IB Analysis and Approaches HL2
Inverse Trigonometric Functions

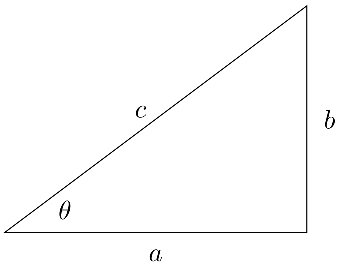
Definition & Purpose

Functions that reverse the effect of the basic trigonometric functions (sine, cosine, and tangent).

If a trigonometric function takes an angle and gives us a ratio, the inverse trigonometric function takes a ratio and gives back the angle.

Triangle Example

Suppose we have a right triangle with an angle θ and sides of length a, b, and c as shown below.

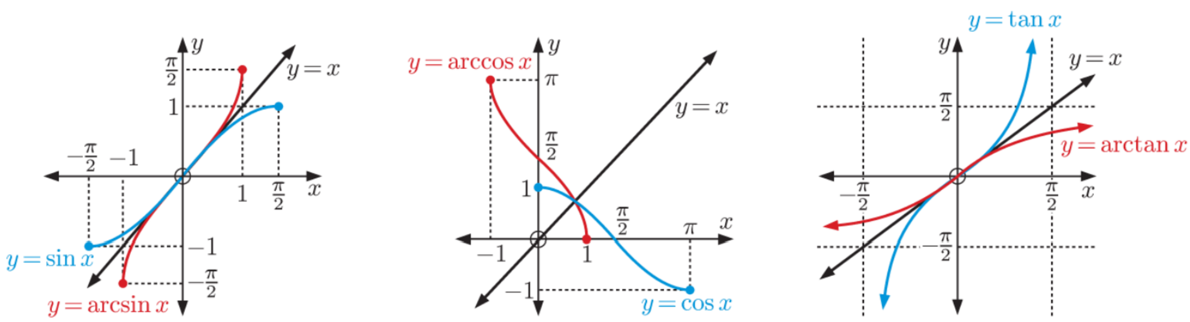


In regular trigonometry:

$$\sin(\theta) = \frac{b}{c},$$
$$\cos(\theta) = \frac{a}{c},$$
$$\tan(\theta) = \frac{b}{a}.$$

Trigonometric Function	**Domain**	**Range**
$\arcsin(x)$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$\arccos(x)$	$-1 \le x \le 1$	$0 \le y \le \pi$
$\arctan(x)$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Domain and Range Visualization



Example Problems

1. **Find, where possible, the exact solutions of**:

   (a) $\arctan x = \frac{\pi}{6}$
   Solution:
   $$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \implies x = \frac{1}{\sqrt{3}}$$

   (b) $\arccos(x-1) = \frac{\pi}{4}$
   Solution:
   $$x - 1 = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$
   Solving for x:
   $$x = \frac{\sqrt{2}}{2} + 1$$

   (c) $\arcsin x = \frac{\pi}{6}$
   Solution:
   $$x = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

2. **Find the invariant point for the inverse transformation from**:

   (a) $y = \sin x$ to $y = \arcsin x$
   Solution: Since both functions need to be equal, we get:
   $$\sin(x) = x$$
   The solution needs to be in the range of $\arcsin(x)$, which is $[-1, 1]$. Therefore, the invariant point is:
   $$(0, 0)$$

   (b) $y = \tan x$ to $y = \arctan x$
   Solution: Since both functions must be equal, we get:
   $$\tan(x) = x$$
   The solution needs to be in the range of $\arctan(x)$, which is $(-\infty, \infty)$. Therefore, the invariant point is:
   $$(0, 0)$$

Key Takeaways

Inverse Trigonometric Functions

- $\arcsin(x)$: Inverse of $\sin(x)$. Domain: $-1 \leq x \leq 1$, Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.
- $\arccos(x)$: Inverse of $\cos(x)$. Domain: $-1 \leq x \leq 1$, Range: $0 \leq y \leq \pi$.
- $\arctan(x)$: Inverse of $\tan(x)$. Domain: $-\infty < x < \infty$, Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$.