FP2 Paper *adapted 2008

1. Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = x$

to obtain y as a function of x. (Total 5 marks)

2. (a) Simplify the expression $\frac{(x+3)(x+9)}{x-1} - (3x-5)$, giving your answer in the form $\frac{a(x+b)(x+c)}{x-1}$, where a, b and c are integers. (4)

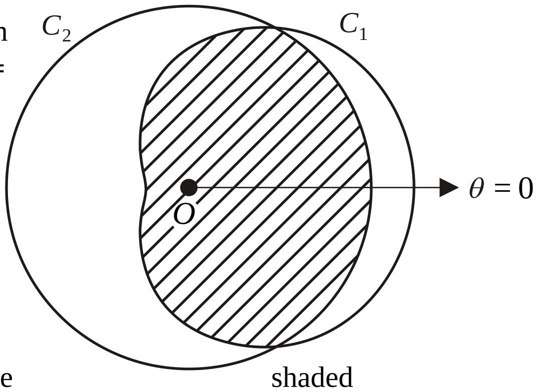
(b) Hence, or otherwise, solve the inequality $\frac{(x+3)(x+9)}{x-1} > 3x-5$ **(4)(Total 8 marks)**

3. (a) Find the general solution of the differential equation $3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = x^2$

(8)

(b) Find the particular solution for which, at $x = 0$, $y = 2$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$.(6)(Total 14 marks)

4. The diagram above shows the curve C_1 which has polar equation $\boldsymbol{r = a(3 + 2\cos\theta)}$, $0 \le \theta < 2\pi$ and the circle C_2 with equation $\boldsymbol{r = 4a}$, $0 \le \theta < 2\pi$, where a is a positive constant.



(a) Find, in terms of a, the polar coordinates of the points where the curve C_1 meets the circle C_2.(4)

The regions enclosed by the curves C_1 and C_2 overlap and this common region R is shaded in the figure.

(b) Find, in terms of a, an exact expression for the area of the region R.(8)

(c) In a single diagram, copy the two curves in the diagram above and also sketch the curve C_3 with polar equation $r = 2a\cos\theta$, $0 \le \theta < 2\pi$ Show clearly the coordinates of the points of intersection of C_1, C_2 and C_3 with the initial line, $\theta = 0$.(3)(Total 15 marks)

5. (a) Find, in terms of k, the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = kt + 5$$, where k is a constant and $t > 0$.(7)

For large values of t, this general solution may be approximated by a linear function.

(b) Given that $k = 6$, find the equation of this linear function.(2)(Total 9 marks)

6. (a) Find, in the simplest surd form where appropriate, the exact values of x for which

$$\frac{x}{2}+3=\left|\frac{4}{x}\right|. \quad (5)$$

(b) Sketch, on the same axes, the line with equation $y=\frac{x}{2}+3$ and the graph of

$y=\left|\frac{4}{x}\right|,\ x\neq 0.$ (3)

(c) Find the set of values of x for which $\frac{x}{2}+3>\left|\frac{4}{x}\right|.$ (2)(Total 10 marks)

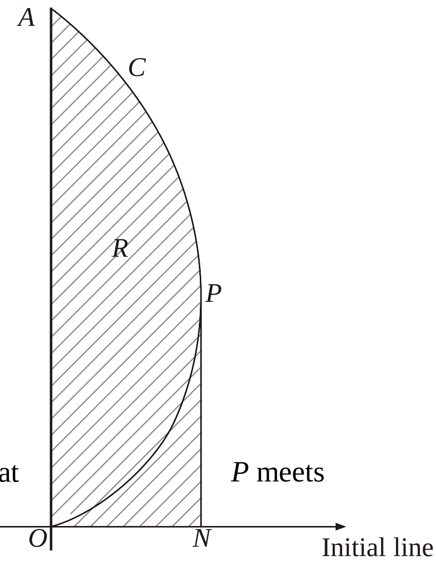
7. (a) Show that the substitution $y = vx$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{y}+\frac{3y}{x},\quad x>0,\quad y>0 \qquad \text{(I)}$$

into the differential equation $\quad x\frac{\mathrm{d}v}{\mathrm{d}x}=2v+\frac{1}{v}.\quad$ (II) (3)

(b) By solving differential equation (II), find a general solution of differential equation (I) in the form $y = \mathrm{f}(x)$. (7)

Given that $y = 3$ at $x = 1$, (c)find the particular solution of differential equation (I).(2)



8. The curve C shown in the diagram above has polar equation

$$r=4(1-\cos\theta),\ 0\leq\theta\leq\frac{\pi}{2}.$$

At the point P on C, the tangent to C is parallel to the line $\theta=\frac{\pi}{2}$.

(a) Show that P has polar coordinates $\left(2,\frac{\pi}{3}\right)$.(5)

The curve C meets the line $\theta=\frac{\pi}{2}$ at the point A. The tangent to C at P meets the initial line at the point N. The finite region R, shown shaded in the diagram above, is bounded by the initial line, the line $\theta=\frac{\pi}{2}$, the arc AP of C and the line PN.

(b) Calculate the exact area of R.

(8)

9. $$(x^2+1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2y^2+(1-2x)\frac{\mathrm{d}y}{\mathrm{d}x} \qquad \text{(I)}$$

(a) By differentiating equation (I) with respect to x, show that

$$(x^2+1)\frac{d^3y}{dx^3} = (1-4x)\frac{d^2y}{dx^2} + (4y-2)\frac{dy}{dx}. \qquad (3)$$

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$,

(b) find the series solution for y, in ascending powers of x, up to and including the term in x_3.(4)

(c) Use your series to estimate the value of y at $x = -0.5$, giving your answer to two decimal places.(1)

10. The point P represents a complex number z on an Argand diagram such that

$$|z - 3| = 2\,|z|.$$

(a) Show that, as z varies, the locus of P is a circle, and give the coordinates of the centre and the radius of the circle.(5)

The point Q represents a complex number z on an Argand diagram such that

$$|z + 3| = |z - i\sqrt{3}|.$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.(5)

(c) On your diagram shade the region which satisfies

$$|z - 3| \geq 2\,|z| \text{ and } |z + 3| \geq |z - i\sqrt{3}|. \qquad (2)$$

11. De Moivre's theorem states that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for $n \in \Re$

(a) Use induction to prove de Moivre's theorem for $n \in \mathbb{Z}^+$. (5)

(b) Show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$ (5)

(c) Hence show that $2\cos\frac{\pi}{10}$ is a root of the equation

$$x^4 - 5x^2 + 5 = 0$$

(3)