

$P(\text{ProgramSyntax})$	$\text{Translation Set of Axioms}$
if B then P1 else P2	$B \rightarrow \varphi$ , for each $\varphi \in \Pi_{P1}^{\vec{X}}$ , $\neg B \rightarrow \varphi$ , for each $\varphi \in \Pi_{P1}^{\vec{X}}$ ,
P1; P2	$\varphi(\vec{X}1/\vec{Y}) \rightarrow \varphi$ , for each $\varphi \in \Pi_{P1}^{\vec{X}}$ , $\varphi(\vec{X}/\vec{Y}) \rightarrow \varphi$ , for each $\varphi \in \Pi_{P2}^{\vec{X}}$ where $\vec{Y} = (Y_1, \dots, Y_k)$ is a tuple of new program variables (array names) such that each $Y_i$ is of the same arity as $X_i$ in $\vec{X}$ , $\varphi(\vec{X}1/\vec{Y})$ is the result of replacing in $\varphi$ each occurrence of $X_i$ by $Y_i$ , and similarly for $\varphi(\vec{X}/\vec{Y})$ . By renaming them if necessary, we assume here that $\Pi_{P1}^{\vec{X}}$ and $\Pi_{P2}^{\vec{X}}$ have no common program variables except those in $\vec{X}$ .
while B do P1	$\varphi(n)$ , for each $\varphi \in \Pi_{P1}^{\vec{X}}$ , $X_i(\vec{x}) = X_i(\vec{x}, 0)$ , for each $X_i \in \vec{X}$ $\text{smallest}(N, n, \neg B(n))$ , $X1_i(\vec{x}) = X_i(\vec{x}, N)$ , for each $X_i \in \vec{X}$ where $n$ is a new natural number variable not already in $\varphi$ , and $N$ a new constant not already used in $\Pi_{P1}^{\vec{X}}$ . For each formula or term $\alpha$ , $\alpha(n)$ is defined inductively as follows: it is obtained from $\alpha$ by performing the following recursive substitutions: <ul style="list-style-type: none"> <li>• for each <math>X_i \in \vec{X}</math>, replace all occurrences of <math>X_i1(e_1, \dots, e_k)</math> by <math>X_i(e_1(n), \dots, e_k(n), n+1)</math>, and</li> <li>• for each program variable <math>X</math> in <math>\alpha</math>, replace all occurrences of <math>X(e_1, \dots, e_k)</math> by <math>X(e_1(n), \dots, e_k(n), n)</math>. Notice that this replacement is for every program variable <math>X</math>, including those not in <math>\vec{X}</math>.</li> </ul> $\text{smallest}(N, n, \neg B(n))$ is a shorthand for the following formula <div style="text-align: right;"> <math>\neg B(N), \quad (1)</math>  <math>\forall n. n &lt; N \rightarrow B(n) \quad (2)</math> </div>

Table 1: Set of rules used by the translation algorithm [1]. Given a program  $P$  and a set  $\vec{X}$  of program variables including all variables used in  $P$ , our system constructs a set of axioms  $\Pi_P^{\vec{X}}$  according to the these presented rules.

## References

- [1] F. Lin, “A formalization of programs in first-order logic with a discrete linear order,” *Artificial Intelligence*, vol. 235, pp. 1 – 25, 2016.