

Esercitazione Sistemi Digitali

08/11/2022

- 1 Calcolare la tabella di verità della funzione:

$$ABC + \bar{A}\bar{B}\bar{C}$$

- 2 Realizzare la funzione tramite un multiplexer a 4 ingressi dati e 2 ingressi di selezione

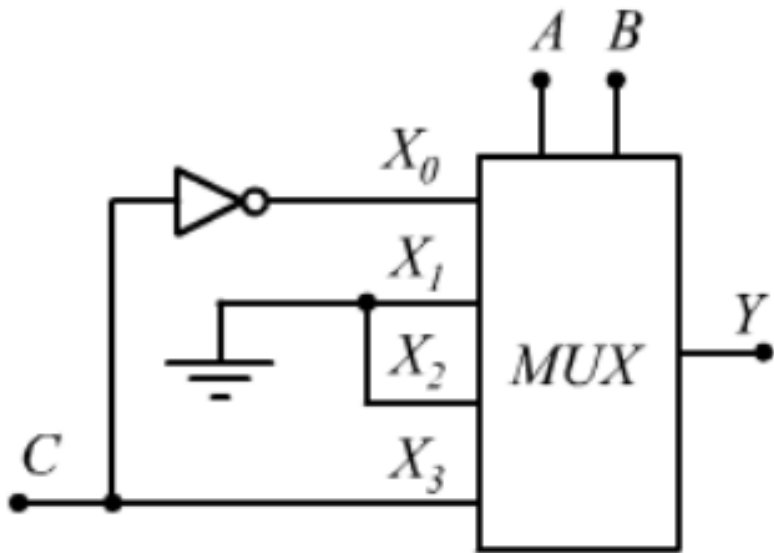
Soluzione esercizio 1 (1)

| A | B | C | ABC | $\bar{A}\bar{B}\bar{C}$ | $ABC + \bar{A}\bar{B}\bar{C}$ |
|-----|-----|-----|-------|-------------------------|-------------------------------|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 |

Osservare che:

- Se $A=0$ e $B=0$ allora $Y=\bar{C}$
- Se $A=0$ e $B=1$ allora $Y=0$
- Se $A=1$ e $B=0$ allora $Y=0$
- Se $A=1$ e $B=1$ allora $Y=C$

Soluzione esercizio 1 (2)



- **Realizzare con un mux a 8 ingressi dati e 3 ingressi di selezione un rilevatore di numeri primi per un sistema binario a 4 bit**

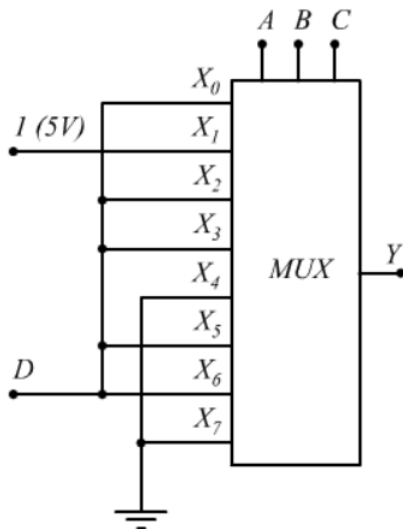
Soluzione esercizio 2 (1)

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>Y</i> |
|----------|----------|----------|----------|----------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

Osservare che:

- Se $A=0$, $B=0$, e $C=0$ allora $Y=D$
- Se $A=0$, $B=0$, e $C=1$ allora $Y=1$
- Se $A=0$, $B=1$, e $C=0$ allora $Y=D$
- Se $A=0$, $B=1$, e $C=1$ allora $Y=D$
- Se $A=1$, $B=0$, e $C=0$ allora $Y=0$
- Se $A=1$, $B=0$, e $C=1$ allora $Y=D$
- Se $A=1$, $B=1$, e $C=0$ allora $Y=D$
- Se $A=1$, $B=1$, e $C=1$ allora $Y=0$

Soluzione esercizio 2 (2)



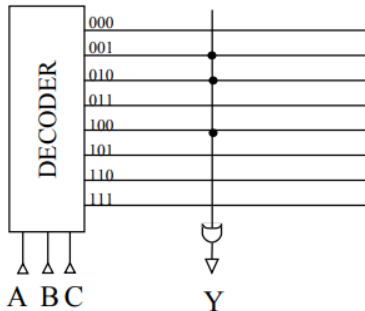
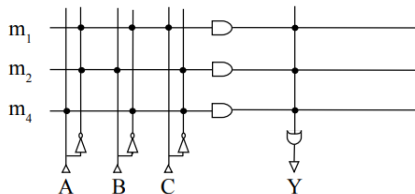
- **Realizza mediante PLA e ROM con 3 ingressi la funzione che vale 1 se e solo se un solo bit in input vale 1**

Esercizio 3- Soluzione

| A | B | C | Y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

FormaSOP

$$Y = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$



- **Realizza mediante PLA e ROM con 4 ingressi la funzione che identifica i multipli di 3**

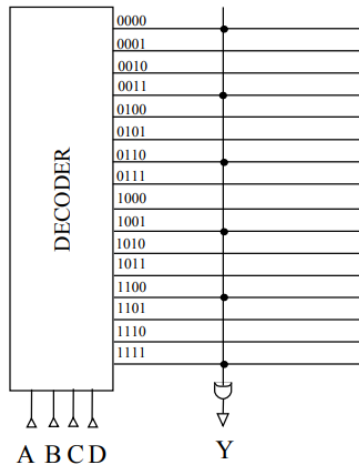
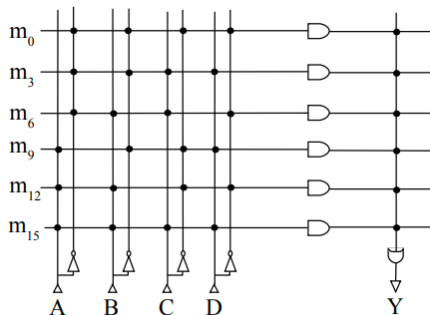
Esercizio 4- Soluzione (1)

| A | B | C | D | Y |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

FormaSOP

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}BC\bar{D} \\ + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + ABCD$$

Esercizio 4- Soluzione (2)



- **Realizza un circuito che calcola il valore opposto di un intero a 4 bits, rappresentandolo in Ca2. Realizzare il circuito nei seguenti modi:**
 - 1 Porte logiche (Costruire mappa di Karnaugh per ogni output e semplificare)
 - 2 ROM
 - 3 PLA
 - 4 MUX (un multiplexer delle opportune dimensioni per ogni output)

Esercizio 5- Soluzione Punto 1 (1)

| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

| | | X_3X_2 | | | |
|----------|----|----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| X_1X_0 | 00 | 0 | 1 | 0 | - |
| | 01 | 1 | 1 | 0 | 0 |
| | 11 | 1 | 1 | 0 | 0 |
| | 10 | 1 | 1 | 0 | 0 |

- $$Y_3 = \bar{X}_3X_0 + \bar{X}_3X_1 + \bar{X}_3X_2 =$$

$$= \bar{X}_3(X_2 + X_1 + X_0)$$

Esercizio 5- Soluzione Punto 1 (2)

| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

| | | X_3X_2 | | | |
|----------|----|----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| X_1X_0 | 00 | 0 | 1 | 1 | - |
| | 01 | 1 | 0 | 0 | 1 |
| | 11 | 1 | 0 | 0 | 1 |
| | 10 | 1 | 0 | 0 | 1 |

$$\begin{aligned}
 Y_2 &= X_2\bar{X}_1\bar{X}_0 + \bar{X}_2X_0 + \bar{X}_2X_1 = \\
 &= \bar{X}_2(X_1 + X_0) + X_2\bar{X}_1\bar{X}_0 = \\
 &X_2 \oplus (X_1 + X_0)
 \end{aligned}$$

Esercizio 5- Soluzione Punto 1 (3)

| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

$X_1 X_0$

| | | $X_3 X_2$ | | | |
|----|--|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| 00 | | 0 | 0 | 0 | - |
| 01 | | 1 | 1 | 1 | 1 |
| 11 | | 0 | 0 | 0 | 0 |
| 10 | | 1 | 1 | 1 | 1 |

$$Y_1 = \bar{X}_1 X_0 + X_1 \bar{X}_0 = X_1 \oplus X_0$$

Esercizio 5- Soluzione Punto 1 (4)

| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

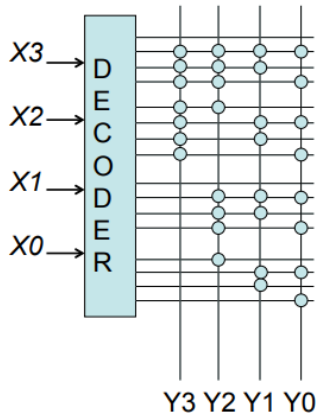
$X_1 X_0$

$Y_0 = X_0$

| | | $X_3 X_2$ | | | |
|-----------|----|-----------|----|----|----|
| | | 00 | 01 | 11 | 10 |
| $X_1 X_0$ | 00 | 0 | 0 | 0 | - |
| | 01 | 1 | 1 | 1 | 1 |
| | 11 | 1 | 1 | 1 | 1 |
| | 10 | 0 | 0 | 0 | 0 |

Esercizio 5- Soluzione Punto 2

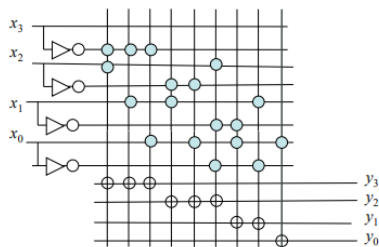
| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |



Esercizio 5- Soluzione Punto 3

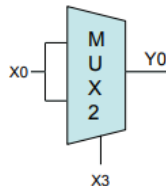
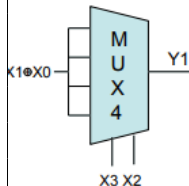
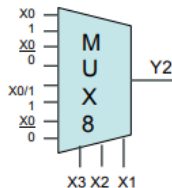
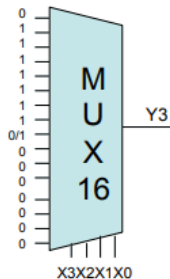
| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |

- $Y_3 = \bar{X}_3 X_0 + \bar{X}_3 X_1 + \bar{X}_3 X_2$
- $Y_2 = X_2 \bar{X}_1 \bar{X}_0 + \bar{X}_2 X_0 + \bar{X}_2 X_1$
- $Y_1 = \bar{X}_1 X_0 + X_1 \bar{X}_0$
- $Y_0 = X_0$



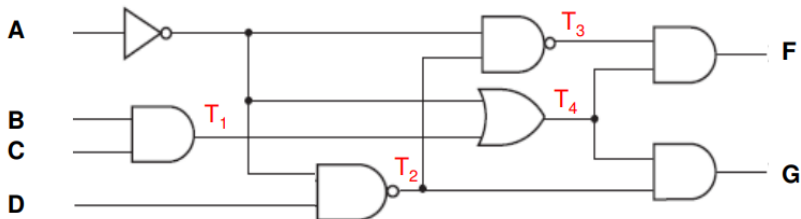
Esercizio 5- Soluzione Punto 4

| X_3 | X_2 | X_1 | X_0 | Y_3 | Y_2 | Y_1 | Y_0 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | — | — | — | — |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 |



Esercizio 7 esercitazione 4- Traccia

- Analizzare il seguente circuito



- 1 Determinare le espressioni booleane per F e G in forma SoP
- 2 Semplificare le espressioni ottenute per F e G tramite le mappe di Karnaugh

Esercizio 7- Soluzione (1)

1

$$T1 = B \cdot C$$

$$T2 = \overline{\bar{A} \cdot D}$$

$$T3 = \overline{\bar{A} \cdot T2}$$

$$T4 = \bar{A} + T1$$

$$\begin{aligned} F &= T3 \cdot T4 = (\overline{\bar{A} \cdot T2}) \cdot (\bar{A} + T1) = \\ &= \overline{(\bar{A} \cdot (\bar{A} \cdot D))} \cdot (\bar{A} + (B \cdot C)) = (A + (\bar{A}D)) \cdot (\bar{A} + (B \cdot C)) = \\ &= (A + D)(\bar{A} + BC) = \textcolor{red}{ABC} + \textcolor{red}{BCD} + \textcolor{red}{\bar{A}D} \end{aligned}$$

$$\begin{aligned} G &= T4 \cdot T2 = (\bar{A} + T1) \cdot (\overline{\bar{A} \cdot D}) = (\bar{A} + (B \cdot C)) \cdot (\overline{\bar{A} \cdot D}) = \\ &= (\bar{A} + (B \cdot C)) \cdot (A + \bar{D}) = \textcolor{red}{\bar{A}\bar{D}} + \textcolor{red}{BC\bar{D}} + \textcolor{red}{ABC} \end{aligned}$$

Esercizio 7- Soluzione (2)

$$F = \overline{A}BC + BCD + A'D$$

| AB \ CD | C | | | |
|---------|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 00 | 0 | 1 | 1 | 0 |
| 01 | 0 | 1 | 1 | 0 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 |

Esercizio 7- Soluzione (2)

$$G = \overline{A}BC + BCD' + A'D'$$

| | | C | | | |
|---|----|----|----|----|----|
| | | D | | | |
| | | 00 | 01 | 11 | 10 |
| A | AB | 00 | 01 | 11 | 10 |
| | 00 | 1 | 0 | 0 | 1 |
| | 01 | 1 | 0 | 0 | 1 |
| | 11 | 0 | 0 | 1 | 1 |
| | 10 | 0 | 0 | 0 | 0 |