

## Histogrammes et Lookup-tables

UPEM - Master 1

Vincent Nozick



## Histogramme

### Histogramme d'une image en niveaux de gris :

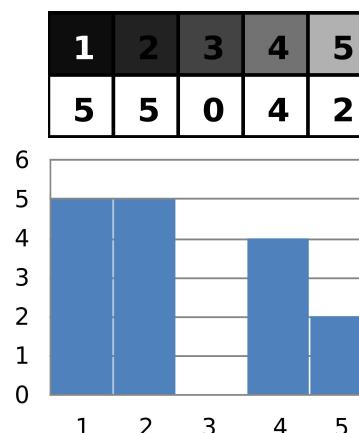
Fonction qui associe à chaque valeur d'intensité le nombre de pixels de l'image ayant cette valeur.

1	5	4	4
1	2	2	1
4	1	2	2
1	4	2	5

1	2	3	4	5
5	5	0	4	2

## Histogramme

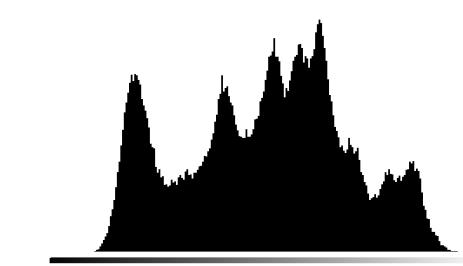
**Histogramme d'une image en niveaux de gris :**  
s'exprime aussi sous forme de graphique.



## Histogramme

### Histogramme d'une image :

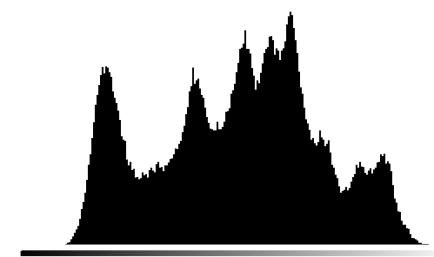
Fonction qui associe à chaque valeur d'intensité le nombre de pixels de l'image ayant cette valeur.



## Histogramme

### Histogramme d'une image :

- fournit des informations statistiques :
  - distribution statistique des niveaux de gris
  - bornes de répartition des niveaux de gris
- mais aucune information spatiale !

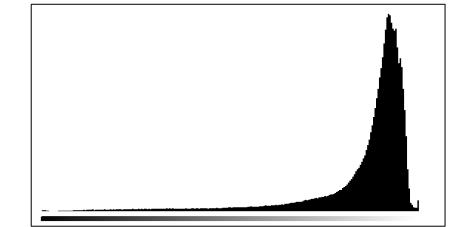


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Histogrammes et Lookup-tables

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## Exemples

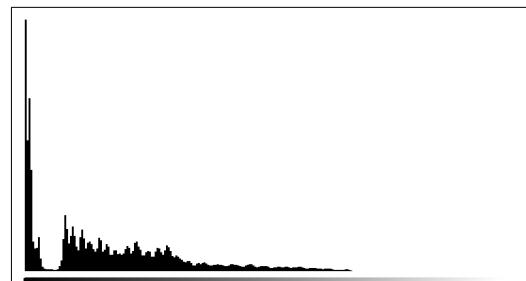


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## Exemples

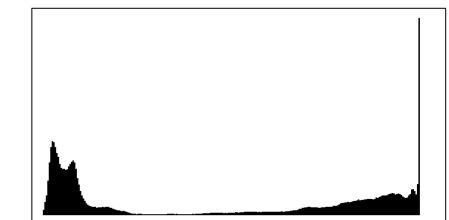


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## Exemples



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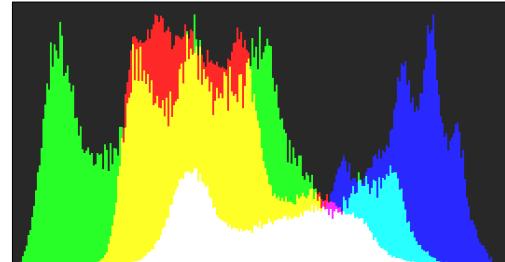
Histogrammes et Lookup-tables

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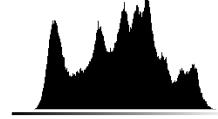
## Histogramme

### Histogramme couleur :

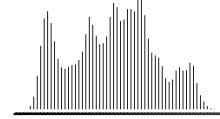
pour une image couleur, on traite chaque canal indépendamment.



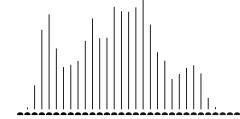
## Nombre de bins



256 bins



64 bins



32 bins

→ extension aux images encodées sur des nombres flottants.

## Histogramme

### Implémentation :

pour une image en niveaux de gris, codée sur 8 bits :

```
uint histogram[256];
histogram.fill(0);

for(x=0; x<width; ++x)
    for(y=0; y<height; ++y)
        histogram[I(x,y)]++;
```

### Remarque :

- pas simple à paralléliser sur GPU.

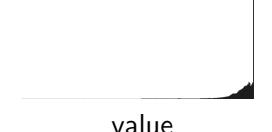
## Autres espaces colorimétriques



hue

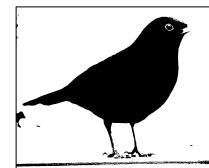
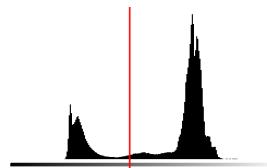


saturation



value

## Applications

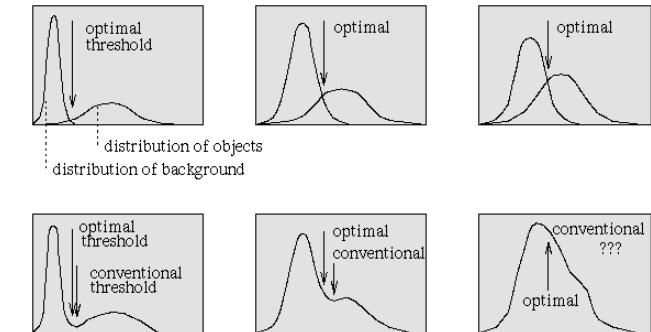


### Seuillage automatique :

- choisir la coupe qui minimise la variance des 2 groupes (méthode d'Otsu)
- ...

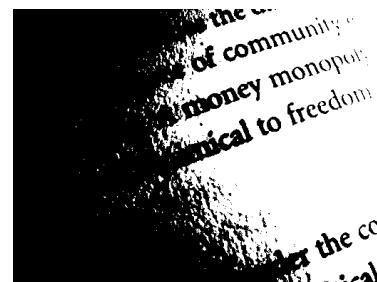
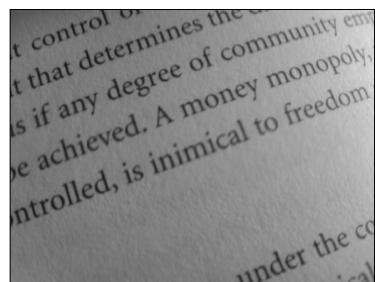
## Applications

### Seuillage adaptatif :



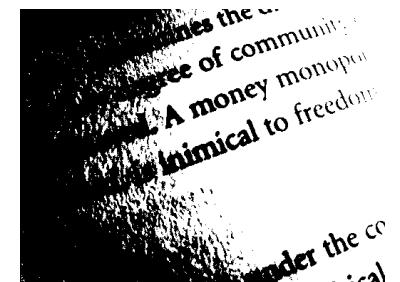
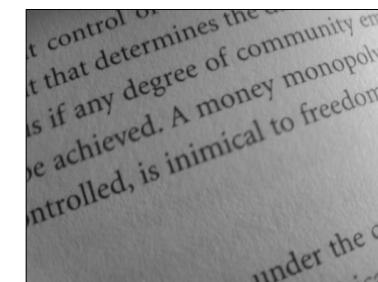
## Applications

### Seuillage adaptatif :



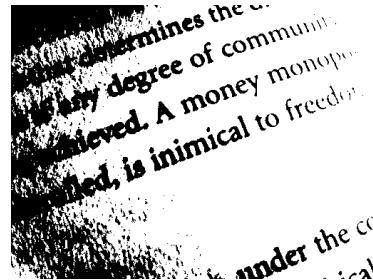
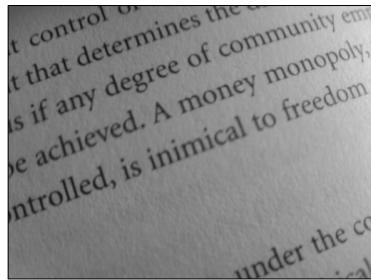
## Applications

### Seuillage adaptatif :



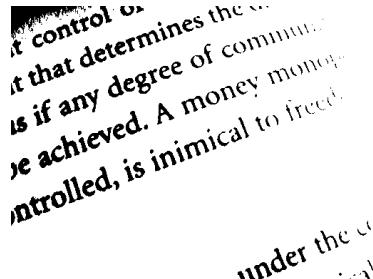
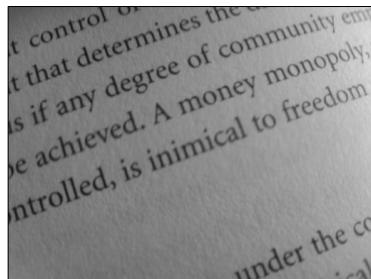
## Applications

Seuillage adaptatif :



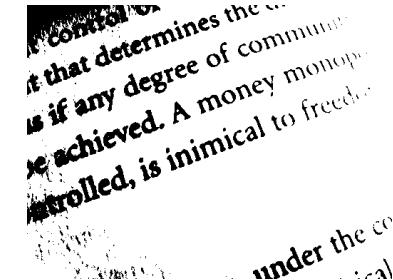
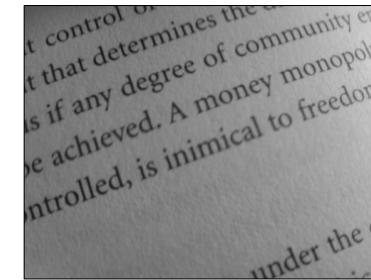
## Applications

Seuillage adaptatif :



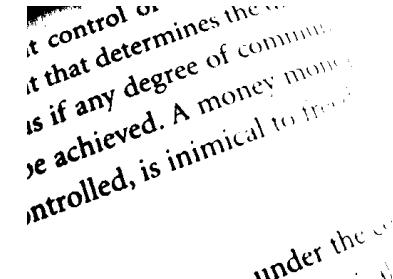
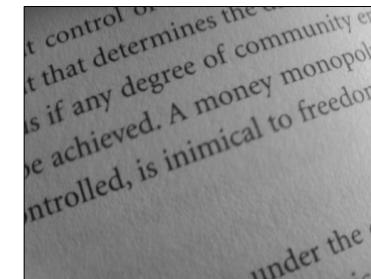
## Applications

Seuillage adaptatif :



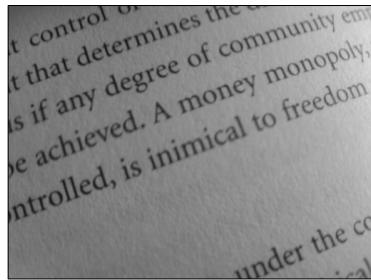
## Applications

Seuillage adaptatif :



## Applications

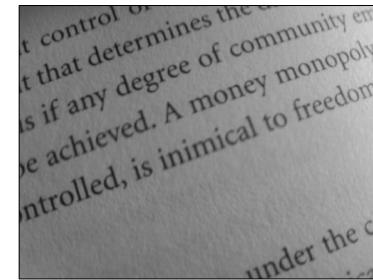
### Seuillage adaptatif :



at control or  
is if any degree of community em-  
be achieved. A money monopoly,  
ntrolled, is inimical to freedom  
under the co

## Applications

### Seuillage adaptatif :

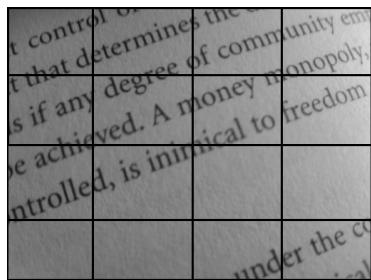


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## Applications

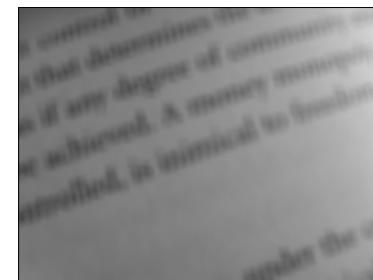
### Seuillage adaptatif : méthode d'Otsu locale



at control or  
is if any degree of community em-  
be achieved. A money monopoly,  
ntrolled, is inimical to freedom  
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## Applications

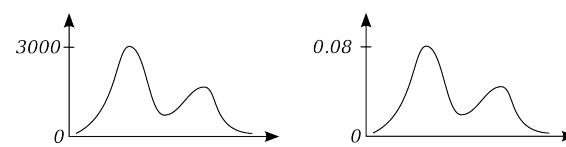
### Seuillage adaptatif : méthode utilisant du flou



at control or  
is if any degree of community em-  
be achieved. A money monopoly,  
ntrolled, is inimical to freedom  
under the co

seuillage du pixel selon que  $I(x, y) > \text{blur}(I(x, y)) - \alpha$

## Histogramme normalisé



### Définition :

on norme l'histogramme par le nombre de pixels dans l'image.

$$H_n(k) = \frac{H(k)}{w \times h}$$

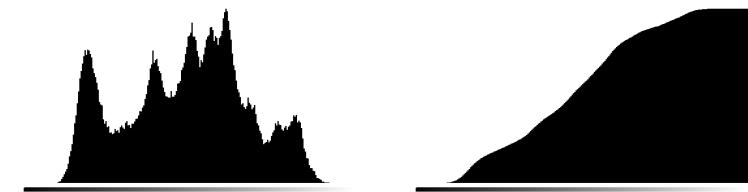
L'histogramme  $H_n(k)$  donne une information statistique sur la probabilité qu'un pixel ait un niveau de gris  $k$ .

## Histogramme cumulé

### Principe :

indique le nombre de pixels dont le niveau de gris est inférieur à  $x$  :

$$H_c(x) = \sum_{i=0}^x H(i)$$

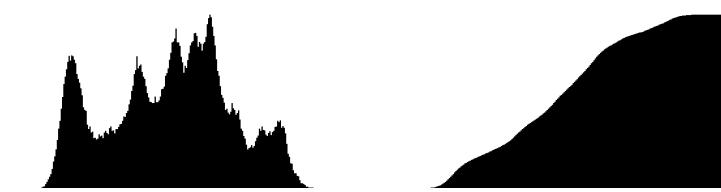


## Histogramme cumulé normalisé

### Principe :

indique le **taux** de pixels dont le niveau de gris est inférieur à  $x$  :

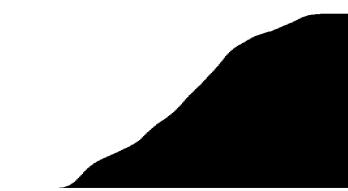
$$H_{cn}(x) = \frac{\sum_{i=0}^x H(i)}{w \times h}$$



## Histogramme cumulé normalisé

### Propriété :

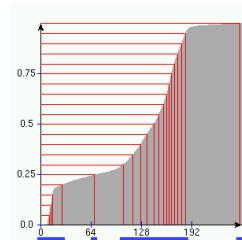
$$H_{cn}(x_m) = \frac{\sum_{i=0}^{x_m} H(i)}{w \times h}$$



le niveau de gris médian est  $x_m$  tel que  $H_{cn}(x_m) = 0.5$

## Applications

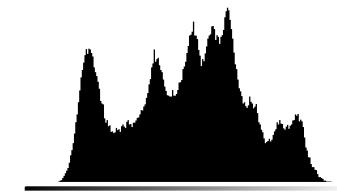
### Segmentation :



images : Antoine Manzanera

## Histogramme : invariances

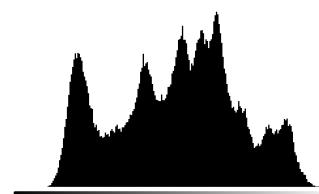
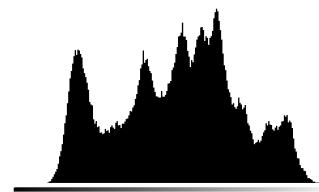
### Invariance spatiale :



du moins en théorie, si on ne tient pas compte de l'interpolation.

## Histogramme : invariances

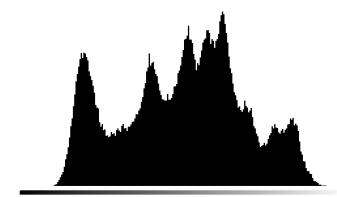
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## Histogramme : invariances

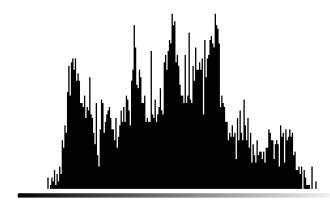
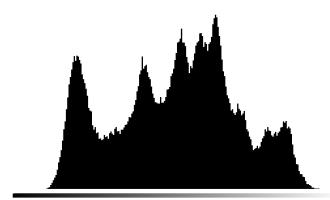
### Résolution : $512 \times 512$ vs. $64 \times 64$



du moins en théorie, si on ne tient pas compte de l'interpolation.

## Histogramme : invariances

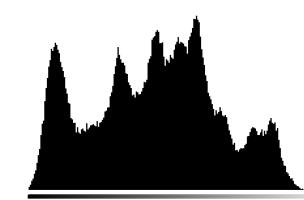
Résolution :  $512 \times 512$  vs.  $64 \times 64$



du moins en théorie, si on ne tient pas compte de l'interpolation.

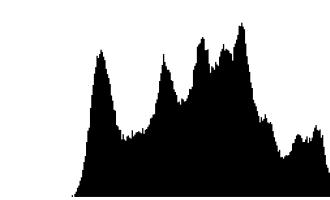
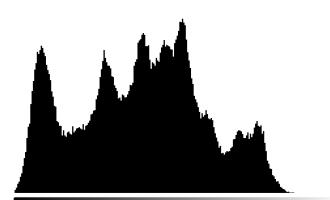
## Histogramme : invariances

Luminance :



## Histogramme : invariances

Luminance :



## Vous avez dit “luminance” ?



comment ça se formalise?

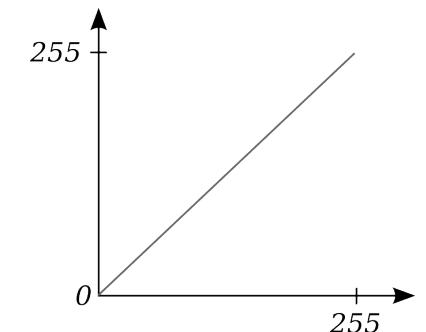
## Vous avez dit “luminance” ?



comment ça se formalise?

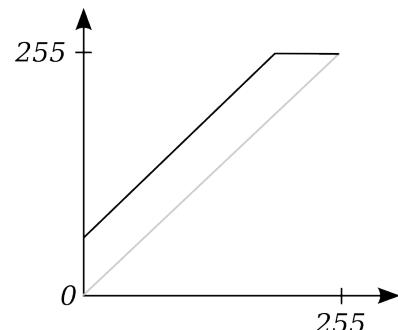
$$I'(x, y) = I(x, y) + \beta$$

## Luminance



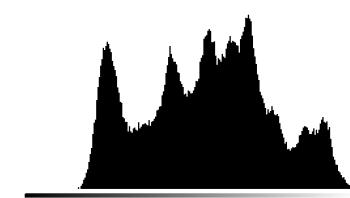
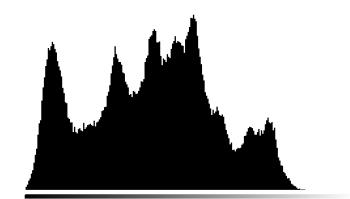
$$I'(x, y) = I(x, y)$$

## Luminance

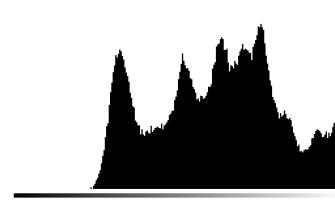
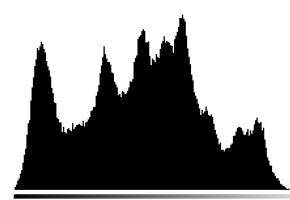


$$I'(x, y) = I(x, y) + \beta$$

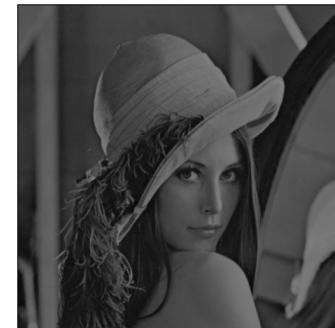
## Luminance



## Luminance : saturation et clamp

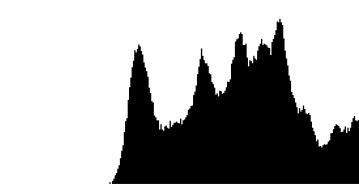
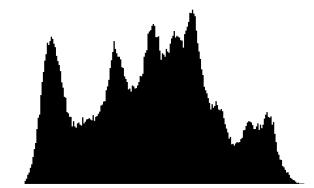


## Contraste



$$I'(x, y) = \alpha \cdot I(x, y)$$

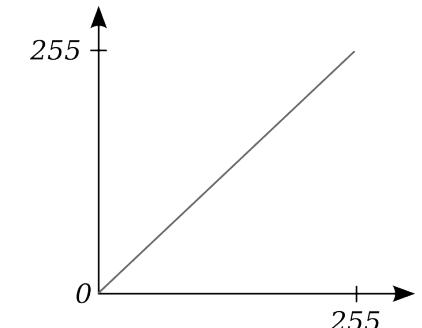
## Luminance : saturation et clamp



**Attention :**

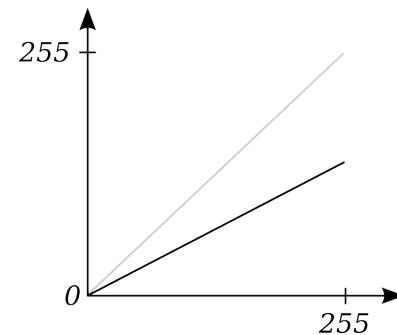
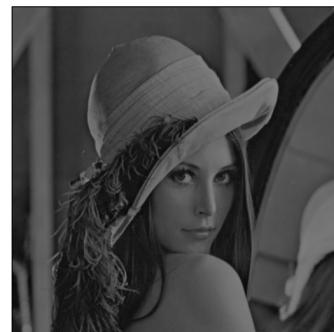
opération non réversible.

## Contraste



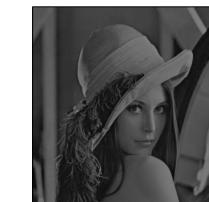
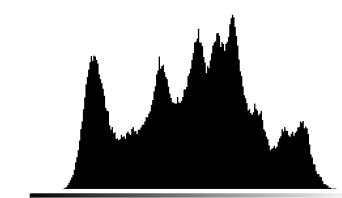
$$I'(x, y) = I(x, y)$$

## Contraste

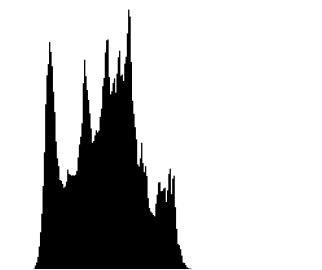
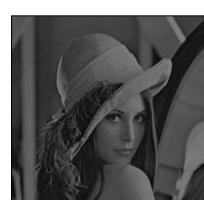
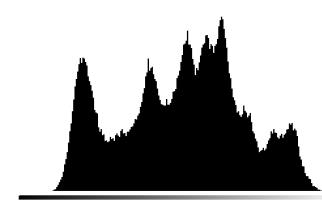


$$I'(x, y) = \alpha \cdot I(x, y)$$

## Contraste



## Contraste

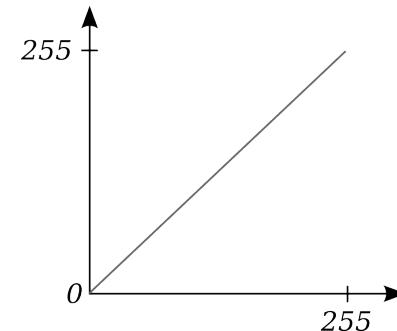


## Luminance + Contraste



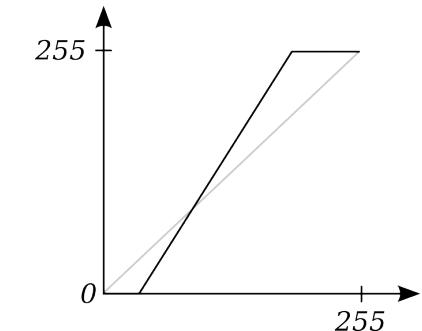
$$I'(x, y) = \alpha \cdot I(x, y) + \beta$$

## Luminance + Contraste



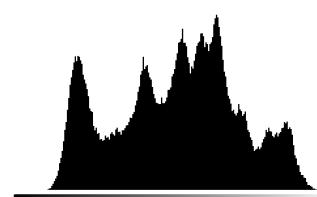
$$I'(x, y) = I(x, y)$$

## Luminance + Contraste

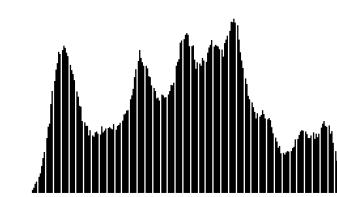
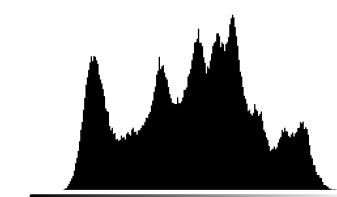


$$I'(x, y) = \alpha \cdot I(x, y) + \beta$$

## Luminance + Contraste

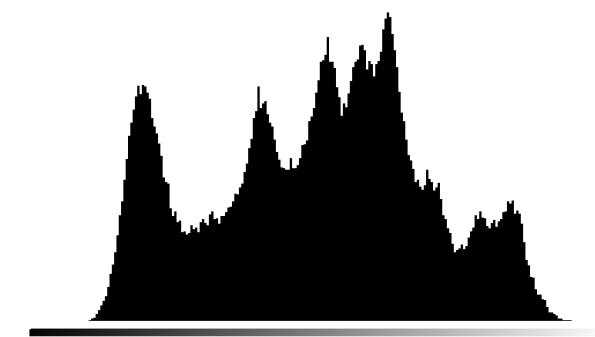
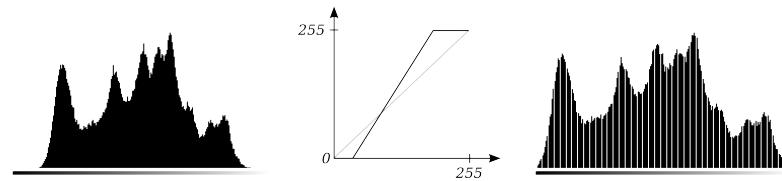


## Luminance + Contraste



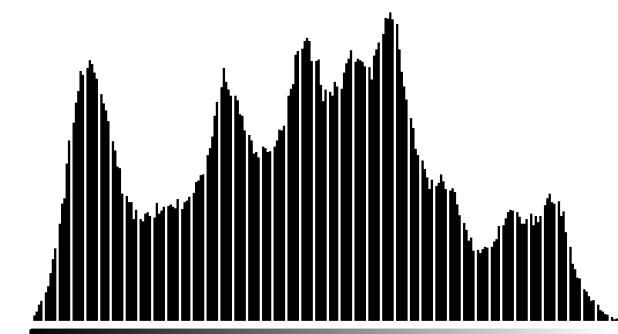
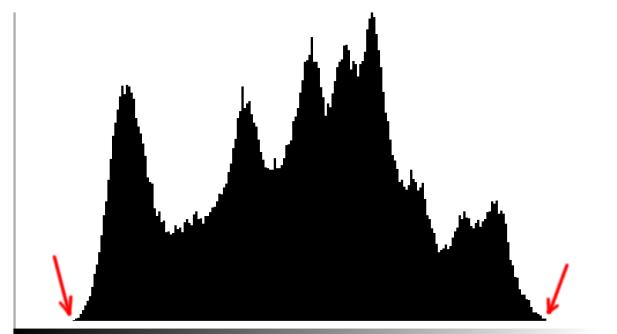
## Luminance + Contraste

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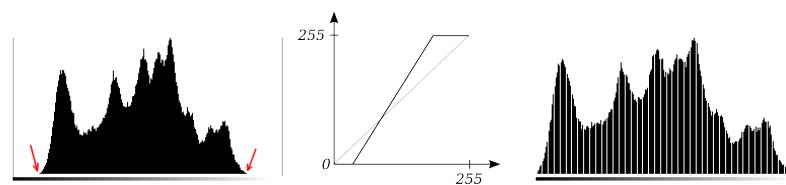


## Luminance + Contraste

## Luminance + Contraste



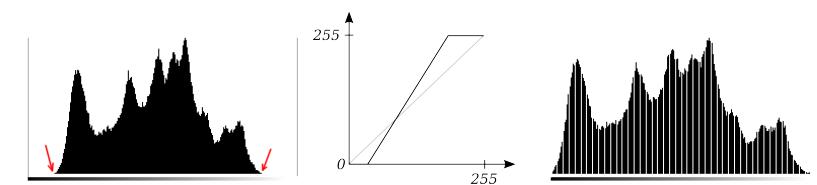
## Luminance + Contraste



$$I'(x,y) = \frac{I(x,y) - I_{\min}}{I_{\max} - I_{\min}} \cdot 255$$

$$I'(x,y) = \underbrace{\frac{255}{I_{\max} - I_{\min}}}_{\alpha} \cdot I(x,y) - \underbrace{\frac{255 \cdot I_{\min}}{I_{\max} - I_{\min}}}_{\beta}$$

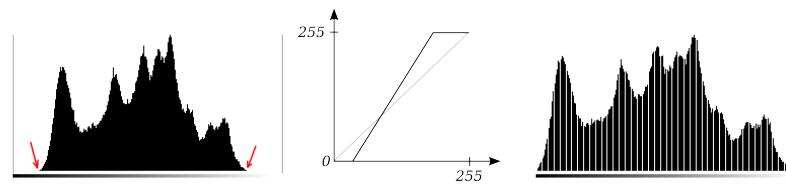
## Luminance + Contraste



$$I'(x,y) = \frac{I(x,y) - I_{\min}}{I_{\max} - I_{\min}} \cdot 255$$

$$I'(x,y) = \underbrace{\frac{255}{I_{\max} - I_{\min}}}_{\alpha} \cdot I(x,y) - \underbrace{\frac{255 \cdot I_{\min}}{I_{\max} - I_{\min}}}_{\beta}$$

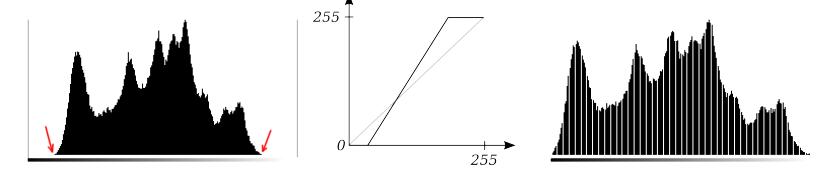
## Luminance + Contraste



$$I'(x,y) = \frac{I(x,y) - I_{\min}}{I_{\max} - I_{\min}} \cdot 255$$

$$I'(x,y) = \underbrace{\frac{255}{I_{\max} - I_{\min}}}_{\alpha} \cdot I(x,y) - \underbrace{\frac{255 \cdot I_{\min}}{I_{\max} - I_{\min}}}_{\beta}$$

## Normalisation d'histogramme



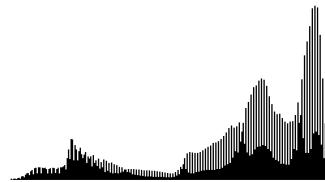
### Normalisation d'histogramme :

- ou *expansion de dynamique*
- transformation affine du niveau de gris des pixels pour générer une image qui utilise toute la dynamique de représentation.

## Normalisation d'histogramme



image initiale

après normalisation  
d'histogramme

## Implémentation

$$I'(x, y) = \frac{I(x, y) - I_{\min}}{I_{\max} - I_{\min}} \cdot 255$$

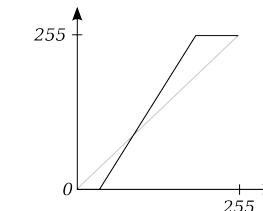
### Version 1 :

```
for(x=0; x<width; ++x)
    for(y=0; y<height; ++y)
        I(x, y) = 255 * (I(x, y)-Imin)/(Imax-Imin)
```

→ ça fait beaucoup d'opérations, à faire pour chaque pixel.

## Implémentation

## Lookup table



### LUT :

- une table de correspondance.
- une fonction injective.

→ à chaque valeur de départ correspond une valeur d'arrivée

### Version 2 : la LookUp Table (LUT)

```
for(i=0; i<255; ++i)
    LUT[i]=clamp( 255*(i-Imin)/(Imax-Imin), 0,255)

for(x=0; x<width; ++x)
    for(y=0; y<height; ++y)
        I(x, y) = LUT[I(x, y)]
```

→ on perd du temps à l'initialisation

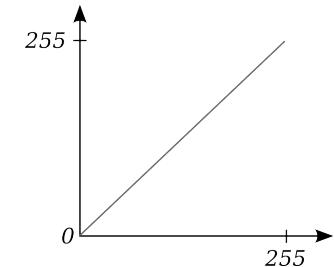
→ qu'on regagne largement pendant le traitement

## Lookup table

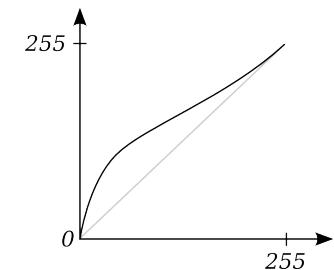
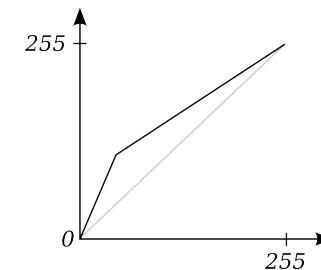
### Avantages :

- plus rapide à appliquer à toute l'image
- on peut cumuler des LUT
  - en travaillant sur des flottant  
→ interpolation linéaire sur la lecture de la LUT
  - en autorisant les dépassement  
→ le clamp est fait tout à la fin
- générique, quelque soit la transformation

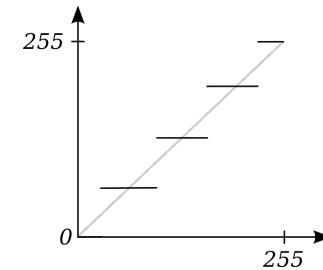
## Transformations linéaires par morceaux



## Transformations linéaires par morceaux



## Transformations non-continues

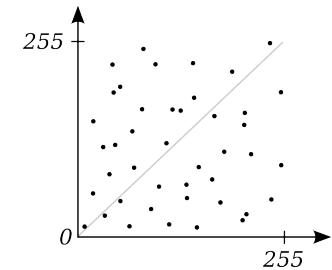


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## Transformations bizarres

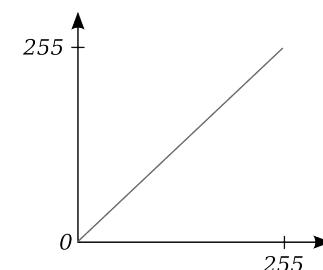


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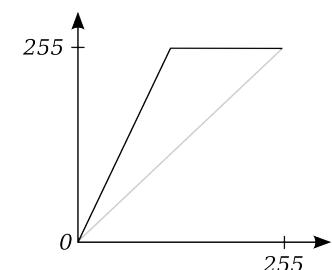
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## Combinaisons de LUT

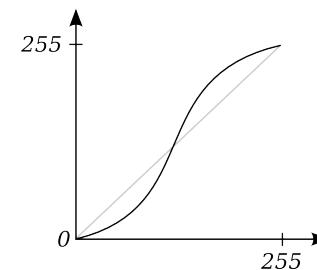
**Image initiale :**

## Combinaisons de LUT

**Augmentation de la dynamique :**

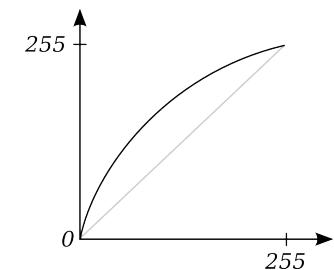
## Combinaisons de LUT

Augmentation du contraste :

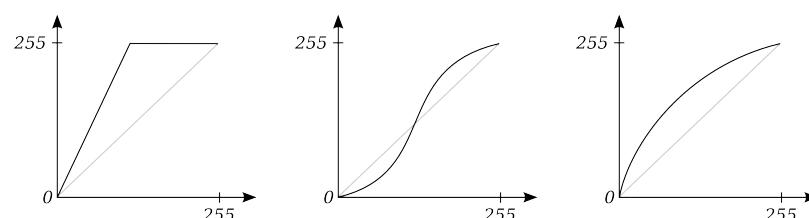


## Combinaisons de LUT

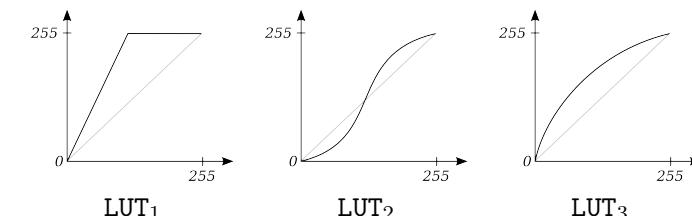
Correction gamma :



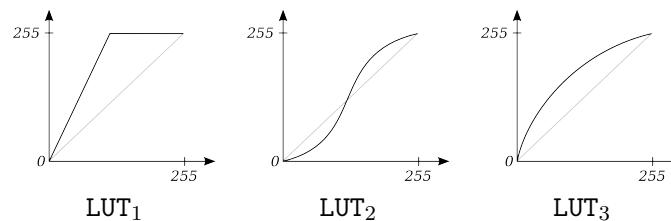
## Combinaisons de LUT



## Combinaisons de LUT

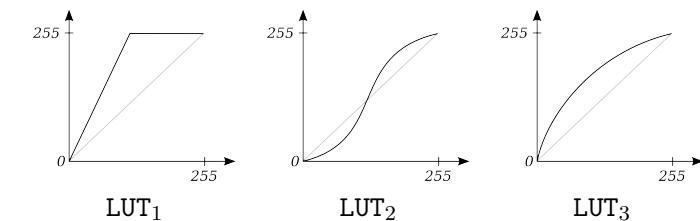


## Combinaisons de LUT

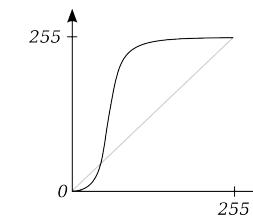


$$\text{LUT}_4 = \text{LUT}_3 \circ \text{LUT}_2 \circ \text{LUT}_1$$

## Combinaisons de LUT



$$\text{LUT}_4 = \text{LUT}_3 \circ \text{LUT}_2 \circ \text{LUT}_1$$



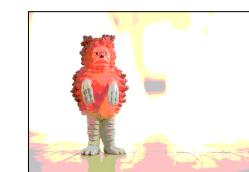
## Quizz : image originale



## Quizz



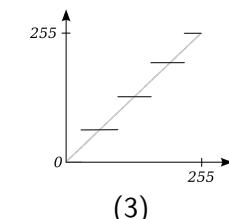
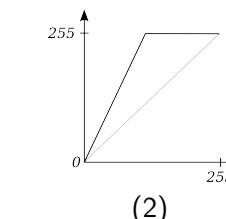
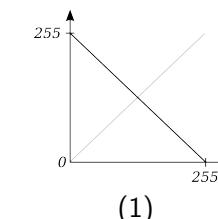
(a)



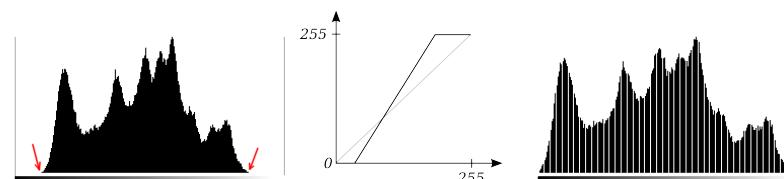
(b)



(c)



## Normalisation d'histogramme



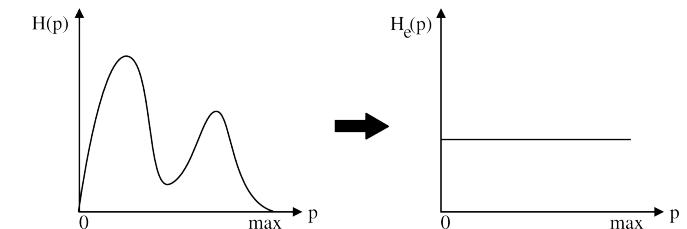
$$I'(x,y) = \frac{I(x,y) - I_{\min}}{I_{\max} - I_{\min}} \cdot 255$$



ne permet pas nécessairement d'équilibrer les tons clairs et les tons foncés.

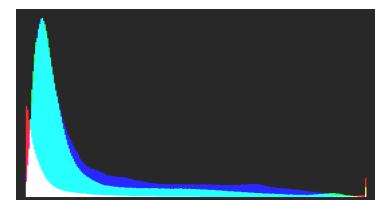
## Egalisation d'histogramme

- Egalisation d'histogramme :**
- transformation des niveaux de gris
  - équilibrer la distribution des intensités des pixels
- ↪ idéalement, on cherche à obtenir un histogramme plat



## Histogramme : égalisation

Image initiale :



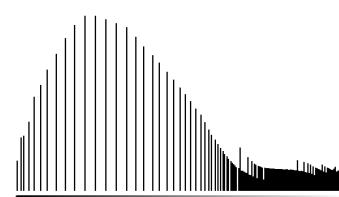
## Histogramme : égalisation

Image initiale : niveaux de gris



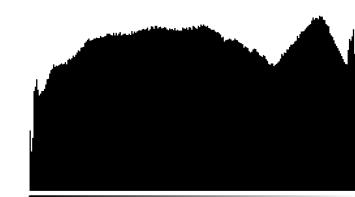
## Histogramme : égalisation

Image égalisée :



## Histogramme : égalisation

Image égalisée + blur :

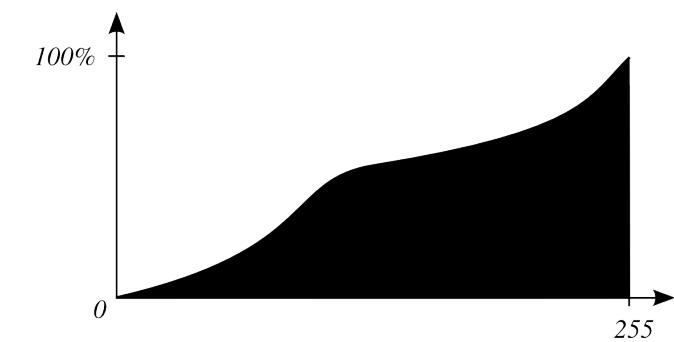


## Histogramme : égalisation : méthode



Histogramme cumulé :  $H_c(x)$

## Histogramme : égalisation : méthode



Histogramme cumulé normalisé :  $H_{cn}(x) = \frac{1}{w \times h} H_c(x)$

$H_{cn}(x)$  = pourcentage de pixels  $y$  tels que  $I(y) \leq I(x)$

## Histogramme : égalisation : méthode



Histogramme cumulé normalisé  $\times 255$  :  $H_{cn}(x) = \frac{255}{w \times h} H_c(x)$

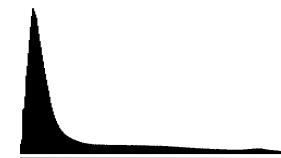
Fonction de transfert où chaque bin devient équiprobable.

## Histogramme : égalisation

**Image originale :**



**histogramme :**



**histogramme cumulé :**



## Histogramme : égalisation

### En pratique :

- calcul de l'histogramme  $H(x)$  de l'image
- calcul de l'histogramme cumulé  $H_c(x)$  correspondant :

$$H_c(x) = \sum_{i=0}^x H(i)$$

- construction d'une fonction de transfert :

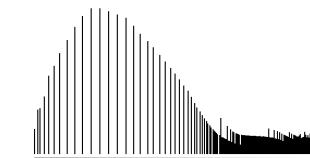
$$LUT(x) = \frac{255}{w \times h} H_c(x)$$

## Histogramme : égalisation

**Image égalisée :**



**histogramme :**



**histogramme cumulé :**



## Histogramme : égalisation

Image couleurs RGB :



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## Histogramme : égalisation

Image couleurs RGB :



Attention :

l'égalisation d'histogrammes d'images couleurs peut "mal se passer" si chacun des canaux est traité indépendamment.

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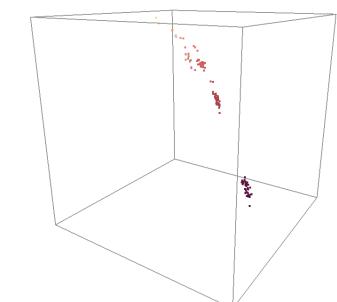
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## Et les images couleurs ?



## Histogramme 3D



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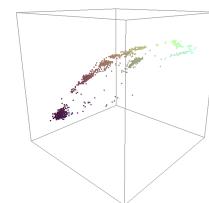
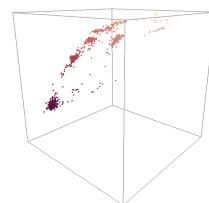
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## Histogramme 3D : visualisation

### Application à la balance des blancs automatique



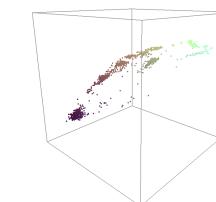
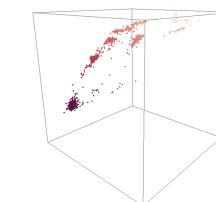
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## Histogramme 3D : visualisation

### Application à la balance des blancs automatique



### The grey world assumption :

on considère que la couleur moyenne d'une image est un gris neutre.

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{bmatrix} \frac{G_{moy}}{R_{moy}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{G_{moy}}{B_{moy}} \end{bmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

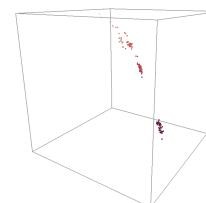
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## Histogramme 3D :

### Application à la segmentation



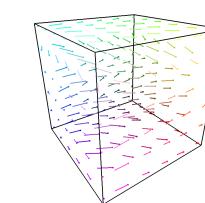
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## LUT 3D :

### Application à la correction colorimétrique

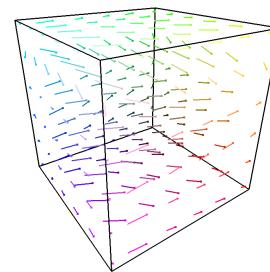


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## Les LUT 3D



### 3 LUT 1D :

- compact
- rapide
- limité

### 1 LUT 3D :

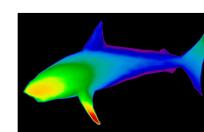
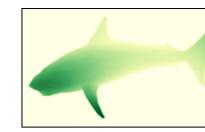
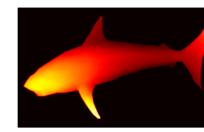
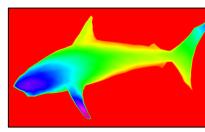
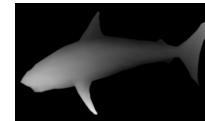
- volumineux (même en  $32 \times 32 \times 32$ )
- rapide
- puissant
- plus complexe à utiliser

## LUT : applications

### Calcul numérique :

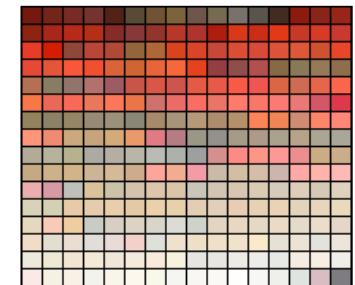
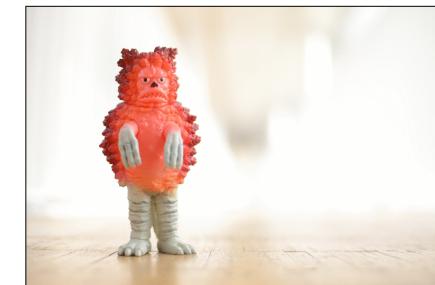
- calcul d'un cos et sin sur GPU
- calcul d'un exp

## Colormaps

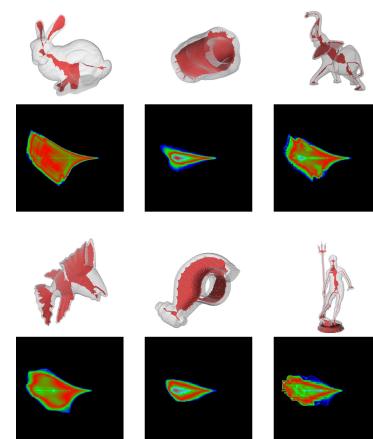


## LUT : applications

### Le format GIF :



## Reconnaissance d'objets 3D

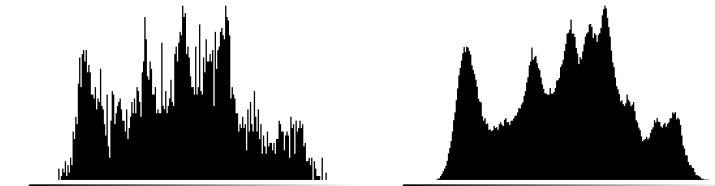


histogramme d'un bi-rapport leur squelette.

## Question ouverte

### Distance d'histogrammes :

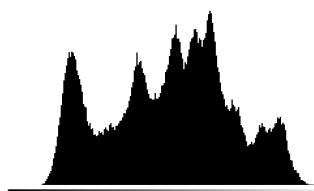
peut-on estimer facilement la ressemblance de 2 histogrammes ?



## Question ouverte

### Est-ce qu'un histogramme est différentiable ?

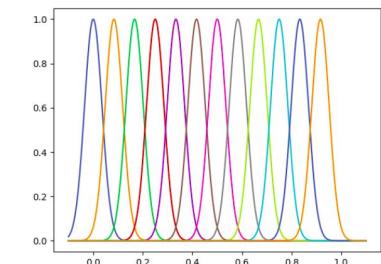
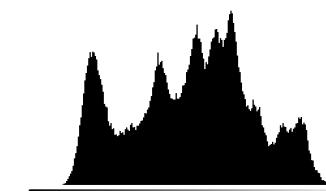
→ pour du deep-learning



## Question ouverte

### Est-ce qu'un histogramme est différentiable ?

→ pour du deep-learning



## Question ouverte

### Histogramme : calcul parallèle

