4.6 (b) $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} 2^n u(-n) e^{-j\omega n}$

$= \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$

$= \frac{2}{2-e^{j\omega}}$

(g) $X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} \sum_{k=0}^{+\infty} (\frac{1}{4})^n \delta(n-3k) e^{-j\omega n}$

$= \sum_{k=0}^{+\infty} (\frac{1}{4})^{3k} e^{-j\omega 3k}$

$= \frac{1}{1-(\frac{e^{-j\omega}}{4})^3}$

(i) $\frac{\sin\frac{\pi n}{3}}{\pi n} \longleftrightarrow X_1(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{3} \\ 0, & \frac{\pi}{3} < |\omega| \le \pi \end{cases}$

$\frac{\sin\frac{\pi n}{4}}{\pi n} \longleftrightarrow X_2(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} < |\omega| \le \pi \end{cases}$

$X(e^{j\omega}) = \frac{1}{2\pi} X_1(e^{j\omega}) \otimes X_2(e^{j\omega})$

$= \begin{cases} \frac{1}{4}, & |\omega| \le \frac{\pi}{12} \\ \frac{1}{2\pi}(\frac{7}{12}\pi - |\omega|), & \frac{\pi}{12} < |\omega| \le \frac{7}{12}\pi \\ 0, & \frac{7}{12}\pi < |\omega| \le \pi \end{cases}$

4.7 (a) $x(n) = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$

$= \frac{1}{2\pi}\left(\delta(n) - 3\delta(n-1) + 2\delta(n+2) + 4\delta(n-4)\right)$

(e) $X(e^{j\omega}) = -\frac{6}{5}\left(\frac{2}{e^{-j\omega}+2} + \frac{3}{e^{-j\omega}-3}\right)$

$\therefore x(n) = -\frac{6}{5}\left((-\frac{1}{2})^n u(n) - (\frac{1}{3})^n u(n)\right)$

4.9 (a) $X(e^{j0}) = \sum_{n=-\infty}^{+\infty} x(n) = 6$

(b) $x(n+2) \longleftrightarrow X(e^{j\omega}) e^{j2\omega}$

$x(n+2)$ 为实偶函数 $\therefore X(e^{j\omega})e^{j2\omega}$ 为实偶函数

$\therefore \theta(\omega) = -2\omega$

(c) $X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x(n) e^{-j\pi n} = \sum_{n=-\infty}^{+\infty} (-1)^n x(n) = 2$

(d) $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega = 2\pi x(0) = 4\pi$

(e) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \sum_{n=-\infty}^{+\infty} |x(n)|^2 = 28\pi$

$\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega = 2\pi \sum_{n=-\infty}^{+\infty} |-jnx[n]|^2 = 316\pi$

4.10 (a) 即奇函数，b, g

(b) 即偶函数，d, e

(c) 即平移α单位后为偶函数，a, b, d, e, f

(d) 即 $x(0)=0$，b, d, e, f, g

(e) 即 $\sum_{n=-\infty}^{+\infty} x(n) = 0$，b, c, g

Scanned with CamScanner

4.14 $H(e^{jw}) = \begin{cases} 1, & |w| \le \frac{\pi}{3} \\ 0, & \frac{\pi}{3} < |w| \le \pi \end{cases}$

(a) $X(e^{jw}) = \frac{1}{1-\frac{3}{4}e^{-jw}}$

$Y(e^{jw}) = \begin{cases} \frac{1}{1-\frac{3}{4}e^{-jw}} & |w| \le \frac{\pi}{3} \\ 0, & \frac{\pi}{3} < |w| \le \pi \end{cases}$

$y(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{jw}) e^{jwn} dw$

$= \frac{1}{2\pi}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{e^{jwn}}{1-\frac{3}{4}e^{-jw}} dw$

$= \frac{e^{j\pi n}(\pi - \arctan 2\sqrt{3})}{\pi}$

(b) $X(e^{jw}) = 2\pi \sum_{k=-\infty}^{+\infty} \delta(w-\pi-2\pi k)$

$Y(e^{jw}) = 0$

$y(n) = 0$

(d) $x(n) = \frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{}$

$X(e^{jw}) = \pi\left(\sum_{k=-\infty}^{+\infty} \delta(w-\frac{\pi}{2}-2\pi k) + \delta(w+\frac{\pi}{2}-2\pi k)\right)$

$Y(e^{jw}) = 0$

$y(n) = 0$

4.22 $x(n)+\frac{1}{4}x(n-1)=y(n)+\frac{1}{3}y(n-1)-\frac{2}{9}y(n-2)$

(1) $X(e^{j\omega})+\frac{1}{4}e^{-j\omega}X(e^{j\omega})=Y(e^{j\omega})+\frac{1}{3}e^{-j\omega}Y(e^{j\omega})-\frac{2}{9}e^{-j2\omega}Y(e^{j\omega})$

$$\Rightarrow H(e^{j\omega})=\frac{Y(e^{j\omega})}{X(e^{j\omega})}=\frac{1+\frac{1}{3}e^{-j\omega}-\frac{2}{9}e^{-j2\omega}}{1+\frac{1}{4}e^{-j\omega}}$$

$$=-\frac{4}{9}\frac{2e^{-j2\omega}-3e^{-j\omega}-9}{e^{-j\omega}+4}$$

$$=-\frac{4}{9}\left(2e^{-j\omega}-11+\frac{35}{e^{-j\omega}+4}\right)$$

$$\therefore h(n)=-\frac{4}{9}\left(2\delta(n-1)-11\delta(n)+\frac{35}{4}\left(-\frac{1}{4}\right)^n u(n)\right)$$

(2) $X_1(e^{j\omega})=\frac{1}{1+e^{-j\omega}}$

$$Y_1(e^{j\omega})=X_1(e^{j\omega})H(e^{j\omega})$$

$$=-\frac{4}{9}\left(\frac{2e^{-j\omega}}{1+e^{-j\omega}}-\frac{11}{1+e^{-j\omega}}+\frac{35}{(e^{-j\omega}+4)(e^{-j\omega}+1)}\right)$$

$$=-\frac{4}{9}\left(2-\frac{\frac{4}{3}}{1+e^{-j\omega}}-\frac{\frac{35}{3}}{e^{-j\omega}+4}\right)$$

$$y_1(n)=-\frac{4}{9}\left(2\delta(n)-\frac{4}{3}(-1)^n u(n)+\frac{35}{12}\left(-\frac{1}{4}\right)^n u(n)\right)$$

(3) $X_2(e^{j\omega})=2\pi\sum_{k=-\infty}^{+\infty}\delta(\omega-\pi-2k\pi)$

$$Y_2(e^{j\omega})=X_2(e^{j\omega})H(e^{j\omega})$$

$$y_2(n)=\frac{1}{2\pi}\int_0^{2\pi}Y_2(e^{j\omega})e^{j\omega n}d\omega$$

$$=\int_0^{2\pi}\delta(\omega-\pi)H(e^{j\omega})e^{j\omega n}d\omega$$

$$=H(e^{j\pi})e^{j\pi n}$$

$$=(-1)^n\cdot\left(-\frac{4}{9}\right)\left(-2-11+\frac{35}{3}\right)$$

$$=\frac{16}{27}(-1)^n$$

4.22 (4) $Y(e^{jw}) = \frac{1}{1-\frac{1}{3}e^{-jw}} + \frac{1}{1+\frac{2}{3}e^{-jw}}$

$= \frac{3(e^{-jw}+6)}{(3-e^{-jw})(2e^{-jw}+3)}$

$X(e^{jw}) = \frac{Y(e^{jw})}{H(e^{jw})} = \frac{3(e^{-jw}+6)}{(3-e^{-jw})(2e^{-jw}+3)} \left(-\frac{9}{4}\right) \frac{e^{-jw}+4}{2e^{-j2w}-3e^{-jw}-9}$

$= \frac{27}{4} \frac{(e^{-jw}+6)(e^{-jw}+4)}{(e^{-jw}-3)^2(2e^{-jw}+3)^2}$

$x(n) = ?$

Scanned with CamScanner