edexcel

Mark Scheme (Results)

Summer 2015

Pearson Edexcel International A Level
in Mechanics 3 (WME03/01)

PEARSON

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ✳ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking
(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

  Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A)   Taking moments about A.

  N2L   Newton's Second Law (Equation of Motion)

  NEL   Newton's Experimental Law (Newton's Law of Impact)

  HL   Hooke's Law

  SHM   Simple harmonic motion

  PCLM   Principle of conservation of linear momentum

  RHS, LHS   Right hand side, left hand side.

June 2015
WME03 M3
Mark Scheme

Question Number	Scheme	Marks
1	(30º or θ for the first 3 lines) $R\sin 30° = mg$ $R\cos 30° = m\left(r\cos 30°\right)\omega^2$ $\omega^2 = \dfrac{R}{mr} = \dfrac{g}{r\sin 30}$ $\omega = \sqrt{\dfrac{2g}{r}}$ Time $= \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{r}{2g}} = \pi\sqrt{\dfrac{2r}{g}}$ $*$	M1A1 M1A1A1 DM1 A1 A1cso [8]

M1   Resolving vertically   30º or θ
A1   Correct equation   30º or θ
M1   Attempting an equation of motion along the radius, acceleration in either form   30º or θ
Allow with *r* for radius
A1   LHS correct   30º or θ
A1   RHS correct, 30º or θ but not *r* for radius
DM1   Obtaining an expression for ω^2 or for v^2 **and** the length of the path   30º or θ Dependent on both previous M marks
A1   Correct expression for ω Must have the numerical value for the trig function now
A1cso   Deducing the GIVEN answer

ALT: Resolve perpendicular to the reaction:
$mg\cos 30 = m\times rad\times\omega^2\cos 60$ M2A1(LHS) A1(RHS)
$\qquad\qquad = mr\cos 30\omega^2\cos 60$ A1
Obtain ω M1A1
Correct time   A1

Question Number	Scheme	Marks
2 (a)	$F = \dfrac{K}{x^2}$ $x = R \Rightarrow F = mg \qquad \therefore mg = \dfrac{K}{R^2}$ $K = mgR^2 \quad *$	M1 A1 (2)
(b)	$\dfrac{mgR^2}{x^2} = -mv\dfrac{\mathrm{d}v}{\mathrm{d}x}$ $g\displaystyle\int \dfrac{R^2}{x^2}\,\mathrm{d}x = -\int v\,\mathrm{d}v$ $-g\dfrac{R^2}{x} = -\dfrac{1}{2}v^2 \ (+c)$ $x = 3R,\ v = V \Rightarrow -g\dfrac{R^2}{3R} = -\dfrac{1}{2}V^2 + c$ $c = -\dfrac{Rg}{3} + \dfrac{1}{2}V^2$ $x = R \Rightarrow \dfrac{1}{2}v^2 = -\dfrac{Rg}{3} + \dfrac{1}{2}V^2 + g\dfrac{R^2}{R}$ $v^2 = V^2 + \dfrac{4Rg}{3}$ $v = \sqrt{V^2 + \dfrac{4Rg}{3}}$	M1 DM1A1ft M1 A1 M1 A1 cso (7)
		[9]

(a) M1   Setting $F = mg$ **and** $x = R$

   A1   Deducing the GIVEN answer

(b) M1   Attempting an equation of motion with acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. The minus sign may be missing.

 DM1   Attempting the integration

 A1ft   Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing

  M1   Substituting $x = 3R, v = V$ to obtain an equation for c

   A1   Correct expression for c

  M1   Substituting $x = R$ and their expression for c

   A1   Correct expression for v, any equivalent form

Question Number	Scheme	Marks
3 (a)	$\dfrac{\mathrm{d}v}{\mathrm{d}t} = -2(t+4)^{-\frac{1}{2}}$	M1
	$v = -\int 2(t+4)^{-\frac{1}{2}}\,\mathrm{d}t$	
	$v = -4(t+4)^{\frac{1}{2}} \quad (+c)$	DM1A1
	$t = 0, v = 8 \Rightarrow c = 16$	M1
	$v = 16 - 4(t+4)^{\frac{1}{2}} \quad (\text{m s}^{-1}) \quad \ast$	A1cso (5)
(b)	$v = 0 \quad 16 = 4(t+4)^{\frac{1}{2}}$	M1
	$16 = t + 4 \quad t = 12$	A1
	$x = 4\int\left(4 - (t+4)^{\frac{1}{2}}\right)\mathrm{d}t$	
	$x = 4\left(4t - \dfrac{2}{3}(t+4)^{\frac{3}{2}}\right) \quad (+d)$	M1A1
	$t = 0, \ x = 0 \quad d = 4 \times \dfrac{2}{3} \times 4^{\frac{3}{2}} = \dfrac{64}{3} \qquad \text{oe}$	A1
	$t = 12 \quad x = 4\left(4 \times 12 - \dfrac{2}{3} \times 16^{\frac{3}{2}}\right) + \dfrac{64}{3} = 42\dfrac{2}{3} \ (\text{m}) \quad \text{oe eg 43 or better}$	DM1A1 (7)
		[12]

(a) **M1**   Attempting an expression for the acceleration in the form $\dfrac{\mathrm{d}v}{\mathrm{d}t}$; minus may be omitted.

DM1   Attempting the integration

A1   Correct integration, constant of integration may be omitted (no ft)

M1   Using the initial conditions to obtain a value for the constant of integration

A1cso   Substitute the value of c and obtain the final GIVEN answer

(b) **M1**   Setting the **given** expression for v equal to 0

A1   Solving to get $t = 12$

M1   Setting $v = \dfrac{\mathrm{d}x}{\mathrm{d}t}$ and attempting the integration wrt t At least one term must clearly be Integrated.

A1   Correct integration, constant may be omitted

A1   Substituting $t = 0$, $x = 0$ and obtaining the correct value of d. Any equivalent number, inc decimals.

DM1   Substituting their value for t and obtaining a value for the required distance. Dependent on The second M mark.

A1   Correct final answer, any equivalent form.

Question Number	Scheme	Marks
4(a)	Energy to top: $\dfrac{1}{2} \times 3m \times u^2 - \dfrac{1}{2} \times 3mv^2 = 3mga$	M1A1
	NL2 at top: $T + 3mg = 3m\dfrac{v^2}{a}$	M1A1
	$T = 3m\dfrac{u^2}{a} - 6mg - 3mg$	DM1
	$T \geqslant 0 \Rightarrow \dfrac{u^2}{a} \geqslant 3g$	M1
	$u^2 \geqslant 3ag$ ＊	A1 cso (7)
(b)	Tension at bottom:	
	$\dfrac{1}{2} \times 3m \times V^2 - \dfrac{1}{2} \times 3mu^2 = 3mga$	M1
	$T_{\max} - 3mg = 3m\dfrac{V^2}{a}$	M1
	$T_{\max} = 3mg + 6mg + 3m\dfrac{u^2}{a}$	A1
	$T_{\min} = 3m\dfrac{u^2}{a} - 9mg$	
	$9mg + 3m\dfrac{u^2}{a} = 3\left(3m\dfrac{u^2}{a} - 9mg\right)$	DM1
	$u^2 = 6ag$ ＊	A1 cso (5)
		[12]

(a) M1   Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of $v^2 = u^2 + 2as$ scores M0

   A1   Correct equation from A to the top

   M1   Attempting an equation of motion along the radius at the top, acceleration in either form

   A1   Correct equation, acceleration in form $\dfrac{v^2}{r}$

DM1   Eliminate v^2 to obtain an expression for T Dependent on both previous M marks

   M1   Use $T \geqslant 0$ at top to obtain an inequality connecting a, g and u

A1cso   Re-arrange to obtain the GIVEN answer

(b) M1   Attempting an energy equation to the bottom, maybe from A or from the top

   M1   Attempting an equation of motion along the radius at the bottom

   A1   Correct expression for the max tension

DM1   Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks

A1cso   Obtaining the GIVEN answer.

Question Number	Scheme	Marks
5 (a)	$T = \dfrac{20e}{2} = \dfrac{15(1.8-e)}{1.2}$	M1A1
	$10e \times 1.2 = 15(1.8-e)$	
	$e = 1$	A1
	$AO = 3\,\text{m} \quad *$	A1cso (4)
(b)	$0.5\ddot{x} = \dfrac{20(1-x)}{2} - \dfrac{15(0.8+x)}{1.2}$	M1A1A1
	$\ddot{x} = -45x \quad \therefore \text{SHM}$	A1cso (4)
(c)	String becomes slack when $x = (-)0.8$ (allow wo sign due to symmetry)	B1
	$v^2 = \omega^2\left(a^2 - x^2\right)$	
	$v^2 = 45\left(1 - 0.8^2\right) \quad (=16.2)$	M1A1ft
	$v = 4.024...\ \text{m s}^{-1}$ (4.0 or better)	A1ft (4)
(d)	$\dfrac{1}{2} \times \dfrac{20y^2}{2} - \dfrac{1}{2} \times \dfrac{20 \times 1.8^2}{2} = \dfrac{1}{2} \times 0.5 \times 16.2 \qquad\qquad$ ft on v	M1A1A1ft
	$20y^2 - 64.8 = 16.2$	
	$y^2 = 4.05 \quad y = 2.012....$	A1
	Distance $DB = \left|5 - 4.012...\right| = 0.988...\text{m}$ (accept 0.99 or better)	A1ft (5) [17]
Alt for d:	Prove SHM with only one string M1A1 (equation)	
	Value ω A1	
	Use $v = a\omega$ to find a A1ft (ft on v)	
	Dist A1ft	

(a) M1 Attempting to obtain and equate the tensions in the two parts of the string.
 A1 Correct equation, extension in AP or BP can be used or use OA as the unknown
 A1 Obtaining the correct extension in either string (ext in BP =0.8 m) or another useful distance
 A1cso Obtaining the correct GIVEN answer
(b) M1 Forming an equation of motion at a general point. There must be a difference of tensions, both with the variable. May have m instead of 0.5 Accel can be a
 A1 A1 Deduct 1 for each error, m or 0.5 allowed, acceleration to be $\ddot{x}$ now
 A1cso Correct equation in the required form, with a concluding statement; m or 0.5 allowed
(c) B1 For $x = \pm0.8$ Need not be shown explicitly
 M1 Using $v^2 = \omega^2\left(a^2 - x^2\right)$ with *their* (numerical) ω and their x
 A1ft Equation with correct numbers ft their ω
 A1ft Correct value for v 2sf or better or exact
(d) M1 Attempting an energy equation with 2 EPE terms and a KE term
 A1 2 correct terms may have $(1.8+x)$ instead of y
 A1ft Completely correct equation, follow through their v from (c)
 A1 Correct value for distance travelled after PB became slack. $x = 0.21$
 A1ft Complete to the distance DB. Follow through their distance travelled after PB became slack.
 Alternatives at end of mark scheme

Question Number	Scheme	Marks
6(a)	$\text{Vol} = \pi \int_0^2 \left(x^2 + 3 \right)^2 \, dx$	M1
	$= \pi \int_0^2 \left(x^4 + 6x^2 + 9 \right) dx$	
	$= \pi \left[\dfrac{1}{5} x^5 + 2x^3 + 9x \right]_0^2$	DM1A1
	$= \dfrac{202}{5} \pi \qquad \text{cm}^3 \quad *$	A1 (4)
(b)	$\pi \int_0^2 x \left(x^2 + 3 \right)^2 dx = \pi \int_0^2 \left(x^5 + 6x^3 + 9x \right) dx$	M1
	$= \pi \left[\dfrac{1}{6} x^6 + \dfrac{3}{2} x^4 + \dfrac{9}{2} x^2 \right]_0^2$	A1
	$= \dfrac{158}{3} \pi$ (Or by chain rule or substitution)	A1
	C of m $= \dfrac{158}{3} \times \dfrac{5}{202}, \ = 1.3036... = 1.30 \quad \text{cm}$	M1A1 (5)
(c)	Mass ratio $\qquad 2 \times \dfrac{202}{5} \pi \qquad \dfrac{1}{3} \pi \times 7^2 \times 6 \qquad \left(\dfrac{404}{5} + 98 \right) \pi$	B1
	Dist from V $\qquad$ 6.7 $\qquad\qquad$ 4.5 $\qquad\qquad\qquad \overline{x}$	B1
	$\dfrac{404}{5} \times 6.7 + 98 \times 4.5 = \left(\dfrac{404}{5} + 98 \right) \overline{x}$	M1A1ft
	$\overline{x} = \dfrac{\dfrac{404}{5} \times 6.7 + 98 \times 4.5}{\left(\dfrac{404}{5} + 98 \right)} = 5.494.... = 5.5 \ \text{cm} \quad \text{Accept 5.49 or better}$	A1 (5)
(d)	$\tan \theta = \dfrac{6 - \overline{x}}{7} = \dfrac{0.5058...}{7}$	M1
	$\alpha = \tan^{-1} \left(\dfrac{6}{7} \right) - \tan^{-1} \left(\dfrac{0.5058...}{7} \right) = 36.468...° = 36° \quad \text{or better}$	M1A1 (3) [17]

Question Number	Scheme	Marks

(a) M1   Using $\pi \int y^2 \mathrm{d}x$ with the equation of the curve, no limits needed

  DM1   Integrating their expression for the volume

  A1   Correct integration inc limits now

  A1   Substituting the limits to obtain the GIVEN answer

(b) M1   Using $(\pi) \int xy^2 \mathrm{d}x$ with the equation of the curve, no limits needed, π can be omitted

  A1   Correct integration, including limits; no substitution needed for this mark

  A1   Correct substitution of limits

  M1   Use of $\dfrac{\pi \int xy^2 \mathrm{d}x}{\pi \int y^2 \mathrm{d}x}$ with their $\pi \int xy^2 \mathrm{d}x$. π must be seen in both numerator and denominator or in neither.

  A1cao   Correct answer. Must be 1.30

(c) B1   Correct mass ratio

  B1   Correct distances, from V or any other point, provided consistent

  M1   Attempting a moments equation

  A1ft   Correct equation, follow through their distances and mass ratio

  A1   Correct distance from V

(d) M1   Attempting the tan of an appropriate angle, numbers either way up

  M1   Attempting to obtain the required angle

  A1   Correct final answer 2sf or more

Alternatives for 5(d)

1   EPE in BP (at release) transferred to EPE in AP (same as MS, except 1 term for first A1)

2

$$0.5a = -10(1.8 + x)$$

$$v\frac{dv}{dx} = -36 - 10x$$

$$\int v\,dv = -\int (36 + 10x)\,dx$$

$$\frac{v^2}{2} = -36x + 5x^2 + c \qquad\qquad \text{M1A1}$$

$$x = 0, v = \frac{9\sqrt{5}}{5} \therefore c = 8.1 \qquad\qquad \text{A1}$$

Then $v = 0$ etc   M1A1