“Climate risk quantification methods and tools”

Handout: DICE model

Gauthier Vermandel

Model Equations (DICE-style)

Time, Technology, Population

$$t(i) = t_0 + \Delta\, i, \tag{1}$$

$$A(i) = \frac{A(i-1)}{1 - g_A \exp(-\delta_A \Delta\, i)}, \tag{2}$$

$$L(i) = L(i-1) \left(\frac{L_\infty}{L(i-1)} \right)^{\ell_g}. \tag{3}$$

Carbon Intensity & Abatement Cost Share

$$\sigma(i) = \sigma(i-1)\, \exp\Big(-\, g_\sigma \left(1 - \delta_\sigma\right)^{\Delta(i-1)} \Delta \Big), \tag{4}$$

$$\theta_1(i) = \frac{p_b}{1000\, \theta_2} \left(1 - \delta_{pb}\right)^{i-1} \sigma(i). \tag{5}$$

Output, Damages/Abatement, Net Output

$$Y(i) = A(i)\, K(i-1)^{\alpha} \left(\tfrac{L(i)}{1000} \right)^{1-\alpha}, \tag{6}$$

$$Q(i) = \Delta\, \frac{\big(1 - \theta_1(i)\, \mu(i)\big)^{\theta_2}}{1 + a_2\, T_{AT}(i)^{a_3}}\, Y(i). \tag{7}$$

Consumption, Investment, Capital Accumulation

$$C(i) = \big(1 - s(i)\big)\, Q(i), \tag{8}$$

$$K(i) = \big(1 - \delta_K\big)^{\Delta} K(i-1) + s(i)\, Q(i). \tag{9}$$

Emissions & Radiative Forcing

$$E(i) = \Delta\Big[\sigma(i)\big(1-\mu(i)\big)Y(i) + E_{\text{Land}}(i)\Big], \tag{10}$$

$$E_{\text{Land}}(i) = E_{L0}\,\big(1-\delta_{EL}\big)^{i}, \tag{11}$$

$$F_{EX}(i) = f_0 + \min\Big(\tfrac{(f_1-f_0)\,i}{t_f},\, f_1 - f_0\Big), \tag{12}$$

$$F(i) = F_{2\times CO_2}\ \log_2\Big(\tfrac{M_{AT}(i-1)}{M_{AT0}}\Big) + F_{EX}(i). \tag{13}$$

Carbon Cycle (Three-Box)

$$M_{AT}(i) = \big(1-\Delta b_{12}\big)\,M_{AT}(i-1) + \Delta b_{12}\,\frac{M_{AT0}}{M_{UP0}}\,M_{UP}(i-1) + \xi\,E(i), \tag{14}$$

$$M_{UP}(i) = \Delta b_{12}\,M_{AT}(i-1) + \Big(1-\Delta b_{12}\tfrac{M_{AT0}}{M_{UP0}} - \Delta b_{23}\Big)\,M_{UP}(i-1) + \Delta b_{23}\,\frac{M_{UP0}}{M_{LO0}}\,M_{LO}(i-1), \tag{15}$$

$$M_{LO}(i) = \Delta b_{23}\,M_{UP}(i-1) + \Big(1-\Delta b_{23}\tfrac{M_{UP0}}{M_{LO0}}\Big)\,M_{LO}(i-1). \tag{16}$$

Temperature Dynamics (Two-Box)

$$T_{AT}(i) = T_{AT}(i-1) + \Delta c_1\,F(i) - \Delta c_1\,\frac{F_{2\times CO_2}}{T_{2\times CO_2}}\,T_{AT}(i-1) - \Delta c_1 c_3\big[T_{AT}(i-1) - T_{LO}(i-1)\big], \tag{17}$$

$$T_{LO}(i) = T_{LO}(i-1) + \Delta c_4\big[T_{AT}(i-1) - T_{LO}(i-1)\big]. \tag{18}$$

Planner's Objective (CRRA over consumption per capita)

$$c_{pc}(i) = 1000\,\frac{C(i)}{L(i)}, \tag{19}$$

$$u(i) = \begin{cases} L(i)\,\dfrac{c_{pc}(i)^{1-\gamma}-1}{1-\gamma}, & \gamma \neq 1, \\ L(i)\,\ln c_{pc}(i), & \gamma = 1, \end{cases} \tag{20}$$

$$W = \sum_{i\in\mathcal{I}} \beta(i)\,u(i), \quad \beta(i) = \Big(\tfrac{1}{1+\rho}\Big)^{\Delta(i-i_0)}. \tag{21}$$

Key Symbols

- Δ time step; ρ pure rate of time preference; γ CRRA.
- A TFP; K capital; L population; α capital share.
- σ carbon intensity; μ abatement rate; θ_1, θ_2 abatement cost params.

- a_2, a_3 damage function params; T_{AT}, T_{LO} temperatures (atmosphere, deep ocean).
- E emissions; E_{Land} land-use emissions; F forcing; F_{EX} exogenous forcing.
- Carbon boxes: M_{AT}, M_{UP}, M_{LO} with preindustrial baselines $M_{AT0}, M_{UP0}, M_{LO0}$.
- Flows b_{12}, b_{23}; conversion ξ; climate response c_1, c_3, c_4; sensitivities $F_{2\times CO_2}$, $T_{2\times CO_2}$.

Table 1: Calibration of Model Parameters

Symbol	**Value**	**Unit**	**Description**
Time grid			
Δ	5	years	Time step
t_0	2015	year	Start year (fixed)
t_T	2100	year	End year of simulations
n_T	–	periods	Number of time steps
Social preferences and production			
ρ	0.015	1/yr	Pure rate of time preference
γ	1.45	–	CRRA coefficient
δ_K	0.10	1/yr	Capital depreciation rate
α	0.30	–	Capital share in production
Technology			
A_0	5.115	–	Initial TFP level
g_A	0.076	1/yr	Initial TFP growth rate
δ_A	0.005	1/yr	TFP growth decline rate
K_0	220.0	trillions USD	Initial capital stock
Population			
ℓ_g	0.134	1/yr	Population growth rate parameter
L_∞	10500	millions	Asymptotic population
L_0	7403	millions	Initial population
Decoupling			
g_σ	0.0152	1/yr	Initial carbon intensity decline
δ_σ	0.00	1/yr	Decline decay rate
σ_0	0.3503	$GtCO_2$/trillion USD	Initial carbon intensity
Abatement			
p_b	550	USD/tCO_2	Initial backstop price
θ_2	2.6	–	Abatement cost exponent
δ_{pb}	0.025	1/yr	Decline rate of backstop price
Climate damages			
a_2	0.00236	–	Damage coefficient
a_3	2	–	Damage exponent

Table 2: Calibration of Model Parameters (cont)

Symbol	Value	Unit	Description
Climate system			
$F_{2\times CO_2}$	3.3613	W/m^2	Forcing from CO_2 doubling
$T_{2\times CO_2}$	3.1	°C	Equilibrium climate sensitivity
ξ	3/11	CO_2 to Carbon	Conversion factor CO2 emissions → carbon
M_{AT}^0	851	GtC	Initial atmospheric carbon
M_{UP}^0	460	GtC	Initial upper-ocean carbon
M_{LO}^0	1740	GtC	Initial deep-ocean carbon
M_{AT}	588	GtC	Preindustrial atmospheric carbon
M_{UP}	360	GtC	Preindustrial upper-ocean carbon
M_{LO}	1720	GtC	Preindustrial deep-ocean carbon
b_{12}	0.024	1/yr	Carbon flow: atmosphere → upper ocean
b_{23}	0.0014	1/yr	Carbon flow: upper ocean → deep ocean
c_1	0.0201	$°C/(W/m^2)$	Climate response coefficient
c_3	0.0176	1/yr	Heat transfer: atmosphere → ocean
c_4	0.005	1/yr	Heat transfer: ocean → atmosphere
T_{AT}^0	0.85	°C	Initial atmospheric temperature anomaly
T_{LO}^0	0.007	°C	Initial deep-ocean temperature anomaly
Exogenous forcing			
f_1	1.00	W/m^2	Final exogenous forcing
f_0	0.50	W/m^2	Initial exogenous forcing
t_f	17.0	years	Transition period for exogenous forcing
Land-use emissions			
E_{L0}	2.6	$GtCO_2/yr$	Initial land-use emissions
δ_{EL}	0.115	1/yr	Decline rate of land-use emissions
Numerical bounds			
$s_{\min}$	0.0	–	Lower bound on saving rate
$s_{\max}$	1.0	–	Upper bound on saving rate