Formative Assessment 4

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Question 5

A geospatial analysis system has four sensors supplying images. The percentage of images supplied by each sensor and the percentage of images relevant to a query are shown in the following table.

Sensor	Percentage of Images Supplied	Percentage of Relevant Images	Total
1	15	50	65
2	20	60	80
3	25	80	105
4	40	85	125
Total	100	275	375

What is the overall percentage of relevant images?

We define:

- Sensor 1 as S_1
- Sensor 2 as S_2
- Sensor 3 as S_3
- Sensor 4 as S_4

The proportion of images that are relevant to a query consists of the sum of the proportions of each sensor that are relevant images. In set notation, if E is the event that an image is relevant, then

$$E = (S_1 \cap E) \cup (S_2 \cap E) \cup (S_3 \cap E) \cup (S_4 \cap E)$$

Since $S_1 \cap E$, $S_2 \cap E$, $S_3 \cap E$, and $S_4 \cap E$ are mutually exclusive, then

$$P(E) = P(S_1 \cap E) + P(S_2 \cap E) + P(S_3 \cap E) + P(S_4 \cap E)$$

$$P(E) = P(S_1)P(E|S_1) + P(S_2)P(E|S_2) + P(S_3)P(E|S_3) + P(S_4)P(E|S_4)$$

$$P(E) = (\tfrac{65}{375})(\tfrac{50}{65}) + (\tfrac{80}{375})(\tfrac{60}{80}) + (\tfrac{105}{375})(\tfrac{80}{105}) + (\tfrac{125}{375})(\tfrac{85}{125})$$

P(E) = 0.7333333

Conclusion

The overall percentage of relevant images is 73.3333333%.

Question 6

A fair coin is tossed twice. Let E1 be the event that both tosses have the same outcome, that is, E1 = (HH, TT). Let E2 be the event that the first toss is a head, that is, E2 = (HH, HT). Let E3 be the event that the second toss is a head, that is, E3 = (TH, HH). Show that E1, E2, and E3 are pairwise independent but not mutually independent

Sample Space

E1: Both tosses have the same outcome (HH, TT).
E2: The first toss is a head (HH, HT).
E3: The second toss is a head (TH, HH).

Our goal is to show that E1,E2, and E3 are pairwise independent but not mutually independent.

```
##   toss1 toss2 outcome
## 1     H     H      HH
## 2     T     H      TH
## 3     H     T      HT
## 4     T     T      TT
```

Defining Events

Next, we define the events based on the sample space.

```
## [1] "HH" "TT"
```

```
## [1] "HH" "HT"
```

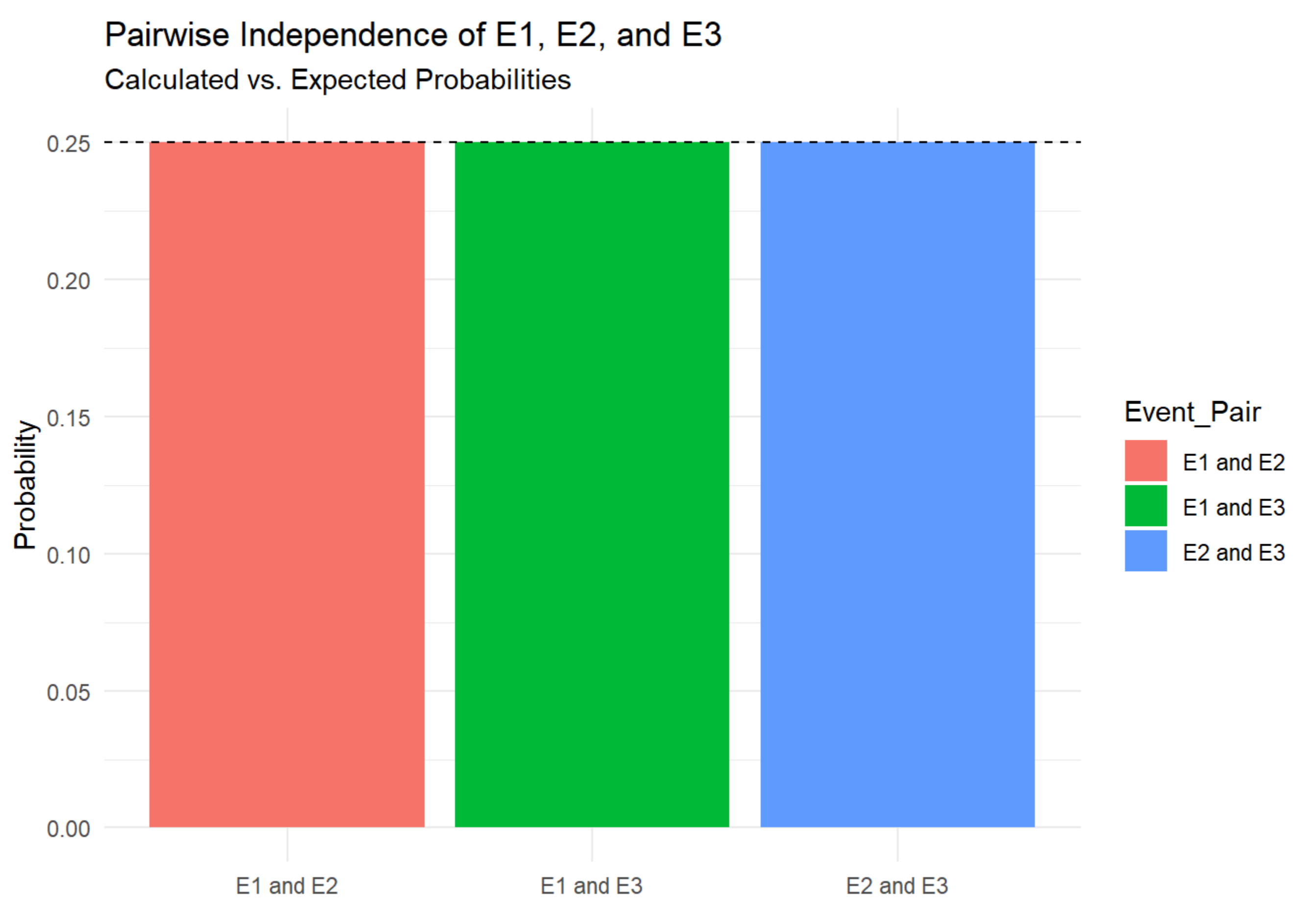
```
## [1] "TH" "HH"
```

Pairwise Independence

Events are pairwise independent if for any two events A and B, the probability of A and B occurring together is the product of their individual probabilities: $P(A \cap B)=P(A) \cdot P(B)$ Let's calculate these probabilities.

```
##   Event_Pair Calculated Expected
## 1  E1 and E2       0.25     0.25
## 2  E1 and E3       0.25     0.25
## 3  E2 and E3       0.25     0.25
```

Demographics// Presentation



Checking Mutual Independence

Events are mutually independent if the probability of all events occurring together equals the product of their individual probabilities.

```
##         Type Probability
## 1 Calculated       0.250
## 2   Expected       0.125
```

Conclusion

The probability calculations show that E1, E2, and E3 are, as the plot indicates, pairwise independent. When examining mutual independence, the three events are not mutually independent since the estimated likelihood of them happening together is not equal to the sum of their individual probabilities.