3.2 (a) $\cos 4t+\sin 6t = \frac{1}{2}e^{j4t}+\frac{1}{2}e^{-j4t}-\frac{1}{2}je^{j6t}+\frac{1}{2}je^{-j6t}$

取 $\Omega_0=2$，则 $A_2=A_{-2}=\frac{1}{2}$，$A_3=-\frac{j}{2}$，$A_{-3}=\frac{j}{2}$

(e) $\Omega_0=\frac{\pi}{2}$，$A_k=\frac{1}{4}\int_{-2}^{2}x(t)e^{-jk\frac{\pi}{2}t}dt$

$$=\frac{1}{4}\left(\int_{-2}^{0}\left(\frac{t}{2}+1\right)e^{-jk\frac{\pi}{2}t}dt+\int_{0}^{2}\left(1-\frac{t}{2}\right)e^{-jk\frac{\pi}{2}t}dt\right)$$

$$=\frac{1}{4}\left(\left(\frac{te^{-jk\frac{\pi}{2}t}}{-jk\pi}+\left(1+\frac{1}{jk\pi}\right)\frac{e^{-jk\frac{\pi}{2}t}}{-jk\frac{\pi}{2}}\right)\Bigg|_{-2}^{0}+\left(\frac{te^{-jk\frac{\pi}{2}t}}{jk\pi}+\left(1-\frac{1}{jk\pi}\right)\frac{e^{-jk\frac{\pi}{2}t}}{-jk\frac{\pi}{2}}\right)\Bigg|_{0}^{2}\right)$$

$$=\frac{1}{4}\left(\left(1+\frac{1}{jk\pi}\right)\frac{1}{-jk\frac{\pi}{2}}-\frac{-2e^{jk\pi}}{-jk\pi}-\left(1+\frac{1}{jk\pi}\right)\frac{e^{jk\pi}}{-jk\frac{\pi}{2}}+\frac{2e^{-jk\pi}}{jk\pi}+\left(1-\frac{1}{jk\pi}\right)\frac{e^{-jk\pi}-1}{-jk\frac{\pi}{2}}\right)$$

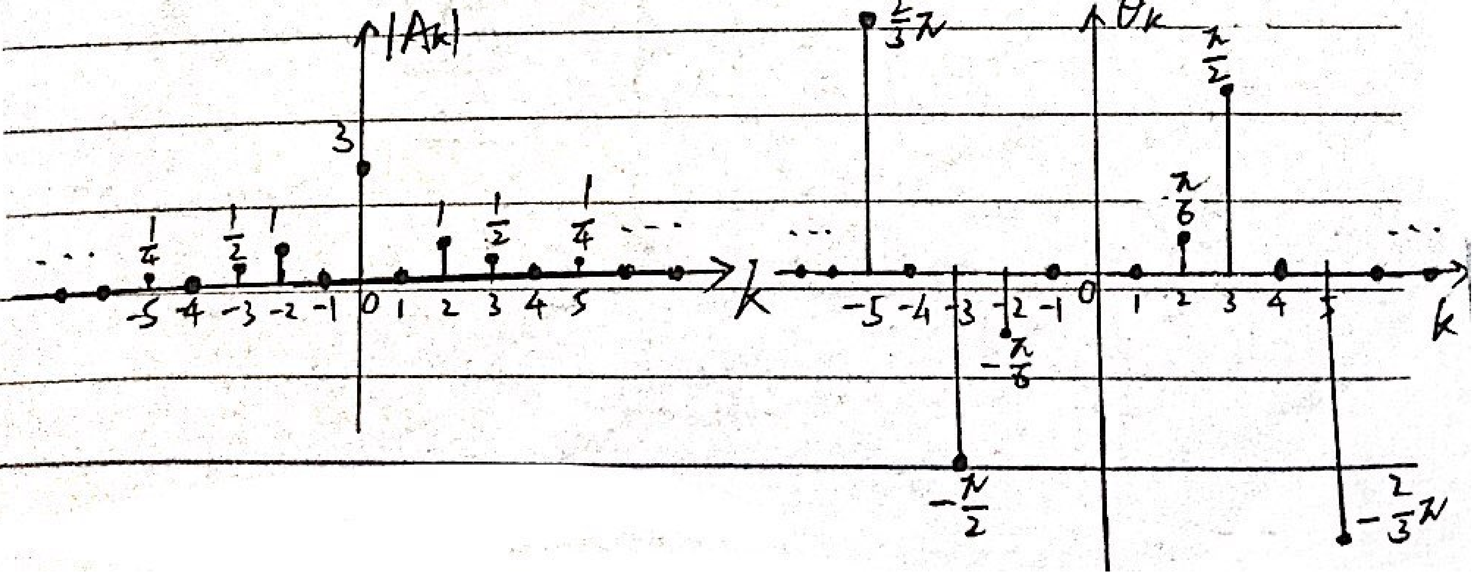
$$=\frac{1}{4}\left(\frac{4}{k^2\pi^2}\left(1-e^{jk\pi}\right)\right)$$

$$=\frac{1-(-1)^k}{k^2\pi^2}\quad(k\neq 0)$$

$$A_0=\frac{1}{2}$$

3.3 $x(t)=3+\sqrt{3}\,\frac{e^{j2t}+e^{-j2t}}{2}+\frac{e^{j2t}-e^{-j2t}}{2j}+\frac{e^{j3t}-e^{-j3t}}{2j}-\frac{1}{2}\,\frac{e^{j(5t+\frac{\pi}{3})}+e^{-j(5t+\frac{\pi}{3})}}{2}$

$$=3+\left(\frac{\sqrt{3}}{2}-\frac{j}{2}\right)e^{j2t}+\left(\frac{\sqrt{3}}{2}+\frac{j}{2}\right)e^{-j2t}+\frac{j}{2}e^{-j3t}-\frac{j}{2}e^{j3t}-\frac{e^{j\frac{\pi}{3}}}{4}e^{j5t}-\frac{e^{-j\frac{\pi}{3}}}{4}e^{-j5t}$$



3.6 (a) $A_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t-t_0) e^{-j\frac{2k\pi}{T_0}t} dt$

$= \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j\frac{2k\pi}{T}(t+t_0)} dt$

$= e^{-j\frac{2k\pi}{T}t_0} \cdot \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j\frac{2k\pi}{T}t}$

$= e^{-j\frac{2k\pi}{T_0}t_0} \dot{A}_k$

(b) $A_k = \frac{1}{T_0}\int_0^{T_0} x(-t) e^{-j\frac{2k\pi}{T}t} = \dot{A}_{-k}$

(c) $A_k = \frac{1}{T_0}\int_0^{T_0} x^*(t) e^{-j\frac{2k\pi}{T}} dt = \left[\frac{1}{T_0}\int_0^{T_0} x(t) e^{j\frac{2k\pi}{T}} dt\right]^* = \dot{A}_k$

(d) $\int_{-\infty}^{t+T_0} x(\tau) d\tau = \int_{-\infty}^{t} x(\tau) d\tau + T_0 \dot{A}_0 = \int_{-\infty}^{t} x(\tau) d\tau$

$\therefore T = T_0$

$A_k = \frac{1}{T_0}\int_0^T \int_{-\infty}^t x(\tau) d\tau e^{-j\frac{2k\pi}{T}t} dt$

$= \frac{1}{T_0}\int_0^T \int_\tau^T x(\tau) e^{-j\frac{2k\pi}{T_0}t} dt\, d\tau$

$= \frac{1}{T_0}\int_0^T x(\tau) d\tau \int_\tau^T e^{-\frac{j2k\pi}{T_0}} d\tau$

$= \frac{1}{T_0}\int_0^T x(\tau) \frac{e^{-j\frac{2k\pi}{T_0}t}}{j\frac{2k\pi}{T}} d\tau$

$= -j\frac{T}{2k\pi}\dot{A}_k$

(e) $\frac{dx(t+T_0)}{dt} = \frac{dx(t)}{dt}$ $\quad \therefore T = T_0$

$A_k = \int_0^T \frac{dx(t)}{dt} e^{-j\frac{2k\pi}{T}t} dt = \int_0^T e^{-j\frac{2k\pi}{T}t} d x(t)$

$= x(t) e^{-j\frac{2k\pi}{T}t}\Big|_0^T + \int_0^T x(t) j\frac{2k\pi}{T_0} e^{-j\frac{2k\pi}{T_0}t} dt$

$= j\frac{2k\pi}{T_0}\dot{A}_k$

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3.8 (c) $X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$

$$=\int_{-2}^{0}(t+2)e^{-j\omega t}dt+\int_{0}^{2}(2-t)e^{-j\omega t}dt$$

$$=\frac{-2e^{2j\omega}+2}{\omega^2}$$

(e) $X(j\omega)=\int_{-1}^{1}(1+\cos\pi t)e^{-j\omega t}dt$

$$=\frac{2\pi^2\sin\omega}{\pi^2\omega-\omega^3}$$

3.9 $x(at+b)e^{j\Omega_0 t}\longleftrightarrow\frac{1}{2\pi}\left(\int_{-\infty}^{\infty}x(at+b)e^{-j\omega t}dt\right)*\left(2\pi\delta(\omega-\Omega_0)\right)$

$$=\left(\int_{-\infty}^{\infty}x(at+b)e^{-j\omega t}dt\right)\Big|_{\omega=\omega-\Omega_0}$$

$$=\left(\int_{-\infty}^{\infty}x(t)e^{-j(\frac{t}{a}-\frac{b}{a})\omega}\frac{dt}{a}\right)\Big|_{\omega=\omega-\Omega_0}$$

$$=\frac{1}{|a|}e^{j\omega\frac{b}{a}}X\left(\frac{j\omega}{a}\right)\Big|_{\omega=\omega-\Omega_0}$$

$$=\frac{1}{|a|}e^{j\frac{b}{a}(\omega-\Omega_0)}X\left(\frac{j(\omega-\Omega_0)}{a}\right)$$

3.11 (b) $x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$

$$=\frac{1}{\pi}\left(e^{jt}+e^{-jt}\right)+\frac{3}{2\pi}\left(e^{j2\pi t}+e^{-j2\pi t}\right)$$

$$=\frac{2}{\pi}\cos t+\frac{3}{\pi}\cos 2\pi t$$

(e) $x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$

$$=\frac{1}{2\pi}\left(\int_{-W}^{0}\left(1+\frac{\omega}{W}\right)e^{j\omega t}d\omega+\int_{0}^{W}\left(1-\frac{\omega}{W}\right)e^{j\omega t}d\omega\right)$$

$$=\frac{1}{2\pi}\left(\frac{2}{t^2W}-\frac{2\cos Wt}{t^2W}\right)$$