

# Growth and climate change: IAMs in Dynare

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## What is an integrated assessment model?

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- IAMs bring together economics, climate science, and energy systems within a single quantitative framework.
- **Purpose:** Provide a tractable way to evaluate long-run interactions between the economy and the climate.
- **Institutional use:**
  - **IPCC** (Intergovernmental Panel on Climate Change) relies on IAMs to define emission scenarios (e.g. SSP pathways).
  - **Central banks, NGFS, OECD, IEA, World Bank** use IAM-based tools for stress testing, policy design, and energy forecasts.
- IAMs are **quantitative tools** used for scenario design, policy evaluation, and to inform international climate negotiations.

## Intro: IAM schematic: economy-climate feedbacks

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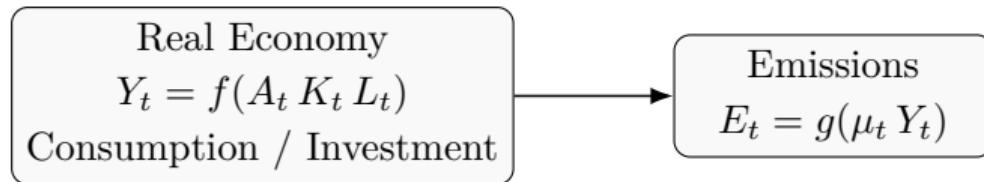
Real Economy

$$Y_t = f(A_t K_t L_t)$$

Consumption / Investment

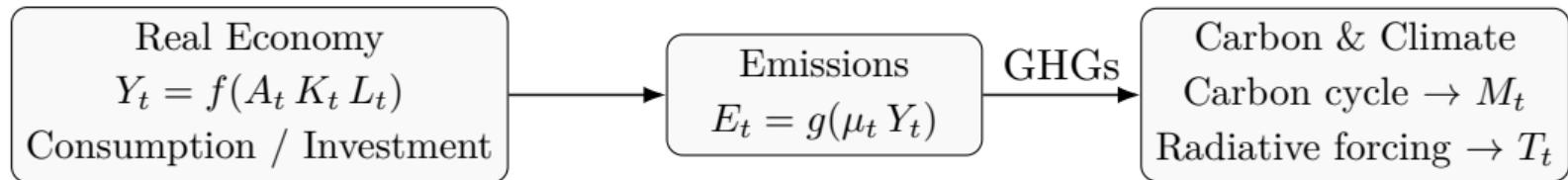
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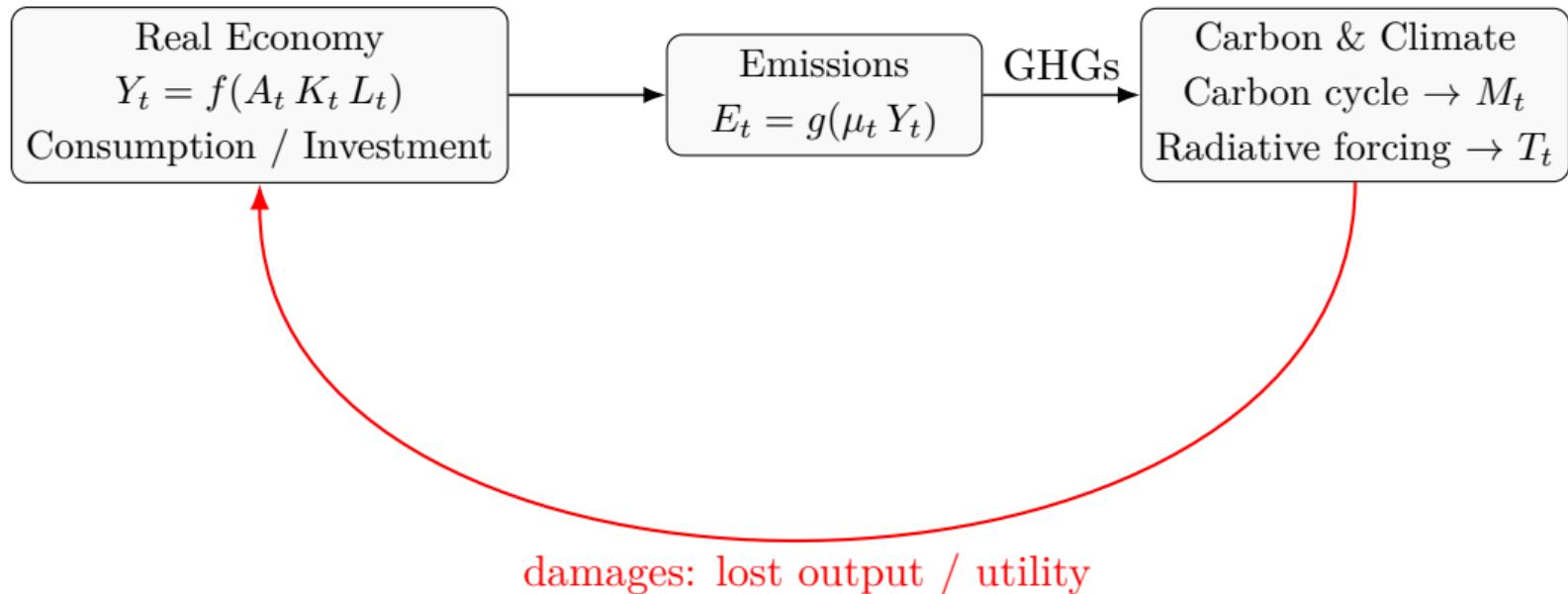


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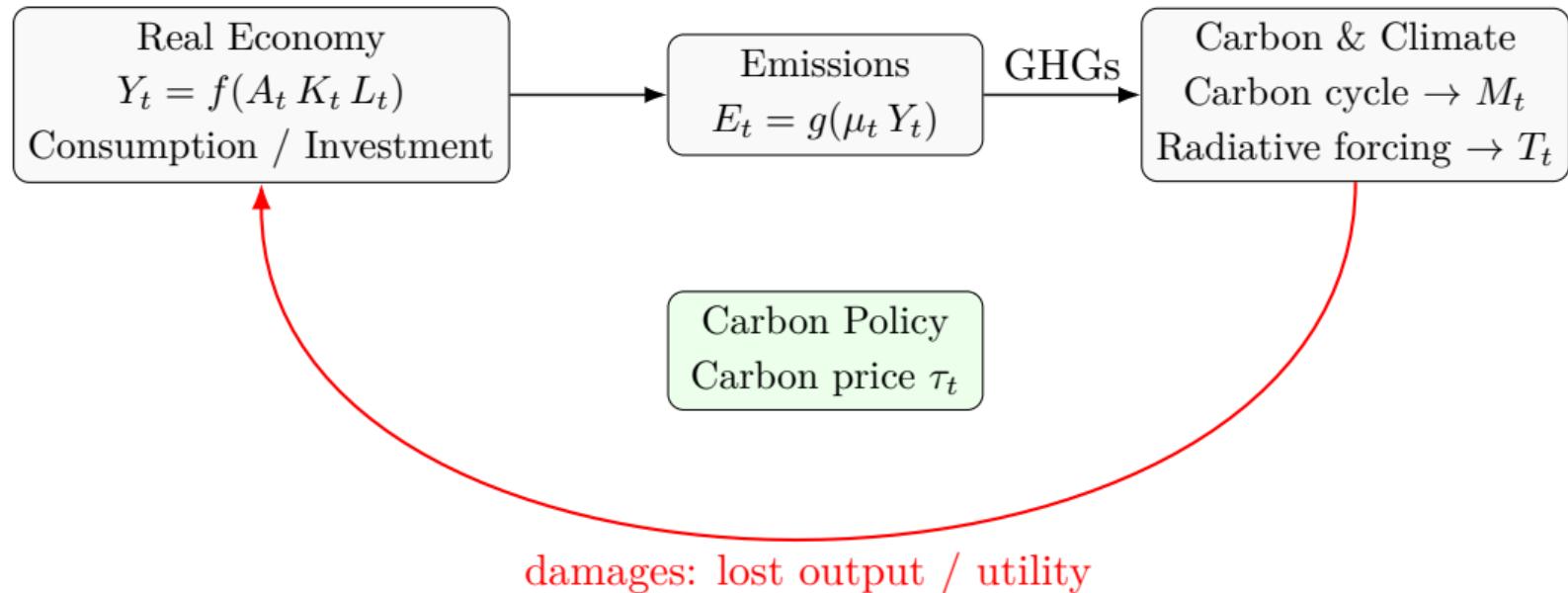
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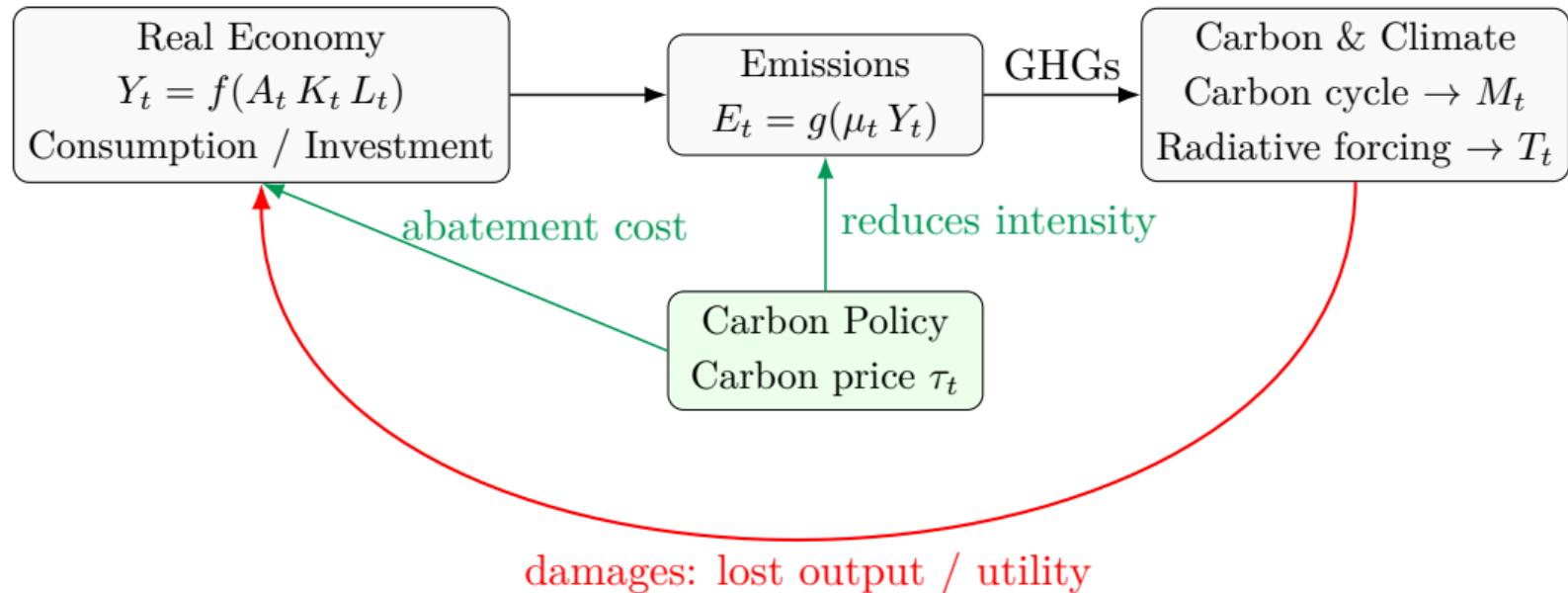
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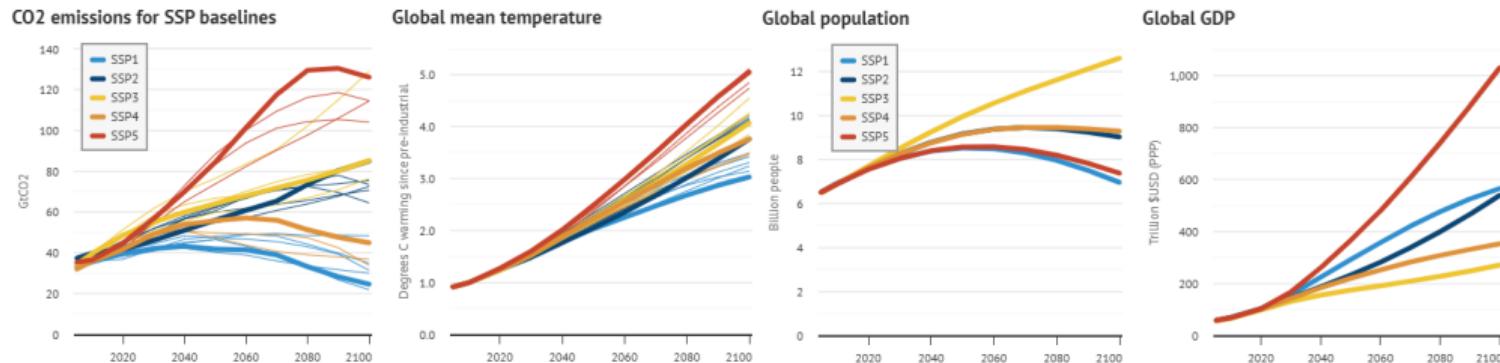
## Intro: IAM schematic: economy-climate feedbacks



## Intro: IAM schematic: economy-climate feedbacks



# Intro: Illustration of IAM output



Notes: Illustrative SSP baseline trajectories across models.

## Branch I: **Engineering-based IAMs**

- Examples: IMACLIM (CIRED), REMIND, GCAM
- Features:
  - Deterministic, large-scale, multi-sectoral
  - Frictionless (nominal rigidities, financial, etc.)
  - Backward-looking, non-optimizing
- But **not immune to Lucas critique**:
  - No expectations or uncertainty
  - No explicit optimization or welfare criterion

### Branch II: Long-run Mainstream Macro IAMs

- Examples:
  - DICE/RICE [[Nordhaus, 1991, 2008, 2017; Barrage and Nordhaus, 2023](#)]
  - Energy-augmented growth models [[Golosov et al. \(2014\); Hassler et al. \(2021\)](#)]
  - Directed technical change [[Acemoglu et al. \(2012, 2016\)](#)]
- Features:
  - Forward-looking, optimizing behavior
  - Deterministic models
  - Long run analysis

### Branch III: E-DSGE models for policy analysis

- Examples:
  - NY Fed multi-sector DSGE model with energy sectors [Del Negro et al., 2023]
  - Medium-scale DSGE with green brown energy [Coenen et al., 2024]
  - IMF energy and Phillips curve [Erceg et al., 2024]
- Features:
  - Forward-looking, optimizing behavior
  - Stochastic in neighborhood of some steady state
  - No climate block, so not truly an IAM

# Dynare: A toolbox for solving (climate) macroeconomic models

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- All these models can be solved efficiently in Dynare.
- Dynare is a **MATLAB/Python/Julia package** that provides a high-level interface to define, solve, and simulate economic models written in *state-space form*.
- Founded by **Michel Juillard**, now hosted by **CEPREMAP** (based at PSE premise).
- Dynare serves as a **front-end** to numerical solvers:
  - *Perturbation methods* (first- or higher-order approximations)
  - *Perfect foresight* and *extended-path* deterministic solvers
  - Bayesian estimation and policy analysis
- **Dynare is the mother tongue of central banks and policy institutions.** Understanding its logic is a valuable skill for applied macro and

## A toy model of the New Classical growth model

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- To fix ideas, let us consider a **toy version of the New Classical growth model**:
- A representative, infinitely-lived household solves

$$\max_{\{C_t, K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

$$s.t. C_t + K_t = A_t K_{t-1}^{\alpha} + (1 - \delta) K_{t-1},$$

- The household chooses  $\{C_t, K_t\}$  taking  $\{A_t\}$  as given. The first-order condition (Euler equation) reads

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[ \frac{1}{C_{t+1}} (1 - \delta + \alpha A_{t+1} K_t^{\alpha-1}) \right].$$

## Model summary

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- The model is a 3 equation model

$$\begin{aligned} -\frac{1}{C_t} + \beta \mathbb{E}_t \left[ \frac{1}{C_{t+1}} (1 - \delta + \alpha A_{t+1} K_t^{\alpha-1}) \right] &= 0 \\ -C_t - K_t + A_t K_{t-1}^\alpha + (1 - \delta) K_{t-1} &= 0 \\ -\log A_t + \rho_A \log A_{t-1} + \varepsilon_t &= 0 \end{aligned}$$

with

- 3 endogenous variables  $y_t = [C_t, K_t, A_t]'$
- 1 exogenous variable  $\eta_t = [\varepsilon_t]$
- 5 structural parameters  $\Theta = \{\beta, \delta, \alpha, \rho_A, \sigma\}$

## A general formulation of a structural model

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- Consider a structural model written in **state-space form**:

$$\mathbb{E}_t\{f_{\Theta}(z_{t-1}, z_t, z_{t+1}, \eta_t)\} = 0$$

- where:
  - $z_t$ : vector ( $N_z \times 1$ ) of endogenous variables,
  - $\eta_t$ : vector ( $N_{\eta} \times 1$ ) of Gaussian innovations,  $\eta_t \sim \mathcal{N}(0, \Sigma)$ ,
  - $f_{\Theta}(\cdot)$ : system of nonlinear equilibrium conditions,
  - $\Theta$ : vector of structural parameters,
  - $\mathbb{E}_t[\cdot]$ : expectation conditional on information available at  $t$ .
- *This representation nests most macroeconomic models: RBC, NK, and E-DSGE.*

## Steady State Determination

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- The **steady state** is a fixed point of the model where all variables are constant and all expectations are realized.
- The steady state satisfies the system of **nonlinear algebraic equations** obtained by removing time indices and shocks:

$$f_{\Theta}(\bar{y}, \bar{y}, \bar{y}, 0) = 0$$

- It plays two key roles in Dynare:
  - As a **fixed point** around which perturbation methods are applied (`stoch_simul`).
  - As the **terminal condition** in perfect-foresight simulations (`perfect_foresight_solver`).
- The steady state can be found:
  - *analytically* (by hand or symbolic computation), or
  - *numerically* (using nonlinear solvers such as `fsolve`).

# A model declaration in Dynare

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- A Dynare model is written in a single `.mod` file.
- It follows a clear and standardized structure:

## 1) Declaration of variables

- `var [list of endogenous variables];`
- `varexo [list of exogenous shocks];`
- `parameters [list of structural parameters];`

## 2) Model definition

- Use the `model; ... end;` block to list the **dynamic equilibrium equations**.
- Equations can be written in **nonlinear (levels)** or **log-linearized** form.

## A model declaration in Dynare (cont.)

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### 3) Provide steady state

- `steady;` compute steady state

### 4) Solution and simulation (to come later)

- `stoch_simul(...);` perturbation methods
- `perfect_foresight_solver;` deterministic simulations

→ Let's work on the exercise 1 of PC1.

2 Short term IAMs in Dynare

3 Long term IAMs in Dynare

## A model declaration in Dynare

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- **E-DSGE** models extend standard DSGE frameworks to incorporate the **climate-economy link** (emissions, damages, and abatement decisions).
- They rely on the same **perturbation methods** used in business-cycle models (e.g. **Smets and Wouters (2007)**), which **linearize the true nonlinear model** around its deterministic steady state.
- The first stochastic E-DSGE implementation is **Heutel (2012)**, who solves an environmental DSGE model by perturbation methods in Dynare.
- **Perturbation methods require:**
  - A **stationary economy** (no unit roots or explosive trends),
  - A well-defined **balanced growth path** (variables scaled by trend growth),
  - **Stationary shocks**, typically AR(1) processes around zero mean.

## A general formulation

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- Consider a function  $f(x)$ . A Taylor expansion around a fixed point  $a$  reads as:

$$f(x) \simeq f(a) + \frac{f'(a)}{1!} (x - a) + \frac{f''(a)}{2!} (x - a)^2 + \dots$$

- Two natural questions:
  1. Which order of approximation to consider?
  2. Which value of  $a$ ?

## A first order taylor expansion

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- Macroeconomics still dominated by linearized models. This lecture will remain on the linear version of the true nonlinear model.
- Consider a first order taylor expansion:

$$F \cdot \mathbb{E}_t\{\hat{z}_{t+1}\} + G \cdot \hat{z}_t + H \cdot \hat{z}_{t-1} + M \cdot \eta_t = 0$$

where

- $F, G, H$  and  $M$  are matrices stacking first order derivatives of  $f_\Theta$
- $\hat{z}_t = z_t - a$  is typically expressed in distance from steady state  $a = z$ . Why? linear model combined with gaussian stochastic shocks  $\eta_t \sim \mathcal{N}(0, \Sigma)$  implies that  $z_t - z \sim \mathcal{N}(0, \Sigma_z)$ . The smallest deviations  $E((z_t - a)^2)$  are obtained when 1st order expansion computed around steady state.

## A first order taylor expansion

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- One looks for a recursive solution that would have the following form:

$$\hat{z}_t = P \cdot \hat{z}_{t-1} + Q \cdot \eta_t$$

where  $P$  and  $Q$  are two unknown matrices.

- To solve the problem, one must determine the expectation scheme. Lucas implements 'rational' expectations, i.e. model-based expectations:  
 $E_t\{\hat{z}_{t+1}\} = P \cdot \hat{z}_t + Q \cdot \eta_{t+1}$ , with conditional expectation  $E_t\{\eta_{t+1}\} = 0$  as  $\eta_t \sim \mathcal{N}(0, \Sigma)$ .
- One can find  $P$  and  $Q$  by substituting  $E_t\{\hat{z}_{t+1}\}$  and  $\hat{z}_t$ :

$$\underbrace{[FP^2 + GP + H]}_{\text{determining } P} \hat{z}_{t-1} + \underbrace{(FPQ + GQ + M)}_{\text{determining } Q} \eta_t = 0$$

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## A first order taylor expansion

---

- Presence of quadratic matrix equation  $FP^2 + GP + H = 0$  with two solutions.  
The usual practice is to calculate solutions for P, pick P that have eigenvalues  $< 1$  absolute terms. If no stable solution, have a look at ?.
- Puzzling aspect of linear rational expectations model: one path of the economy not stable.
- Once solved, a linear DSGE model has similar features as VAR(1) model (useful when it comes to its estimation).

→ Let's work on the exercise 2 of PC1.

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## The first E-DSGE

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- Heutel (2012) develops the first fully stochastic **E-DSGE model**, built as a **stationarized version of DICE**.
- The framework introduces business-cycle dynamics into climate economics:
  - Short-run productivity shocks affect output, consumption, and emissions.
  - The model studies how optimal climate policy reacts to these fluctuations.
- A **centralized social planner** chooses consumption, investment, and abatement effort to maximize welfare, while internalizing the externality from the carbon stock.
- The model is **stationary** and solved by **perturbation method**.

## The central planner's problem

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$$\max_{\{z_t, i_t, c_t, y_t, \mu_t, x_t, k_t\}} E_t \sum_{s=0}^{\infty} \beta^s U(c_{t+s})$$

s.t.

$$\begin{cases} c_t + i_t + z_t \leq y_t, \\ k_t = (1 - \delta_K)k_{t-1} + i_t, \\ x_t = (1 - \delta_X)x_{t-1} + e_t + e_t^{row}, \\ e_t = (1 - \mu_t)h(y_t), \\ z_t = g(\mu_t)y_t, \\ y_t = (1 - d(x_t))a_t f(k_{t-1}). \end{cases}$$

## Key shadow values

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- The planner's value for carbon reads as:

$$\lambda_t^x = \mathbf{1}_{\{\text{policy on}\}} \underbrace{\lambda_t^y \frac{d'(x_t)}{1 - d(x_t)} y_t}_{\text{marginal damage}} + (1 - \delta_X) \beta \underbrace{\frac{c_{t+1}^{-\phi}}{c_t^{-\phi}} \lambda_{t+1}^x}_{\text{continuation value}}$$

where  $\mathbf{1}_{\{\text{policy on}\}}$  planner internalizes externality  $\rightarrow$  carbon price computed.

- Firms in response engage into abatement spending:

$$\mu_t = \left[ \frac{\sigma}{\theta_1 \theta_2} \lambda_t^x y_t^{-\gamma} \right]^{1/(\theta_2 - 1)}$$

$\rightarrow$  Let's work on the exercise 3 of PC1.

## Some observations

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- Main contribution of Heutel (2012)
  - Introduces **business-cycle frequency** into climate-economy models.
  - Quantifies the **co-movement between output and emissions**.
  - Identifies an **optimal carbon price path** that is **procyclical**: rising in booms, falling in recessions.
- Limitations:
  - **Stationary treatment of carbon stock and emissions:** implies mean reversion, which leads to flawed medium- and long-term predictions.
  - The **carbon stock is highly abstract:** emissions are treated as a dimensionless flow, abatement = 1 yields 0 carbon stock!
  - As a result, the model is well suited for **short-run cyclical analysis**, but not for long-run climate dynamics.

- 2 Short term IAMs in Dynare
- 3 Long term IAMs in Dynare

## Integrated Assessment Models (IAMs)

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- **Integrated Assessment Models (IAMs)** combine economic growth, the energy system, and the climate into a single dynamic framework.
- They study the **long-run interaction** between the economy and the climate under a **deterministic** environment - no stochastic shocks, no uncertainty.
- **Key characteristics:**
  - Deterministic **transition dynamics** toward a steady state.
  - Driven by **trends** - population, technology, productivity.
  - Often feature **unbalanced growth** between capital, output, and damages.
  - Carbon price and climate change implies structural change.
- Canonical example: the **DICE model** (Nordhaus).

- Our model, with no stochastic shocks, reads as:

$$f_{\Theta}(z_{t+1}, z_t, z_{t-1}) = 0$$

- The problem becomes a two-point boundary value problem:
  - An **initial condition**  $z_0$  given by the current state of the economy
  - A **terminal condition** requiring convergence to the steady state
- The algorithm searches for the entire **transition path** that connects these two bounds: the path of endogenous variables consistent with  $f_{\Theta}(\cdot)$ .
- In Dynare, this is solved using the `perfect_foresight_solver;` command:

A sketch of the numeric problem:

- Finite horizon problem for  $t = 0, 2, \dots T + 1$ ;
- Terminal  $z_{T+1}$  and initial  $z_0$  conditions are given → need to numerically get  $z_1, z_2, \dots z_T$ ;
- In absence of stochastic variables → deterministic problem → perfect foresight setup where any variable in  $z_{t+1}$  corresponds to the realized variable in  $t + 1$ ;

- Over the time horizon  $t = 1, 2, \dots, T$ , stacking  $F(Y)$  over time yields

$$F(Y) = \begin{bmatrix} f(z_2, z_1, z_0) \\ f(z_3, z_2, z_1) \\ \vdots \\ f(z_{T+1}, z_T, z_{T-1}) \end{bmatrix}$$

with  $Y = [z'_t, z'_{t+1}, \dots, z'_T]'$  and  $F : \mathbb{R}^{NT} \rightarrow \mathbb{R}^{NT}$

- $Y$  and  $F(Y)$  are two vectors of size  $NT \times 1$ .

## Numeric solution

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- The goal is to numerically solve:

$$Y^* = \arg \min_{\{Y\}} |F(Y)|$$

- How? Newton–Raphson method very efficient as shown by [Laffargue \(1990\)](#), [Boucekkine \(1995\)](#) and [Juillard et al. \(1996\)](#). Basic idea:
  - Set an initial value  $Y^{(0)}$ .
  - $n^{th}$  Newton iterations:

$$Y^{(n)} = Y^{(n-1)} - J_F \left( Y^{(n-1)} \right)^{-1} F(Y^{(n-1)})$$

where  $J_F(Y^{(n-1)})$  is Jacobian matrix of  $F$  of dimensions  $NT \times NT$ .

- Stop the iterations if  $|F(Y^{(n-1)})| < \varepsilon$ .

## Numeric solution

---

- Each iteration requires to solve:

$$\begin{bmatrix} J_{1,1} & J_{1,2} & \dots & 0_N & 0_N \\ J_{2,1} & J_{2,2} & \dots & 0_N & 0_N \\ \dots & \dots & \dots & \dots & \dots \\ 0_N & 0_N & \dots & J_{T-1,T-2} & J_{T-1,T} \\ 0_N & 0_N & \dots & J_{T-1,T-1} & J_{T,T} \end{bmatrix} \begin{bmatrix} \Delta y_1^{(n)} \\ \Delta y_2^{(n)} \\ \dots \\ \Delta y_{T-1}^{(n)} \\ \Delta y_T^{(n)} \end{bmatrix} = - \begin{bmatrix} f_1 \\ f_2 \\ \dots \\ f_{T-1} \\ f_T \end{bmatrix}$$

- The (inefficient) bruteforce way:

$$\begin{bmatrix} \Delta y_1^{(n)} \\ \Delta y_2^{(n)} \\ \dots \\ \Delta y_{T-1}^{(n)} \\ \Delta y_T^{(n)} \end{bmatrix} = -J_F \left( \begin{bmatrix} \Delta y_1^{(n-1)} \\ \Delta y_2^{(n-1)} \\ \dots \\ \Delta y_{T-1}^{(n-1)} \\ \Delta y_T^{(n-1)} \end{bmatrix} \right)^{-1} \begin{bmatrix} f_1 \\ f_2 \\ \dots \\ f_{T-1} \\ f_T \end{bmatrix}$$

## Numeric solution

- Laffargue (1990) proposes using a triangular expression of  $J_F(Y^{(n)})$  to allow backward induction.
- Linear algebra yields the following:

$$\begin{bmatrix} I_N & g_{1,2} & \dots & g_{1,T-1} & g_{1,T} \\ 0_N & I_N & \dots & g_{2,T-1} & g_{2,T} \\ \dots & \dots & \dots & \dots & \dots \\ 0_N & 0_N & \dots & I_N & g_{T-1,T} \\ 0_N & 0_N & \dots & 0_N & I_N \end{bmatrix} \begin{bmatrix} \Delta y_1^{(n)} \\ \Delta y_2^{(n)} \\ \dots \\ \Delta y_{T-1}^{(n)} \\ \Delta y_T^{(n)} \end{bmatrix} = - \begin{bmatrix} d_1 \\ d_2 \\ \dots \\ d_{T-1} \\ d_T \end{bmatrix} \quad (1)$$

- Principle: once matrices  $g_{\tau,t}$  and  $d_t$  (for  $\tau, t \in [1, T]$ ) are obtained, easy to get  $\Delta y_t^{(n)}$  recursively by starting by last row of problem (1).

- Thanks to this transformation of the Jacobian, each Newton step is fast and is done through a loop.
- This allows macroeconomists to solve their models in a second, without suffering much the curse of dimensionality!

→ Let's work on the exercise 4 of PC1.

## A baby DICE

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- To illustrate transition dynamics with climate, let's us consider a baby DICE model:
  - Productivity grows exogenously but converges to a steady state.
  - Output is affected by climate damages.
  - Planner allocates resources between consumption and abatement.
- The presence of large nonlinear transition dynamics in productivity, damages, and abatement renders perturbation methods unreliable. Perfect-foresight simulation in Dynare provides an appropriate global solution.

$$\frac{A_t}{A_{t-1}} = \exp \left[ (1 - e^{-\delta}) \left( \frac{g}{\delta} - \log \left( \frac{A_{t-1}}{A_0} \right) \right) \right].$$

- $A_0$ : initial productivity.
- $g$ : initial growth rate,  $\delta$ : rate of decline.
- $\Rightarrow$  Productivity converges to steady state  $\lim_{t \rightarrow \infty} A_t = \bar{A}$ .

# Climate Damage and Output

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**Endowment output:**

$$y_t = A_t(1 - \varphi(m_t)).$$

**Damage function:**

$$\varphi(m_t) = \xi (m_t - \bar{m})^2,$$

where:

- $\bar{m}$ : pre-industrial carbon stock,
- $\xi$ : elasticity of damage to excess carbon.

⇒ Higher  $m_t$  permanently reduces effective output.

## Carbon accumulation:

$$m_t - \bar{m} = \gamma(m_{t-1} - \bar{m}) + e_t,$$

- $\gamma$ : persistence / rate of transfer to deep oceans,
- $e_t$ : new emissions (in Gt of carbon).

## Emissions:

$$e_t = \sigma(1 - \mu_t)A_t a_t,$$

- $\mu_t$ : abatement effort ( $\mu_t = 1$  means zero emissions),
- $\sigma$ : carbon intensity of production,
- $a_t$ : exogenous activity index.

# Planner's Resource Constraint

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## Resource allocation:

$$y_t = c_t + \theta_1 \mu_t^{\theta_2} A_t,$$

- $c_t$ : consumption,
- $\theta_1 \mu_t^{\theta_2}$ : abatement (green investment) cost

## Preferences:

$$\begin{aligned} & \max_{\{c_t, m_t, \mu_t\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \log(c_{t+\tau}) \\ & + \lambda_{t+\tau} \left[ A_t \left( 1 - \theta_1 \mu_t^{\theta_2} - \varphi(m_t) \right) - c_t \right] \\ & + \lambda_{t+\tau} q_{t+\tau} [m_t - \gamma m_{t-1} - \sigma(1 - \mu_t) A_t] \end{aligned}$$

## Planner's Resource Constraint

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- Therefore the first main equation is the social cost of carbon (ie the shadow value) that reflects the discounted sum of marginal damages of one carbon emission along its lifetime:

$$q_t = A_t \varphi'(m_t) + \gamma \beta \frac{\lambda_{t+1}}{c_{t+1}} q_{t+1} \quad (2)$$

- In addition, the optimal abatement rate reads as:

$$\theta_2 \theta_1 \mu_t^{\theta_2 - 1} = \sigma q_t \quad (3)$$

The marginal cost of reducing carbon must be equal to the expected saved output.

→ Let's work on the exercise 5 of PC1.

Thank you for your attention

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