Formative Assessment 9

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2024-05-06

Problem 1

A random variable *X* is said to have the gamma distribution, or to be gamma distributed, if the density function is $f(x) = \begin{cases} \frac{x\alpha^{-1}e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)} & x > 0 \\ 0 & x \leq 0 \end{cases}$ $(\alpha, \beta > 0)$, where Γ(α) is the gamma function. Show that the mean and variance of the gamma distribution are given by (a) $\mu = \alpha\beta$, (b) $\sigma^2 = \alpha\beta^2$.

```
alpha <- 2
beta <- 3

meanofgamma<- alpha * beta

varofgamma <- alpha * beta^2

cat("Mean (μ) of the gamma distribution:", meanofgamma, "\n")
```

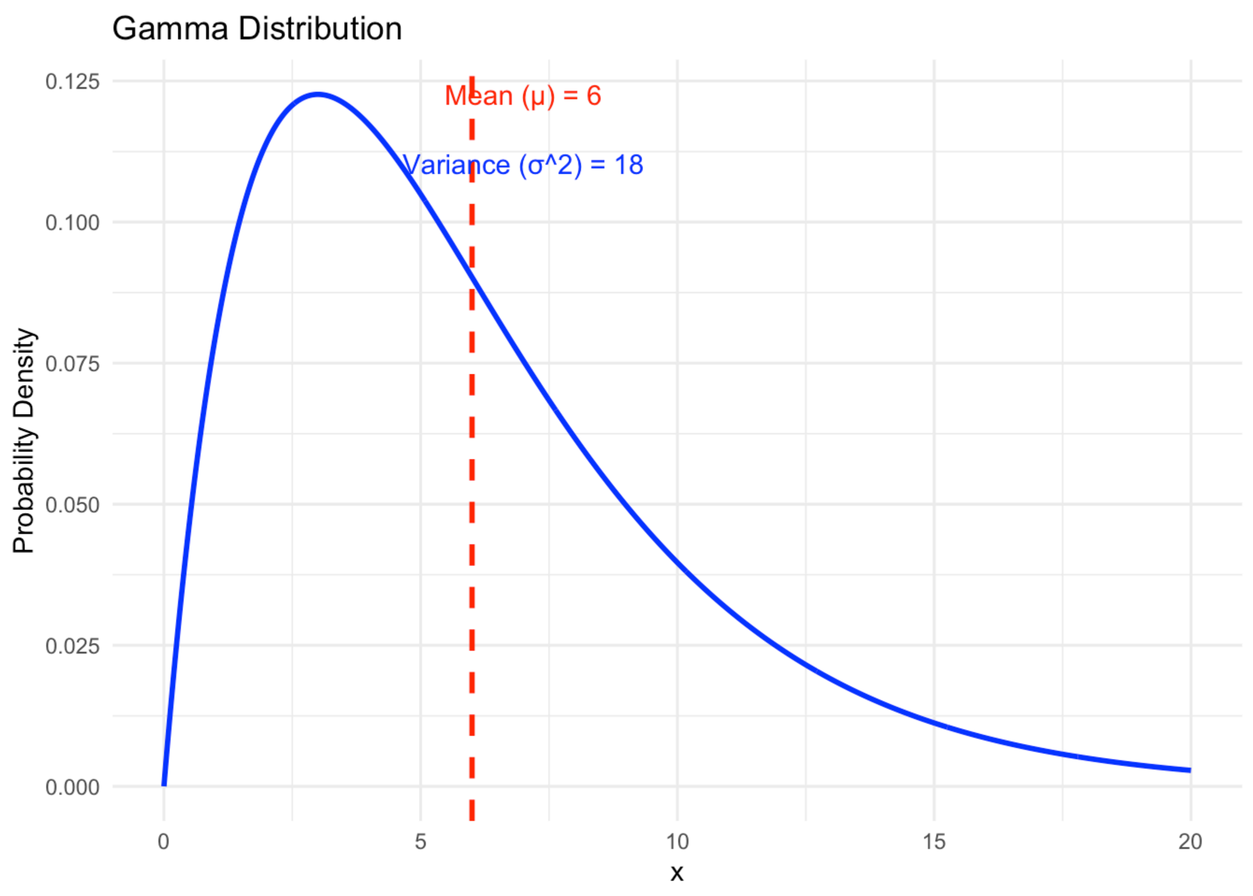
```
## Mean (μ) of the gamma distribution: 6
```

```
cat("Variance (σ^2) of the gamma distribution:", varofgamma, "\n")
```

```
## Variance (σ^2) of the gamma distribution: 18
```

```
library(ggplot2)

x <- seq(0, 20, length.out = 1000)

pdf <- dgamma(x, shape = alpha, scale = beta)

mean_gamma <- alpha * beta
var_gamma <- alpha * beta^2

df <- data.frame(x = x, pdf = pdf)

ggplot(df, aes(x)) +
  geom_line(aes(y = pdf), color = "blue", size = 1) +
  geom_vline(xintercept = mean_gamma, linetype = "dashed", color = "red", size = 1) +
  annotate("text", x = mean_gamma + 1, y = max(pdf), label = paste("Mean (μ) =", round(mean_gamma, 2)), color =
"red") +
  annotate("text", x = mean_gamma + 1, y = max(pdf) * 0.9, label = paste("Variance (σ^2) =", round(var_gamma,
2)), color = "blue") +
  labs(title = "Gamma Distribution", x = "x", y = "Probability Density") +
  theme_minimal()
```

```
## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## ℹ Please use `linewidth` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```



Problem 2

Prove that the mean and variance of a binomially distributed random variable are, respectively $\mu = np$ and and $\sigma^2 = npq$.

```
n <- 10
p <- 0.5

binomial_samples <- rbinom(10000, size = n, prob = p)

meanempirical <- mean(binomial_samples)
varempirical <- var(binomial_samples)

cat("Empirical Mean (μ) of the binomial distribution:", meanempirical, "\n")
```

```
## Empirical Mean (μ) of the binomial distribution: 4.9977
```

```
cat("Empirical Variance (σ^2) of the binomial distribution:", varempirical, "\n")
```

```
## Empirical Variance (σ^2) of the binomial distribution: 2.472142
```

```
meantheoretical <- n * p
vartheoretical <- n * p * (1 - p)

cat("Theoretical Mean (μ) of the binomial distribution:", meantheoretical, "\n")
```

```
## Theoretical Mean (μ) of the binomial distribution: 5
```

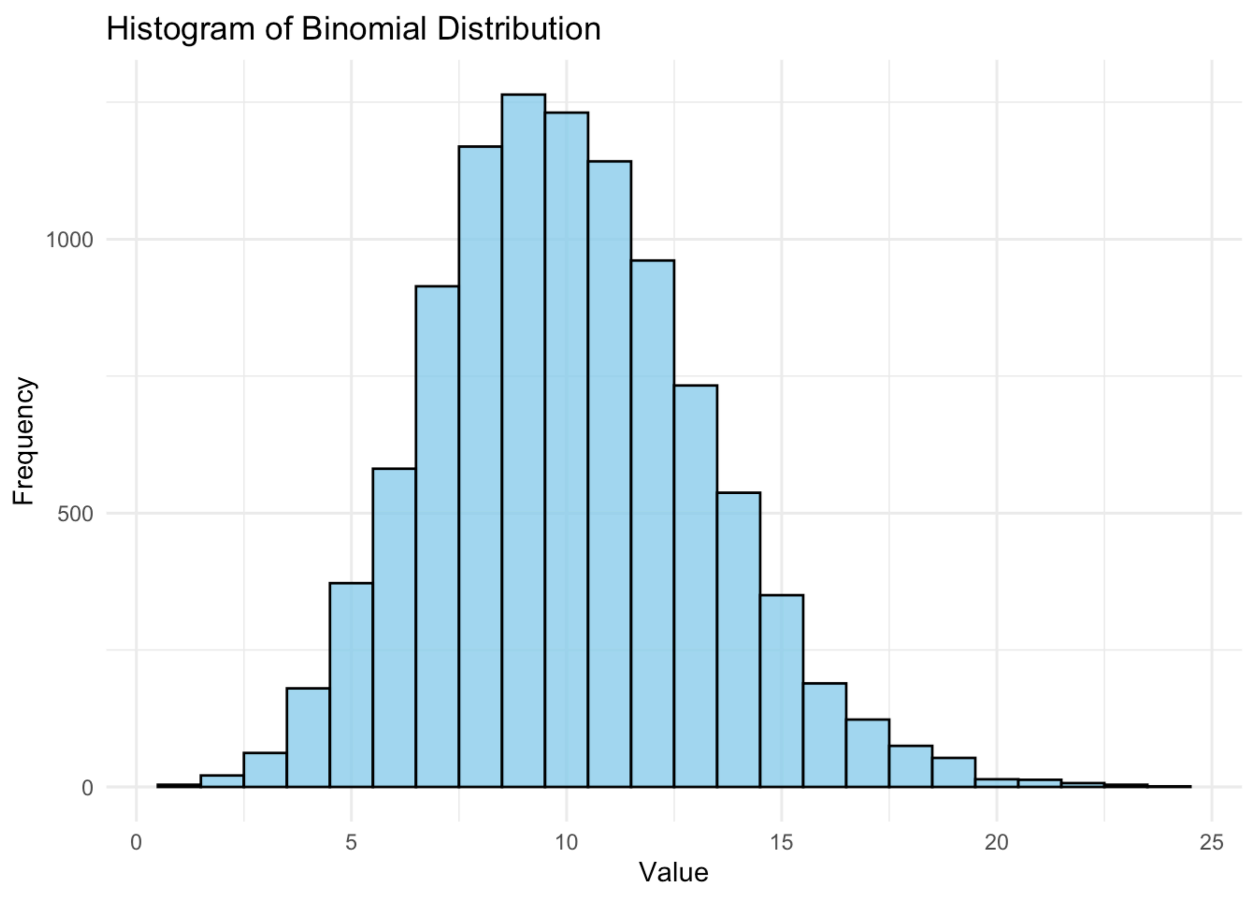
```
cat("Theoretical Variance (σ^2) of the binomial distribution:", vartheoretical, "\n")
```

```
## Theoretical Variance (σ^2) of the binomial distribution: 2.5
```

Problem 3

Establish the validity of the Poisson approximation to the binomial distribution.

```
library(ggplot2)

n <- 1000
p <- 0.01

# Generate data from binomial distribution
samplesofbinomial <- rbinom(10000, size = n, prob = p)

# Calculate the mean and variance of the binomial distribution
meanbinomial <- n * p
varbinomial <- n * p * (1 - p)

# Plot histogram of binomial samples
ggplot(data.frame(x = samplesofbinomial), aes(x)) +
  geom_histogram(binwidth = 1, fill = "skyblue", color = "black", alpha = 0.8) +
  ggtitle("Histogram of Binomial Distribution") +
  labs(x = "Value", y = "Frequency") +
  theme_minimal()
```




```
# Generate data from Poisson distribution with the same mean
lambda <- meanbinomial
poissonsamples <- rpois(10000, lambda)

# Plot histogram of Poisson samples
ggplot(data.frame(x = poissonsamples), aes(x)) +
  geom_histogram(binwidth = 1, fill = "salmon", color = "black", alpha = 0.8) +
  ggtitle("Histogram of Poisson Distribution") +
  labs(x = "Value", y = "Frequency") +
  theme_minimal()
```