8.14 (a) $sY(s) + 2Y(s) = X(s)$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2} \quad , Re[s] > -2$$

(b) $h(t) = e^{-2t}u(t)$

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau = u(t)\int_0^t e^{-2\tau}d\tau = \frac{1-e^{-2t}}{2}u(t)$$

(c) $$y_1(t) = \int_{-\infty}^{+\infty} x_1(\tau)h(t-\tau) = \int_{-\infty}^{+\infty} e^{-\tau}e^{2\tau-2t}u(t-\tau)d\tau$$

$$= \int_{-\infty}^{t} e^{\tau-2t}d\tau$$

$$= e^{-t}$$

(d) $X_2(s) = \frac{1}{s+1} \quad , Re[s] > -1$

$$Y_2(s) = X_2(s)H(s) = \frac{1}{s+1} - \frac{1}{s+2} \quad , Re[s] > -1$$

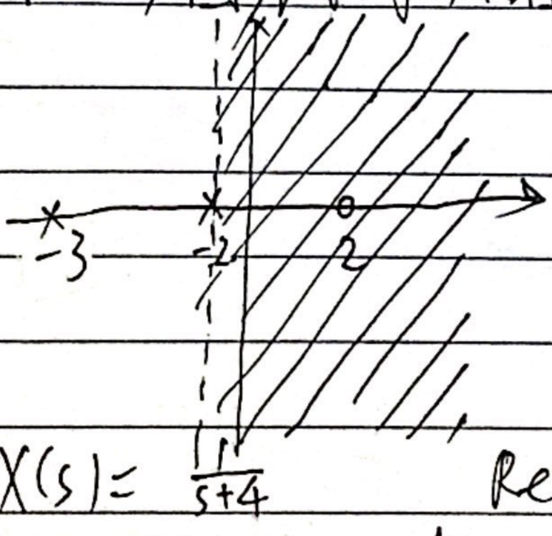
$$y_2(t) = e^{-t}u(t) - e^{-2t}u(t)$$

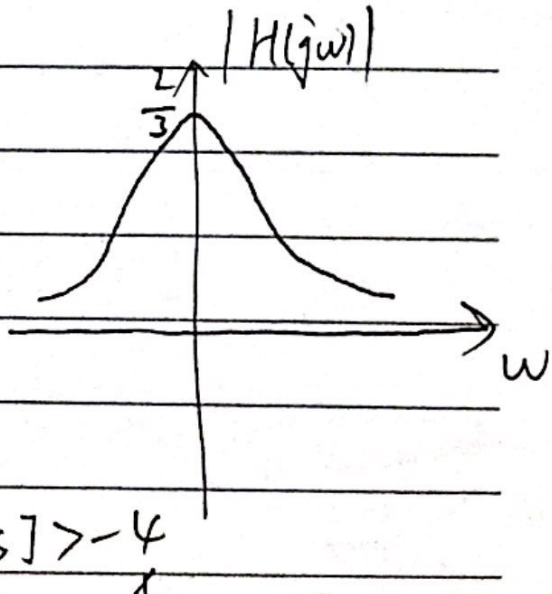
8.16 (a) $$2\frac{X(s)}{s} - 4\frac{X(s)}{s^2} = Y(s) + 5\frac{Y(s)}{s} + 6\frac{Y(s)}{s^2}$$

$$\Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{2s-4}{s^2+5s+6} \quad Re[s] > -2$$

(b) 稳定，因为 $H(j\omega)$ 存在

(c)





(d) $X(s) = \frac{1}{s+4} \quad Re[s] > -4$

$$Y(s) = X(s)H(s) = \frac{10}{s+3} - \frac{6}{s+4} - \frac{4}{s+2} \quad Re[s] > -2$$

$$y(t) = 10e^{-3t}u(t) - 6e^{-4t}u(t) - 4e^{-2t}u(t)$$

9.4 (a) $x(n)=-\left(-\frac{1}{2}\right)^n u(-n-1)$

(c) $X(z)=\frac{z^{-1}}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}$

$=\frac{2}{1-z^{-1}}-\frac{2}{1-\frac{1}{2}z^{-1}}$ $\quad \frac{1}{2}<|z|<1$

$x(n)=-u(-n-1)-\left(\frac{1}{2}\right)^n u(n)$

(e) $X(z)=\frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$

$=\frac{6}{1-\frac{1}{2}z^{-1}}-\frac{6}{1-\frac{1}{3}z^{-1}}$ $\quad \frac{1}{3}<|z|<\frac{1}{2}$

$x(n)=-\left(\frac{1}{2}\right)^n u(-n-1)-\left(\frac{1}{3}\right)^n u(n)$

9.5 (a) 1° $|z|<\frac{1}{3}$; 2° $\frac{1}{3}<|z|<\frac{4}{3}$, 3° $|z|>\frac{4}{3}$

(b) $X(z)=\frac{3}{5}\left(\frac{1}{1-\frac{4}{3}z^{-1}}-\frac{1}{1+\frac{1}{3}z^{-1}}\right)$

1° $x(n)=\frac{3}{5}\left(-\left(\frac{4}{3}\right)^n u(-n-1)+\left(-\frac{1}{3}\right)^n u(-n-1)\right)$

2° $x(n)=\frac{3}{5}\left(-\left(\frac{4}{3}\right)^n u(-n-1)-\left(-\frac{1}{3}\right)^n u(n)\right)$

3° $x(n)=\frac{3}{5}\left(\left(\frac{4}{3}\right)^n u(n)-\left(-\frac{1}{3}\right)^n u(n)\right)$

(c) 2°存在，因为其ROC包含单位圆；其他不存在

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9.6 (a)

$$1-\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}-\frac{1}{8}z^{-3}$$

$$1+\frac{1}{2}z^{-1}\,\overline{)\,1}$$

$$1+\frac{1}{2}z^{-1}$$

$$-\frac{1}{2}z^{-1}$$

$$-\frac{1}{2}z^{-1}-\frac{1}{4}z^{-2}$$

$$\frac{1}{4}z^{-2}$$

$$\frac{1}{4}z^{-2}+\frac{1}{8}z^{-3}$$

$$-\frac{1}{8}z^{-3}$$

$$-\frac{1}{8}z^{-3}-\frac{1}{16}z^{-4}$$

$$\frac{1}{16}z^{-4}$$

$x(0)=1 \quad x(1)=-\frac{1}{2} \quad x(2)=\frac{1}{4} \quad x(3)=-\frac{1}{8}$

(b)

$$\frac{1}{8}-\frac{3}{32}z+\frac{23}{128}z^{2}-\frac{63}{512}z^{3}$$

$$8z^{-2}+6z^{-1}+1\,\overline{)\,z^{-2}\quad +1}$$

$$z^{-2}+\frac{3}{4}z^{-1}+\frac{1}{8}$$

$$-\frac{3}{4}z^{-1}+\frac{7}{8}$$

$$-\frac{3}{4}z^{-1}-\frac{9}{16}-\frac{3}{32}z$$

$$\frac{23}{16}+\frac{3}{32}z$$

$$\frac{23}{16}+\frac{69}{64}z+\frac{23}{128}z^{2}$$

$$-\frac{63}{64}z-\frac{23}{128}z^{2}$$

$$-\frac{63}{64}z-\frac{189}{256}z^{2}-\frac{63}{512}z^{3}$$

$$-\frac{143}{256}z^{2}+\frac{63}{512}z^{3}$$

$x(0)=\frac{1}{8}$, $x(-1)=-\frac{3}{32}$, $x(-2)=\frac{23}{128}$, $x(-3)=-\frac{63}{512}$

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(c)

$$1-2z^{-1}+5z^{-2}-11z^{-3}$$

$$1+3z^{-1}+2z^{-2} \;\big)\; 1+z^{-1}+z^{-2}$$

$$1+3z^{-1}+2z^{-2}$$

$$-2z^{-1}-z^{-2}$$

$$-2z^{-1}-6z^{-2}-4z^{-3}$$

$$5z^{-2}+4z^{-3}$$

$$5z^{-2}+15z^{-3}+10z^{-4}$$

$$-11z^{-3}-10z^{-4}$$

$$-11z^{-3}-33z^{-4}-22z^{-5}$$

$$23z^{-4}+22z^{-5}$$

$x(0)=1,\ x(1)=-2,\ x(2)=5,\ x(3)=-11$

(d)

$$-4z+32z^{2}-160z^{3}+704z^{4}$$

$$\frac{1}{8}z^{-2}+\frac{3}{4}z^{-1}+1 \;\big)\; -\frac{1}{2}z^{-1}+1$$

$$-\frac{1}{2}z^{-1}-3-4z$$

$$4+4z$$

$$4+24z+32z^{2}$$

$$-20z-32z^{2}$$

$$-20z-120z^{2}-160z^{3}$$

$$88z^{2}+160z^{3}$$

$x(0)=0\quad x(-1)=-4,\ x(-2)=32,\ x(-3)=-160,\ x(-4)=704$

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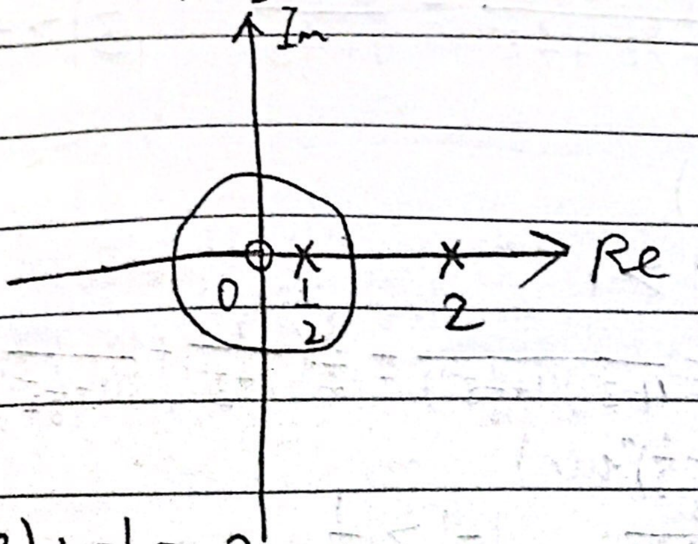
9.9 $H(z)=\frac{-3z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}}=-3\frac{z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}=-3\frac{z}{(z-\frac{1}{2})(z-2)}$

$=\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}z^{-1}}-\frac{1}{1-2z^{-1}}\right)$



(a) $|z|>2$

$h(n)=\frac{1}{2}\left((\frac{1}{2})^n u(n)-2^n u(n)\right)$

(b) $|z|<\frac{1}{2}$

$h(n)=\frac{1}{2}\left(-(\frac{1}{2})^n u(-n-1)+2^n u(-n-1)\right)$

(c) $\frac{1}{2}<|z|<2$

$h(n)=\frac{1}{2}\left((\frac{1}{2})^n u(n)+2^n u(-n-1)\right)$

9.12 (a) $Y(z)+\frac{5}{6}z^{-1}Y(z)+\frac{1}{6}z^{-2}Y(z)=X(z)+\frac{1}{2}z^{-1}X(z)$

$\Rightarrow H(z)=\frac{Y(z)}{X(z)}=\frac{1+\frac{1}{2}z^{-1}}{1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}=\frac{1}{1+\frac{1}{3}z^{-1}} \quad |z|>\frac{1}{3}$

(b) $h(n)=\left(-\frac{1}{3}\right)^n u(n)$

(c) $X(z)=\frac{1}{1-z^{-1}} \quad |z|>1$

$Y(z)=X(z)H(z)=\frac{1}{(1-z^{-1})(1+\frac{1}{3}z^{-1})}=\frac{3}{4}\left(\frac{1}{1-z^{-1}}+\frac{1}{3(1+\frac{1}{3}z^{-1})}\right) \quad |z|>1$

$y(n)=\frac{3}{4}u(n)+\frac{1}{4}\left(-\frac{1}{3}\right)^n u(n)$

(d) $Y(z)=\frac{2}{1+\frac{1}{3}z^{-1}}-\frac{3}{1+\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$

$X(z)=\frac{Y(z)}{H(z)}=2-\frac{3+z^{-1}}{1+\frac{1}{2}z^{-1}}=-\frac{1}{1+\frac{1}{2}z^{-1}} \quad |z|>\frac{1}{2}$

$x(n)=-\left(-\frac{1}{2}\right)^n u(n)$

9.14 (a) $H(z) = \frac{z^2}{(z-1)(z+\frac{1}{3})}$ $|z| > 1$

(b) $H(z) = \frac{1}{(1-z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{1}{4}\left(\frac{1}{1+\frac{1}{3}z^{-1}} + \frac{3}{1-z^{-1}}\right)$ $|z| > 1$

$h(n) = \frac{1}{4}\left((-\frac{1}{3})u(n) + 3u(n)\right)$

(c) $\frac{Y(z)}{X(z)} = \frac{1}{1-\frac{2}{3}z^{-1}-\frac{1}{3}z^{-2}}$

$Y(z) - \frac{2}{3}z^{-1}Y(z) - \frac{1}{3}z^{-2}Y(z) = X(z)$

$y(n) - \frac{2}{3}y(n-1) - \frac{1}{3}y(n-2) = x(n)$

(d) $X(z) = \frac{1}{1+2z^{-1}}$ $|z| > 2$

$Y(z) = X(z)H(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})(1+\frac{1}{3}z^{-1})}$

$= \frac{4}{5}\frac{1}{1+2z^{-1}} + \frac{1}{4}\frac{1}{1-z^{-1}} - \frac{1}{20}\frac{1}{1+\frac{1}{3}z^{-1}}$

$y(n) = \frac{4}{5}(-2)^n u(n) + \frac{1}{4}u(n) - \frac{1}{20}(-\frac{1}{3})^n u(n)$

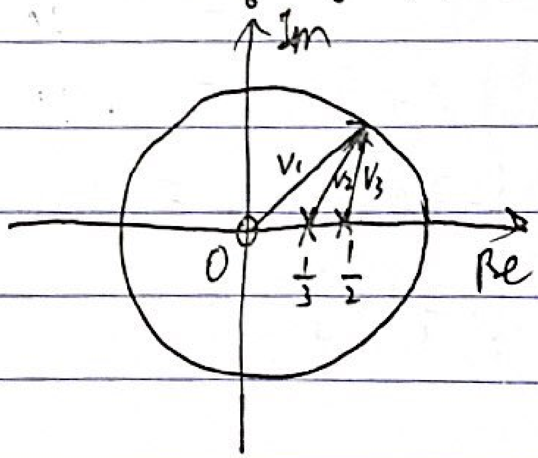
9.17 (a)
$$\begin{cases} W(z) = X(z) + \frac{5}{6}z^{-1}W(z) - \frac{1}{6}z^{-2}W(z) \\ Y(z) = z^{-1}W(z) \end{cases}$$

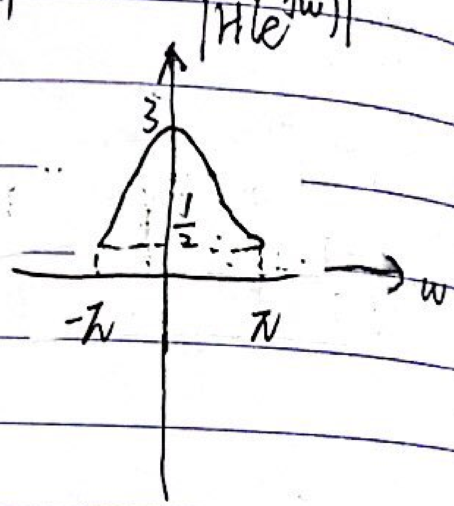
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \quad \left(|z| > \frac{1}{2}\right)$$

$$= 6\left(\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-\frac{1}{3}z^{-1}}\right)$$

(b) $h(n) = 6\left(\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{3}\right)^n u(n)\right)$

(c) $H(z) = \frac{z}{z^2-\frac{5}{6}z+\frac{1}{6}} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})}$





9.18 (a)
$$\begin{cases} W(z) = X(z) + \frac{1}{4}z^{-1}W(z) + \frac{1}{8}z^{-2}W(z) \\ Y(z) = 8X(z) + 2z^{-1}W(z) + z^{-2}W(z) \end{cases}$$

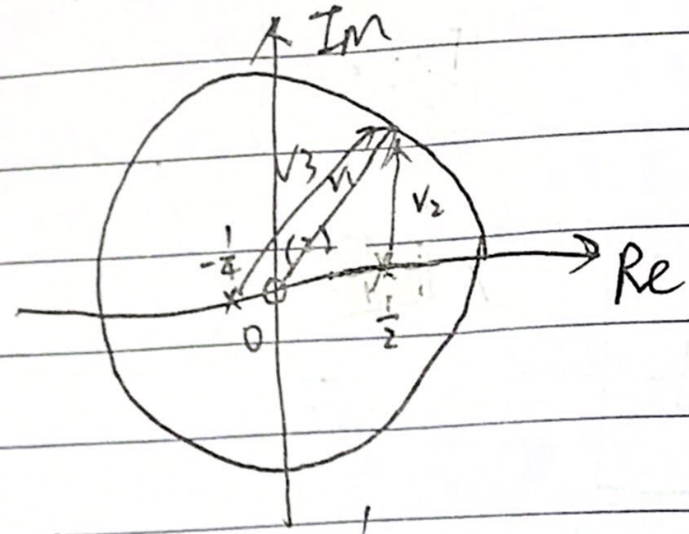
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = 8 + \frac{2z^{-1}+z^{-2}}{1-\frac{1}{4}z^{-1}-\frac{1}{8}z^{-2}} = \frac{8}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$$

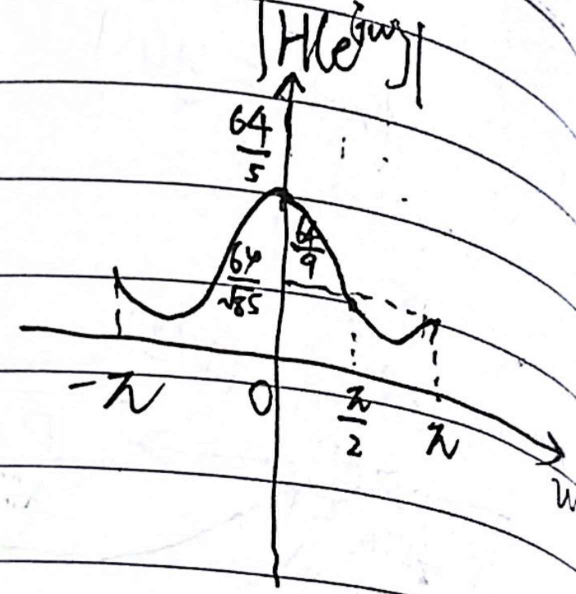
$$= \frac{8}{3}\left(\frac{2}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{4}z^{-1}}\right)$$

$$\left(|z| > \frac{1}{2}\right)$$

(b) 稳定，因为其收敛域包含单位圆

9.18 (c) $H(z) = \dfrac{8z^2}{(z-\frac{1}{2})(z+\frac{1}{4})}$





(d) $X(z) = \dfrac{1}{1-\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$

$$Y(z) = X(z)H(z) = \frac{8}{(1-\frac{1}{2}z^{-1})^2(1+\frac{1}{4}z^{-1})}$$

$$= \frac{16}{3}\frac{1}{(1-\frac{1}{2}z^{-1})^2} + \frac{8}{9}\left(\frac{2}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{4}z^{-1}}\right)$$

$$y(n) = \frac{16}{3}(n+1)\left(\frac{1}{2}\right)^n u(n) + \frac{16}{9}\left(\frac{1}{2}\right)^n u(n) + \frac{8}{9}\left(-\frac{1}{4}\right)^n u(n)$$