# VC's for Functions and Pointers

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# Where are we? Recap

- The trickiest case is the one for while loops
- While loops require loop invariants, that is formulas that hold before and after each loop iteration
- Not all invariants work for verification! The proof rule requires an inductive invariant
- Such an invariant allows us to prove that, if the invariant holds *before* the loop iteration it also holds *after* the loop iteration.
- That is, for invariant I, loop condition b and loop body s, we can prove

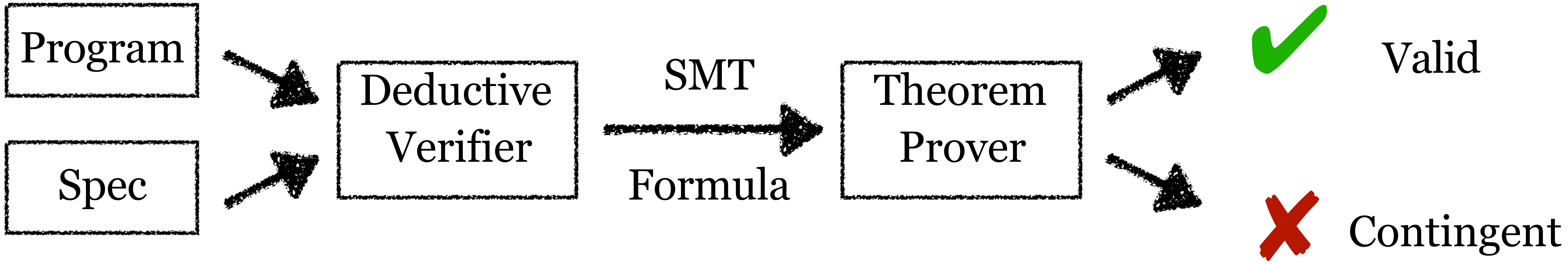
  $\vdash \{I \land b\}\ \mathrm{s}\ \{I\}$
- Finding inductive invariant is the most important problem in deductive verification

# Where are we? Recap

- Next, we looked at automating proofs in Hoare logic using weakest preconditions
- **wp**($s$, Q) denotes the weakest formula that needs to hold before $s$,
  to ensure that $Q$ holds *after* $s$.

- Most rules are straight forward from the Hoare proof rules
- Again, the most difficult part is <u>loops</u>
- The weakest precondition of a loop is its invariant
- But, we still need to check that the invariant is inductive
- This requires side-conditions, which we generate via function vc

# Program Verification




- Spec: Assume, Assert, Loop invariants
- Verification condition: An SMT formula ϕ s.t. program is correct iff ϕ is valid

# Where are we? Recap

- Now: two important extensions:
  - Functions
  - Pointers
- Next lectures:
  - Horn clauses (good way to represent conditions on loop invariants)
  - Algorithms to find invariants automatically

# Extensions & Plan

- Nano is missing many features of real programming languages
- We will now look at two extensions:
    - Functions
    - Pointers
- After that, we'll look at techniques to discover loop invariants (semi-) automatically

# Assertions: Syntax

- But first, we will add three new statements to our language
  - The statement **assert**(F) **fails** if F evaluates to $\perp$
  - The statement **assume**(F) **tells us** that F evaluates to $\top$
  - The statement x := **havoc**() assigns a **non-deterministic value** to variable x

# Assertions: Semantics

- Add new failure state **fail**, i.e., our state is now either a function σ or **fail**

$$(\text{assert-}\top)\quad \frac{\sigma \models F}{\langle\, \mathbf{assert}(F), \sigma \rangle \Downarrow \sigma} \qquad\qquad (\text{assert-}\bot)\quad \frac{\sigma \not\models F}{\langle\, \mathbf{assert}(F), \sigma \rangle \Downarrow \mathbf{fail}}$$

- Assert behaves like **skip**, in case the assertion holds, and otherwise enters the failure state

$$(\text{assume-}\top)\quad \frac{\sigma \models F}{\langle\, \mathbf{assume}(F), \sigma \rangle \Downarrow \sigma}$$

- Assume only needs a single rule
- If the assumption holds, it behaves like **skip**
- Else, the execution gets “stuck”, (but doesn’t fail)
    - Thus, when proving partial correctness, we can ignore such executions

# Assertions: Semantics

- Finally, x := **havoc**() resets variable x to some non-deterministically chosen value

$$\text{(havoc)}\quad \frac{n \in \mathbb{Z}}{\langle x := \textbf{havoc}(),\ \sigma\rangle \Downarrow \sigma[x \mapsto n]}$$

**Quiz:**

- After including **havoc** in our language, is $\langle s, \sigma\rangle \Downarrow \sigma'$ still a (partial) function?

- **havoc** introduces non-determinism!

# Assertions: Proof Rules

- Proof rule for assertions:

$$\frac{P \Rightarrow F}{\vdash \{P\}\ \mathbf{assert}(\mathrm{F})\ \{P \wedge F\}}$$

- Proof rule for assumption:

$$\frac{}{\vdash \{P\}\ \mathbf{assume}(\mathrm{F})\ \{P \wedge F\}}$$

- Proof rule for havoc:

$$\frac{}{\vdash \{\forall \mathrm{y}.\mathrm{Q}[\mathrm{y}/\mathrm{x}]\}\ \mathrm{x} := \mathbf{havoc}()\ \{Q\}}$$

- If Q holds, no matter what we choose for x, then Q holds after.

# Assertions

**Quiz:**

- What's wp( **assert**$(P),Q)$?

- What's wp( **assume**$(P),Q)$?

- Given statement s, can we transform it into a statement s' such that $\{P\}$ s $\{Q\}$ holds if and only if $\{\top\}$ s' $\{\top\}$ holds?

# Functions

- Let's add functions to our language

**Program:** $P \ni Prog ::= F^+$ (one or more functions)

**Function:** $F \ni Fun ::=$ **fun** $f(x_1, \ldots, x_n)\{s;$ **return** $e;\}$

**Statement:** $s \ni Stmt ::= x := f(e_1, \ldots, e_n) \mid \ldots$

- Aside: we can use the name functions, procedures, method calls
- Often, using procedure or method call is done to indicate that the functions have side-effects

# Handling Functions

- How do we generate VCs if we encounter function calls?

$$x := f(e_1, \ldots, e_n)$$

- Just like we asked programmer to provide loop invariants,
  also ask them for method <u>pre-</u> and <u>post-conditions</u>

- Preconditions specify what is expected of f's arguments

- Postconditions describe f's <u>return value</u> and its possible <u>side-effects</u>

# Pre- and post- Example

**Quiz:**

- Consider a function *get* that takes an array $a$ of size $n$ and index $i$ and returns the $i$'th element
- What would be a good pre-condition on inputs $a$,$n$, and $i$?
- What would be a good post-condition for return value *ret*?
- Together, pre-, and post-condition are also called <u>function contract</u>

# Generating VCs for method calls

- Contracts make verification modular, that is, we can verify one function at a time
- But how can we use a contract for verification?
- There are two questions we need to answer:

  1. How do we verify that a method satisfies its contracts?
  2. How to use the contract when generating VCs for method calls?

# 1. Verifying Contracts

- Consider the following function declaration:

```
fun f(x1, ..., xn)
  { requires(Pre)
  ;
  ensures(Post);
  s;
  return e;
}
```

Quiz:

- Let's assume post refers to return value e using the name *ret*
- Which Hoare triple do we have to prove for statement s; ret:=e; ?

# 2. Verifying Calls

- Which verification conditions should we generate if we encounter a function call?

$$x := f(e_1, \ldots, e_n)$$

- Say our function has arguments $x_1, \ldots, x_n$ and pre-condition *Pre* and post-condition *Post*

**Quiz:**

- What needs to hold *before* the function call?
- What holds *after* the function call?

# 2. Verifying Calls

- Which verification conditions should we generate if we encounter a function call?

$$x := f(e_1, \ldots, e_n)$$

- Say our function has arguments $x_1, \ldots, x_n$ and pre-condition *Pre* and post-condition *Post*

- We can replace the function call by the following code, where *tmp* is a fresh variable

$$\textbf{assert}(Pre[e_1/x_1, \ldots, e_n/x_n]);\ \ \textbf{assume}(Post[tmp/ret, e_1/x_1, \ldots, e_n/x_n]);\ \ x := tmp$$

**Quiz:**

- Why do we need the last assignment?
- Why does *tmp* have to be fresh?

# Modular Verification

- When verifying a function definition:
  - When **assume** the precondition
  - And **assert** the postcondition
- When verifying a function call:
  - When **assert** the precondition
  - When **assume** the postcondition
- This is crucial for modular verification –
  decompose verification tasks into individual functions

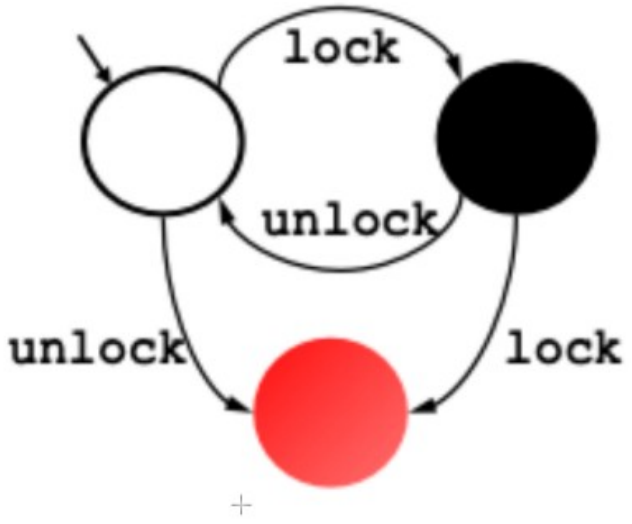
# Modular Verification

**Quiz:**

- Say we don’t have function pre and postconditions
- Is there still some way we could verify programs with functions?
- What’s the downside?
- What’s the downside of modular verification?

# Modular Verification

Exercise:




Quiz:

*“An attempt to re-acquire an acquired lock or release a released lock will cause a **deadlock**.”*

- Suppose we represent locks as integers – 0 means locked; 1 means unlocked
- What are the contracts for methods lock and unlock?

# Modular Verification

<u>Exercise:</u>

- Eliminate the function calls in the following caller of lock and unlock

```
assume(b=0 ∨ b=1);
l:= b;
if(b ≠ 0) l := lock(l); else l := unlock(l);
if(b = 0) l:= lock(l) else l:= unlock(l);
```

- Is the program correct? If not, point out the assertion that fails.

```
fun lock(l)
  { requires(l=1
  );
  ensures(l=0);
  l := 0;
  return l;
}

fun unlock(l)
  { requires(l=0
  );
  ensures(l=1);
  l := 1;
  return l;
}
```

# Global variables

**Quiz:**

- So far, we assumed our function call doesn't have side-effects
- But suppose our function $f$ has access to some global variable $g$
- Does our method, as presented so far, still work?

# Global variables

- To deal with global variables, we will make use of x := **havoc**()

- Extend method contracts:

```
fun f(x₁, …, xₙ)
  { requires(Pre);
  ensures(Post);
  modifies(v₁, v₂, …);
  s;
  return e;
}
```

- Need to check that indeed only $v_1$, $v_2$, … are modified

# Global variables

- Given such a contract, we can translate a function call as follows:

$$x := f(e_1, \ldots, e_n)$$

**assert**$(Pre[e_1/x_1, \ldots, e_n/x_n])$;**havoc**$(v_1, v_2, \ldots)$; $x := tmp$; **assume**$(Post[tmp/ret, e_1/x_1, \ldots, e_n/x_n])$;

**Quiz:**

- What happens if we leave out the havoc statement?

# Adding Pointers

- Next, let’s add pointers to Nano

**Statement:** $s \ni \text{Stmt} ::= \; y := *x \text{ (load)} \mid *x := e \text{ (store)} \mid \ldots$

**Quiz:**

- How would we have to modify our state in order to add pointers to our semantics?

- Does the old Hoare rule for assignments still work?

# Old Rule: Counterexample

<u>Example:</u> $x := y$; $*y := 3$; $*x := 2$; $z := *y$; assert($z = 3$)

**Quiz:**

- Should the post-condition hold?
- What's the weakest pre-condition?
- What's the problem with our old proof rule?

# Verification with Pointers

- As the previous example shows, the old rule for assignments doesn't work!
- Problem: Due to aliasing, an assignment `*x := e` can affect values of expressions beyond `*x`

- Treat the heap as a gigantic array $\mu$ that maps addresses to values
- That means, we need the theory of arrays & new rules for store and load

# Rules for Loads and Stores

- Loads

$$\frac{}{\vdash \{Q[\mu[y]/x]\}\ x := *y\ \{Q\}}$$

- Stores

$$\frac{}{\vdash \{Q[\mu\langle x \lhd e\rangle/\mu]\}\ *x := e\ \{Q\}}$$

# Revisiting our example: New Rules

Example: x := y; *y := 3; *x := 2; z := *y; assert(z = 3)

Quiz:

- What's the weakest pre-condition with our new rules?
- What if we change our assertion to assert(z=2)?

$$\frac{}{\vdash \{Q[\mu[y]/x]\}\ x := *y\ \{Q\}}$$

$$\frac{}{\vdash \{Q[\mu\langle x \triangleleft e\rangle/\mu]\}\ *x := e\ \{Q\}}$$

# Verification with Pointers

**Quiz:**

- How do our array rules reason about aliasing?
- Why is this computationally expensive?

# Verification with Pointers

- Optimization: use pointer analysis to partition μ into several smaller arrays that can't alias
- What about data-structures like linked-lists & trees?
- There's another logic for that: separation logic! A primer on this below:
- http://www0.cs.ucl.ac.uk/staff/p.ohearn/papers/Marktoberdorf11LectureNotes.pdf
- Unfortunately, is undecidable, so automation is hard.
- However, successfully applied by Facebook/Meta: see https://fbinfer.com/
- More reading on VCs with pointers: https://github.com/barghouthi/cs704/blob/master/notes/cs704-lec-04-19-2010.pdf

# What’s next

- Next lectures:
  - Finding inductive loop invariants!
  - First: Horn clauses (good way to represent conditions on loop invariants)
  - Algorithms to solve Horn clauses = find invariants automatically