

$$9.4 (a) x(n) = -\left(-\frac{1}{2}\right)^n u(-n-1)$$

$$(c) X(z) = \frac{z^{-1}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

$$= \frac{2}{1-z^{-1}} - \frac{2}{1-\frac{1}{2}z^{-1}} \quad \frac{1}{2} < |z| < 1$$

$$x(n) = -u(-n-1) - \left(\frac{1}{2}\right)^n u(n)$$

$$(e) X(z) = \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$= \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{1}{3}z^{-1}} \quad \frac{1}{3} < |z| < \frac{1}{2}$$

$$x(n) = -\left(\frac{1}{2}\right)^n u(-n-1) - \left(\frac{1}{3}\right)^n u(n)$$

$$9.5 (a) 1^{\circ} |z| < \frac{1}{3}; 2^{\circ} \frac{1}{3} < |z| < \frac{4}{3}; 3^{\circ} |z| > \frac{4}{3}$$

$$(b) X(z) = \frac{3}{5} \left(\frac{1}{1-\frac{4}{3}z^{-1}} - \frac{1}{1+\frac{1}{3}z^{-1}} \right)$$

$$1^{\circ} x(n) = \frac{3}{5} \left(-\left(\frac{4}{3}\right)^n u(-n-1) + \left(-\frac{1}{3}\right)^n u(-n-1) \right)$$

$$2^{\circ} x(n) = \frac{3}{5} \left(-\left(\frac{4}{3}\right)^n u(-n-1) - \left(-\frac{1}{3}\right)^n u(n) \right)$$

$$3^{\circ} x(n) = \frac{3}{5} \left(\left(\frac{4}{3}\right)^n u(n) - \left(-\frac{1}{3}\right)^n u(n) \right)$$

(c) 2° 存在, 因为其 ROC 包含单位圆; 其他不存在

$$9.6 \text{ (a)} \quad \frac{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2} - \frac{1}{8}z^{-3}}{1 + \frac{1}{2}z^{-1}}$$

$$\frac{1 + \frac{1}{2}z^{-1}}{1 + \frac{1}{2}z^{-1}}$$

$$- \frac{1}{2}z^{-1}$$

$$- \frac{1}{2}z^{-1} - z^{-2}$$

$$\frac{1}{4}z^{-2}$$

$$\frac{1}{4}z^{-2} + \frac{1}{8}z^{-3}$$

$$- \frac{1}{8}z^{-3}$$

$$- \frac{1}{8}z^{-3} - \frac{1}{16}z^{-4}$$

$$\frac{1}{16}z^{-4}$$

$$x(0) = 1 \quad x(1) = -\frac{1}{2} \quad x(2) = \frac{1}{4} \quad x(3) = -\frac{1}{8}$$

$$1b) \quad \frac{\frac{1}{8} - \frac{3}{32}z + \frac{23}{128}z^2 - \frac{63}{512}z^3}{8z^2 + 6z + 1}$$

$$\frac{1}{8} - \frac{3}{32}z + \frac{23}{128}z^2 - \frac{63}{512}z^3$$

$$z^{-2} + \frac{3}{4}z^{-1} + \frac{1}{8}$$

$$- \frac{3}{4}z^{-1} + \frac{7}{8}$$

$$- \frac{3}{4}z^{-1} - \frac{9}{16} - \frac{3}{32}z$$

$$\frac{23}{16} + \frac{3}{32}z$$

$$\frac{23}{16} + \frac{69}{64}z + \frac{23}{128}z^2$$

$$- \frac{63}{64}z - \frac{23}{128}z^2$$

$$- \frac{63}{64}z - \frac{189}{256}z^2 - \frac{63}{512}z^3$$

$$x(0) = \frac{1}{8}, \quad x(-1) = -\frac{3}{32}, \quad x(-2) = \frac{23}{128}, \quad x(-3) = -\frac{63}{512}$$

$$\begin{array}{r}
 (c) \quad 1 - 2z^{-1} + 5z^{-2} - 11z^{-3} \\
 \hline
 1 + 3z^{-1} + 2z^{-2} \overline{) 1 + z^{-1} + z^{-2}} \\
 \underline{1 + 3z^{-1} + 2z^{-2}} \\
 -2z^{-1} - z^{-2} \\
 \underline{-2z^{-1} - 6z^{-2} - 4z^{-3}} \\
 5z^{-2} + 4z^{-3} \\
 \underline{5z^{-2} + 15z^{-3} + 10z^{-4}} \\
 -11z^{-3} - 10z^{-4} \\
 \underline{-11z^{-3} - 33z^{-4} - 22z^{-5}} \\
 23z^{-4} + 22z^{-5}
 \end{array}$$

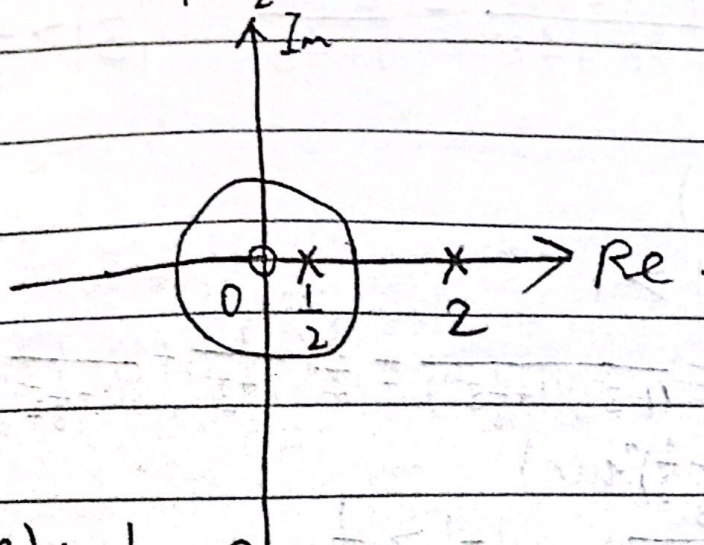
$$x(0)=1, x(1)=-2, x(2)=5, x(3)=-11$$

$$\begin{array}{r}
 (d) \quad -4z + 32z^2 - 160z^3 + 704z^4 \\
 \hline
 \frac{1}{8}z^{-2} + \frac{3}{4}z^{-1} + 1 \overline{) -\frac{1}{2}z^{-1} + 1} \\
 \underline{-\frac{1}{2}z^{-1} - 3 - 4z} \\
 4 + 4z \\
 \underline{4 + 24z + 32z^2} \\
 -20z - 32z^2 \\
 \underline{-20z - 120z^2 - 160z^3} \\
 88z^2 + 160z^3
 \end{array}$$

$$x(0)=0, x(-1)=-4, x(-2)=32, x(-3)=-160, x(-4)=704$$

$$9.9 \quad V(z) = \frac{-3z^{-1}}{1 - \frac{1}{2}z^{-1} + z^{-2}} = -3 \frac{z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - 2z^{-1})} = -3 \frac{z}{(z - \frac{1}{2})(z - 2)}$$

$$= \frac{1}{2} \left(\frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - 2z^{-1}} \right)$$



(a) $|z| > 2$

$$h(n) = \frac{1}{2} \left(\left(\frac{1}{2}\right)^n u(n) - 2^n u(n) \right)$$

(b) $|z| < \frac{1}{2}$

$$h(n) = \frac{1}{2} \left(-\left(\frac{1}{2}\right)^n u(-n-1) + 2^n u(-n-1) \right)$$

(c) $\frac{1}{2} < |z| < 2$

$$h(n) = \frac{1}{2} \left(\left(\frac{1}{2}\right)^n u(n) + 2^n u(-n-1) \right)$$

$$9.12 \text{ (a)} \quad Y(z) + \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = X(z) + \frac{1}{2}z^{-1}X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = \frac{1}{1 + \frac{1}{3}z^{-1}} \quad |z| > \frac{1}{3}$$

$$(b) \quad h(n) = \left(-\frac{1}{3}\right)^n u(n)$$

$$(c) \quad X(z) = \frac{1}{1-z^{-1}} \quad |z| > 1$$

$$Y(z) = X(z)H(z) = \frac{1}{(1-z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{3}{4} \left(\frac{1}{1-z^{-1}} + \frac{1}{3(1+\frac{1}{3}z^{-1})} \right) \quad |z| > 1$$

$$y(n) = \frac{3}{4}u(n) + \frac{1}{4}\left(-\frac{1}{3}\right)^n u(n)$$

$$(d) \quad Y(z) = \frac{2}{1+\frac{1}{3}z^{-1}} - \frac{3}{1+\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$

$$X(z) = \frac{Y(z)}{H(z)} = 2 - \frac{3+z^{-1}}{1+\frac{1}{2}z^{-1}} = -\frac{1}{1+\frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$$

$$x(n) = -\left(-\frac{1}{2}\right)^n u(n)$$

$$9.14 \text{ (a)} H(z) = \frac{z^2}{(z-1)(z+\frac{1}{3})} \quad |z| > 1$$

$$\text{(b)} H(z) = \frac{1}{(1-z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{1}{4} \left(\frac{1}{1+\frac{1}{3}z^{-1}} + \frac{3}{1-z^{-1}} \right) \quad |z| > 1$$

$$h(n) = \frac{1}{4} \left(\left(-\frac{1}{3}\right)^n u(n) + 3u(n) \right)$$

$$\text{(c)} \frac{Y(z)}{X(z)} = \frac{1}{1-\frac{2}{3}z^{-1}-\frac{1}{3}z^{-2}}$$

$$Y(z) - \frac{2}{3}z^{-1}Y(z) - \frac{1}{3}z^{-2}Y(z) = X(z)$$

$$y(n) - \frac{2}{3}y(n-1) - \frac{1}{3}y(n-2) = x(n)$$

$$\text{(d)} X(z) = \frac{1}{1+2z^{-1}} \quad |z| > 2$$

$$Y(z) = X(z)H(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})(1+\frac{1}{3}z^{-1})}$$

$$= \frac{4}{5} \frac{1}{1+2z^{-1}} + \frac{1}{4} \frac{1}{1-z^{-1}} - \frac{1}{20} \frac{1}{1+\frac{1}{3}z^{-1}}$$

$$y(n) = \frac{4}{5} (-2)^n u(n) + \frac{1}{4} u(n) - \frac{1}{20} \left(-\frac{1}{3}\right)^n u(n)$$

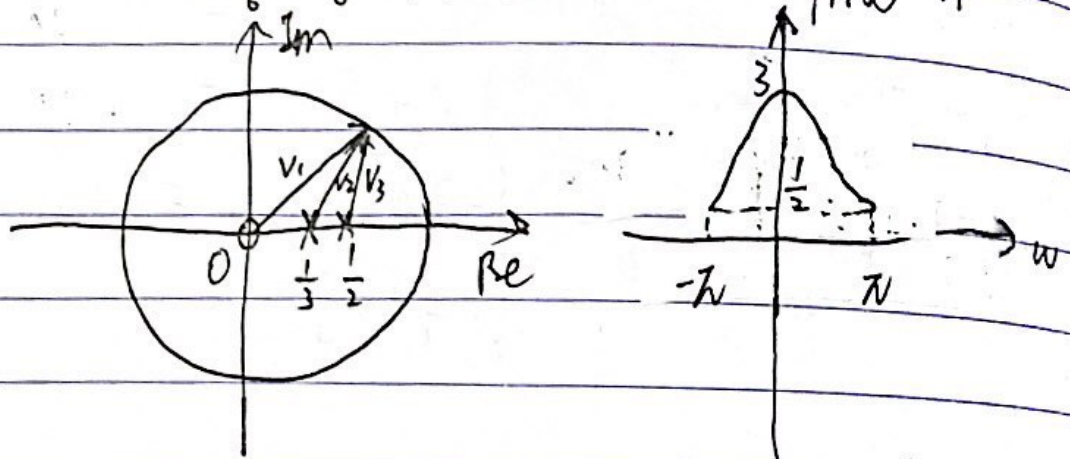
$$9.17^{(a)} \quad \begin{cases} W(z) = X(z) + \frac{5}{6}z^{-1}W(z) - \frac{1}{6}z^{-2}W(z) \\ Y(z) = z^{-1}W(z) \end{cases}$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} \quad (|z| > \frac{1}{2})$$

$$= 6 \left(\frac{1}{1 - \frac{1}{2}z^{-1}} - \frac{1}{1 - \frac{1}{3}z^{-1}} \right)$$

$$(b) \quad h[n] = 6 \left(\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{3}\right)^n u[n] \right)$$

$$(c) \quad H(z) = \frac{z}{z^2 - \frac{5}{6}z + \frac{1}{6}} = \frac{z}{(z - \frac{1}{2})(z - \frac{1}{3})}$$



$$9.18^{(a)} \quad \begin{cases} W(z) = X(z) + \frac{1}{4}z^{-1}W(z) + \frac{1}{8}z^{-2}W(z) \\ Y(z) = 8X(z) + 2z^{-1}W(z) + z^{-2}W(z) \end{cases}$$

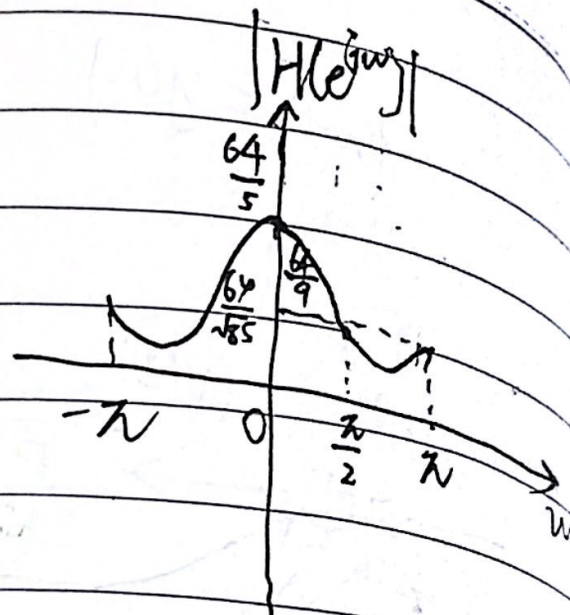
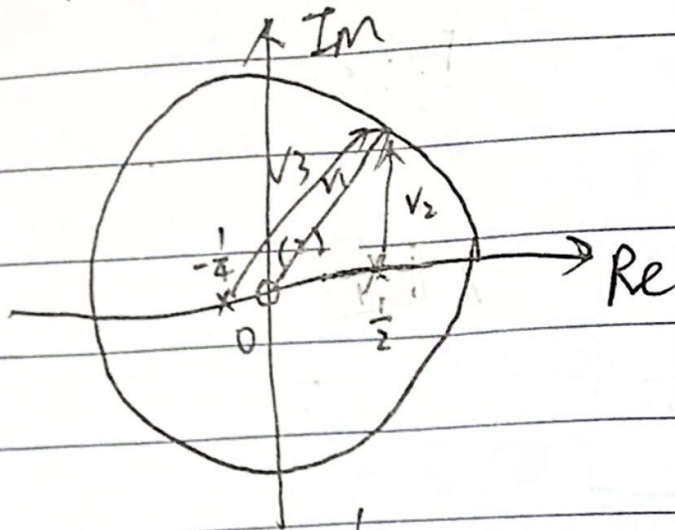
$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = 8 + \frac{2z^{-1} + z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} = \frac{8}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})}$$

$$= \frac{8}{3} \left(\frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 + \frac{1}{4}z^{-1}} \right)$$

$$(|z| > \frac{1}{2})$$

(b) 稳定, 因为其收敛域包含单位圆

9.18(c) $H(z) = \frac{8z^2}{(z - \frac{1}{2})(z + \frac{1}{4})}$



(a) $X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad |z| > \frac{1}{2}$

$$Y(z) = X(z)H(z) = \frac{8}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})}$$

$$= \frac{16}{3} \frac{1}{(1 - \frac{1}{2}z^{-1})^2} + \frac{8}{9} \left(\frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 + \frac{1}{4}z^{-1}} \right)$$

$$y(n) = \frac{16}{3} (n+1) \left(\frac{1}{2}\right)^n u(n) + \frac{16}{9} \left(\frac{1}{2}\right)^n u(n) + \frac{8}{9} \left(-\frac{1}{4}\right)^n u(n)$$