

Modélisation, estimation, simulation des risques climatiques

# A Quick Introduction to Parametric Uncertainty in IAMs

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## Schedule of the day

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### Objectives

- Understanding the role of parametric uncertainty quantification in IAMs.

### Materials:

- DICE\_Notebook1-3\_Uncertainty.ipynb *notebook exercises about the quantification of parametric uncertainty.*

# Introduction

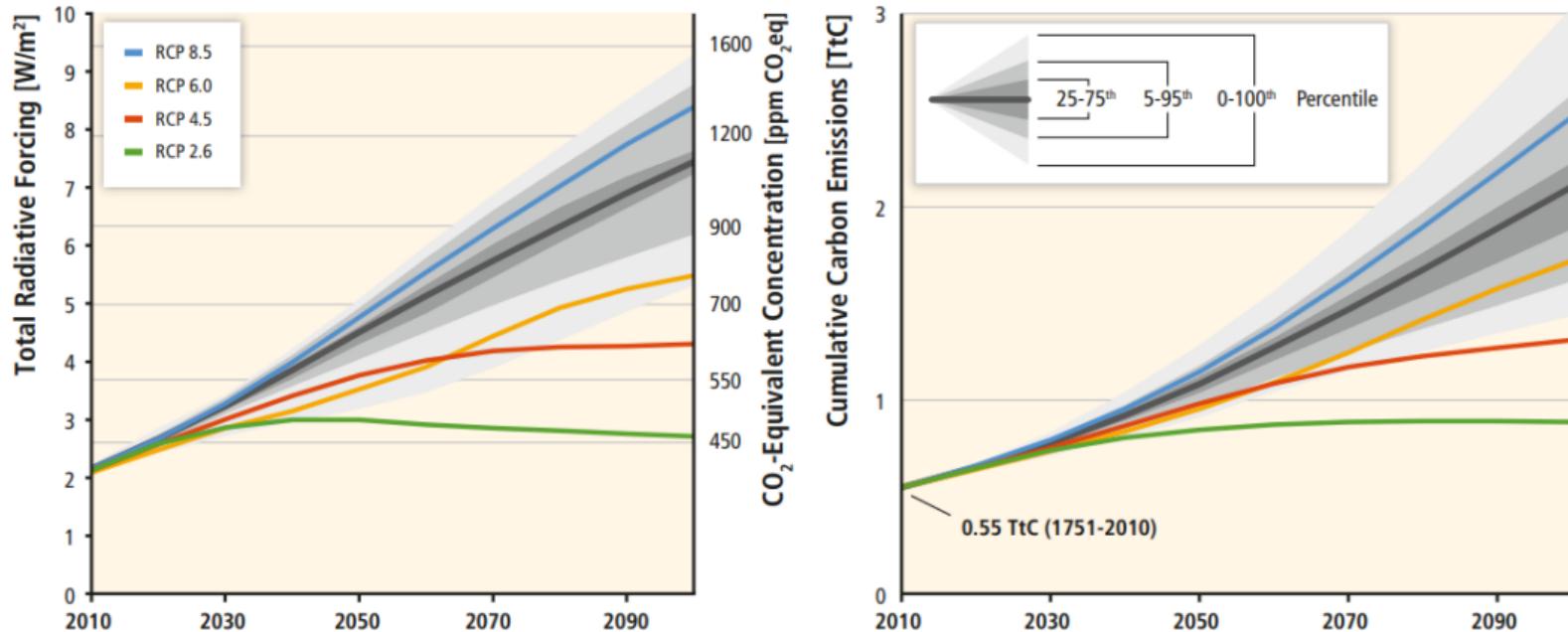
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## Introduction: Why uncertainty matters

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- Climate policy is a **risk management problem** (IPCC, WG3 Ch.2 [Kunreuther et al. \(2014\)](#)).
- Large uncertainty in key drivers:
  - Climate sensitivity (ECS), damages, mitigation costs.
  - Socioeconomic pathways (SSPs, population, growth).
- Standard DICE baseline: *deterministic path*.
- Reality: **distribution of possible futures**

# Introduction: Why uncertainty matters



**Figure 2.3** | Total radiative forcing (left panel) and cumulative carbon emissions since 1751 (right panel) in baseline scenario literature compared to RCP scenarios. Forcing was estimated ex-post from models with full coverage using the median output from the MAGICC results. Secondary axis in the left panel expresses forcing in CO<sub>2</sub>eq concentrations. Scenarios are depicted as ranges with median emboldened; shading reflects interquartile range (darkest), 5th–95th percentile range (lighter), and full extremes (lightest). Source: Figure 6.6 from WGIII AR5.

## Introduction: Taxonomy of uncertainty

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- **Parametric:** uncertain parameter values (ECS, damages).
- **Paradigmatic (or model-structural) uncertainty:** different IAM structures, climate models.
- **Scenario:** socioeconomic, policy, emissions pathways.
- **Trend vs. cycle:** long-run drivers vs short-term shocks.
- **Ambiguity:** situations where probabilities themselves are unclear or contested.

# Introduction: Metrics for reporting uncertainty

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## IPCC guidance: two complementary scales

- **Confidence** = level of certainty in a finding
  - Combines: **evidence** (amount, quality, consistency of data)  $\times$  **agreement** (consensus across studies).
  - Expressed qualitatively: low / medium / high / very high confidence.
  - Example: "High confidence that warming exceeds natural variability."
- **Likelihood** = assessed probability of an outcome
  - Mapped to calibrated terms:
    - Very likely =  $> 90\%$ , likely =  $> 66\%$ , about as likely as not =  $33\text{--}66\%$ , etc.
  - Example: "It is *likely* that ECS is between  $1.5\text{--}4.5^\circ\text{C}$ ."

## In modeling practice:

- Report probability distributions, fan charts, quantile bands.
- Highlight **tail risks**: low-probability but high-impact outcomes.

## Introduction: Focus in this course: parametric uncertainty

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- Treat parameters as **random variables**.
- Examples in DICE from [Nordhaus \(2018\)](#):
  - Equilibrium climate sensitivity ( $T_{2xCO_2}$ ).
  - Damage function coefficients ( $a_2, a_3$ ).
  - Backstop technology cost ( $p_b$ ).
- Goal: generate an ensemble of plausible trajectories.

## Parametric Uncertainty

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# Parametric Uncertainty: Sampling the population of calibrations

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## Monte Carlo sampling

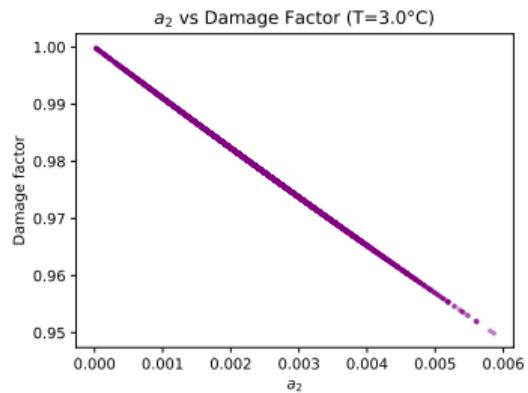
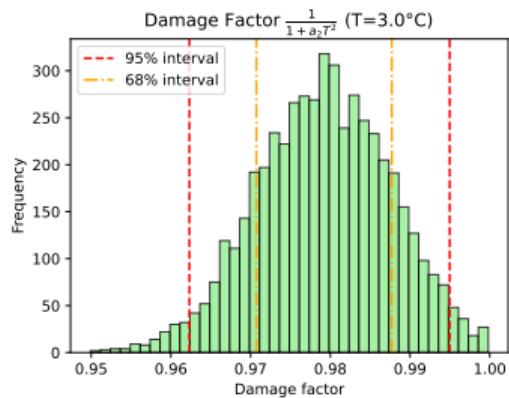
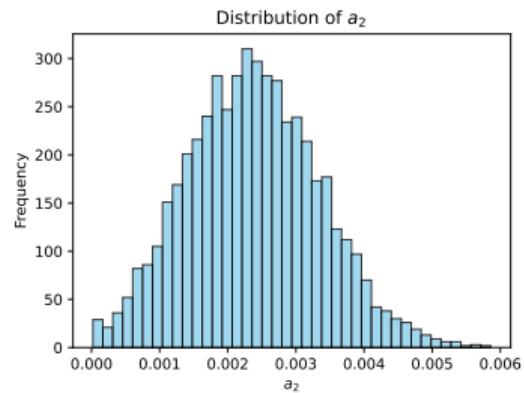
- Draw  $N$  parameter sets from a prior (here: truncated Gaussian for  $a_2$ ).
- Each draw  $\Rightarrow$  one implied outcome; in a full run, one DICE trajectory.
- Contrast *priors* (literature-based) vs *posteriors* (after calibration/estimation).

## Very simple illustration (single-parameter): quadratic damages

$$\text{Damage factor at temperature } T : \quad \frac{1}{1 + a_2 T^2}$$

- Sample  $a_2 \sim \mathcal{N}^+(0.00236, 0.001^2)$  (truncated at  $a_2 > 0$ ), fix  $T = 3^\circ C$ .
- Report empirical quantiles (5–95%) for  $a_2$  and for the damage factor.
- Visuals: histogram of  $a_2$ , histogram of damage factor, and  $a_2$  vs damage-factor scatter.

# Parametric Uncertainty: Illustration



## Parametric Uncertainty: Interpretation

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- Distributions summarize **parametric uncertainty**.
- Quantile bands and fan charts generalize to full DICE trajectories (temperature, welfare, carbon price).
- Not a point prediction: a **range of plausible outcomes**; policy should pay attention to tails.

## Parametric Uncertainty: Interpretation

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### Example: damage function

- At  $T = 3^\circ\text{C}$ , the quadratic damage factor  $\frac{1}{1+a_2T^2}$  varies substantially across draws.
- In our Monte Carlo exercise:
  - 68% interval  $\approx [0.72, 0.84]$
  - 95% interval  $\approx [0.68, 0.87]$
- Interpretation: depending on  $a_2$ , GDP losses at  $3^\circ\text{C}$  range from  $\sim 13\%$  to  $\sim 32\%$ .  $\Rightarrow$  **Uncertainty quantification of damage parameter translates into uncertainty on damages themselves.**

# Parametric Uncertainty: Decision under uncertainty

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## Two levels of uncertainty:

- **Uncertainty quantification (UQ):**
  - Given model structure, explore parametric distributions (Monte Carlo, ensembles, fan charts).
  - Output: ranges of outcomes (temperature, damages, welfare).
- **Uncertainty in the planner's problem:**
  - Planner internalizes future risks  $\Rightarrow$  **precautionary motive**.
  - Early work: [Cai and Lontzek \(2019\)](#), stochastic DICE.
  - Uncertainty directly affects optimal abatement, carbon tax.

## Parametric Uncertainty: Decision under uncertainty (cont'd)

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### Challenges:

- Stochastic DICE  $\Rightarrow$  high-dimensional state space (climate, capital, shocks).
- Requires advanced computational methods (stochastic dynamic programming, adaptive sparse grids, GPUs).
- Trade-off: richer treatment of uncertainty vs. feasibility for teaching / policy use.

Thank you!

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## References

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Cai, Y. and Lontzek, T. (2019). The social cost of carbon with economic and climate risks. *Journal of Political Economy*, 127:2684–2734.

Kunreuther, H., Gupta, S., Bosetti, V., Cooke, R., Dutt, V., Ha-Duong, M., Held, H., Llanes-Regueiro, J., Patt, A., Shittu, E., et al. (2014). Integrated risk and uncertainty assessment of climate change response policies. In *Climate Change 2014: Mitigation of Climate Change: Working Group III Contribution to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*, pages 151–206. Cambridge University Press.

Nordhaus, W. (2018). Projections and uncertainties about climate change in an era of minimal climate policies. *American Economic Journal: Economic Policy*, 10:333–360.