

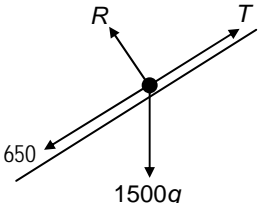
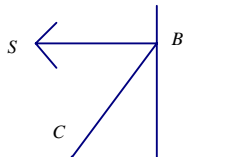
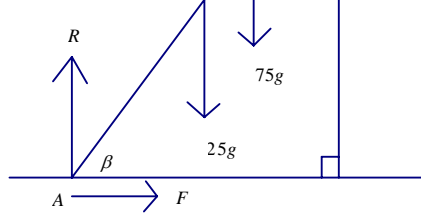
Mark Scheme (Results)

January 2009




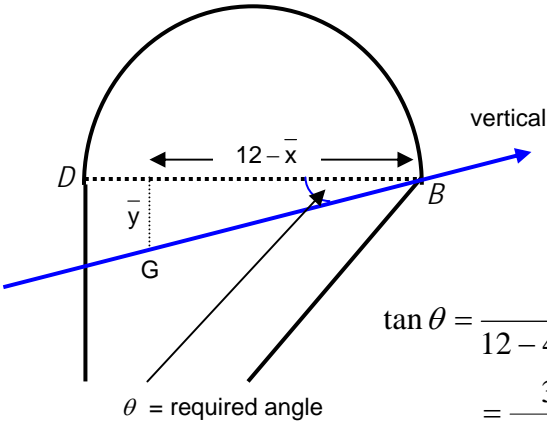
GCE

GCE Mathematics (6678/01)

January 2009
6678 Mechanics M2
Mark Scheme

Question Number	Scheme		Marks
1		$F = ma$ parallel to the slope, $T - 1500g \sin \theta - 650 = 1500a$ Tractive force, $30000 = T \times 15$ $a = \frac{\frac{30000}{15} - 1500(9.8)(\frac{1}{14}) - 650}{1500}$ $0.2 \text{ (m s}^{-2}\text{)}$	M1* A1 M1* d*M1 A1 (5) [5]
2	<div style="display: flex; flex-direction: column; align-items: flex-start;"> <div style="margin-bottom: 20px;"> (a)  </div> <div> (b)  </div> </div>	$R(\uparrow): R = 25g + 75g (= 100g)$ $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ $= 44g (= 431)$ $M(A):$ $25g \times 2 \cos \beta + 75g \times 2.8 \cos \beta$ $= S \times 4 \sin \beta$ $R(\leftrightarrow): F = S$ $176g \sin \beta = 260g \cos \beta$ $\beta = 56(^{\circ})$	B1 M1 A1 (3) M1 A2, 1, 0 M1A1 A1 (6) B1 [10]
	(c)	So that Reece's weight acts directly at the point C.	

Question Number	Scheme	Marks
<p>3 (a)</p> <div data-bbox="292 331 603 577" data-label="Diagram"> </div>	$R(\uparrow) : R = 10g$ $F = \mu R \Rightarrow F = \frac{4}{7}(10g) = 56$ $\therefore \text{WD against friction} = \frac{4}{7}(10g)(50)$ $2800(\text{J})$ <p>(b)</p> $70(50) - "2800" = \frac{1}{2}(10)v^2 - \frac{1}{2}(10)(2)^2$ $700 = 5v^2 - 20, \quad 5v^2 = 720 \Rightarrow v^2 = 144$ <p>Hence, $v = \underline{12} \text{ (m s}^{-1}\text{)}$</p> <p>Or (b)</p> $\text{N2L}(\rightarrow): 70 - \frac{4}{7}R = 10a$ $70 - \frac{4}{7} \times 10g = 10a, \quad (a = 1.4)$ $\text{AB}(\rightarrow): v^2 = (2)^2 + 2(1.4)(50)$ <p>Hence, $v = \underline{12} \text{ (m s}^{-1}\text{)}$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(4)</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>(4)</p> <p>M1*</p> <p>A1ft</p> <p>d*M1</p> <p>A1 cao</p> <p>(4)</p> <p>[8]</p>
<p>4 (a)</p>	$v = 10t - 2t^2, \quad s = \int v dt$ $= 5t^2 - \frac{2t^3}{3} (+C)$ $t = 6 \Rightarrow s = 180 - 144 = \underline{36} \text{ (m)}$ <p>(b)</p> $\underline{s} = \int v dt = \frac{-432t^{-1}}{-1} (+K) = \frac{432}{t} (+K)$ $t = 6, s = "36" \Rightarrow 36 = \frac{432}{6} + K$ $\Rightarrow K = -36$ $\text{At } t = 10, s = \frac{432}{10} - 36 = \underline{7.2} \text{ (m)}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>d*M1</p> <p>A1</p> <p>(5)</p> <p>[8]</p>

Question Number	Scheme				Marks
5	(a)	 MR 108	 18π	 $108 + 18\pi$	B1
		$x_i (\rightarrow)$ from AD 4	6	\bar{x}	B1
		$y_i (\downarrow)$ from BD 6	$-\frac{8}{\pi}$	\bar{y}	
		$AD(\rightarrow): 108(4) + 18\pi(6) = (108 + 18\pi)\bar{x}$ $\bar{x} = \frac{432 + 108\pi}{108 + 18\pi} = 4.68731... = \underline{4.69} \text{ (cm) (3 sf) AG}$			M1 A1 (4)
(b)		$y_i (\downarrow)$ from BD 6	$-\frac{8}{\pi}$	\bar{y}	B1 oe
		$BD(\downarrow): 108(6) + 18\pi(-\frac{8}{\pi}) = (108 + 18\pi)\bar{y}$ $\bar{y} = \frac{504}{108 + 18\pi} = 3.06292... = \underline{3.06} \text{ (cm) (3 sf)}$			M1 A1ft A1 (4)
	(c)	 $\tan \theta = \frac{\bar{y}}{12 - 4.68731..}$ $= \frac{3.06392..}{12 - 4.68731..}$			M1 dM1 A1 A1 (4) [12]
		$\theta = 22.72641... = \underline{23} \text{ (nearest degree)}$			

Question Number	Scheme	Marks
6	<p>(a) Horizontal distance: $57.6 = p \times 3$ $p = 19.2$</p> <p>(b) Use $s = ut + \frac{1}{2}at^2$ for vertical displacement. $-0.9 = q \times 3 - \frac{1}{2}g \times 3^2$ $-0.9 = 3q - \frac{9g}{2} = 3q - 44.1$ $q = \frac{43.2}{3} = 14.4$ *AG*</p> <p>(c) initial speed $\sqrt{p^2 + 14.4^2}$ (with their p) $= \sqrt{576} = \underline{24} \text{ (m s}^{-1}\text{)}$</p> <p>(d) $\tan \alpha = \frac{14.4}{p} (= \frac{3}{4})$ (with their p)</p> <p>(e) When the ball is 4 m above ground: $3.1 = ut + \frac{1}{2}at^2$ used $3.1 = 14.4t - \frac{1}{2}gt^2$ o.e. ($4.9t^2 - 14.4t + 3.1 = 0$) $\Rightarrow t = \frac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied $t = \frac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389... \text{ or } 2.70488...$ awrt 0.23 and 2.7 duration = $2.70488... - 0.023389...$ $= 2.47 \text{ or } 2.5 \text{ (seconds)}$</p>	<p>M1 A1 (2)</p> <p>M1 A1 A1 cso (3)</p> <p>M1 A1 cao (2)</p> <p>B1 (1)</p> <p>M1 A1 M1 A1 (6)</p>
or 6	<p>(e) M1A1M1 as above $t = \frac{14.4 \pm \sqrt{146.6}}{9.8}$ Duration $2 \times \frac{\sqrt{146.6}}{9.8}$ o.e. $= 2.47 \text{ or } 2.5 \text{ (seconds)}$</p> <p>(f) Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball, The ball is not a particle.</p>	<p>A1 M1 A1 (6)</p> <p>B1 (1)</p>

[15]

Question Number	Scheme	Marks
7 (a)	<p>Before $\xrightarrow{2u}$ \xleftarrow{u}</p> <p>P $(3m)$ Q $(2m)$</p> <p>After \xrightarrow{x} \xrightarrow{y}</p> <p>Correct use of NEL</p> <p>$y - x = e(2u + u)$ o.e.</p> <p>CLM (\rightarrow): $3m(2u) + 2m(-u) = 3m(x) + 2m(y)$ ($\Rightarrow 4u = 3x + 2y$)</p> <p>Hence $x = y - 3eu$, $4u = 3(y - 3eu) + 2y$, ($u(9e + 4) = 5y$)</p> <p>Hence, speed of $Q = \frac{1}{5}(9e + 4)u$ AG</p>	<p>M1*</p> <p>A1</p> <p>B1</p> <p>d*M1</p> <p>A1 cso</p> <p>(5)</p>
(b)	<p>$x = y - 3eu = \frac{1}{5}(9e + 4)u - 3eu$</p> <p>Hence, speed $P = \frac{1}{5}(4 - 6e)u = \frac{2u}{5}(2 - 3e)$ o.e.</p> <p>$x = \frac{1}{2}u = \frac{2u}{5}(2 - 3e) \Rightarrow 5u = 8u - 12eu, \Rightarrow 12e = 3$ & solve for e</p> <p>gives, $e = \frac{3}{12} \Rightarrow e = \frac{1}{4}$ AG</p>	<p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p>(4)</p>
Or (b)	<p>Using NEL correctly with given speeds of P and Q</p> <p>$3eu = \frac{1}{5}(9e + 4)u - \frac{1}{2}u$</p> <p>$3eu = \frac{9}{5}eu + \frac{4}{5}u - \frac{1}{2}u$, $3e - \frac{9}{5}e = \frac{4}{5} - \frac{1}{2}$ & solve for e</p> <p>$\frac{6}{5}e = \frac{3}{10} \Rightarrow e = \frac{15}{60} \Rightarrow e = \frac{1}{4}$.</p>	<p>M1#</p> <p>A1</p> <p>d#M1</p> <p>A1</p> <p>(4)</p>
(c)	<p>Time taken by Q from A to the wall $= \frac{d}{y} = \left\{ \frac{4d}{5u} \right\}$</p> <p>Distance moved by P in this time $= \frac{u}{2} \times \frac{d}{y} (= \frac{u}{2} \left(\frac{4d}{5u} \right) = \frac{2}{5}d)$</p> <p>Distance of P from wall $= d - x \left(\frac{d}{y} \right); = d - \frac{2}{5}d = \frac{3}{5}d$ AG</p>	<p>M1[†]</p> <p>A1</p> <p>d[†]M1;</p> <p>A1 cso</p> <p>(4)</p>
or (c)	<p>Ratio speed P:speed $Q = x:y = \frac{1}{2}u : \frac{1}{5} \left(\frac{9}{4} + 4 \right)u = \frac{1}{2}u : \frac{5}{4}u = 2:5$</p> <p>So if Q moves a distance d, P will move a distance $\frac{2}{5}d$</p> <p>Distance of P from wall $= d - \frac{2}{5}d; = \frac{3}{5}d$ AG</p>	<p>M1[†]</p> <p>A1</p> <p>d[†]M1; A1</p> <p>(4)</p>

Question Number	Scheme	Marks
(d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>Time for P, $T_{AB} = \frac{\frac{3d}{5} - x}{\frac{1}{2}u}$, Time for Q, $T_{WB} = \frac{x}{\frac{1}{4}u}$ from their y</p> <p>Hence $T_{AB} = T_{WB} \Rightarrow \frac{\frac{3d}{5} - x}{\frac{1}{2}u} = \frac{x}{\frac{1}{4}u}$</p> <p>gives, $2\left(\frac{3d}{5} - x\right) = 4x \Rightarrow \frac{3d}{5} - x = 2x, 3x = \frac{3d}{5} \Rightarrow x = \frac{1}{5}d$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1 cao</p> <p>(4)</p>
or (d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>speed $P = x = \frac{1}{2}u$, speed P: new speed $Q = \frac{1}{2}u : \frac{1}{4}u = 2:1$ from their y</p> <p>Distance of B from wall = $\frac{1}{3} \times \frac{3d}{5} = \frac{d}{5}$ their $\frac{1}{2+1}$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1; A1</p> <p>(4)</p>
2 nd or (d)	<p>After collision with wall, speed $Q = \frac{1}{5}y = \frac{1}{5}\left(\frac{5u}{4}\right) = \frac{1}{4}u$ their y</p> <p>Combined speed of P and $Q = \frac{1}{2}u + \frac{1}{4}u = \frac{3}{4}u$</p> <p>Time from wall to 2nd collision = $\frac{\frac{3d}{5}}{\frac{3u}{4}} = \frac{3d}{5} \times \frac{4}{3u} = \frac{4d}{5u}$ from their y</p> <p>Distance of B from wall = (their speed)x(their time) = $\frac{u}{4} \times \frac{4d}{5u} = \frac{1}{5}d$</p>	<p>B1ft</p> <p>B1ft</p> <p>M1; A1</p> <p>(4)</p> <p>[17]</p>