8. Radius of Curvature

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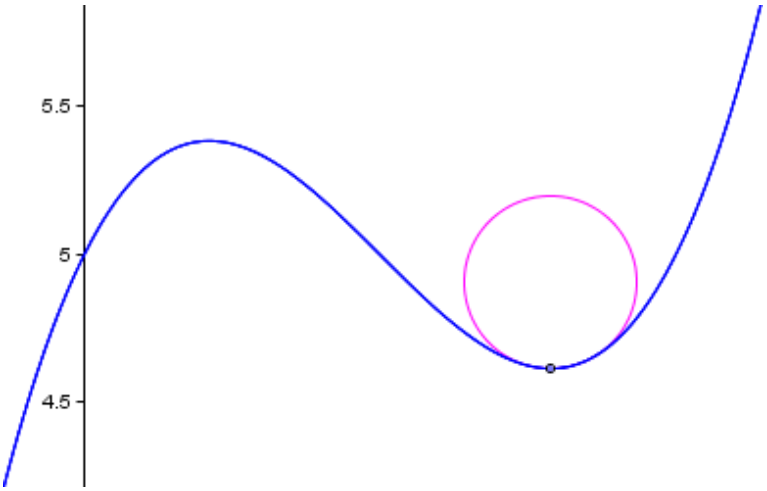
We can draw a circle that closely fits nearby points on a local section of a curve, as follows.



We say the curve and the circle **osculate** (which means "to kiss"), since the 2 curves have the same tangent and curvature at the point where they meet.

The **radius of curvature** of the curve at a particular point is defined as the radius of the approximating circle. This radius changes as we move along the curve.

How do we find this changing radius of curvature?

The formula for the radius of curvature at any point x for the curve $y = f(x)$ is given by:

$$\text{Radius of curvature} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Proof

Example 1

Find the radius of curvature for the cubic

$$y = 2x^3 - x + 3$$

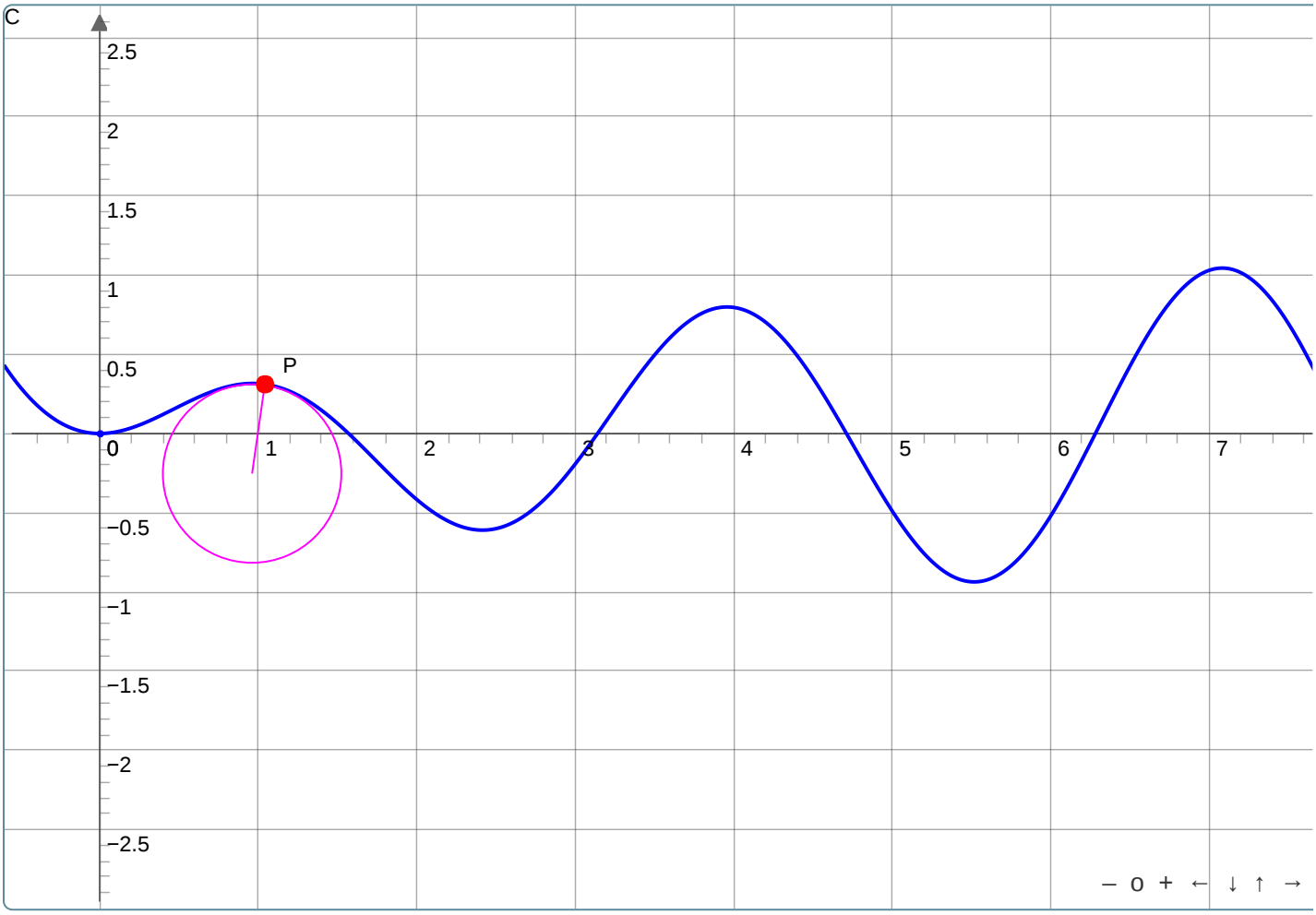
at the point $x = 1$.

Show answer

Exploration

In the following interactive graph you can explore what "changing radius of curvature" means.

Slowly drag the point "P" around the curve to see the changing radius of curvature (segment CP). It works best if you use a left-right motion - don't worry about following the up-down of the graph.



You'll notice at the **point of inflexion** there is interesting behavior. The circle changes from being below the curve to above (when moving left to right). When we are right on the point of inflexion, what does the circle become?

Example 2

[This example was supplied by a reader.]

We have a curve which is defined by data points and we don't know the function for this data. How can we find the radius of curvature?

We take any 3 data points to illustrate ways of solving this. I chose the points $(1, 1)$, $(2, 3)$ and $(3,8)$.

We'll do this in 3 different ways, just for fun (and for learning about how different math approaches can be used)!

Method 1: Approximation Using a Parabolic Fit and Calculus Methods

Show answer

Method 2: Using Linear Approximations and Calculus Methods

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Method 3: Finding the Radius of the Circle through our 3 Points

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