

# Mark Scheme (Results)

## January 2008

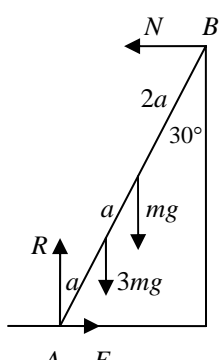
GCE

GCE Mathematics (6678/01)

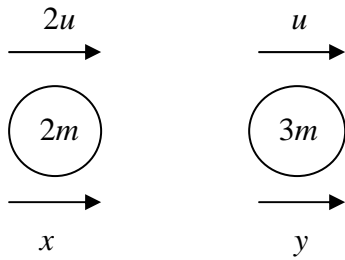
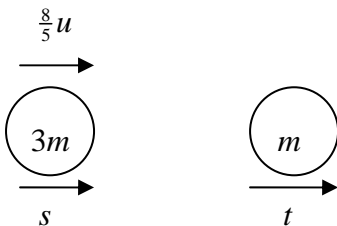
January 2008  
6678 Mechanics M2  
Mark Scheme

Question Number	Scheme	Marks
<b>1.</b>	<p>(a) KE lost is <math>\frac{1}{2} \times 2.5 \times 8^2 = 80 \text{ (J)}</math></p> <p>(b) Work energy <math>80 = R \times 20</math> ft their (a) <math>R = 4</math></p> <p>Alternative to (b) <math>0^2 = 8^2 - 2 \times a \times 20 \Rightarrow a = (-)1.6</math> N2L <math>R = 2.5 \times 1.6</math> ft their <math>a</math> <math>= 4</math></p>	<p>M1 A1 <b>(2)</b></p> <p>M1 A1 ft A1 <b>(3)</b> <b>[5]</b></p> <p>M1 A1ft A1 <b>(3)</b></p>
<b>2.</b>	<p>(a) <math>\dot{\mathbf{p}} = (6t - 6)\mathbf{i} + (9t^2 - 4)\mathbf{j} \text{ (ms}^{-1}\text{)}</math></p> <p>(b) <math>9t^2 - 4 = 0</math> <math>t = \frac{2}{3}</math></p> <p>(c) <math>t = 1 \Rightarrow \dot{\mathbf{p}} = 5\mathbf{j}</math> ft their <math>\dot{p}</math> (+/-) <math>2\mathbf{i} - 6\mathbf{j} = 0.5(\mathbf{v} - 5\mathbf{j})</math> <math>\mathbf{v} = 4\mathbf{i} - 7\mathbf{j} \text{ (ms}^{-1}\text{)}</math></p>	<p>M1 A1 <b>(2)</b></p> <p>M1 DM1 A1 <b>(3)</b></p> <p>B1ft M1 M1 A1 <b>(4)</b> <b>[9]</b></p>

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3.	<p>(a) <math>20000 = 16F</math> (<math>F = 1250</math>)  <math>\nearrow</math> <math>F = 550 + 1000 \times 9.8 \sin \theta</math> ft their <math>F</math>  Leading to <math>\sin \theta = \frac{1}{14}</math> * cso</p> <p>(b) N2L <math>\nearrow</math> <math>550 + 1000 \times 9.8 \times \sin \theta = 1000a</math>  <math>(550 + 1000 \times 9.8 \times \frac{1}{14} = 1000a)</math>  or <math>1250 = 1000a</math>  <math>(a = (-)1.25)</math>  <math>v^2 = u^2 + 2as \Rightarrow 16^2 = 2 \times 1.25 \times y</math>  <math>y \approx 102</math> accept <b>102.4</b>, 100</p> <p>Alternative to (b)  Work-Energy <math>\frac{1}{2} \times 1000 \times 16^2 - 1000 \times 9.8 \times \frac{1}{14} y = 550y</math>  <math>y \approx 102</math> accept <b>102.4</b>, 100</p>	<p>M1 A1  M1 A1ft  A1 (5)</p> <p>M1 A1</p> <p>M1  A1 (4)  [9]</p> <p>M1 M1 A1  A1 (4)</p>
4.	<p>(a) Triangle Circle S  Mass ratio 126 <math>9\pi</math> <math>126 - 9\pi</math>  <math>\bar{x}</math> 7 5 <math>\bar{x}</math>  <math>\bar{y}</math> 4 5 <math>\bar{y}</math> 4, 7 seen</p> <p><math>126 \times 7 = 9\pi \times 5 + (126 - 9\pi) \times \bar{x}</math> ft their table values  <math>\bar{x} \approx 7.58</math> (<math>\frac{882 - 45\pi}{126 - 9\pi}</math>) awrt 7.6</p> <p><math>126 \times 4 = 9\pi \times 5 + (126 - 9\pi) \times \bar{y}</math> ft their table values  <math>\bar{y} \approx 3.71</math> (<math>\frac{504 - 45\pi}{126 - 9\pi}</math>) awrt 3.7</p> <p>(b) <math>\tan \theta = \frac{\bar{y}}{21 - \bar{x}}</math> ft their <math>\bar{x}, \bar{y}</math>  <math>\theta \approx 15^\circ</math></p>	<p>B1 B1ft</p> <p>B1</p> <p>M1 A1ft  A1</p> <p>M1 A1ft  A1 (9)</p> <p>M1 A1ft  A1 (3)  [12]</p>

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5.	<p>(a)</p>  <p>M(A) <math>N \times 4a \cos 30^\circ = 3mg \times a \sin 30^\circ + mg \times 2a \sin 30^\circ</math></p> <p><math>N = \frac{5}{4}mg \tan 30^\circ (= \frac{5}{4\sqrt{3}}mg = 7.07...m)</math></p> <p><math>\rightarrow F_r = N, \quad \uparrow R = 4mg</math></p> <p>Using <math>F_r = \mu R</math></p> <p><math>\frac{5}{4\sqrt{3}}mg = \mu R</math> for their <math>R</math></p> <p><math>\mu = \frac{5}{16\sqrt{3}}</math> awrt 0.18</p> <p>Alternative method:</p> <p>M(B): <math>mg \times 2a \sin 30 + 3mg \times 3a \sin 30 + F \times 4a \cos 30 = R \times 4a \sin 30</math></p> <p><math>11mga \sin 30 + F \times 4a \cos 30 = R \times 4a \sin 30</math></p> <p><math>\frac{11mg}{2} + F \frac{4\sqrt{3}}{2} = 2R</math></p> <p><math>\uparrow R = 4mg,</math></p> <p>Using <math>F_r = \mu R</math></p> <p><math>8\mu\sqrt{3} = \frac{5}{2}, \quad \mu = \frac{5}{16\sqrt{3}}</math></p>	<p>M1 A2(1,0)</p> <p>DM1 A1</p> <p>B1, B1</p> <p>B1</p> <p>M1</p> <p>A1 (10)</p> <p>[10]</p> <p>M1A3(2,1,0)</p> <p>DM1A1</p> <p>B1</p> <p>B1</p> <p>M1 A1</p>

6.	<p>(a)</p> $\begin{aligned} &\rightarrow 30 = 2ut \\ &\uparrow -47.5 = 5ut - 4.9t^2 \\ &-47.5 = 75 - 4.9t^2 \quad \text{eliminating } u \text{ or } t \\ &t^2 = \frac{75 + 47.5}{4.9} (= 25) \\ &t = 5 \quad * \quad \text{cso} \end{aligned}$ <p>(b)</p> $30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$ <p>(c)</p> $\begin{aligned} &\uparrow \dot{y} = 5u - 9.8t = -34 \quad \text{M1 requires both} \\ &\rightarrow \dot{x} = 2u = 6 \quad \dot{x} \text{ and } \dot{y} \\ &v^2 = 6^2 + (-34)^2 \\ &v \approx 34.5 \text{ (ms}^{-1}\text{)} \quad \text{accept 35} \end{aligned}$ <p>Alternative to (c)</p> $\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m \times g \times 47.5 \quad \text{with} \quad v_A^2 = 6^2 + 15^2 = 261$ $v_B^2 = 261 + 2 \times 9.8 \times 47.5 (= 1192)$ $v_B \approx 34.5 \text{ (ms}^{-1}\text{)} \quad \text{accept 35}$ <p>BEWARE : Watch out for incorrect use of <math>v^2 = u^2 + 2as</math></p>	<p>B1 M1 A1 DM1 DM1 A1 (6)</p> <p>M1 A1 (2)</p> <p>M1 A1 A1 DM1 A1 (5)</p> <p>[13]</p> <p>M1 A(2,1,0)</p> <p>DM1 A1 (5)</p>
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7.	<p>(a)</p>  <p>LM <math>4mu + 3mu = 2mx + 3my</math>  NEL <math>y - x = \frac{1}{2}u</math>  Solving to <math>y = \frac{8}{5}u</math> *</p> <p>(b)</p> <p><math>x = \frac{11}{10}u</math> or equivalent  Energy loss <math>\frac{1}{2} \times 2m \left( (2u)^2 - \left( \frac{11}{10}u \right)^2 \right) + \frac{1}{2} \times 3m \left( u^2 - \left( \frac{8}{5}u \right)^2 \right)</math>  <math>= \frac{9}{20}mu^2</math></p> <p>(c)</p>  <p>LM <math>\frac{24}{5}mu = 3ms + mt</math>  NEL <math>t - s = \frac{8}{5}eu</math>  Solving to <math>s = \frac{2}{5}u(3 - e)</math></p> <p>For a further collision <math>\frac{11}{10}u &gt; \frac{2}{5}u(3 - e)</math>  <math>e &gt; \frac{1}{4}</math> ignore <math>e \leq 1</math></p>	<p>M1 A1  B1  M1 A1 (5)</p> <p>B1  M1 A(2,1,0)  A1 (5)</p> <p>M1 A1  B1  M1 A1</p> <p>M1  A1 (7)  [17]</p>