3.12 $X(j\omega) = \int_0^1 (1-t) e^{-j\omega t} dt$

$= -\frac{j}{\omega} + \frac{1-e^{-j\omega}}{\omega^2}$

$x_1(t) = x(t-1) \longleftrightarrow e^{-j\omega} X(j\omega)$

$x_2(t) = x(-t-1) \longleftrightarrow e^{j\omega} X(-j\omega)$

$x_3(t) = 2x(1-\frac{t}{2}) \longleftrightarrow 4e^{-2j\omega} X(-\frac{j\omega}{2})$

$x_4(t) = x(t) + x(-t) \longleftrightarrow X(j\omega) + X(-j\omega)$

$x_5(t) = x(t) + x(1-t) \longleftrightarrow X(j\omega) + e^{-j\omega} X(-j\omega)$

$x_6(t) = (x(t) + x(-t)) \cos \Omega_0 t$

$= (x(t)+x(-t)) \frac{e^{j\Omega_0 t}}{2} + (x(t)+x(-t)) \frac{e^{-j\Omega_0 t}}{2}$

$\longleftrightarrow \frac{1}{2\pi}(X(j\omega) + X(-j\omega)) * \pi\delta(\omega - \Omega_0)$

$+ \frac{1}{2\pi}(X(j\omega) + X(-j\omega)) * \pi\delta(\omega + \Omega_0)$

$= \frac{1}{2} X(j(\omega - \Omega_0)) + \frac{1}{2} X(j(\Omega_0 - \omega))$

$+ \frac{1}{2} X(j(\omega + \Omega_0)) + \frac{1}{2} X(j(-\Omega_0 - \omega))$

Scanned with CamScanner

3.13 (a) $X(0) = \int_{-\infty}^{+\infty} x(t)dt = 7$

(b) $\int_{-\infty}^{\infty} X(j\omega)d\omega = 2\pi x(0) = 1$

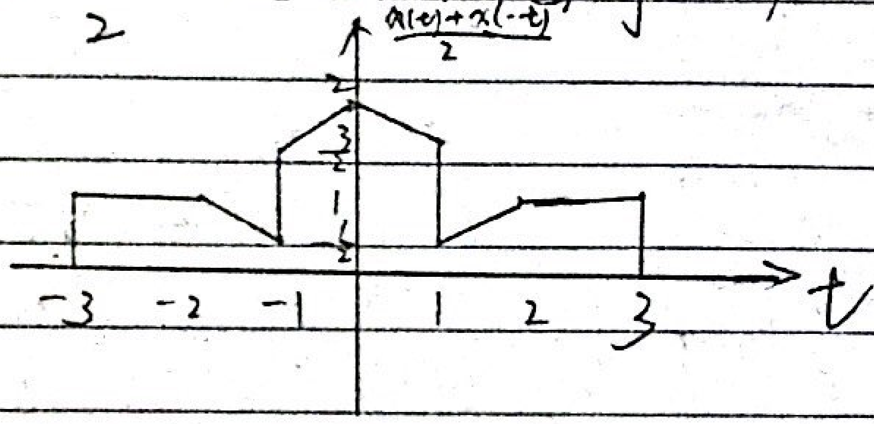
(c) $\int_{-\infty}^{\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega = 2\pi\left(x(t)*(u(t+1)-u(t-1))\right)\Big|_{t=2}$

$= 2\pi\int_{1}^{3} x(t)dt$

$= 7\pi$

(d) $\int_{-\infty}^{\infty}|X(j\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty}|x(t)|^2 dt = \frac{76}{3}\pi$

(e) $\frac{x(t)+x(-t)}{2} \longleftrightarrow Re(X(j\Omega))$



3.15 (a) $Re(X(j\Omega))=0 \Leftrightarrow x(t)$ 为奇函数

$\therefore$ 图 (a), (b) 满足

(b) $Im(X(j\Omega))=0 \Leftrightarrow x(t)$ 为偶函数

$\therefore$ 图 (d), (f) 满足

(c) 即平移 a 后为偶函数

$\therefore$ 图 (a)(c)(d)(f) 满足

(d) 即 $x(0)=0$

$\therefore$ 图 (a)(b)(c)(e)(f) 满足

(e) $x'(0)=0$

$\therefore$ 图 (c)(d)(e)(f) 满足

(f) 即由冲激函数线性组合得到

$\therefore$ 图 (c) 满足

3.17 $H(jw)=u(w-2\pi)-u(w+2\pi)$

(1) $X(jw)=\pi(\delta(w-\pi)+\delta(w+\pi))+\frac{2\pi}{j}\left(\delta(w-\frac{3}{2}\pi)-\delta(w+\frac{3}{2}\pi)\right)$

$Y(jw)=X(jw)H(jw)=X(jw)$

$\therefore y(t)=x(t)=\cos\pi t+2\sin\frac{3}{2}\pi t$

(2) $X(jw)=\frac{1}{2\pi}\pi(\delta(w-\pi)+\delta(w+\pi))*\pi(\delta(w-2\pi)+\delta(w+2\pi))$

$=\frac{\pi}{2}\left(\delta(w-3\pi)+\delta(w-\pi)+\delta(w+\pi)+\delta(w+3\pi)\right)$

$Y(jw)=X(jw)H(jw)=\frac{\pi}{2}(\delta(w-\pi)+\delta(w+\pi))$

$\therefore y(t)=\frac{1}{2}\cos\pi t$

(3) $y(t)=\sum_{k=-\infty}^{+\infty}\frac{\sin 2\pi(t-\frac{10}{3}k)}{\pi(t-\frac{10}{3}k)}$ (?)

3.28 (a) $j\omega Y(j\omega)+2Y(j\omega)=X(j\omega)$

$$\Rightarrow H(j\omega)=\frac{1}{2+j\omega}$$

$$h(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}H(j\omega)e^{j\omega t}d\omega=e^{-2t}u(t)$$

(b) $X(j\omega)=\frac{1}{1+j\omega}$

$$Y(j\omega)=X(j\omega)H(j\omega)=\frac{1}{1+j\omega}-\frac{1}{2+j\omega}$$

$$\therefore y(t)=e^{-t}u(t)-e^{-2t}u(t)$$

(c) (1) $Y(j\omega)=\frac{1+j\omega}{(2+j\omega)^2}=\frac{1}{2+j\omega}-\frac{1}{(2+j\omega)^2}$

$$\therefore y(t)=e^{-2t}u(t)-(e^{-2t}u(t))*(e^{-2t}u(t))$$

$$=e^{-2t}u(t)-te^{-2t}u(t)$$

(2) $Y(j\omega)=\frac{3+j\omega}{(1+j\omega)(2+j\omega)}=\frac{2}{1+j\omega}-\frac{1}{2+j\omega}$

$$\therefore y(t)=2e^{-t}u(t)-e^{-2t}u(t)$$

(3) $Y(j\omega)=\frac{1}{(1+j\omega)(2+j\omega)^2}=\frac{1}{2+j\omega}\left(\frac{1}{1+j\omega}-\frac{1}{2+j\omega}\right)$

$$=\frac{1}{1+j\omega}-\frac{1}{2+j\omega}-\frac{1}{(2+j\omega)^2}$$

$$\therefore y(t)=e^{-t}u(t)-e^{-2t}u(t)-te^{-2t}u(t)$$

补 3.17 (3)

$$x(t)=\sum_{k=-\infty}^{+\infty}\delta(t-\frac{10}{3}k)=\sum_{k=-\infty}^{+\infty}A_k\mathrm{e}^{jk\omega_0 t}$$

$$T_0=\frac{10}{3},\omega_0=\frac{2\pi}{\omega_0}=\frac{3}{5}\pi,A_k=\frac{3}{10}\int_0^{\frac{10}{3}}\delta(t)\mathrm{e}^{-jk\omega_0 t}=\frac{3}{10}$$

$$\Rightarrow x(t)=\frac{3}{10}\sum_{k=-\infty}^{+\infty}\mathrm{e}^{jk\frac{3\pi}{5}t}$$

$$\Rightarrow X(j\omega)=\frac{3}{5}\pi\sum_{k=-\infty}^{+\infty}\delta(\omega-\frac{3\pi}{5}k)$$

$$\Rightarrow Y(j\omega)=X(j\omega)H(j\omega)=\frac{3}{5}\pi(\delta(\omega-\frac{9}{5}\pi)+\delta(\omega-\frac{6}{5}\pi)+\delta(\omega-\frac{3}{5}\pi)+\delta(\omega)+\delta(\omega+\frac{3}{5}\pi)+\delta(\omega-\frac{6}{5}\pi)+\delta(\omega+\frac{9}{5}\pi))$$

$$\Rightarrow y(t)=\frac{3}{5}(\frac{1}{2}+\cos\frac{3}{5}\pi t+\cos\frac{6}{5}\pi t+\cos\frac{9}{5}\pi t)$$

4.1 (a) $x(n) = \frac{e^{j\frac{2\pi}{3}n}+e^{-j\frac{2\pi}{3}n}}{2} + \frac{e^{j\frac{2\pi}{7}n}-e^{-j\frac{2\pi}{7}n}}{2j}$

$N=21$, $A_7 = A_{14} = \frac{1}{2}$, $A_3 = \frac{1}{2j}$, $A_{18} = -\frac{1}{2j}$

(c) $N=4$

$$A_k = \frac{1}{4}\sum_{n=0}^{3} x[n] e^{-j\frac{\pi}{2}kn}$$

$$= \frac{1}{4}\sum_{n=0}^{3} e^{-j\frac{\pi}{2}kn} - \frac{1}{8j}\sum_{n=0}^{3} e^{j\frac{\pi n}{4}} e^{-j\frac{\pi}{2}kn} + \frac{1}{8j}\sum_{n=0}^{3} e^{-j\frac{\pi n}{4}} e^{-j\frac{\pi}{2}kn}$$

$$= \frac{1}{4}\frac{1-e^{-j2\pi k}}{1-e^{-j\frac{\pi}{2}k}} - \frac{1}{8j}\frac{1-e^{4(-j\frac{\pi}{2}k+j\frac{\pi}{4})}}{1-e^{-j\frac{\pi}{2}k+j\frac{\pi}{4}}} + \frac{1}{8j}\frac{1-e^{4(-j\frac{\pi}{2}k-j\frac{\pi}{4})}}{1-e^{-j\frac{\pi}{2}k-j\frac{\pi}{4}}}$$

$(k=1,2,3)$

$$A_0 = \frac{1}{4}\sum_{n=0}^{3} x[n] = \frac{1}{4}\left(1+1-\frac{\sqrt{2}}{2}+1-\frac{\sqrt{2}}{2}\right) = \frac{3-\sqrt{2}}{4}$$

(e) $N=6$

$$A_k = \frac{1}{6}\sum_{n=0}^{5} x[n] e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1}{6}\sum_{n=0}^{3} e^{-j\frac{\pi}{3}kn}$$

$$= \frac{1}{6}\frac{1-e^{-j\frac{4}{3}\pi k}}{1-e^{-j\frac{\pi}{3}k}} \quad (k=1,2,3,4,5)$$

$$A_0 = \frac{4}{6} = \frac{2}{3}$$

4.4 (a) $x(n-n_0) = \sum_{k=\langle N\rangle} A_k e^{j\frac{2\pi k(n-n_0)}{N}}$

$$= \sum_{k=\langle N\rangle} e^{-j\frac{2\pi k n_0}{N}} A_k e^{j\frac{2\pi kn}{N}}$$

$$\therefore A_k' = e^{-j\frac{2\pi k n_0}{N}} A_k$$

(c) $x^*(-n) = \left(\sum_{k=\langle N\rangle} A_k e^{j\frac{2\pi k(-n)}{N}}\right)^*$

$$= \sum_{k=\langle N\rangle} A_k^* e^{j\frac{2\pi kn}{N}}$$

$$\therefore A_k' = A_k^*$$