

Mark Scheme (Results) Summer 2010

GCE

Further Pure Mathematics FP2 (6668)



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June 2010 Further Pure Mathematics FP2 6668 Mark Scheme

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| Question Number | Scheme | Marks |
| 1(a) | $\frac{1}{3r-1} - \frac{1}{3r+2}$ | M1 A1 (2) |
| (b) | $\sum_{r=1}^{n} \frac{3}{(3r-1)(3r+2)} = \frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \dots + \frac{1}{3n-1} - \frac{1}{3n+2}$ | M1 A1ft |
| | $= \frac{1}{2} - \frac{1}{3n+2} = \frac{3n}{2(3n+2)} $ | A1 (3) |
| (c) | Sum = f(1000) - f(99) $\frac{3000}{6004} - \frac{297}{598} = 0.00301 \text{or } 3.01 \times 10^{-3}$ | M1 A1 (2) |
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| Question Number | Scheme | Marks |
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| 2 | $f''(t) = -x - \cos x,$ $f''(0) = -1$ | B1 |
| | $f'''(t) = (-1 + \sin x) \frac{dx}{dt}, \qquad f'''(0) = -0.5$ | M1A1 |
| | $f(t) = f(0) + tf'(0) + \frac{t^2}{2}f''(0) + \frac{t^3}{3!}f'''(0) + \dots$ $= 0.5t - 0.5t^2 - \frac{1}{12}t^3 + \dots$ | M1 A1 5 |
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| Question Number | Scheme | Mark | s |
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| 3(a) | $(x+4)(x+3)^2 - 2(x+3) = 0$, $(x+3)(x^2+7x+10) = 0$ so $(x+2)(x+3)(x+5) = 0$ or alternative method including calculator | M1 | |
| | Finds critical values –2 and -5 | A1 A1 | |
| | Establishes $x > -2$ | A1ft | |
| | Finds and uses critical value -3 to give $-5 < x < -3$ | M1A1 | (6) |
| (b) | x > -2 | B1ft | (1) |
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| Question Number | Scheme | Marks |
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| 4(a) | Modulus = 16 | B1 |
| | Argument = $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$ | M1A1 (3) |
| (b) | $z^{3} = 16^{3} \left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)^{3} = 16^{3} \left(\cos 2\pi + i\sin 2\pi\right) = 4096 \text{ or } 16^{3}$ | M1 A1 (2) |
| (c) | $w = 16^{\frac{1}{4}} \left(\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})\right)^{\frac{1}{4}} = 2\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right) \left(=\sqrt{3} + i\right)$ | M1 A1ft |
| | OR $-1+\sqrt{3}i$ OR $-\sqrt{3}-i$ OR $1-\sqrt{3}i$ | M1A2(1,0) (5) |
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| Question Number | Scheme | Marks |
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| 5(a) | $1.5 + \sin 3\theta = 2 \to \sin 3\theta = 0.5 \therefore 3\theta = \frac{\pi}{6} \left(\text{or } \frac{5\pi}{6} \right),$ | M1 A1, |
| | and $:: \theta = \frac{\pi}{18}$ or $\frac{5\pi}{18}$ | A1 (3) |
| (b) | Area = $\frac{1}{2} \left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (1.5 + \sin 3\theta)^2 d\theta \right], -\frac{1}{9}\pi \times 2^2$ | - M1, M1 |
| | $= \frac{1}{2} \left[\int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} (2.25 + 3\sin 3\theta + \frac{1}{2}(1 - \cos 6\theta)) d\theta \right] - \frac{1}{9}\pi \times 2^{2}$ | - M1 |
| | $= \frac{1}{2} \left[(2.25\theta - \cos 3\theta + \frac{1}{2}(\theta - \frac{1}{6}\sin 6\theta)) \right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \frac{1}{9}\pi \times 2^{2}$ | -M1 A1 |
| | $= \frac{13\sqrt{3}}{24} - \frac{5\pi}{36}$ | M1 A1 (7) |
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| Question Number | Scheme | Marks | | |
| 6(a) | Re(z) = 3 Real axis Vertical Straight line Through 3 on real axis | B1 B1 | 2) | |
| (b) | These are points where line $x = 3$ meets the circle centre (3, 4) with radius 5. The complex numbers are $3 + 9i$ and $3 - i$. | M1 A1 A1 | (3) | |
| (c) | $ z-6 = z \Rightarrow \left \frac{30}{w} - 6 \right = \left \frac{30}{w} \right $ $\therefore 30 - 6w = 30 \Rightarrow \therefore 5 - w = 5 $ This is a circle with Cartesian equation $(u-5)^2 + v^2 = 25$ | | 5) | |

| Question Number | Scheme | Mark | S |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-----|
| 7(a) | $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$ and $\frac{dy}{dz} = 2z$ so $\frac{dy}{dx} = 2z \cdot \frac{dz}{dx}$ | M1 M1 | A1 |
| | Substituting to get $2z \cdot \frac{dz}{dx} - 4z^2 \tan x = 2z$ and thus $\frac{dz}{dx} - 2z \tan x = 1$ | M1 A1 | (5) |
| (b) | $I.F. = e^{\int -2\tan x dx} = e^{2\ln \cos x} = \cos^2 x$ | M1 A1 | |
| | $\therefore \frac{\mathrm{d}}{\mathrm{d}x} \left(z \cos^2 x \right) = \cos^2 x \ \therefore z \cos^2 x = \int \cos^2 x dx$ | M1 | |
| | $\therefore z \cos^2 x = \int \frac{1}{2} (\cos 2x + 1) dx = \frac{1}{4} \sin 2x + \frac{1}{2} x + c$ $\therefore z = \frac{1}{2} \tan x + \frac{1}{2} x \sec^2 x + c \sec^2 x$ | M1 A1 | |
| | $\therefore \zeta - \frac{1}{2} \tanh x + \frac{1}{2} x \sec x + c \sec x$ | A1 | (6) |
| (c) | $\therefore y = (\frac{1}{2} \tan x + \frac{1}{2} x \sec^2 x + c \sec^2 x)^2$ | B1ft | (1) |
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| Question Number | Scheme | Mark | S |
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| 8(a) | Differentiate twice and obtaining $\frac{dy}{dx} = \lambda \sin 5x + 5\lambda x \cos 5x \text{ and } \frac{d^2y}{dx^2} = 10\lambda \cos 5x - 25\lambda x \sin 5x$ | M1 A1 | |
| | Substitute to give $\lambda = \frac{3}{10}$ | M1 A1 | (4) |
| (b) | Complementary function is $y = A\cos 5x + B\sin 5x$ or $Pe^{5ix} + Qe^{-5ix}$ | M1 A1 | |
| | So general solution is $y = A\cos 5x + B\sin 5x + \frac{3}{10}x\sin 5x$ or in exponential form | A1ft | (3) |
| (c) | y=0 when $x=0$ means $A=0$ | B1 | |
| | $\frac{dy}{dx} = 5B\cos 5x + \frac{3}{10}\sin 5x + \frac{3}{2}x\cos 5x \text{ and at } x = 0 $ $\frac{dy}{dx} = 5 \text{ and so } 5 = 5A$ | M1 M1 | |
| | So $B = 1$ | A1 | |
| | So $y = \sin 5x + \frac{3}{10}x\sin 5x$ | A1 | (5) |
| (d) | "Sinusoidal" through O amplitude becoming larger Crosses x axis at | B1 | |
| | The crosses x axis at $\frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}$ | B1 | |
| | ind | | (2) |
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