1.13 (a). $y(t)=e^{x(t)}$.

~~成立：~~

瞬时 √ ：根据定义.

时不变 √ ：瞬时系统⇒时不变.

线性 X ：$2y(t) \neq e^{2x(t)}$.

因果 √ ：瞬时 ⇒ 因果.

稳定 √ ：若 $\forall t, |x(t)|<M$, 则 $\forall t, |y(t)|<e^M$.

(i) $y(n)=x(2n)$.

瞬时 X ：定义.

时不变 X ：$\cancel{x(n) \rightarrow x(n+b)}$

$\cancel{y(n) \rightarrow x(2n+b)} \neq$

$x(n) \rightarrow x'(n)=x(n+b)$

$y(n)=x(2n) \rightarrow y'(n)=x'(2n)=x(2n+b)$

$y(n+b)=x(2n+2b) \neq x(2n+b)=y'(n)$.

线性 √ ：

$\cancel{x(n)=}$

$y_1(n)=x_1(2n)$

$y_2(n)=x_2(2n)$

$\cancel{y(n)=}$

$x(n)=a_1x_1(n)+a_2x_2(n)$.

$y(n)=x(2n)$

$=a_1x_1(2n)+a_2x_2(2n)$.

$=a_1y_1(n)+a_2y_2(n)$.

0 3 3 3 3 0 0.
0 -2 -4 2 -2 0 0
2 -2 2 -2 2 -2 0
2 -1 1 3 3 -2 0.

因果 X ：

$x_1(t)=\begin{cases} -1, & t=-2 \\ 0, & \text{otherwise.} \end{cases}$

$x_2(t)=\begin{cases} -2, & t=-2 \\ 0, & \text{otherwise.} \end{cases}$

$\forall t \geq -1,\ x_1(t)=x_2(t)$.

$y_1(-1)=x_1(-2)=-1$.

$y_2(-1)=x_2(-2)=-2$.

$y_1(-1) \neq y_2(-1)$.

稳定 √ ：$\forall t, |x(t)|<M \Rightarrow \forall t, |y(t)|=|x(2t)|<M$.

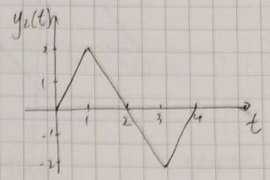
1.16.

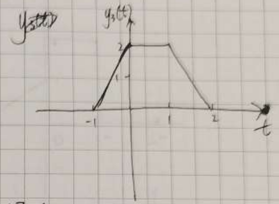
$x_2(t)=x_1(t)-x_1(t-2)$

$y_2(t)=y_1(t)-y_1(t-2)$

$x_3(t)=x_1(t)+x_1(t+1)$

$y_3(t)=y_1(t)+y_1(t+1)$.





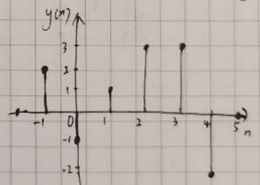
1.17. (a)

$\cancel{x(t)=2x_3}$

$x(n)=\cancel{2x_3(n)+3}$

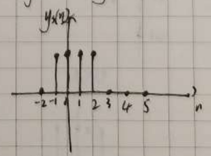
$3x_1(n)-2x_2(n)+2x_3(n)$

$y(n)=3y_1(n)-2y_2(n)+2y_3(n)$



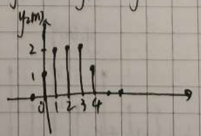
(b) $x_3(n)=x_1(n+1)$

$y_3(n)=y_1(n+1)$.



$x_2(n)=x_1(n)+x_1(n-1)$

$y_2(n)=y_1(n)+y_1(n-1)$.



2.2. (c) $x(n)=(-\frac{1}{2})^n u(n-4)$

$h(n)=4^n u(2-n)$

$y(n)=x(n)*h(n)$

$$=\sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

$$=\sum_{k=4}^{n-2}(-\frac{1}{2})^k\cdot 4^{n-k}$$

$$=\sum_{k=4}^{n-2}4^n\cdot(-\frac{1}{2}\cdot\frac{1}{4})^k$$

$$=4^n\cdot\frac{1-(-\frac{1}{8})^{n-5}}{1+\frac{1}{8}}\cdot(-\frac{1}{8})^4$$

$$=4^n\cdot\frac{8}{9}\cdot\cancel{\frac{1}{2048}}\frac{1}{4096}\cdot\left[1-(-\frac{1}{8})^{n-5}\right]$$

$$=\frac{4^{n-8}}{9}\cdot\left[1-(-8)^{5-n}\right]$$

(d) $x(n)=h(n)=[-2\le n\le 2]$

$\cancel{y(n)=x(n)*h(n)}$

$\cancel{=\sum_{k=-\infty}^{+\infty}x(k)h(n-k)}$

$\cancel{=\sum_{k=-2}^{2}[-2\le n-k\le 2]}$

$\cancel{=\sum_{k=-2}^{2}[n-2\le k\le n+2]}$

$y(n)=x(n+2)+x(n+1)+x(n)+x(n-1)+x(n-2)$

$$\begin{bmatrix}y(-4)\\y(-3)\\y(-2)\\y(-1)\\y(0)\\y(1)\\y(2)\\y(3)\\y(4)\end{bmatrix}=\begin{bmatrix}1\\1\\1\\1\\1\\0\\0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\1\\1\\1\\1\\1\\0\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\1\\1\\1\\1\\1\\0\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\\1\\1\\1\\1\\1\\0\end{bmatrix}+\begin{bmatrix}0\\0\\0\\0\\1\\1\\1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\2\\3\\4\\5\\4\\3\\2\\1\end{bmatrix}$$

$\arg\max_n y(n)=0$

$\max_n y(n)=5$

2.4 (a)

若 $\alpha=\beta$

$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$

$$=\int_{-\infty}^{+\infty}e^{\alpha\tau}u(\tau)e^{\beta(t-\tau)}u(t-\tau)d\tau$$

$$=\int_{-\infty}^{+\infty}e^{\alpha t}\cdot[0\le\tau\le t]d\tau$$

$$=\int_0^t e^{\alpha t}d\tau$$

$$=te^{\alpha t}$$

若 $\alpha\ne\beta$

$y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$

$$=\int_0^t e^{\alpha\tau}\cdot e^{\beta(t-\tau)}d\tau$$

$$=e^{\beta t}\int_0^t e^{(\alpha-\beta)\tau}d\tau$$

$$=e^{\beta t}\left[e^{(\alpha-\beta)t}-e^{(\alpha-\beta)\cdot 0}\right]$$

$$=e^{\alpha\tau-\beta\tau+\beta t}-e^{\beta t}$$

(c) $\cancel{x(t)=u(t)-u(t-2)}$

$\cancel{h(t)=u(t)-2u(t-2)+u(t-4)}$

$\cancel{y(t)=x(t)*h(t)}$

$\cancel{=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau}$

$\cancel{=\int_{-\infty}^{+\infty}[u(\tau)-u(\tau-2)][u(t-\tau)-2u(t-\tau-2)+u(t-\tau-4)]d\tau}$

$\cancel{=\int_{-\infty}^{+\infty}[u(\tau)u(t-\tau)-u(\tau-2)u(t-\tau)-2u(\tau)u(t-\tau-2)+2u(\tau-2)u(t-\tau-2)+u(\tau)u(t-\tau-4)-u(\tau-2)u(t-\tau-4)]d\tau}$

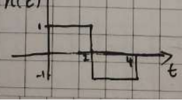
$\cancel{=t-(t-2)-2(t-2)+2(t-4)+(t-4)-(t-6)}$

$\cancel{=0}$

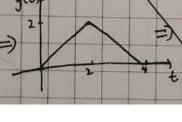
(c) $x=u(t)-u(t-2)=\begin{cases}1, & 0\le t\le 2\\ 0\end{cases}$

$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}h(\tau)x(t-\tau)d\tau$

$$=\int_{t-2}^{t}h(\tau)d\tau$$



$\Rightarrow$



$\Rightarrow$ $y(t)=tu(t)+(4-2t)u(t-2)+(t-4)u(t-4)$