



- (a) Give a reason why the maximum speed of  $P$  occurs when  $x = 30$ .

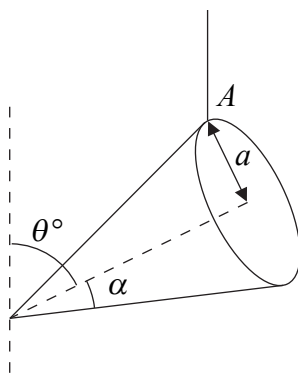
(1)

(b) find an expression for  $v^2$  in terms of  $x$ .

(5)

[illegible]

### Figure 1



Find, to one decimal place, the value of  $\theta$ .

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

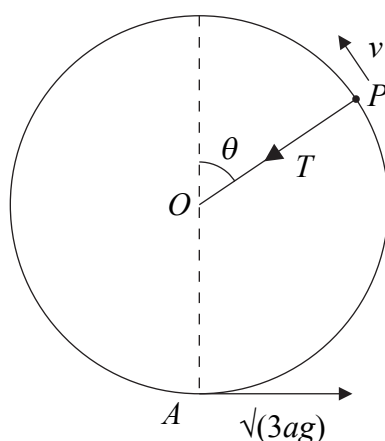
- (3)**

(6)

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4.

Figure 2



A particle  $P$  of mass  $m$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a point  $O$ . The point  $A$  is vertically below  $O$ , and  $OA = a$ . The particle is projected horizontally from  $A$  with speed  $\sqrt{3ag}$ . When  $OP$  makes an angle  $\theta$  with the upward vertical through  $O$  and the string is still taut, the tension in the string is  $T$  and the speed of  $P$  is  $v$ , as shown in Figure 2.

- (a) Find, in terms of  $a$ ,  $g$  and  $\theta$ , an expression for  $v^2$ .

(3)

- (b) Show that  $T = (1 - 3 \cos \theta)mg$ .

(3)

The string becomes slack when  $P$  is at the point  $B$ .

- (c) Find, in terms of  $a$ , the vertical height of  $B$  above  $A$ .

(2)

After the string becomes slack, the highest point reached by  $P$  is  $C$ .

- (d) Find, in terms of  $a$ , the vertical height of  $C$  above  $B$ .

(5)

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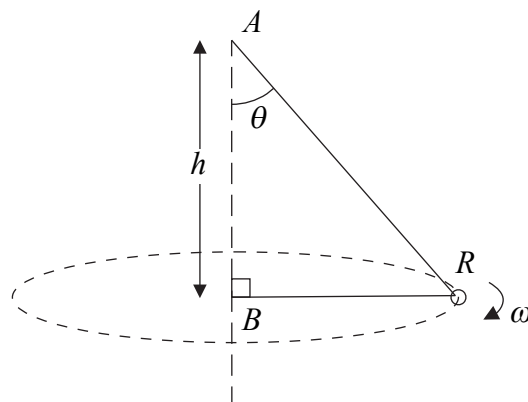
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### Figure 3



(a) Show that  $\omega^2 = \frac{g}{h} \left( \frac{1 + \sin \theta}{\sin \theta} \right)$ . (7)

(b) Deduce that  $\omega > \sqrt{\frac{2g}{h}}$ . (2)

Given that  $\omega = \sqrt{\frac{3g}{h}}$ ,

(c) find, in terms of  $m$  and  $g$ , the tension in the string.

**(4)**

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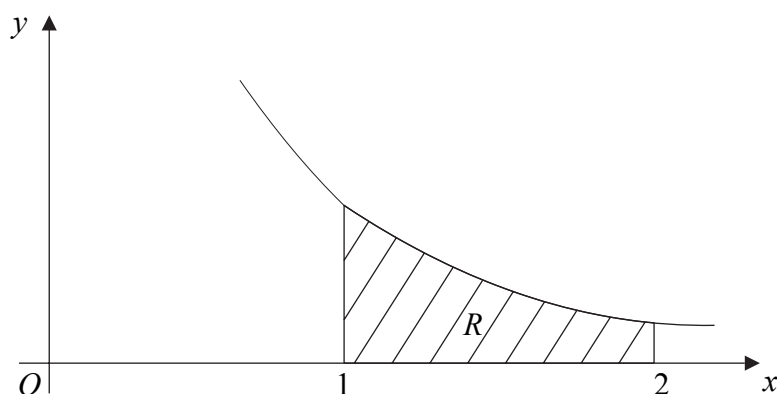
**Question 5 continued**

[illegible]



6.

Figure 4

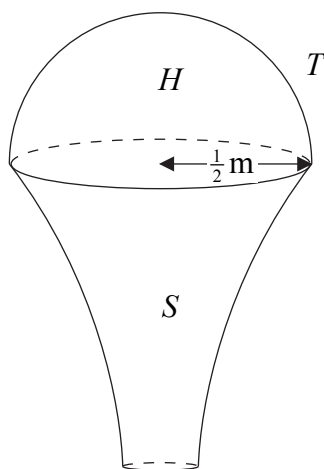


The shaded region  $R$  is bounded by the curve with equation  $y = \frac{1}{2x^2}$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 2$ , as shown in Figure 4. The unit of length on each axis is 1 m. A uniform solid  $S$  has the shape made by rotating  $R$  through  $360^\circ$  about the  $x$ -axis.

- (a) Show that the centre of mass of  $S$  is  $\frac{2}{7}$  m from its larger plane face.

(6)

Figure 5



A sporting trophy  $T$  is a uniform solid hemisphere  $H$  joined to the solid  $S$ . The hemisphere has radius  $\frac{1}{2}$  m and its plane face coincides with the larger plane face of  $S$ , as shown in Figure 5. Both  $H$  and  $S$  are made of the same material.

- (b) Find the distance of the centre of mass of  $T$  from its plane face.

(7)

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**Question 6 continued**

[illegible]

- (a) Show that  $\lambda = 39.2$ .

(2)

(b) Prove that, while the string remains stretched,  $P$  moves with simple harmonic motion of period  $\frac{\pi}{7}$  s. (6)

(6)

- (c) Calculate the speed of  $P$  at the instant when the string first becomes slack.

(3)

(d) Find, to 3 significant figures, the time taken for  $P$  to move from  $B$  to  $C$ .

(5)

[illegible]

