Climate Change and Growth

IAMs in Dynare

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Exercise 1: writing the dynamic model

Our economy is characterized by a representative household which consumes, produces goods, accumulates physical capital involved in the production. The household face the following maximization problem:

$$\max_{\{C_t,K_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

$$s.t.\ C_t + K_t = A_t K_{t-1}^{\alpha} + (1-\delta) K_{t-1}, \tag{1}$$

Concerning the productivity shock, it is assumed to follow a simple AR(1) process given by:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \eta_t^A, \tag{2}$$

where $0 \leq \rho_A < 1$ is the autocorrelation coefficient of the shock and η_t^A is an innovation distributed normally with zero mean and finite variance σ_A^2 such that $\eta_t^A \sim \mathcal{N}(0, \sigma_A^2)$.

The household chooses $\{C_t, K_t\}$ taking $\{A_t\}$ as given. The first-order condition (Euler equation) reads

$$\frac{1}{C_t} = \beta \mathbb{E}_t \left[\frac{1}{C_{t+1}} \left(1 - \delta + \alpha A_{t+1} K_t^{\alpha-1}\right) \right]. \tag{3}$$

The simple New Classical growth model comprises three equations (1)-(2)-(3).

1. Create first a file called RBC.mod and open it in explorer.

We want to write the model into a form Dynare can interpret. In the first part of the Dynare code, we need to define the endogenous variables which appear in the model. There are three equations, so we have three variables: c_t, k_t and a_t. The syntax for writing this in Dynare is var [variable1 variable2, ...]; with the line ended by a semicolumn. For our model, we write:

```
var k a c;
```

Next we write the parameters of the model. In our model, there are four structural parameters (α, β, δ and ρ_A) to declare. The syntax for writing this in Dynare is parameters parameter1 parameter2, ... ; with the line ended by a semicolumn. Here we note:

```
parameters  alpha beta delta rho_A;
```

Next, we write the exogenous variables in the model. We only have one: η_t. The syntax for writing this in Dynare is varexo variable1 variable2, ... ; with the line ended by a semicolumn. For our set-up, we write:

```
varexo e_a;
```

Finally, we give the parameters values using our calibration $\alpha = 0.40$, $\beta = 0.99$, $\delta = 0.025$, $\rho_A = 0.95$. The syntax for giving parameters values is parameter = value; which is similar to normal Matlab code. We write:

```
alpha        = 0.40;
beta            = 0.99;
delta            = 0.025;
rho_A        = 0.95;
Kss         = ((1/beta-(1-delta))/alpha)^(1/(alpha-1));
Css         = Kss^alpha - delta*Kss;
```

2. Write in your Dynare file the parameters, endogenous and exogenous variable declarations, as well as calibration table.

We now write the dynamic equations of the model. Those equations are usually written in the same way as they appear in the paper. The equation section starts with model; and ends with end;.[1] A few syntax considerations:

- Variables which are multiplied (divided) by a parameter are separated by the parameter by the symbol * (/).

- Contemporaneous variables do not have a time subscript in the Dynare code. Lagged variables x_{t-i} are written as x(i), while expectational variables $\mathbb{E}_t x_{t+i}$ are written as x(+i).

- Equations are ended with a semicolumn ;.

The equations in Dynare syntax then are:

```
model;
        [name='Budget constraint']
        c + k = a*k(-1)^alpha + (1-delta)*k(-1);
        [name='Euler equation']
        1/c = beta*1/c(+1)*(1-delta+alpha*a(+1)*k^(alpha-1));
        [name='Productivity shock']
        log(a) = rho*log(a(-1))+e_a;
end;
```

3. Write the dynamic equation block.

[1] Note that the tag model (linear); includes the option 'linear' because the model is already linearized. For fully non-linear model, just write model ;.

We now need to specify the steady state of the model. Consider:

$$\bar{A} = 1$$
$$\bar{K} = \left[\left(\frac{1}{\beta} - (1-\delta)\right)\frac{1}{\alpha A}\right]^{1/(\alpha-1)}$$
$$\bar{C} = \bar{A}\bar{K}^{\alpha} - \delta\bar{K}$$

which can be given to Dynare as:

```
steady_state_model;
a             = 1;
k         = ((1/beta-(1-delta))/(a*alpha))^(1/(alpha-1));
c         = a*k^alpha - delta*k;
end;
```

One can next check the residuals and compute the steady state:

```
resid;
steady;
```

4. Write the steady state block and the two additional commands.

To run the file, you also need to set Matlab's current directory where your Dynare file is. To initiate Dynare, in Matlab's command line, type:

console

```
dynare RBC
```

5. Give a try and run your model, and have a look at the error messages that you get and debug it until it works.

Recall that production and investment are:

$$Y_t = A_t K_{t-1}^{\alpha}$$
$$I_t = K_t - (1-\delta)K_{t-1}$$

6. Add these two variables, dynamic equations and steady state and run again to see if that works.

Exercise 2: perturbing the model

Make a copy of RBC.mod called stochasticRBC.mod. After specifying the equations, we define the shock, and set its standard deviation. We begin the section with a tag shocks; and end it with end;. The syntax for specifying each shock in Dynare is var shock = shock variance;. For the model, the section is:

```
shocks;
        var e_a;  stderr .007;
end
```

Next, we can perform a diagnostic of the model stationarity with respect to the Blanchard-Kanh condition. In Dynare, we add:

```
check;
```

Finally, we solve the model and perform stochastic simulations. To do so, we employ the function stoch_simul(options=value) variable 1 variable 2;. Here we want a 30-period system response of consumption, capital and productivity after a technological shock. We write accordingly:

```
stoch_simul(irf=30) c k a;
```

There many options available for the stoch_simul() function, see directly the Dynare user guide Adjemian et al. (2011) for details about options.

1. Include the shock variance, the Blanchard-Kanh command, and the stochastic solver.

2. Change the calibration to get a non-stationnary system response. What is Dynare telling you in the command window? Does increasing return to scale makes the model non-stationnary?

Exercise 3: the Heutel Model

Consider downloading `Heutel.mod` as well as `heutel_ss.m`. Let us consider the model of planner in Heutel's paper:

$$
\begin{aligned}
L = {} & \beta^s \frac{c_{t+s}^{1-\phi}}{1-\phi} \\
& + \beta^s \lambda_{t+s}^c \left[y_{t+s} \left(1 - \theta_1 \mu_{t+s}^{\theta_2}\right) - c_{t+s} - k_{t+s} + (1-\delta_K)\, k_{t-1+s} \right] \\
& + \beta^s \lambda_{t+s}^c \lambda_{t+s}^y \left[(1 - d\,(x_{t+s}))\, a_{t+s} k_{t-1+s}^{\alpha} - y_{t+s} \right] \\
& + \beta^s \lambda_{t+s}^c \lambda_{t+s}^x \left[x_{t+s} - (1-\delta_X)\, x_{t-1+s} - \sigma\,(1-\mu_{t+s})\, y_{t+s}^{1-\gamma} - e^* \right]
\end{aligned}
$$

We get the following first order conditions:

$$
\begin{aligned}
&y_t : \lambda_t^c \left(1-\theta_1\mu_t^{\theta_2}\right) - \lambda_t^c\lambda_t^y - \lambda_t^c\lambda_t^x\sigma\left(1-\gamma\right)\left(1-\mu_t\right)y_t^{-\gamma} = 0\\
&c_t : c_t^{-\phi} - \lambda_t^c = 0\\
&k_t : -\lambda_t^c + \left(1-\delta_K\right)\beta\lambda_{t+1}^c + \beta\lambda_{t+1}^c\lambda_{t+1}^y\alpha y_{t+1}/k_t = 0\\
&x_t : -\lambda_t^c\lambda_t^y\frac{d'\left(x_t\right)}{1-d\left(x_t\right)}y_t + \lambda_t^c\lambda_t^x - \beta\left(1-\delta_X\right)\lambda_{t+1}^c\lambda_{t+1}^x = 0\\
&\mu_t : -\lambda_t^c y_t\theta_1\theta_2\mu_t^{\theta_2-1} + \lambda_t^c\lambda_t^x\sigma y_t^{1-\gamma}
\end{aligned}
$$

We can rewrite those first order conditions as follows:

$$
\begin{aligned}
\lambda_t^y &= 1-\theta_1\mu_t^{\theta_2} - \lambda_t^x\left(1-\gamma\right)e_t/y_t\\
c_t^{-\phi} &= \beta c_{t+1}^{-\phi}\left[\left(1-\delta_K\right) + \lambda_{t+1}^y\alpha y_{t+1}/k_t\right]\\
\lambda_t^x &= \mathbf{1}_{\{\text{policy on}\}}\lambda_t^y\frac{d'\left(x_t\right)}{1-d\left(x_t\right)}y_t + \left(1-\delta_X\right)\beta\frac{c_{t+1}^{-\phi}}{c_t^{-\phi}}\lambda_{t+1}^x\\
\mu_t &= \left[\frac{\sigma}{\theta_1\theta_2}\lambda_t^x y_t^{-\gamma}\right]^{1/(\theta_2-1)}
\end{aligned}
$$

1. Interpret each of the equations.
2. Why do we need a numerical solver to get the steady state?
3. Do you think the level of abatement is realistic? How could we increase it?
4. What is the effect of a carbon tax on the correlation between emissions and GDP?
5. Simulate the model for 500 periods and plot it. Plot emission and the stock of carbon. Do they make sense?

Exercise 4: Solving the Koopman model

Make a copy of `RBC.mod` that you will call `pfRBC.mod`. We now configure the perfect-foresight problem to compute the economy's transition dynamics. Specifically, we consider an initial capital stock below its steady-state level and solve for the adjustment path back to the steady state. This transition illustrates the capital accumulation process, akin to the convergence dynamics in a Solow-type model, but without exogenous growth.

Recall our model is given by:

$$
\begin{aligned}
C_t + K_t &= A_t K_{t-1}^{\alpha} + (1-\delta)K_{t-1}\\
\frac{1}{C_t} &= \beta\,\mathbb{E}_t\left[\frac{1}{C_{t+1}}\left(1-\delta+\alpha A_{t+1}K_t^{\alpha-1}\right)\right]\\
\log\left(A_t\right) &= \rho_A\log\left(A_{t-1}\right) + \eta_t^A
\end{aligned}
$$

Stacking in

$$f_\Theta\left(z_{t+1}, z_t, z_{t-1}\right) = 0$$

We want to solve the transition from $z_0 = [0.5\bar{K}, \bar{C}, \bar{A}]$ to the steady state $z_{T+1} = [\bar{K}, \bar{C}, \bar{A}]$, with $T = 200$ periods.

After the steady state declaration and residuals checks, we need firm to compute the starting point:

```
k0 = 0.5*((1/beta - (1-delta))/(alpha))^(1/(alpha-1)); // 50
```

which can next be preloaded with other state variables:

```
// Historical (predetermined) state at t=-1:
histval;
k(0) = k0;           // this sets k(-1)
a(0) = 1;
end;
```

1. Write them in Dynare file `pfRBC.mod` to set initial conditions.

In what follows, we initialize the path/guess of endogenous variables $\{z_t\}_{t=0}^{T+1}$ with the Dynare command `perfect_foresight_setup(periods=200);` for $T = 200$ periods. Next we solve the path by minimizing the residuals with `perfect_foresight_solver;`.

```
// Length of the transition and solve
perfect_foresight_setup(periods=200);
perfect_foresight_solver;
```

2. Write them in Dynare file `pfRBC.mod` to initialize and solve the path.

Plot then the results, or use `oo_.endo_simul` if you want to do it manually.


```
// Length of the transition and solve
perfect_foresight_setup(periods=200);
perfect_foresight_solver;
```


3. Plot the results and interpret the transition.

Exercise 5: A toy DICE with transition dynamics

Download the code `toyDICE.mod`. The planner's problem reads as:

$$\begin{aligned}
&\max_{\{c_t,m_t,\mu_t\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \log(c_{t+\tau}) \\
&+ \lambda_{t+\tau} \left[A_t \left(1 - \theta_1 \mu_t^{\theta_2} - \varphi\left(m_t\right) \right) - c_t \right] \\
&+ \lambda_{t+\tau} q_{t+\tau} \left[m_t - \gamma m_{t-1} - \chi_m \sigma (1-\mu_t) A_t \right]
\end{aligned}$$

First order conditions are given by:

$$c_t : \frac{1}{c_t} - \lambda_t = 0 \tag{4}$$

$$m_t : -\lambda_t A_t \varphi'(m_t) - \gamma\beta\lambda_{t+1} q_{t+1} + \lambda_t q_t = 0 \tag{5}$$

$$\mu_t : -\lambda_t A_t \theta_2 \theta_1 \mu_t^{\theta_2 - 1} + \chi_m \sigma \lambda_t q_t A_t \tag{6}$$

We match the model to replicate forecasts in 2019. $A_0 = 125$ trillions USD, $e_0 = 40$ gigatonnes CO2, while the current stock of carbon in the atmosphere $m_0 = 950$. The TFP growth is about 3%, $g = 0.03$ with decay rate $\delta = 0.035$. The cost of abatement is capped up to 3.5% of output $\theta_1 = 0.035$ while curvature $\theta_2 = 2.6$. $\chi_m = 3/11$ is a conversion factor of CO2 into carbon. A warming of $4°C$, consistent with a carbon stock of 2,000 Gt generates a 10% loss in output. We thus match the elasticity ξ in the the damage accordingly imposing $\varphi(2,000) = 0.1$. The persistence of carbon in the atmosphere is about 80 years, $\gamma = 1 - 1/80$. Finally, carbon intensity is given by $\sigma = e_0/A_0$.

The set of equations is given by:

$$\begin{aligned}
\frac{A_t}{A_{t-1}} &= \exp\left((1-\exp(-\delta))\left(\frac{g}{\delta} - \log\left(\frac{A_{t-1}}{A_0}\right)\right)\right) \\
c_t &= A_t\left(1 - \theta_1 \mu_t^{\theta_2} - \varphi(m_t)\right) \\
m_t &= \gamma m_{t-1} + \sigma(1-\mu_t)A_t \\
q_t &= A_t \varphi'(m_t) + \gamma\beta E_t\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-1} q_{t+1}\right\} \\
\mu_t &= \left(\frac{\sigma}{\theta_2\theta_1} q_t\right)^{1/(\theta_2-1)}
\end{aligned}$$

1. Interpret the results.
2. Add temperatures equation $T_t = T_{t-1} + \chi_T \cdot m_t$, where as in Dietz et al. (2021), captures the transient climate response $\chi_T = 0.0022$.
3. Increase the damage function, such as we get a 30% damage instead of 10% and reports the implication on the carbon price policy. Report the abatement curves and carbon prices in the plots.

Additional exercise

Assume now that our economy has a government. The government finances public spending by charging a lump-sum tax T_t on households. The total amount of public spending reads as, $P_t G_t$. The balance sheet of government writes:

$$P_t G_t = T_t. \tag{7}$$

The budget constraint of the household is now:

$$C_t + I_t = A_t K_{t-1}^{\alpha} - \frac{T_t}{P_t}, \tag{8}$$

while the resource constraint/the GDP through the expenditure approach is now given by:

$$Y_t = C_t + I_t + G_t. \tag{9}$$

The decision of increasing/decreasing spendings is decided exogenously by the governement. In absence of an explicit modelling of political cycles, the spending is assumed to vary around a fixed value of output:

$$G_t = \bar{G}\varepsilon_t^G = g^y \bar{Y} \varepsilon_t^G, \tag{10}$$

where spending follows an $AR(1)$ process:

$$\log\left(\varepsilon_t^G\right) = \rho_G \log\left(\varepsilon_{t-1}^G\right) + \eta_t^G, \tag{11}$$

with $0 \leq \rho_G < 1$ is the autocorrelation coefficient of the demand shock and η_t^G is an innovation distributed normally with zero mean and finite variance σ_G^2 such that $\eta_t^G \sim \mathcal{N}(0, \sigma_G^2)$. Parameter g^y is the spending-to-GDP ratio. We borrow the values of Smets and Wouters (2007) and assume that $\rho_G = 0.97$, $\sigma_G = 0.053$ and $g^y = 0.20$. Implicitely in steady state, the shock is normalized to one $\bar{\varepsilon}^G = 1$.

1. Compute the new steady state of the model. *Tips:* (i) define a new parameter, $g^y = 0.2$, which affects the new steady state; (ii) since the budget is balanced, you can substitute real taxes T_t/P_t by spending G_t.
2. Let's now calibrate the demand shock process, show the IRF after the realization of the demand shock.
3. According to Keynesian economics, is this shock consistent with this theory? Why do households are "Ricardian"? Is this behaviour observed in the data?
4. Analyse the variance decomposition, what is the main driver of output?
5. Is this RBC model now able to replicate another business cycle fact which says output is more volatile than consumption?

References

Adjemian, S., Bastani, H., Juillard, M., Mihoubi, F., Perendia, G., Ratto, M., Villemot, S., 2011. Dynare: Reference manual, version 4.

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Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: A Bayesian DSGE approach. American Economic Review 97, 586–606.