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CONTENTS

KAS 101/201 : PHYSICS

1 UNIT

Relativistic Mechanics

PART-1

Frame of Reference Inertial and Non-inertial Frames, Galilean Transformations, Michelson-Morley Experiment, Postulates of Special Theory of Relativity.

CONCEPT OUTLINE : PART-1

Frame of Reference : It is that coordinate system which is used to identify the position or motion of an object.

Types of Frame of Reference :

a. Inertial frame of reference, and
b. Non-inertial frame of reference.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.1. Show that the frame of reference moving with constant velocity v is an inertial frame of reference.

Answer

1. Let s is a frame of reference which is in rest to an observer and s' is another frame of reference moving with constant velocity v in the positive x direction with respect to the same observer.
2. o and o' are origin of frame s and s' respectively.

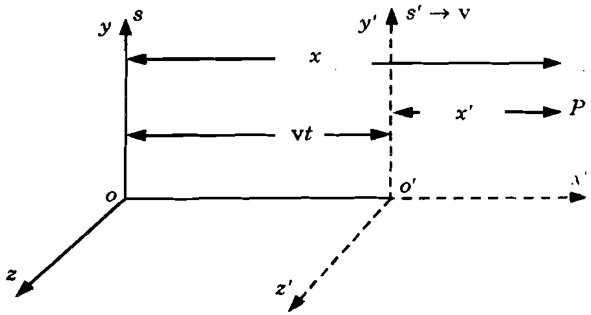


Fig. 1.1.1.

3. Initially o and o' coincide with each other at time $t = t' = 0$, where t and t' = time measured in s and s' frames respectively.

4. Suppose P be a point in the space.
5. Now from Fig. 1.1.1, $x = x' + vt$ ...(1.1.1)

$$y = y' \quad ...(1.1.2)$$
$$z = z' \quad \text{No relative motion} \quad ...(1.1.3)$$
$$t = t' \quad ...(1.1.4)$$

6. Eq. (1.1.1) to eq. (1.1.4) are position and time transformation equations in s and s' frame.
7. Differentiating eq. (1.1.1) w.r.t. t on both sides,

$$\frac{dx}{dt} = \frac{dx'}{dt} + \frac{v\,dt'}{dt} \quad ...(1.1.5)$$

$$\frac{dx}{dt} = \frac{dx'}{dt'} + \frac{v\,dt'}{dt'} \qquad (\because \ t = t' \ \therefore \ dt = dt')$$

$$\Rightarrow \quad u_x = u'_x + v \quad ...(1.1.6)$$

8. Differentiating eq. (1.1.2) w.r.t. t,

$$\frac{dy}{dt} = \frac{dy'}{dt} \quad ...(1.1.7)$$

$$\frac{dy}{dt} = \frac{dy'}{dt'} \qquad (\because \ t = t')$$

$$\Rightarrow \quad u_y = u'_y \quad ...(1.1.8)$$

9. Similarly, $u_z = u'_z$ ...(1.1.9)
10. Now differentiating eq. (1.1.6) w.r.t. t, we get,

$$\frac{du_x}{dt} = \frac{du'_x}{dt} + \frac{dv}{dt}$$

$$\frac{du_x}{dt} = \frac{du'_x}{dt} \qquad (\because \ v = \text{constant})$$

$$\frac{du_x}{dt} = \frac{du'_x}{dt'} \qquad (\because \ t = t')$$

$$\Rightarrow \quad a_x = a_x' \quad ...(1.1.10)$$

11. Similarly on differentiating eq. (1.1.8) and eq. (1.1.9), we get

$$a_y = a'_y \quad ...(1.1.11)$$
$$a_z = a'_z \quad ...(1.1.12)$$

12. Eq. (1.1.10), eq. (1.1.11) and eq. (1.1.12) shows that the acceleration is invariant in both frames.
13. So a frame of reference moving with constant velocity is an inertial frame.

Que 1.2. **Derive the Galilean transformation equations and show that its acceleration components are invariant.**

AKTU 2015-16, Marks 05

Answer

1. Suppose we are in an inertial frame of reference s and the coordinates of some event that occurs at the time t are x, y, z as shown in Fig. 1.2.1.

2. An observer located in a different inertial frame s' which is moving with respect to s at the constant velocity $\vec{v}$, will find that the same event occurs at time t' and has the position coordinates x', y' and z'.

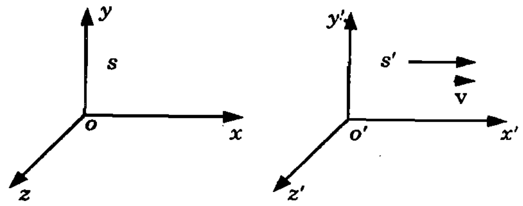


Fig. 1.2.1. Galilean transformation.

3. Assume that $\vec{v}$ is in positive x direction.
4. When origins of s and s' coincide, measurements in the x direction made in s is greater than those of s' by $\vec{v}t$ (distance).
5. Hence,

$$x' = x - vt \quad ...(1.2.1)$$
$$y' = y \quad ...(1.2.2)$$
$$z' = z \quad ...(1.2.3)$$
$$t' = t \quad ...(1.2.4)$$

These set of equations are known as Galilean transformations.

6. Differentiating eq. (1.2.1) with respect to t, we get

$$\frac{dx'}{dt} = \frac{dx}{dt} - v\frac{dt}{dt} \Big\} \text{ no relative motion}$$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - v \qquad (\because \ t = t' \ \therefore \ dt' = dt)$$

7. Similarly $\frac{dy'}{dt'} = \frac{dy}{dt}$

and $\frac{dz'}{dt'} = \frac{dz}{dt}$

8. Since, $dx'/dt' = u_x'$, the x-component of the velocity measured in s', and $dx/dt = u_x$, etc., then,

$$u_x' = u_x - v \quad ...(1.2.5)$$
$$u_y' = u_y \quad ...(1.2.6)$$
$$u_z' = u_z \quad ...(1.2.7)$$

9. Eq. (1.2.5), eq. (1.2.6) and eq. (1.2.7) can be written collectively in the vector form as

$$\vec{u}' = \vec{u} - \vec{v} \quad ...(1.2.8)$$

10. To obtain the acceleration transformation, we differentiate the eq. (1.2.5), eq. (1.2.6) and eq. (1.2.7) with respect to time such that

$$\frac{du_x'}{dt'} = \frac{d}{dt}(u_x - v) = \frac{du_x}{dt}$$

Similarly, $\frac{du_y'}{dt'} = \frac{du_y}{dt}$ and $\frac{du_z'}{dt'} = \frac{du_z}{dt}$

Since, $\frac{du_x'}{dt'} = a_x'$; $\frac{du_y'}{dt'} = a_y'$; $\frac{du_z'}{dt'} = a_z'$

$\frac{du_x}{dt} = a_x$; $\frac{du_y}{dt} = a_y$; $\frac{du_z}{dt} = a_z$

Then we get

$$a_x' = a_x \quad ...(1.2.9)$$
$$a_y' = a_y \quad ...(1.2.10)$$
$$a_z' = a_z \quad ...(1.2.11)$$

or writing these equations collectively, $\vec{a}' = \vec{a}$

3. The measured components of acceleration of a particle are independent of the uniform relative velocity of the reference frames.
4. In other words, acceleration remains invariant when passing from one inertial frame to another that is in uniform relative translational motion.

Que 1.3. **Show that the distance between points is invariant under Galilean transformations.**

Answer

Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the coordinate of two points P and Q in rest frame s. Then the distance between them will be

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Now the distance between them measured in moving frame s' is

$$d' = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}$$

Using Galilean transformation

$$x_2' = x_2 - v_x t,\ y_2' = y_2 - v_y t \text{ and } z_2' = z_2 - v_z t$$
$$x_1' = x_1 - v_x t,\ y_1' = y_1 - v_y t \text{ and } z_1' = z_1 - v_z t$$

4. Hence

$$d' = \sqrt{[(x_2 - v_x t) - (x_1 - v_x t)]^2 + [(y_2 - v_y t) - (y_1 - v_y t)]^2 + [(z_2 - v_z t) - (z_1 - v_z t)]^2}$$
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$
$$\therefore \quad d' = d$$

Que 1.4. **Discuss the objective and outcome of Michelson-Morley experiment.**

Answer

A. Objective of Michelson-Morley Experiment :

1. The main objective of conducting the Michelson–Morley experiment was to confirm the existence of stationary ether.
2. According to Morley if there exist some imaginary medium like 'ether' in the earth atmosphere, there should be some time difference between relative motion of body with respect to earth and against the motion of earth.

3. Due to this time difference there exist some path difference and if such path difference occurs, Huygens concept is correct and if it does not occur then Huygens concept is wrong.

B. Michelson-Morley Experiment :

1. In Michelson-Morley experiment there is a semi-silvered glass-plate P and two plane mirrors M_1 and M_2 which are mutually perpendicular and equidistant from plate P.
2. There is a monochromatic light source in front of glass plate P.
3. The whole arrangement is fixed on a wooden stand and that wooden stand is dipped in a mercury pond. So it becomes easy to rotate.
4. Let v be the speed of imaginary medium (ether) w.r.t. earth and c is the velocity of light, so time taken to move the light ray from plate P to M_2 and reflected back,

$$T_1 = \frac{d}{c+v} + \frac{d}{c-v} = d\left[\frac{2c}{(c^2 - v^2)}\right]$$

$$T_1 = \frac{2dc}{c^2\left[1 - \frac{v^2}{c^2}\right]} = \frac{2d}{c\left[1 - \frac{v^2}{c^2}\right]}$$

5. Expanding binomially,

$$T_1 = \frac{2d}{c}\left[1 + \frac{v^2}{c^2}\right] \quad ...(1.4.1)$$

[Neglecting higher power term]

6. Time taken to move a light ray from plate P to M_1 and to reflect back,

$$T_2 = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2d}{c}\left[1 - \frac{v^2}{c^2}\right]^{-\frac{1}{2}}$$

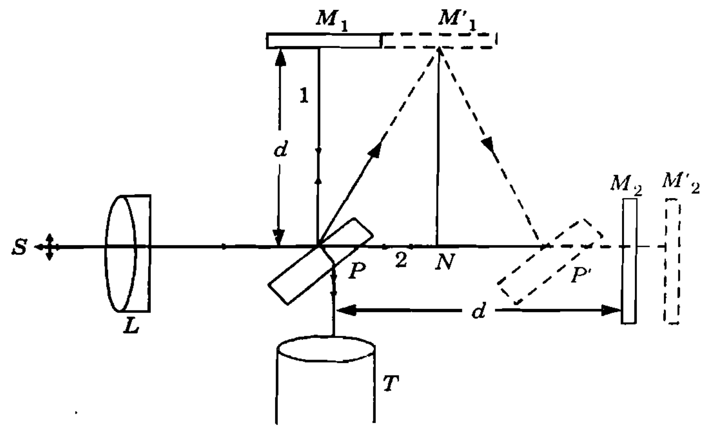


Fig. 1.4.1. The Michelson–Morley experiment.

7. Expanding binomially, $T_2 = \dfrac{2d}{c}\left[1+\dfrac{v^2}{2c^2}\right]$ ...(1.4.2)

[Neglecting higher power term]

8. So, time difference,

$$\Delta t = T_1 - T_2 = \frac{2d}{c}\left[1+\frac{v^2}{c^2}\right]-\frac{2d}{c}\left[1+\frac{v^2}{2c^2}\right]$$

$$= \frac{2d}{c}\left[1+\frac{v^2}{c^2}-1-\frac{v^2}{2c^2}\right]=\frac{2d}{c}\left[\frac{v^2}{2c^2}\right]=\frac{d\,v^2}{c^3} \quad ...(1.4.3)$$

9. Now, the apparatus is rotated by 90° so that the position of mirror M_1 and M_2 gets interchange. So time taken from P to M_1 is,

$$T_1' = \frac{2d}{c}\left[1+\frac{v^2}{2c^2}\right]$$

and time taken from P to M_2 is,

$$T_2' = \frac{2d}{c}\left[1+\frac{v^2}{c^2}\right]$$

10. Time difference, $\Delta t' = T_1' - T_2'$

$$= \frac{2d}{c}\left[1+\frac{v^2}{2c^2}-\frac{v^2}{c^2}-1\right]=-\frac{2d\,v^2}{2c^3}=-\frac{d\,v^2}{c^3} \quad ...(1.4.4)$$

11. So, total time difference,

$$\Delta T = \Delta t - \Delta t' = \frac{d\,v^2}{c^3}-\left(-\frac{d\,v^2}{c^3}\right)=\frac{2d\,v^2}{c^3}$$

12. Since, path difference = speed of light × ΔT

$$\Delta = c \times \Delta T = c \times \frac{2d\,v^2}{c^3} = \frac{2d\,v^2}{c^2} \quad ...(1.4.5)$$

13. If λ is the wavelength of light used then the path difference in terms of the number of fringes is given by,

$$n = \Delta/\lambda = \frac{2d\,v^2}{c^2\lambda}$$

15. Taking, $d = 11$ m

Velocity of earth, $v = 3 \times 10^4$ m/s

Velocity of light, $c = 3 \times 10^8$ m/s

$\lambda = 6000$ Å [1 Å = 10^{-10} m]

$$n = \frac{2\times 11\times 9\times 10^8}{9\times 10^{16}\times 6000\times 10^{-10}}=\frac{22\times 10^2}{6000}=\frac{22}{60}=\frac{11}{30}$$

$n = 0.36$

16. If such type of medium like 'ether' exists in the atmosphere, there must be a fringe shift of 0.36.

17. Michelson–Morley performed that experiment several times in different situations, in different weather conditions but no fringe shift was obtained hence Huygens concept of 'ether drag' is wrong. This is known as negative result.

Que 1.5. **What will be the expected fringe shift on the basis of stationary ether hypothesis in Michelson-Morley experiment ? If the effective length of each part is 8 m and wavelength used is 8000 Å ?**

Answer

Given : $d = 8$ m, $\lambda = 8000 \times 10^{-10}$ m
To Find : Fringe shift.

1. We know that fringe shift is given by

$$n = \frac{2d\,v^2}{c^2\lambda} = \frac{2\times 8\times(3\times 10^4)^2}{(3\times 10^8)^2\times 8\times 10^{-7}}=0.2$$

[∵ $v = 3 \times 10^4$ m/s and $c = 3 \times 10^8$ m/s]

Que 1.6. **State Einstein's postulates of special theory of relativity. Explain why Galilean relativity failed to explain actual results of Michelson-Morley experiment.**

AKTU 2013-14, Marks 05

Answer

A. Einstein's Postulates : There are two postulates of the special theory of relativity proposed by Einstein :

i. Postulate 1 :

The laws of physics are the same in all inertial frames of reference moving with a constant velocity with respect to one another.

Explanation :

1. If the laws of physics had different forms for observers in different frames in relative motion, one could determine from these differences which objects are stationary in space and which are moving.
2. As there is no universal frame of reference, therefore this distinction between objects cannot be made. Hence universal frame of reference is absent.

ii. Postulate 2 :

The speed of light in free space has the same value in all inertial frames of reference. This speed is 2.998×10^8 m/s.

Explanation :

1. This postulate is directly followed from the result of Michelson-Morley experiment.

B. Reason for Failure Galilean transformation to Explain Actual Results of Michelson Morley Experiment :

1. In Galilean transformations the speed of light was not taken to be constant in all inertial frames.
2. These equations were based on absolute time and absolute space.
3. The above two assumptions contradict the Einstein postulates.

4. So the Galilean transformation failed to explain the actual results of Michelson-Morley experiment.

PART-2

Lorentz Transformations, Length Contraction, Time Dilation, Velocity Addition Theorem.

CONCEPT OUTLINE : PART-2

Lorentz Transformations : The equation relating the coordinates of a particle in the two inertial frame on the basis of special theory of relativity are called Lorentz transformations.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.7. What are Lorentz transformation equations ? Derive the expression for it.

Answer

1. The equations in special theory of relativity, which relate to the space and time coordinates of an event in two inertial frames of reference moving with a uniform velocity relative to one another, are called Lorentz transformations.
2. Let us consider two frames of reference s and s' in which frame s' is moving with velocity v along x-axis. The coordinates of frame s are (x, y, z, t) while the coordinates of frame s' are (x', y', z', t').
3. According to first postulates of special theory of relativity in frame s',

$$x' \propto (x - \text{v}t) \Rightarrow x' = k(x - \text{v}t) \quad ...(1.7.1)$$

where, k = Proportionality constant.

In frame s, $x \propto (x' + \text{v}t')$

$$x = k(x' + \text{v}t') \quad ...(1.7.2)$$

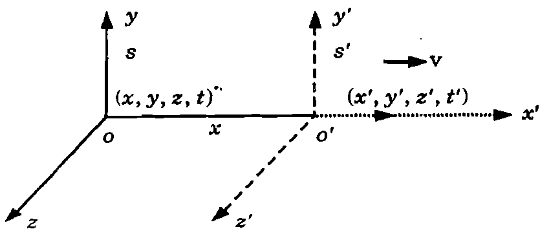


Fig. 1.7.1.

5. Putting x' in eq. (1.7.2),

$$x = k\,[k(x - \text{v}t) + \text{v}t'] = [k^2(x - \text{v}t) + \text{v}\,kt']$$
$$x = k^2x - k^2\text{v}t + k\text{v}t'$$
$$k\text{v}t' = (1 - k^2)x + k^2\text{v}t$$
$$t' = \frac{(1-k^2)x + k^2\text{v}t}{k\text{v}} = \frac{(1-k^2)x}{k\text{v}} + kt \quad ...(1.7.3)$$

6. According to second postulate of special theory of relativity, speed of light is a constant quantity.

In frame s, $x = ct$ ...(1.7.4)

In frame s', $x' = ct'$ ...(1.7.5)

7. Putting the value of x' and t' from eq. (1.7.1) and eq. (1.7.3) in eq. (1.7.5),

$$k(x - \text{v}t) = \frac{cx}{\text{v}k}(1-k^2) + ckt$$
$$x\left[k - \frac{c}{\text{v}k}(1-k^2)\right] = [ck + \text{v}k]\,t$$
$$x = \frac{[ck + \text{v}k]t}{\left[k - \frac{c}{\text{v}k}(1-k^2)\right]} \quad ...(1.7.6)$$

9. On comparing eq. (1.7.6) with eq. (1.7.4),

$$c = \frac{(ck + \text{v}k)}{\left[k - \frac{c}{\text{v}k}(1-k^2)\right]}$$
$$ck + \text{v}k = ck - \frac{c^2}{\text{v}k}(1-k^2)$$
$$\text{v}k = -\frac{c^2}{\text{v}k}(1-k^2)$$
$$\text{v}^2k^2 = -c^2 + c^2k^2$$
$$k^2(\text{v}^2 - c^2) + c^2 = 0$$
$$k^2(c^2 - \text{v}^2) = c^2$$
$$\frac{k^2}{c^2}[c^2 - \text{v}^2] = 1$$
$$k^2\left[1 - \frac{\text{v}^2}{c^2}\right] = 1$$
$$k^2 = \frac{1}{1 - \frac{\text{v}^2}{c^2}}$$

or
$$k = \frac{1}{\sqrt{1 - \frac{\text{v}^2}{c^2}}}$$

10. Putting the value of k in eq. (1.7.1),

$$x' = \frac{1}{\sqrt{1 - \frac{\text{v}^2}{c^2}}}[x - \text{v}t]$$ (First Lorentz transformation equation)

$y' = y$ (Second Lorentz transformation equation)

$z' = z$ (Third Lorentz transformation equation)

11. Now,
$$t' = \frac{x(1-k^2)}{k\mathrm{v}} + kt = \frac{x}{\mathrm{v}}\left[\frac{1}{k} - k\right] + kt$$
$$= \frac{x}{\mathrm{v}}\left[\frac{1}{\dfrac{1}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}} - \frac{1}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right] + t\frac{1}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$$
$$= \frac{x}{\mathrm{v}}\left[\sqrt{1-\frac{\mathrm{v}^2}{c^2}} - \frac{1}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right] + \frac{t}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$$
$$= \frac{x}{\mathrm{v}}\left[\frac{1-\dfrac{\mathrm{v}^2}{c^2}-1}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right] + \frac{t}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}} = \frac{x}{\mathrm{v}}\left[\frac{-\dfrac{\mathrm{v}^2}{c^2}}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right] + \frac{t}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$$
$$= -\frac{x}{c^2}\left[\frac{\mathrm{v}}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right] + \frac{t}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}} = \frac{-\mathrm{v}x + c^2 t}{c^2\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$$
$$t' = \frac{t - \dfrac{\mathrm{v}x}{c^2}}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$$

12. Similarly putting the value of k in eq. (1.7.2), inverse Lorentz transformation equations,
$$x = \frac{1}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}[x' + \mathrm{v}t']$$
$$y = y'$$
$$z = z'$$
$$t = \frac{t' + \dfrac{\mathrm{v}x'}{c^2}}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$$

Que 1.8. **Deduce the Lorentz transformation equations from Einstein's postulates. Also show that at low velocities, the Lorentz transformations reduce to Galilean transformations.**

AKTU 2014-15, Marks 05

Answer

A. Lorentz Transformation Equations from Einstein's Postulates : Refer Q. 1.7, Page 1–9A, Unit-1.

B. Condition at which Lorentz Transformations Reduce to Galilean Transformations :

1. Lorentz transformation equation,
$$x = \frac{x' + \mathrm{v}t'}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}} \qquad ...(1.8.1)$$
2. At low velocities means v <<< c

Thus, $1 - \dfrac{\mathrm{v}^2}{c^2} \simeq 1$

So eq. (1.8.1) reduces to, $x = x' + \mathrm{v}t'$ ...(1.8.2)

Eq. (1.8.2) is a Galilean transformation equation.

3. It means at low velocities, the Lorentz transformation reduces to Galilean transformation.

Que 1.9. **Show that the space time interval $x^2 + y^2 + z^2 - c^2t^2 = 0$ is invariant under Lorentz transformation.**

Answer

1. The Lorentz transformation are
$$x = \frac{x' + \mathrm{v}t'}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}},\ y = y',\ z = z' \text{ and } t = \frac{t' + \dfrac{\mathrm{v}x'}{c^2}}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}} \qquad ...(1.9.1)$$
2. Using eq. (1.9.1) in given equation
$$x^2 + y^2 + z^2 = c^2t^2$$
$$\Rightarrow \left[\frac{x' + \mathrm{v}t'}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right]^2 + y'^2 + z'^2 = c^2\left[\frac{t' + \dfrac{\mathrm{v}x'}{c^2}}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right]^2$$
$$\Rightarrow y'^2 + z'^2 = \frac{1}{1-\dfrac{\mathrm{v}^2}{c^2}}\left[c^2t'^2 + \frac{\mathrm{v}^2x'^2}{c^2} - x'^2 - \mathrm{v}^2t'^2\right]$$
$$\Rightarrow y'^2 + z'^2 = \frac{1}{1-\dfrac{\mathrm{v}^2}{c^2}}\left[(c^2t'^2 - x'^2)\left(1-\frac{\mathrm{v}^2}{c^2}\right)\right]$$
$$\Rightarrow x'^2\left(1-\frac{\mathrm{v}^2}{c^2}\right) + y'^2\left(1-\frac{\mathrm{v}^2}{c^2}\right) + z'^2\left(1-\frac{\mathrm{v}^2}{c^2}\right) = c^2t'^2\left(1-\frac{\mathrm{v}^2}{c^2}\right)$$

$$\Rightarrow \quad x'^2 + y'^2 + z'^2 = c^2 t'^2$$

3. Hence $x^2 + y^2 + z^2 = c^2 t^2$ is invariant under Lorentz transformation.

Que 1.10. **As measured by O a bulb goes off at $x = 100$ km, $y = 10$ km, $z = 1$ km and $t = 5 \times 10^{-4}$ sec. What are the coordinates x', y', z' and t' of this event as determined by a second observer O' moving relative to O at $-0.8\,c$ along the common x-x' axis ?**

Answer

Given : $x = 100$ km, $y = 10$ km, $z = 1$ km
$v = -0.8 \times 3 \times 10^5$ km/s, $t = 5 \times 10^{-4}$ s
To Find : Coordinates x', y', z' and t'.

1. Since,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \frac{100 - (-0.8 \times 3 \times 10^5) \times 5 \times 10^{-4}}{\sqrt{1 - (0.8)^2}} = 366.66 \text{ km}$$

$$[\because \; c = 3 \times 10^5 \text{ km/s}]$$

2.
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{5 \times 10^{-4} - \frac{(-0.8 \times 3 \times 10^5) \times 100}{(3 \times 10^5)^2}}{\sqrt{1 - (0.8)^2}}$$

$$= 12.77 \times 10^{-4} \text{ s}$$

3.
$$y' = y = 10 \text{ km}$$
$$z' = z = 1 \text{ km}$$

Que 1.11. **Show that a moving circle will appeared to be an ellipse if it is seen from a frame which is at rest.**

Answer

1. The equation of circle is $x^2 + y^2 = a^2$.
2. Putting the values from Lorentz transformation,

$$\left[\frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}\right]^2 + y'^2 = a^2$$

$$\frac{x'^2 + v^2 t'^2 + 2x'.vt'}{\left(1 - \frac{v^2}{c^2}\right)} + y'^2 = a^2$$

$$x'^2 + v^2 t'^2 + 2x'\,vt' + y'^2 - y'^2\frac{v^2}{c^2} = a^2 - \frac{a^2 v^2}{c^2}$$

$$x'^2 + y'^2\left(1 - \frac{v^2}{c^2}\right) = a^2\left(1 - \frac{v^2}{c^2}\right) - v^2 t'^2 - 2x'vt'$$

$$\frac{x'^2}{\left(1 - \frac{v^2}{c^2}\right)} + \frac{y'^2}{1} = a^2 - \frac{v^2 t'^2 + 2x'vt'}{\left(1 - \frac{v^2}{c^2}\right)} \quad \text{...(1.11.1)}$$

3. Let, $P^2 = 1 - \dfrac{v^2}{c^2}$ and $Q^2 = a^2 - \dfrac{v^2 t'^2 + 2x'vt'}{\left(1 - \frac{v^2}{c^2}\right)}$

Hence, eq. (1.11.1) becomes,

$$\frac{x'^2}{P^2} + \frac{y'^2}{1^2} = Q^2$$

$$\frac{x'^2}{(QP)^2} + \frac{y'^2}{Q^2} = 1 \quad \text{...(1.11.2)}$$

4. Eq. (1.11.2) represents an equation of ellipse.

Que 1.12. **What is length contraction ? Find out its equation using Lorentz transformation.**

Answer

1. The appeared decrease in the length of a body in the direction of motion is called length contraction.
2. Let us consider two frame of reference s and s' in which frame s' is moving with velocity v along x-axis.
3. A rod of length 'L_0' is moving horizontally in frame s'.

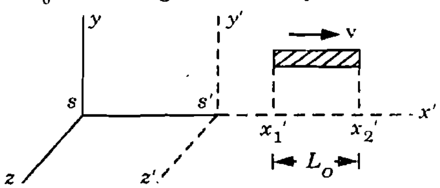


Fig. 1.12.1.

3. According to Lorentz transformation,

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

4. $$x_2 - x_1 = L \quad \text{(apparent length)}$$
$$x_2' - x_1' = L_0$$
$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

Que 1.13. **What do you mean by proper length ? Derive the expression for relativistic length. Calculate the percentage contraction of a rod moving with a velocity of 0.6 c in a direction inclined at 30° to its own length.** **AKTU 2015-16, Marks 10**

Answer

A. **Proper Length :** The length of a body measured by an observer in the frame in which the body is at rest called proper length or actual length.

B. **Expression for Relativistic Length :** Refer Q. 1.12, Page 1–14A, Unit-1.

C. **Numerical :**

Given : v = 0.6c, inclination angle of rod = 30°.
To Find : Percentage contraction of rod.

1. Suppose the proper length of rod is L_0.
2. Observed length, along the direction of motion,
$$L_x = L_0 \cos 30° \times \sqrt{1-(0.6)^2} = 0.7L_0$$
3. Observed length, perpendicular to the direction of motion,
$$L_y = L_0 \sin 30° = \frac{L_0}{2}$$

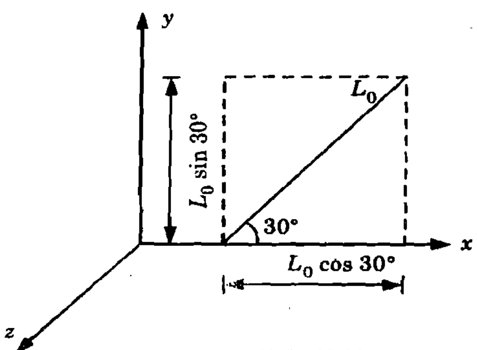


Fig. 1.13.1.

4. Length of moving rod,
$$L = \sqrt{L_x^2 + L_y^2} = \left[(0.7L_0)^2 + \left(\frac{L_0}{2}\right)^2\right]^{1/2} = 0.86L_0$$
5. Percentage contraction in length $= \frac{L_0 - L}{L_0} \times 100$
$$= \frac{L_0 - 0.86L_0}{L_0} \times 100 = 14\ \%$$

Que 1.14. **What will be the apparent length of rod of length 5 m and inclined at an angle 60° to horizontal. This rod is moving with a speed of 3×10^7 m/s.**

Answer

Given : L_0 = 5 m, inclination angle of rod = 60°, v = 3×10^7 m/s
To Find : Apparent length.

1. Since, frame of reference is moving along x-direction. So length of rod appears to change in x-direction only.
2. So, new length in x-direction,
$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = 5\cos 60°\sqrt{1 - \left(\frac{3 \times 10^7}{3 \times 10^8}\right)^2}$$
$$L_x = 2.487 \text{ m}$$
3. New apparent length of rod,
$$= \sqrt{L_x^2 + (5\sin 60°)^2}$$
$$= \sqrt{(2.487)^2 + (5\sin 60°)^2} = 4.99 \text{ m}$$

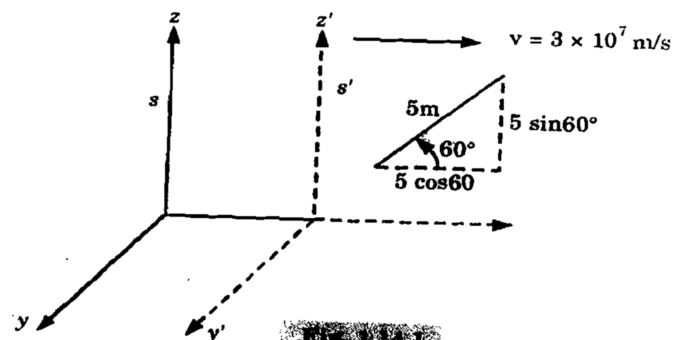


Fig. 1.14.1.

Que 1.15. **How much time does a metre stick moving at 0.1 *c* relative to an observer take to pass the observer ? The metre stick is parallel to its motion.**

Answer

Given : $L_0 = 1$ m, v = 0.1 c
To Find : Time of stick to pass observer.

1. Since, $L = L_0\sqrt{1-\frac{v^2}{c^2}} = L_0\sqrt{1-0.01} = 1\times\sqrt{0.99} = 0.994$ m
2. $\text{Time} = \frac{L}{v} = \frac{0.994}{0.1\times 3\times 10^8} = 3.31 \times 10^{-8}$ sec

Que 1.16. **What is time dilation ? Find out its equation using Lorentz transformation and give an example to show that time dilation is a real effect.**

Answer

A. Time Dilation :
1. In the special theory of relativity, the moving clock is found to run slower than a clock at rest does. This effect is known as time dilation.
2. Suppose *s* and *s'* are two frames of references. Frame *s'* is moving with constant velocity v in the positive *x* direction w.r.t. frame *s*.
3. If (t_1', t_2') be the times of occurrence of two events measured by the clock in frame *s'* and t_o be the corresponding time interval, then we have

$$t_o = t'_2 - t'_1 \quad ...(1.16.1)$$

4. If (t_1, t_2) be the times of occurrence of the same events measured by the another clock in the stationary frame *s* and *t* be the corresponding time interval, then we have

$$t = t_2 - t_1$$

5. Using Lorentz transformation equation,

$$t = \frac{t'_2+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t'_1+\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t'_2+\frac{vx'}{c^2}-t'_1-\frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t'_2-t'_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} \quad ...(1.16.2)$$

6. From eq. (1.16.1) and eq. (1.16.2), we get, $t > t_o$
So the relativistic interval of time is more than proper interval of time.

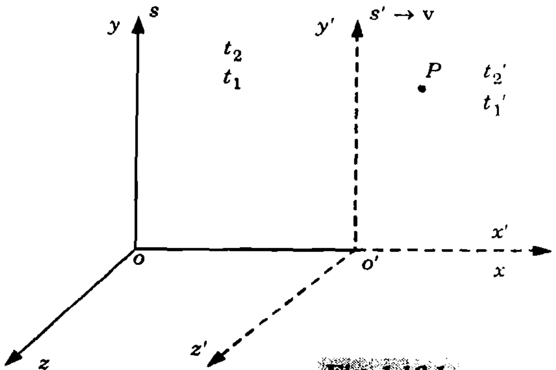


Fig. 1.16.1.

7. Therefore a moving clock appears to go slow (*i.e.*, take more time to complete a rotation compared to a rest clock).
8. If v = c, then, $t = \infty$
9. If v is very less than *c* then $t = t_o$

B. Time Dilation is a Real Effect :
1. The cosmic ray particles called μ-meson are created at high altitude about 10 km above earth's atmosphere and are projected towards earth surface with velocity 2.994×10^8 m/s.
2. It decays into e^+ (Positron) with an average life time of about 2×10^{-6} sec.
3. Therefore, in its life time μ-meson can travel a distance,

$$d = vt = 2.994 \times 10^8 \times 2 \times 10^{-6} = 600 \text{ m}$$

4. But it is found at earth surface also. It is possible because of time dilation effect.

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2\times10^{-6}}{\sqrt{1-\left(\frac{2.994\times10^8}{3\times10^8}\right)^2}} \quad [\because t_o = 2\times10^{-6} \text{ sec.}]$$

$$= 3.17 \times 10^{-5} \text{ sec}$$

5. In this time a μ-meson can travel a distance

$$d = 2.994 \times 10^8 \times 3.17 \times 10^{-5} = 9490.98 \text{ m} \approx 10 \text{ km}$$

6. This shows that time dilation is a real effect.

Que 1.17. **The proper mean life time of + μ meson is 2.5×10^{-8} sec. Calculate :**
1. **Mean life time of + μ meson travelling with the velocity 2.4×10^8 m/s.**
2. **The distance travelled by this + μ meson during one mean life time.**
3. **The distance travelled without relativistic effect.**

Answer

Given : $t_o = 2.5 \times 10^{-8}$ s, v = 2.4×10^8 m/s
To Find : i. Mean life time.
ii. Distance travelled in one mean life time.
iii. Distance travelled without relativistic effect.

1. Mean life time, $$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.5\times10^{-8}}{\sqrt{1-\left(\frac{2.4\times10^8}{3\times10^8}\right)^2}} = \frac{2.5\times10^{-8}}{\sqrt{1-(0.8)^2}}$$
$$= \frac{2.5\times10^{-8}}{0.6} = 4.17 \times 10^{-8} \text{ sec}$$
2. The distance travelled = $(2.4 \times 10^8) \times (4.166 \times 10^{-8}) = 10$ m
3. The distance travelled without relativistic effect
$$= (2.4 \times 10^8) \times (2.5 \times 10^{-8}) = 6 \text{ m.}$$

Que 1.18. **The half life of a particular particle as measured in the laboratory comes out to be 4.0×10^{-8} sec, when its speed is 0.8 *c* and 3.0×10^{-8} sec, when its speed is 0.6 *c*. Explain this.**

Answer

1. The time interval in motion is given by
$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{(where, } t_0 = \text{proper time interval)}$$
2. The proper half life of the given particle is
$$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$$
3. In the first case $t = 4.0 \times 10^{-8}$ sec and v = 0.8 *c*
$$t_0 = 4.0 \times 10^{-8}\sqrt{1-\left(\frac{0.8\,c}{c}\right)^2} = 2.4 \times 10^{-8} \text{ sec}$$
4. As proper half life is independent of velocity, therefore half life of the particle when speed is 0.6 *c* must be given by
$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.4\times10^{-8}}{\sqrt{1-\left(\frac{0.6\,c}{c}\right)^2}} = \frac{2.4\times10^{-8}}{0.8} = 3\times10^{-8} \text{ sec}$$
which is actual observation.
5. Thus the variation of half life of given particle is due to relativistic time dilation.

Que 1.19. **At what speed should a clock be moved so that it may appear to lose 1 minute in each hour ?**

Answer

Given : Time loss = 1 min
To Find : Speed of clock.

1. Since, rest clock takes 60 minutes for *T* time interval.
∴ Rest clock takes 1 minute for *T*/60 time interval.
2. Now, moving clock takes 59 minutes for same *T* time interval.
∴ Moving clock takes 1 minute for *T*/59 time interval.
3. Here, $$t_0 = \frac{T}{60} \text{ and } t = \frac{T}{59}$$
4. From time dilation formula, $t = \dfrac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$
$$\frac{T}{59} = \frac{\frac{T}{60}}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \sqrt{1-\frac{v^2}{c^2}} = \frac{59}{60}$$
$$v = 5.45 \times 10^7 \text{ m/s}$$

Que 1.20. **Deduce the relativistic velocity addition theorem. Show that it is consistent with Einstein's second postulate.**

AKTU 2014-15, Marks 05
AKTU 2017-18, Marks 07

Answer

A. Relativistic Velocity Addition Theorem :
1. Let *s* and *s'* be two frame of references in which *s'* is moving with a constant velocity v in the *x* direction w.r.t. frame *s*.
2. Let *P* be a point having coordinate (*x*, *y*, *z*, *t*) and (*x'*, *y'*, *z'*, *t'*) in frame *s* and *s'* at any instant of time.
3. In these two frames the components of the velocities of that particle along *x*, *y* and *z* axis will be given by
$$u_x = \frac{dx}{dt} \text{ and } u_x' = \frac{dx'}{dt'}$$
$$u_y = \frac{dy}{dt} \text{ and } u_y' = \frac{dy'}{dt'}$$
$$u_z = \frac{dz}{dt} \text{ and } u_z' = \frac{dz'}{dt'}$$
4. Now using the Lorentz transformation equation,
$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z' \text{ and } t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

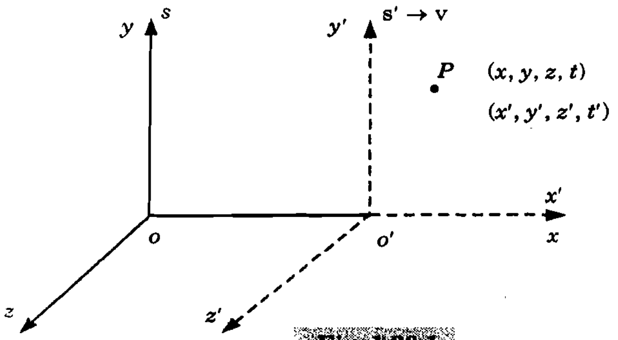


Fig. 1.20.1.

5. Differentiating above equations,

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ...(1.20.1)$$

$$dy = dy' \quad ...(1.20.2)$$

$$dz = dz' \quad ...(1.20.3)$$

$$dt = \frac{dt' + \frac{v}{c^2}dx'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad ...(1.20.4)$$

6. Dividing eq. (1.20.1) by eq. (1.20.4),

$$\frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}dx'}$$

Multiply and divide by $\frac{1}{dt'}$ on R.H.S.

$$\frac{dx}{dt} = \frac{\frac{dx'}{dt'} + v}{\frac{dt'}{dt'} + \frac{v}{c^2}\frac{dx'}{dt'}} \Rightarrow u_x = \frac{u_x' + v}{1 + \frac{v}{c^2}u_x'} \quad ...(1.20.5)$$

7. Similarly dividing eq. (1.20.2) by eq. (1.20.4),

$$\frac{dy}{dt} = \frac{dy'\sqrt{1 - \frac{v^2}{c^2}}}{dt' + \frac{v}{c^2}dx'} \times \frac{\frac{1}{dt'}}{\frac{1}{dt'}}$$

$$u_y = \frac{u_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2}u_x'} \quad ...(1.20.6)$$

9. Similarly,

$$u_z = \frac{u_z'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v}{c^2}u_x'} \quad ...(1.20.7)$$

10. If motion of object is only in x direction then $u = u_x$ and equation becomes

$$u = \frac{u' + v}{1 + \frac{vu'}{c^2}}$$

B. Consistency with Einstein's Second Postulate :

Case–1 : If $u' = c$, then

$$u = \frac{c + v}{1 + \frac{v}{c^2}c} \Rightarrow u = \frac{(c+v).c}{(c+v)} = c$$

Case–2 : If $v = c$ then

$$u = \frac{u' + c}{1 + \frac{c.u'}{c^2}} = c$$

Case–3 : If $v = c$ and $u' = c$ then

$$u = \frac{c + c}{1 + \frac{c.c}{c^2}} = c$$

1. So the velocity of any object cannot be greater than 'c', whatever be the velocity of moving frame or velocity of object in that frame.
2. Therefore, the relativistic velocity addition theorem is consistent with the Einstein's second postulate of special theory of relativity.

Que 1.21. Rocket A travels towards the right and rocket B travels towards the left with velocity 0.8c and 0.6c respectively relative to the earth. What is the velocity of rocket :

a. A, measured from B, and

b. B, measured from A ?

Answer

Given : Velocity of rocket A and B = 0.8c and 0.6c respectively.

To Find : i. Velocity of rocket A, measured from B.
ii. Velocity of rocket B, measured from A.

1. The velocity addition formula,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

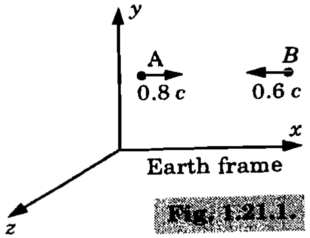


Fig. 1.21.1.

2. Velocity of earth w.r.t. B, $v = 0.6\,c$
3. Velocity of A w.r.t. earth, $u' = 0.8\,c$
4. Velocity of A w.r.t. B,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{0.8c + 0.6c}{1 + \frac{(0.8c)(0.6c)}{c^2}}$$

$$\Rightarrow \quad u = \frac{1.4c}{1.48} = 0.946c$$

5. Velocity of earth w.r.t. A,

$$v = -0.8\,c$$

6. Velocity of B w.r.t. earth,

$$u' = -0.6\,c$$

7. Velocity of B w.r.t. A,

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}} = \frac{(-0.6c) + (-0.8c)}{1 + \frac{(-0.8)(-0.6)c^2}{c^2}}$$

$$u = -0.946\,c$$

8. Negative sign indicates that velocity of B w.r.t. A is towards left.

Que 1.22. **Show that no particle can attain a velocity larger than velocity of light.**

Answer

1. Let $\quad v_x' = c$ and $v = c$
2. we know, $\quad v_x = \dfrac{v_x' + v}{1 + \dfrac{v_x' v}{c^2}}$

$$v_x = \frac{c + c}{1 + \frac{c^2}{c^2}} = \frac{2c}{2} = c$$

PART-3

Variation of Mass with Velocity, Einstein's Mass Energy Relation, Relativistic Relation between Energy and Momentum, Massless Particle.

CONCEPT OUTLINE : PART-3

Variation of Mass with Velocity : Mass is a function of the velocity of the body. It increases with increasing velocity represented by the relation :

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Mass Energy Equivalence : The variation of mass with velocity has modified the idea of energy, so that, a relationship can be established between mass and energy.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 1.23. **Deduce expression for variation of mass with velocity.**

OR

Show that the relativistic invariance of the law of conservation of momentum leads to the concept of variation of mass with velocity.

AKTU 2015-16, Marks 10

Answer

1. Suppose s and s' are two frame of references in which s' is moving with a constant velocity 'v' w.r.t. observer o.
2. Two identical bodies A and B having same mass m are moving with velocity u' but in opposite direction in s' frame.
3. After some time both collides and stick together and momentarily comes to rest in s' frame.
4. Now from velocity addition theorem,

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \qquad \text{...(1.23.1)}$$

$$u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \qquad \text{...(1.23.2)}$$

Where, $\quad u_1$ and u_2 = velocity of A and B in s frame before collision and

m_1 and m_2 = their masses in s frame.

5. From the law of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\,v \qquad \text{...(1.23.3)}$$

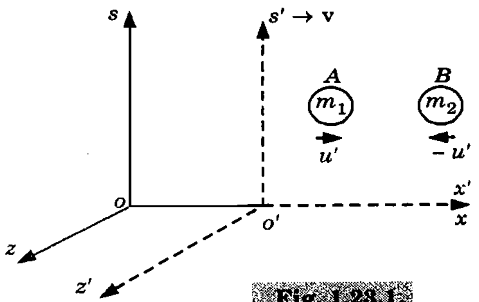


Fig. 1.23.1.

Putting the value of u_1 and u_2 from eq. (1.23.1) and eq. (1.23.2), in eq. (1.23.3) we get

$$\Rightarrow \quad m_1\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)+m_2\left(\frac{-u'+v}{1-\frac{u'v}{c^2}}\right)=(m_1+m_2)v$$

$$\Rightarrow \quad m_1\left(\frac{u'+v}{1+\frac{u'-v}{c^2}}\right)=m_2\left(v-\frac{-u'+v}{1-\frac{u'v}{c^2}}\right)$$

$$\Rightarrow \quad m_1\left[\frac{u'+v-v-\frac{u'v^2}{c^2}}{1+\frac{u'v}{c^2}}\right]=m_2\left[\frac{v-\frac{u'v^2}{c^2}+u'-v}{1-\frac{u'v}{c^2}}\right]$$

$$\Rightarrow \quad m_1 u'\frac{\left[1-\frac{v^2}{c^2}\right]}{1+\frac{u'v}{c^2}}=m_2 u'\frac{\left[1-\frac{v^2}{c^2}\right]}{1-\frac{u'v}{c^2}}$$

$$\Rightarrow \quad \frac{m_1}{m_2}=\frac{1+\frac{u'v}{c^2}}{1-\frac{u'v}{c^2}} \qquad ...(1.23.4)$$

Now, squaring eq. (1.23.1), we get

$$u_1^2=\left(\frac{u'+v}{1+u'v/c^2}\right)^2$$

$$1-\frac{u_1^2}{c^2}=1-\frac{1}{c^2}\left(\frac{u'+v}{1+\frac{u'v}{c^2}}\right)^2=\frac{\left(1+\frac{u'v}{c^2}\right)^2-\frac{(u'+v)^2}{c^2}}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$=\frac{1+\frac{u'^2v^2}{c^4}+\frac{2u'v}{c^2}-\left(\frac{u'^2}{c^2}+\frac{v^2}{c^2}+\frac{2u'v}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$1-\frac{u_1^2}{c^2}=\frac{\left(1-\frac{v^2}{c^2}\right)-\frac{u'^2}{c^2}\left(1-\frac{v^2}{c^2}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$1+\frac{u'v}{c^2}=\sqrt{\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{1-\frac{u_1^2}{c^2}}} \qquad ...(1.23.5)$$

7. Similarly, we can take eq. (1.23.2) and proceed in the same manner,

$$1-\frac{u'v}{c^2}=\sqrt{\frac{\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{u'^2}{c^2}\right)}{1-\frac{u_2^2}{c^2}}} \qquad ...(1.23.6)$$

8. Putting eq. (1.23.5) and eq. (1.23.6) in eq. (1.23.4),

$$\frac{m_1}{m_2}=\sqrt{\frac{1-\frac{u_2^2}{c^2}}{1-\frac{u_1^2}{c^2}}}$$

$$m_1\sqrt{1-\frac{u_1^2}{c^2}}=m_2\sqrt{1-\frac{u_2^2}{c^2}}=m_o=\text{constant}$$

9. If body B is at rest in stationary frame s that is $u_2 = 0$ before collision and $m_2 = m_0$ in frame s.
10. As bodies A and B are identical and have same mass in s'. So, $m_1 = m$ (relativistic mass) for $u_1 = v$.

Therefore, $$m=\frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Que 1.24. **Derive Einstein mass energy relation $E = mc^2$ and discuss it. Give some evidence showing its validity.**

Answer

A. Einstein Mass Energy Relation :

1. Suppose a force 'F' is acting on a body of mass 'm' in the same direction as its velocity 'v'.
2. The gain in K.E. in the body is in the form of work done.
3. If a force 'F' displaces the particle through a small distance 'ds', then work done,

$$dW = dK = F.\,ds \qquad ...(1.24.1)$$

4. According to Newton's Law of motion,

$$F=\frac{dp}{dt}=\frac{d(m\,v)}{dt}$$

$$\Rightarrow \quad F = \frac{m\,d\mathrm{v}}{dt} + \mathrm{v}\frac{dm}{dt}$$

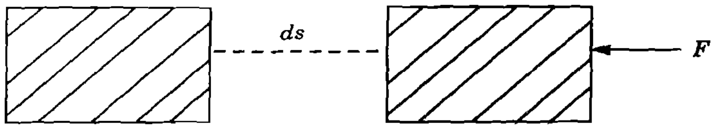


Fig. 1.24.1.

5. Multiplying 'ds' on both sides, we get

$$\Rightarrow \quad F.ds = m\,\frac{ds}{dt}.d\mathrm{v} + \mathrm{v}\frac{ds}{dt}.dm$$

6. From eq. (1.24.1), $dK = m\,\mathrm{v}\,d\mathrm{v} + \mathrm{v}^2\,dm$ ...(1.24.2) $\left(\because \frac{ds}{dt} = \mathrm{v}\right)$

7. But we know that, $m = \dfrac{m_0}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$ (m_0 = rest mass of particle)

On differentiating, we get

$$dm = m_0\left(-\frac{1}{2}\right)\left(1-\frac{\mathrm{v}^2}{c^2}\right)^{-\frac{3}{2}}\left(\frac{-2\mathrm{v}}{c^2}\right)d\mathrm{v} = \frac{m_0\,\mathrm{v}\,d\mathrm{v}}{c^2\left(1-\dfrac{\mathrm{v}^2}{c^2}\right)^{\frac{3}{2}}}$$

$$dm = \frac{m\,\mathrm{v}\,d\mathrm{v}}{c^2\left(1-\dfrac{\mathrm{v}^2}{c^2}\right)} \qquad \left(\because \quad m = \frac{m_0}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}\right)$$

$$dm\,(c^2 - \mathrm{v}^2) = m\mathrm{v}\,d\mathrm{v} \qquad ...(1.24.3)$$

8. Now putting the value of eq. (1.24.3) in eq. (1.24.2), we get

$$dK = (c^2 - \mathrm{v}^2)\,dm + \mathrm{v}^2\,dm = c^2\,dm$$

9. If the change in kinetic energy of a particle be K when it's mass changes from rest mass m_0 to relativistic mass m, then

$$\int_0^K dK = \int_{m_0}^{m} c^2\,dm$$

$$K = c^2\,(m - m_0) = c^2\,(\Delta m)$$

10. Total energy of particle,

$$E = \text{Relativistic K.E.} + \text{Rest mass energy}$$

$$E = (m - m_0)\,c^2 + m_0c^2 = mc^2$$

This is Einstein's mass energy-relation, which states mass energy equivalence.

B. Evidence of its Validity :

1. In nuclear reaction such as fission and fusion. These reactions take place in nuclear reactor and during the explosion of atom bomb. The cause of production of energy in stars and some other processes becomes known today only due to the discovery of this important mass energy relation.

2. In process of annihilation of matter, an electron and a positron give up all its mass into two photons. The entire mass is converted into energy. This verifies mass-energy relation.

Que 1.25. **Derive the relation**

a. $E^2 = P^2c^2 + m_0^{\,2}c^4$, **and**

b. $P = \sqrt{\dfrac{K^2}{c^2} + 2m_0K}$ **where, K is kinetic energy.**

Answer

a. Derivation for $E^2 = P^2c^2 + m_0^{\,2}c^4$:

1. Total energy of a particle is, $E = mc^2$ ...(1.25.1)

2. The relativistic mass, $m = \dfrac{m_0}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}}$ ...(1.25.2)

3. Putting the value of eq. (1.25.2) in eq. (1.25.1),

$$E = \frac{m_0c^2}{\sqrt{1-\dfrac{\mathrm{v}^2}{c^2}}} = \frac{m_0c^2}{\sqrt{1-\dfrac{m^2\mathrm{v}^2}{m^2c^2}}} = \frac{m_0\,c^2}{\sqrt{1-\dfrac{m^2\mathrm{v}^2c^2}{m^2c^4}}}$$

$$\Rightarrow \quad E = \frac{m_0c^2}{\sqrt{1-\dfrac{P^2c^2}{m^2c^4}}}$$

$$\Rightarrow \quad E^2 = \frac{m_0^2c^4}{1-\dfrac{P^2c^2}{m^2c^4}} = \frac{m_0^2c^4}{1-\dfrac{P^2c^2}{E^2}} \qquad [\because \quad E = mc^2]$$

$$\Rightarrow \quad E^2\left(1-\frac{P^2c^2}{E^2}\right) = m_0^{\,2}c^4$$

$$\Rightarrow \quad E^2 - P^2c^2 = m_0^{\,2}c^4$$

$$\Rightarrow \quad E^2 = P^2c^2 + m_0^{\,2}c^4$$

b. Derivation for $P = \sqrt{\dfrac{K^2}{c^2} + 2m_0K}$:

1. Total energy, E = relativistic kinetic energy + rest mass energy

$$\Rightarrow \quad E = K + m_0c^2$$

$$\Rightarrow \quad K = E - m_0c^2$$

$$\Rightarrow \quad K = \sqrt{m_0^2c^4 + P^2c^2} - m_0c^2 \quad \left(\because E = \sqrt{m_0^2c^4 + P^2c^2}\right)$$

$$\Rightarrow \quad K + m_0c^2 = \sqrt{m_0^2c^4 + P^2c^2} \qquad ...(1.25.3)$$

2. On squaring both side of eq. (1.25.3), we get

$$\Rightarrow \quad K^2 + m_0^{\,2}c^4 + 2Km_0c^2 = m_0^{\,2}c^4 + P^2c^2$$

$$\Rightarrow \quad K^2 + 2Km_0c^2 = P^2c^2$$

$$\Rightarrow \quad P^2 = 2Km_0 + \frac{K^2}{c^2}$$

$$\Rightarrow \quad P = \sqrt{\frac{K^2}{c^2} + 2m_0K}$$

Que 1.26. **Show that the relativistic form of Newton's second law, when $\bar{F}$ is parallel to $\bar{v}$ is**

$$\bar{F} = m_o \frac{d\bar{v}}{dt}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}$$

Answer

1. Newton's second law,

$$\bar{F} = \frac{d\bar{P}}{dt} = \frac{d}{dt}(m\bar{v})$$

But
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

So,
$$\bar{F} = \frac{d}{dt}\left[\frac{m_o \bar{v}}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

$$\bar{F} = m_o\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{d\bar{v}}{dt} + \frac{\bar{v}\left(-\frac{1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{-1/2}\left(\frac{-2\bar{v}}{c^2}\right)}{\left(1 - \frac{v^2}{c^2}\right)}\frac{d\bar{v}}{dt}\right]$$

$$= m_o\left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\frac{d\bar{v}}{dt} + \frac{\frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}\frac{d\bar{v}}{dt}\right]$$

$$\bar{F} = m_o\frac{d\bar{v}}{dt}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}\left[\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2}{c^2}\right] = m_o\frac{d\bar{v}}{dt}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}$$

Que 1.27. **The mass of a moving electron is 11 times its rest mass. Find its kinetic energy and momentum.** **AKTU 2011-12, Marks 05**

Answer

Given : $m = 11\, m_0$
To Find : i. Kinetic energy.
ii. Momentum

1. Since,
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

So
$$11\, m_o = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{11} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{121}$$

$$\frac{v}{c} = 0.995$$

$$v = 0.995 \times 3 \times 10^8 = 2.985 \times 10^8 \text{ m/s}$$

2. Kinetic Energy,
$$\text{K.E.} = (m - m_0)\, c^2$$
$$= (11\, m_0 - m_0)\, c^2 = 10\, m_0c^2$$
$$= 10 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 = 8.19 \times 10^{-13} \text{ J}$$

3. Momentum,
$$P = mv = 11\, m_0v = 11 \times 9.1 \times 10^{-31} \times 2.985 \times 10^8$$
$$P = 2.987 \times 10^{-21} \text{ N-s}$$

Que 1.28. **The total energy of a moving meson is exactly twice its rest energy. Find the speed of meson.** **AKTU 2012-13, Marks 05**

Answer

Given : $E = 2E_o$
To Find : Speed of meson.

1. As given
$$E = 2E_0$$
$$mc^2 = 2m_oc^2$$
or
$$m = 2m_o$$

2. Since,
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$2m_0 = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$2 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow v = 0.866\,c$$

$$= 0.866 \times 3 \times 10^8 = 2.59 \times 10^8 \text{ m/s}$$

Que 1.29. **Show that the relativistic K.E. will convert into classical K.E. if v << *c*.**

Answer

1. The expression for relativistic K.E. is

$$K = (m - m_0)\,c^2 = \left(\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - m_0\right)c^2 = m_0c^2\left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right]$$

2. Expanding using binomial theorem,

$$K = m_0c^2\left[1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \ldots - 1\right]$$

3. Since, $v << c$ *i.e.*, $v/c << 1$, so, higher terms may be neglected.

Thus, $K = m_0c^2\left[1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right] \approx \frac{1}{2}m_0v^2$ .... (Classical K.E.)

4. Therefore, if $v << c$ then relativistic K.E. will convert into classical K.E.

Que 1.30. **Calculate the workdone to increase speed of an electron of rest energy 0.5 MeV from 0.6c to 0.8c.** **AKTU 2014-15, Marks 05**

Answer

Given : Initial velocity = 0.6c, Final velocity = 0.8c, m_0c^2 = 0.5 MeV.
To Find : Amount of workdone.

1. K.E. of electron, $K = m - m_0c^2 = \left[\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} - m_0\right]c^2$

$$K = m_0c^2\left[\left\{1-\left(\frac{v}{c}\right)^2\right\}^{-\frac{1}{2}} - 1\right]$$

2. Now initial kinetic energy,

$$K_1 = m_0c^2\left[\left\{1-\left(\frac{0.6c}{c}\right)^2\right\}^{-\frac{1}{2}} - 1\right] = 0.25\, m_0c^2$$

$\Rightarrow$ $K_1 = 0.25 \times 0.5 \times 10^6$ eV $= 1.25 \times 10^5$ eV

3. Final K.E.,

$$K_2 = m_0c^2\left[\left\{1-\left(\frac{0.8c}{c}\right)^2\right\}^{-\frac{1}{2}} - 1\right]$$

$K_2 = 0.666 \times 0.5 \times 10^6$ eV $= 3.33 \times 10^5$ eV

4. Amount of work $= K_2 - K_1$
$= 3.33 \times 10^5 - 1.25 \times 10^5 = 2.08 \times 10^5$ eV

$= 2.08 \times 10^5 \times 1.6 \times 10^{-19}$ J [$\because$ 1 eV = 1.6×10^{-19} J]
$= 3.328 \times 10^{-14}$ J

Que 1.31. **A charged particle shows an acceleration of 4.2×10^{12} cm/s² under an electric field at low speed. Compute the acceleration of the particle under the same field when the speed has reached a value 2.88×10^{10} cm/s. The speed of light is 3×10^{10} cm/s.**

Answer

Given : $a_0 = 4.2 \times 10^{12}$ cm/s², $v = 2.88 \times 10^{10}$ cm/s, $c = 3 \times 10^{10}$ cm/s.
To Find : Acceleration of particle.

1. At low speed $v << c$, the effective mass $m = m_o$
2. Acceleration at low speed,

$$a_0 = \frac{F}{m_o} = 4.2 \times 10^{12}\ \text{cm/s}^2$$

3. Now,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{m_o}{\sqrt{1-\left(\frac{2.88\times10^{10}}{3\times10^{10}}\right)^2}} = \frac{m_o}{0.28}$$

4. Now, acceleration at high speed,

$$a = \frac{F}{m} = \frac{F}{m_o/0.28} = \frac{0.28F}{m_o}$$

$$a = 0.28 \times 4.2 \times 10^{12} = 1.176 \times 10^{12}\ \text{cm/s}^2$$

Que 1.32. **If the kinetic energy of a body is twice its rest mass energy, find its velocity.** **AKTU 2015-16, Marks 05**

Answer

Given : K.E. = $2\,m_0c^2$
To Find : Velocity of body.

1. Kinetic energy, K.E. = 2 × rest mass energy

$$\frac{1}{2}mv^2 = 2 \times m_0c^2$$

2. We know that, $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$\therefore$ So, $$\frac{1}{2}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\cdot v^2 = 2\times m_0c^2$$

$$v^2 = 4c^2\sqrt{1-\frac{v^2}{c^2}}$$

$$v^4 = 16\,c^4\left(1-\frac{v^2}{c^2}\right)$$

$$v^4 = 16\,c^4 - 16\,c^2v^2$$

$$v^4 + 16\,c^2\,v^2 - 16\,c^4 = 0$$

$$v^2 = \frac{-16\,c^2 \pm \sqrt{256\,c^4 + 4\times 16\,c^4}}{2}$$

$$v^2 = \frac{-16\,c^2 \pm 17.89\,c^2}{2}$$

$$v = 0.972\,c \qquad \text{[Taking positive sign]}$$

Que 1.33. **Show that the rest mass of photon is zero.**

Answer

A photon travels with the velocity of light. Its momentum is given by,

$$P = \frac{m_0\,v}{\sqrt{1-\frac{v^2}{c^2}}}$$

But the momentum of a photon of radiation of wavelength λ is

$$P = \frac{h}{\lambda} \qquad (h \text{ is Planck's constant})$$

$\therefore$ $$\frac{h}{\lambda} = \frac{m_0\,v}{\sqrt{1-\frac{v^2}{c^2}}}$$

or $$m_0 = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{v\lambda}$$

But for photon $v = c$,

So, $m_0 = 0$

Therefore, the rest mass of a photon is zero.

Que 1.34. **Calculate the length of one meter rod moving parallel to its length when its mass is 1.5 times of its rest mass.**

AKTU 2013-14, Marks 05

Answer

Given : $m = 1.5m_0$, $L_0 = 1$ m.
To Find : Length of rod.

1. As we know $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$\therefore$ $$1.5\,m_0 = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{2}{3}$$

2. According to length contraction formula

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$L = 1\times\frac{2}{3} = \frac{2}{3} = 0.667 \text{ m}$$

☺☺☺

2
UNIT

Electromagnetic Field Theory

PART-1

Continuity Equation for Current Density, Displacement Current, Modifying Equation for the Curl of Magnetic Field to Satisfy Continuity Equation, Maxwell's Equation in Vacuum and in Non Conducting Medium.

CONCEPT OUTLINE : PART-1

Electromagnetic : It is the study of the mutual interactions between electric charges. Charges may be stationary, they may move with constant velocity or they may be in accelerated motion.

Equation of Continuity : It states, "The net change in the total amount (of the conserved quantity) inside any region is equal to the resultant of the amount that flows in and out of the region through the boundary".

Maxwell's Electromagnetic Theory : Maxwell proposed the theory of light to explain the different phenomenon (e.g., reflection, refraction, total internal reflection, polarization, etc.). This theory is known as Maxwell's electromagnetic theory.

Maxwell's Equations in Free Space or in Non Conducting Medium :

In differential form	In integral form
1. div $\vec{D} = 0$	$\int \vec{D}\cdot d\vec{S} = 0$
2. div $\vec{B} = 0$	$\int \vec{B}\cdot d\vec{S} = 0$
3. curl $\vec{E} = -\frac{\partial B}{\partial t}$	$\int_C \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\left(\int_S \vec{B}\cdot d\vec{S}\right)$
4. curl $\vec{H} = \frac{\partial \vec{D}}{\partial t}$	$\int_C \vec{H}\cdot d\vec{l} = \int_S \frac{\partial \vec{D}}{\partial t}.d\vec{S}$

Displacement Current : It is defined as ϵ_0 times to the rate of change of electric flux through the surface.

$$I_d = \epsilon_0 \frac{\partial \phi}{\partial t}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.1. Derive the equation of continuity. Also write its physical significance.

Answer

A. **Equation of Continuity :**

1. The equation of continuity represents the conservation law of charge in electromagnetism.
2. If I is the corresponding current due to decrease of charge inside the given region, then we have

$$I = -\frac{dq}{dt} \quad \text{...(2.1.1)}$$

3. If ρ is the volume charge density of that region and $\vec{J}$ the corresponding surface current density, then eq. (2.1.1) can be expressed as

$$\int_S \vec{J}.d\vec{s} = -\frac{d}{dt}\int_V \rho dV \quad \text{...(2.1.2)}$$

4. Using Gauss' divergence theorem

$$\int_S \vec{A}.d\vec{s} = \int_V \text{div}\,\vec{A}\,dV$$

On left hand side of eq. (2.1.2), we get

$$\int_V \text{div}\,\vec{J}\,dV = \frac{-d}{dt}\int_V \rho dV$$

or
$$\int_V \text{div}\,\vec{J}\,dV = \int_V \left(\frac{\partial \rho}{\partial t}\right) dV$$

or
$$\int_V \left(\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t}\right) dV = 0 \quad \text{...(2.1.3)}$$

5. Since V is an arbitrary volume, eq. (2.1.3) holds only if its integrand will be zero, *i.e.*,

$$\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0$$

This expression is known as equation of continuity.

B. **Physical Significance :**

1. The equation of continuity is expressed as,

$$\text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0 \quad \text{...(2.1.4)}$$

2. According to the Gauss's theorem in electrostatic

$$\text{div}\,\vec{D} = \vec{\nabla}.\vec{D} = \rho$$

where $\vec{D}$ is the displacement vector.

3. Thus, eq. (2.1.2), becomes

$$\text{div}\,\vec{J} + \frac{\partial}{\partial t}(\text{div}\,\vec{D}) = 0$$

or $$\text{div}\,\vec{J} + \text{div}\left(\frac{\partial \vec{D}}{\partial t}\right) = 0 \quad (\because \text{ Divergence is independent of time})$$

or $$\text{div}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0 \quad \text{...(2.1.5)}$$

4. The solenoidal vector is very important *e.g.*, we consider the discharge of a capacitor through resistance R.
5. If we consider a closed surface S_1 on a part of the circuit Fig. 2.1.1, then the current leaving S_1 is equal to the current entering it. Same cannot be said to the closed surface S_2, until one admits a displacement current ($\partial \vec{D}/\partial t$) as equivalent to the conduction current $\vec{J}$.
6. But with this equivalence we have a closed circuit, which takes the form of conduction current in the wire and a displacement current in the capacitor dielectric between the plates.
7. Eq. (2.1.5) signifies the treatment of the capacitor as a circuit element.

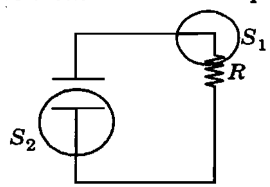


Fig. 2.1.1.

Que 2.2. Derive the modified equation for the curl of magnetic field to satisfy continuity equation.

Answer

1. Ampere's law is given as

$$\nabla \times \vec{H} = \vec{J} \quad \text{...(2.2.1)}$$

Taking the divergence of both sides of eq. (2.2.1), we get

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

2. But according to vector identity, divergence of a curl is zero, therefore,

$$\nabla \cdot (\nabla \times \vec{H}) = 0$$

i.e., $$\nabla \cdot \vec{J} = 0$$

3. This resultant is not consistent with the continuity equation $\left(\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t}\right)$

i.e., statement of Ampere's law is inconsistent and some modification is required in it.

4. Suppose we add an unknown term $\vec{G}$ to eq. (2.2.1), then

$$\nabla \times \vec{H} = \vec{J} + \vec{G} \quad \text{...(2.2.2)}$$

5. Taking divergence of both sides of eq. (2.2.2), we have

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot (\vec{J} + \vec{G}) = 0$$

i.e., $$\nabla \cdot \vec{J} + \nabla \cdot \vec{G} = 0$$

or $$\nabla \cdot \vec{G} = -\nabla \cdot \vec{J}$$

$$\nabla \cdot \vec{G} = -\left(-\frac{\partial \rho_v}{\partial t}\right) = \frac{\partial \rho_v}{\partial t} \quad \left[\because \nabla . \vec{J} = -\frac{\partial \rho_v}{\partial t}\right] \quad \text{...(2.2.3)}$$

6. We know the point form of Gauss's law as, $\nabla \cdot \vec{D} = \rho_v$

Taking differentiation of both sides $\nabla \cdot \frac{\partial \vec{D}}{\partial t} = \frac{\partial \rho_v}{\partial t}$

Putting this value in eq. (2.2.3), we get

$$\nabla \cdot \vec{G} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

or $$\vec{G} = \frac{\partial \vec{D}}{\partial t} \quad \text{...(2.2.4)}$$

7. Using eq. (2.2.2) and eq. (2.2.4) we get,

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Que 2.3. **Derive the Maxwell's equation in differential form.**

Answer

A. Derivation of Maxwell's First Relation :

1. According to Gauss' law of electrostatics :

"The net electric flux (ϕ_E) passing through a closed surface is equal to ($1/\varepsilon_0$) times the total charge q contained in the surface".

Thus, $$\phi_E = \frac{q}{\varepsilon_0} \quad \text{...(2.3.1)}$$

2. The electric flux can also be expressed as

$$\phi_E = \int_S \vec{E} . d\vec{s} \quad \text{...(2.3.2)}$$

where $\vec{E}$ is the strength of the electric field and $\vec{E} . d\vec{s}$ is the electric flux passing through a surface elements $d\vec{s}$ of a closed surface S.

3. From eq. (2.3.1) and (2.3.2), we have

$$\int_S \vec{E} . d\vec{s} = \frac{q}{\varepsilon_0} \quad \text{...(2.3.3)}$$

4. If V be the volume enclosed by the surface S and ρ be the volume charge density, then the total charge enclosed in the closed surface can be expressed as

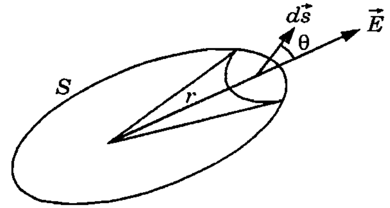


Fig. 2.3.1. A closed surface enclosing charge q.

$$q = \int_V \rho \, dV \quad \text{...(2.3.4)}$$

5. From eq. (2.3.3) and eq. (2.3.4), we get

$$\int_S \vec{E} . d\vec{s} = \frac{1}{\varepsilon_0} \int_V \rho \, dV$$

or $$\varepsilon_0 \int_S \vec{E} . d\vec{s} = \int_V \rho \, dV$$

or $$\int \varepsilon_0 \vec{E} . d\vec{s} = \int_V \rho \, dV$$

or $$\int_S \vec{D} . d\vec{s} = \int_V \rho \, dV \quad \text{...(2.3.5)}$$

where $\vec{D} = \varepsilon_0 \vec{E}$ is the electric displacement vector in the presence of free space.

6. Using Gauss' divergence theorem on the left hand side of eq. (2.3.5), we get,

$$\int_V \text{div}\, \vec{D} \, dV = \int_V \rho \, dV \quad \left(\because \int_S \vec{A} \cdot d\vec{S} = \int_V \text{div}\, \vec{A} \, dV\right)$$

or $\int_V (\text{div}\,\overline{D}\,dV - \rho dV) = 0$ ...(2.3.6)

7. Since V is an arbitrary volume, eq. (2.3.6) holds only if its integrand will be zero, *i.e.*,

$$(\text{div}\,\vec{D} - \rho) = 0$$

i.e., $$\text{div}\,\vec{D} = \rho$$

This is the Maxwell's first relation or equation.

8. In free space, volume charge density, $\rho = 0$

So, $$\text{div}\,\vec{D} = 0$$

B. Derivation of Maxwell's Second Relation :

1. We know that an isolated magnetic pole does not exist in the nature. Hence, the net magnetic flux Φ_M passing through a closed surface is zero.

Thus, $$\Phi_M = 0 \qquad ...(2.3.7)$$

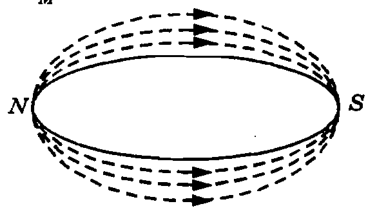


Fig. 2.3.2. Magnetic flux through a closed surface.

2. The magnetic flux can be expressed as

$$\Phi_M = \int_S \overline{B}\,.\,d\vec{s} \qquad ...(2.3.8)$$

where $\overline{B}$ is the strength of the magnetic field and $\overline{B}\,.\,d\vec{s}$ the net magnetic flux passing through a surface elements $d\vec{s}$ of a closed surface S.

3. From eq. (2.3.7) and eq. (2.3.8), we get

$$\int_S \overline{B}\,.\,d\vec{s} = 0 \qquad ...(2.3.9)$$

4. Using Gauss' divergence theorem on the left hand side of eq. (2.3.9) we get,

$$\int_V \text{div}\,\overline{B}\,dV = 0 \qquad \left(\because \int_S \vec{A}\,.\,\vec{ds} = \int_V \text{div}\,\vec{A}\,dV\right) \qquad ...(2.3.10)$$

5. Since V is an arbitrary volume, holds only if its integrand will be zero, *i.e.*,

$$\text{div}\,\vec{B} = 0$$

This is the Maxwell's second relation or equation.

6. This relation states that there are no magnetic monopoles in the world.

C. Derivation of Maxwell's Third Relation :

1. According to Faraday's law of electromagnetic induction :
"The induced emf (electromotive force) produced in a current carrying coil is equal to the negative time-rate of magnetic flux Φ_M associated with the coil".

Thus, $$e = -\frac{d}{dt}(\Phi_M) \qquad ...(2.3.11)$$

2. If $\vec{E}$ is the strength of the electric field corresponding to the induced emf e, then the induced emf can be expressed as line-integral of $\vec{E}$ around the coil Fig. 2.3.3, *i.e.*,

$$e = \int_C \overline{E}\cdot d\overline{l} \qquad ...(2.3.12)$$

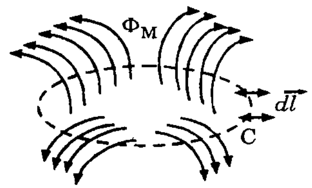


Fig. 2.3.3. Induced emf in a current carrying coil.

3. Comparing eq. (2.3.11) and eq. (2.3.12), we have

$$\int_C \overline{E}\cdot d\overline{l} = -\frac{d}{dt}(\Phi_M) \qquad ...(2.3.13)$$

4. The magnetic flux can be expressed as

$$\Phi_M = \int_S \overline{B}\cdot d\vec{s} \qquad ...(2.3.14)$$

5. Substituting the value of Φ_M in eq. (2.3.13), we get

$$\int_C \overline{E}\cdot d\overline{l} = -\frac{d}{dt}\left(\int_C \overline{B}\cdot d\vec{s}\right)$$

or $$\int_C \overline{E}\,.\,d\overline{l} = -\int_S \frac{\partial}{\partial t}(\overline{B}\,.\,d\vec{s}) \qquad ...(2.3.15)$$

6. Using the Stokes curl theorem on the left hand side of eq. (2.3.15), we get

$$\int_S \text{curl}\,\overline{E}\,.\,d\vec{s} = -\int_S \frac{\partial}{\partial t}(\overline{B}\,.\,d\vec{s}) \qquad \left(\because \int_C \vec{A}\,.\,\vec{dl} = \int_S \text{curl}\,\vec{A}\cdot\vec{ds}\right)$$

$$\int_S \left[\text{curl}\,\vec{E}\,.\,d\vec{s} + \frac{\partial}{\partial t}(\vec{B}\,.\,d\vec{s})\right] = 0$$

$$\int_S \left[\text{curl}\,\vec{E} + \frac{\partial \vec{B}}{\partial t}\right]\cdot(d\vec{s}) = 0 \quad ...(2.3.16)$$

7. Since, S is an arbitrary surface, eq. (2.3.16) holds only if its integrand is zero, *i.e.*,

$$\text{curl}\,\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

or
$$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

This is the Maxwell's third relation.

D. Derivation of Maxwell's Fourth Relation :

1. According to Ampere's circuit law :

"The line-integral of magnetic induction vector $\vec{B}$ around a closed current carrying loop is equal to μ_0 times the current flowing in the loop".

2. Thus,
$$\int_C \vec{B}\,.\,d\vec{l} = \mu_0 I \quad ...(2.3.17)$$

3. Since, $\vec{B} = \mu_0 \vec{H}$, we have
$$\int_C \mu_0 \vec{H}\,.\,d\vec{l} = \mu_0 I$$

or
$$\int_C \vec{H}\,.\,d\vec{l} = I \quad ...(2.3.18)$$

4. Let us consider a small surface elements $d\vec{s}$ of the surface S bounded by the closed loop C. If $\vec{J}$ be the surface current density of the loop, then the current flowing in the closed loop can be expressed as

$$I = \int_S \vec{J}\,.\,d\vec{s} \quad ...(2.3.19)$$

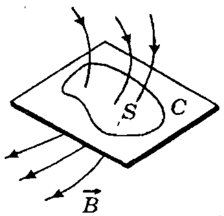


Fig. 2.3.4. A closed current-carrying loop.

5. Substituting the value of I from eq. (2.3.19) in eq. (2.3.18), we have

$$\int_C \vec{H}\,.\,d\vec{l} = \int_S \vec{J}\,.\,d\vec{s} \quad ...(2.3.20)$$

6. Using Stokes curl theorem on left hand side of eq. (2.3.20), we get

$$\int_S \text{curl}\,\vec{H}\,.\,d\vec{s} = \int_S \vec{J}\,.\,d\vec{s} \qquad \left(\because \int_C \vec{A}\,.\,d\vec{l} = \int_S \text{curl}\,\vec{A}\cdot d\vec{s}\right)$$

or
$$\text{curl}\,\vec{H} = \vec{J} \quad ...(2.3.21)$$

7. Using eq. (2.3.21) in the equation of continuity, we get

$$\text{div}\,(\text{curl}\,\vec{H}) + \frac{\partial \rho}{\partial t} = 0 \qquad \left(\because \text{div}\,\vec{J} + \frac{\partial \rho}{\partial t} = 0\right)$$

i.e.,
$$\frac{\partial \rho}{\partial t} = 0 \qquad [\because \text{div}\,(\text{curl}\,\vec{H}) = 0] \quad ...(2.3.22)$$

8. Eq. (2.3.22) is applicable for the steady-state conditions in which the charge density is not changing with time. This shows that for time varying fields, Ampere's law should be modified. For this, Maxwell suggested that eq. (2.3.21) should be modified as follows :

$$\text{curl}\,\vec{H} = \vec{J} + \vec{J}_D \quad ...(2.3.23)$$

where $\vec{J}_D$ is known as the displacement current density.

9. Taking divergence on both sides of eq. (2.3.23), we get

$$\text{div}\,(\text{curl}\,\vec{H}) = \text{div}\,(\vec{J} + \vec{J}_D)$$

or
$$0 = \text{div}\,(\vec{J} + \vec{J}_D) \qquad [\because \text{div}\,(\text{curl}\,\vec{H}) = 0]$$

or
$$\text{div}\,\vec{J} = -\,\text{div}\,\vec{J}_D$$

or
$$\frac{\partial \rho}{\partial t} = \text{div}\,\vec{J}_D \qquad \left[\because \text{div}\,(\vec{J}) + \frac{\partial \rho}{\partial t} = 0\right]$$

or
$$\frac{\partial(\text{div}\,\vec{D})}{\partial t} = \text{div}\,\vec{J}_D$$

or
$$\text{div}\left(\frac{\partial \vec{D}}{\partial t}\right) = \text{div}\,\vec{J}_D$$

or
$$\frac{\partial \vec{D}}{\partial t} = \vec{J}_D \quad ...(2.3.24)$$

10. From eq. (2.3.23) and eq. (2.3.24), we get

$$\text{curl}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

This is the Maxwell's fourth relation or equation.

11. From this relation, it is clear that the displacement current density relates the electric field vector $\vec{E}$ as ($\vec{D} = \varepsilon \vec{E}$) to the magnetic field vector $\vec{H}$.

12. In free space, surface current density $\vec{J} = 0$

So, $$\text{curl}(\vec{H}) = \frac{\partial \vec{D}}{\partial t}$$

Que 2.4. **Derive the Maxwell's equation in integral form.**

Answer

A. Maxwell's First Relation in Integral Form :

1. The Maxwell's first relation is

$$\text{div } \vec{D} = \rho \quad \text{...(2.4.1)}$$

2. Integrating this over an arbitrary volume V bounded by a closed surface S, we have

$$\int_V (\text{div } \vec{D})\, dV = \int_V \rho\, dV \quad \text{...(2.4.2)}$$

3. Using Gauss' divergence theorem on the left hand side of eq. (2.4.2), we get

$$\int_S \overline{D} \cdot d\vec{s} = \int_V \rho\, dV \qquad \left(\because \int_S A \cdot d\vec{s} = \int_V \text{div } \vec{A}\, dV\right)$$

where S is the surface which bounds the volume V.

4. In free space, volume charge density, $\rho = 0$.

So, $$\int_S \vec{D} . d\vec{s} = 0$$

This is the integral form of the Maxwell's first relation.

B. Maxwell's Second Relation in Integral Form :

1. The Maxwell's second relation is

$$\text{div } \vec{B} = 0 \quad \text{...(2.4.3)}$$

2. Integrating this over an arbitrary volume V bounded by a closed surface S, we have

$$\int_V \text{div } \vec{B}\, dV = 0 \quad \text{...(2.4.4)}$$

3. Using Gauss' divergence theorem on the left hand side of eq. (2.4.4), we get

$$\int_S \vec{B}\, d\vec{s} = 0 \qquad \left(\because \int_S \vec{A} . d\vec{s} = \int_V \text{div } \vec{A}\, dV\right) \quad \text{...(2.4.5)}$$

Where S is the surface which bounds the volume V.

4. This is the integral form of Maxwell's second relation.

C. Maxwell's Third Relation in Integral Form :

1. The third Maxwell's relation is

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{...(2.4.6)}$$

2. Integrating this over an arbitrary surface S bounded by a closed loop C, we have

$$\int_S \text{curl } \overline{E} . d\vec{s} = -\int_S \frac{\partial}{\partial t} (\overline{B} . d\vec{s}) \quad \text{...(2.4.7)}$$

3. Using Stoke's curl theorem on left hand side of eq. (2.4.7), we get

$$\int_C \overline{E} . d\bar{l} = -\int_S \frac{\partial}{\partial t} (\overline{B} . d\vec{s}) \qquad \left(\because \int_C \vec{A} . d\vec{l} = \int_S \text{curl } \vec{A} \cdot d\vec{s}\right)$$

i.e., $$\int_C \overline{E} . d\bar{l} = \frac{-d}{dt}\left(\int_S \overline{B} . d\vec{s}\right)$$

Here C is the closed loop which bounds surface S.

4. This is the integral form of Maxwell's third relation.

D. Maxwell's Fourth Relation in Integral Form :

1. The Maxwell's fourth relation is

$$\text{curl}(\vec{H}) = J + \frac{\partial \vec{D}}{\partial t} \quad \text{...(2.4.8)}$$

2. Integrating this over an arbitrary surface S bounded by a closed loop C, we have

$$\int_S \text{curl } \overline{H} . d\vec{s} = \int_S \left(\overline{J} + \frac{\partial D}{\partial t}\right) \cdot d\vec{s} \quad \text{...(2.4.9)}$$

3. Using Stoke's curl theorem on left hand side of eq. (2.4.9), we get

$$\int_C \overline{H} . d\bar{l} = \int_S \left(\overline{J} + \frac{\partial \overline{D}}{\partial t}\right) \cdot d\vec{s} \qquad \left(\because \int_C \vec{A} . d\vec{l} = \int_S \text{curl } \vec{A} \cdot d\vec{s}\right)$$

or $$\int_C \overline{H} . d\bar{l} = \int_S (\overline{J} + \overline{J}_D) \cdot d\vec{s}$$

Here, C is the closed loop which bounds surface S.

4. This is the integral form of Maxwell's fourth relation.

5. In free space, surface current density, $\vec{J} = 0$

So, $$\int_C \vec{H}.d\vec{l} = \int_S \vec{J}_D\, d\vec{s}$$

or $$\int_C \vec{H}.d\vec{l} = \int_S \frac{\partial \vec{D}}{\partial t}.d\vec{s}$$

Que 2.5. **Given the physical significance of Maxwell's equations.**

Answer

A. Physical Significance of Maxwell's First Equation :

1. It signifies that the total flux of electric displacement through a closed surface enclosing a volume is equal to the net charge $q\left(=\int_V \rho\, dV\right)$ contained within that volume.

B. Physical Significance of Maxwell's Second Equation :

1. It signifies that the net outward flux of magnetic induction through a surface enclosing a volume is equal to zero.
2. This shows the non-existence of monopoles in nature.

C. Physical Significance of Maxwell's Third Equation :

1. It signifies that the emf $\left(=\int_C \vec{E}\,.\,d\vec{l}\right)$ induced around a closed path is equal to the negative rate of change of magnetic flux $\left(=-\int_S \vec{B}\,.\,d\vec{s}\right)$ linked with that closed path.

D. Physical Significance of Maxwell's Fourth Equation :

1. It signifies that the mmf $\left(=\int_C \vec{H}\,.\,d\vec{l}\right)$ around a closed path is equal to the sum of the conduction current and displacement current linked with that closed path.

Que 2.6. **Using Maxwell equation Curl $B = \mu_0\left[J + \frac{\partial D}{\partial t}\right]$, prove that div $D = \rho$.**

Answer

1. The Maxwell's equation is given by :

$$\text{Curl } \vec{B} = \mu_0 \left\{\vec{J} + \frac{\partial \vec{D}}{\partial t}\right\}$$

But we know, $\vec{B} = \mu_0 \vec{H}$

So, $$\text{Curl } H = \left\{\vec{J} + \frac{\partial \vec{D}}{\partial t}\right\} \quad ...(2.6.1)$$

2. Taking the divergence of eq. (2.6.1), we obtain

$$\text{div(Curl } H) = \text{div}\left\{\vec{J} + \frac{\partial \vec{D}}{\partial t}\right\} = 0$$

3. Since the div of any vector quantity is zero *i.e.*,
div (Curl H) = 0

Therefore, $$\text{div}\left\{\vec{J} + \frac{\partial \vec{D}}{\partial t}\right\} = 0$$

$$\text{div } \vec{J} + \text{div}\left\{\frac{\partial \vec{D}}{\partial t}\right\} = 0$$

$$\text{div } \vec{J} + \left\{\frac{\partial\, \text{div } \vec{D}}{\partial t}\right\} = 0 \quad ...(2.6.2)$$

4. From continuity equation, we have div $\vec{J} + \frac{\partial \rho}{\partial t} = 0$

i.e., $$\text{div } \vec{J} = -\frac{\partial \rho}{\partial t} \quad ...(2.6.3)$$

5. Using eq. (2.6.3) in eq. (2.6.2), we obtain

$$-\frac{\partial \rho}{\partial t} + \frac{\partial\, \text{div } \vec{D}}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial\, \text{div } \vec{D}}{\partial t}$$

$$\text{div } \vec{D} = \rho$$

Que 2.7. **Explain the concept of displacement current and show how it led the modification of Ampere's law.**

Answer

1. Let us consider an electric circuit consisting of a battery B, resistance R, key K and a capacitor C in series as shown in Fig. 2.7.1.

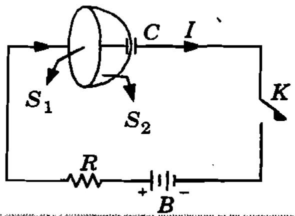


Fig. 2.7.1. Electric circuit showing the concept of displacement current.

2. When we close the circuit by pressing the key K, the charging of the capacitor starts.
3. Consider a circular surface S_1 and a semispherical surface S_2 such that both are bound by the same closed path. During charging, there is no actual flow of charge between the plates of the capacitor.
4. Hence, the current flows through surface S_1 but not through S_2.
5. Applying Ampere's law for the surface S_1, we have

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \quad ...(2.7.1)$$

Applying Ampere's law for the surface S_2, we have

$$\oint \vec{B}\cdot d\vec{l} = 0 \quad ...(2.7.2)$$

6. Since the results of eq. (2.7.1) and eq. (2.7.2) contradict each other, these equations cannot be corrected.
7. Maxwell tried to improve the contradiction between the two equations by adding an additional term $\mu_0 I_D$ on the right-hand side of eq. (2.7.1).
8. Thus, the modified Amperes law can be expressed as

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (I + I_D) \quad ...(2.7.3)$$

where I_D is known as displacement current.

9. The displacement current is given by

$$I_D = A\, J_D \quad ...(2.7.4)$$

where A is the area of the plates of capacitor and J_D is the displacement current density.

10. According to Maxwell's fourth relation, the displacement current density is given by

$$J_D = \frac{\partial D}{\partial t} \quad ...(2.7.5)$$

11. From eq. (2.7.4) and eq. (2.7.5), we have

$$I_D = A\frac{\partial D}{\partial t}$$

or
$$I_D = A\frac{\partial(\varepsilon_0 E)}{\partial t} \quad [\because D = \varepsilon_0 E]$$

or
$$I_D = A\varepsilon_0 \frac{\partial (E)}{\partial t} \quad ...(2.7.6)$$

12. If σ is the surface charge density of the plates of capacitor and q is the charge on each plate, then we know that

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0} \quad [\because \sigma = (q/A)]$$

13. Using this relation in eq. (2.7.6), we get

$$I_D = A\varepsilon_0 \frac{\partial}{\partial t}\left(\frac{q}{A\varepsilon_0}\right)$$

i.e.,
$$I_D = \frac{\partial q}{\partial t} = I$$

14. This shows that the displacement current in the space between the plates of capacitor during charging is equal to the conduction current.
15. Thus, the concept of displacement current needs a modification to Ampere's law.

Que 2.8. **Explain the characteristics of displacement current.**

Answer

1. The displacement current is the current only in the sense that it produces a magnetic field. It has none of the other properties of current since it is not linked with the motion of charges.
2. The magnitude of displacement current is equal to the rate of change of magnitude of electric displacement vector, *i.e.*, $J_D = (\partial D/\partial t)$.
3. It serves the purpose to make the total current continuous across the discontinuity in conduction current. As an example, a battery charging a capacitor produces a closed current loop in terms of total current $I_{total} = I + I_D$.
4. The displacement current in a good conductor is negligible compared to the conduction current at any frequency less than the optical frequencies ($\approx 10^{15}$ Hz).

PART-2

Energy in an Electromagnetic Field, Poynting Vector and Poynting Theorem, Plane Electromagnetic Waves in Vacuum and their Transverse Nature, Relation Between Electric and Magnetic Fields of an Electromagnetic Waves.

CONCEPT OUTLINE : PART-2

Poynting Vector : The magnitude and the direction of flow of energy per unit area per unit time in an EM-wave travelling in free space (or vacuum) can be expressed by a vector known as Poynting vector $\vec{S}$.

Electromagnetic Waves : Electromagnetic waves are coupled electric and magnetic oscillations that move with speed of light and exhibit typical wave behaviour.

Energy Density of Electromagnetic Wave : The total electromagnetic energy density

$$u = \varepsilon_0 E^2$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.9. **What is Poynting vector ?**

Answer

A. Poynting Vector :

1. The magnitude and the direction of flow of energy per unit area per unit time in an EM- wave travelling in free space (or vacuum) can be expressed by a vector known as Poynting vector $\vec{S}$.

B. Expression for Poynting Vector :

1. Let us consider a small volume element $dV = A\,dx$, where A is its cross sectional area and dx its length along X-axis.
2. If u is the EM-energy density of the EM-waves, then

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \quad \text{...(2.9.1)}$$

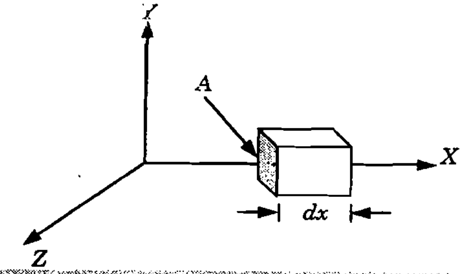


Fig. 2.9.1. Illustration of concept of poynting vector.

3. Hence, the energy associated with volume element dV is given by

$$U = u\,dV = \left(\frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2\right) A\,dx \quad \text{...(2.9.2)}$$

4. The relation between the magnitudes of field vectors $\vec{E}$ and $\vec{H}$ is

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \mu_0 C = \frac{1}{\varepsilon_0 C} \quad \text{...(2.9.3)}$$

5. Using eq. (2.9.3) and eq. (2.9.2), we have

$$U = \left(\frac{1}{2}\varepsilon_0 E \frac{H}{\varepsilon_0 C} + \frac{1}{2}\mu_0 H \frac{E}{\mu_0 C}\right) A\,dx$$

or $$U = \left(\frac{1}{2C} + \frac{1}{2C}\right) EHA\,dx$$

or $$U = \frac{EHA\,dx}{(dx/dt)} \qquad \left[\because C = \frac{dx}{dt}\right]$$

or $$U = EHAdt \quad \text{...(2.9.4)}$$

6. Hence, by definition, the EM-energy passing through per unit area per unit time with EM-wave, *i.e.*, magnitude of Poynting vector is

$$S = \frac{U}{Adt} = \frac{EHA\,dt}{A\,dt} = EH$$

7. In vector form, this expression can be expressed as

$$\vec{S} = \vec{E} \times \vec{H}$$

Que 2.10. **Derive Poynting theorem and explain its physical significance.**

Answer

A. **Poynting Theorem :**

1. According to this theorem, "The time rate of EM-energy within a certain volume plus the time rate of EM-energy flowing out through the boundary surface is equal to the power transferred into the EM-field".

B. **Derivation of Poynting Theorem :**

1. Suppose we have some charges and current, which at time t produces fields $\vec{E}$ and $\vec{B}$.

2. Then, the work done on an element charge dq contained in a volume element dV due to its displacement $d\vec{l}$ in time dt under the influence of Lorentz force $\vec{F}$ is

$$\vec{F}\cdot d\vec{l} = dq\,(\vec{E} + \vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\vec{F}\cdot d\vec{l} = dq\,(\vec{E} + \vec{v}\times\vec{B})\cdot\vec{v}dt \qquad \left[\because \quad \vec{v} = \frac{dl}{dt}\right]$$

$$= dq\,(\vec{E}\cdot\vec{v})\,dt + dq\,\{(\vec{v}\times\vec{B})\cdot\vec{v}\}\,dt$$

$$= (\vec{E}\cdot\vec{v})\,dq\,dt$$

$$[\text{since } (\vec{v}\times\vec{B})\cdot\vec{v} = [\vec{v}\,\vec{B}\,\vec{v}] = 0] \quad ...(2.10.1)$$

3. If ρ and $\vec{J}$ be the volume charge density and current density respectively, then we have

$$dq = \rho\,dV \text{ and } \rho\,\vec{v} = \vec{J} \quad ...(2.10.2)$$

4. Using eq. (2.10.2) in eq. (2.10.1), we have

$$\vec{F}\cdot d\vec{l} = \left(\vec{E}\cdot\frac{\vec{J}}{\rho}\right)\cdot\rho dV\,dt = (\vec{E}\cdot\vec{J})\,dV\,dt$$

5. Hence, the total work done per unit time on all the charges in some volume V is given by

$$\frac{dW}{dt} = \int_V (\vec{E}\cdot\vec{J})\,dV \quad ...(2.10.3)$$

6. Maxwell's fourth relation is

$$\text{curl}\,\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

or
$$\text{curl}\,\vec{H} = \vec{J} + \varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad [\because \vec{D} = \varepsilon_0\vec{E}] \quad ...(2.10.4)$$

7. Taking the scalar product of $\vec{E}$ on both sides of the above eq.(2.10.4), we have

$$\vec{E}\cdot\text{curl}\,\vec{H} = \vec{E}\cdot\left[J + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right]$$

or
$$\vec{E}\cdot\text{curl}\,\vec{H} = \vec{E}\cdot\vec{J} + \varepsilon_0\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

or
$$\vec{E}\cdot\vec{J} = \vec{E}\cdot\text{curl}\,\vec{H} - \varepsilon_0\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

or
$$\vec{E}\cdot\vec{J} = \vec{H}\cdot\text{curl}\,\vec{E} - \text{div}\,(\vec{E}\times\vec{H}) - \varepsilon_0\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} \quad ...(2.10.5)$$

$$[\because \text{div}\,(\vec{A}\times\vec{B}) = \vec{B}\cdot\text{curl}\,\vec{A} - \vec{A}\cdot\text{curl}\,\vec{B}]$$

8. Maxwell's third relation is

$$\text{curl}\,\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

or
$$\text{curl}\,\vec{E} = -\mu_0\frac{\partial\vec{H}}{\partial t} \quad ...(2.10.6)$$

9. Using eq. (2.10.6) in eq. (2.10.5), we have

$$\vec{E}\cdot\vec{J} = \vec{H}\left(-\mu_0\frac{\partial\vec{H}}{\partial t}\right) - \text{div}\,(\vec{E}\times\vec{H}) - \varepsilon_0\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

or
$$\vec{E}\cdot\vec{J} = -\mu_0\vec{H}\frac{\partial\vec{H}}{\partial t} - \varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} - \text{div}\,(\vec{E}\times\vec{H})$$

or
$$\vec{E}\cdot\vec{J} = -\frac{\partial}{\partial t}\left(\frac{\mu_0}{2}H^2 + \frac{\varepsilon_0}{2}E^2\right) - \text{div}\,(\vec{E}\times\vec{H})$$

or
$$\vec{E}\cdot\vec{J} = -\frac{\partial}{\partial t}\left(\frac{\mu_0}{2}H^2 + \frac{\varepsilon_0}{2}E^2\right) - \text{div}\,(\vec{S}) \quad ...(2.10.7)$$

where $\vec{S} = (\vec{E}\times\vec{H})$ is the Poynting vector.

10 Using eq. (2.10.7) in eq. (2.10.3), we have

$$\frac{dW}{dt} = \int_V\left[-\frac{\partial}{\partial t}\left(\frac{\mu_0}{2}H^2 + \frac{\varepsilon_0}{2}E^2\right) - \text{div}\,(\vec{S})\right]dV$$

or
$$\frac{dW}{dt} = \frac{-d}{dt}\int_V\left(\frac{\mu_0}{2}H^2 + \frac{\varepsilon_0}{2}E^2\right)dV - \int_V \text{div}\,(\vec{S})\,dV \quad ...(2.10.8)$$

11. Since, dW/dt represents the power density that is transferred into EM-field and increases the mechanical energy (kinetic, potential or whatever) of the charges, therefore, if u_M denotes the mechanical energy, then we have

$$\frac{dW}{dt} = \frac{d}{dt}\int_V u_M\,dV \quad ...(2.10.9)$$

12. Also, the EM-energy density is given by

$$u_{em} = \frac{\mu_0}{2} H^2 + \frac{\varepsilon_0}{2} E^2 \quad ...(2.10.10)$$

13. Using the eq. (2.10.9) and eq. (2.10.10) in eq. (2.10.8), we obtain

$$\frac{d}{dt}\int_V u_M \, dV = \frac{-d}{dt}\int_V u_{em}\, dV - \int_V \text{div}\,(\vec{S})\, dV$$

or $$\int_V \text{div}\,(\vec{S})\, dV = -\frac{d}{dt}\int_V (u_M + u_{em})\, dV$$

or $$\int_V \text{div}\,(\vec{S})\, dV = \int_V \frac{\partial}{\partial t}(u_M + u_{em})\, dV$$

or $$\text{div}\,\vec{S} = -\frac{\partial}{\partial t}(u_M + u_{em})$$

C. **Physical Significance :**

The Poynting vector $\vec{S}$ describes the flow of energy in the same way as the current density vector $\vec{J}$ describes the flow of charge. Since the equation of continuity expresses the conservation of charge, the Poynting theorem represents the conservation of energy.

Que 2.11. A 500 watt lamp radiates power uniformly in all directions. Calculate the electric and magnetic field intensities at 1 m distance from the lamp.

Answer

Given : Energy of the lamp = 500 watt
To Find : i. Electric field intensity at 1 m distance from lamp.
ii. Magnetic field intensity at 1 m distance from lamp.

1. Area illuminated = $4\pi r^2 = 4\pi\,(1)^2 = 4\pi$ m^2
2. Therefore, Energy radiated per unit area per second = $\frac{500}{4\pi}$
3. Hence, from Poynting theorem

$$|\vec{S}| = |\vec{E} \times \vec{H}| = EH = \frac{500}{4\pi} \quad ...(2.11.1)$$

and $$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega \quad ...(2.11.2)$$

4. Multiplying eq. (2.11.1) and eq. (2.11.2), we get

$$E^2 = \frac{500}{4\pi} \times 377 = 15000.36$$

or $$E = 122.475 \text{ V/m}$$

and $$H = \frac{E}{377} = 0.325 \text{ A/m}$$

Que 2.12. Calculate the magnitude of Poynting vector at the surface of the sun. Given that power radiated by the sun = 3.8×10^{26} Watts and radius of the sun = 7×10^8 m.

Answer

Given : Power radiated by the sun, Power = 3.8×10^{26} Watts
Radius of the sun, $r = 7 \times 10^8$ m
To Find : Magnitude of Poynting vector at the surface of the sun.

1. The Poynting vector is given by :

$$S = \frac{\text{Power}}{4\pi r^2} = \frac{3.8 \times 10^{26}}{4\pi(7 \times 10^8)^2} = \frac{3.8 \times 10^{26}}{1.96\pi \times 10^{18}}$$
$$= 6.171 \times 10^7 \text{ W/m}^2$$

Que 2.13. Define EM waves. State a few properties of electromagnetic waves.

Answer

1. EM waves are coupled electric and magnetic oscillations that move with speed of light and exhibit typical wave behaviour.
2. The properties of electromagnetic waves are as follows :
   i. In free space or vacuum, the EM wave travel with speed of light.
   ii. The electrostatic energy density is equal to the magnetic energy density.
   iii. These waves carry both energy and momentum, which can be delivered to a surface.
   iv. EM waves are transverse in nature.
   v. Electromagnetic waves of different frequencies can exist.

Que 2.14. Derive electromagnetic wave equation in free space.

OR

Using Maxwell's equations, derive electromagnetic wave equations in vacuum and prove that wave propagate with speed of light.

Answer

1. Maxwell's relations are given by,

$$\left.\begin{aligned} \text{div } \overline{D} &= \rho \\ \text{div } \overline{B} &= 0 \\ \text{curl } \overline{E} &= -\frac{\partial \overline{B}}{\partial t} \\ \text{curl } \overline{H} &= \overline{J} + \frac{\partial \overline{D}}{\partial t} \end{aligned}\right\} \quad ...(2.14.1)$$

2. For the propagation of EM-waves in free space (or vacuum), we have

$$\sigma = 0,\ \rho = 0,\ \varepsilon_r = 1,\ \mu_r = 1$$

i.e., $\overline{J} = \sigma\overline{E} = 0$, $\varepsilon = \varepsilon_0\,\varepsilon_r = \varepsilon_0$ and $\mu = \mu_0\,\mu_r = \mu_0$

3. Thus, the Maxwell's relations for the propagation of EM-wave in free space (or vacuum) are given by

$$\left.\begin{aligned} \text{div } \overline{D} &= 0 \\ \text{div } \overline{B} &= 0 \\ \text{curl } \overline{E} &= -\frac{\partial \overline{B}}{\partial t} \\ \text{curl } \overline{H} &= \frac{\partial \overline{D}}{\partial t} \end{aligned}\right\} \quad ...(2.14.2)$$

4. For free space, $\overline{D} = \varepsilon_0\overline{E}$ and $\overline{B} = \mu_0\overline{H}$, so we have

$$\left.\begin{aligned} \text{div } \overline{E} &= 0 \\ \text{div } \overline{H} &= 0 \\ \text{curl } \overline{E} &= -\mu_0\frac{\partial \overline{H}}{\partial t} \\ \text{curl } \overline{H} &= \varepsilon_0\frac{\partial \overline{E}}{\partial t} \end{aligned}\right\} \quad ...(2.14.3)$$

5. Taking curl on both sides of the third relation in eq. (2.14.3), we have

$$\text{curl (curl } \overline{E}) = \text{curl}\left(-\mu_0\frac{\partial \overline{H}}{\partial t}\right)$$

$$\text{grad (div } \overline{E}) - \nabla^2\overline{E} = -\mu_0 \text{ curl}\left(\frac{\partial \overline{H}}{\partial t}\right)$$

$$[\because \ \text{grad (div } \overline{A}) - \nabla^2\overline{A} = \text{curl (curl } \overline{A})]$$

$$-\nabla^2\vec{E} = -\mu_0\left(\frac{\partial}{\partial t}\right)(\text{curl } \overline{H}) \qquad [\because \ \text{div } \vec{E} = 0]$$

$$-\nabla^2\vec{E} = -\mu_0\frac{\partial}{\partial t}\left(\varepsilon_0\frac{\partial \overline{E}}{\partial t}\right) \qquad \left[\because \ \text{curl } \vec{H} = \varepsilon_0\frac{\partial \vec{E}}{\partial t}\right]$$

$$-\nabla^2\vec{E} = \varepsilon_0\,\mu_0\left(\frac{\partial^2 E}{\partial t^2}\right) \quad ...(2.14.4)$$

6. Now taking curl on both sides of fourth relation in eq. (2.14.3), we can show that

$$\nabla^2\vec{H} = \varepsilon_0\mu_0\left(\frac{\partial^2 \overline{H}}{\partial t^2}\right) \quad ...(2.14.5)$$

7. Comparing eq. (2.14.4) and eq. (2.14.5) with the general wave eq.,

$$\nabla^2\vec{\psi} = \frac{1}{v^2}\left(\frac{\partial^2\psi}{\partial t^2}\right),\ \text{v is the wave velocity}$$

we get
$$\varepsilon_0\,\mu_0 = \frac{1}{v^2}$$

i.e.,
$$v = \frac{1}{\sqrt{\varepsilon_0\,\mu_0}} = \frac{1}{\sqrt{(4\pi\,\varepsilon_0)\left(\frac{\mu_0}{4\pi}\right)}}$$

$$v = \frac{1}{\sqrt{\left(\frac{1}{9\times10^9}\right)(10^{-7})}} = \frac{1}{\sqrt{\left(\frac{1}{9\times10^{16}}\right)}}$$

$(\because \ 1/4\,\pi\varepsilon_0 = 9\times10^9$ N-m^2/C^2 and $\mu_0/4\pi = 10^{-7}$ Wb/A-m$)$

$$v = \sqrt{9\times10^{16}} = 3\times10^8 \text{ m/s} = c \text{ (speed of light)}$$

This shows that EM-wave travel with the speed of light in free space (or vacuum).

Que 2.15. Show that *E*, *H* and direction of propagation form a set of orthogonal vectors.

Answer

1. Considering *EM*-waves as plane waves, the electric and magnetic vectors $\vec{E}$ and $\vec{H}$ can be expressed as

$$\vec{E} = \vec{E}_0 \exp[i(\vec{K}\cdot\vec{r} - \omega t)]$$

$$\vec{H} = \vec{H}_0 \exp[i(\vec{K}\cdot\vec{r} - \omega t)]$$

2. These two equations represent the plane-wave solutions of Maxwell's relations for the propagation of *EM*-waves in free space (or vacuum). In this condition, the del operator (∇) and partial time derivative operator $\left(\frac{\partial}{\partial t}\right)$ can be expressed as

$$\left.\begin{aligned} \nabla &\equiv iK \\ \text{and} \quad \left(\frac{\partial}{\partial t}\right) &= -i\omega \end{aligned}\right\} \quad ...(2.15.1)$$

where K is the propagation constant and ω the angular frequency of EM-wave.

3. From eq. (2.14.3) and eq. (2.15.1), we get

$$\vec{K} \cdot \vec{E} = 0$$

$$\vec{K} \cdot \vec{H} = 0$$

$$i\vec{K} \times \vec{E} = -\mu_0(-i\omega\vec{H})$$

$$\text{and} \quad i\vec{K} \times \vec{H} = \varepsilon_0(-i\omega\vec{E})$$

$$\text{or} \quad \left.\begin{aligned} \vec{K} \cdot \vec{E} &= 0 \\ \vec{K} \cdot \vec{H} &= 0 \\ \vec{K} \times \vec{E} &= \mu_0\omega\vec{H} \\ \vec{K} \times \vec{H} &= -\varepsilon_0\omega\vec{E} \end{aligned}\right\} \quad ...(2.15.2)$$

4. From the set of eq. (2.15.2), it is clear that $\vec{E}$, $\vec{K}$ and $\vec{H}$ vectors are perpendicular to each other. This indicates that EM-waves propagating in free space (or vacuum) are transverse in nature. This fact is shown in Fig. 2.15.1.

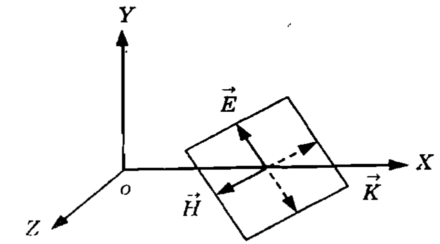


Fig. 2.15.1. Transverse nature of EM-waves in free space (or vacuum).

Que 2.16. **Discuss the energy density of electromagnetic wave.**

Answer

1. When the electromagnetic waves travel in free space, the electrostatic energy density u_e and magnetostatic energy density u_m are given by

$$u_e = \frac{1}{2}\varepsilon_0 E^2$$

$$u_m = \frac{1}{2}\mu_0 H^2$$

2. Total energy density of EM-wave is given by

$$u = u_e + u_m = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2$$

3. But for a plane electromagnetic wave in free space

$$\frac{E}{\mu} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \text{ or } H = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,E$$

4. So,

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 \cdot \frac{\varepsilon_0}{\mu_0}E^2$$

$$u = \varepsilon_0 E^2$$

This is the total electromagnetic energy density.

PART-3

Energy and Momentum Carried by Electromagnetic Waves, Resultant Pressure, Skin Depth.

CONCEPT OUTLINE : PART-3

Skin Depth : It is defined as the depth of a conductor from the surface at which the amplitude of the wave is $1/e$ of the amplitude of surface.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 2.17. **Derive the relation between energy and momentum carried by electromagnetic waves.**

Answer

1. The momentum of a particle of mass m moving with velocity v is given by

$$p = mv \quad ...(2.17.1)$$

2. According to Einstein's mass-energy relation

Energy, $U = mc^2$ or $m = \frac{U}{c^2}$

$$\therefore \quad \vec{p} = \frac{U}{c^2}\vec{v} \qquad ...(2.17.2)$$

3. The energy density in plane electromagnetic wave in free space is given by

$$u = \varepsilon_0 E^2 \qquad ...(2.17.3)$$

where E is the magnitude of electric field.

4. Thus, the momentum density or momentum per unit volume associated with an electromagnetic wave is

$$\vec{p} = \frac{u}{c^2}\vec{v} \qquad ...(2.17.4)$$

5. If the electromagnetic waves are propagating along X-axis, then

$$\vec{v} = c\hat{i}$$

$$\therefore \quad \vec{p} = \frac{u}{c}\hat{i} \qquad ...(2.17.5)$$

6. The Poynting vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

$$\therefore \quad \vec{S} = \frac{E^2}{\mu_0 c}\hat{i} \qquad \left[\because \vec{E} \times \vec{B} = \frac{E^2}{c}\hat{i}\right] \qquad ...(2.17.6)$$

7. Substituting the value of E^2 from eq. (2.17.3) in eq. (2.17.6), we get

$$\vec{S} = \frac{u}{\varepsilon_0 c \mu_0}\hat{i} = uc\,\hat{i} \qquad \left(\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right)$$

or $$u\,\hat{i} = \frac{\vec{S}}{c} \qquad ...(2.17.7)$$

8. Putting the value of $u\,\hat{i}$ from eq. (2.17.7) in eq. (2.17.5), we get

$$\vec{p} = \frac{\vec{S}}{c^2} = \frac{1}{\mu_0 c^2}(\vec{E} \times \vec{B})$$

or $$\vec{p} = \varepsilon_0(\vec{E} \times \vec{B}) \qquad \left(\because \frac{1}{\mu_0 c^2} = \varepsilon_0\right) \qquad ...(2.17.8)$$

9. Eq. (2.17.8) represents momentum per unit volume for an electromagnetic wave. The value of this momentum is,

$$p = \frac{u}{c} \quad \text{or} \quad u = pc$$

i.e., Energy density = wave momentum × wave velocity.

Que 2.18. Define radiation pressure. Derive the relation between radiation pressure and energy density.

Answer

A. Radiation Pressure :

1. When electromagnetic wave strikes a surface, its momentum changes. The rate of change of momentum is equal to the force. This force acting on the unit area of the surface exerts a pressure, called radiation pressure.

B. Relation between Radiation Pressure and Energy Density :

1. Let a plane electromagnetic wave incident normally on a perfectly absorbing surface of area A for a time t.
2. If energy U is absorbed during this time, the momentum p delivered to the surface is given, according to Maxwell's prediction, by

$$p = \frac{U}{c}$$

3. If S is the energy passing per unit area per unit time, then

$$U = SA\,t$$

$$\therefore \quad p = \frac{S\,At}{c}$$

where S is the magnitude of Poynting vector.

4. But $\frac{S}{c} = u$ (energy density)

$$\therefore \quad p = uAt$$

5. From Newton's law average force F acting on the surface is equal to the average rate at which momentum is delivered to the surface. Therefore,

$$F = \frac{p}{t} = uA$$

6. The radiation pressure p_{rad} exerted on the surface.

$$p_{rad} = \frac{F}{A} = u$$

Hence, the radiation pressure exerted by a normally incident plane electromagnetic wave on a perfect absorber is equal to the energy density in the wave.

7. For a perfect reflector or for a perfectly reflecting surface, the radiation after reflection has a momentum equal in magnitude but opposite in direction to the incident radiation. The momentum imparted to the surface will therefore be twice as on perfect absorber. That is,

$$p_{rad} = 2u$$

Que 2.19. Discuss depth of penetration or skin depth.

Answer

1. The depth of penetration is defined as the depth in which the strength of electric field associated with the electromagnetic wave reduces to $1/e$ times of its initial value.
2. Depth of penetration or skin depth is a very important parameter in describing conductor behaviour in electromagnetic field and in radio communication.

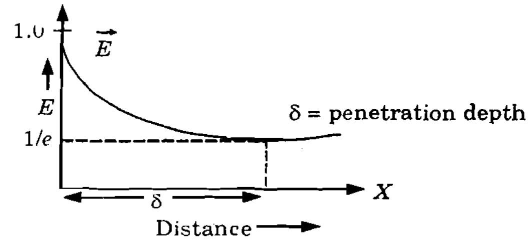


Fig. 2.19.1.

3. The reciprocal of attenuation constant is called skin depth or depth of penetration *i.e.*,

$$\delta = \frac{1}{\text{attenuation constant}}$$

4. For good conductors, penetration depth decreases with increase in frequency and is given as

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

5. For poor conductors, skin depth is independent of frequency.

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$$

Que 2.20. **Show that for a poor conductor, the skin depth can be expressed as**

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Answer

1. We know that for a conducting medium, the propagation constant can be expressed as

$$K = \alpha + i\beta$$

Here,
$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2}$$

and
$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} + 1\right]^{1/2}$$

and skin depth is given by

$$\delta = \frac{1}{\alpha}$$

2. For a poor conductor $\sigma << \varepsilon\omega$, so we can approximate the first term in square root bracket of right hand side of expression of α using the binomial theorem as

$$\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} = \left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)^{\frac{1}{2}} = 1+\frac{\sigma^2}{2\varepsilon^2\omega^2} - \frac{\sigma^4}{8\,\varepsilon^4\omega^4} + \ldots = 1+\frac{\sigma^2}{2\varepsilon^2\omega^2}$$

i.e.,
$$\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1 = \frac{\sigma^2}{2\,\varepsilon^2\,\omega^2}$$

3. The expression of α therefore reduces to

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\frac{\sigma^2}{2\,\varepsilon^2\omega^2}\right]^{1/2}$$

or
$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\,\frac{\sigma}{\sqrt{2}\,\varepsilon\omega} \quad \text{or} \quad \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

4. Hence, the skin depth for a poor conductor can be expressed as

$$\delta = \frac{1}{\alpha} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Que 2.21. **For silver, $\mu = \mu_o$ and $\sigma = 3 \times 10^7$ mhos/m. Calculate the skin depth at 10^8 Hz frequency. Given, $\mu_o = 4\pi \times 10^7$ N/A^2.**

Answer

Given : $\mu = \mu_0 = 4\pi \times 10^{-7}$ N/m^2, $\sigma = 3 \times 10^7$ mhos/m, $f = 10^8$ Hz
To Find : Skin depth.

1. Since silver is a good conductor, therefore, the skin depth is given by,

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{(2\pi f)\,\mu_0\sigma}}$$

$$\delta = \sqrt{\frac{2}{(2\pi \times 10^8) \times 4\pi \times 10^{-7} \times 3 \times 10^7}}$$

$$\delta = 9.19 \times 10^{-6}\ \text{m}$$

☺☺☺

3 UNIT

Quantum Mechanics

PART-1

Black Body Radiation, Stefan's Law, Wien's Law, Rayleigh-Jeans Law and Plank's Law, Wave Particle Duality, Matter Waves.

CONCEPT OUTLINE : PART-1

Wave Particle Duality :
According to Einstein, the energy of light is concentrated in small bundles called photon. Hence, light behaves as a wave on one hand and as a particle on the other hand. This nature of light is known as dual nature, while this property of light is known as wave particle duality.

de-Broglie Wave or Matter Waves :
de-Broglie wavelength is given by

$$\lambda = \frac{h}{p}$$

de-Broglie wavelength in terms of temperature is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.1. **Explain black body radiation and discuss energy distribution in the spectrum of a black body radiation.**

Answer

A. Black Body :

1. A body which absorbs completely all the radiations incident upon it, reflecting none and transmitting none, is called a black body.
2. Absorptivity of a black body is unity for all wavelengths.
3. It appears black whatever the wavelength of incident radiation is.
4. When a black body is heated to a suitable high temperature it emits total radiation which is known as black body radiations.
5. From the energy point of view, black body radiation is equivalent to the radiation of an infinitely large number of non-interacting harmonic oscillations, the so called radiation oscillations.

6. No actual body is a perfect black body, it is only an ideal conception.
7. Lamp black is the nearest approach to black body which absorbs nearly 99 % of the incident radiation.
8. Platinum black is another example of a black body.

B. Energy Distribution :

1. Results of the studies of black body radiation spectra are shown in Fig. 3.1.1 in which variation of intensity with wavelength for various temperatures are shown.

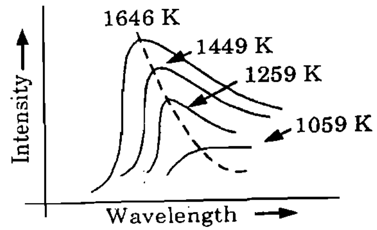


Fig. 3.1.1.

2. The energy distribution in the radiation spectrum of black body is not uniform. As the temperature of the body rises, the intensity of radiation for each wavelength increases.
3. At a given temperature, the intensity of radiation increases with increase in wavelength and becomes maximum at a particular wavelength. With further increase in wavelength, the intensity of radiation decreases.
4. The point of maximum energy shifts towards the shorter wavelengths as the temperature increases.
5. For a given temperature the total energy of radiation is represented by the area between the curve and the horizontal axis and the area increases with increase of temperature, being directly proportional to the fourth power of absolute temperatures.

Que 3.2. Discuss Stefan's law.

Answer

1. The total amount of heat (E) radiated by a perfectly black body per second per unit area is directly proportional to the fourth power of its absolute temperature (T), *i.e.*,

$$E \propto T^4 \quad \text{or} \quad E = \sigma T^4$$

where, σ = Universal constant and is called Stefan's constant.

2. This law is also called as Stefan's fourth power law.
3. If a black body at absolute temperature T is surrounded by another black body at absolute temperature T_0 then the net amount of heat (E) lost by the former per second per cm^2 is given by

$$E = \sigma(T^4 - T_0^4)$$

4. This law is also known as Stefan-Boltzmann law.

Que 3.3. Explain Wien's laws of energy distribution.

Answer

A. Fifth Power Law :

1. The total amount of energy emitted by a black body per unit volum an absolute temperature T and contained in the spectral region me within the wavelength λ and $\lambda + d\lambda$ is given as,

$$u_\lambda \, d\lambda = \frac{A}{\lambda^5} f(\lambda T) \, d\lambda \quad ...(3.3.$$

where, A = Constant, and

$f(\lambda T)$ = A function of the product λT and is given a

$f(\lambda T) = e^{-ch/\lambda kT} = e^{-a/\lambda T}$

$\therefore \quad u_\lambda \, d\lambda = A\lambda^{-5} e^{-a/\lambda T} d\lambda$

2. For $\lambda = \infty$, $u_\lambda = 0$ and for $\lambda = 0$, $u_\lambda = 0$.
3. Thus eq. (3.3.2) shows that no energy is emitted by a wave of infini wavelength as well as by a wave of zero wavelength.
4. For $T = \infty$, eq. (3.3.2) reduces to : $u_\lambda \, d\lambda = A\lambda^{-5}$, which is finite quantity and is in open contradiction with the Stefan's fourth power law (σT^4).
5. Wien's law of energy distribution, however, explains the energy distribution at short wavelengths at higher temperature and fails for long wavelengths.

B. Displacement Law :

1. As the temperature of the body is raised the maximum energy tends to be associated with shorter wavelength, *i.e.*,

$$\lambda_m T = \text{constant} \quad ...(3.3.3)$$

where, λ_m = wavelength at which the energy is maximum , and

T = absolute temperature.

2. Thus, if radiation of a particular wavelength at a certain temperature is adiabatically altered to another wavelengths then temperature changes in the inverse ratio.

Que 3.4. Discuss Rayleigh-Jean's Law.

Answer

1. The total amount of energy emitted by a black body per unit volume at an absolute temperature T in the wavelength range λ and $\lambda + d\lambda$ is given as,

$$u_\lambda \, d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

where, k = Boltzmann's constant = 1.381×10^{-23} J /K.

2. The energy radiated in a given wavelength range λ and $\lambda + d\lambda$ increases rapidly as λ decreases and approaches infinity for very short wavelengths which however can't be true.
3. This law, thus, explains the energy distribution at longer wavelengths at all temperatures and fails totally for shorter wavelengths.
4. The energy distribution curves of black body show a peak while going towards the ultra-violet wavelength (shorter λ) and then fall while Rayleigh-Jean's law indicate continuous rise only.
5. This is the failure of classical physics and this failure was often called as the "Ultra-violet catastrophe" of classical physics.

Que 3.5. **Derive Planck's radiation law and show how this law successfully explained observed spectrum of black body radiation.**

Answer

A. Planck's Radiation Law :

1. According to Planck's quantum hypothesis the exchange of energy by radiations with matters do not take place continuously but discontinuously and discretely as an integral multiple of an elementary quantum of energy represented by the relation

$$E = h\nu$$

where, ν = frequency of radiations, and
h = Planck's constant.

2. Thus, the resonators can oscillate only with integral energy values $h\nu$, $2h\nu$, $3h\nu$, ... , $nh\nu$ or in general $E_n = n\,h\nu$ (n = 1, 2, 3,...).
3. Hence, emission and absorption of energy by the particles of a radiating body interchanging energy with the radiation oscillation occur discretely, not in a continuous sequence.
4. In relation $E_n = n\,h\nu$, n is called a quantum number and the energies of the radiators are said to be quantised and allowed energy states are called quantum states.
5. On the basis of his assumptions Planck derived a relation for energy density (u_ν) of resonators emitting radiation of frequency lying between ν and $\nu + d\nu$ which is given as follows :

$$u_\nu\, d\nu = \frac{8\pi h\nu^3}{c^3}\frac{d\nu}{e^{h\nu/kT}-1} \quad ...(3.5.1)$$

or

$$u_\lambda\, d\lambda = \frac{8\pi\, hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1} \quad ...(3.5.2)$$

6. The eq. (3.5.1) and eq. (3.5.2) are known as Planck's radiation law.

B. Experimental Verification of Planck's Radiation Law : According to Planck's radiation law expression for energy density is given as

$$u_\lambda\, d\lambda = \frac{8\pi\, hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

a. Wien's Law from Planck's Radiation Law :

1. For shorter wavelengths λT will be small and hence $e^{hc/\lambda kT} >> 1$
2. Hence, for small values of λT Planck's formula reduces to

$$u_\lambda\, d\lambda = \frac{8\pi\, hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}} = 8\pi\, hc\, \lambda^{-5}\, e^{-hc/\lambda kT} d\lambda$$

or $$u_\lambda\, d\lambda = A\lambda^{-5}\, e^{-a/\lambda T}\, d\lambda \quad ...(3.5.3)$$

where, A = Constant (= $8\pi\, hc$), and
a = Constant (= hc/k).

3. Eq. (3.5.3) is Wien's law.
4. This result shows that at shorter wavelengths Planck's law approaches Wien's law and hence at shorter wavelengths Planck's law and Wien's law agrees (Fig. 3.5.1).

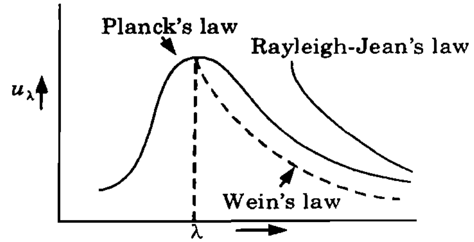


Fig. 3.5.1.

b. Rayleigh-Jean's Law :

1. For longer wavelengths $e^{hc/\lambda kT}$ is small and can be expanded as follows :

$$e^{hc/\lambda kT} = 1 + \frac{hc}{\lambda\, kT} \approx \frac{hc}{\lambda\, kT}$$

2. Hence, for longer wavelengths Planck's formula reduces to

$$u_\lambda d\lambda = \frac{8\pi\, hc}{\lambda^5}\frac{\lambda kT}{hc}\, d\lambda$$

or $$u_\lambda d\lambda = \frac{8\pi\, kT}{\lambda^4}\, d\lambda \quad ...(3.5.4)$$

3. Eq. (3.5.4) shows that for longer wavelengths Planck's law approaches to Rayleigh-Jean's law and thus at longer wavelengths Planck's law and Rayleigh-Jean's law agree (Fig. 3.5.1).
4. Thus, it is concluded that the Planck's radiation law successfully explained the entire shape of the curves giving the energy distribution in black body radiation.

Que 3.6. **Discuss the wave particle duality.**

Answer

1. According to the Planck's theory of thermal radiation; Einstein's explanation of photoelectric effect; emission and absorption of radiation by substance; black body radiation etc., the electromagnetic radiation consist of discrete indivisible packets of energy ($h\nu$) called photons which manifest particle character of radiation. On the other hand, macroscopic optical phenomena like interference, diffraction and polarisation reveal and firmly confirm the wave character of electromagnetic radiation. Therefore, we conclude that the electromagnetic radiation has dual character, in certain situation it exhibits wave properties and in other it acts like a particle.
2. The particle and wave properties of radiation can never be observed simultaneously. To study the path of a beam of monochromatic radiation, we use the wave theory, while to calculate the amount of energy transactions of the same beam, we have to recourse to the photon or particle theory.
3. It has been found impossible to separate the particle and wave aspects of electromagnetic radiation.

Que 3.7. **What are de-Broglie's waves or matter waves ?**

AKTU 2017-18

Answer

1. When a material particle moves in a medium, a group of waves is associated with it due to which it shows the wave particle duality. These waves are known as matter waves or de-Broglie waves.
2. According to de-Broglie's concept, each material particle in motion behaves as waves, having wavelength 'λ' associated with moving particle of momentum p.

$$\lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$$

$$\text{Wave nature} \propto \frac{1}{\text{Particle nature}}$$

Que 3.8. **Deduce expression for wavelength of de-Broglie wave.**

Answer

1. Let a photon having energy,

$$E = h\nu = \frac{hc}{\lambda} \qquad ...(3.8.1)$$

2. If a photon possesses mass, it is converted into energy.
3. Now according to Einstein's law,

$$E = mc^2 \qquad ...(3.8.2)$$

4. From eq. (3.8.1) and eq. (3.8.2),

$$mc^2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{mc^2}$$

$$\lambda = \frac{h}{mc} \Rightarrow \lambda = \frac{h}{p} \qquad [\because mc = p]$$

5. In place of photon, we take material particle having mass 'm' moving with velocity 'v'. The momentum,

$$p = m\text{v}$$

6. The wavelength of wave associated with particle is,

$$\lambda = \frac{h}{m\text{v}} = \frac{h}{p}$$

This is de-Broglie's wavelength.

7. If E_k is kinetic energy of material particle of mass 'm' moving with velocity 'v' then,

$$E_k = \frac{1}{2} m\text{v}^2$$

$$E_k = \frac{m^2 \text{v}^2}{2m}$$

or $$E_k = \frac{(m\text{v})^2}{2m} = \frac{p^2}{2m}$$

or $$p = \sqrt{2mE_k}$$

8. The de-Broglie's wavelength, $\lambda = \dfrac{h}{\sqrt{2mE_k}}$
9. According to kinetic theory of gases, the average kinetic energy (E_k) of the material particle is given as

$$E_k = \frac{3}{2} KT$$

10. The de-Broglie's wavelength,

$$\lambda = \frac{h}{\sqrt{2m \times \frac{3KT}{2}}} = \frac{h}{\sqrt{3mKT}}$$

where, $K = 1.38 \times 10^{-23}$ J/K

T = temperature (K).

11. Suppose material particle is accelerated by potential difference of V volt then,

$$E_k = qV$$

where, q = charge of particle.

12. The de-Broglie's wavelength,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Que 3.9. Derive an expression for de-Broglie wavelength of helium atom having energy at temperature *T* K.

Answer

1. According to kinetic theory of gases, the average kinetic energy (E_k) of the material particle is given as

$$E_k = \frac{3}{2} KT$$

2. The de-Broglie's wavelength,

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2m \times \frac{3KT}{2}}} = \frac{h}{\sqrt{3mKT}}$$

where, $K = 1.38 \times 10^{-23}$ J/K, and

T = temperature (K).

Que 3.10. The kinetic energy of an electron is 4.55×10^{-25} J. Calculate the velocity, momentum and wavelength of the electron.

Answer

Given : $E_k = 4.55 \times 10^{-25}$ J

To Find :
i. Velocity
ii. Momentum
iii. Wavelength of electron.

1. If m_o is the rest mass of electron, v is the velocity of the electron, then its kinetic energy (E_k) is given by

$$E_k = \frac{1}{2} m_o v^2$$

$$v = \sqrt{\frac{2E_k}{m_o}} = \sqrt{\frac{2 \times 4.55 \times 10^{-25}}{9.1 \times 10^{-31}}} = 1 \times 10^3 \text{ m/s}$$

[$\because$ $m_o = 9.1 \times 10^{-3}$ kg]

2. Momentum of electron,

$$p = m_o v = 9.1 \times 10^{-31} \times 10^3 = 9.1 \times 10^{-28} \text{ kg m/s}$$

3. Wavelength of electron,

$$\lambda = h/p = (6.63 \times 10^{-34})/(9.1 \times 10^{-28}) = 7.29 \times 10^{-7} \text{ m}$$

Que 3.11. Find the de-Broglie wavelength of neutron of energy 12.8 MeV (given that $h = 6.625 \times 10^{-34}$J–s, mass of neutron ($m_n$) = 1.675×10^{-27} kg and 1 eV = 1.6×10^{-19} Joule).

Answer

Given : E_k = 12.8 MeV, $h = 6.625 \times 10^{-34}$ J-s, $m_n = 1.675 \times 10^{-27}$ kg, 1 eV = 1.6×10^{-19} J.

To Find : de-Broglie wavelength.

1. Rest mass energy of neutron is given as

$$m_o c^2 = 1.675 \times 10^{-27} \times (3 \times 10^8)^2$$
$$= 1.5075 \times 10^{-10} \text{ J}$$
$$= \frac{1.507 \times 10^{-10}}{1.6 \times 10^{-19}} = 942.18 \text{ MeV}$$

2. The given energy 12.8 MeV is very less compared to the rest mass energy of neutron, therefore relativistic consideration in this case is not applicable.

3. Now de-Broglie wavelength of the neutron is given as

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

$$E_k = 12.8 \times 10^6 \times (1.6 \times 10^{-19}) \text{ J}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 12.8 \times 10^6 \times 1.6 \times 10^{-19}}}$$

$$= \frac{6.625 \times 10^{-34}}{8.28 \times 10^{-20}}$$

$$= 8 \times 10^{-15} \text{ m}$$
$$= 8 \times 10^{-5} \text{ Å}$$

Que 3.12. Calculate the de-Broglie's wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}$ th of light velocity.

Answer

Given : v = (1/20) $c = \frac{1}{20} \times 3 \times 10^8$ m/s = 1.5×10^7 m/s

To Find : de-Broglie wavelength

1. Formula for de-Broglie's wavelength :

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1.5 \times 10^{7}} = 2.646 \times 10^{-14}\ \text{m}$$

$[\because\ m = 1.67 \times 10^{-27}$ kg and $h = 6.63 \times 10^{-34}$ J-s$]$

$= 2.646 \times 10^{-4}$ Å

PART-2

Time Dependent and Time Independent Schrodinger's Wave Equation, Born Interpretation of Wave Function, Solution to Stationary State, Schrodinger Wave Equation for One-Dimensional Particle in a Box, Compton Effect.

CONCEPT OUTLINE : PART-2

Wave Function and its Significance :

The wave function ψ is described as mathematical function whose variation builds up matter waves. $|\psi|^2$ defines the probability density of finding the particle within the given confined limits. ψ is defined as probability amplitude and $|\psi|^2$ is defined as probability density.

Schrodinger's Wave Equation :

This wave equation is a fundamental equation in quantum mechanics and describes the variation of wave function ψ in space and time.

Compton Effect : The phenomenon in which the wavelength of the incident X-rays increases and hence the energy decreases due to scattering from an atom is known as Compton effect.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 3.13. **What is Schrodinger wave equation ? Derive time independent and time dependent Schrodinger wave equations.**

AKTU 2015-16, 2016-17

Answer

1. Schrodinger's equation, which is the fundamental equation of quantum mechanics, is a wave equation in the variable ψ.

A. Time Independent Schrodinger Wave Equation :

1. Consider a system of stationary wave to be associated with particle an... the position coordinate of the particle (x, y, z) and ψ is the periodic displacement at any instant time 't'.
2. The general wave equation in 3-D in differential form is given as

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad ...(3.1...)$$

where, v = velocity of wave, and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \text{Laplacian operator.}$$

3. The wave function may be written as

$$\psi = \psi_o e^{-i\omega t} \quad ...(3.13.2)$$

4. Differentiate eq. (3.13.2) w.r.t. time, we get,

$$\frac{\partial\psi}{\partial t} = -i\,\omega\,\psi_o e^{-i\omega t} \quad ...(3.13...)$$

5. Again differentiating eq. (3.13.3), we get

$$\frac{\partial^2\psi}{\partial t^2} = i^2\omega^2\psi_o e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi \quad ...(3.13...)$$

6. Putting these value in eq. (3.13.1),

$$\nabla^2\psi = \frac{-\omega^2}{v^2}\psi \quad ...(3.1...)$$

7. But $\omega = 2\pi\nu = \dfrac{2\pi v}{\lambda} \Rightarrow \dfrac{\omega}{v} = \dfrac{2\pi}{\lambda}$ ...(3.13.6)
8. From eq. (3.13.6) and eq. (3.13.5), we get

$$\nabla^2\psi = -\frac{4\pi^2}{\lambda^2}\psi \quad ...(3.13.7)$$

9. From de-Broglie's wavelength, $\lambda = \dfrac{h}{mv}$

then,

$$\nabla^2\psi = \frac{-4\pi^2m^2v^2}{h^2}\psi \quad ...(3.13.8)$$

10. If E and V are the total energy and potential energy of a particle and E_k is kinetic energy, then,

$$E_k = E - V \quad \text{or} \quad \frac{1}{2}mv^2 = E - V$$

$$m^2v^2 = 2m(E - V) \quad ...(3.13.9)$$

11. From eq. (3.13.8) and eq. (3.13.9), we get

$$\nabla^2\psi = \frac{-4\pi^2 2m[E-V]\psi}{h^2}$$

$$\nabla^2\psi + \frac{2m[E-V]\psi}{\hbar^2} = 0 \qquad \left[\text{where, } \hbar = \frac{h}{2\pi}\right] \qquad ...(3.13.10)$$

This is required time-independent Schrodinger wave equation.

12. For free particle, $V = 0$

$$\therefore \quad \nabla^2\psi + \frac{2m}{\hbar^2} E\psi = 0$$

B. Time Dependent Schrodinger Wave Equation :

1. We know that wave function is $\psi = \psi_o e^{-i\omega t}$
2. On differentiating w.r.t. time, we get,

$$\frac{\partial\psi}{\partial t} = -i\omega\,\psi_o e^{-i\omega t}$$

or
$$\frac{\partial\psi}{\partial t} = -i\,(2\pi\nu)\,\psi \qquad ...(3.13.11)$$

3. But $\quad E = h\nu \;\Rightarrow\; \nu = \dfrac{E}{h}$
4. So, eq. (3.13.11) becomes,

$$\frac{\partial\psi}{\partial t} = -i2\pi\left(\frac{E}{h}\right)\psi$$

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}E\psi \qquad \left[\because \; \hbar = \frac{h}{2\pi}\right]$$

or
$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$$

or
$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad ...(3.13.12)$$

5. Now time independent Schrodinger wave equation is,

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}[E\psi - V\psi] = 0$$

6. Using eq. (3.13.12), we get,

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right] = 0$$

$$\nabla^2\psi - \frac{2m}{\hbar^2}V\psi = -\frac{2m}{\hbar^2}i\hbar\frac{\partial\psi}{\partial t}$$

$$\left(\nabla^2 - \frac{2m}{\hbar^2}V\right)\psi = -\frac{2m}{\hbar^2}i\hbar\frac{\partial\psi}{\partial t}$$

or
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

This is required time dependent Schrodinger wave equation.

7. $\quad -\dfrac{\hbar^2}{2m}\nabla^2 + V = H \rightarrow$ is known as Hamiltonian operator.

and, $\quad i\hbar\dfrac{\partial\psi}{\partial t} = E\psi \rightarrow$ energy operator.

Then, $\quad H\psi = E\psi$

Que 3.14. **Discuss the Born's interpretation of wave function.**

Answer

1. Max Born interpreted the relation between the wave function $\psi(x, t)$ and the location of the particle by drawing an analogy between the intensity of light or photon beam and the intensity of electron beam.
2. Consider a beam of light (EM-wave) incident normally on a screen. The magnitude of electric field vector $\vec{E}$ of the beam is given by

$$E = E_0 \sin(kx - \omega t)$$

where, $\quad E_0$ = Amplitude of the electric field.

3. For an EM-wave, the intensity I at a point due to a monochromatic beam of frequency ν is given by

$$I = c\varepsilon_0 \langle E^2 \rangle \qquad ...(3.14.1)$$

Here, $\langle E^2 \rangle$ = Average of the square of the instantaneous magnitudes of the electric field vector of the wave over a complete cycle,
c = Velocity of light in free space, and
ε_0 = Electric permittivity of free space.

4. The intensity may also be interpreted as the number N of photons each of energy $h\nu$ crossing unit area in unit time at the point under consideration normal to the direction of the photons.

Thus, $$I = N h\nu \qquad ...(3.14.2)$$

5. Comparing eq. (3.14.1) and eq. (3.14.2), we get,

$$N h\nu = c\varepsilon_0 \langle E^2 \rangle$$

or
$$N = \frac{c\varepsilon_0}{h\nu}\langle E^2 \rangle$$

or
$$N \propto \langle E^2 \rangle \qquad ...(3.14.3)$$

This relation is valid only when a large number of photons are involved, i.e., the beam has the large intensity.

6. If we consider the scattering of only a single photon by a crystal or the passage of only a single photon through a narrow slit, then it is impossible to observe the usual pattern of intensity variation or diffraction.
7. In this situation, we can only say that the probability of photon striking the screen is highest at places where the wave theory predicts a maximum and lowest at places where the wave theory predicts a minimum.
8. Eq. (3.14.3) shows that $< E^2 >$ is a measure of the probability of photon crossing unit area per second at the point under consideration. Hence, in one dimension, $<E^2>$ is a measure of the probability per unit length of finding the photon at the position x at time t.

Que 3.15. **The wave function of a particle confined to a box of length L is**

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \qquad 0 < x < L$$

and $\psi(x) = 0$ everywhere else.
Calculate probability of finding the particle in region $0 < x < \frac{L}{2}$.

Answer

1. The probability of finding the particle in interval dx at distance x is
$$p(x)dx = |\psi|^2 dx = \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx$$
2. The probability in region $0 < x < \frac{L}{2}$ is
$$P = \int_0^{L/2} p(x)dx = \int_0^{L/2} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx$$
$$= \frac{1}{L}\int_0^{L/2}\left(1 - \cos\frac{2\pi x}{L}\right)dx$$
$$= \frac{1}{L}\cdot\frac{L}{2} = \frac{1}{2}$$

Que 3.16. **Discuss the stationary state solutions in brief.**

Answer

1. A state of the system in which probability distribution function $\psi\psi^*$ is independent of time is called stationary state of the system.
where, ψ = wave function, and
ψ^* = complex conjugate of wave function.
2. Let the probability distribution function $\psi\psi^*$ for a system in the state is given by the wave function

$$\psi(x, y, z, t) = \sum_{n=1}^{\infty} a_n f_n(x, y, z)\, e^{-iE_n t/\hbar} \qquad ...(3.16.1)$$

The complex conjugate of eq. (3.16.1) is,

$$\psi^*(x, y, z, t) = \sum_{m=1}^{\infty} a_m{}^* f_m{}^*(x, y, z)\, e^{iE_m t/\hbar} \qquad ...(3.16.2)$$

The product of ψ and ψ^* or probability distribution function $\psi\psi^*$ is given by

$$\psi\psi^* = \left[\sum_{n=1}^{\infty} a_n f_n(x, y, z)\, e^{-iE_n t/\hbar}\right]\left[\sum_{m=1}^{\infty} a_m^* f_m^*(x, y, z)\, e^{iE_m t/\hbar}\right]$$
$$= \sum a_n a_n^* f_n(x, y, z) f_n^*(x, y, z)$$
$$+ \sum_m \sum_n a_n a_m^* f_n(x, y, z) f_m^*(x, y, z)\, e^{i(E_m - E_n)t/\hbar} \qquad ...(3.16.3)$$

3. The probability distribution function that is, $\psi\psi^*$ will be independent of time only when a_n's are zero for all values except for one value of E_n. In such case the wave function contains only a single term and expressed as
$$\psi_n(x, y, z, t) = f_n(x, y, z)\, e^{-iE_n t/\hbar} \qquad ...(3.16.4)$$
4. Since $\psi\psi^* = f_n f_n^*$ is independent of time, the solution represented by eq. (3.16.4) is stationary state solution.

Que 3.17. **Write the Schrodinger wave equation for the particle in a box and solve it to obtain the eigen value and eigen function.**

AKTU 2013-14, 2017-18

Answer

1. Let a particle is confined in one-dimensional box of length 'L'. The particle is free *i.e.*, no external force, so potential energy inside box is zero ($V = 0$).

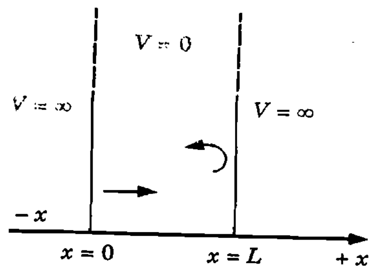


Fig. 3.17.1.

$V = 0$ for $0 < x < L$
$V = \infty$ for $x < 0$ and $x > L$
i.e., outside the box the potential energy is infinite.
2. The particle cannot exist outside the box.

$$\psi = 0 \text{ for } x = L \text{ and } x = 0$$

3. Schrodinger time independent equation for free particle ($V = 0$),

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E\psi = 0 \qquad ...(3.17.1)$$

or $$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0 \qquad \left[\text{where, } k^2 = \frac{2mE}{\hbar^2}\right] \qquad ...(3.17.2)$$

4. Solution of eq. (3.17.2) is

$$\psi(x) = A \sin kx + B \cos kx \qquad ...(3.17.3)$$

5. Using boundary condition, $\psi = 0$ at $x = 0$

$$0 = A \sin 0 + B \cos 0$$

or $$B = 0$$

and $$\psi = 0 \text{ at } x = L$$

$$0 = A \sin kL + B \cos kL$$

or $$A \sin kL = 0 \quad \text{or} \quad \sin kL = \sin n\pi \qquad [\because \; B = 0]$$

$$kL = n\pi, \quad n = 1, 2, 3, \ldots \text{ But } n \neq 0$$

$$k = \frac{n\pi}{L}$$

6. Now eq. (3.17.3) becomes,

$$\psi_n(x) = A \sin \frac{n\pi x}{L} \qquad \text{[Eigen function]}$$

and $$\frac{n^2\pi^2}{L^2} = \frac{2mE}{\hbar^2} \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

7. Then, $$E_n = \frac{n^2h^2}{8mL^2} \qquad \text{[Eigen energy]} \qquad \left[\hbar = \frac{h}{2\pi}\right]$$

For $n = 1$, $E_1 = \dfrac{h^2}{8mL^2}$, it is ground state energy of particle.

Que 3.18. **A particle is in motion along a line between $x = 0$ and $x = L$ with zero potential energy. At points for which $x < 0$ and $x > L$, the potential energy is infinite. The wave function for the particle in n^{th} state is given by :**

$$\psi_n = A \sin \frac{n\pi x}{L}$$

Find the expression for the normalized wave function.

OR

Derive normalization wave function.

Answer

1. The eigen function is,

$$\psi_n(x) = A \sin \frac{n\pi x}{L} \qquad ...(3.18.1)$$

2. Now applying normalization condition to find constant A,

$$\int_0^L |\psi_n(x)|^2 \, dx = 1$$

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\frac{A^2}{2}\int_0^L \left(1 - \cos\frac{2n\pi}{L}x\right) dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{\sin\frac{2n\pi x}{L}}{\frac{2n\pi}{L}}\right]_0^L = 1$$

$$\frac{A^2}{2} L = 1$$

$$A = \sqrt{\frac{2}{L}}$$

3. So, eq. (3.18.1) becomes, $\psi_n(x) = \sqrt{\dfrac{2}{L}} \sin\left(\dfrac{n\pi x}{L}\right)$.

This is normalization function.

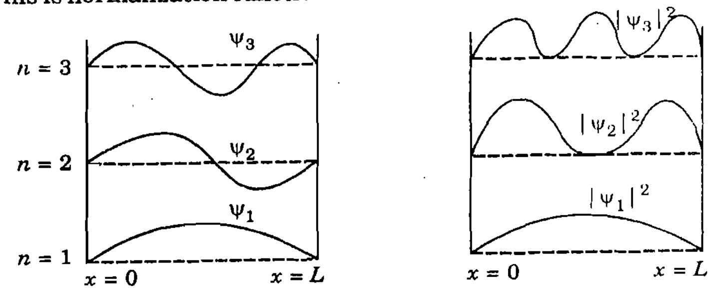


Fig. 3.18.1.

Que 3.19. **An electron is bound in one dimensional potential box which has width 2.5×10^{-10} m. Assuming the height of the box to be infinite, calculate the lowest two permitted energy values of the electron.**

AKTU 2014-15, Marks 05

Answer

Given : $L = 2.5 \times 10^{-10}$ m, $n = 1, 2$

To Find : Lowest two permitted energy values of electron.

1. We know that,

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_n = \frac{n^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$[\because \; h = 6.63 \times 10^{-34} \text{ J-s}, m = 9.1 \times 10^{-31} \text{ kg}]$

$$= 9.66 \times 10^{-19} n^2 \text{ J}$$

$$= \frac{9.66 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 6.037\, n^2 \text{ eV}$$

2. For $n = 1$, $E_1 = 6.037$ eV
   $n = 2$, $E_2 = 24.15$ eV

Que 3.20. Compute the energy difference between the ground state and first excited state of an electron in one-dimensional box of length 10^{-8} m.

Answer

Given : $L = 10^{-8}$m.
To Find : Energy difference between ground state and first excited state.

1. We know that eigen energy, $E_n = \dfrac{n^2h^2}{8mL^2}$

$$E_n = n^2 \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-8})^2} = 0.60 \times 10^{-21} n^2 \text{ J}$$

$$= \frac{0.6 \times 10^{-21} n^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$E_n = 3.75 \times 10^{-3} n^2 \text{ eV}$$

2. For ground state ($n = 1$), $E_1 = 3.75 \times 10^{-3}$ eV
3. First excited state ($n = 2$), $E_2 = 0.015$ eV
4. Difference between first excited and ground state,

$$E_2 - E_1 = (15 - 3.75)\, 10^{-3} \text{ eV} = 11.25 \text{ meV}$$

Que 3.21. A particle confined to move along x-axis has the wave function $\psi = ax$ between $x = 0$ and $x = 1.0$, and $\psi = 0$ elsewhere. Find the probability that the particle can be found between $x = 0.35$ to $x = 0.45$. Also, find the expectation value $< x >$ of particle's position.

Answer

Given : $\psi = ax$.
To Find : i. Probability of particle can be found between $x = 0.35$ to $x = 0.45$.
ii. Expectation value.

1. The probability of finding the particle between x_1 and x_2 when it is in n^{th} state is,

$$P = \int_{x_1}^{x_2} |\psi_n|^2 dx$$

2. Here, $x_1 = 0.35$ and $x_2 = 0.45$

Therefore,
$$P = \int_{0.35}^{0.45} (ax)^2 dx = a^2 \int_{0.35}^{0.45} x^2 dx$$

$$P = \frac{a^2}{3}\left[x^3\right]_{0.35}^{0.45} = \frac{a^2}{3}\,[(0.45)^3 - (0.35)^3]$$

$$= \frac{a^2}{3}\,[0.091125 - 0.042875] = 0.0161\, a^2$$

3. The expectation value of the position of particle is given by

$$< x > = \int_{-\infty}^{\infty} x|\psi_n(x)|^2 dx$$

4. Since, the particle is confined in a box having its limit $x = 0$ to $x = 1$ then,

$$< x > = \int_0^1 x.(ax)^2 dx = a^2 \int_0^1 x^3 dx$$

$$< x > = \frac{a^2}{4} = 0.25\, a^2$$

Que 3.22. Determine the probabilities of finding a particle trapped in a box of length L in the region from $0.45\,L$ to $0.55\,L$ for the ground state. **AKTU 2017-18**

Answer

Given : $x_1 = 0.45L$, $x_2 = 0.55L$
To Find : Probabilities of finding a particle trapped in a box.

1. The eigen function of particle trapped in a box of length L is

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

2. Probability,
$$P = \int_{x_1}^{x_2} |\psi_n(x)|^2 dx = \frac{2}{L}\int_{x_1}^{x_2} \sin^2 \frac{n\pi x}{L}\, dx$$

$$P = \frac{2}{L}\int_{x_1}^{x_2} \frac{1}{2}\left(1 - \cos\frac{2\pi nx}{L}\right) dx = \frac{1}{L}\left[x - \frac{L}{2\pi n}\sin\frac{2n\pi x}{L}\right]_{x_1}^{x_2}$$

Since, $x_1 = 0.45\,L$ and $x_2 = 0.55\,L$, for ground state, $n = 1$

$$P = \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right]_{0.45L}^{0.55L}$$

$$= \frac{1}{L}\left[\left(0.55L - \frac{L}{2\pi}\sin 1.1\pi\right) - \left(0.45L - \frac{L}{2\pi}\sin 0.9\pi\right)\right]$$

$$= \left[\left(0.55 - \frac{1}{2\pi}\sin 198°\right) - \left(0.45 - \frac{1}{2\pi}\sin 162°\right)\right]$$

$$P = 0.198362 = 19.8\ \%$$

Que 3.23. **Discuss Compton effect and derive an expression for Compton shift.**

OR

Derive an expression for Compton shift showing dependency on angle of scattering.

Answer

1. When a monochromatic beam of high frequency radiation is scattered by a substance, the scattered radiation contain two components-one having a lower frequency or greater wavelength and the other having the same frequency or wavelength.

   The radiation of unchanged frequency in the scattered beam is known as 'unmodified radiation' while the radiation of lower frequency or slightly higher wavelength is called as 'modified radiation'.

   This phenomenon is known as 'Compton effect'.
4. Let a photon of energy $h\nu$ collides with an electron at rest.
5. During the collision it gives a fraction of energy to the free electron. The electron gains kinetic energy and recoil as shown in Fig. 3.23.1.

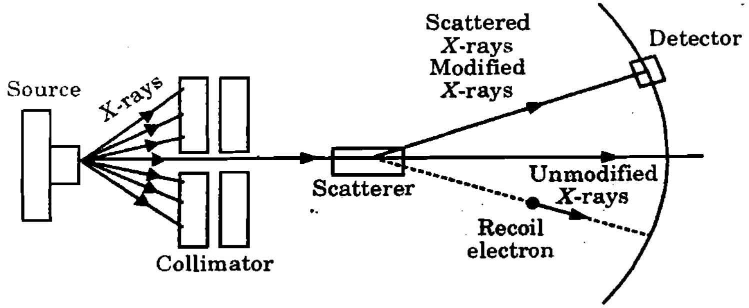
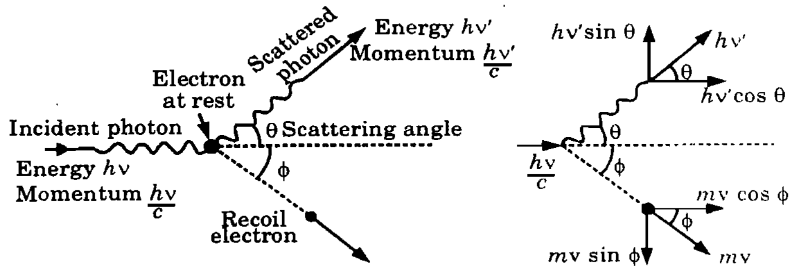


Fig. 3.23.1. Compton effect.



(*a*) Geometry of Compton scattering (*b*) Components of momentum before and after collision

Fig. 3.23.2.

a. Before collision :
   i. Energy of incident photon = $h\nu$
   ii. Momentum of incident photon = $\frac{h\nu}{c}$
   iii. Rest energy of electron = m_0c^2
   iv. Momentum of rest electron = 0
b. After collision :
   i. Energy of scattered photon = $h\nu'$
   ii. Momentum of scattered photon = $\frac{h\nu'}{c}$
   iii. Energy of electron = mc^2
   iv. Momentum of recoil electron = $m\nu$

6. According to the principle of conservation of energy,
$$h\nu + m_0c^2 = h\nu' + mc^2 \quad ...(3.23.1)$$
7. Again using the principle of conservation of momentum along and perpendicular to the direction of incident, we get,
   Momentum before collision = Momentum after collision
$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + m\nu\cos\phi \quad ...(3.23.2)$$
$$0 + 0 = \frac{h\nu'}{c}\sin\theta - m\nu\sin\phi \quad ...(3.23.3)$$
8. From eq. (3.23.2), we get,
$$m\nu c\cos\phi = h\nu - h\nu'\cos\theta \quad ...(3.23.4)$$
9. From eq. (3.23.3), we get,
$$m\nu c\sin\phi = h\nu'\sin\theta \quad ...(3.23.5)$$
10. Squaring eq. (3.23.4) and eq. (3.23.5) and then adding, we get,
$$m^2\nu^2c^2 = (h\nu - h\nu'\cos\theta)^2 + (h\nu'\sin\theta)^2$$
$$= h^2\nu^2 - 2h^2\nu\nu'\cos\theta + h^2\nu'^2\cos^2\theta + h^2\nu'^2\sin^2\theta$$
$$= h^2[\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta] \quad ...(3.23.6)$$
11. From eq. (3.23.1), we get,
$$mc^2 = h(\nu - \nu') + m_0c^2$$

Squaring, $m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')\, m_0c^2 + m_0^2c^4$ ...(3.23.7)

12. Subtracting eq. (3.23.6) from eq. (3.23.7), we have,

$$m^2c^4 - m^2\nu^2c^2 = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')\, m_0c^2 + m_0^2c^4$$

or $$m^2c^2(c^2 - \nu^2) = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')\, m_0c^2 + m_0^2c^4$$

or $$\frac{m_0^2c^2}{1 - \frac{\nu^2}{c^2}}(c^2 - \nu^2) = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')\, m_0c^2 + m_0^2c^4$$

$$\left[\because\ m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\right]$$

or $$m_0^2c^4 = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')\, m_0c^2 + m_0^2c^4$$

$\therefore$ $$2h(\nu - \nu')\, m_0c^2 = 2h^2\nu\nu'(1 - \cos\theta) \quad ...(3.23.8)$$

or $$\frac{\nu - \nu'}{\nu\nu'} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta) \quad ...(3.23.9)$$

13. Eq. (3.23.9) shows that $\nu' < \nu$ as m_0, c, h are the constants with positive values and the maximum value of $\cos\theta = 1$. This shows that the scattered frequency is always smaller than the incident frequency.

14. From eq. (3.23.9), we have,

$$\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0c}(1 - \cos\theta)$$

or $$\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)$$

or $$\Delta\lambda = \frac{2h}{m_0c}\sin^2\frac{\theta}{2} \quad ...(3.23.10)$$

15. From eq. (3.23.10), it is noted that Compton shift depends on angle of scattering.

Que 3.24. **Explain the experimental verification of Compton effect.**

AKTU 2016-17

Answer

1. The apparatus used by Compton for experimental verification of Compton effect is shown in Fig. 3.24.1.

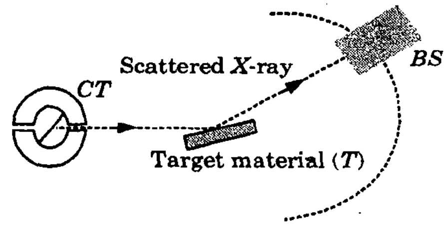


Fig. 3.24.1. Experimental verification of Compton effect.

2. Monochromatic X-rays of wavelength λ from a Coolidge tube CT are allowed to fall on a target material T such as a small block of carbon.
3. The scattered X-rays of wavelength λ' are received by a Bragg spectrometer BS, which can move along the arc of a circle.
4. The wavelength of the scattered X-rays is measured for different values of the scattering angle.

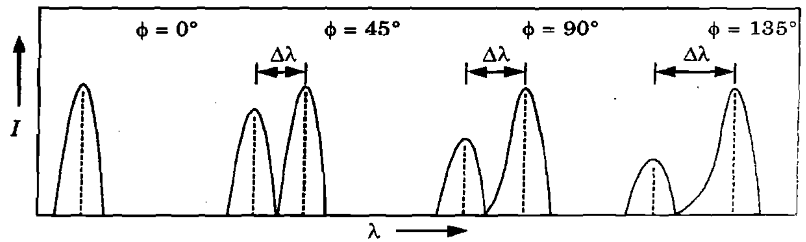


Fig. 3.24.2. Intensity of scattered X-rays.

5. The plots of intensity of scattered X-rays against their wavelength are shown in Fig. 3.24.2 for $\phi = 0°, 45°, 90°$ and $135°$.
6. These plots show the two peaks for non-zero values of ϕ, which is the indication of the presence of two distinct lines in the scattered X-ray radiation.
7. One of these lines is known as the unmodified line which has the same wavelength as the incident radiation and the other line is known as the modified line which has the comparatively longer wavelength. The Compton shift $\Delta\lambda$ is found to vary with the angle of scattering.

Que 3.25. **Why Compton shift is not observed with visible light ?**

Answer

1. The energy of a visible light photon say of wavelength λ = 6000 Å ($= 6 \times 10^{-7}$ m) is given by

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(6.6 \times 10^{-34} \times 3 \times 10^{8})}{(6 \times 10^{-7})} \text{ J}$$

$$= \frac{(6.6 \times 10^{-34} \times 3 \times 10^{8})}{(1.6 \times 10^{-19} \times 6 \times 10^{-7})} \text{ eV}$$

$$= 2.06 \text{ eV} \approx 2 \text{ eV}$$

Whereas the energy of X-ray photon, say of wavelength $\lambda = 1$ Å $= 10^{-10}$ m will be more than 1000 times the above value.

3. The binding energy of the electron in the atoms is of the order of the 10 eV. For example, the binding energy of the electron in the hydrogen atom is,

$$E_b = \frac{2\pi^2 k^2 Z^2 m_0 e^4}{h^2}$$

Where,
$k = 1/4\pi\varepsilon_0 = 9 \times 10^9$ N-m²/C²,
$m_0 = 9.1 \times 10^{-31}$ kg,
$h = 6.6 \times 10^{-34}$ J-s,
$e = 1.6 \times 10^{-19}$ C,
$Z = 1$.

Thus,
$$E_b = \frac{2\pi^2 (9 \times 10^9)^2 (9.1 \times 10^{-31})(1.6 \times 10^{-19})^4}{(6.6 \times 10^{-34})^2} \text{ J}$$

$$= \frac{2\pi^2 (81 \times 10^{18})(9.1 \times 10^{-31})(1.6 \times 10^{-19})^4}{(6.6 \times 10^{-34})^2 (1.6 \times 10^{-19})} \text{ eV}$$

$$= 13.68 \text{ eV}$$

Hence, this electron can be treated as free when X-rays are incident but this electron cannot be treated as free for visible light. So, the Compton effect cannot be observed for visible light.

☺☺☺

4 UNIT

Wave Optics

PART-1

Coherent Sources, Interference in Uniform and Wedge Shaped Thin Films, Necessity of Extended Sources, Newton's Rings and its Applications.

CONCEPT OUTLINE : PART-1

Interference : The non-uniform distribution of the light intensity due to the superposition of two waves is called interference.

Necessary Conditions for Interference :

1. Light sources must be coherent in nature.
2. Light waves should be of same frequency.
3. The sources of light must be very close to each other.
4. Light sources should be monochromatic in nature.
5. The light waves must propagate along the same direction.

Types of Interference :

1. **Constructive Interference :** At certain points the resultant intensity (I) is greater than the sum of individual intensity of two waves. The interference produced at this point is known as constructive interference, it results into bright fringe. At constructive interference,
$$I > I_1 + I_2$$
2. **Destructive Interference :** At certain points the resultant intensity (I) is less than the sum of individual intensity of two waves. The interference produced at this point is known as destructive interference and it results into dark fringe. At destructive interference,
$$I < I_1 + I_2$$

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.1. What do you understand by coherent sources ? How are these obtained in practice ?

Answer

A. Coherent Sources :

1. Two sources are said to be coherent if they emit continuous light waves of the same frequency or wavelength, nearly the same amplitude and

having sharply defined phase difference that remains constant with time.

B. Production of Coherent Sources :

1. If two sources are derived from a single source by some device, then any phase-change in one is simultaneously accompanied by the same phase change in the other. Thus the phase difference between the two sources remains constant.
2. Following are the devices for creating coherent sources of light :

a. Young's Double Slit :

1. In this device, two narrow slits S_1 and S_2 receive light from same narrow slit S.
2. Hence S_1 and S_2 act as coherent sources, as shown in Fig. 4.1.1.

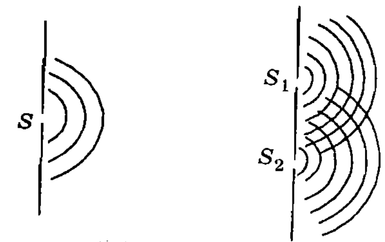


Fig. 4.1.1.

b. Lloyd's Mirror :

1. In this device, a slit S and its virtual image S' formed by reflection at a mirror are the coherent sources, as shown in Fig. 4.1.2.

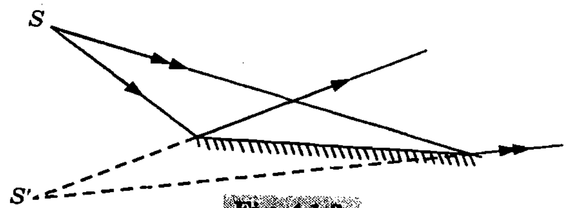


Fig. 4.1.2.

c. Fresnel's Double Mirror :

1. In this device two virtual images S_1 and S_2 of a single slit S, formed by reflection at two plane mirrors M_1 and M_2 inclined at a small angle to each other, are the coherent sources as shown in Fig. 4.1.3.

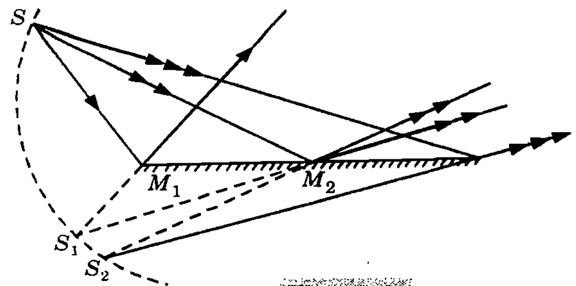


Fig. 4.1.3.

Fresnel's Biprism :

1. In this device, S_1 and S_2, which are the images of a slit S formed by refraction through a biprism, act as coherent sources, as shown in Fig. 4.1.4.

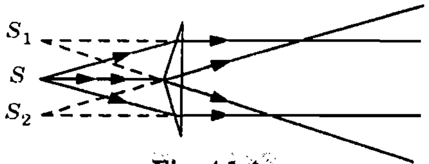


Fig. 4.1.4.

Michelson's Interferometer :

1. In this device, a single beam is broken into two light waves perpendicular to each other, one by reflection and the other by refraction.
2. The two beams, when reunite produce interference fringes. Here these two beams act as coherent sources.

Que 4.2. **Explain theory of interference by two waves.**

OR

Explain Young's double slit experiment.

Answer

Let us consider two superimposed waves travelling with same frequency $\left(\frac{\omega}{2\pi}\right)$ and having constant phase difference in the same region.

If a_1 and a_2 are amplitude of two waves, the displacement of two waves at any instant t is given by

$$y_1 = a_1 \sin \omega t \quad \text{...(4.2.1)}$$
$$y_2 = a_2 \sin (\omega t + \delta) \quad \text{...(4.2.2)}$$

where, δ = initial phase difference

$$= \frac{2\pi}{\lambda} \times \text{Path difference}$$

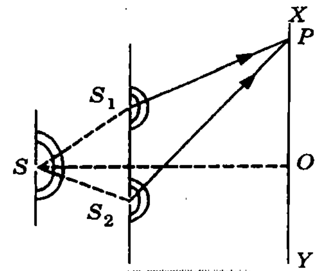


Fig. 4.2.1.

3. According to principle of superposition, the resultant displacement at point P is

$$y = y_1 + y_2$$
$$y = a_1 \sin \omega t + a_2 \sin (\omega t + \delta)$$
$$= a_1 \sin \omega t + a_2 [\sin \omega t \cos \delta + \cos \omega t \sin \delta]$$

or $$y = \sin \omega t\,(a_1 + a_2 \cos \delta) + (a_2 \sin \delta) \cos \omega t \quad \text{...(4.2.3)}$$

4. Let us take,

$$A \cos \phi = a_1 + a_2 \cos \delta \quad \text{...(4.2.4)}$$
$$A \sin \phi = a_2 \sin \delta \quad \text{...(4.2.5)}$$

Then, eq. (4.2.3) becomes,

$$y = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t$$

or $$y = A \sin (\omega t + \phi) \quad \text{...(4.2.6)}$$

5. Since, eq. (4.2.6) the resultant wave equation having amplitude A this can be obtained by squaring eq. (4.2.4) and eq. (4.2.5) and adding,

$$A^2 = a_1^2 + a_2^2 \cos^2 \delta + 2a_1a_2 \cos \delta + a_2^2 \sin^2 \delta$$
$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$$

6. By the definition, intensity is directly proportional to the square of amplitude,

$$I \propto A^2$$

or $$I = KA^2 \qquad [K = 1, \text{ in arbitrary unit}]$$

$\therefore$ $$I = A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$$

A. Condition for Maximum Intensity (Constructive Interference, I_{max}) :

1. If $\cos \delta = 1$ *i.e.*, $\delta = 2n\pi$

where, $n = 0, 1, 2, 3, \ldots$

2. $$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$
$$I_{max} = (a_1 + a_2)^2$$

3. So, $$I_{max} > I_1 + I_2$$

4. $$\text{Path difference} = \frac{\lambda}{2\pi} \times \text{Phase difference}$$
$$= \frac{\lambda}{2\pi} \times 2n\pi = 2n\,\frac{\lambda}{2}$$
$$= \text{even multiple of } \lambda/2.$$

B. Condition for Minimum Intensity (Destructive Interference, I_{min}) :

1. If $\cos\delta = -1$ *i.e.*, $\delta = (2n+1)\pi$
   where, $n = 0, 1, 2, 3...$
2. $$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$
3. Hence, $I_{min} < I_1 + I_2$
4. $$\text{Path difference} = \frac{\lambda}{2\pi} \times \text{Phase difference}$$
   $$= \frac{\lambda}{2\pi} \times (2n+1)\pi = (2n+1)\frac{\lambda}{2}$$
   $= \text{odd multiple of } \lambda/2.$

Que 4.3. **Discuss the interference in thin film due to reflected light. What happens when film is excess thin ?**

AKTU 2013-14, Marks 05

OR

Explain the phenomenon of interference in thin films due to reflected light and transmitted light.

Answer

1. Consider a parallel sided transparent thin film of thickness t and refractive index $\mu > 1$.
2. Let SA a monochromatic light of wavelength λ be incident on the upper surface of the film at an angle i. This ray gets partially reflected along AB and partially refracted along AC direction.
3. Now at point C it again gets reflected along CD and transmitted along DE.

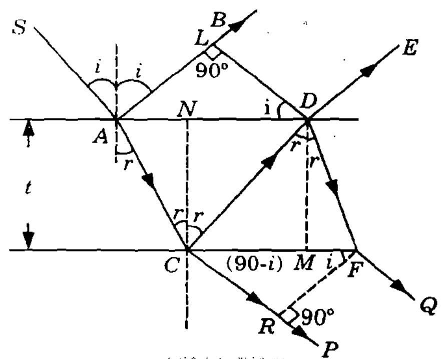


Fig. 4.3.1.

A. Interference in a Thin Film by Reflected Light :

1. According to Fig. 4.3.2, the path difference between AB and DE rays,
   $$\Delta = \text{path } ACD \text{ in film} - \text{path } AL \text{ in air}$$
   $$\Delta = \mu\,(AC + CD) - AL \qquad ...(4.3.1)$$
2. Now in ΔANC and ΔNCD,
   $$\cos r = \frac{CN}{AC} = \frac{CN}{CD}$$
   $$AC = CD = \frac{t}{\cos r}$$
3. Now in ΔALD,
   $$\sin i = \frac{AL}{AD} \Rightarrow AL = AD \sin i$$
   $$AL = (AN + ND)\sin i$$
4. But, from ΔANC and ΔNCD,
   $$AN = t\tan r \quad \text{and} \quad ND = t\tan r$$
   So, $$AL = 2t\tan r \sin i$$
5. Putting the values of AC, CD, and AL in eq. (4.3.1),
   $$\Delta = \mu\left(\frac{2t}{\cos r}\right) - 2t\tan r.\sin i$$
   $$= \frac{2\mu t}{\cos r} - 2t\frac{\sin r}{\cos r}.\sin i$$
   $$\therefore \quad \Delta = \frac{2\mu t}{\cos r}[1 - \sin^2 r] \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$
   $$\Delta = 2\mu t \cos r$$
6. Since, the ray AB is reflected at the surface of a denser medium. Therefore, it undergoes a phase change of π or path difference of $\frac{\lambda}{2}$. The effective path difference between AB and DE is
   $$\Delta = 2\mu t\cos r + \frac{\lambda}{2} \qquad ...(4.3.2)$$

a. Condition for Maxima :

1. If $$\Delta = 2n\frac{\lambda}{2}$$
   where, $n = 0, 1, 2, 3...$
2. Then, $$2\mu t\cos r + \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$
   $$2\mu t\cos r = (2n-1)\frac{\lambda}{2} \qquad ...(4.3.3)$$

b. Condition for Minima :

1. $$\Delta = (2n + 1)\frac{\lambda}{2}$$

where, $n = 0, 1, 2, 3...$

2. Then, $$2\mu t \cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \quad ...(4.3.4)$$

B. Interference in a Thin Film by Transmitted Light :

1. From Fig. 4.3.1, the path difference between two transmitted rays, CP and FQ,

$$\Delta = \text{Path } CDF \text{ in film} - \text{path } CR \text{ in air}$$

$$\Delta = \mu(CD + DF) - CR \quad ...(4.3.5)$$

2. Now in ΔCDM, $\cos r = \dfrac{DM}{CD} \Rightarrow CD = \dfrac{t}{\cos r}$ and $CM = t \tan r$

3. In ΔDMF, $\cos r = \dfrac{DM}{DF} \Rightarrow DF = \dfrac{t}{\cos r}$ and $MF = t \tan r$

4. In ΔCRF, $\sin i = \dfrac{CR}{CF} \Rightarrow CR = CF \sin i$

or $$CR = (CM + MF)\sin i$$

$$CR = 2t \tan r \sin i$$

5. On putting the value of CD, DF and CR in eq. (4.3.5),

$$\Delta = \frac{2\mu t}{\cos r} - 2t \tan r \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t\frac{\sin r}{\cos r}\mu \sin r \qquad \left[\because \mu = \frac{\sin i}{\sin r}\right]$$

$$\Delta = \frac{2\mu t}{\cos r}[1 - \sin^2 r]$$

$$\Delta = 2\mu t \cos r \quad ...(4.3.6)$$

a. Condition for Maxima :

1. If $$\Delta = 2n\frac{\lambda}{2} \qquad [\text{where, } n = 0, 1, 2, 3...]$$

2. Then, $$2\mu t \cos r = 2n\frac{\lambda}{2}$$

or $$2\mu t \cos r = n\lambda \quad ...(4.3.7)$$

b. Condition for Minima :

1. If $$\Delta = (2n + 1)\frac{\lambda}{2} \qquad [\text{where, } n = 0, 1, 2, 3...]$$

2. Then, $$2\mu t \cos r = (2n + 1)\frac{\lambda}{2} \quad ...(4.3.8)$$

C. Condition for Excess Thin Film :

1. When the film is excessively thin such that its thickness t is very small as compared to the wavelength of light, then $2\mu t \cos r$ is almost zero.

   Hence effective path difference becomes $\dfrac{\lambda}{2}$.

2. Thus every wavelength will be absent and film will appear black in reflected light.

Que 4.4. **Discuss the formation of interference fringes due to a wedge-shaped thin film seen by normally reflected monochromatic light and obtain an expression for the fringe width.**

AKTU 2017-18, Marks 07

Answer

A. Formation of Interferences Fringes :

1. A wedge shaped thin film is one whose plane surfaces OA and OB are slightly inclined to each other at a small angle θ and encloses a film of transparent material of refractive index μ as shown in Fig. 4.4.1.
2. The thickness of the film increases gradually from O to A. At the point of contact thickness is zero.
3. When the upper surface OB of the film is illuminated by a parallel beam of monochromatic light and the surface is viewed by reflected light, then the interference between the two rays; one PQ reflected from the upper surface of the film (glass to film boundary) and the other FG obtained by internal reflection (film to glass boundary) at the back surface and consequent transmissions at the film surface AB.

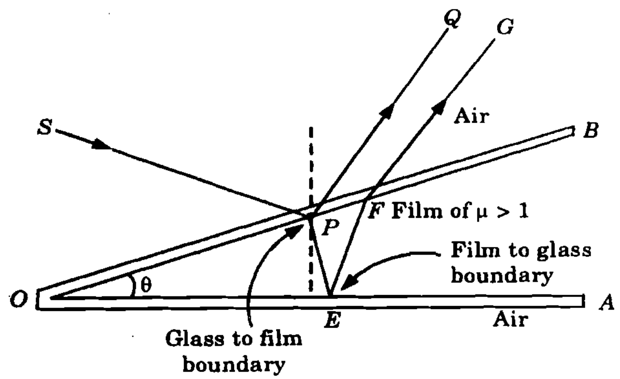


Fig. 4.4.1.

4. Since both the rays PQ and FG (or $PEFG$) are derived from the same incident ray SP (by the division of amplitude) they are coherent and on overlapping produce a system of equidistant bright and dark fringes. The fringes are straight and parallel to the contact edge of the wedge. With white light coloured fringes are observed.
5. When a beam of monochromatic light is incident normally at point P on the upper surface of the film, the path difference between the rays reflected from the upper and lower surfaces of the film is $2\mu t$, where t is the thickness of the film at P.
6. At point P, reflection occurs from the interface between the optically denser medium and optically rarer medium, therefore, there occurs an additional path difference of $\lambda/2$ or phase change of π. Thus an additional path difference of $\lambda/2$ is introduced in the ray reflected from the upper surface.
7. Hence the effective path difference between the two rays

$$= 2\mu t + \frac{\lambda}{2}$$

8. The condition for bright fringe or maximum intensity is

$$2\mu t + \lambda/2 = 2n\lambda/2$$

or $\quad 2\mu t = (2n-1)\lambda/2,$

where, $\quad n = 1, 2, 3.....$ ...(4.4.1)

9. Similarly,, the condition for dark fringe or minimum intensity is

$$2\mu t + \frac{\lambda}{2} = (2n+1)\lambda/2$$

or $\quad 2\mu t = n\lambda,$

where, $\quad n = 0, 1, 2, ...$ ...(4.4.2)

B. Expression for Fringe Width : Fringe width ω or the separation between the two successive bright fringes or between two successive dark fringes may be obtained as follows :

1. Let x_n be the distance of n^{th} dark fringe from the edge O of the film as shown in Fig. 4.4.1.

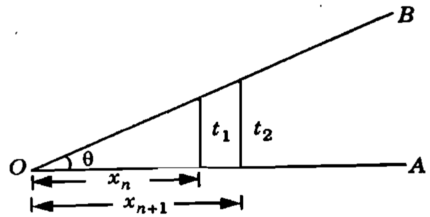


Fig. 4.4.1.

Then, $\quad \tan\theta = \frac{t_1}{x_n} \quad$ or $\quad t_1 = x_n \tan\theta$

2. Putting this value of t_1 in eq. (4.4.2), we get,

$$2\mu x_n \tan\theta = n\lambda \quad ...(4.4.3)$$

3. Similarly, if x_{n+1} is the distance of $(n+1)^{th}$ dark fringe, then

$$2\mu x_{n+1} \tan\theta = (n+1)\lambda \quad ...(4.4.4)$$

4. Subtracting eq. (4.4.3) from eq. (4.4.4), we get,

$$2\mu (x_{n+1} - x_n) \tan\theta = \lambda$$

or $$x_{n+1} - x_n = \frac{\lambda}{2\mu \tan\theta} \quad ...(4.4.5)$$

5. For very small value of θ, $\tan\theta \approx \theta$

$\therefore$ Fringe width, $$\omega = x_{n+1} - x_n = \frac{\lambda}{2\mu\,\theta} \quad ...(4.4.6)$$

where, θ is measured in radian.

6. Similarly, we can obtain same formula for the fringe width of bright fringes, that is the fringe width of bright fringe is expressed as,

$$\omega = \frac{\lambda}{2\mu\theta} \quad ...(4.4.7)$$

7. It is clear from eq. (4.4.6) and eq. (4.4.7) that for a given wedge angle θ, the fringe width of dark or bright fringes is constant (as λ and μ is constant). It means that the interference fringes are equidistant from one another.

Que 4.5. **White light falls normally on a film of soapy water whose thickness is 1.5×10^{-5} cm and refractive index is 1.33. Which wavelength in the visible region will be reflected most strongly ?**

Answer

Given : $\mu = 1.33$, $t = 1.5 \times 10^{-5}$ cm, $r = 0°$, white light on the film falls normally
To Find : Most strongly reflected wavelength in visible region.

1. Since, $$2\mu t \cos r = \frac{(2n-1)\lambda}{2}$$

$$2 \times 1.33 \times 1.5 \times 10^{-5} \times 1 = (2n-1)\lambda/2$$

$$\lambda = \frac{4 \times 1.33 \times 1.5 \times 10^{-5}}{2n-1} = \frac{7.98 \times 10^{-5}}{2n-1} \text{ cm}$$

$$\lambda = \frac{7.98 \times 10^{-7}}{(2n-1)} \text{ m}$$

2. For $n = 1$, $\quad \lambda_1 = 7980 \times 10^{-10} = 7980$ Å (visible region).
3. For $n = 2$, $\quad \lambda_2 = \frac{7.98 \times 10^{-7}}{(2n-1)} = \frac{7.98 \times 10^{-7}}{3} = 2660 \times 10^{-10}$ $= 2660$ Å (not in visible region).
4. Hence, 7980 Å is most strongly reflected wavelength in visible region.

Que 4.6. **A man whose eyes are 150 cm above the oil film on water surface observes greenish colour at a distance of 100 cm from his feet. Find the thickness of the film.**

$$(\mu_{oil} = 1.4, \mu_{water} = 1.33, \lambda_{green} = 5000 \text{ Å})$$

Answer

Given : $\mu_{oil} = 1.4$, $\mu_{water} = 1.33$, $\lambda_{green} = 5000$ Å

To Find : Thickness of the film.

1. The condition for maxima,

$$2\mu t \cos r = (2n - 1)\frac{\lambda}{2} \qquad \text{[where, } n = 1, 2, 3....]$$

or
$$t = \frac{(2n-1)\lambda}{4\mu \cos r}$$

2. From Fig. 4.6.1,

$$\tan i = \frac{100}{150} = \frac{2}{3}$$

$$\sin i = \frac{2}{\sqrt{13}}$$

3. Since,
$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\frac{2}{\sqrt{13}}}{1.4} = 0.3962$$

and
$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{[1 - (0.3962)^2]} = 0.9182$$

4. Therefore,
$$t = \frac{(2n-1)\lambda}{4\mu \cos r} = \frac{(2n-1)5\times10^{-7}}{4\times 1.4 \times 0.9182}$$

$$= (2n-1) \times 9.724 \times 10^{-8} \text{ m}$$

Putting $n = 1, 2, 3......$ value of t is calculated.

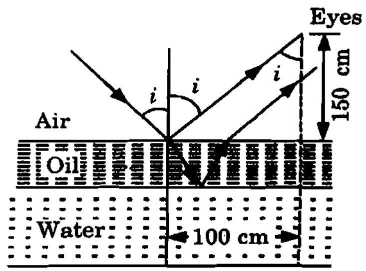


Fig. 4.6.1.

Que 4.7. **Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of $\mu = 1.42$. What is the least thickness of this film that will appear :**

a. dark

b. bright

Answer

Given : $\lambda = 5893$ Å, $\mu = 1.42$

To Find : Least thickness of this film that will appear :

i. Dark

ii. Bright

i. Least Thickness of Dark Film :

1. Since, the condition for the dark film in reflected system is

$$2\mu t \cos r = n\lambda$$

2. For normal incidence, $r = 0$ and $\cos r = 1$

$$\therefore \qquad 2\mu t = n\lambda \quad \text{or} \quad t = n\lambda/2\mu$$

3. For least thickness of the film, $n = 1$

$$t = \frac{\lambda}{2\mu}$$

$$t = \frac{5893\times10^{-8}}{2\times1.42} = 2.075\times10^{-5} \text{ cm}$$

ii. Least Thickness of Bright Film :

1. The condition for bright film

$$2\mu t \cos r = (2n - 1)\frac{\lambda}{2}$$

2. For normal incidence, $r = 0$ and $\cos r = 1$

$$\therefore \qquad 2\mu t = (2n - 1)\frac{\lambda}{2}$$

3. For least thickness, $n = 1$

$$2\mu t = (2 \times 1 - 1)\frac{\lambda}{2} \quad \text{or} \quad 2\mu t = \frac{\lambda}{2}$$

and
$$t = \frac{\lambda}{4\mu} = \frac{5893\times10^{-8}}{4\times1.42} = 1.0375\times10^{-5} \text{ cm}$$

Que 4.8. **White light is incident on a soap film at an angle $\sin^{-1}\frac{4}{5}$ and the reflected light is observed with spectroscope. It is found that two consecutive dark bands correspond to wavelength 6.1×10^{-5} cm and 6.0×10^{-5} cm. If the μ of the film be 4/3, calculate its thickness.**

Answer

Given : $i = \sin^{-1} 4/5$, $\lambda_1 = 6.1 \times 10^{-5}$ cm, $\lambda_2 = 6.0 \times 10^{-5}$ cm, $\mu = 4/3$
To Find : Thickness of film.

1. Since, the condition for dark band is

$$2\mu t \cos r = n\lambda \quad ...(4.8.1)$$

2. If n and $(n + 1)$ are the orders for dark bands for wavelengths λ_1 and λ_2 respectively, then,

$$2\mu t \cos r = n\lambda_1 \quad ...(4.8.2)$$

and $$2\mu t \cos r = (n + 1)\lambda_2 \quad ...(4.8.3)$$

or $$2\mu t \cos r = n\lambda_1 = (n + 1)\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

3. On putting the value of n in eq. (4.8.2),

$$2\mu t \cos r = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} \text{ or } t = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2) 2\mu \cos r}$$

4. But,

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \left(\frac{\sin i}{\mu}\right)^2} \quad \left(\because \mu = \frac{\sin i}{\sin r}\right)$$

$$\cos r = \sqrt{1 - \left(\frac{4/5}{4/3}\right)^2} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

5. Now,

$$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 \times 10^{-5} - 6.0 \times 10^{-5}) \times 2 \times \frac{4}{3} \times \frac{4}{5}}$$

$$= 0.0017 \text{ cm.}$$

Que 4.9. **Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges, in sodium light of wavelength, λ = 5890 Å of normal incidence, find the diameter of the wire.**

Answer

Given : $N = 20$, $\lambda = 5890$ Å
To Find : Diameter of wire.

1. Let the diameter of the wire be 'd' and the length of the wedge be 'l'.
2. The wedge angle is given as

$$\tan\theta = \frac{d}{l}$$

$$\tan\theta \approx \theta \text{ (As } \theta \text{ is small)}$$

$$\therefore \quad \theta = \frac{d}{l}$$

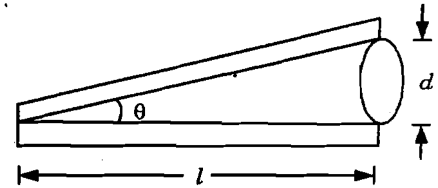


Fig. 4.9.1.

3. Now, fringe-width in air wedge is

$$\omega = \frac{\lambda}{2\mu\theta} = \frac{\lambda}{2\theta} \quad (\because \mu = 1 \text{ for air wedge})$$

or $$\omega = \frac{l\lambda}{2d}$$

4. If N fringes are seen,

$$l = N\omega$$

Thus, $$\omega = \frac{\lambda N\omega}{2d}$$

$$d = \frac{N\lambda}{2}$$

$$d = \frac{20 \times 5890}{2}$$

$$d = 58900 \text{ Å}$$

$$d = 5.89 \times 10^{-4} \text{ cm}$$

Que 4.10. **Discuss the necessity of an extended source ?**

Answer

1. When a thin film is illuminated with monochromatic light from a point source and is viewed with a lens of small aperture, the light reflected from all corresponding point on the film does not reach the eye simultaneously as shown in Fig. 4.10.1(a). Thus only a small portion of the film will be visible.
2. To see the whole film, the eye will have to be moved from one position to the other. Hence, with a point source the entire film cannot be viewed at a glance.
3. If we employ an extended source, the light reflected by every point of the film reaches the eye (as shown in Fig. 4.10.1(b)). Hence, the entire film can be viewed simultaneously by keeping the eye at one place only.
4. Hence an extended source of light is necessary to view a film simultaneously.

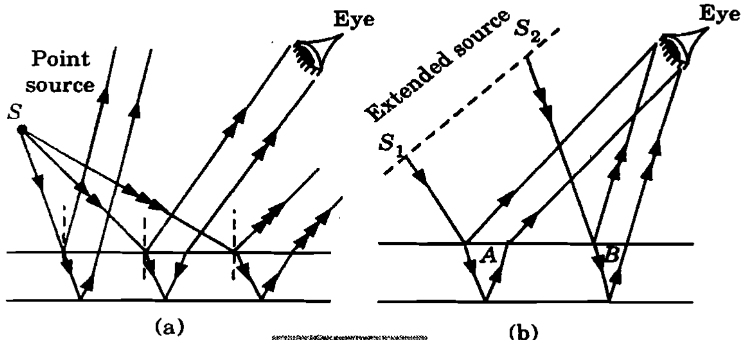


Fig. 4.10.1.

Que 4.11. **What are Newton's rings ? Explain with diagram.**

Answer

1. When a plano-convex lens of large radius of curvature is placed on a plane glass plate with convex surface in contact, a thin film between the lower surface of the lens and the upper surface of glass plate is formed.
2. The thickness of this film is very small (or zero) at the contact point and gradually increases from contact point to outward.
3. When a monochromatic light falls on the film, we get dark and bright concentric circular fringes having uniform thickness.
4. These rings are first investigated by Newton and are called Newton's rings.

A. Experimental Arrangement :

1. According to Fig. 4.11.1, S is a monochromatic source of light placed at the focus of lens L_1.
2. A horizontal beam of light fall on the glass plate G placed at 45° to the incident beam.
3. This beam is partly reflected from glass plate G.
4. This reflected beam fall normally on the lens L, placed on glass plate E.
5. Hence, the interference occurs between the rays reflected from the upper and lower surface of the film.
6. Interference rings are seen with the help of lower power microscope M.
7. The fringes are circular because the air film is symmetrical about the point of contact with lens and glass plate.

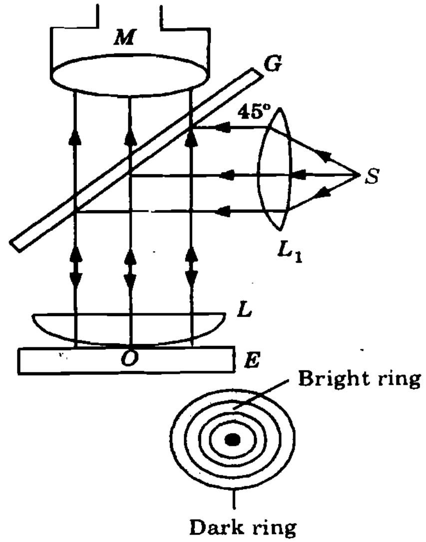


Fig. 4.11.1.

B. Explanation :

1. According to Fig. 4.11.2, rays (1) and (2) are reflected interfering rays corresponding to incident ray SP.

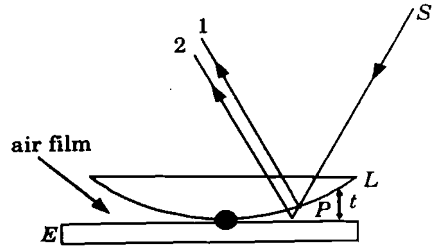


Fig. 4.11.2.

2. Now the effective path difference between (1) and (2) rays is given as :

$$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$$

where, μ = Refractive index, and

t = thickness.

3. For normal incidence, $r = 0$, therefore

$$\cos \theta = 1$$

4. Hence, $$\Delta = 2\mu t + \frac{\lambda}{2}$$

5. At point of contact (O) of the lens,

$$t = 0$$

$$\Rightarrow \quad \Delta = \frac{\lambda}{2}$$

6. This is condition of minimum intensity hence the central spot of the ring is dark.

a. Condition for Maximum Intensity (Bright Rings) :

1. If path difference, $\Delta = 2n\,\dfrac{\lambda}{2}$ [where, $n = 0, 1, 2, 3...$]
2. Hence, $$2\mu t + \frac{\lambda}{2} = 2n\,\frac{\lambda}{2} \quad \text{or} \quad 2\mu t = (2n - 1)\,\frac{\lambda}{2}$$
3. For air, $$\mu = 1 \Rightarrow 2t = (2n - 1)\,\frac{\lambda}{2}$$

b. Condition for Minimum Intensity (Dark Rings) :

1. If path difference, $\Delta = (2n + 1)\,\dfrac{\lambda}{2}$ [where, $n = 0, 1, 2, 3...$]
2. Hence, $$2\mu t + \frac{\lambda}{2} = (2n + 1)\,\frac{\lambda}{2}$$ $$2\mu t = n\lambda$$
3. For air $$\mu = 1 \Rightarrow 2t = n\lambda$$

Que 4.12. Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.

AKTU 2014-15, Marks 05

OR

What are Newton's rings ? Prove that in reflected light diameters of the bright rings are proportional to the square root of odd natural number.

Answer

A. Newton's Rings : Refer Q. 4.11, Page 4–16A, Unit-4.

B. Diameter of Rings :

1. Let R is radius of curvature of lens 'L' and 't' is thickness of air film at point 'P'.
2. From the geometrical properties of circle as shown in Fig. 4.12.1,

$$AP \times AB = AO \times AF$$

$$r \times r = t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

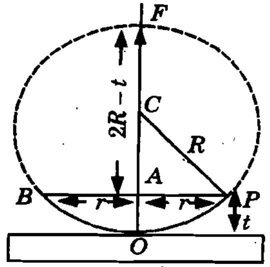


Fig. 4.12.1.

3. In actual, R is quite large and t is very small. So, t^2 is neglected. Hence, $$r^2 = 2Rt$$ $$t = \frac{r^2}{2R} \qquad ...(4.12.1)$$

a. For Bright Rings :

1. Since, we know that, $$2t = (2n - 1)\,\frac{\lambda}{2}$$ On putting the value of t from eq. (4.12.1), $$2\,\frac{r^2}{2R} = (2n - 1)\frac{\lambda}{2} \quad \text{or} \quad r^2 = (2n - 1)\,\frac{\lambda R}{2}$$
2. If radius of n^{th} bright ring is r_n, Then, $$r_n^2 = \frac{(2n-1)\lambda R}{2} \qquad ...(4.12.2)$$
3. D_n is diameter of n^{th} bright ring, $$\therefore \quad r_n^2 = \left(\frac{D_n}{2}\right)^2$$ Now, from eq. (4.12.2), $$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2}$$ or $$D_n^2 = 2(2n - 1)\,\lambda R$$ or $$D_n = \sqrt{2(2n-1)\lambda R}$$
4. Let, $$K = \sqrt{2\lambda R}$$ $$D_n = K\sqrt{2n-1} \qquad \text{[where, } n = 0, 1, 2, 3...$$ $$D_n \propto \sqrt{2n-1}$$
5. Diameter of bright ring is proportional to the square root of odd natural numbers.

b. For Dark Rings :

1. Since, we know, $2t = n\lambda$

   Substituting the value of 't' from eq. (4.12.1),

$$2\frac{r^2}{2R} = n\lambda \text{ or } r^2 = n\lambda R$$

2. If radius of n^{th} dark ring is r_n.

   Then, $r_n^2 = n\lambda R$

3. D_n is diameter of n^{th} dark ring,

$$r_n = D_n / 2$$

   then, $$\left(\frac{D_n}{2}\right)^2 = n\lambda R \quad \text{or} \quad D_n^2 = 4n\lambda R$$

$$D_n = \sqrt{4n\lambda R}$$

4. Let, $$K = \sqrt{4\lambda R}$$

   Therefore, $$D_n = K\sqrt{n} \text{ or } D_n \propto \sqrt{n}$$

5. Diameter of dark ring is proportional to the square root of natural number.

Que 4.13. Explain the formation of Newton's ring ? If in a Newton's ring experiment, the air in the interspaces is replaced by a liquid of refractive index 1.33, in what proportion would the diameter of the rings changed ?

AKTU 2015-16, Marks 10

Answer

A. **Formation of Newton's Ring :** Refer Q. 4.11, Page 4–16A, Unit-4.

B. **Numerical :**

Given : $\mu = 1.33$ (refractive index of liquid)

To Find : Proportion of change in diameter.

1. $$\frac{\text{Diameter of a ring in liquid film}}{\text{Diameter of the same ring in air film}} = \frac{1}{\sqrt{\mu}}$$

$$= \frac{1}{\sqrt{1.33}} = 0.867$$

2. So, the diameter of rings decreased by the portion of 0.867 of natural diameter.

Que 4.14. Show that the diameter D_n of the n^{th} Newton's ring, when two surface of radius R_1 and R_2 are placed in contact is given by the relation : $\frac{1}{R_1} \pm \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$.

Answer

Newton's Rings formed by two Curved Surfaces :

a. **Case I :**

1. When a planoconvex lens of radius of curvature R_1 is placed on the planoconcave lens of radius R_2.
2. Let at point A the thickness of air film is 't' and n^{th} dark ring is passing through A and its radius is r_n.
3. According to Fig. 4.14.1.

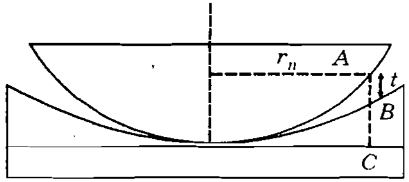


Fig. 4.14.1.

$$t = t_1 - t_2 \qquad [\because AC = t_1 \text{ and } BC = t_2]$$

$$= \frac{r_n^2}{2R_1} - \frac{r_n^2}{2R_2} \text{ or } t = \frac{r_n^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$2t = r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ...(4.14.1)$$

4. For dark ring,

$$2\mu t = n\lambda$$

$$2t = n\lambda \qquad [\because \mu = 1 \text{ for air}] \qquad ...(4.14.2)$$

5. From eq. (4.14.1) and eq. (4.14.2),

$$r_n^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = n\lambda$$

or $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{n\lambda}{r_n^2}$$

6. But, $$D_n = 2r_n$$

$$\therefore \quad \frac{1}{R_1} - \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

b. **Case II :**

1. Let both the lenses are planoconvex and their curved surface is in contact.
2. Let 't' is thickness of air film at point A and R_1 and R_2 are radius of curvature of lenses respectively as shown in Fig 4.14.2.

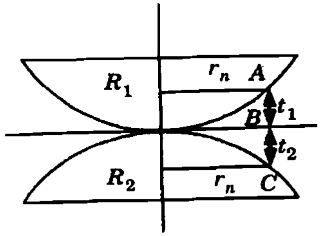


Fig. 4.14.2.

3. Since, $t = t_1 + t_2$

$$= \frac{r_n^2}{2R_1} + \frac{r_n^2}{2R_2} \Rightarrow \frac{r_n^2}{2}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

or $$2t = r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

4. For dark ring, $2t = n\lambda$

$$\therefore \quad r_n^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = n\lambda$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{n\lambda}{r_n^2},$$

5. But, $r_n = D_n/2$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

6. Hence, $$\frac{1}{R_1} \pm \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$

Que 4.15. **Describe how Newton's ring experiment can be used to determine the refractive index of a liquid ?**

Answer

1. The transparent liquid whose refractive index is to be determined is introduced between lens and glass plate.
2. Since, diameter of n^{th} dark ring is given by

$$D_n^2 = \frac{4n\lambda R}{\mu} \quad \text{...(4.15.1)}$$

3. Similarly, for $(n+p)^{th}$ dark ring,

$$D_{n+p}^2 = \frac{4(n+p)\lambda R}{\mu} \quad \text{...(4.15.2)}$$

4. On subtracting eq. (4.15.1) from eq. (4.15.2),

$$D_{n+p}^2 - D_n^2 = \frac{4p\lambda R}{\mu} \quad \text{(For liquid)} \quad \text{...(4.15.3)}$$

5. For air, $\mu = 1$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R \quad \text{...(4.15.4)}$$

6. On dividing eq. (4.15.4) by eq. (4.15.3),

$$\mu = \frac{(D_{n+p}^2 - D_n^2)_{\text{air}}}{(D_{n+p}^2 - D_n^2)_{\text{liquid}}}$$

Que 4.16. **Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15^{th} bright ring is 0.590 cm and the diameter of the 5^{th} ring is 0.336 cm, what is the wavelength of light used ?**

AKTU 2014-15, Marks 05

Answer

Given : $D_{15} = 0.590$ cm, $D_5 = 0.336$ cm, $p = 15 - 5 = 10$, $R = 100$ cm.
To Find : Wavelength of light.

1. If D_{n+p} and D_n be the diameter of $(n+p)^{th}$ and n^{th} bright ring,

then, $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4\,pR}$$

$$\therefore \quad \lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= 5.88 \times 10^{-5} \text{ cm} = 5880 \text{ Å}$$

Que 4.17. **Newton's rings are observed normally in reflected light of wavelength 6000 Å. The diameter of the 10^{th} dark ring is 0.50 cm. Find the radius of curvature of lens and thickness of the corresponding air film.**

AKTU 2013-14, Marks 05

Answer

Given : $D_n = 0.50$ cm, $\lambda = 6000$ Å $= 6.0 \times 10^{-5}$ cm, $n = 10$
To Find : i. Radius of curvature of lens
ii. Thickness of film.

1. The diameter of n^{th} dark ring is given by

$$D_n^2 = 4n\lambda R \quad \text{or} \quad R = \frac{D_n^2}{4n\lambda}$$

$$\therefore \quad R = \frac{0.50 \times 0.50}{4 \times 10 \times 6.0 \times 10^{-5}} = 104.17 \text{ cm}$$

2. If t is the thickness of the film corresponding to a ring of D_n diameter, then,

$$2t = \frac{D_n^2}{4R} \text{ or } t = \frac{D_n^2}{8R} = \frac{0.50 \times 0.50}{8 \times 104.17} = 2.99 \times 10^{-4} \text{ cm}$$

PART-2

Fraunhofer Diffraction at Single Slit and at Double Slit, Absent Spectra, Diffraction Grating, Spectra with Grating, Dispersive Power, Rayleigh's Criterion of Resolution, Resolving Power of Grating.

CONCEPT OUTLINE : PART-2

Diffraction : Diffraction of light is a phenomenon of bending of light and spreading out towards the geometrical shadow when passed through an obstruction.
Rayleigh's Criteria : The spectral lines of equal intensity are said to be resolved, if the position of the principal maxima of one spectral line coincide with first minima of the other spectral line.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 4.18. What is meant by diffraction of light ? Write name of the two classes of diffraction and explain it.

Answer

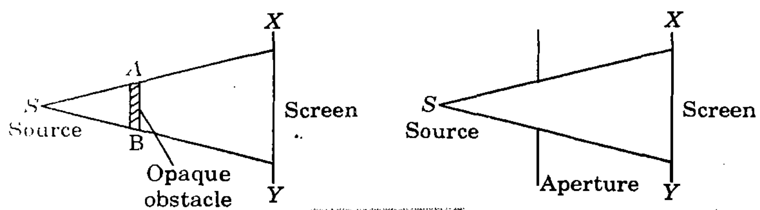


Fig. 4.18.1.

The departure of light path from true rectilinear path or the bending of light around corners of an obstacle is called diffraction of light.

2. When light passes through small apertures or by the side of small obstacles it does not follow rectilinear path strictly, but bends at round corners of the obstacles.
3. Diffraction phenomena can be divided into two types :

a. **Fresnel Diffraction :**
   1. In Fresnel diffraction, the source of light and the screen, both are placed at finite distance from the diffraction element (obstacle or apertures) in which incident wavefront is either spherical or cylindrical and no lens are used.

b. **Fraunhofer Diffraction :**
   1. In Fraunhofer diffraction, source of light and the screen both are placed at infinite distance from diffraction element in which incident wavefront is often plane.
   2. Here convex lenses are used for focusing diffracted light.

Que 4.19. Describe Fraunhofer diffraction due to a single slit and show that relative intensities of successive maxima are nearly $1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121}$ .....

OR

Discuss the phenomenon of diffraction at a single slit and show that the relative intensities of successive maxima are nearly

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} ......$$

AKTU 2014-15, Marks 05

OR

Explain the diffraction pattern obtained with diffraction at single slit. By what fraction the intensity of second maximum reduced from principal maximum ? **AKTU 2013-14, Marks 05**

Answer

A. **Fraunhofer Diffraction due to a Single Slit :**

1. The light from a monochromatic source S is converted into parallel beam of light by convex lens L_1.
2. Now this beam is incident normally on a slit AB of width 'e'.
3. Now according to Huygen's wave theory, every point in AB sends out secondary waves which are superimposed to give diffraction pattern on screen XY.
4. In this diffraction pattern, a central bright band is obtained because the rays from AB reach at C in same phase and here the intensity is maximum.
5. The rays which are directed through an angle θ are focused at point 'P'.

6. To find intensity at P let us draw normal AK.
7. Path difference of rays meeting at P is

$$BK = e \sin \theta$$

and, phase difference $= \left(\frac{2\pi}{\lambda}\right) e \sin \theta$

8. Let AB be divided into large number of equal parts. The secondary waves originating from these parts will be of equal amplitude 'a' (say).
9. Then phase difference between two successive waves will be

$$\delta = \frac{1}{n}\left(\frac{2\pi}{\lambda}\right) e \sin \theta$$

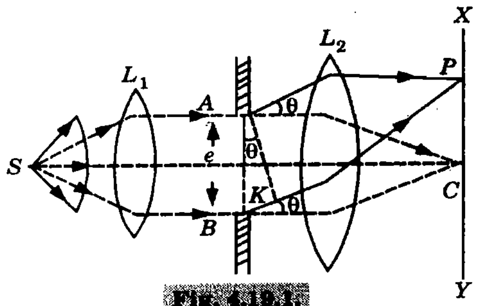


Fig. 4.19.1.

10. Now, according to n simple harmonic waves,

$$R = \frac{a \sin \frac{n\delta}{2}}{\sin \delta / 2} = \frac{a \sin\left(\frac{\pi e \sin \theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin \theta}{n\lambda}\right)}$$

11. Let $\quad n\delta = 2\alpha$

and, $\quad \alpha = \frac{\pi}{\lambda} e \sin \theta$

Then, $\quad R = \frac{a \sin \alpha}{\sin\left(\frac{\alpha}{n}\right)} \quad \left[\because \frac{\alpha}{n} \text{ is very small. So, } \sin \frac{\alpha}{n} = \frac{\alpha}{n}\right]$

or $\quad R = na \frac{\sin \alpha}{\alpha} \Rightarrow R = \frac{A_o \sin \alpha}{\alpha} \quad [\because na = A_o]$

12. Intensity at point 'P',

$$I = R^2 = \frac{A_o^2 \sin^2 \alpha}{\alpha^2} \qquad ...(4.19.1)$$

a. Position of Maxima :

1. $\frac{\sin \alpha}{\alpha} = 1$ when $\alpha \to 0$

$$\lim_{\alpha \to 0} \frac{\sin \alpha}{\alpha} = \lim_{\alpha \to 0} \frac{1}{\alpha}\left(\alpha - \frac{\alpha^3}{\underline{|3}} + \frac{\alpha^5}{\underline{|5}} - ...\right) = 1$$

2. $I = I_o(1)^2 \Rightarrow I = I_o$
3. $\alpha = \frac{\pi}{\lambda} e \sin \theta$

$$\frac{\pi}{\lambda} e \sin\theta = 0 \Rightarrow \sin\theta = 0$$
$$\theta = 0$$

4. **Point C is central maxima or principal maxima.**

b. Position of Minima :

1. If $\frac{\sin \alpha}{\alpha} = 0$

$\Rightarrow \quad \sin \alpha = 0$

$\alpha \neq 0$

$\alpha = \pm m\pi \quad$ [where, $m = 1, 2, 3....$]

2. Hence, $\quad \frac{e\pi \sin \theta}{\lambda} = \pm m\pi$

$e \sin \theta = \pm (m\lambda) \quad$ [where, $m = 1, 2, 3.....$] $\quad ...(4.19.2)$

3. **Eq. (4.19.2) gives the direction of first, second, third .... minima and this equation is called diffraction equation.**

c. Secondary Maxima :

1. **The condition of secondary maxima may be obtained by differentiating eq. (4.19.1) w.r.t. α and equating it to zero.**

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A_o^2\left(\frac{\sin \alpha}{\alpha}\right)^2\right] = A_o^2 2 \frac{\sin \alpha}{\alpha}\left[\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2}\right]$$

2. Either $\quad \frac{\sin \alpha}{\alpha} = 0 \Rightarrow \sin \alpha = 0$

or $\quad \alpha \cos \alpha - \sin \alpha = 0$

$\alpha = \tan \alpha$

3. **$\sin \alpha = 0$, gives position of principal minima and position of secondary maxima is given by**

$$\alpha = \tan \alpha \qquad ...(4.19.3)$$

4. **Eq. (4.19.3) can be solved graphically by plotting the curves**

$$y = \alpha \quad \text{and} \quad y = \tan \alpha$$

5. **According to curves, the point of intersection of these two curves gives the value of α satisfying the equation $\alpha = \tan \alpha$. These points correspond to the value of**

$$\alpha = 0, \pm \frac{3\pi}{2}, \frac{\pm 5\pi}{2}, \pm \frac{7\pi}{2} ....$$

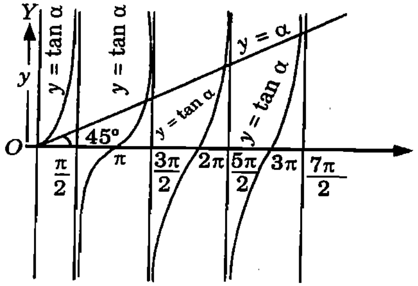


Fig. 4.19.2.

6. At $\alpha = 0$, the position of principal maxima,

$$I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_o$$

7. At $\alpha = \frac{3\pi}{2}$, the intensity of first secondary maxima,

$$I_1 = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2 \Rightarrow I_o\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{4I_o}{9\pi^2}$$

8. At $\alpha = \frac{5\pi}{2}$, the intensity of second secondary maxima,

$$I_2 = I_o\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 = \frac{4I_o}{25\pi^2}$$

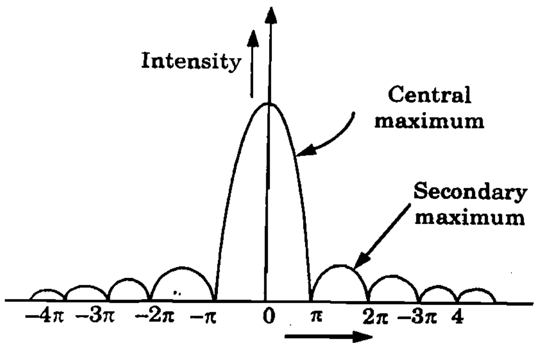


Fig. 4.19.3.

9. At $\alpha = \frac{7\pi}{2}$, $\quad I_3 = \frac{4I_o}{49\pi^2}$

10. The ratio of relative intensity as : $1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : .....$

or, $\quad 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : .....$

11. Direction of secondary maxima is given by

$$e \sin\theta = \pm (2m + 1)\frac{\lambda}{2}$$

12. But, $\quad \alpha = \frac{\pi e}{\lambda}\sin\theta \quad$ or $\quad \alpha = \frac{\pi}{\lambda}.(2m+1)\frac{\lambda}{2}$

$$\alpha = (2m + 1)\frac{\pi}{2} \quad m = 1, 2, 3, 4.....$$

Hence, $\quad \alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2} .....$

Que 4.20. A light of wavelength 6000Å falls normally on a straight slit of width 0.10 mm. Calculate the total angular width of the central maxima and also the linear width as observed on a screen placed 1 meter away.

Answer

Given : $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm, $e = 0.10$ mm $= 0.1$ cm, $D = 1$ m $= 100$ cm

To Find : i. Total angular width of central maxima.
ii. Linear width of central maxima.

1. Since, $\quad e \sin\theta = n\lambda$
2. For $n = 1$ and for small θ,

$$\theta = \frac{\lambda}{e} = \frac{6 \times 10^{-5}}{0.01} = 6 \times 10^{-3} \text{ rad.}$$

3. Total angular width, $2\theta = 2 \times 6 \times 10^{-3}$
$= 1.2 \times 10^{-2}$ rad or 0.688°
4. Linear half width $= \theta D = 6 \times 10^{-3} \times 100 = 0.6$ cm
5. Total linear width $= 2\theta D = 2 \times 0.6 = 1.2$ cm

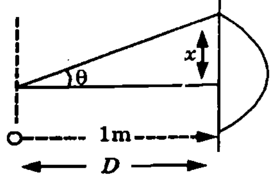


Fig. 4.20.1.

Que 4.21. **Discuss Fraunhofer diffraction at a double slit.**

Answer

1. Consider a parallel beam of monochromatic light having wavelength λ incident normally on two parallel slits AB and CD as shown in Fig. 4.21.1.
2. Width of each slit is 'e' and are separated by distance 'd'. Distance between S_1 and S_2 point is '$e + d$'.
3. Now each slit diffracts the light at an angle θ to incident direction.

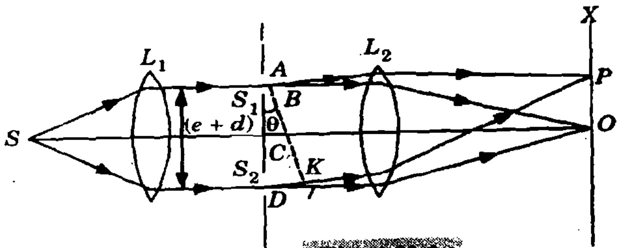


Fig. 4.21.1.

5. From the theory of diffraction due to single slit we know that, resultant amplitude is

$$R = A\frac{\sin\alpha}{\alpha} \quad \text{and} \quad \alpha = \frac{\pi}{\lambda} e \sin\theta$$

6. Let S_1 and S_2 are two coherent sources, each sending wavelets of amplitude $A\frac{\sin\alpha}{\alpha}$ in the direction of θ.
7. Therefore, the resultant amplitude due to interference of these two waves at point P can be calculated as :
   i. Draw perpendiculars S_1K on S_2K.
   ii. Path difference, $S_2K = (e + d)\sin\theta$ ...(4.21.1)
   and, phase difference,

$$\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta \qquad ...(4.21.2)$$

   iii. If R' is resultant amplitude at point P then, according to Fig. 4.21.2.

$$OB^2 = OA^2 + AB^2 + 2AB.OA\cos\delta$$
$$R'^2 = R^2 + R^2 + 2RR\cos\delta$$
$$= 2R^2 + 2R^2\cos\delta$$
$$R'^2 = 2R^2(1+\cos\delta) = 2R^2\left(2\cos^2\frac{\delta}{2}\right)$$

$$R'^2 = 4R^2\cos^2\frac{\delta}{2}$$

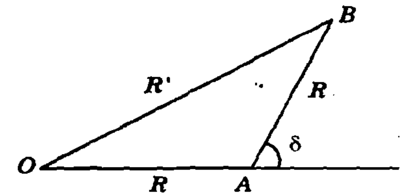


Fig. 4.21.2.

   iv. But, $R = A\frac{\sin\alpha}{\alpha}$

   then, $R'^2 = 4\frac{A^2\sin^2\alpha}{\alpha^2}\cos^2\frac{\delta}{2}$

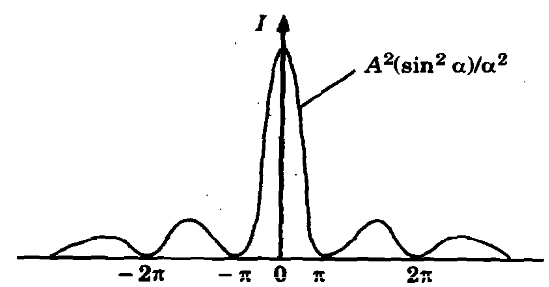
   v. Let, $\beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta$

   then, $R'^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}.\cos^2\beta$

8. Now, by definition,

$$I = R'^2 = \frac{4A^2\sin^2\alpha.\cos^2\beta}{\alpha^2}$$

9. Hence, the resultant intensity depends upon following two factors :
   i. $\frac{A^2\sin^2\alpha}{\alpha^2}$ due to diffraction, and
   ii. $\cos^2\beta$, due to interference.



(a)

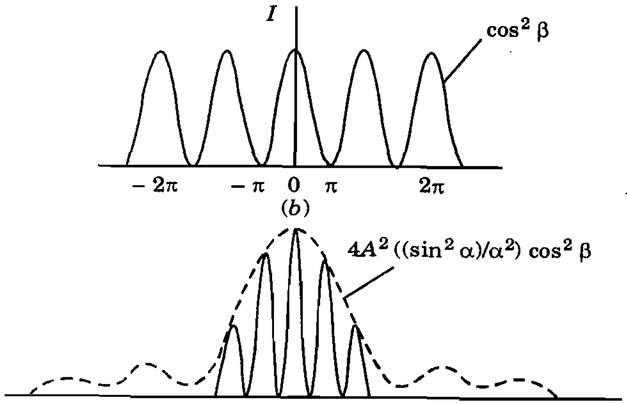


Fig. 4.21.3.

A. **Conditions for Maxima :**

If, $\cos^2 \beta = 1$ or $\beta = \pm n\pi$ [where, $n = 0, 1, 2, 3,...$]

$\therefore$ But,
$$\beta = \frac{\pi}{\lambda}(e+d)\sin\theta$$
$$\pm n\pi = \frac{\pi}{\lambda}(e+d)\sin\theta$$
$$(e + d) \sin \theta = \pm n\lambda \quad \text{[where, } n = 0, 1, 2, 3,....\text{]}$$

B. **Conditions for Minima :**

If, $\cos^2 \beta = 0$ or $\cos \beta = 0$
$$\beta = \pm (2n + 1)\frac{\pi}{2} \quad \text{[where, } n = 0, 1, 2, 3,...\text{]}$$

Then, $(e + d) \sin \theta = \pm (2n + 1)\dfrac{\lambda}{2}$

Que 4.22. **What do you mean by a diffraction grating ? Derive expression of Fraunhofer diffraction due to N slits.**

OR

Give the construction and theory of plane transmission grating. Explain the formation of spectra by it. **AKTU 2017-18, Marks 07**

Answer

1. The diffraction grating consists of a large number (N) of parallel slits having equal width and separated by an equal opaque space.

2. It is constructed by rolling a large number of parallel and equidistant lines on a glass plate with a diamond point.

A. **Explanation :**

1. Let a parallel beam of monochromatic light of wavelength 'λ' be incident on 'N' slits.
2. This light diffracted at an angle θ is focused at point P on the screen by lens L_2 having same amplitude.
$$R = A\,\frac{\sin\alpha}{\alpha}$$
3. Let e be the width of each slit and 'd' be the opaque space between two slits, then $(e + d)$ is called grating element.
4. Path difference = $(e + d) \sin \theta$
and, phase difference,
$$2\beta = \frac{2\pi}{\lambda}(e+d)\sin\theta.$$
5. Therefore, as we pass from one vibration to another, the path goes on increasing by an amount $(e + d) \sin \theta$ and phase goes on increasing by an amount $\dfrac{2\pi}{\lambda}(e+d)\sin\theta$. Thus, phase increases in arithmetic progression.
6. Now, the resultant amplitude and intensity at point P due to N slits can be obtained by vector polygon method,
$$R' = R\,\frac{\sin\dfrac{Nd}{2}}{\sin\dfrac{d}{2}}$$

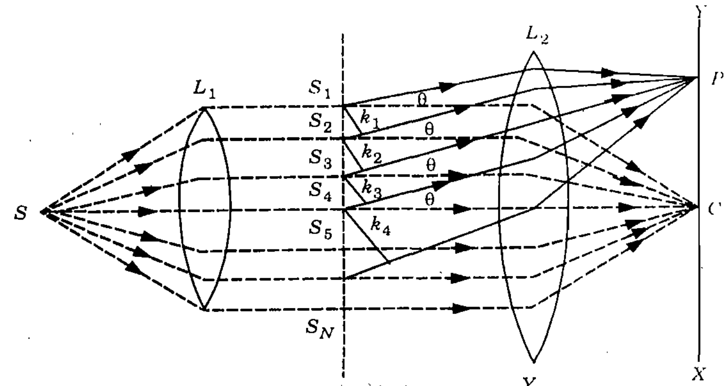


Fig. 4.22.1.

7. But, $d = 2\beta$

Hence, $$R' = \frac{R \sin \frac{2N\beta}{2}}{\sin \frac{2\beta}{2}}$$

or $$R' = R \frac{\sin N\beta}{\sin \beta} \quad ...(4.22.1)$$

where, $$R = A \frac{\sin \alpha}{\alpha} \text{ (due to single slit)}$$

8. Intensity, $$I = R'^2 = A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\sin \beta}\right)^2 \quad ...(4.22.2)$$

9. Hence, intensity distributed is product of two terms I[st] term $A^2\left(\frac{\sin \alpha}{\alpha}\right)^2$ represents diffraction pattern due to single slit and II[nd] term $\left(\frac{\sin N\beta}{\sin \beta}\right)^2$ represents interference pattern due to N slits.

a. Condition and Intensity of Principal Maxima :

1. When, $\sin \beta = 0$

$$\beta = \pm n\pi \quad \text{[where, } n = 0, 1, 2, 3...] \quad ...(4.22.3)$$

2. Then, $\sin N\beta = 0$

Hence, $\frac{\sin N\beta}{\sin \beta} = \frac{0}{0}$ is indeterminate form.

It is solved by L-Hospital rule.

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin \beta)} = \lim_{\beta \to \pm n\pi} \frac{N \cos N\beta}{\cos \beta} = N$$

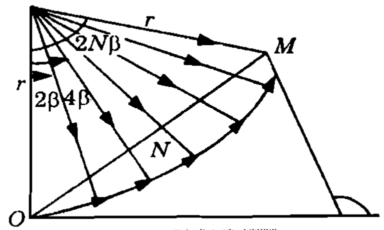


Fig. 4.22.2.

3. Putting the value of $\frac{\sin N\beta}{\sin \beta} = N$ in eq. (4.22.2), we get,

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} . N^2 \quad ...(4.22.4)$$

4. The direction of principal maxima is given by, $\sin \beta = 0$, $\beta = \pm n\pi$

5. We know, $$\beta = \frac{\pi}{\lambda}(e + d) \sin \theta$$

So, $$\pm n\pi = \frac{\pi}{\lambda}(e+d)\sin\theta \Rightarrow (e+d)\sin\theta = \pm n\lambda \quad ...(4.2$$

6. For $n = 0$ we get, $\theta = 0$ and get zero order principal maxima. $n = 1, 2, 3, ...$ correspond to I[st], II[nd], III[rd] order principal maxi

b. Condition for Minima :

1. The intensity is minimum when, $\sin N\beta = 0$
2. But, $\sin \beta \neq 0$

Hence, $$N\beta = \pm m\pi \quad \text{or} \quad \beta = \pm \frac{m\pi}{N}$$

or, $$\frac{N\pi}{\lambda}(e+d)\sin\theta = \pm m\pi \quad \text{or} \quad N(e+d)\sin\theta = \pm m\lambda \quad ...(4.2$$

where, m can take all integral values except 0, N, $2N$, $3N$ .... $m = 0$ gives maxima and $m = 1, 2, 3, ... (N - 1)$ give minima.

B. Secondary Maxima :

1. There are $(N - 1)$ minima between two consecutive principal ma therefore, there are $(N - 2)$ other maxima coming alternatively w minima between two successive principal maxima.
2. Position of secondary maxima is obtained by differentiating eq. (4. w.r.t. β and equating it to zero.

$$\frac{dI}{d\beta} = A^2 \frac{\sin^2 \alpha}{\alpha^2} 2\left[\frac{\sin N\beta}{\sin \beta}\right]\left[\frac{N \cos N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta}\right] = 0$$

or, $N \cos N\beta \sin \beta - \sin N\beta \cos \beta = 0$

and, $\tan N\beta = N \tan \beta$

3. From Fig. 4.22.3,

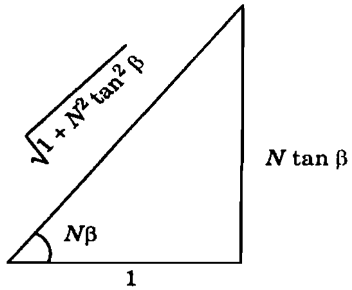


Fig. 4.22.3.

$$\sin N\beta = \frac{N \tan \beta}{\sqrt{1 + N^2 \tan^2 \beta}}$$

Squaring both sides and dividing by $\sin^2 \beta$,

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2 \tan^2 \beta}{[(1 + N^2 \tan^2 \beta)\sin^2 \beta]} = \frac{N^2}{[1 + (N^2 - 1)\sin^2 \beta]}$$

Putting the value of $\dfrac{\sin^2 N\beta}{\sin^2 \beta}$ in eq. (4.22.2),

$$I' = A_o^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{N^2}{[1+(N^2-1)\sin^2\beta]} \qquad ...(4.22.8)$$

$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{I'}{I} = \frac{1}{[1+(N^2-1)\sin^2\beta]}$$

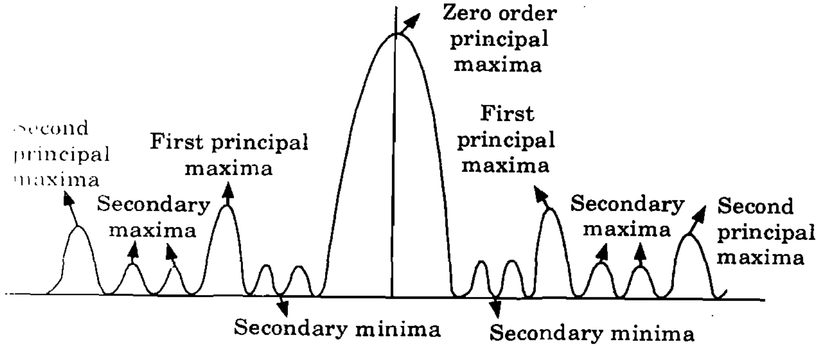


Fig. 4.22.4.

Que 4.23. **What is diffraction grating ? Show that its dispersive power can be expressed as $\dfrac{1}{\sqrt{\left(\dfrac{e+d}{n}\right)^2 - \lambda^2}}$ where all terms have usual meanings.**

AKTU 2013-14, Marks 05

Answer

Diffraction Grating : Refer Q. 4.22, Page 4–32A, Unit-4.

Proof :

Since, $(e+d)\sin\theta = n\lambda \Rightarrow \sin\theta = \dfrac{n\lambda}{(e+d)}$ ...(4.23.1)

Differentiating eq. (4.23.1) w.r.t. λ, we get,

$$(e+d)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$d\theta = \frac{nd\lambda}{(e+d)\cos\theta} = \frac{nd\lambda}{(e+d)\sqrt{1-\sin^2\theta}} \qquad ...(4.23.2)$$

Putting the valve of $\sin\theta$ from eq. (4.23.1) in eq. (4.23.2), we get

$$d\theta = \frac{nd\lambda}{(e+d)\sqrt{1-\dfrac{n^2\lambda^2}{(e+d)^2}}} = \frac{nd\lambda}{\sqrt{(e+d)^2 - n^2\lambda^2}} = \frac{d\lambda}{\sqrt{\left(\dfrac{e+d}{n}\right)^2 - \lambda^2}}$$

4. Hence, dispersive power, $\dfrac{d\theta}{d\lambda} = \dfrac{1}{\sqrt{\left(\dfrac{e+d}{n}\right)^2 - \lambda^2}}$

Que 4.24. **What do you understand by missing order spectrum ? What particular spectra would be absent if the width of transparencies and opacities of the grating are equal ? Show that only first order spectra is possible if the width of the grating element is more than wavelength of light and less than twice the wavelength of light.**

OR

What are the conditions absent spectra in the grating ?

Answer

1. Sometime for a particular angle of diffraction 'θ' satisfying the relation $(e+d)\sin\theta = n\lambda$, there is no visible spectrum obtained. This phenomenon is known as missing order spectrum.
2. We know that condition for a minima in a single slit is given by
$$e\sin\theta = m\lambda \qquad ...(4.24.1)$$
and the condition for the principal maxima in the n^{th} order spectrum is given by
$$(e+d)\sin\theta = n\lambda \qquad ...(4.24.2)$$
3. If both conditions are simultaneously satisfied, the diffracted rays from all transparencies are superimposed upon each other but the resultant intensity is zero, *i.e.* the spectrum is absent.
4. From eq. (4.24.1) and eq. (4.24.2), we have, $\dfrac{e+d}{e} = \dfrac{n}{m}$
This is the condition for absent spectra.
5. If $e = d$ then, $n = 2m$
So that 2^{nd}, 4^{th}, 6^{th}, ... order of the spectra will be missing corresponding to $m = 1, 2, 3, ...$
6. When $d = 2e$ then, $n = 3$ m
Hence, 3^{rd}, 6^{th}, 9^{th} ... spectra will be missing corresponding to $m = 1, 2, 3$ ...
7. The maximum number of spectra available with a diffraction grating in the visible region can be evaluated by using the grating equation for normal incidence as
$$(e+d)\sin\theta_n = n\lambda \quad \text{or} \quad n = \frac{(e+d)\sin\theta_n}{\lambda}$$
8. The maximum possible value of θ is 90^0.
$$n_{max} = \frac{(e+d)\sin 90}{\lambda} = \frac{(e+d)}{\lambda}$$
9. If the grating element $(e+d)$ lies between λ and 2λ or grating element $(e+d) < 2\lambda$
Then, $$n_{max} < \frac{2\lambda}{\lambda} < 2$$

10. Therefore, for normal incidence only first order will be obtained.
11. Hence, if the width of grating element is less than twice the wavelength of light, then only first order is possible.

Que 4.25. What is dispersive power of grating and resolving power of an optical instrument ? Explain Rayleigh's criterion of resolution.

Answer

A. Dispersive Power of a Diffraction Grating :

1. The dispersive power of a diffraction grating is defined as the rate of change of the diffraction angle with the wavelength. It is expressed as $\frac{d\theta}{d\lambda}$.
2. For a grating, $(e + d)\sin\theta = n\lambda$
   Differentiating w.r.t. λ, we get,
   $$(e + d)\cos\theta \frac{d\theta}{d\lambda} = n$$
   or
   $$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

B. Resolving Power of an Optical Instrument :

1. The ability of an optical instrument to produce the separate images of two objects placed very close to each other is known as resolving power.

C. Rayleigh's Criteria of Resolution :

1. According to Rayleigh's criterion, the spectral lines of equal intensity are said to be resolved, if the position of the principal maxima of one spectral line coincide with first minima of the other spectral line.

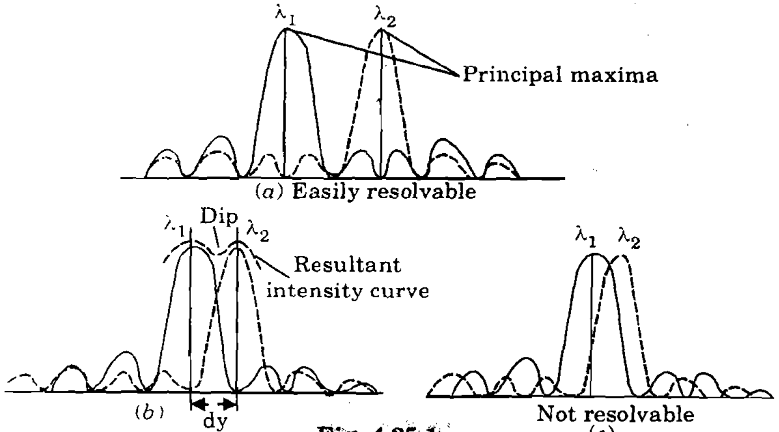


Fig. 4.25.1.

Que 4.26. What do you mean by resolving power of grating ? Deriv the necessary expression.

Answer

A. Resolving Power of Grating :

1. It is defined as the ratio of wavelength (λ) of any spectral line to the smallest difference of two wavelengths ($d\lambda$), for which the spectral line can be resolved at the wavelength λ.
   $$\text{Resolving power of grating} = \frac{\lambda}{d\lambda}$$

B. Expression :

1. Let a light consisting of two wavelengths λ_1 and λ_2 is incident normal on a grating element $(e + d)$ and the spectral lines corresponding to λ_1 and λ_2 are formed on screen P_1 and P_2.
2. These spectral lines, just resolve if they satisfy the Rayleigh's criterion the direction of n^{th} principal maxima for wavelength (λ_1) is given by,
   $$(e + d)\sin\theta = n\lambda_1$$
   $$N(e + d)\sin\theta = Nn\lambda_1 \quad ...(4.26.1)$$
3. And I^{st} minima in direction $(\theta + d\theta)$ is
   $$N(e + d)\sin(\theta + d\theta) = m\lambda_1 \quad ...(4.26.2)$$
   except ($m = 0, N, 2N$ ..... or 1, 2, 3..., $N - 1$)

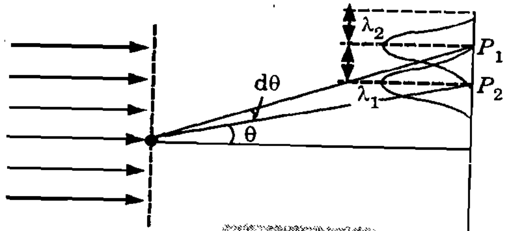


Fig. 4.26.1.

4. When, $m = (nN + 1)$, eq. (4.26.2) becomes
   $$N(e + d)\sin(\theta + d\theta) = (nN + 1)\lambda_1 \quad ...(4.26.3)$$
5. The principal maxima due to wavelength λ_2 in direction $(\theta + d\theta)$ is
   $$(e + d)\sin(\theta + d\theta) = n\lambda_2$$
   $$N(e + d)\sin(\theta + d\theta) = Nn\lambda_2 \quad ...(4.26.4)$$
6. Comparing eq. (4.26.3) and eq. (4.26.4), we get,
   $$(nN + 1)\lambda_1 = Nn\lambda_2 \quad ...(4.26.5)$$
7. If $\lambda_1 = \lambda$, $\lambda_2 = \lambda + d\lambda$, $d\lambda = \lambda_2 - \lambda_1$ eq. (4.26.5) becomes
   $$(nN + 1)\lambda = Nn(\lambda + d\lambda)$$
   or
   $$\lambda = Nnd\lambda \quad \text{or} \quad \frac{\lambda}{d\lambda} = nN$$
   But,
   $$(e + d)\sin\theta = n\lambda \quad \text{or} \quad n = \frac{(e+d)\sin\theta}{\lambda}$$
   $$\frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta}{\lambda}$$

Que 4.27. **Find the angular separation of 5048 Å and 5016 Å wavelengths in second order spectrum obtained by a plane diffraction grating having 15000 lines per cm.**

Answer

Given : $n = 2$, $\lambda_1 = 5048$ Å $= 5048 \times 10^{-10}$ m, $\lambda_2 = 5016$ Å $= 5016 \times 10^{-10}$ m,

$(e + d) = \dfrac{2.54}{15000} = 1.693 \times 10^{-4}$ cm $= 1.693 \times 10^{-6}$ m

To Find : Angular separation.

Note : The value of $(e + d)$ or diffraction grating can't be 15000 lines per cm due to this value of square root comes out to be negative, hence, considering the $(e + d)$ value as 15000 lines per inch.

1. Since, $d\lambda = \lambda_1 - \lambda_2$

$= 5048 - 5016 = 32$ Å $= 32 \times 10^{-10}$ m

2. $$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5048 + 5016}{2} = 5032 \text{ Å} = 5032 \times 10^{-10} \text{ m}$$

3. Now, angular separation is given by,

$$d\theta = \frac{d\lambda}{\sqrt{\left[\left(\frac{e+d}{n}\right)^2 - \lambda^2\right]}} = \frac{32 \times 10^{-10}}{\sqrt{\left[\left(\frac{1.696 \times 10^{-6}}{2}\right)^2 - (5032 \times 10^{-10})^2\right]}}$$

$$= \frac{32 \times 10^{-10}}{\sqrt{71.65 \times 10^{-14} - 25.32 \times 10^{-14}}} = \frac{32 \times 10^{-10}}{\sqrt{10^{-14}(71.65 - 25.32)}}$$

$$= \frac{32 \times 10^{-10}}{4.63 \times 10^{-6}} = 6.91 \times 10^{-4} \text{ rad}$$

Que 4.28. **A diffraction grating used at normal incidence gives a yellow line (λ = 6000 Å) in a certain spectral order superimposed on a blue line (λ = 4800 Å) of next higher order. If the angle of diffraction is $\sin^{-1}$ (3/4), calculate the grating element.**

AKTU 2015-16, Marks 05

Answer

Given : $\lambda_1 = 6000$ Å $= 6000 \times 10^{-8}$ cm, $\lambda_2 = 4800$ Å $= 4800 \times 10^{-8}$ cm,
$\theta = \sin^{-1}(3/4) = 48.6°$

To Find : Grating element.

1. $$\lambda_1 - \lambda_2 = (6000 - 4800) \times 10^{-8}$$

$$= 1200 \times 10^{-8} \text{ cm}$$

2. Since grating element,

$$e + d = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2) \sin \theta}$$

$$e + d = \frac{6000 \times 10^{-8} \times 4800 \times 10^{-8}}{1200 \times 10^{-8} \times 0.75}$$

$$= 3.2 \times 10^{-4} \text{ cm.}$$

Que 4.29. **A diffraction grating used at normal incidence gives a yellow line (λ = 6000 Å) in a certain spectral order superimposed on a blue line (λ = 4800 Å) of next higher order. If the angle of diffraction is 60°, calculate the grating element.**

AKTU 2013-14, Marks 05

Answer

Same as Q. 4.28, Page 4–40A, Unit-4. (**Ans.** 2.77×10^{-4} cm)

Que 4.30. **Find out if a diffraction grating will resolve the lines 8037.20 Å and 8037.50 Å in the second order given that the grating is just able to resolve two lines of wavelengths 5140.34 Å and 5140.85 Å in the first order.**

AKTU 2014-15, Marks 05

Answer

Given : Wavelengths of two lines in first order = 5140.34 Å and 5140.85 Å, wavelengths of two lines in second order = 8037.20 Å and 8037.50 Å

To Find : Ability to resolve by a diffraction grating.

1. The resolving power of a grating is given by $\dfrac{\lambda}{d\lambda} = nN$

Therefore, $$N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$$

where, $$\lambda = \frac{5140.34 + 5140.85}{2} = 5140.595 \text{ Å}$$

$d\lambda = 5140.85 - 5140.34 = 0.51$ Å and $n = 1$

$$\therefore \quad N = \frac{1}{1}\left(\frac{5140.595}{0.51}\right) = 10080$$

2. Hence, the resolving power of a grating in second order

$$\lambda/d\lambda = nN = 2 \times 10080 = 20160.$$

3. The resolving power required to resolve the lines 8037.20 Å and 8037.50 Å in the second order is $\lambda / d\lambda$

In this case $\lambda' = \dfrac{8037.20 + 8037.50}{2} = 8037.35$ Å

$d\lambda' = 8037.50 - 8037.20 = 0.30$

Resolving power, $\dfrac{\lambda'}{d\lambda'} = \dfrac{8037.35}{0.30} = 26791.17$

4. Thus, the grating will not be able to resolve the lines 8037.20 Å and 8037.50 Å in the second order because the required resolving power (26791.17) is greater than the actual resolving power (20160).

Que 4.31. **A diffraction grating used at normal incidence gives a green line (5400 Å) in a certain order *n* superimposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°, find the value of *n*, also find how many lines per cm there in the grating.**

Answer

Given : $\lambda_1 = 5400$ Å $= 5400 \times 10^{-8}$ cm, $\lambda_2 = 4050 \times 10^{-8}$ cm, $\theta = 30°$,
$\lambda_1 - \lambda_2 = 1350 \times 10^{-8}$ cm

To Find : i. Order of spectrum, *n*.
ii. Grating lines per cm.

1. The direction of principal maxima for normal incidence is given by

$$(e + d) \sin \theta = n\lambda$$

2. Let n^{th} order maxima of λ_1, coincide with $(n + 1)^{\text{th}}$ order maxima of λ_2, we have,

$$(e + d) \sin \theta = n\lambda_1 = (n + 1)\lambda_2$$

or $$n\lambda_1 = (n + 1)\lambda_2 \quad \text{or} \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$n = \frac{4050 \times 10^{-8}}{1350 \times 10^{-8}} = 3$$

3. Now, $(e + d) \sin \theta = \dfrac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$ or $e + d = \dfrac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2) \sin \theta}$

$$e + d = \frac{5400 \times 10^{-8} \times 4050 \times 10^{-8}}{1350 \times 10^{-8} \times \sin 30°}$$

4. Number of lines per cm,

$$N = \frac{1}{e + d} = \frac{1350 \times 10^{8}}{5400 \times 4050 \times 2} = 3086$$

☺☺☺

5 UNIT

Fiber Optics and Laser

PART-1

Introduction to Fibre Optics, Acceptance Angle, Numerical Aperture, Normalized Frequency, Classification of Fibre, Attenuation and Dispersion in Optical Fibre.

CONCEPT OUTLINE : PART-1

Optical Fibre : Optical fibre consists of a core surrounded by a cladding and a sheath. It is a thin, transparent and flexible strand. It is made up of glass or plastic. It works on the principle of total internal reflection.
Acceptance Angle : It is defined as the maximum angle that a light ray can have relative to the axis of the fibre and propagates down the fibre.
Numerical Aperture : It is a dimensionless number that characterizes the range of angles over which the fibre can accept or emit light.
Dispersion : The amplitude of the optical signal propagating in an optical fibre attenuates due to losses in fibres as well as it spreads during its propagation. Thus, the output signal received at the end becomes wider compared to the input signal. This type of distortion arises due to dispersion effect in optical fibres.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.1. **What is optical fibre ?**

Answer

1. Optical fibre is a long, thin transparent dielectric material made up of glass or plastic, which carries electromagnetic waves of optical frequencies (visible to infrared) from one end of the fibre to the other by means of multiple total internal reflection.
2. Optical fibres work as wave guides in optical communication systems.
3. An optical fibre consists of an inner cylindrical material made up of glass or plastic called core.
4. The core is surrounded by a cylindrical shell of glass or plastic called the cladding.
5. The refractive index of core (n_1) is slightly larger than the refractive index of cladding (n_2), (*i.e.*, $n_1 > n_2$).

6. The cladding is enclosed in a polyurethane jacket as shown in Fig. 5.1.1. This layer protects the fibre from the surrounding atmosphere.
7. Many fibres are grouped to form a cable. A cable may contain one to several hundred such fibres.

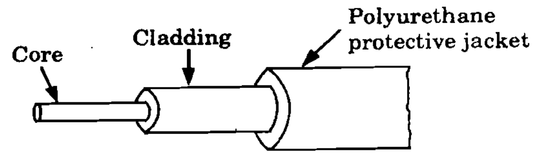


Fig. 5.1.1.

Que 5.2. **Explain the principle of optical fibre.**

Answer

1. The working of optical fibre is based on the principle of total internal reflection.
2. Total internal reflection is the phenomenon in which a light ray reflects completely in the first medium, when it is incident on the boundary of two different mediums.
3. When a light ray is incident on a high to low refractive index interface, then from Snell's law, Fig. 5.2.1(*a*).

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1} \quad ...(5.2.1)$$

where, n_1 and n_2 = Refractive indices of denser and rarer mediums respectively.

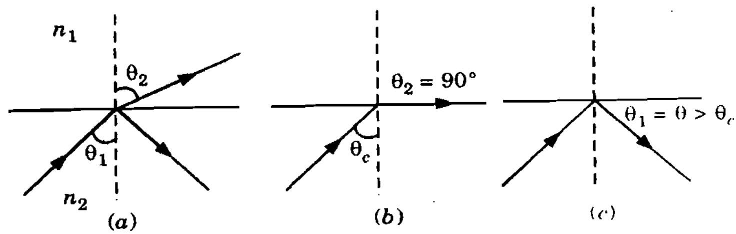


Fig. 5.2.1. Principle of optical fibre.

4. Since $n_1 > n_2$, so from eq. (5.2.1), we have

$$\frac{\sin \theta_1}{\sin \theta_2} < 1$$

i.e., $\sin \theta_1 < \sin \theta_2$

i.e., $\theta_1 < \theta_2$ ...(5.2.2)

5. When the refracted light ray emerges along the interface, *i.e.*, when the angle of refraction becomes 90°, *i.e.* $\theta_2 = 90°$, the corresponding value of the angle of incidence is called the critical angle and denoted by θ_c, Fig. 5.2.1(*b*).
6. Thus for $\theta_1 = \theta_c$, $\theta_2 = 90°$, eq. (5.2.1) becomes

$$\sin\theta_c = \frac{n_2}{n_1} \quad ...(5.2.3)$$

7. If $\theta_1 = \theta > \theta_c$, the incident light ray is totally reflected back into the originating medium with high reflection efficiency 99.9 %. This event is known as total internal reflection, Fig. 5.2.1(*c*).
8. The necessary conditions for total internal reflection of a light ray in an optical fibre are therefore as follows :
   i. The refractive index of fibre core should be higher than that of the cladding.
   ii. The light ray should be incident between the core-cladding interface and the normal to the core-cladding interface at an angle greater than the critical angle.
   iii. The respective refractive indices n_1 and n_2 of core and cladding materials of the fibre should be related to the critical angle by the relation given in eq. (5.2.3).

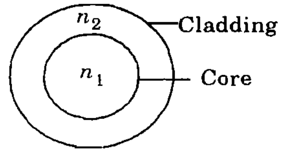


Fig. 5.2.2.

Que 5.3. What do you mean by acceptance angle and numerical aperture ? Obtain the expression for acceptance angle and numerical aperture.

Answer

A. Acceptance Angle : It is defined as the maximum angle of incidence at the end face of an optical fibre for which the ray can be propagated in the optical fibre. This angle is also called acceptance cone half-angle.

B. Numerical Aperture :

1. It represents the light-gathering capacity of an optical fibre.
2. Light-gathering capacity is proportional to the acceptance angle, θ_0.
3. So, numerical aperture can be the sine of acceptance angle of the fibre *i.e.*, $\sin\theta_0$.

C. Expression for Acceptance Angle :

1. Applying Snell's law at points *B* in Fig. 5.3.1.

$$n_1 \sin(90° - \theta_1) = n_2 \sin 90°$$
$$n_1 \cos\theta_1 = n_2$$
$$\cos\theta_1 = \frac{n_2}{n_1}$$

or
$$\sin\theta_1 = \sqrt{1-\cos^2\theta_1} = \sqrt{1-\frac{n_2^2}{n_1^2}} \quad ...(5.3.1)$$

2. Snell's law at *O*,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

or
$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \quad ...(5.3.2)$$

3. On substituting eq. (5.3.1) in eq. (5.3.2),

$$\sin\theta_0 = \frac{n_1}{n_0}\sqrt{1-\frac{n_2^2}{n_1^2}} = \frac{\sqrt{n_1^2-n_2^2}}{n_0} \quad ...(5.3.3)$$

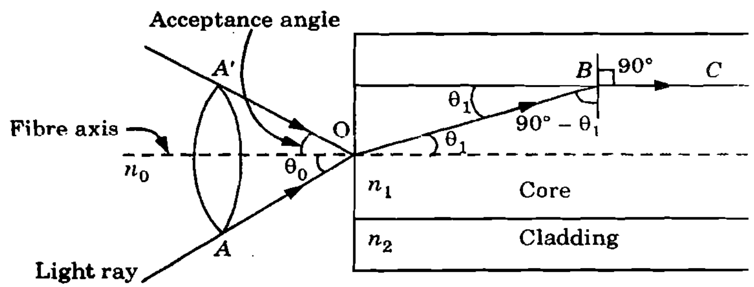


Fig. 5.3.1. Light propagation in an optical fibre.

4. As the fibre is in air. So, the refractive index $n_0 = 1$
   The eq. (5.3.3) becomes,

$$\sin\theta_0 = \sqrt{n_1^2 - n_2^2} \quad ...(5.3.4)$$

This is the equation for acceptance angle.

D. Expression for Numerical Aperture (NA) :

1. According to the definition for numerical aperture (NA),

$$NA = \sin\theta_0 = \sqrt{n_1^2 - n_2^2} \quad ...(5.3.5)$$

2. Let the fractional change in the refractive index (Δ) be the ratio between the difference in refractive indices of core and cladding to the refractive index of core.

i.e., $$\Delta = \frac{n_1 - n_2}{n_1} \quad ...(5.3.6)$$

or $$n_1 - n_2 = \Delta n_1 \quad ...(5.3.7)$$

3. Eq. (5.3.5) can be written as :

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 - n_2)(n_1 + n_2)} \quad ...(5.3.8)$$

4. Substituting eq. (5.3.7) in eq. (5.3.8), we get

$$NA = \sqrt{\Delta n_1 (n_1 + n_2)}$$

5. Since, $n_1 = n_2$; so, $n_1 + n_2 \approx 2n_1$

$\therefore$ $$NA = \sqrt{2\Delta n_1^2} = n_1 \sqrt{2\Delta} \quad ...(5.3.9)$$

6. Numerical aperture can be increased by increasing 'Δ' and thus enhances the light-gathering capacity of the fibre.
7. We cannot increase Δ to a very large value because it leads to intermodal dispersion, which cause signal distortion.

Que 5.4. **Write the classification of optical fibres.**

OR

Discuss the different types of optical fibre. Why graded index fibre is better than multimode step index fibre. **AKTU 2014-15, Marks 05**

Answer

A. Classification of Optical Fibres :

i. Classification of Optical Fibre Depending on Material used :

a. Glass Fibres :

1. These fibres consist of glass as the core and also glass as the cladding.
2. These are the most widely used fibres.
3. To reduce the refractive index of cladding, impurities such as Germanium, Boron, Phosphorous or Fluoride are added to the pure glass.

b. Plastic Clad Silica or P.C.S. Fibres :

1. By replacing the cladding with a plastic coating of the refractive index lower than that of core, a plastic clad fibre is achieved.
2. Its advantage is only that the replacement of the glass cladding with plastic offers the saving in cost.
3. The limitations are :
   i. Losses are more than the glass fibres.
   ii. Refractive index varies with temperature.
   iii. Fibre life is small, mainly in humid environment.

c. Plastic Fibres :

1. These fibres consist of both core and cladding of the plastic material.
2. These fibres are cheaper in comparison to the above fibres.
3. But these fibres have high losses and low bandwidth.
4. Also life of these fibres is small and refractive index varies with temperature.
5. These fibres don't need protective coating and they are more flexible.
6. Attenuation of plastic fibres is more than glass or silica fibres but even then they are frequently used for short distance computer applications.

ii. Classification of Optical Fibres Depending on Number of Modes :

a. Monomode or Single Mode Fibre :

1. In this, fibre is capable of transmitting only one mode.
2. Suppose we make the core of the fibre for any small ray of order of 2 to 8 μm, then only one ray of light can enter the core and get guided by total internal reflection.
3. Major advantage of single mode fibre is that it exhibits minimum dispersion loss and hence, the highest transmission bandwidth.
4. Only high-quality laser sources that produce a very focused beam of nearly monochromatic light can be used for single-mode operation.
5. Because of the superior transmission characteristics, such fibres are extensively used for long-distance applications.

b. Multimode Fibres :

1. In this, the fibre is capable of transmitting more than one mode, so the name multimode fibre.
2. The multimode fibre has the core diameter of the order of 50 μm *i.e.*, larger than the monomode fibre.
3. As the core radius is large enough, it accommodates many different rays of light or modes, each entering the core at different angles.
4. Since the different mode have different groups velocities, there exists considerable broadening of transmitted light pulses.
5. Hence, dispersion losses are more and bandwidth length product is small of order of 1 GHz-km.
6. These fibres are useful for moderate distances.
7. The loss of information capacity however is compensated by certain benefits of multimode fibres over monomode fibre such as :
   i. Incoherent optical source can be used in multimode fibre due to large core diameter and large acceptance angle.
   ii. Ease of splicing or joining.
   iii. Lower tolerance requirements on fibre connectors.

iii. Classification of Optical Fibres Depending on the Index Profile :

a. Multimode Step Index Fibre (MMSIF) :

1. It consists of a core material surrounded by a concentric layer of cladding material with a uniform index of refraction n_2 that is only slightly less than that of core of refractive index n_1.
2. If the refractive index is plotted against the radial distance from the core, the refractive index abruptly changes at the core-cladding surface creating a step, hence the name step index.

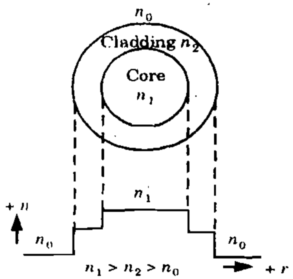


Fig. 5.4.1.

3. The name step index is due to this index profile and the term multimode is due to its feature of propagating a number of modes.
4. Its manufacturing is such that its core radius is large enough to accommodate many different rays of light or mode each entering the core at different angles.

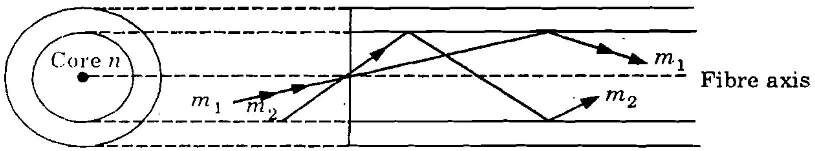


Fig. 5.4.2. Propagation in multimode step index fibre.

b. Multimode Graded Index Fibre (MMGIF) :

1. In this, the material in the core is modified so that the refractive index profile does not exhibit step index change but a parabolic refractive index profile which is maximum at the fibre axis.
2. In this fibre, index of refraction has a maximum value n_1 at the axis and lesser values falling off gradually and hence the name graded index is given to this fibre.
3. Since the light travels faster in a medium with lower refractive index, the light ray, which is farther from the fibre axis travels faster than the ray which is nearer to the axis.
4. As the refractive index is continuously changing across the fibre axis, the light ray is bent towards the fibre axis in almost sinusoidal fashion.
5. Light rays are curved towards the fibre axis by refraction.

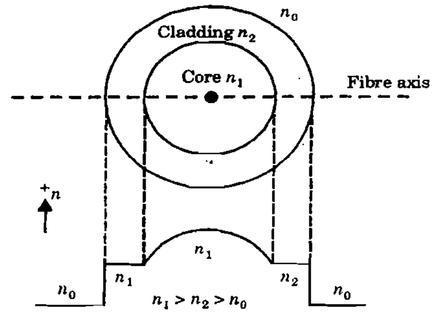


Fig. 5.4.3.

6. Light rays periodically diverge and converge along the length of fibre.

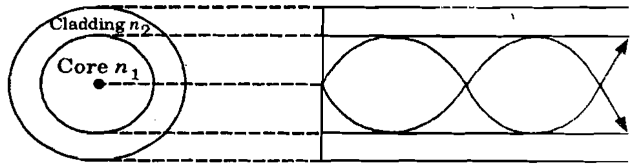


Fig. 5.4.4. Propagation in a multimode graded index fibre.

c. Single Mode Step Index Fibre (SMSIF) :

1. In this fibre, the core of a fibre is made so small that only one light can enter the core and get guided by the total internal re hence the name single mode.
2. This will be the only ray of light or mode that can enter the such a shallow angle.

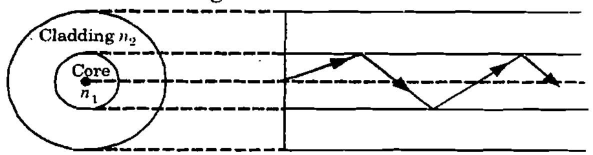


Fig. 5.4.5. Propagation in a single mode step index fibre.

3. Major advantage of this fibre is that modal dispersion is eliminated and because of this, such fibres are extensively u long distance communication.
4. Different fibre designs have a specific wavelength called cu wavelength above which it carries only one mode.
5. Single mode step index fibre has a superior transmission over other fibre types of the above because of the absence of dispersion.

B. Graded Index Fibre is Better than Multimode Step Index Fibre : In graded index fibre, the index of refraction in the core decreases continuously while in multimode step index fibres the refractive index of a core has a constant value. Therefore graded index fibre is better than multimode step index fibre.

Que 5.5. What are advantages of optical fibre over copper wire ?

Answer

1. The information carrying capacity of a fibre is much greater than the microwave radio system.
2. Attenuation in optical fibre is much lower than that of coaxial cable or twisted pair.
3. Smaller in size and lighter in weight.
4. The life of fibre is longer than corresponding copper wire.
5. Fibre communication system is more reliable as it can better withstand environmental conditions.
6. The cost per channel is lower than that of metal counterpart.
7. Handling and installation cost of optical fibre system is very nominal.

Que 5.6. Differentiate between single mode fibres and multimode fibres.

Answer

S. No.	Single Mode Fibres	Multimode Fibres
1.	In single mode fibres there is only one path for ray propagation.	In multimode fibres, large numbers of paths are available for light ray propagation.
2.	Single mode step index fibres have less core diameter (< 10 μm) and the difference between the refractive indices of core and cladding is very small.	Multimode step index fibres have larger core diameter (50 to 200 μm) and the difference between the refractive indices of core and cladding is large.
3.	In single mode fibres, there is no dispersion.	There is signal distortion and dispersion takes place in multimode fibres.
4.	Signal transmission capacity is less but the single mode fibres are suitable for long distance communication.	Signal transmission capacity is more in multimode fibres. Because of large dispersion and attenuation, they are less suitable for long distance transmission.
5.	Launching of light into single mode fibres is difficult.	Launching of light into multi mode fibres is easy.
6.	Fabrication cost is very high.	Fabrication cost is less.

Que 5.7. If refractive indices of core and cladding of an optical fibre are 1.50 and 1.45 respectively determine the values of numerical aperture, acceptance angle and critical angle of the fibre.

AKTU 2014-15, Marks 05

Answer

Given : $n_1 = 1.50$, $n_2 = 1.45$

To Find :
i. Numerical aperture.
ii. Acceptance angle.
iii. Critical angle of fibre.

1. Numerical aperture,

$$\text{NA} = n_1\sqrt{(2\Delta)}$$

where,
$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.50 - 1.45}{1.50} = 0.033$$

So,
$$\text{NA} = 1.50\sqrt{(2 \times 0.033)} = 1.50 \times 0.257 = 0.385$$

2. Acceptance angle, $\theta_0 = \sin^{-1}(\text{NA}) = \sin^{-1}(0.385) = 22.64°$
3. According to Snell's law,

$$\sin\theta_c = \frac{n_2}{n_1} \quad \text{or} \quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.45}{1.50}\right)$$
$$= 75.16°$$

Que 5.8. A step index fibre has core and cladding refractive indices 1.466 and 1.460 respectively. If the wavelength of light 0.85 μm is propagated through the fibre of core diameter 50 μm, find the normalized frequency and the number of mode supported by the fibre.

Answer

Given : $n_1 = 1.466$, $n_2 = 1.460$, $\lambda = 0.85$ μm, $a = \frac{d}{2} = \frac{50}{2} = 25$ μm

To Find :
i. Normalized frequency.
ii. Number of mode.

1. Normalized frequency is given by,

$$v = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$
$$= \frac{2 \times \pi \times 25}{0.85}\sqrt{(1.466)^2 - (1.460)^2} = 24.48 \text{ Hz}$$

2. Number of guided modes,

$$N = \frac{v^2}{2} = \frac{(24.48)^2}{2} = 299.635 \approx 300$$

Que 5.9. Describe the basic principle of communication of wave in optical fibre. A step index fibre has core refractive index 1.468, cladding refractive index 1.462. Compute the maximum radius allowed for a fibre, if it supported only one mode at a wavelength 1300 nm. **ARTU 2015-16, Marks 10**

Answer

A. **Basic principle of c/0ommunication of wave in optical fibre :** Refer Q. 5.2, Page 5–3A, Unit-5.

B. **Numerical :**

Given : $N = 1$, $\lambda = 1300$ nm, $n_1 = 1.468$, $n_2 = 1.462$
To Find : Maximum radius allowed for a fibre.

1. Number of modes supported, $N = \frac{v^2}{2}$

$$1 = \frac{v^2}{2}$$

$$v = 1.414$$

2. Let, a is radius allowed for a fibre.

$$\therefore \quad v = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2}$$

$$1.414 = \frac{2\pi \times a}{1300 \times 10^{-9}}\sqrt{(1.468)^2 - (1.462)^2}$$

$$292.56 \times 10^{-9} = a\sqrt{(1.468)^2 - (1.462)^2}$$

$$a = 2.2 \times 10^{-6}\,\text{m} = 2.2\,\mu\text{m}$$

Que 5.10. Discuss the different types of losses in optical fibre.

Answer

A. **Absorption Losses :**

1. Absorption is the most prominent factor causing the attenuation in optical fibre.
2. The absorption of light is caused by the following three different mechanisms :

   i. **Intrinsic Absorption :**

      1. It is the absorption of light by the material of the core itself.

      2. The intrinsic absorption is a material property of glass itself.
      3. There is a tendency of the fibre material to absorb a sm... amount of light energy.

   ii. **Extrinsic Absorption :**

      1. The presence of impurities in the fibre material is a main source of loss in practical fibres. This is known as extrinsic absorption.

   iii. **Absorption by Atomic Defects :**

      1. Atomic defects in the fibre material are also responsible for the loss of light energy.
      2. The atomic defects are created in the manufacture of the fib...
      3. These defects are also created when the fibre is exposed to rays, γ-rays, neutrons and electron beams.

B. **Scattering :**

1. It is the loss of optical energy due to imperfections in the fibre.
2. Due to this phenomenon, the light is scattered in all directions wh... causes the loss of the optical power in the forward direction.
3. This loss is known as Rayleigh scattering loss.
4. Rayleigh scattering loss is found to be inversely proportional to the fourth power of the light wavelength.

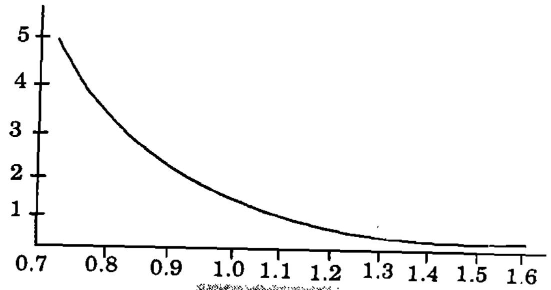


Fig. 5.10.2.

C. **Bending Losses :**

1. The bending losses occur due to bends present in the fibre structure.
2. These are of two types :

   i. **Micro Bending :**

      1. Micro bending losses are caused either during the manufacturing or during the cabling process.
      2. Microbends may not be visible with the naked eyes.
      3. During the manufacturing the microscopic bending of the c... of the fibre occurs due to thermal contraction between the core and cladding.

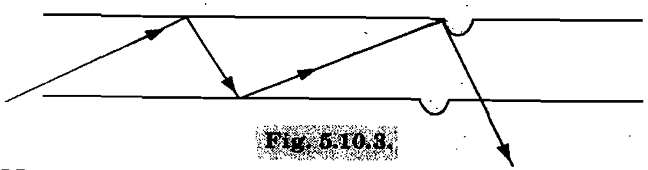


Fig. 5.10.3.

i. **Macro Bending :**

1. Excessive bending of the cable or fibre may result in loss known as macrobend loss.
2. The fibre is sharply bent so that light traveling down the fibre can't make turn and is lost in the cladding.

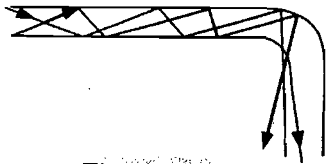


Fig. 5.10.4.

Que 5.11. Define attenuation. Explain attenuation constant.

Answer

A. **Attenuation :**

1. It is defined as the reduction in amplitude (or power) and intensity of signal as it is guided through optical fibre.
2. It is mainly due to absorption and scattering.

B. **Attenuation Constant :**

If P_i is the optical power launched at the input end of the fibre, then the power P_0 at a distance L down the fibre is given by

$$P_0 = P_i\, e^{-\alpha L} \qquad ...(5.11.1)$$

where, α = Attenuation constant.

Taking logarithms on both the sides of eq. (5.11.1),

$$\alpha = \frac{1}{L} \ln \frac{P_i}{P_0} \qquad ...(5.11.2)$$

In units of dB/km, α is defined through the equation

$$\alpha_{dB/km} = \frac{10}{L} \log \frac{P_i}{P_0}$$

In case of an ideal fibre, $P_0 = P_i$ and the attenuation would be zero.

Que 5.12. Write a short note on dispersion.

Answer

1. Dispersion is the time distortion of an optical signal that results from the time of flight differences of different component of that signal, typically resulting in the pulse broadening.
2. In digital transmission, dispersion limits the maximum data rate, the maximum distance, or the information carrying capacity of a single-mode fibre link.
3. In analog transmission, dispersion can cause a waveform to become significantly distorted and can result in unacceptable levels of composite second order distortion (CSO).
4. When no overlapping of light pulses takes place, the digital bit rate BT must be less than the reciprocal of the broadened (through dispersion) pulse duration (2τ).
5. Dispersion in optical fibres can be classified into three main types :
   i. Material dispersion,
   ii. Waveguide dispersion, and
   iii. Modal dispersion.

Que 5.13. A communication system uses a 10 km fibre having a loss of 2.5 dB/km. Compute the output power if the input power is 500 μW.

Answer

Given : L = 10 km, α = 2.5 dB/km, $P_i = 500 \times 10^{-6}$ W
To Find : Output power.

1. We know that loss in fibre,

$$\alpha_{dB/km} = \frac{10}{L} \log_{10} \frac{P_i}{P_o}$$

$$2.5 = \frac{10}{10} \log_{10} \frac{500}{P_o} \times 10^{-6}$$

$$(10)^{2.5} = \frac{500 \times 10^{-6}}{P_o}$$

$$\Rightarrow \quad P_o = \frac{500 \times 10^{-6}}{(10)^{2.5}} = 1.58\ \mu\text{W}$$

Que 5.14. Explain dispersion and attenuation in optical fibre. The optical power, after propagating through a 500 m long fibre, reduced to 25 % of its original value. Calculate fibre loss in dB/km.

AKTU 2017-18, Marks 07

Answer

A. **Dispersion :** Refer Q. 5.12, Page 5–14A, Unit-5.
B. **Attenuation :** Refer Q. 5.11, Page 5–14A, Unit-5.
C. **Numerical :**

Given : L = 500 m = 0.5 km, P_0 = 75 % of P_i
To Find : Fibre loss in dB/km.

1. The attenuation is given as,

$$\alpha = \frac{10}{L}\log_{10}\left(\frac{P_i}{P_0}\right)$$

$$\alpha = \frac{10}{0.5}\log_{10}\left[\frac{P_i}{0.75\,P_i}\right]$$

$$\alpha = 20 \times 0.1249$$

$$\alpha = 2.498 \text{ dB/km} \approx 2.5 \text{ dB/km}$$

PART-2

Laser : Absorption of Radiation, Spontaneous and Stimulated Emission of Radiation, Einstein's Coefficient, Population Inversion, Various Levels of Laser, Ruby Laser, He-Ne Laser, Laser Application.

CONCEPT OUTLINE : PART-2

Laser : It is an acronym for Light Amplification by Stimulated Emission of Radiation.

Spontaneous Emission : It takes place when excited atoms make transition to lower energy level without any external stimulation.

Stimulated Emission : It takes place when a photon of energy ($h\nu = E_2 - E_1$) stimulates an excited atom to make transition to lower energy level.

Einstein's Coefficients : $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$

Population Inversion : The phenomenon is which the number of atoms in the higher energy state becomes comparatively greater than the number of atoms in the lower energy state is known as population inversion.

Questions-Answers

Long Answer Type and Medium Answer Type Questions

Que 5.15. **Explain LASER and different types of process of radiations.**

Answer

A. **LASER :**
1. LASER stands for "Light Amplification by Stimulated Emission of Radiation".
2. It is a device used to produce a strong, monochromatic, collimated and highly coherent beam of light and it depends on the phenomenon of "stimulated emission".

B. **Processes of Radiation :**

a. **Absorption of Radiation :**
1. When an atom is in its ground state and a photon of energy $h\nu$ is incident over it, it comes to its excited state after absorbing that photon. This process is known as absorption of radiation.

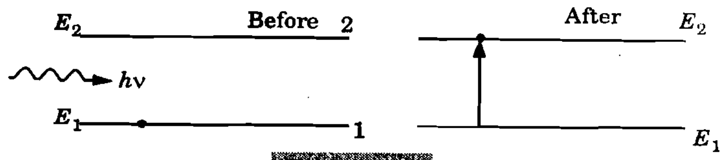


Fig. 5.15.1.

2. The probability of absorption of radiation is given by :

$$P_{12} = B_{12}\,\rho(\nu)$$

where, B_{12} = Einstein's coefficient of absorption of radiation and
$\rho(\nu)$ = Energy density.

b. **Spontaneous Emission of Radiation :**
1. When an atom is in its excited state, it can remain there only for 10^{-8} sec. After that it comes to its ground state and releases a photon of energy $h\nu$. This process is called spontaneous emission of radiation.

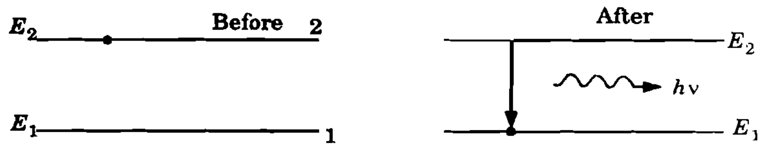


Fig. 5.15.2.

2. The probability of spontaneous emission of radiation is given by :

$$(P_{21})_{\text{spontaneous}} = A_{21}$$

where, A_{21} = Einstein's coefficient of spontaneous emission of radiation.

c. **Stimulated Emission (Induced Emission) of Radiation :**
1. When an atom is in its excited state and a photon of energy $h\nu$ is incident over it, atom comes to its ground state.

2. But now instead of one, two photons of energy $h\nu$ each are released.
3. When these two photons of energy $h\nu$ each are incident on another two excited state atoms, four photons of energy $h\nu$ each are released.

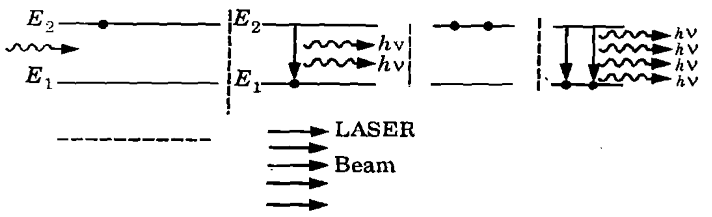


Fig. 5.15.3.

4. This process goes on continuously and as a result, a monochromatic, unidirectional beam of photon is released, which is known as stimulated emission of radiation.
5. The probability of stimulated emission of radiation is given by :

$$(P_{21})_{\text{stimulated}} = B_{21}\rho(\nu)$$

where, B_{21} = Einstein's coefficient of stimulated emission of radiation, and

$\rho(\nu)$ = Energy density.

Que 5.16. **Differentiate between spontaneous emission and stimulated emission.**

Answer

S. No.	Spontaneous Emission	Stimulated Emission
1.	It is a natural transition in which an atom is de-excited after the end of its life-time in the higher energy level.	It is an artificial transition which occurs due to de-excitation of an atom before the end of its life-time in the higher energy level.
2.	The photon emitted due to spontaneous emission can move in any direction.	The photon emitted due to stimulated emission can move only in the direction of the incident photon.
3.	The probability of spontaneous emission depends only on the properties of the two energy levels between which the transition occurs.	The probability of stimulated emission depends on the properties of the two energy levels involved in the transition as well as on the energy density of incident radiation.

Que 5.17. **Discuss necessary condition to achieve laser action.**

Answer

1. There are three conditions to achieve laser action as follows :
   i. The number of atoms in higher energy state must be greater than that in lower energy state so that the rate of emission becomes greater than the rate of absorption.
   ii. The radiation must be coherent so the probability of spontaneous emission should be negligible in comparison to the probability of stimulated emission.
   iii. The coherent beam of light must be sufficiently amplified.

Que 5.18. **What are Einstein's coefficients A and B ? Establish a relation between them.**

Answer

A. Einstein's Coefficients A and B :

1. The probability that an absorption transition occurs is given by

$$P_{12} = B_{12}\,\rho(\nu) \quad ...(5.18.1)$$

where, B_{12} = Constant of proportionality known as the Einstein's coefficient for induced absorption.

2. The probability that a spontaneous transition occurs is given by

$$(P_{21})_{\text{spontaneous}} = A_{21} \quad ...(5.18.2)$$

where, A_{21} = Constant known as the Einstein's coefficient for spontaneous emission

3. The probability that a stimulated transition occurs is given by

$$(P_{21})_{\text{stimulated}} = B_{21}\rho(\nu) \quad ...(5.18.3)$$

where, B_{21} = Constant of proportionality known as the Einstein's coefficient for stimulated emission.

B. Relation Between Einstein's Coefficients A and B :

1. Under thermal equilibrium, the mean population N_1 and N_2 in the lower and upper energy levels respectively must remain constant.
2. This condition requires that the number of transitions from E_2 to E_1 must be equal to the number of transitions from E_1 to E_2.

Thus,

$$\begin{pmatrix}\text{The number of atoms absorbing} \\ \text{photons per second per unit volume}\end{pmatrix} = \begin{pmatrix}\text{The number of atoms emitting} \\ \text{photons per unit volume}\end{pmatrix}$$

3. The number of atoms absorbing photons per second per unit volume

$$= B_{12}\rho(\nu)N_1$$

4. The number of atoms emitting photons per second per unit volume

$$= A_{21}N_2 + B_{21}\rho(\nu)N_2$$

5. As the number of transitions from E_1 to E_2 must equal the number of transitions from E_2 to E_1, we have

$$B_{12}\rho(\nu)N_1 = A_{21}N_2 + B_{21}\rho(\nu)N_2 \quad ...(5.18.4)$$

$$\rho(\nu)\,[B_{12}N_1 - B_{21}N_2] = A_{21}N_2$$

$$\therefore \quad \rho(\nu) = \frac{A_{21}N_2}{[B_{12}N_1 - B_{21}N_2]} \quad ...(5.18.5)$$

6. By dividing both the numerator and denominator on the right hand side of the eq. (5.18.5) with $B_{12}N_2$, we obtain,

$$\rho(\nu) = \frac{A_{21}/B_{12}}{\left[\frac{N_1}{N_2} - \frac{B_{21}}{B_{12}}\right]} \quad ...(5.18.6)$$

7. But, according to Boltzmann distribution law,

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

As $\quad E_2 - E_1 = h\nu,$

$$\therefore \quad \frac{N_2}{N_1} = e^{-h\nu/kT} \quad \text{or} \quad \frac{N_1}{N_2} = e^{h\nu/kT}$$

$$\therefore \quad \rho(\nu) = \frac{A_{21}}{B_{12}}\left[\frac{1}{e^{h\nu/kT} - B_{21}/B_{12}}\right] \quad ...(5.18.7)$$

8. To maintain thermal equilibrium, the system must release energy in the form of electromagnetic radiation.
9. It is required that the radiation be identical with black body radiation and be consistent with Planck's radiation law for any value of T.
10. According to Planck's law,

$$\rho(\nu) = \left(\frac{8\pi h\nu^3}{c^3}\right)\left[\frac{1}{e^{h\nu/kT} - 1}\right] \quad ...(5.18.8)$$

where, $\quad c$ = Velocity of light in free space.

11. Energy density $\rho(\nu)$ given by eq. (5.18.7) will be consistent with Planck's law given by eq. (5.18.8), only if

$$\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3} \quad ...(5.18.9)$$

and, $$\frac{B_{21}}{B_{12}} = 1 \quad \text{or} \quad B_{12} = B_{21} \quad ...(5.18.10)$$

12. The eq. (5.18.9) and eq. (5.18.10) are known as the Einstein's relations. The coefficients B_{12}, B_{21} and A_{21} are known as Einstein's coefficients.
13. It follows that the coefficients are related through

$$B_{12} = B_{21} = \frac{c^3}{8\pi h\nu^3} A_{21} \quad ...(5.18.11)$$

14. The relation (5.18.11) shows that the ratio of coefficients of spontaneous versus stimulated emission is proportional to the third power of frequency of the radiation. This is why it is difficult to achieve laser action in higher frequency ranges such as X-rays.

Que 5.19. **What is population inversion ?**

Answer

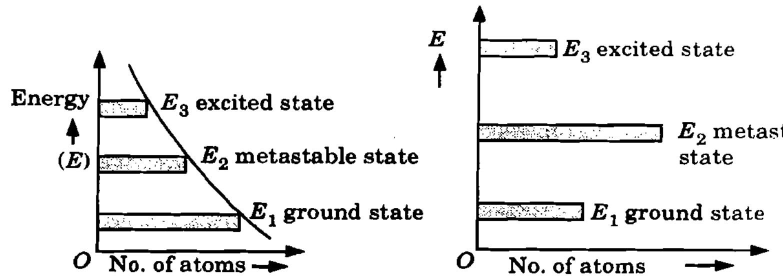
1. The phenomenon is which the number of atoms in the higher energy state becomes comparatively greater than the number of atoms in the lower energy state is known as population inversion.
2. According to Boltzmann's equation, if N_1 and N_2 are the number of atoms in the ground and excited states,

then, $$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT}$$

or $$\frac{N_2}{N_1} = e^{-\Delta E/kT}$$

where, ΔE = Energy difference between the ground state and excited state,
k = Boltzmann's constant, and
T = Absolute temperature.

3. But for atomic radiation ΔE is much greater than kT. Therefore, in thermal equilibrium the population of higher state is very much smaller than the ground state *i.e.*, $N_1 > N_2$.
4. As a result the numbers of stimulated emissions are very little as compared to absorption. Therefore laser action will not take place.
5. If somehow the number of atoms in excited state are made larger than in the ground state *i.e.*, $N_2 > N_1$, the process of stimulated emission dominates and the laser action can be achieved.



(*a*) In thermal equilibrium (*b*) After population inversion

Fig. 5.19.1.

Que 5.20. **Explain the concept of 3 and 4 level laser.**

OR

Discuss the principal pumping schemes.

Answer

A. Three Level Pumping Scheme :

1. A typical three level pumping scheme is shown in Fig. 5.20.1.
2. The state E_1 is the ground level; E_3 is the pump level and E_2 is the metastable upper lasing level.

3. When the medium is exposed to pump frequency radiation, a large number of atoms will be excited to E_3 level.
4. They do not stay at that level but rapidly undergo downward transitions to the metastable level E_2 through transitions.

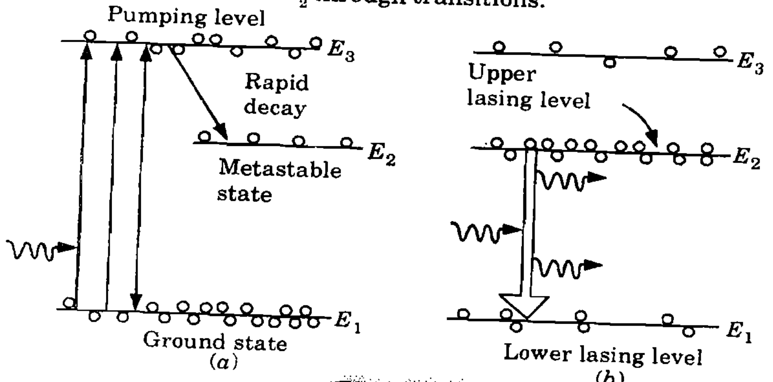


Fig. 5.20.1.

5. The atoms are trapped at this level as spontaneous transition from the level E_2 to the level E_1 is forbidden.
6. The pumping continues and after a short time there will be a large accumulation of atoms at the level E_2.
7. When more than half of the ground level atoms accumulate at E_2, the population inversion condition is achieved between the two levels E_1 and E_2.
8. Now a photon can trigger stimulated emission.

B. Four Level Pumping Scheme :

1. A typical four-level pumping scheme is shown in Fig. 5.20.2.
2. The level E_1 is the ground level, E_4 is the pumping level, E_3 is the metastable upper lasing level and E_2 is the lower lasing level. E_2, E_3 and E_4 are the excited levels.

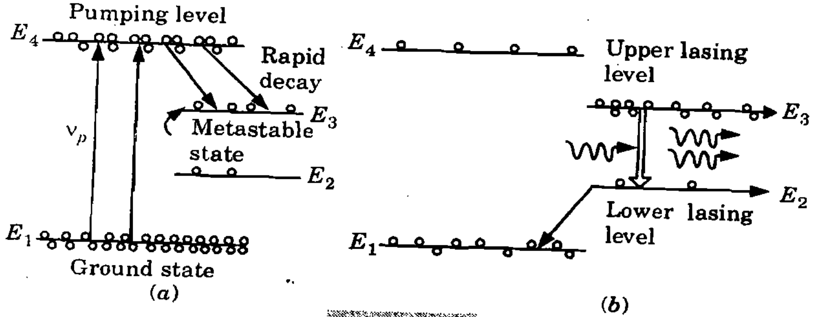


Fig. 5.20.2.

3. When light of pump frequency ν_p is incident on the lasing medium, the active centers are readily excited from the ground level to the pumping level E_4.

4. The atoms stay at the E_4 level for only about 10^{-8} sec, and quickly drop to the metastable level E_3.
5. As spontaneous transitions from the level E_3 to level E_2 cannot take place, the atoms get trapped at the level E_3. The population at the level E_3 grows rapidly.
6. The level E_2 is well above the ground level such that $(E_2 - E_1) > kT$. Therefore, at normal temperature atoms cannot jump to level E_2 on the strength of thermal energy.
7. As a result, the level E_2 is virtually empty. Therefore, population inversion is attained between the levels E_3 and E_2.
8. A photon of energy $h\nu = (E_3 - E_2)$ emitted spontaneously can start a chain of stimulated emissions, bringing the atoms to the lower laser level E_2.
9. From the level E_2 the atoms subsequently undergo non-radiative transitions to the ground level E_1 and will be once again available for excitation.

Que 5.21. Show that two level laser system has no practical significance for lasing. Explain the principle of three level lasers.

AKTU 2013-14, Marks 05

Answer

A. Two Level Laser System has no Practical Significance for Lasing :

1. The two level laser system has no practical significance because in two level system pumping is not suitable for obtaining population inversion.
2. The time span Δt, for which atoms have to stay at the upper level E_2, must be longer for achieving population inversion condition.
3. Hence, in two level system condition of population inversion will not achieve because $(N_1 = N_2)$.
4. Thus stimulated emission will not take place and laser amplification will not occur.

B. Principle of Three Level Laser : Refer Q. 5.20, Page 5–21B, Unit-5.

Que 5.22. Explain Ruby laser with its construction and working. Also explain its draw backs.

Answer

A. Ruby Laser :

1. Ruby is basically Al_2O_3 (silica) crystal containing about 0.05 % (by weight) of chromium atoms.
2. The Al^{3+} ions in the crystal lattice are substituted by Cr^{3+} ions.
3. Cr^{3+} ions constitute the active centres whereas the aluminium and oxygen atoms are inert.
4. The chromium ions give the transparent Al_2O_3 crystal a pink or red colour depending upon its concentration.

B. Construction :

1. The construction for generating Ruby laser is shown in Fig. 5.22.1.
2. Active material is a small cylinder of pink synthetic ruby, about 0.5 cm in diameter and few centimetres long.
3. Two parallel mirrors are used, one mirror M_1 is fully silvered and the other mirror M_2 is partly silvered so as to enable the coherent light radiation to be emitted through that end.
4. The mirrors must be separated by a distance that is an exact number of half wavelengths apart.

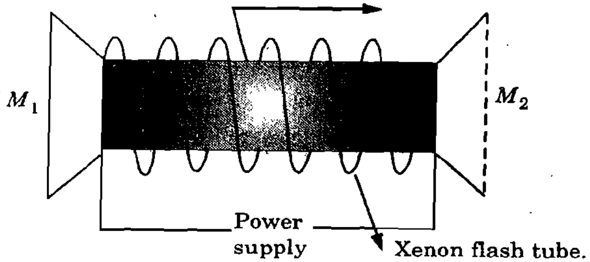


Fig. 5.22.1. Construction of Ruby laser.

5. Cooling is used to keep the ruby at a constant temperature.
6. Since quite a lot of the energy pumped into it, is dissipated into heat, pumping process is carried with the help of a xenon flash tube.

C. Working :

1. Chromium ions are excited by the optical pumping, which is achieved by the xenon flash tube and raised to higher energy states H.
2. The excited atoms return to the lower state L from higher state H in two steps as shown in Fig. 5.22.2.
3. First they return to meta-stable state M.
4. This transition is radiationless transition and energy of this transition is passed to the crystal lattice as heat loss due to collisions.
5. The chromium ions that returned to M level can remain in this state for several milli-seconds.

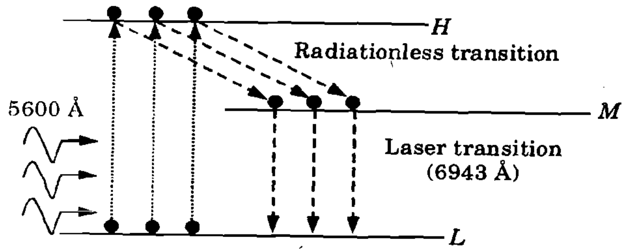


Fig. 5.22.2. Energy level diagram of Ruby laser.

6. Thus, the accumulation of the coming excited atoms at M level from H level increases its population.

7. When a chromium ion falls to the lower level L from the meta-stal level M by spontaneous emission, it emits a photon of 6943 Å.
8. This photon travels along the axis of ruby rod and is reflected back a forth by the silvered mirrors as shown in Fig. 5.22.3.
9. The photon travelling parallel to the axis of the tube will start phot multiplication by stimulated emission of other chromium ions of M lev
10. When the photon beam becomes sufficiently intense, it emerges throu the partially silvered end of the ruby rod in the form of laser pulses.
11. The laser beam is red in colour and corresponds to a wavelength (6943 Å).

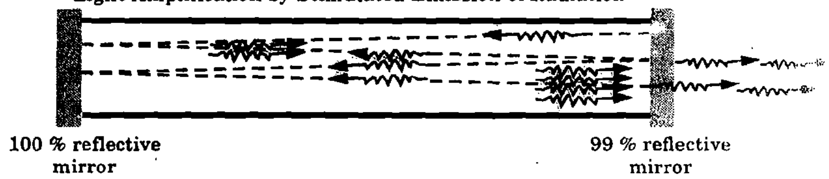


Fig. 5.22.3. Photon multiplication in Ruby laser.

D. Drawbacks :

1. The output of the laser is not continuous but occurs in the form of pu of microsecond's duration.
2. The efficiency of ruby lasers is very less.
3. It requires greater excitation in order to achieve population inver

Que 5.23. **Discuss the He-Ne laser with necessary diagrams. Giv its superiority over ruby laser.** **AKTU 2014-15, Marks 05**

Answer

A. He-Ne Laser :

1. Helium-neon (He-Ne) laser is a gaseous laser.
2. The laser action of this laser is based on a four level pumping sch
3. The population inversion is achieved through inelastic atom-at collisions. This is the basic principle of He-Ne laser.

B. Construction :

1. The construction for generating He-Ne laser is shown in Fig. 5.23.1
2. It consists of a long discharge tube of length about 50 cm and diame 1 cm.
3. The tube is filled with a mixture of He and Ne gases in the ratio 80
4. Helium is the pumping medium and Neon is the lasing medium.
5. Electrodes are provided to produce a discharge in the gas and the connected to a high voltage power supply.
6. On the axis of the tube, two reflectors M_1 (fully silvered) and M_2 (parti silvered) are fixed.
7. The distance between the mirrors is adjusted such that it equals m

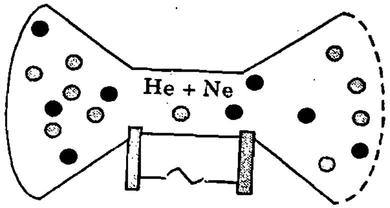


Fig. 5.23.1. Construction of He-Ne laser.

C. Working :

The energy level diagram is shown in Fig. 5.23.2.

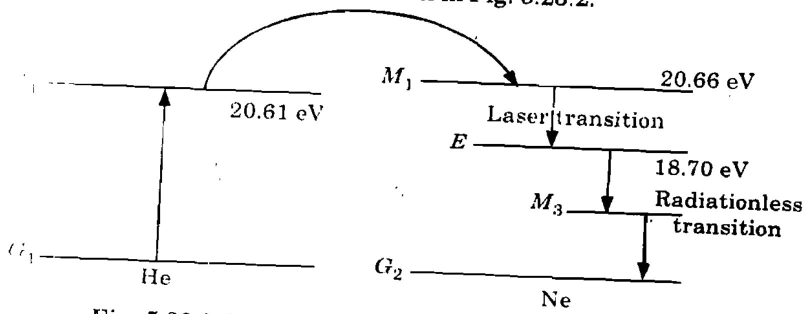


Fig. 5.23.2. Energy level diagram of He-Ne laser.

Pumping is achieved by using electrical discharge in the helium-neon mixture.

Electrons and ions in this discharge collide with He atoms raising them to a level M_1, which is meta-stable.

The He atoms are more readily excitable than Ne atom because they are lighter. Excitation level M_1 = 20.61 eV of He is very close to excitation level M_2 = 20.66 eV of Ne.

Some of the excited He atoms transfer their energy to Ne atoms of ground states by collisions between helium and neon atoms.

Thus, the excited He atoms return to ground state by transferring their energy to Ne atoms through collisions.

The kinetic energy of helium atoms provides the additional 0.05 % for exciting the neon atom.

This is the main pumping scheme of He-Ne system.

Thus, neon atoms are active centres.

The helium gas in the laser tube provides the pumping medium to attain the necessary population inversion for laser action. This population inversion is maintained because :

The meta-stability of level E_3 ensures a ready supply of Ne atoms in level E_3.

The Ne atoms from level E decay rapidly to the neon ground state G_2.

11. In this way, M_2 state of Ne can become more highly populated than level E.
12. The laser transition occurs when Ne atoms fall from level M_2 to E through stimulated emission.
13. A red laser light of wavelength 6328 Å is obtained in He-Ne laser.

E. Superiority of He-Ne laser over Ruby laser :

1. He-Ne laser produces continuous laser beam while ruby laser produces light in the form of pulses.
2. He-Ne laser employs a four level pumping scheme while ruby laser employs a three level pumping scheme.

Que 5.24. What is the advantage of four level laser systems over three level laser systems ? Describe the construction and working of ruby laser. **AKTU 2017-18, Marks 07**

Answer

A. Advantages of Four Level Laser System over Three Level Systems :

1. It is easy to achieve population inversion with four level system than with a three level system.
2. In the four level laser, the transition does not terminate at the ground state, the pumping power needed for the excitation of atoms is much lower than in a three level laser.
3. The efficiency of four level laser is much better than that of a three level laser.
4. In order to get population inversion in three level laser systems more than 50 % of the atoms in the ground level E_1 must be lifted to level E_2 while it is not necessary in case of four level lasers.
5. The threshold pump power required for population inversion in three level lasers is larger than in four level laser

B. Construction and Working of Ruby Laser : Refer Q. 5.22, Page 5–23A, Unit-5.

Que 5.25. What are various applications of LASER beam ?

Answer

1. The laser beam is used for drilling, welding and melting of hard materials like diamonds, iron, steel, etc.
2. It is used in heat treatments for hardening or annealing in metallurgy.
3. The laser beam is used in delicate surgery like cornea grafting and in the treatment of kidney stone, cancer and tumor.
4. Laser is used in holography, fibre optics and nonlinear optics.
5. During war-time, lasers are used to detect and destroy enemy missiles.
6. Now, laser-pistols, laser-rifles and laser bombs are also being made, which can be aimed at the enemy in the night.

7. Laser is very useful in science and research areas.
8. Laser is used for communications and measuring large distances.
9. Semiconductor laser is used for recording and erasing of data on compact disks.
10. Semiconductor lasers and helium-neon lasers are used to scan the universal barcodes to identify products in supermarket scanners.

Que 5.26. **In a Ruby laser, total number of Cr^{+3} is 2.8×10^{19}. If the laser emits radiation of wavelength 7000 Å, calculate the energy of the laser pulse.** **AKTU 2015-16, Marks 05**

Answer

Given : $n = 2.8 \times 10^{19}$, $\lambda = 7000$ Å $= 7000 \times 10^{-10}$ m
To Find : Energy of the laser pulse.

1. We know that,

$$\text{Energy} = \frac{nhc}{\lambda}$$

$$\therefore \quad \text{Energy} = \frac{2.8 \times 10^{19} \times 6.63 \times 10^{-34} \times 3 \times 10^{8}}{7000 \times 10^{-10}}$$

$$[\because \; h = 6.63 \times 10^{-34} \text{ J-s and } c = 3 \times 10^{8} \text{ m/s}]$$

$$= 7.956 \text{ J}$$

or $$\text{Energy} = \frac{7.956}{1.67 \times 10^{-19}} \text{ eV}$$

$$= 4.764 \times 10^{19} \text{ eV}$$

☺☺☺

1 UNIT

Relativistic Mechanics (2 Marks Questions)

1.1. What is frame of reference ?
Ans. A coordinate system with respect to which we measure the position of a point object of an event is called a frame of reference.

1.2. Define inertial frame of reference.
Ans. Inertial frame of reference is defined as the frame in which a body is at rest or moving with uniform velocity and is not under any force.

1.3. What are non-inertial frames ?
Ans. The frames of reference with respect to which an unaccelerated body appears accelerated are called non-inertial frames.

1.4. What was the aim of Michelson-Morley experiment ?
Ans. The aim of this experiment was to prove the existence of the ether and to test whether the ether is fully or partially dragged with bodies moving in it.

1.5. What are the conclusions of Michelson-Morley experiment ?
Ans.
a. There is no existence of hypothetical medium ether.
b. The velocity of light in all inertial frames of reference remain constant.

1.6. What are the Einstein postulates of special theory of relativity ?
Ans. **Postulate I :** The principle of equivalence.
Postulate II : The principle of constancy of the speed of light.

1.7. What do you understand by variant and invariant under the Galilean transformation ?
Ans. Variant means the physical quantities which change from one frame of reference to another frame of reference, e.g. velocity. Invariant means the physical quantities which do not change from one frame of reference to another frame of reference. Example: distance between two points is invariant in true inertial frames

1.8. What do you mean by Lorentz transformation ?

Ans. The equations in special theory of relativity, which relate to the space and time coordinates of an event in two inertial frames of reference moving with a uniform velocity relative to one-another, are called Lorentz transformations.

1.9. What is the conclusion of Lorentz transformation ?

Ans. The conclusion of Lorentz transformation is that it limits the maximum velocity of the material bodies.

1.10. Write down the inverse Lorentz transformation equations.

Ans. Lorentz inverse transformation equations are :

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}, y = y', z = z' \text{ and } t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}}$$

1.11. Define length contraction.

Ans. The length of a moving rod will appear to be contracted if it is seen from a frame of reference which is at rest. This decrease in length in the direction of motion is called length contraction.

1.12. What do you understand by time dilation ?

AKTU 2013-14, Marks 02

Ans. In the special theory of relativity, the moving clock is found to run slower than a clock at rest does. This effect is known as time dilation.

1.13. Give the Einstein's mass-energy relation.

Ans.
$$E = mc^2$$
This relation is known as Einstein's mass-energy relation.

1.14. Give some examples of mass-energy equivalence.

Ans. Some important examples of the mass-energy equivalence are as follows :

a. Pair production phenomenon,
b. Annihilation phenomenon (production of γ-rays photon),
c. Nuclear fusion, and
d. Nuclear fission.

1.15. What are massless particles ? **AKTU 2013-14, Marks 02**

Ans. A particle which has zero rest mass (m_0) is called a massless particle. The velocity of the massless particle is same as that of light in free space.

1.16. Which frames are known as accelerated frames ?

Ans. Non-inertial frames are known as accelerated frames.

1.17. Find relativistic relation between energy and momentum.

AKTU 2015-16, Marks 02

Ans. As the relativistic total energy E of a particle of rest mass (m_0) in terms of its momentum p may be expressed as

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$$\therefore \quad E = pc \qquad [\because \quad m_0 = 0]$$

1.18. Why are Galilean transformations used ?

Ans. Galilean transformations are used to convert the laws of mechanics from one frame of reference to another frame, moving with constant velocity with respect to the first frame.

1.19. Is earth an inertial frame of reference or not ?

Ans. According to Newton's assumption earth is an inertial frame because for the study of any particle or body on earth, we can take earth as inertial frame of reference.

But on other hand earth rotates about its axis as well as revolves around the sun in its orbit so it can also be treated as non-inertial frame of reference.

1.20. How the negative results of Michelson-Morley experiment interpreted ? **AKTU 2015-16, Marks 02**

Ans. Following explanations were offered to interpret the negative results of Michelson-Morley experiment :

a. Ether drag hypothesis,
b. Fitzgerald-Lorentz contraction hypothesis, and
c. Constancy of speed of light hypothesis.

1.21. What is proper length of a rod ?

AKTU 2016-17, Marks 02

Ans. The length of the rod measured by an observer in the frame in which the rod is at rest is called proper length or actual length of rod.

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2
UNIT

Electromagnetic Field Theory (2 Marks Questions)

2.1. What is displacement current ?

Ans. The changing electric field in vacuum or dielectric is equivalent to a current which produces the same magnetic effect as an ordinary current in a conductor. This equivalent current is known as displacement current.

2.2. Explain mathematically displacement current.

Ans.

$$I_d = \frac{\partial}{\partial t} \oint_s \vec{D} \cdot d\vec{s}$$

$$I_d = \oint_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$= \oint_s \vec{J}_d \cdot d\vec{s}$$

Where, $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$ is displacement current density.

2.3. State Gauss' law in electrostatics.

Ans. Gauss' law states that the electric flux passing through any closed hypothetical surface of any shape drawn in an electric field is equal to $1/\epsilon_0$ times the total charge enclosed by the surface.

i.e., $$\phi = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

Where, q = Total charge.

2.4. Define Gauss' law in magnetostatics.

Ans. Gauss' law states that the magnetic flux around a closed surface is equal to zero.

$$\phi = \oint \vec{B} \cdot d\vec{s} = 0$$

2.5. State the Ampere's circuital law.

Ans. Ampere's circuital law states that line integral of magnetic induction $\vec{B}$ for a closed path is numerically equals to μ_0 times the current through the area bounded by the path.

i.e., $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Where, μ_0 = Permeability of free space.

2.6. State Gauss' divergence theorem.

Ans. It states that the flux of vector field $\vec{F}$ over any closed surface 's' is equal to volume integral of the divergence of the vector field enclosed by the surface 's'.

$$\iint_s \vec{F} \cdot d\vec{s} = \iiint_V \text{div}\,\vec{F}\, dV$$

It is used to transform surface integral into volume integral.

2.7. Give the physical interpretation of the Maxwell's equation.

Ans.
a. First equation represents the Gauss' law in electrostatics for the static charge.
b. Second equation represents Gauss' law for magnetism.
c. Third equation represents Faraday's law in electromagnetic induction.
d. Fourth equation represents the generalized form of Ampere's law.

2.8. What do you understand by equation of continuity ?

Ans. The equation of continuity expresses the fact that the electric charges can neither be created nor be destroyed in macroscopic quantities.
It is given as

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial P}{\partial t} = 0$$

For static field, $\frac{\partial P}{\partial t} = 0$

$$\vec{\nabla} \cdot \vec{J} = 0$$

2.9. Write the physical significance of equation of continuity.

Ans. The current diverging from a small volume element must be equal to the rate of decrease of charge within the volume.

2.10. Write Maxwell's equation in free space.

Ans. For free space, $\rho = 0$ and conductivity, $\sigma = 0$ therefore $J = 0$. Hence the Maxwell's equations in differential form become :

a. $\vec{\nabla}.\vec{D} = 0$ or $\vec{\nabla}.\vec{E} = 0$

b. $\vec{\nabla}.\vec{B} = 0$ or $\vec{\nabla}.\vec{H} = 0$

c. $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$

d. $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

2.11. What do you mean by electromagnetic waves ?

Ans. Electromagnetic waves are the coupled electric and magnetic oscillations that move with speed of light and exhibit wave behaviour.

2.12. State Poynting theorem.

Ans. According to this theorem, the time rate of EM-energy within a certain volume plus the time rate of EM-energy flowing out through the boundary surface is equal to the power transferred into the EM-field.

2.13. Write wave equation in free space.

Ans. The wave equations in free space are as follows :

$$\nabla^2 \vec{H} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

2.14. Write the mathematical expression of skin depth.

Ans. The reciprocal of attenuation constant is called skin depth or depth of penetration.

$$\delta = \frac{1}{\text{Attenuation constant}}$$

For good conductor $\delta = \sqrt{\frac{2}{\omega\mu\sigma}}$

For poor conductor $\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$

2.15. The electric field intensity $\vec{E} = 250 \sin 10^{10}t$ V/m for a field propagating in the medium whose $\sigma = 5.0$ S/m and $\epsilon_r = 1.0$. Calculate the displacement current density J_d.

Ans.

Given : $\vec{E} = 250 \sin 10^{10}t$ V/m, $\sigma = 5.0$ s/m, $\epsilon_r = 1.0$

To Find : Displacement current density, J_d.

1. The displacement current density J_d is given by

$$J_d = \frac{dD}{dt} = \frac{\epsilon\, dE}{dt} \quad (\because \epsilon = \epsilon_r \epsilon_0 = 1 \times 8.85 \times 10^{-12})$$

$$J_d = 8.85 \times 10^{-12} \frac{d}{dt}[250 \sin 10^{10}t]$$

$$= 8.85 \times 10^{-12} \times 250 \times 10^{10} \cos 10^{10} t$$

$$= 22.125 \cos 10^{10} t \text{ A/m}^2$$

2.16. If a plane electromagnetic wave in free space has magnitude of H as 1 A/m. What is the magnitude of E ?

Ans.

Given : $H_0 = 1$ A/m

To Find : Magnitude of E_0

1. For free space, the characteristics impedance,

$$Z_0 = \frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$E_0 = H_0 \sqrt{\frac{\mu_0}{\epsilon_0}} = 1 \times \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 376.72 \text{ Vm}^{-1}$$

2.17. Calculate the skin depth for a frequency of 10^{10} Hz for silver. Given $\sigma = 2 \times 10^7$ S/m and $\mu = 4\pi \times 10^{-7}$ H/m.

Ans.

Given : $\nu = 10^{10}$ Hz, $\sigma = 2 \times 10^7$ S/m, $\mu = 4\pi \times 10^{-7}$ H/m

To Find : Skin depth.

1. We know that, $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$

$$= \sqrt{\frac{2}{4\pi \times 10^{-7} \times 2 \times 10^7 \times 2 \times 3.14 \times 10^{10}}} \quad [\because \omega = 2\pi\nu]$$

$= 1.125$ μm

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3 UNIT

Quantum Mechanics (2 Marks Questions)

3.1. What is a black body ?

Ans. A body which absorbs completely all the radiations incident upon it, reflecting none and transmitting none, is called a black body.

3.2. Define black body radiations.

Ans. When a black body is heated to a suitable high temperature it emits total radiations which are known as black body radiations.

3.3. Which body is assumed to be perfectly black ?

Ans. Lamp black is the nearest approach to black body which absorbs nearly 99 % of the incident radiation.

3.4. Define Wien's law.

Ans. Wien showed that the maximum energy, E_m of the emitted radiation from black body is proportional to fifth power of absolute temperature (T^5).

$$E_m \propto T^5$$

or $$E_m = \text{Constant} \times T^5$$

3.5. What is Rayleigh-Jean's law ?

Ans. Rayleigh-Jean's law states that the total amount of energy emitted by a black body per unit volume at an absolute temperature T in the wavelength range λ and $\lambda + d\lambda$ is given as

$$u_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

3.6. Define Planck's law.

Ans. Planck's derived an equation for the energy per unit volume of black body in the entire spectrum of black body radiation. It is given by

$$u_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

3.7. What do you mean by wave particle duality ?

Ans. According to Einstein, the energy of light is concentrated in small bundles called photons. Hence, light behave as a wave on one hand and as a particle on the other hand. This nature of light is known dual nature, while this property of light is known as wave particle duality.

3.8. What are the properties of matter waves ?

Ans. Following are the properties of matter waves :

a. Each wave of the group travel with a velocity known as phase velocity.

b. These waves cannot be observed.

c. The wavelength of these waves, $\lambda = \frac{h}{p}$.

3.9. What is matter or de-Broglie waves ?

Ans. According to de-Broglie, a particle of mass m, moving with velocity v is associated with a wave called matter wave or de-Broglie wave

$$\lambda = \frac{h}{p} = \frac{h}{m\text{v}}$$

3.10. If uncertainty in the position of a particle is equal to de-Broglie wavelength, what will be uncertainty in the measurement of velocity ? AKTU 2015-16, Marks 02

Ans. Uncertainty in the position of a particle = de-Broglie wavelength

$$\Delta x = \lambda = \frac{h}{p} \quad \text{...(3.10.1)}$$

Where, h = Planck's constant, and

p = momentum of particle.

According to Heisenberg's uncertainty principle,

$$\Delta x \ \Delta p \geq \frac{h}{2\pi} \quad \text{...(3.10.2)}$$

Let Δv be the uncertainty in velocity,

$$\Delta p = m.\Delta \text{v} \quad \text{...(3.10.3)}$$

Putting eq. (3.10.1) and eq. (3.10.3) in eq. (3.10.2)

$$\frac{h}{p} m \, \Delta \text{v} = \frac{h}{2\pi}$$

So, uncertainty in velocity,

$$\Delta \text{v} = \frac{\text{v}}{2\pi} \qquad [\because p = m\text{v}]$$

3.11. Define wave function.

Ans. The quantity whose variation builds matter wave is called wave function (ψ).

3.12. What are eigen values and eigen functions ?

Ans. The values of energy for which Schrodinger's steady state equation can be solved are called eigen values and the corresponding wave functions are called eigen functions.

3.13. Write the characteristics of wave function.

AKTU 2015-16, Marks 02

Ans.

a. The wave function Ψ contains all the measurable information about the particle.

b. It can interfere with itself. This property explains the phenomenon of electron diffraction.

c. The wave function Ψ permits the calculation of most probable value of a given variable.

3.14. How can we obtain a perfect black surface ?

Ans. An ideal model of a perfectly black surface is obtained if a small hole is made in the opaque walls of a closed hollow cavity.

3.15. What is the physical significance of wave function ?

Ans. The wave function Ψ itself has no physical significance but the square of its absolute magnitude $|\Psi|^2$ gives the probability of finding the particle at that time.

3.16. Why Compton effect is not observable for visible light ?

Ans. Compton effect is not observable for visible light because the maximum value of Compton effect $\Delta\lambda = 0.04852$ Å (when $\theta = 180°$) is very small (about 0.001%) as compared to the mean value of wavelength of visible light (~5000 Å). Compton effect is observable only with X-rays and not with visible light.

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UNIT 4

Wave Optics (2 Marks Questions)

4.1. Define interference.

Ans. The modification in the intensity of light resulting from the superposition of two (or more) waves of light is called interference.

4.2. Explain interference in thin films.

Ans. When a thin film of transparent material like oil drop spread over the surface of water is exposed to an extended source of light, it appears coloured. This phenomenon can be explained as interference of thin films.

4.3. Explain the factor responsible for changing fringe width in wedge shaped film. **AKTU 2016-17, Marks 02**

Wedge angle 'θ' is the factor responsible for changing fringe width in wedge shaped film because as the wedge angle is gradually decrease, the fringe width increases and finally fringes disappear when $\theta \approx 0$ or when the faces of the film become parallel.

4.4. Define Newton's rings.

Ans. When a monochromatic light falls on the film, we get dark and bright concentric fringes having uniform thickness, these rings are called Newton's rings.

4.5. Write down the conditions for bright and dark rings.

Ans. **Condition for Bright Rings :**

$$2\mu t = (2m - 1)\lambda / 2$$

Condition for Dark Rings :

$$2\mu t = m\lambda$$

4.6. In Newton's ring experiment fringe width decreases with the increase in the order of fringe. Explain why ?

AKTU 2013-14, Marks 02

Ans. In Newton's ring experiment fringe width decreases with the increase in the order of fringe because as the order of fringe increases the thickness of the air film is also increased.

4.7. Why are fringes circular in Newton's ring experiment ? Explain.

AKTU 2014-15, Marks 02

Ans. In a Newton's ring set up the air film is enclosed below the convex lens. The thickness of the film is constant over a circle having centre at the center of the lens. Hence the fringes are circular.

4.8. On which factor the condition of brightness or darkness depends ?

Ans. The condition of brightness or darkness depends on the path difference between the two reflected rays.

4.9. Define diffraction.

Ans. Diffraction of light is a phenomenon of bending of light and spreading out towards the geometrical shadow when passed through an obstruction.

4.10. What are the types of diffraction ?

Ans. There are two types of diffraction :

a. Fresnel diffraction, and

b. Fraunhofer diffraction.

4.11. What happens to diffraction pattern when slit width of single slit experiment increases ?

AKTU 2016-17, Marks 02

Ans. If we increase the width of the slit, diffraction pattern gets narrower. Increasing the size of the opening reduces the spread in the pattern.

4.12. Differentiate between Fresnel and Fraunhofer diffractions.

Ans.

S. No.	Fresnel Diffraction	Fraunhofer Diffraction
a.	Lateral distances are important.	The angular inclinations are important.
b.	Observed pattern is a projection of a diffracting element.	Observed pattern is an image of the source.
c.	The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel zones.	The centre of the diffraction pattern is always bright for all paths parallel to the axis of the lens.

4.13. Define diffraction grating.

Ans. Diffraction grating is an arrangement consisting of a large number of close parallel, straight, transparent and equidistant slits, each of equal width e, with neighbouring slits being separated by an opaque region of width d.

4.14. What do you mean by dispersive power of a plane diffraction grating ?

Ans. Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines.

4.15. How the dispersive power related to order of the spectrum

Ans. The dispersive power is directly proportional to the order of spectrum, *i.e.*, higher is the order greater is the dispersive power.

4.16. What is Rayleigh's criterion of resolution ?

AKTU 2015-16, Marks 02

Ans. According to Rayleigh, the two point sources or two equally intense spectral lines are just resolved by an optical instrument when the central maximum of the diffraction pattern due to one source falls exactly on the first minimum of the diffraction pattern of the other and vice-versa.

4.17. Differentiate between the dispersive power and resolving power of grating.

Ans.

S. No.	Dispersive Power	Resolving Power
a.	It is defined as the rate of change of angle of diffraction with the wavelength used.	It is defined as the ratio of the wavelength of any spectral line to the smallest wavelength difference between neighbouring lines for which the spectral lines can be just resolved.
b.	Dispersive power is given by, $\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$	The resolving power of a grating is given by $\frac{\lambda}{d\lambda} = nN$
c.	Dispersive power depends upon the grating element	Resolving power is independent of grating element.

4.18. Differentiate the interference and diffraction.

Ans.

S. No.	Interference	Diffraction
a.	In this, the interference occurs between the two separate wavefront emanating from two coherent sources.	In this, the interference occurs between the innumerable secondary wavelets produced by the unobstructed portion of the same wavefront.
b.	The interference fringes are usually equally spaced.	The diffraction fringes are never equally spaced.
c.	In an interference pattern all the bright fringes are of equal intensity.	In diffraction pattern the intensity of central maximum is maximum and goes on decreasing as the order of maxima increases on either side of the central maxima.

4.19. Why the centre of Newton's ring is dark ?

AKTU 2015-16, Marks 02

Ans. At the point of contact of lens and glass plate, the path difference is zero and phase change 'π' takes place due to reflection on glass. Hence dark spot will be formed at the centre of ring system.

4.20. What are the applications of thin film interference ?

Ans. Applications of thin film interference are as follows :

a. Measurement of small displacements,
b. Testing of surface finish,
c. Testing of a lens surface, and
d. Thickness of a thin film coating.

4.21. Two independent sources could not produce interference. Why ?

Ans. Two independent sources could not produce interference because there will be phase difference development between the two waves and hence sustained interference will not develop.

4.22. What will be the effect on the intensity of principal maxima of diffraction pattern when single slit is replaced by double slit ?

Ans. In single slit diffraction, intensity of principal maxima,

$$I = R^2 = \frac{A^2 \sin^2 \alpha}{\alpha^2}$$

By replacing single slit by double slit, the resultant intensity at any point on the screen is given by :

$$I = \frac{4A^2 \sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

So, the intensity of principal maxima becomes 4 times.

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5 UNIT

Fibre Optics and Laser (2 Marks Questions)

5.1. Define fibre optics.

Ans. Fibre optics is a technology in which signals are converted from electrical into optical signals, transmitted through a thin glass fibre and reconverted into electrical signals.

5.2. What is optical fibre ?

Ans. An optical fibre is a cylindrical wave guide made of transparent dielectric, which guides light waves along its length by total internal reflection.

5.3. What do you understand by total internal reflection ?

Ans. Total internal reflection is the phenomenon in which there is complete reflection of light within the medium or there is no refracted ray. It occurs when angle of incidence is greater than critical angle.

5.4. Write down the advantages of optical fibres.

Ans. The advantages of optical fibres include :

a. High data transmission rates and bandwidth,
b. Low losses,
c. Small cable size and weight, and
d. Data security.

5.5. What are the functions of cladding ?

Ans. The cladding performs the following important functions :

a. Protects the fibre from physical damage and absorbing surface contaminants.
b. Prevents leakage of light energy from the fibre through evanescent waves.

5.6. What is critical ray ?

Ans. The ray incident, at the core-cladding boundary, at the critical angle is called a critical ray.

5.7. Define acceptance angle.

Ans. Acceptance angle is the maximum angle that a light ray can have relative to the axis of the fibre and propagate down the fibre.

5.8. Define modes.

Ans. The light ray paths along which the waves are in phase inside the fibre are known as modes.

5.9. What is attenuation ?

Ans. The attenuation is defined as the reduction in amplitude or power and intensity of a signal as it is guided through optical fibre.

5.10. Define acceptance cone.

Ans. A cone obtained by rotating a ray at the end face of an optical fibre, around the fibre axis with acceptance angle is known as acceptance cone.

5.11. What is dispersion ?

Ans. The amount by which a pulse broadens as it passes through a multimode fibre is commonly known as dispersion.

5.12. Give full form of LASER.

Ans. LASER is the acronym for Light Amplification by Stimulated Emission of Radiation.

5.13. Differentiate between the ordinary beam and laser beam.

Ans.

S. No.	Ordinary Beam	Laser Beam
a.	It is not monochromatic.	It is monochromatic.
b.	Ordinary light is produced by spontaneous emission.	Laser beam is produced by stimulated beam.
c.	It is incoherent.	It is coherent.

5.14. Define spontaneous emission and stimulated emission.

OR

What is stimulated emission of radiation in a laser ?

AKTU 2014-15, Marks 02

Ans. **Spontaneous Emission :** The process in which photon emission occurs without any interaction with external radiation is called spontaneous emission.

Stimulated Emission : The phenomena of forced emission of photons are called induced emission or stimulated emission.

5.15. What do you mean by population inversion ?

AKTU 2013-14, Marks 02

Ans. The phenomenon in which the number of atoms in the higher energy state becomes comparatively greater than the number of atoms in the lower energy state is known as population inversion.

5.16. Define pumping.

Ans. The process of supplying energy to the medium to transfer it into the state of population inversion is known as pumping.

5.17. Define metastable state. **AKTU 2015-16, Marks 02**

Ans. Metastable state is particular excited state of an atom, nucleus, or other system that has a longer lifetime than ordinary excited states and that generally has a shorter lifetime than the lowest energy state, called the ground state.

5.18. Give few important applications of optical fibre ?

AKTU 2015-16, Marks 02

Ans. Following are the important applications of optical fibre :

a. In communication,
b. In optical sensors,
c. In illumination applications, and
d. In imaging optics.

5.19. Which fibres are generally used under sea water ?

Ans. Single mode fibres are generally used under sea water.

5.20. Write down some applications of laser.

Ans. Following are the applications of laser :

a. In surgery,
b. In holography,
c. In communications,
d. In computer industry, and
e. Laser printing.

5.21. How can be modal dispersion minimized ?

Ans. Modal dispersion can be minimized by using single mode step, monomode step index fibres or graded index multi mode fibre.

5.22. Why does ruby laser emits red or pink colour ?

Ans. Ruby laser emits red or pink colour due to the presence of chromium ions depending upon its concentration.

5.23. How is population inversion achieved in He-Ne laser ?

Ans. In He-Ne laser, the population inversion is achieved through metastable atom-atom collisions.

5.24. What precautions are needed to minimize material dispersion ? **AKTU 2016-17, Marks 02**

Ans. Material dispersion can be minimized either by choosing sources with narrow spectral range or by operating at longer wavelength.

5.25. Differentiate between spontaneous and stimulated emission of radiation.

Ans.

S. No.	Spontaneous Emission	Stimulated Emission
a.	In this, light emitted is not monochromatic.	Monochromatic light is emitted.
b.	Not controllable from outside.	Controllable from outside.
c.	Incoherent photons are emitted.	Coherent photons are emitted.
d.	No amplification of light.	Amplified beam is achieved.

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B.Tech.
(SEM. I) ODD SEMESTER THEORY EXAMINATION, 2013-14
ENGINEERING PHYSICS-I

Time : 3 Hours **Max. Marks : 100**

SECTION – A

Note : There are three Sections A, B and C in this paper. Questions are to be done from all three Sections.

1. Attempt **all** parts. Give answer of each part in short : **(2 × 5 = 10)**

a. **What do you understand by time dilation ?**

Ans. Refer Q. 1.12, Page SQ–2A, 2 Marks Questions, Unit-1.

b. **What are massless particles ?**

Ans. Refer Q. 1.15. Page SQ–2A, 2 Marks Questions, Unit-1.

c. **In Newton's ring experiment fringe width decreases with the increase in the order of fringe. Explain why ?**

Ans. Refer Q. 4.6, Page SQ–11A, 2 Marks Questions, Unit-4.

d. **How the unpolarized light and circularly polarized light distinguish?**

Ans. In unpolarized light, the light is passed through a single plane while in circularly polarized light, the light is passed through two planes.

e. **What do you mean by population inversion?**

Ans. Refer Q. 5.15, Page SQ–16A, 2 Marks Questions, Unit-5.

SECTION – B

2. Attempt any three parts. All parts carry equal marks : **(5 × 3 = 15)**

a. **Calculate the length of one meter rod moving parallel to its length when its mass is 1.5 times of its rest mass.**

Ans. Refer Q. 1.34, Page1–33A, Unit-1.

b. **The speed of an electron is measured to be 5.0×10^3 m/s to an accuracy of 0.003 %. Find the uncertainty in determining the position of this electron (mass of electron is 9.1×10^{-31} kg and Planck's constant is 6.62×10^{-34} J-s).**

Ans.

Given : $v = 5.0 \times 10$ m/s, accuracy = 0.083 %, $m_e = 9.1 \times 10^{-31}$ kg, $h = 6.62 \times 10^{-34}$ J s

To Find : Uncertainty in position of electron.

1. Uncertainty in velocity, $\Delta v = \dfrac{0.003}{100} \times 5.0 \times 10^3 = 0.15$ m/s

2. $$\Delta x\,\Delta p = \frac{h}{2\pi} \Rightarrow \Delta x = \frac{h}{2\pi \times \Delta p}$$

$$\Delta x = \frac{h}{2\pi m(\Delta v)} = \frac{6.62 \times 10^{-34}}{2 \times 3.14 \times 9.1 \times 10^{-31} \times 0.15}$$

$$\Delta x = 7.72 \times 10^{-4}\ \text{m}.$$

c. **Newton's rings are observed in reflected light with wavelength 6000 Å. If the diameter of the 10^{th} dark ring is 0.5 cm, find the radius of curvature of the lens and the thickness of the corresponding air film.**

Ans. Refer Q. 4.17, Page 4–23A, Unit-4.

d. **A diffraction grating used at normal incidence gives a yellow line (λ = 6000 Å) in a certain spectral order superimposed on a blue line (λ = 4800 Å) of next higher order. If the angle of diffraction is 60°, calculate the grating element.**

Ans. Refer Q. 4.29, Page 4–41A, Unit-4.

e. **The refractive indices of quartz for polarized light μ_e and μ_o are 1.5508 and 1.5418 respectively. Calculate the phase retardation for λ = 5000 Å when the plate thickness is 0.032 mm.**

Ans.

Given : $\mu_e = 1.5508$, $\mu_o = 1.5418$, $t = 0.032$ mm = 0.0032 cm, $\lambda = 5000$ Å $= 5000 \times 10^{-8}$ cm

To Find : Phase retardation.

1. The phase retardation $= \dfrac{2\pi}{\lambda}\Delta = \dfrac{2\pi}{\lambda}(\mu_e - \mu_o)t$

$$= \frac{2 \times 3.14(1.5508 - 1.5418)}{5000 \times 10^{-8}} \times 0.0032$$

$$= 3.617 \text{ rad.}$$

SECTION – C

Note : Attempt **all** the question of this section. **All** questions carry equal marks.

3. Attempt any **one** part of the following : **(5 × 1 = 5)**

a. **State Einstein's postulates of special theory of relativity. Explain why Galilean relativity failed to explain actual results of Michelson-Morley experiment.**

Ans. Refer Q. 1.6, Page 1–8A, Unit-1.

b. **Show that the relativistic invariance of the law of conservation of momentum leads to the concept of variation of mass with velocity and equivalence of mass and energy.**

Ans.

A. **Variation of Mass with Velocity :** Refer Q. 1.23, Page 1–24A, Unit-1.

B. **Equivalence of Mass and Energy :** Refer Q. 1.24, Page 1–26A, Unit-1.

4. Attempt any **one** part of the following : **(5 × 1 = 5)**

a. **Deduce a relation between phase velocity and group velocity in a medium where wave velocity is frequency dependent. What happens if the phase velocity is independent of frequency ?**

Ans.

1. As phase velocity $v_p = \frac{dx}{dt} = \frac{\omega}{K}$
2. For the amplitude of wave packet to be constant

$$\frac{\Delta\omega t}{2} - \frac{\Delta K}{2}x = \text{Constant}$$

3. Hence, group velocity

$$v_g = \frac{dx}{dt} = \frac{\frac{\Delta\omega}{2}}{\frac{\Delta K}{2}} = \frac{\Delta\omega}{\Delta K}$$

$$v_g = \lim_{\Delta K \to 0} \frac{\Delta\omega}{\Delta K} = \frac{d\omega}{dK}$$

$$v_g = \frac{d}{dK}(Kv_p) = v_p + K\frac{dv_p}{dK}$$

$$\left[\because \frac{\omega}{K} = v_p \text{ and } \omega = Kv_p\right]$$

$$v_g = v_p + \left(\frac{2\pi}{\lambda}\right)\frac{dv_p}{d\left(\frac{2\pi}{\lambda}\right)}$$

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

4. This is a relation between group velocity v_g and wave velocity v_p in a dispersive medium in which wave velocity is frequency dependent.

5. If $\frac{dv_p}{d\lambda} = 0$, then the phase velocity does not depends on frequen[cy] and become independent

$$v_p = v_g$$

b. **A particle of mass m is confined to a one-dimensional box of length L. Derive an expression for wave function and energy.**

Ans. Refer Q. 3.17, Page 3–16A, Unit-3.

5. Attempt any **one** part of the following : (5 × 1 = 5)

a. **Discuss the interference in thin film due to reflected light. What happens when film is excess thin ?**

Ans. Refer Q. 4.3, Page 4–6A, Unit-4.

b. **Explain the diffraction pattern obtained with diffraction at single slit. By what fraction the intensity of second maximum reduced from principal maximum ?**

Ans. Refer Q. 4.19, Page 4–25A, Unit-4.

6. Attempt any **one** part of the following : (5 × 1 = 5)

a. **What is diffraction grating ? Show that its dispersive power can be expressed as $\frac{1}{\sqrt{\left(\frac{e+d}{n}\right)^2 - \lambda^2}}$ where all terms have their usual meanings.**

Ans. Refer Q. 4.23, Page 4–36A, Unit-4.

b. **What do you mean by double refraction ? Explain the working principle of Nicol Prism.**

Ans.

A. **Double refraction :**

1. When a beam of unpolarized light is incident on the surface of an anisotropic crystal such as calcite or quartz, it is found that it will separate into two rays that travel in different directions. This phenomenon is called birefringence or double refraction.

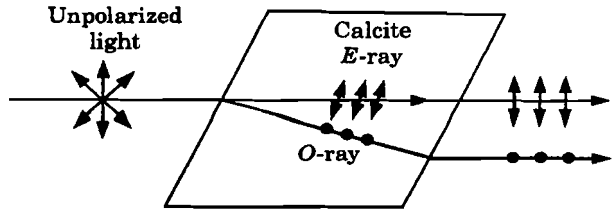

Fig. 1.

2. The two rays are known as ordinary ray (*O*-ray) and extraordinary ray (*E*-ray), which are linearly polarized in mutually perpendicular directions.

B Working Principle of Nicol Prism :

1. When an unpolarized light ray *SM* parallel to the face *DC'* is incident on the face *A' D*, it splits into *O*-ray and *E*-ray.
2. The *O*-ray is going from calcite to Canada Balsam travels from optically denser medium ($\mu_O = 1.66$) to a rarer medium ($\mu_{CB} = 1.55$).
3. The refractive index of *O*-ray w.r.t. Canada Balsam $= \mu_{CB}/\mu_O$
4. The critical angle for *O*-ray, $\theta_c = \sin^{-1}(\mu_{CB}/\mu_O) = \sin^{-1}(1.55/1.66) = 69°$
5. Since the length of the prism is sufficient, the angle of incidence of *O*-ray at Canada Balsam layer becomes greater than its critical angle. Hence the *O*-ray is totally reflected from the Canada Balsam layer.
6. On the other hand, the *E*-ray is going from calcite to Canada Balsam travels from an optically rarer medium ($\mu_E = 1.49$) to a denser medium ($\mu_{CB} = 1.55$).
7. In this way, the light emerging from the Nicol prism is plane polarized with vibrations parallel to the principal section.

7. Attempt any **one** part of the following : **(5 × 1 = 5)**

a. **Show that two level laser system has no practical significance for lasing. Explain the principle of three level lasers.**

Ans. Refer Q. 5.21, Page 5–23A, Unit-5.

b. **Discuss different types of optical fibre. Why graded index fibre is better than multimode step index fibre ?**

Ans. Refer Q. 5.4, Page 5–6A, Unit-5.

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B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION, 2014-15

ENGINEERING PHYSICS-I

Time : 3 Hours **Max. Marks : 100**

SECTION – A

1. Attempt **all** parts of this question. Each part carries 2 marks. **(2 × 5 = 10)**

a. **What are inertial and non-inertial frames of reference ?**

Ans. **Inertial Frame :** Refer Q. 1.2, Page SQ–1A, 2 Marks Questions, Unit-1.

Non-inertial Frame : Refer Q. 1.3, Page SQ–1A, 2 Marks Questions, Unit-1.

b. **What is double refraction ?**

Ans. The phenomenon in which we get two refracting plane polarized rays corresponding to one incident polarized light ray is called double refraction.

c. **Why are fringes circular in Newton's ring experiment ? Explain.**

Ans. Refer Q. 4.7, Page SQ–12A, 2 Marks Questions, Unit-4.

d. **What is stimulated emission of radiation in a laser ?**

Ans. Refer Q. 5.14, Page SQ–16A, 2 Marks Questions, Unit-5.

e. **What do you know about acceptance angle and cone in a fiber ?**

Ans. **Acceptance Angle :** Refer Q. 5.7, Page SQ–15A, 2 Marks Questions, Unit-5.

Acceptance Cone : Refer Q. 5.10, Page SQ–16A, 2 Marks Questions, Unit-5.

SECTION – B

2. Attempt any **three** of this question. Each part carries 5 marks. **(5 × 3 = 15)**

a. **Calculate the work done to increase speed of an electron of rest energy 0.5 MeV from 0.6c to 0.8c.**

Ans. Refer Q. 1.30, Page 1–31A, Unit-1.

b. **An electron is bound in one dimensional potential box which has width 2.5×10^{-10} m. Assuming the height of the box to be infinite, calculate the lowest two permitted energy values of the electron.**

Ans. Refer Q. 3.19, Page 3–18A, Unit-3.

c. **Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of**

the 15th bright ring is 0.590 cm and the diameter of the 5th ring is 0.336 cm, what is the wavelength of light used ?

Ans. Refer Q. 4.16, Page 4–23A, Unit-4.

d. **Find out if a diffraction grating will resolve the lines 8037.20 Å and 8037.50 Å in the second order given that the grating is just able to resolve two lines of wavelengths 5140.34 Å and 5140.85 Å in the first order.**

Ans. Refer Q. 4.30, Page 4–41A, Unit-4.

e. **If refractive indices of core and cladding of an optical fibre are 1.50 and 1.45 respectively determine the values of numerical aperture, acceptance angle and critical angle of the fibre.**

Ans. Refer Q. 5.7, Page 5–11A, Unit-5.

SECTION – C

3. Attempt any **one** part of all questions. Each question carries 5 marks. **(5 × 5 = 25)**

a. **Deduce the Lorentz transformation equations from Einstein's postulates. Also show that at low velocities, the Lorentz transformations reduce to Galilean transformations.**

Ans. Refer Q. 1.8, Page 1–11A, Unit-1.

b. **Deduce the relativistic velocity addition theorem. Show that it is consistent with Einstein's second postulate.**

Ans. Refer Q. 1.20, Page 1–20A, Unit-1.

4. a. **Explain group velocity. Establish a relation between group velocity and phase velocity and show that these velocities are equal in non-dispersive medium.**

Ans.

A. **Group Velocity :** The velocity with which a wave packet moves forward in the medium is called group velocity.

B. **Relation between Group Velocity and Phase Velocity :**

1. A wave packet consists of a group of waves slightly differing in their wavelength, velocities, phase and amplitude as shown in Fig. 1.

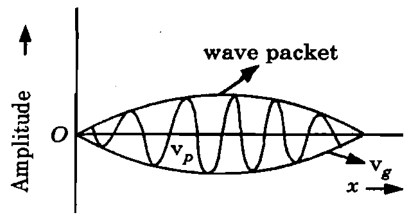


Fig. 1.

2. Such a wave packet moves with its own velocity called group velocity (v_g) or particle velocity and velocity of individual waves forming the wave packet is called phase velocity (v_p).

3. We know that $v_p = \nu\lambda$
   and the de-Broglie's wavelength

$$\lambda = \frac{h}{mv}$$

and $$E = h\nu = mc^2 \Rightarrow \nu = mc^2/h$$

4. Eq. (1) becomes,

$$v_p = \left(\frac{mc^2}{h}\right)\left(\frac{h}{mv}\right) = \frac{c^2}{v}$$

5. Since group velocity (v_g) is equal to particle velocity (v) *i.e.*, v_g =

Then $$v_p = \frac{c^2}{v_g} \quad \text{or} \quad v_p \cdot v_g = c^2$$

C. **Velocities in Non-Dispersive Medium :**

1. The angular frequency ω of de-Broglie's waves associated with particle of rest mass m_o and moving with velocity v is given by,

$$\omega = 2\pi\nu \text{ and } E = h\nu$$

$$\Rightarrow \quad \nu = \frac{E}{h} \text{ and } E = mc^2$$

$$\omega = \frac{2\pi E}{h} = \frac{2\pi mc^2}{h} = \frac{2\pi m_o c^2}{h \times \sqrt{1-\frac{v^2}{c^2}}} \quad \left[\because m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$\omega = \frac{2\pi m_o c^2}{h}\left(1-\frac{v^2}{c^2}\right)^{\frac{-1}{2}}$$

2. Differentiate both sides of above equation w.r.t. v, we get

$$\frac{d\omega}{dv} = \frac{2\pi m_o c^2}{h}\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{\frac{-3}{2}}\left(-\frac{2v}{c^2}\right)$$

$$\frac{d\omega}{dv} = \frac{2\pi m_o v}{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad ...(2)$$

3. Also, the wave number k of de-Broglie's wave is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi mv}{h} = \frac{2\pi m_o v}{h \times \sqrt{1-\frac{v^2}{c^2}}}$$

4. Differentiate both sides of above equation w.r.t. v, we get

$$\frac{dk}{dv} = \frac{2\pi m_o}{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \quad ...(3)$$

5. We know that group velocity,

$$v_g = \frac{d\omega}{dk}$$

From eq. (2) and eq. (3), we get

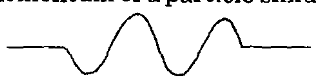
$$v_g = \frac{2\pi m_o v}{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \cdot \frac{h\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}{2\pi m_o} = v$$

b. Explain Heisenberg's uncertainty principle ? Describe Heisenberg's gamma ray microscope.

Ans.

A. Heisenberg's Uncertainty Principle :

1. According to this principle, "It is impossible to determine the exact position and momentum of a particle simultaneously".



(*a*) Δp large, Δx small

(*b*) Δx large, Δp small

Fig. 2. Wave-packet : (a) Narrow, and (b) Wide.

2. If Δx and Δp are the uncertain position and momentum of particle then according to this principle

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$

or $\quad \Delta x \, \Delta p \geq \hbar$

The product of uncertainty position and uncertainty momentum of particle is greater than or equal to $h/2\pi$.

B. Heisenberg's Gamma-ray Microscope :

1. Let us try to measure the position and linear momentum of an electron using an imaginary microscope with a very high resolving power as shown in Fig. 3.
2. The electron can be observed if at least one photon is scattered by it into the microscope lens.
3. The limit of resolution of the microscope is given by the relation.

$$d = \frac{\lambda}{2\sin\theta}$$

Here *d* represents the distance between the two points which can be just resolved by the microscope.

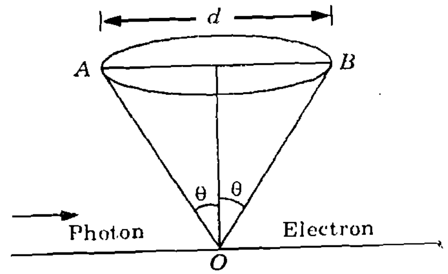


Fig. 3. Measurement of position and linear momentum of an electron.

4. This is the range in which the electron would be visible when disturbed by the photon.
5. Therefore, the uncertainty in the measurement of the position of the electron is

$$\Delta x = d = \frac{\lambda}{2\sin\theta} \quad ...(1)$$

6. However, the incoming photon will interact with the electron through the Compton effect.
7. To see this electron, the scattered photon should enter the microscope within the angle 2θ.
8. The momentum imparted by the photon to the electron during the impact is of the order of h/λ.
9. The component of this momentum along *OA* is $(-h/\lambda \sin\theta)$ and that along *OB* is $(h/\lambda \sin\theta)$.
10. Hence the uncertainty in the measurement of the momentum of the electron is

$$\Delta p = \left(\frac{h}{\lambda}\sin\theta\right) - \left(-\frac{h}{\lambda}\sin\theta\right) = 2\frac{h}{\lambda}\sin\theta \quad ...(2)$$

11. Multiplying eq. (1) by eq. (2), we obtain

$$\Delta x \, \Delta p = \frac{\lambda}{2\sin\theta} \, 2\frac{h}{\lambda}\sin\theta = h$$

12. A more sophisticated approach will show that $\Delta x \, \Delta p$

5. a. Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light the diameter of bright rings are proportional to the square roots of odd natural numbers.

Ans. Refer Q. 4.12, Page 4–18A, Unit-4.

b. Discuss the phenomenon of diffraction at a single slit and show that the relative intensities of the successive maximum are nearly 1 : 4/9π² : 4/25π² ...

Ans. Refer Q. 4.19, Page 4–25A, Unit-4.

6. a. Describe the construction, working and application of Nicol prism.

Ans.

A. Construction :

1. It is a calcite crystal with a principal section $ABCD$ whose length is three times its breadth as shown in Fig. 4.

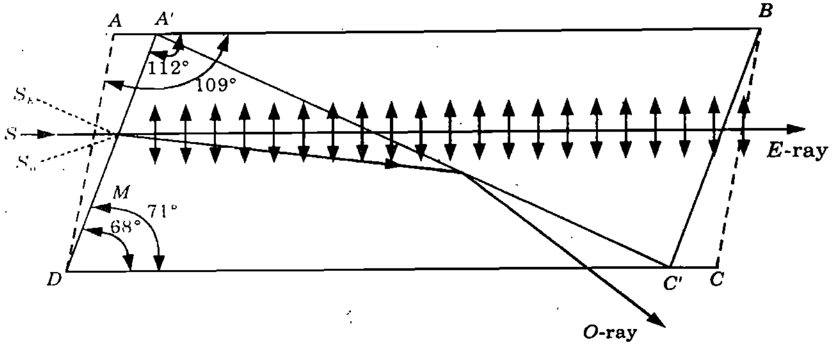


Fig. 4. Construction of a calcite crystal.

2. In this situation, the new blunt corners will A' and C'.
3. The crystal is then cut into two pieces from one blunt corner A' to other blunt corner C' along a plane $A'C'$ perpendicular to the principal section $ABCD$ and perpendicular to both the faces $A'D'$ and BC'.
4. The two cut surfaces so obtained are polished optically flat and joined by a transparent medium called Canada balsam.
5. Canada Balsam is a transparent liquid whose refractive index lies between the refractive indices of calcite for the O-ray and E-ray.

B. Working : Refer Q. 6(b), Page SP–4A, Solved Paper 2013-14.

C. Application :

1. Use in microscopy and polarimetry.
2. Used for observing the sample placed between orthogonally oriented polarizers.

b. Discuss the He-Ne laser with necessary diagrams. Give its superiority over ruby laser.

Ans. Refer Q. 5.23, Page 5–25A, Unit-5.

7. a. Explain single mode and multimode fibres. Also give the characteristics of each type of mode.

Ans. Refer Q. 5.4, Page 5–6A, Unit-5.

b. Explain the process of a hologram construction with necessary diagrams. Also give some applications of hologram.

Ans.

A. Hologram Construction :

1. The monochromatic light from a laser has been passed through a 50 % beam splitter so that the amplitude division of the incident beam into two beams takes place.

2. One beam falls on mirror M_1 and the light reflect from M_1 falls on the object. This beam is known as an object beam.
3. The object scatters this beam in all directions, so that a part of the scattered beam falls on the holographic plate.
4. The other beam is reflected by mirror M_2 and falls on the holographic plate. This beam is known as reference beam.
5. Superposition of the scattered rays from the object and the reference beam takes place on the plane of the holographic plate, so that interference pattern is formed on the plate and it is recorded.
6. The recorded interference pattern contains all the information of the scattered rays *i.e.*, the phases and intensities of the scattered rays.
7. For proper recording, the holographic plate has to be exposed to the interference pattern for a few seconds.
8. After exposing, the holographic plate is to be developed and fixed as like in the case of ordinary photograph.
9. The recorded holographic plate is known as hologram.
10. The hologram does not contain a distinct image of the object. It contains information in the form of interference pattern.
11. Fig. 5 shows the method of recording an image on a holographic plate.

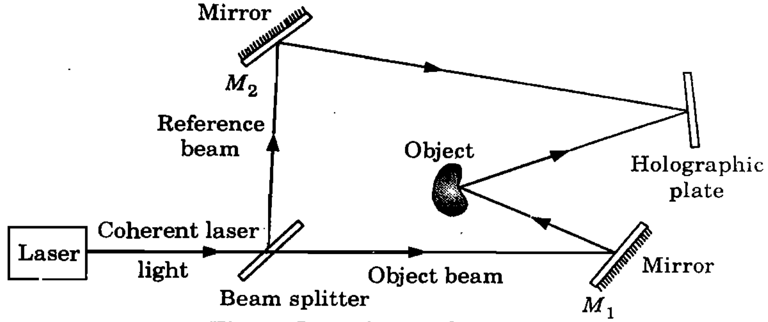


Fig. 5. Recording of hologram.

B. Applications :

1. The 3-D images produced by holograms have been used in various fields, such as technical, educational also in advertising, artistic display etc.
2. In hospitals, holography can be used to view the working of inner organs three dimensionally.
3. Holographic interferometry is used in non-destructive testing of materials to find flaws in structural parts and minute distortions due to stress or vibrations, etc. in the objects.
4. Holography is used in information coding.
5. Many museums have made holograms of valuable articles in their collections.

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B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION, 2015-16

ENGINEERING PHYSICS-I

Time : 3 Hours **Max. Marks : 100**

SECTION - A

1. Attempt **all** parts. All sections carry equal marks. Write answer of each part in short : **(2 × 10 = 20)**

a. How the negative results of Michelson-Morley experiment interpreted ?

Ans. Refer Q. 1.20, Page SQ–3A, 2 Marks Questions, Unit-1.

b. Find relativistic relation between energy and momentum.

Ans. Refer Q. 1.17, Page SQ–3A, 2 Marks Questions, Unit-1.

c. If uncertainty in the position of a particle is equal to de-Broglie wavelength, what will be uncertainty in the measurement of velocity ?

Ans.

Given : Uncertainty in the position of a particle = de-Broglie wavelength

$$\Delta x = \lambda = \frac{h}{p} \quad ...(1)$$

To Find : Uncertainty in the velocity.

1. According to Heisenberg's uncertainty principle,

$$\Delta x \,.\, \Delta p \geq \frac{h}{2\pi} \quad ...(2)$$

2. Let Δv be the uncertainty in velocity,

$$\Delta p = m\Delta v \quad ...(3)$$

3. Putting eq. (1) and eq. (3) in eq. (2)

$$\frac{h}{p} \cdot m\,\Delta v = \frac{h}{2\pi} \quad [\because\ p = mv]$$

So, uncertainty in velocity,

$$\Delta v = \frac{v}{2\pi}$$

d. Write the characteristics of wave function.

Ans. Refer Q. 3.13, Page SQ–10A, 2 Marks Questions, Unit-3.

e. Why the center of Newton's ring is dark ?

Ans. Refer Q. 4.19, Page SQ–14A, 2 Marks Questions, Unit-4.

f. Define plane of polarization and plane of vibration.

Ans. **Plane of Polarization :** The plane containing the direction of propagation of the light, but containing no vibrations is called the plane of polarization.

Plane of Vibration : The plane containing the direction of vibration and direction of propagation of light is called the plane of vibration.

g. Define optic axis of doubly refracting crystal.

Ans. A certain direction in a doubly refracting crystal along which the speed of light of two refracted light rays remains the same, is known as optic axis of that doubly refracting crystal.

h. What is Rayleigh's criterion of resolution ?

Ans. Refer Q. 4.16, Page SQ–13A, 2 Marks Questions, Unit-4.

i. Define metastable state.

Ans. Refer Q. 5.17, Page SQ–16A, 2 Marks Questions, Unit-5.

j. Give few important applications of optical fibre.

Ans. Refer Q. 5.18, Page SQ–17A, 2 Marks Questions, Unit-5.

SECTION - B

Note : Attempt any **five** questions : **(10 × 5 = 50)**

2. What do you mean by proper length ? Derive the expression for relativistic length. Calculate the percentage contraction of a rod moving with a velocity of 0.8c in a direction inclined at 30° to its own length.

Ans. Refer Q. 1.13, Page 1–15A, Unit-1.

3. Show that the relativistic invariance of the law of conservation of momentum leads to the concept of variation of mass with velocity.

Ans. Refer Q. 1.23, Page 1–24A, Unit-1.

4. State Heisenberg's uncertainty principle. Prove that electron cannot exist inside the nucleus and proton can exist.

Ans.

A. Heisenberg's Uncertainty Principle : Refer Q. 4(b), Page SP–9A, Solved Paper 2014-15.

B. Non-existence of Electrons in the Nucleus :

1. We know that the radius of nucleus is the order of 10^{-14} m.

2. If an electron is confined within nucleus the uncertainty position of electron is

$$\Delta x = 2 \times 10^{-14} \text{ m}$$

3. Now according to uncertainty principle,

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$

and
$$\Delta p = \frac{h}{2\pi \Delta x} = \frac{6.63 \times 10^{-34}}{2 \times \pi \times 2 \times 10^{-14}} = 5.276 \times 10^{-21} \text{ kg m/s}$$

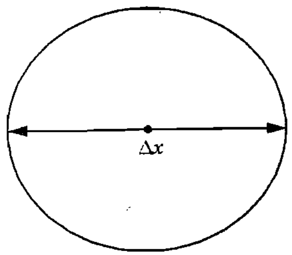

Fig. 1.

4. Using relativistic formula for the energy of the electron

$$E^2 = p^2 c^2 + m_o^2 c^4$$

5. As the rest energy m_oc^2 of an electron is of the order of 0.511 MeV, which is much smaller than the value of first term. Hence the second term is neglected therefore,

$$E^2 = p^2 c^2$$
$$E = pc = (5.276 \times 10^{-21}) \times (3 \times 10^8) \text{ J}$$
$$E = \frac{5.276 \times 10^{-21} \times 3 \times 10^8}{1.6 \times 10^{-19}} \text{ eV} \approx 97 \text{ MeV}$$

6. Thus, if an electron exists inside the nucleus then its energy should be of the order of 97 MeV. But the experiment shows that no electron in the atom possesses kinetic energy greater than 4 MeV.
7. Hence, no electron can exist inside the nucleus.

C. Existence of Proton in Nucleus :

1. We know that the radius of an atom is of the order of 10^{-14} m. Thus, if a proton exists inside the nucleus then the maximum uncertainty in its position is given by

$$(\Delta x)_{max} = 2 \times 10^{-14} \text{ m}$$

2. Using the uncertainty relation,

$$(\Delta p)(\Delta x) = h$$

The minimum uncertainty in the momentum of proton is given by

$$(\Delta p)_{min} = \frac{h}{(\Delta x)_{max}} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}} = 5.275 \times 10^{-21} \text{ kg-m/s}$$

3. Since the minimum uncertainty in the momentum of a proton should be equal to its momentum, *i.e.*,

$$p = (\Delta p)_{min} = 5.275 \times 10^{-21} \text{ kg-m/s}$$

4. The corresponding energy of the proton is given by

$$E = \frac{p^2}{2m} = \frac{(5.275 \times 10^{-21})^2}{2 \times 1.6 \times 10^{-27}} \text{ J}$$

i.e., $$E = \frac{(5.275 \times 10^{-21})^2}{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-19}} \text{ eV}$$

i.e., $$E = 52 \text{ keV}$$

5. Thus, if a proton exists inside the nucleus then its energy should be of the order of 52 keV. The experiment shows that a proton in the atom possesses kinetic energy of the order of 52 keV. Hence proton can exist inside the nucleus.

5. Explain the physical significance of wave function. Derive Schrodinger's time independent wave equation.

Ans. **Physical Significance of Wave Function :** Refer Concept Outline-2, Page 3–11A, Unit-3.

Schrodinger's Time Independent Wave Equation : Refer Q. 3.13, Page 3–11A, Unit-3.

6. Explain the formation of Newton's ring ? If in a Newton's ring experiment, the air in the interspaces is replaced by a liquid of refractive index 1.33, in what proportion would the diameter of the rings changed ?

Ans. Refer Q. 4.13, Page 4–20A, Unit-4.

7. Discuss the phenomenon of diffraction at a single slit and show that intensities of successive maxima are :

$$1 : \frac{4}{9\pi^2} : \frac{4}{15\pi^2} : \frac{4}{49\pi^2}$$

Ans. Refer Q. 4.19, Page 4–25A, Unit-4.

8. Discuss the construction and working of a He-Ne laser. Compare it with ruby laser.

Ans. Refer Q. 5.23, Page 5–25A, Unit-5.

9. Describe the basic principle of communication of wave in optical fibre. A step index fibre has core refractive index 1.468, cladding refractive index 1.462. Compute the maximum radius allowed for a fibre, if it supported only one mode at a wavelength 1300 nm.

Ans. Refer Q. 5.9, Page 5–12A, Unit-5.

SECTION – C

Note : Attempt any **two** questions from this section : **(15 × 2 = 30)**

10. a. Derive the Galilean transformation equations and show that its acceleration components are invariant.

Ans. Refer Q. 1.2, Page 1–3A, Unit-1.

b. If the kinetic energy of a body is twice its rest mass energy, find its velocity.

Ans. Refer Q. 1.32, Page 1–32A, Unit-1.

c. Explain de-Broglie's hypothesis. Discuss the outcome of Davisson-Germer's experiment in detail.

Ans.

A. de-Broglie's Hypothesis : Refer Q. 3.7, Page 3–7A, Unit-3.

B. Outcomes of Davisson-Germer's Experiment :

1. Davisson and Germer calculated the de-Broglie wavelength using two different approaches.
2. In the first approach, Davisson and Germer used de-Broglie's hypothesis.
3. They plotted the variation in the intensity of electron beam against scattering angle for different accelerating voltages to study the effect of increasing electron energy on the scattering angle φ.
4. They found that a bump begins to appear in the curve for $V = 44$ volts.
5. With increasing potential, the bump moves upward, and becomes more prominent in the curve for $V = 54$ volts at $\varphi = 50°$, thereby indicating the maximum suffering in electron beam for $V = 54$ volts as shown in Fig. 2.

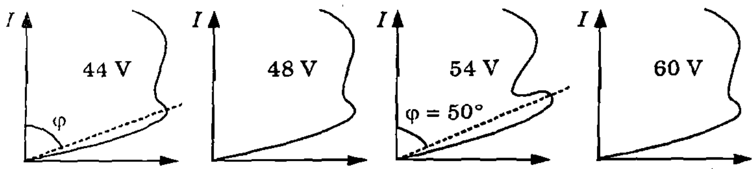

Fig. 2. Plots of intensity of electron beam against scattering angle for different values of accelerating voltage.

6. Thus, for $V = 54$ V, the de-Broglie wavelength of the electrons is

$$\lambda = \frac{12.24}{\sqrt{V}} = \frac{12.24}{\sqrt{54}} = 1.66 \text{ Å} \quad ...(1)$$

7. In the second approach, Davisson and Germer calculated the de-Broglie wavelength by treating the electron beam as a wave.
8. They used Bragg's equation, $n\lambda = 2d \sin \theta$.
9. For nickel crystal, $d = 0.91$ Å. Also, $\theta = 65°$. Hence for the first order ($n = 1$) reflection, we have

$$\lambda = 2d \sin \theta = 2 \times 0.91 \times \sin 65° = 1.65 \text{ Å} \quad ...(2)$$

10. Eq. (1) and eq. (2) show an excellent agreement between the two approaches.
11. Thus, the Davisson–Germer experiment provides a direct verification of wave nature of electrons and hence it also verifies the de Broglie's hypothesis.

11. a. Explain the phenomenon of interference in thin film due to reflected rays.

Ans. Refer Q. 4.3, Page 4–6A, Unit-4.

b. A diffraction grating used at normal incidence gives a yellow line (λ = 6000 Å) in a certain spectral order superimposed on a blue line (λ = 4800 Å) of next higher order. If the angle of diffraction is $\sin^{-1}$ (3/4), calculate the grating element.

Ans. Refer Q. 4.28, Page 4–40A, Unit-4.

c. Describe the construction and working of Nicol prism.

Ans. Refer Q. 6(b), Page SP–4A, Solved Paper 2013-14.

12. a. Prove that $v_p \times v_g = c^2$, where v_p = phase velocity and v_g = group velocity.

Ans. Refer Q. 4(a), Page SP–7A, Solved Paper 2014-15.

b. Discuss the different types of optical fiber in detail.

Ans. Refer Q. 5.4, Page 5–6A, Unit-5.

c. In a Ruby Laser, total number of Cr^{+3} is 2.8×10^{19}. If the Laser emits radiation of wavelength 7000 Å, calculate the energy of the Laser pulse.

Ans. Refer Q. 5.26, Page 5–28A, Unit-5.

Physical Constants :

Mass of electron	$m_0 = 9.1 \times 10^{-31}$ kg
Mass of proton	$m_p = 1.67 \times 10^{-27}$ kg
Speed of light	$c = 3 \times 10^8$ m/s
Planck's constant	$h = 6.63 \times 10^{-34}$ J/s
Charge of electron	$e = 1.67 \times 10^{-19}$ C
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ m^2 kg s^{-2} k^{-1}

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B.Tech.
(SEM. I) ODD SEMESTER THEORY EXAMINATION, 2016-17
ENGINEERING PHYSICS-I

Time : 3 Hours **Total Marks : 70**

Note : **A, B and C are three sections in this question paper. Attempt all seven parts from section A, any three parts from section B and all questions from section C.**

Section – A

1. Attempt **all** parts of this section. **(2 × 7 = 14)**

a. **What is proper length of a rod ?**

Ans. Refer Q. 1.21, Page SQ–3A, 2 Marks Questions, Unit-1.

b. **Explain the concept of rest mass of photon.**

Ans. The rest mass is the mass of a particle (photon) as measured by an observer who sees the particle still and with zero speed. In other words, the particle is at rest as far as this observer is concerned. Thus comes the term "rest mass". But according to special relativity, light always travels with the light speed c, and is never at rest and so particle has zero rest mass.

c. **What is Wien's law ?**

Ans. Refer Q. 3.4, Page SQ–8A, 2 Marks Questions, Unit-3.

d. **Explain the factor responsible for changing fringe width in wedge shaped film.**

Ans. Refer Q. 4.3, Page SQ–11A, 2 Marks Questions, Unit-4.

e. **What happens to diffraction pattern when slit width of single slit experiment increases ?**

Ans. Refer Q. 4.11, Page SQ–12A, 2 Marks Questions, Unit-4.

f. **What are metastable states ?**

Ans. Refer Q. 5.17, Page SQ–16A, 2 Marks Questions, Unit-5.

g. **What precautions are needed to minimize material dispersion ?**

Ans. Refer Q. 5.24, Page SQ–17A, 2 Marks Questions, Unit-5.

Section – B

2. Attempt any **three** parts : **(7 × 3 = 21)**

a. **Describe Michelson - Morley experiment and explain the outcome of the experiment.**

Ans. Refer Q. 1.4, Page 1–5A, Unit-1.

b. **Derive time independent Schrodinger wave equation and give physical interpretation of wave function. Also explain eigen value and eigen function.**

Ans.

A. **Time Independent Schrodinger Wave Equation :** Refer Q. 3.13, Page 3–11A, Unit-3.

B. **Physical Interpretation of Wave Function :** Refer Concept Outline : Part 2, Page 3–11A, Unit-3.

C. **Eigen Value and Eigen Function :**

1. The values of energy E_n for which Schrodinger's steady-state equation can be solved are called eigen values and the corresponding wave functions Ψ_n are called eigen functions.
2. The discrete energy levels of the hydrogen atom are an example of a set of eigen values.
3. Eigen value equation

$$\hat{G}\,\Psi_n = G_n \Psi_n \quad ...(1)$$

Where $\hat{G}$ is the operator that corresponds to G and each G_n is a real number.

4. When eq. (1) holds for the wave function of a system, it is a fundamental postulate of quantum mechanics that any measurement of G can only yield one of the values of G_n.
5. If measurements of G are made on a number of identical systems all in states described by the particular eigen function Ψ_k, each measurement will yield the single value G_k.

c. **What do you understand by Newton's ring ? Explain their experimental arrangement. How can you determine the wavelength of light with this experiment ?**

Ans.

A. **Newton's Ring and their Experimental Arrangement :** Refer Q. 4.11, Page 4–16A, Unit-4.

B. **Wavelength of Light :** Refer Q. 4.15, Page 4–22A, Unit-4.

d. **What is the concept of four level laser systems ? Give the construction and working of He-Ne laser.**

Ans.

A. **Four Level Laser System :** Refer Q. 5.20, Page 5–21A, Unit-5.

B. **He-Ne Laser :** Refer Q. 5.23, Page 5–25A, Unit-5.

e. **What do you understand by modes of an optical fibre ? Discuss propagation of light in single mode, multimode and graded index fibres.**

Ans.

A. **Modes of an Optical Fibre :** Refer Q. 5.8, Page SQ–15A, 2 Marks Questions, Unit-5.

B. **Propagation of Light in Single Mode, Multimode and Graded Index Fibres :** Refer Q. 5.4, Page 5–6A, Unit-5.

Section – C

3. Attempt any **two** parts : **(3.5 × 2 = 7)**

a. **What do you mean by length contraction ? Explain it.**

Ans. Refer Q. 1.12, Page 1–14A, Unit-1.

b. **Deduce and discuss Einstein's mass-energy relation, $E = mc^2$**

Ans. Refer Q. 1.24, Page 1–26A, Unit-1.

c. Calculate the percentage contraction of a rod moving with a velocity of 0.8 c in a direction at 60° to its own length.

Ans. Same as Q. 1.13, Page 1–15A, Unit-1. (**Ans.** = 8.34%)

4. Attempt any **two** parts : **(3.5 × 2 = 7)**

a. Describe energy distribution in black body radiation.

Ans. Refer Q. 3.1, Page 3–2A, Unit-3.

b. Explain the modified and unmodified radiations in Compton scattering ?

Ans. Refer Q. 3.24, Page 3–23A, Unit-3.

c. Calculate the wavelength of an electron associated with kinetic energy of 6.95 × 10^{-25} Joules.

Ans. Same as Q. 3.10, Page 3–9A, Unit-3. (**Ans.** = 5.895 × 10^{-7} m)

5. Attempt any **two** parts : **(3.5 × 2 = 7)**

a. Explain the missing orders in the spectra of a plane transmission grating.

Ans. Refer Q. 4.24, Page 4–37A, Unit-4.

b. Explain Rayleigh criterion of resolution.

Ans. Refer Q. 4.25, Page 4–38A, Unit-4.

c. A plane transmission grating has 15000 lines per inch. Find the resolving power of grating and the smallest wavelength difference that can be resolved with a light of wavelength 6000 A° in the second order.

Ans.

Given : N = 15000, n = 2, λ = 6000 Å

To Find : i. Resolving power.
ii. Smallest wavelength difference.

1. The resolving power of a grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

$$\left(\frac{\lambda}{d\lambda}\right) = 2 \times 15000 = 30{,}000$$

2. The smallest wavelength difference is given by

$$\frac{\lambda}{d\lambda} = 30{,}000$$

$$d\lambda = \frac{\lambda}{30{,}000} = \frac{6000 \times 10^{-10}}{30{,}000} = 0.2 \text{ Å}$$

6. Attempt any **two** parts : **(3.5 × 2 = 7)**

a. Show that the plane polarized and circularly polarized light are the special cases of elliptically polarized light.

Ans.

1. Suppose that a plane polarized light ray of amplitude A is incident on a uniaxial crystal at an angle of θ as shown in Fig. 1.

2. Let A cos θ and A sin θ be the amplitudes of E-ray and O-ray respectively. If δ be the phase difference between the two emergent rays, then their vibrations can be expressed as

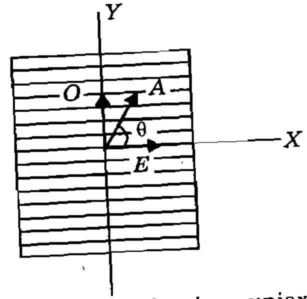

Fig. 1. Double refraction in a uniaxial crystal.

$$x = A \cos\theta \sin(\omega t + \delta) = a \sin(\omega t + \delta) \quad ...(1)$$

[For E-ray] $y = A \sin\theta \sin\omega t = b \sin\omega t$ ...(2)

[For O-ray] $y = A \cos\theta \sin\omega t = a \sin\omega t$

Where, $a = A \cos\theta$ and $b = A \sin\theta$

3. From eq. (2), we have

$$\sin\omega t = y/b \quad ...(3)$$

Hence, $\cos\omega t = [1 - \sin^2\omega t]^{1/2} = [1 - (y/b)^2]^{1/2}$ ...(4)

4. Now from eq. (1), we have

$$x = a \sin(\omega t + \delta) = a(\sin\omega t \cos\delta + \cos\omega t \sin\delta) \quad ...(5)$$

5. Using eq. (3) and eq. (4) in eq. (5), and rearranging, we have

$$(x/a)^2 + (y/b)^2 - [(2xy \cos\delta)/ab] = \sin^2\delta \quad ...(6)$$

Eq. (6) is the general eq. of an ellipse. Now we consider the following three cases :

Case I :

1. When δ = 0, then from eq. (6), we have

$$(x/a)^2 + (y/b)^2 - [(2xy \cos 0)/ab] = \sin^2 0$$

$$(x/a)^2 + (y/b)^2 - [(2xy)/ab] = 0$$

$$[(x/a) - (y/b)]^2 = 0$$

$$y = (b/a)\,x$$

2. This is the equation of a straight line as shown in Fig. 1(a). In this case, the emergent light is plane polarized.

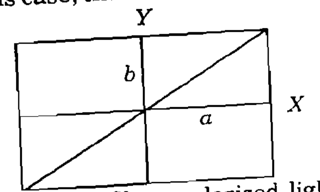

Fig. 1. (a) Plane polarized light.

Case II :

1. When $\delta = \pi/2$, then from eq. (6), we have

$$(x/a)^2 + (y/b)^2 - [(2xy \cos\pi/2)] = \sin^2\pi/2$$

$$(x/a)^2 + (y/b)^2 = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

2. This is the equation of a symmetrical ellipse as shown in Fig. 1(b). In this case, the emergent light is elliptically polarized.

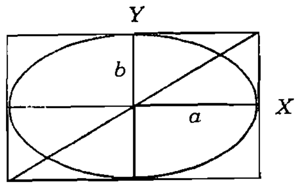

Fig. 1. (b) Elliptically polarized light.

Case III :

1. When $\delta = \pi/2$ and $a = b$, then from eq. (6), we have

$$(x/a)^2 + (y/a)^2 - [(2xy \cos \pi/2)a^2] = \sin^2 \pi/2$$
$$(x/a)^2 + (y/a)^2 = 1$$
$$x^2 + y^2 = a^2$$

2. This is the equation of a circle as shown in Fig. 1(c). In this case, the emergent light is circularly polarized.

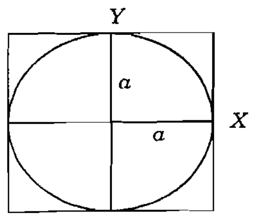

Fig. 1. (c) Circularly polarized light.

6. From the above discussion, it is clear that the plane polarized light and the circularly polarized light are the special case of elliptically polarized light.

b. What are Einstein's coefficients ? Obtain a relation between them.

Ans. Refer Q. 5.18, Page 5–19A, Unit-5.

c. A certain length of 5 % solution causes the optical rotation of 20°. How much length of 10 % solution of the same substance will cause 35° rotation ?

Ans.

Given : $\theta_1 = 20°$ and $\theta_2 = 35°$

To Find : Length of 10 % solution.

1. Let length of 5 % solution = l_1, and Length of 10 % solution = l_2
2. Since substance is same,

$$\therefore \quad \text{Specific rotation } S = \frac{\theta_1}{l_1 C_1} = \frac{\theta_2}{l_2 C_2}$$

$$\frac{20°}{l_1 \times 5\ \%} = \frac{35°}{l_2 \times 10\ \%}$$

$$l_2 = 7/8\, l_1$$

7. Attempt any **two** parts : **(3.5 × 2 = 7)**

a. Describe different types of losses in optical fibre.

Ans. Refer Q. 5.10, Page 5–12A, Unit-5.

b. Explain the construction and reconstruction of image in holography.

Ans.

A. Construction : Refer Q. 7(b), Page SP–11A, Solved Paper 2014-15.

B. Reconstruction :

1. As shown in Fig. 2, the hologram is exposed to the laser beam from one side and it can be viewed from the other side.

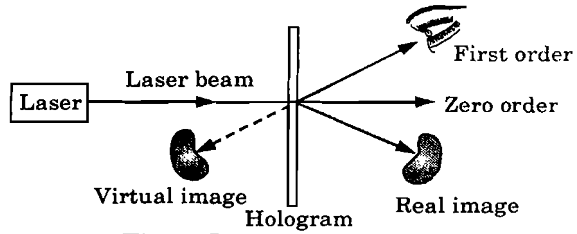

Fig. 2. Image reconstruction.

2. This beam is known as reconstruction beam.
3. The reconstruction beam illuminates the hologram at the same angle as the reference beam.
4. The hologram acts as a diffraction grating, so constructive interference takes place in some directions and destructive interference takes place in other direction.
5. A real image is formed in front of the hologram and a virtual image is formed behind the hologram.
6. It is identical to the object and hence it appears as if the object is present. The 3-D effect in the image can be seen by moving the head of the observer.
7. During recording, the secondary waves from every point of the object reach complete plate.
8. So, each bit of the plate contains complete information of the object.
9. Hence, image can be constructed using a small piece of hologram.

c. Calculate the acceptance angle and numerical aperture of the optical fibre if the refractive index of core and cladding are 1.50 and 1.45 respectively.

Ans. Refer Q. 5.7, Page 5–11A, Unit-5.

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B. Tech.
(SEM. I) ODD SEMESTER THEORY EXAMINATION, 2017-18
ENGINEERING PHYSICS-I

Time : 3 Hours **Max. Marks : 70**

Note : Attempt **all** sections. If require any missing data : then choose suitably.

Section-A

1. Attempt **all** questions in brief. **(2 × 7 = 14)**

a. **Is earth an inertial or non-inertial frame of reference ? Justify your answer.**

Ans. Refer Q. 1.19, Page SQ–3A, 2 Marks Questions, Unit-1.

b. **What is Wien's displacement law ?**

Ans. As the temperature of the body is raised the maximum energy tends to be associated with shorter wavelength, *i.e.*,

$$\lambda_m T = \text{Constant}$$

c. **What do you mean by group velocity ?**

Ans. The velocity with which the wave packet obtained by superposition of wave travelling in group is called group velocity

$$v_g = \frac{\Delta\omega}{\Delta K}$$

d. **Define dispersive power of a plane transmission diffraction grating.**

Ans. Refer Q. 4.14, SQ–13A, 2 Marks Questions, Unit-4.

e. **Differentiate between spontaneous and stimulated emission of radiation.**

Ans. Refer Q. 5.25, Page SQ–17A, 2 Marks Questions, Unit-5.

f. **What do you mean by specific rotation ?**

Ans. The specific rotation of an optically active substance at a given temperature for a given wavelength of light is defined as the rotation (in degrees) produced by a path of one decimeter length in a substance of unit density.

g. **What do you mean by acceptance angle ?**

Ans. Refer Q. 5.7, Page SQ–15A, 2 Marks Questions, Unit-5.

Section-B

2. Attempt any **three** parts of the following : **(7 × 3 = 21)**

a. **Obtain Galilean transformation equations. Show that length and acceleration are invariant under Galilean transformations.**

Ans.
A. **Galilean Transformation Equation and Acceleration Component are Invariant :** Refer Q. 1.2, Page 1–3A, Unit-1.
B. **Length Component are Invariant :** Refer Q. 1.3, Page 1–5A, Unit-1.

b. **Derive Planck's radiation law. Show that Planck's formula for the energy distribution in a thermal spectrum is applicable for all wavelengths.**

Ans. Refer Q. 3.5, Page 3–5A, Unit-3.

c. **Give the construction and theory of plane transmission grating. Explain the formation of spectra by it.**

Ans. Refer Q. 4.23, Page 4–36A, Unit-4.

d. **What is the advantage of four level laser systems over three level laser systems ? Describe the construction and working of ruby laser.**

Ans. Refer Q. 5.24, Page 5–27A, Unit-5.

e. **What is holography ? Explain the basic principle of holography using construction and reconstruction of image.**

Ans.
A. **Holography :** It is a method of producing a three dimensional image of an object employing the coherence properties of a laser beam.
B. **Construction and Reconstruction of Image :** Refer Q. 7(a), Page SP–24A, Solved Paper 2016-17.

Section-C

3. Attempt any **one** of the following : **(7 × 1 = 7)**

a. **Deduce the relativistic velocity addition theorem. Show that it is consistent with Einstein's second postulate.**

Ans. Refer Q. 1.20, Page 1–20A, Unit-1.

b. **What do you mean by time dilation ? Establish a relation for it. At what speed should a clock be moved so that it may appear to lose 1 min each hour ?**

Ans.
A. **Time Dilation :** Refer Q. 1.16, Page 1–17A, Unit-1.
B. **Numerical :** Refer Q. 1.19, Page 1–19A, Unit-1.

4. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. **What is the concept of de-Broglie matter waves ? Describe Davisson-Germer experiment and prove that electrons possess wave nature.**

Ans.
A. **De-Broglie Matter Waves :** Refer Q. 3.7, Page 3–7A, Unit-3.
B. **Davisson-Germer Experiment :** Refer Q. 10(c), Page SP–17A, Solved Paper 2015-16.

b. **Find an expression for the energy states of a particle in a one-dimensional box. Determine the probability of finding a particle trapped in a box of length L in the region from 0.45 L to 0.55 L for the ground state.**

Ans.

A. **Expression for the Energy States of a Particle in a One-dimensional Box :** Refer Q. 3.17, Page 3–16A, Unit-3.

B. **Numerical :** Refer Q. 3.22, Page 2–20A, Unit-3.

5. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. **Discuss the formation of interference fringes due to a wedge-shaped thin film seen by normally reflected monochromatic light and obtain an expression for the fringe width.**

Ans. Refer Q. 4.4, Page 4–9A, Unit-4.

b. **Obtain an expression for the intensity distribution due to Fraunhofer diffraction at a single slit. A light of wavelength 6000 Å falls normally on a slit of width 0.10 mm. Calculate the total angular width of the central maximum.**

Ans.

A. **Fraunhofer Diffraction at Single Slit :** Refer Q. 4.19, Page 4–25A, Unit-4.

B. **Numerical :** Refer Q. 4.20, Page 4–29A, Unit-4.

6. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. **Explain the phenomenon of double refraction and discuss the various characteristics of ordinary and extraordinary rays. Find the thickness of a quarter wave plate of quartz for light of wavelength 5893 Å. The refractive indices for ordinary and extraordinary rays are 1.544 and 1.553 respectively.**

Ans.

A. **Phenomenon of Double Refraction :** Refer Q. 6(b), Page SP–4A, Solved Paper 2013-14.

B. **Characteristics of Ordinary Ray :**

1. Ordinary ray travels with the same velocity in all directions of the crystal.
2. Velocity of *O*-ray is same along different directions of the crystal.
3. Ordinary ray obeys the law of reflection.
4. Vibrational vectors of *O*-ray are perpendicular to the optic axis.

C. **Characteristics of Extra Ordinary Ray :**

1. Extraordinary ray travels with different velocities along different directions of the crystal.
2. Velocity of *E*-ray is different along different directions of the crystal.
3. Vibrational vectors of *E*-ray are in the principal plane.

D. **Numerical :**

Given : $\mu_0 = 1.544$, $\mu_e = 1.553$ and for sodium light

$\lambda = 5893$ Å $= 5.893 \times 10^{-5}$ cm

To Find : Thickness of quarter wave plate.

1. The thickness of a quarter wave plate for positive crystal like quartz is

$$t = \frac{\lambda}{4(\mu_e - \mu_0)}$$

$$\therefore \quad t = \frac{5.893 \times 10^{-5}}{4(1.553 - 1.544)} = \frac{5.893 \times 10^{-5}}{4 \times 9 \times 10^{-3}} = 1.624 \times 10^{-3} \text{ cm}$$

b. **What do you mean by optical activity ? Give Fresnel's theory of optical activity and derive the necessary expression for the optical rotation.**

Ans.

A. **Optical Activity :**

1. When plane-polarized light passes through certain substances the plane of polarization (or plane of vibration) of light is rotated about the direction of propagation of light through a certain angle.
2. This property is known as optical activity and these substances are said to be optically active and this phenomenon is called optical rotation.

B. **Fresnel's Theory of Optical Activity :**

1. When beam of plane-polarized light incident on an optically active substance along its optic axis it splits into two oppositely directed circular motion, one is in clockwise direction, while another is in anticlockwise direction.
2. The velocity of two circularly polarized beams is different for optically active substance, and same for optically inactive substance.
3. Due to different velocities the phase difference occurs between them.
4. In dextrorotatory substance, the velocity of right handed component is greater than left handed component $v_R > v_L$.
5. In laevorotatory substance, the velocity of $v_L > v_R$
6. On emergence from the substance, these two circular motions recombine to produce a plane polarized light.

C. **Mathematical Explanation :**

1. Let a plane polarized beam be incident normally on a doubly refracting crystal like quartz plate with its faces perpendicular to the optic axis.
2. These beam divided into two parts, clockwise and anticlockwise directions, in circular motion.
3. The circular motions travelling along the optic axis have different velocities.

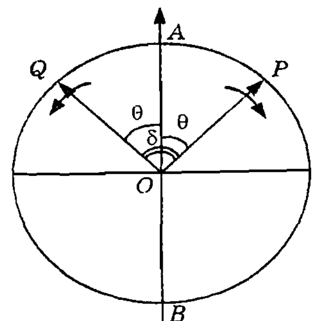

Fig. 1.

4. Therefore a phase difference (δ) occurs between them.
5. Now considering the case of dextrorotatory ($v_R > v_L$).
   AB – Optic axis.
   OP and OQ – Two circular motions rotating in opposite directions.
6. At the time of emergence these vibration are represented as
   $x_1 = a \cos(\omega t + \delta), y_1 = a \sin(\omega t + \delta)$ clockwise.
   $x_2 = -a \cos(\omega t), y_2 = a \sin(\omega t)$ anticlockwise.
7. By superposition theorem the resultant of x and y component are

$$X = x_1 + x_2 = a \cos(\omega t + \delta) - a \cos \omega t$$

$$X = 2a \sin\frac{\delta}{2} \cdot \sin\left(\omega t + \frac{\delta}{2}\right) \quad ...(1)$$

and

$$Y = y_1 + y_2 = a \sin(\omega t + \delta) + a \sin \omega t$$

$$Y = 2a \cos\frac{\delta}{2} \cdot \sin\left(\omega t + \frac{\delta}{2}\right) \quad ...(2)$$

8. The resulting vibrations along X-and Y-axis are in same period and phase.
9. Now dividing eq. (1) by eq. (2), we get

$$\frac{X}{Y} = \tan\frac{\delta}{2} \text{ or } X = Y \tan\frac{\delta}{2}$$

   This equation is of a straight line having slope $\tan \delta/2$ with Y-axis.
10. If μ_R and μ_L are refractive indices of right and left handed light and t is thickness of crystal,
    path difference = $(\mu_L - \mu_R)t$
    and phase difference = $\delta = \frac{2\pi}{\lambda}(\mu_L - \mu_R)t$
    and angle of rotation $2\theta = \delta$

$$2\theta = \frac{2\pi}{\lambda}(\mu_L - \mu_R)t$$

$$\theta = \frac{\pi}{\lambda}(\mu_L - \mu_R)t$$

7. Attempt any **one** part of the following : **(7 × 1 = 7)**

a. **Explain single mode and multimode fibres. Differentiate between characteristic properties of single mode and multimode fibres.**

Ans.

A. **Single Mode and Multimode Fibres :** Refer Q. 5.4, Page 5–6A, Unit-5.

B. **Difference between Single Mode and Multimode Fibres :** Refer Q. 5.6, Page 5–10A, Unit-5.

b. **Explain dispersion and attenuation in optical fibre. The optical power, after propagating through a 500 m long fibre, reduced to 25 % of its original value. Calculate fibre loss in dB/km.**

Ans. Refer Q. 5.14, Page 5–15A, Unit-5.

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