01
## 6        6       9  7.5        0.02
## 7        9       9  9.0        0.04
## 8       12       9 10.5        0.08
## 9       15       9 12.0        0.04
## 10      18       9 13.5        0.02
## 11       6      12  9.0        0.04
## 12       9      12 10.5        0.08
## 13      12      12 12.0        0.16
## 14      15      12 13.5        0.08
## 15      18      12 15.0        0.04
## 16       6      15 10.5        0.02
## 17       9      15 12.0        0.04
## 18      12      15 13.5        0.08
## 19      15      15 15.0        0.04
## 20      18      15 16.5        0.02
## 21       6      18 12.0        0.01
## 22       9      18 13.5        0.02
## 23      12      18 15.0        0.04
## 24      15      18 16.5        0.02
## 25      18      18 18.0        0.01
```