IB Analysis and Approaches HL2
Inverse Trigonometric Functions

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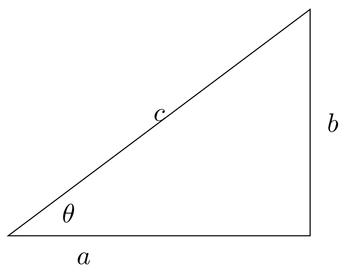
Introduction

Inverse trigonometric functions are crucial for determining angles from given trigonometric ratios. They are the inverse operations of the basic trigonometric functions.

Understanding Trigonometry

Suppose we have a right triangle with an angle θ and sides of length a, b, and c as shown below.



In regular trigonometry:

$$\sin(\theta) = \frac{b}{c},$$
$$\cos(\theta) = \frac{a}{c},$$
$$\tan(\theta) = \frac{b}{a}.$$

Inverse Trigonometric Functions

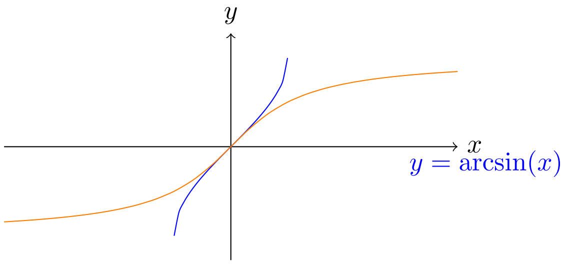
Inverse trigonometric functions return the angle for a given trigonometric ratio, essentially reversing the operations shown above.

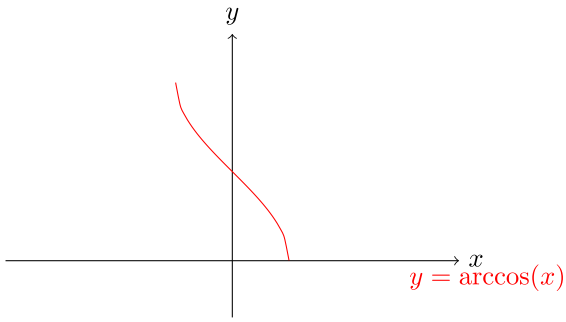
Key Functions and Their Properties

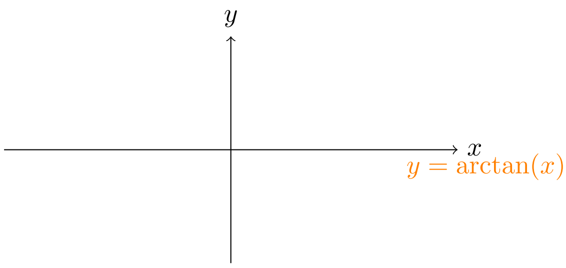
Function	Definition	Domain	Range
$\arcsin(x)$	$\sin(\arcsin(x)) = x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq \arcsin(x) \leq \frac{\pi}{2}$
$\arccos(x)$	$\cos(\arccos(x)) = x$	$-1 \leq x \leq 1$	$0 \leq \arccos(x) \leq \pi$
$\arctan(x)$	$\tan(\arctan(x)) = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < \arctan(x) < \frac{\pi}{2}$

Graphical Representations

The graphs below help visualize the behavior and transformation from standard trigonometric functions to their inverses.







Example Problems

Here are some examples to practice:
1. **Calculate** $\arcsin(\frac{1}{2})$. 2. **Determine** $\arccos(-1)$. 3. **Find** $\arctan(1)$.