Mark Scheme (Results)

Summer 2015

Pearson Edexcel International A Level in Further Pure Mathematics F2 (WFM02/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\lfloor\,\rfloor$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

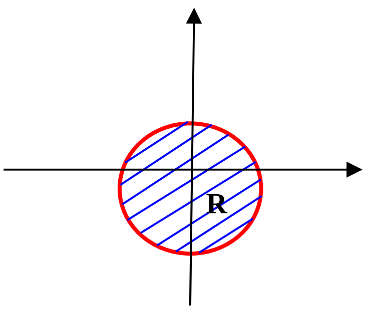
June 2015
WFM02 Further Pure Mathematics F2
Mark Scheme

Question Number	Scheme	Notes	Marks
1.	$\frac{x}{x+2} < \frac{2}{x+5}$		
	Critical Values -2 and -5	Seen anywhere in solution Both correct B1B1; one correct B1B0	B1, B1
	$\frac{x}{x+2} - \frac{2}{x+5} < 0$		
	$\frac{x^2+3x-4}{(x+2)(x+5)} < 0$		
	$\frac{(x+4)(x-1)}{(x+2)(x+5)} < 0$	Attempt single fraction and factorise numerator or use quad formula	M1
	Critical values -4 and 1	Correct critical values May be seen on a graph or number line.	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4) \cup (-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0 Award on basis of the inequalities seen - ignore any and/or between them Set notation answers do not need the union sign.	dM1A1,A1
			(7)
ALT	Critical Values -2 and -5	Seen anywhere in solution	B1, B1
	$\frac{x}{x+2} < \frac{2}{x+5} \Rightarrow x(x+5)^2(x+2) < 2(x+2)^2(x+5)$		
	$\Rightarrow (x+5)(x+2)\left[x(x+5)-2(x+2)\right] < 0$		
	$\Rightarrow (x+5)(x+2)\left[(x-1)(x+4)\right] < 0$	Multiply by $(x+5)^2(x+2)^2$ and attempt to factorise a quartic or use quad formula	M1
	Critical values -4 and 1	Correct critical values	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4) \cup (-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0	dM1A1,A1
			(7)

Any solutions with no algebra (eg sketch graph followed by critical values with no working) scores max B1B1

Question Number	Scheme	Notes	Marks
	$\frac{1}{(r+6)(r+8)}$		
2(a)	$\frac{1}{2(r+6)}-\frac{1}{2(r+8)}$ oe	Correct partial fractions, any equivalent form	B1
			(1)
(b)	$=\left(2\times\frac{1}{2}\right)\left(\frac{1}{7}-\frac{1}{9}+\frac{1}{8}-\frac{1}{10}+\frac{1}{9}-\frac{1}{11}...+\frac{1}{n+5}-\frac{1}{n+7}+\frac{1}{n+6}-\frac{1}{n+8}\right)$ Expands at least 3 terms at start and 2 at end (may be implied) The partial fractions obtained in (a) can be used without multiplying by 2. Fractions may be $\frac{1}{2}\times\frac{1}{7}-\frac{1}{2}\times\frac{1}{9}$ etc These comments apply to both M1 and A1		M1
	$=\frac{1}{7}+\frac{1}{8}-\frac{1}{n+7}-\frac{1}{n+8}$	Identifies the terms that do not cancel	A1
	$=\frac{15(n+7)(n+8)-56(2n+15)}{56(n+7)(n+8)}$	Attempt common denominator Must have multiplied the fractions from (a) by 2 now	M1
	$=\frac{n(15n+113)}{56(n+7)(n+8)}$		A1cso
			(4)
			Total 5

Question Number	Scheme	Notes	Marks
3	$\frac{dy}{dx}+2xy=xe^{-x^2}y^3$		
(a)	$z=y^{-2}\Rightarrow y=z^{-\frac{1}{2}}$		
	$\frac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	M1: $\frac{dy}{dx}=kz^{-\frac{3}{2}}\frac{dz}{dx}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}+\frac{2x}{z^{\frac{1}{2}}}=xe^{-x^2}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1cso
			(4)
	(a) Alternative 1		
	$\frac{dz}{dy}=-2y^{-3}$ oe	M1: $\frac{dz}{dy}=ky^{-3}$ A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
	(a) Alternative 2		
	$\frac{dz}{dx}=-2y^{-3}\frac{dy}{dx}$	M1: $\frac{dz}{dx}=ky^{-3}\frac{dy}{dx}$ inc chain rule A1: Correct differentiation	M1A1
	$-\frac{1}{2}y^3\frac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\frac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
(b)	$I=e^{\int -4x\,dx}=e^{-2x^2}$	M1: $I=e^{\int \pm 4x\,dx}$ A1: e^{-2x^2}	M1A1
	$ze^{-2x^2}=\int -2xe^{-3x^2}\,dx$	$z\times I=\int -2xe^{-x^2}I\,dx$	dM1
	$\frac{1}{3}e^{-3x^2}\ (+c)$	$\int xe^{qx^2}\,dx=pe^{qx^2}\ (+c)$	M1
	$z=ce^{2x^2}+\frac{1}{3}e^{-x^2}$	Or equivalent	A1
			(5)
(c)	$\frac{1}{y^2}=ce^{2x^2}+\frac{1}{3}e^{-x^2}\Rightarrow y^2=\frac{1}{ce^{2x^2}+\frac{1}{3}e^{-x^2}}$	$y^2=\frac{1}{(b)}\left(=\frac{3e^{x^2}}{1+ke^{3x^2}}\right)$	B1ft
			(1)
			Total 10

Question Number	Scheme	Notes	Marks
	$w=\dfrac{z-1}{z+1}$		
4(a)	$w=\dfrac{z-1}{z+1}\Rightarrow wz+w=z-1\Rightarrow z=...$	Attempt to make *z* the subject	M1
	$z=\dfrac{w+1}{1-w}$	Correct expression in terms of *w*	A1
	$=\dfrac{u+iv+1}{1-u-iv}\times\dfrac{1-u+iv}{1-u+iv}$	Introduces "*u* + i*v*" and multiplies top and bottom by the complex conjugate of the bottom	M1
	$x=\dfrac{-u^2-v^2+1}{....},\quad y=\dfrac{2v}{....}$		
	$y=2x\Rightarrow 2v=-2u^2-2v^2+2$	Uses real and imaginary parts and *y* = 2*x* to obtain an equation connecting "*u*" and "*v*" Can have the 2 on the wrong side.	M1
	$u^2+\left(v+\frac{1}{2}\right)^2-\frac{1}{4}=1$	Processes their equation to a form that is recognisable as a circle ie coefficients of u^2 and v^2 are the same and no *uv* terms	M1
	Centre (0, $-\frac{1}{2}$), radius $\dfrac{\sqrt{5}}{2}$	A1: Correct centre (allow -½i) A1: Correct radius	A1,A1
			(7)
	Special Case:		
	$w=\dfrac{x+\text{i}y-1}{x+\text{i}y+1}=\dfrac{(x-1)+2x\text{i}}{(x+1)+2x\text{i}}\times\dfrac{(x+1)-2x\text{i}}{(x+1)-2x\text{i}}$	M1: rationalise the denominator, may have 2*x* or *y*	
	$=\dfrac{(x^2-1)+4x^2+2x\text{i}(x+1-(x-1))}{(x+1)^2+4x^2}$	A1: Correct result in terms of *x* only. Must have rational denominator shown, but no other simplification needed	
(b)		B1ft: Their circle correctly positioned provided their equation does give a circle B1: Completely correct sketch and shading	B1ft B1
			(2)
			Total 9

Question Number	Scheme	Notes	Marks
5	$y = \cot x$		
(a)	$\dfrac{dy}{dx} = -\text{cosec}^2 x$		
	$\dfrac{d^2y}{dx^2} = (-2\text{cosec}\, x)(-\text{cosec}\, x \cot x)$	M1: Differentiates using the chain rule or product/quotient rule A1: Correct derivative	M1A1
	$= 2\text{cosec}^2 x \cot x = 2\cot x + 2\cot^3 x$ *	A1: Correct completion to printed answer $1+\cot^2 x = \text{cosec}^2 x$ or $\cos^2 x + \sin^2 x = 1$ must be used Full working must be shown	A1cso*
			(3)
	Alternative:		
	$y = \dfrac{\cos x}{\sin x} \to \dfrac{dy}{dx} = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\dfrac{1}{\sin^2 x}$		
	$\dfrac{d^2y}{dx^2} = -\left(-2\sin^{-3} x \cos x\right) = \ldots$		M1A1
	A1: Correct completion to printed answer see above		A1
(b)	$\dfrac{d^3y}{dx^3} = -2\text{cosec}^2 x - 6\cot^2 x\, \text{cosec}^2 x$	Correct third derivative	B1
	$= -2\left(1+\cot^2 x\right) - 6\cot^2 x\left(1+\cot^2 x\right)$	Uses $1+\cot^2 x = \text{cosec}^2 x$	M1
	$= -6\cot^4 x - 8\cot^2 x - 2$	cso	A1
			(3)
(c)	$f\left(\dfrac{\pi}{3}\right) = \dfrac{1}{\sqrt{3}}, f'\left(\dfrac{\pi}{3}\right) = -\dfrac{4}{3}, f''\left(\dfrac{\pi}{3}\right) = \dfrac{8}{3\sqrt{3}}, f'''\left(\dfrac{\pi}{3}\right) = -\dfrac{16}{3}$ M1: Attempts all 4 values at $\dfrac{\pi}{3}$ No working need be shown		M1
	$(y =)\dfrac{1}{\sqrt{3}} - \dfrac{4}{3}\left(x - \dfrac{\pi}{3}\right) + \dfrac{4}{3\sqrt{3}}\left(x - \dfrac{\pi}{3}\right)^2 - \dfrac{8}{9}\left(x - \dfrac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x - \dfrac{\pi}{3}\right)^3$ A1: Correct expression Must start $y =$ .... or $\cot x$ $f(x)$ allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698.., so accept 0.77) 0.889		M1A1
			(3)
			Total 9

Question Number	Scheme	Notes	Marks
6(a)	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=2\sin x$		
	AE: $m^2-2m-3=0$		
	$m^2-2m-3=0\Rightarrow m=...(-1,3)$	Forms Auxiliary Equation and attempts to solve (usual rules)	M1
	$(y=)\,Ae^{3x}+Be^{-x}$	Cao	A1
	PI: $(y=)\,p\sin x+q\cos x$	Correct form for PI	B1
	$(y'=)\,p\cos x-q\sin x$ $(y''=)-p\sin x-q\cos x$		
	$-p\sin x-q\cos x-2(p\cos x-q\sin x)-3p\sin x-3q\cos x=2\sin x$ Differentiates twice and substitutes		M1
	$2q-4p=2,\ 4q+2p=0$	Correct equations	A1
	$p=-\frac{2}{5},\ q=\frac{1}{5}$	A1A1 both correct A1A0 one correct	A1A1
	$y=\frac{1}{5}\cos x-\frac{2}{5}\sin x$		
	$y=Ae^{3x}+Be^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Follow through their p and q and their CF	B1ft
			(8)
(b)	$y'=3Ae^{3x}-Be^{-x}-\frac{1}{5}\sin x-\frac{2}{5}\cos x$	Differentiates their GS	M1
	$0=A+B+\frac{1}{5},\ 1=3A-B-\frac{2}{5}$	M1: Uses the given conditions to give two equations in A and B A1: Correct equations	M1A1
	$A=\frac{3}{10},\ B=-\frac{1}{2}$	Solves for A and B Both correct	A1
	$y=\frac{3}{10}e^{3x}-\frac{1}{2}e^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Sub their values of A and B in their GS	A1ft
			(5)
			Total 13

Question Number	Scheme	Notes	Marks
7(a)	$\theta=\frac{\pi}{3}\Rightarrow r=\sqrt{3}\sin\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Attempt to verify coordinates in at least one of the polar equations	M1
	$\theta=\frac{\pi}{3}\Rightarrow r=1+\cos\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Coordinates verified in both curves (Coordinate brackets not needed)	A1
			(2)
	Alternative:		
	Equate rs: $\sqrt{3}\sin\theta=1+\cos\theta$ and verify (by substitution) that $\theta=\frac{\pi}{3}$ is a solution or solve by using $t=\tan\frac{\theta}{2}$ or writing $\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta=\frac{1}{2}$ $\sin\left(\theta-\frac{\pi}{6}\right)=\frac{1}{2}$ $\theta=\frac{\pi}{3}$ Squaring the original equation allowed as θ is known to be between 0 and π		M1
	Use $\theta=\frac{\pi}{3}$ in either equation to obtain $r=\frac{3}{2}$		A1
(b)	$\frac{1}{2}\int(\sqrt{3}\sin\theta)^2\,\mathrm{d}\theta,\quad \frac{1}{2}\int(1+\cos\theta)^2\,\mathrm{d}\theta$	Correct formula used on at least one curve (1/2 may appear later) Integrals may be separate or added or subtracted.	M1
	$=\frac{1}{2}\int 3\sin^2\theta\,\mathrm{d}\theta,\quad \frac{1}{2}\int(1+2\cos\theta+\cos^2\theta)\,\mathrm{d}\theta$		
	$=\left(\frac{1}{2}\right)\int\frac{3}{2}(1-\cos 2\theta)\,\mathrm{d}\theta,\quad \left(\frac{1}{2}\right)\int\left(1+2\cos\theta+\frac{1}{2}(1+\cos 2\theta)\right)\mathrm{d}\theta$ Attempt to use $\sin^2\theta$ or $\cos^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos 2\theta$ on either integral **Not** dependent 1/2 may be missing		M1
	$=\frac{3}{4}\left[\theta-\frac{1}{2}\sin 2\theta\right]_{(0)}^{\left(\frac{\pi}{3}\right)},\quad \frac{1}{2}\left[\frac{3}{2}\theta+2\sin\theta+\frac{1}{4}\sin 2\theta\right]_{\left(\frac{\pi}{3}\right)}^{(\pi)}$ Correct integration (ignore limits) A1A1 or A1A0		A1, A1
	$R=\frac{3}{4}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}(-0)\right]+\frac{1}{2}\left[\frac{3\pi}{2}-\left(\frac{\pi}{2}+\sqrt{3}+\frac{\sqrt{3}}{8}\right)\right]$	Correct use of limits for both integrals Integrals must be added. Dep on both previous M marks	**dd**M1
	$=\frac{3}{4}\left(\pi-\sqrt{3}\right)$	Cao No equivalents allowed	A1
			(6)
			Total 8

Question Number	Scheme	Notes	Marks
8(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3$		
	$=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1: Attempt to expand A1: Correct expansion	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors seen	A1
			(3)
(a) ALT	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3}$, $\left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$		M1A1
	M1: Attempt to expand both cubic brackets A1: Correct expansions		
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors	A1
			(3)
(b)(i)(ii)	$z^n=\cos n\theta+i\sin n\theta$	Correct application of de Moivre	B1
	$z^{-n}=\cos(-n\theta)+i\sin(-n\theta)=\pm\cos n\theta\pm\sin n\theta$ but must be different from their z^n	Attempt z^{-n}	M1
	$z^n+\frac{1}{z^n}=2\cos n\theta*$, $z^n-\frac{1}{z^n}=2i\sin n\theta*$	$z^{-n}=\cos n\theta-i\sin n\theta$ must be seen	A1*
			(3)
(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2i\sin\theta)^3$		B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2i\sin 6\theta-6i\sin 2\theta$	Follow through their k in place of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta=2i\sin 6\theta-6i\sin 2\theta$	Equating right hand sides and simplifying $2^3\times(2i)^3$ (B mark needed for each side to gain M mark)	M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *		A1cso
			(4)
(d)	$\int_0^{\frac{\pi}{8}}\cos^3\theta\sin^3\theta\,d\theta=\int_0^{\frac{\pi}{8}}\frac{1}{32}(3\sin 2\theta-\sin 6\theta)\,d\theta$		
	$=\frac{1}{32}\left[-\frac{3}{2}\cos 2\theta+\frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$	M1: $p\cos 2\theta+q\cos 6\theta$ A1: Correct integration Differentiation scores M0A0	M1A1
	$=\frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}}-\frac{1}{6\sqrt{2}}\right)-\left(-\frac{3}{2}+\frac{1}{6}\right)\right]=\frac{1}{32}\left(\frac{4}{3}-\frac{5\sqrt{2}}{6}\right)$	dM1: Correct use of limits – lower limit to have non-zero result. Dep on previous M mark A1: Cao (oe) but must be exact	dM1A1
			(4)
			Total 14