Modélisation, estimation, simulation des risques climatiques

The Social Cost of Carbon

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Objectives

- Formulate the social planner's **optimal control** problem (Ramsey setup).
- See how the **SCC emerges as a shadow price** and maps to an optimal carbon tax (USD/tCO_2).

Materials:

- DICE_Notebook2_optimization.ipynb *notebook exercises about scenarii simulations.*
- DICE_Notebook2-2_discounting_tipping.ipynb notebook bonus on uncertainty.
- Handout_DICE.pdf a synthetic presentation of the DICE model.

Introduction

Introduction: From what could happen to what should happen

- In the previous lecture: we built **transition scenarios** (SSPs, forcing pathways) — describing *possible futures*.
- But in principle, we may also ask: **what should the transition look like?**
  - Formalized as an **optimal control problem**:

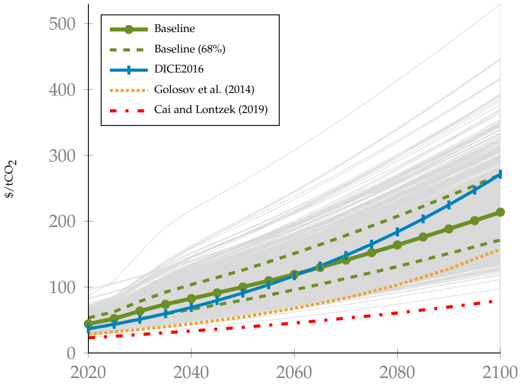
$$\max_{\{\mu_t,\dots\}} \sum_{t=0}^{\infty} \beta^t \, U(c_t) \quad \text{s.t. climate–economy dynamics}$$

  - Find the sequence of decisions that maximizes social welfare.
- Advantages:
  - Reduces an **infinite set of possible paths** to **one “optimal” path**.
  - Provides a **normative benchmark** for policy: what should be done.

Introduction: From Optimal Abatement to Optimal Carbon Price

- The key policy metric is not the abatement rate itself, but the **optimal carbon price** (or **Social Cost of Carbon, SCC**).
- Definition: the SCC is the **monetary value (USD per ton of CO_2)** of the marginal damage from one additional ton of emissions.
- Why it matters:
    - Abstract concept (model-based), but provides guidance on **what carbon price should be today**.
    - Forms the basis for actual carbon tax design.
    - Example: in the US, the Biden Administration used **five IAM-based SCC estimates** to guide federal policy.

Introduction: Example



Note: The figure displays the social cost of carbon (SCC) on the vertical axis. The baseline model from Jondeau et al. (2022) is represented by a solid green line. Uncertainty interval, computed from 2,000 simulations using alternative parameter draws from the Metropolis-Hastings algorithm, are depicted by solid gray lines. Alternative SCC are also presented: (i) DICE2016 (solid blue with vertical markers), (ii) Golosov et al. (2014) (orange dotted line), and (iii) Cai and Lontzek, 2019 with a benchmark calibration assuming 1.5% total factor productivity growth (red dashed line).

Outline

The Optimal Control Problem

Control Problem: the objective function

- **Individual preferences.** Utility from consumption $u(c)$ with $u'(c) > 0$ and $u''(c) < 0$ (diminishing marginal utility). We use CRRA:

$$u(c) = \begin{cases} \dfrac{c^{1-\gamma} - 1}{1-\gamma}, & \gamma \neq 1, \\ \log c, & \gamma = 1, \end{cases} \qquad \gamma > 0.$$

- **Aggregation and units.** Let C_t be aggregate consumption (trillion USD) and L_t population (million). Per-capita consumption in *thousand USD* is

$$c_t \;=\; 1,000 \times \frac{C_t}{L_t}.$$

  Instantaneous social welfare is $L_t \, u(c_t)$.

- **Intertemporal welfare.** With pure rate of time preference $\rho > 0$,

$$W_t = \sum_{i=0}^{\infty} \frac{L_{t+\Delta i} \, u(c_{t+\Delta i})}{(1+\rho)^{\Delta i}}.$$

Control Problem: the solution toolkit

Planner's problem. Consider a government that chooses decisions variables (savings/investment S_t and abatement μ_t) to maximize

$$\max_{\{s_t,\mu_t\}_{i\geq 0}} \sum_{i=0}^{\infty} \frac{L_{t+\Delta i}\, u(c_{t+\Delta i})}{(1+\rho)^{\Delta i}} \quad \text{s.t. resource, capital, ...}$$

How to solve the problem?

1. **Numerical solver:** use solver to find sequences $\{S_t, \mu_t\}_{t=0}^{T}$ that maximizes objective.
2. **Ramsey solution:** maximize social welfare based on N_y equations and $N_y + N_z$ control variables.
3. **Dynamic programming:** use Bellman equation and value function iterations or projections possibly with fancy neural networks.

Control Problem: model summary (a recap)

- **Exogenous variables** each driven by an exogenous process (growth/decline path).

$$x_t = \{A_t,\ \sigma_t,\ \theta_{1,t},\ F_t^{EX},\ E_t^{\text{land}},\ L_t\}$$

- **Endogenous state/flow variables**

$$y_t = \{M_t^{AT},\ M_t^{UP},\ M_t^{LO},\ T_t^{AT},\ T_t^{LO},\ Y_t,\ K_t,\ C_t\}$$

- **Decision variables (controls)**

$$z_t = \{\mu_t,\ s_t\}$$

- **Horizon and time step:** $t \in \{2015, 2020, 2025, \ldots, T\}$, $\Delta = 5$ years (quinqennial).

Control Problem: a sketch of the numerical solution

- Because the social welfare is forward looking, the system is both forward and backward looking as follows:

$$z_t = \arg \min_{\{s_t, \mu_t\}_{i \geq 0}} - \sum_{i=0}^{\infty} (1+\rho)^{-\Delta i} L_{t+\Delta i}\, u(c_{t+\Delta i})$$

- When a sequence $\{z_{t+\Delta i}\}_{i=0}^{\infty}$ is found, one can compute:

$$\begin{aligned} x_t &= g(x_{t-1}), & g &: \ \mathbb{R}^{N_x} \to \mathbb{R}^{N_x}, \\ y_t &= f(y_{t-1},\, x_t,\, z_t), & f &: \ \mathbb{R}^{N_y} \times \mathbb{R}^{N_x} \times \mathbb{R}^2 \to \mathbb{R}^{N_y}. \end{aligned}$$

- $x_t \in \mathbb{R}^{N_x}$: **exogenous variables** $(A_t,\ \sigma_t,\ \ldots, L_t)$.
- $y_t \in \mathbb{R}^{N_y}$: **endogenous states/flows** $(M_t^{AT}, M_t^{UP}, \ldots, Y_t, K_t)$.
- $z_t = [\mu_t, s_t]' \in \mathbb{R}^2$: **decision variables** (abatement, saving).

Control Problem: a sketch of the numerical solution

- **Idea.** Approximate the **infinite-horizon** objective with a **finite** one. Let $\beta_\Delta \equiv (1+\rho)^{-\Delta}$. Because $\lim_{i\to\infty} \beta_\Delta^i = 0$, the tail contributes little.
- **Pick the horizon $I^\star$** so that the tail weight is below a tolerance ε:

$$\beta_\Delta^{I^\star} \leq \varepsilon$$

  Trade-off: smaller $I^\star \Rightarrow$ faster solve but larger truncation error.
- **Finite-horizon problem** with $T = t + I^\star\Delta$, solve

$$\max_{\{s_{t+i},\mu_{t+i}\}_{i=0}^{I^\star}} \sum_{i=0}^{I^\star} \beta_\Delta^i \, L_{t+i\Delta} \, u\left(1000 \times \frac{C_{t+i\Delta}}{L_{t+i\Delta}}\right)$$

  s.t. dynamics (resource, capital, emissions, climate) and bounds.

Control Problem: Two Policies

- The mitigation policy:

$$\max_{\{s_{t+i},\mu_{t+i}\}_{i=0}^{I^\star}} \sum_{i=0}^{I^\star} \beta_\Delta^i \, L_{t+i\Delta} \, u\left(1000 \times \frac{C_{t+i\Delta}}{L_{t+i\Delta}}\right)$$

s.t. dynamics (resource, capital, emissions, climate) and bounds.

- The *laisser-faire* / Business As Usual (BAU) policy:

$$\max_{\{s_{t+i},\}_{i=0}^{I^\star}} \sum_{i=0}^{I^\star} \beta_\Delta^i \, L_{t+i\Delta} \, u\left(1000 \times \frac{C_{t+i\Delta}}{L_{t+i\Delta}}\right)$$

s.t. dynamics (resource, capital, emissions, climate) and bounds.

$$s.t.\ \mu_t = 0.03$$

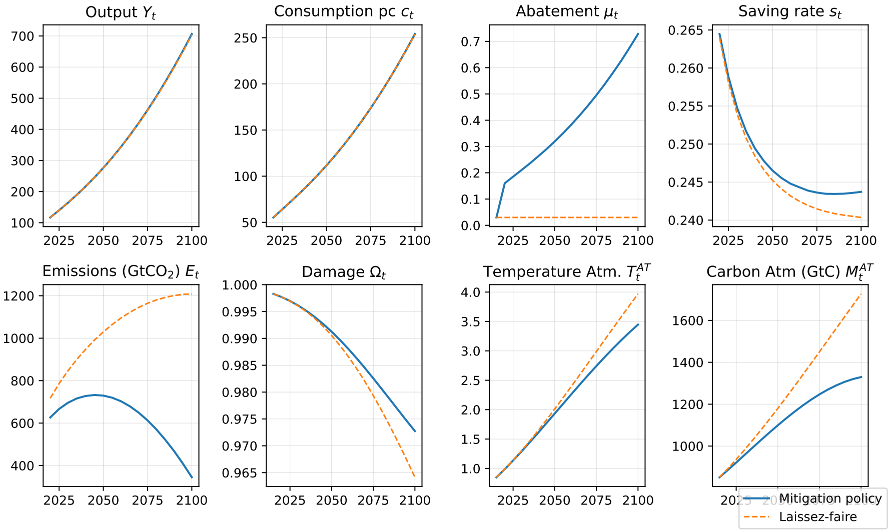


Figure. Simulations under two policy scenarios

Some striking but controversial results from DICE:

- "Optimal" to warm the planet up to $3.3°C$;
- Mitigation policy saves about 1% of climate damage;
- Net zero emission optimal by 2130, 80 years later than in Paris-Agreement;
- Main critic from DICE's policy output: those results have scientifically grounded climate inaction in policy circles...

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The SSC in the Ramsey Problem

Social cost of carbon

- Why is it optimal to warm so much the planet according to DICE?
- An optimal carbon tax = how much the society is willing to pay to reduce emissions ($ per ton of carbon).
- This is usually referred to as the Social Cost of Carbon (SCC) in literature.
- SCC reflects the society's gains and losses from implementing the carbon tax.
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The Ramsey (1927) problem:

$$
\begin{aligned}
&\max_{\{c_t, Y_t, K_t, T_t, M_t, \mu_t, s_t\}} \sum_{i=0}^{\infty} \beta_{\Delta}^{i} L_{t+\Delta i}\, u(c_{t+\Delta i}) \\
&+ \beta_{\Delta}^{i} \lambda_{1,t+\Delta i} \left[(1 - s_{t+\Delta i})\, Y_{+\Delta it} \left(1 - \theta_{1,t+\Delta i} \mu_{t+\Delta i}^{\theta_2}\right) \Omega \left(T_{t+(i-1)\Delta}\right) - L_{t+\Delta i} c_{t+\Delta i}/1,000 \right] \\
&+ \beta_{\Delta}^{i} \lambda_{2,t+\Delta i} \left[K_{t+\Delta i} - (1 - \delta_K)\, K_{t+(i-1)\Delta} - s_{t+\Delta i} Y_{t+\Delta i} \left(1 - \theta_{1,t+\Delta i} \mu_{t+\Delta i}^{\theta_2}\right) \Omega \left(T_{t+(i-1)\Delta}\right) \right] \\
&+ \beta_{\Delta}^{i} \lambda_{3,t+\Delta i} \left[Y_{t+\Delta i} - A_{t+\Delta i} K_{t+(i-1)\Delta}^{\gamma} L_{t+\Delta i}^{1-\gamma} \right] \\
&+ \beta_{\Delta}^{i} \lambda_{4,t+\Delta i} \left[T_{t+\Delta i} - \Phi_T T_{t+(i-1)\Delta} - \Xi_T F_{t+\Delta i} \left(M_{t+\Delta i}\right) \right] \\
&+ \beta_{\Delta}^{i} \lambda_{5,t+\Delta i} \left[M_{t+\Delta i} - \Phi_M M_{t+(i-1)\Delta} - \Xi_M \left(\sigma_{t+\Delta i} \left(1 - \mu_{t+\Delta i}\right) Y_{t+\Delta i} + E_{t+\Delta i}^{\text{land}} \right) \right]
\end{aligned}
$$

Summary: optimal control problem with 7 control variables, 5 constraints $\rightarrow$ 12 variables/equations.

(note that climate variables include more equations + Lagrangian multipliers)

- Recall that compact form for climate block with

$$T_t = [T_t^{AT}, T_t^{LO}]^\top \text{ and } M_t = [M_t^{AT}, M_t^{UP}, M_t^{LO}]^\top$$

- The corresponding multipliers:

$$\lambda_{4,t} = [\lambda_{5,t}^{AT}, \lambda_{5,t}^{LO}]^\top \quad \text{and } \lambda_{5,t} = [\lambda_{5,t}^{AT}, \lambda_{5,t}^{UP}, \lambda_{5,t}^{LO}]^\top$$

- Damages and forcing functions as function of vectors T_t and M_t :

$$\Omega(T_t) = 1/\left(1 + a_2 \left(T_t^{AT}\right)^2\right)$$

$$F_t(M_t) = \eta \, \log\left(M_t^{AT}/M_{1750}\right)/\log(2) + F_t^{EX}$$

- Corresponding gradients:

$$\nabla_T \Omega(T_t) = \begin{bmatrix} -\Omega(T_t)^2 \, 2a_2 \left(T_t^{AT}\right) & 0 \end{bmatrix}^\top$$

$$\nabla_M F(M_t) = \begin{bmatrix} \eta/(M_t^{AT} \ln 2) & 0 & 0 \end{bmatrix}^\top$$

Social cost of carbon

- Get back to FOCs of the Ramsey-planner:

$$T_t : \lambda_{4,t} = \beta_\Delta \left[\Phi_T^\top \lambda_{4,t+\Delta} + \lambda_{2,t+\Delta} s_{t+\Delta} Y_{t+\Delta} \left(1 - \theta_{1,t+\Delta} \mu_{t+\Delta}^{\theta_2}\right) \nabla_T \Omega \left(T_t\right) \right]$$
$$M_t : \lambda_{5,t} = \beta_\Delta \left[\Phi_M^\top \lambda_{5,t+\Delta} + \left(\Xi_T^\top \lambda_{4,t+\Delta} \right) \nabla_M F \left(M_t\right) \right]$$

  $\lambda_{5,t}$ ($\lambda_{4,t}$) is the marginal loss from carbon (temperature) increase in each climate box.

- SCC expresses the social loss into numeraire equivalents (here consumption):

$$SSC_t = -1000 \times \lambda_{5,t}^{AT} / \lambda_{1,t} \simeq -1000 \times \left(\partial W_t / \partial M_t^{AT} \right) / \left(\partial W_t / \partial C_t \right)$$

  where $\lambda_{1,t}$ is marginal utility of consumption while $\lambda_{5,t}^{AT}$ measures the planner's value for a marginal change in atmospheric carbon.

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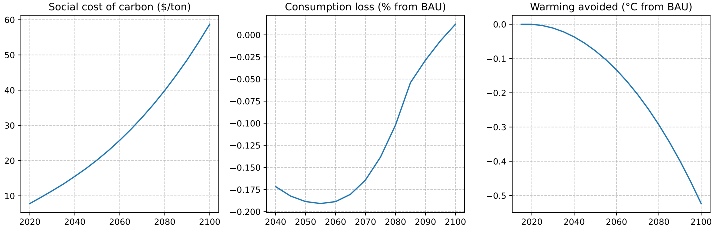
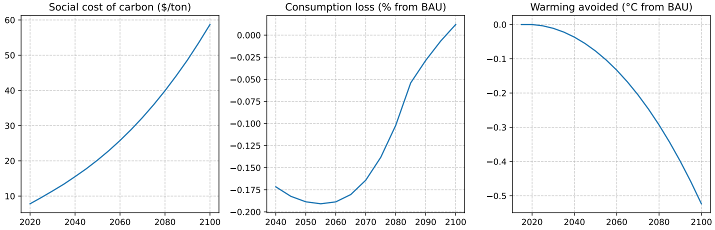


- SCC reflect the planner's relative gains in terms of welfare from controlling M_t against its consumption losses C_t from such action;
- Optimal to cut consumption now by 0.2% in order to avoid $0.55°C$ of warming in 2100;
- As well off between scenarios by 2090 on current consumption grounds (but not in welfare terms).



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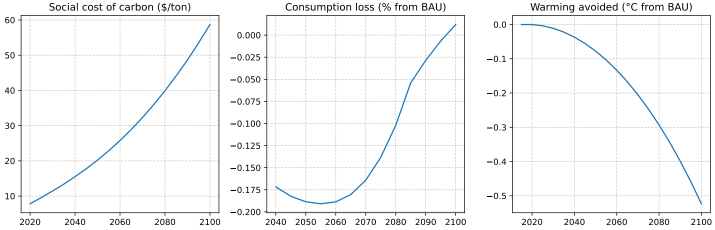

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- How to retrieve the SCC if we don't have any lagrangian multiplier?
- One can use the firm's static problem (given output Y_t):

$$\min_{\mu_t \in [0,1]} \quad \underbrace{\theta_{1,t}\, \mu_t^{\theta_2}\, Y_t}_{\text{abatement cost}} \; + \tau_t \quad \underbrace{\sigma_t (1 - \mu_t)\, Y_t}_{\text{taxed residual emissions}} \quad .$$

- First-order condition in μ_t:

$$\theta_2\, \theta_{1,t}\, \mu_t^{\theta_2 - 1} Y_t \;=\; \tau_t\, \sigma_t\, Y_t \quad \Rightarrow \quad \boxed{\tau_t \;=\; \frac{\theta_2\, \theta_{1,t}}{\sigma_t}\, \mu_t^{\theta_2 - 1}}$$

- This tax is in trillion USD per gigatonne carbon. The carbon tax $\tau_t^{\$}$ in US$/tCO$_2$:

$$\tau_t^{\$} = \frac{10^{12}}{10^9} \times \tau_t = 1,000 \times \tau_t$$

as $1GtC = 10^9$ and $1\ \textit{trillion}\ \$ = 10^{12}$.

IAMs Controversies

How relevant are IAM models?

- **IAM-believiers:** they are relevant at least on normative grounds to provide discussion basis on mitigation scenarios.

- **IAM-deniers:** they are useless to guide policy scenarios → the Pindyck (2013, 2017) critique enumerates a list of flaws:

    1. "*certain inputs [parameters] are arbitrary, but have huge effects on the SCC estimates the models produce*";
    2. "*the models' descriptions of the impact of climate change are completely ad hoc, with no theoretical or empirical foundation*";
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The role of inputs: the discount factor

- In 2006, the main view on climate change among economists was the Nordhaus (1992)'s view: optimal to warm up to $3.5°C$ the planet.
- The Stern (2006)'s view shook this general agreement on many aspects:
  - the core argument: the price of inaction would be extraordinary and the cost of action modest ;
  - irreversible impacts from climate change threatening access to water, food production, health, and use of land and the environment;
  - IAM supports such actions in particular by changing discount factor → debate.

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The role of inputs: the discount factor

- What's discount factor? Welfare function as a discounted sum of future gains:

$$\mathcal{W}_t = L_t U\left(c_t\right) + \beta_\Delta \mathcal{W}_{t+\Delta}$$

- β_Δ is a deep parameter that depends on several factors:

  1. psychological aspects (time-preference $\rightarrow \beta_\Delta$ low),
  2. political aspects (how policymakers weight the future $\rightarrow \beta_\Delta$ low/high),
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The role of inputs: the discount factor

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- Introducing a one-period financial claim:

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The discount factor

- The Stern (2006)'s report provides a scientific motivation for the Paris-Agreement.

- How? Consider FOC on T:

$$\begin{aligned}
\lambda_{4,t} &= \beta_\Delta \left[\Phi_T^\top \lambda_{4,t+\Delta} + \lambda_{2,t+\Delta} s_{t+\Delta} Y_{t+\Delta} \left(1 - \theta_{1,t+\Delta} \mu_{t+\Delta}^{\theta_2}\right) \nabla_T \Omega\left(T_t\right) \right] \\
&= \sum_{i=1}^{\infty} \left(\beta_\Delta \Phi_T^\top\right)^i s_{t+\Delta i} Y_{t+\Delta i} \left(1 - \theta_{1,t+\Delta i} \mu_{t+\Delta i}^{\theta_2}\right) \nabla_T \Omega\left(T_{t+\Delta(i-1)}\right)
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Tipping points

- Tipping point are critical threshold beyond which a system reorganizes abruptly & irreversibly.

- Natural system tipping points well identified:
    - Antarctic & Artic ice sheet disintegration: complete disintegration (at +10°C) would raise the global sea levels by 53.3 metres;
    - Amazon Rainforest destruction: a warming planet →rainforest may transform into a dry savanna landscape;
    - Sibera's Permafrost thaw: unfreezing permafrost would release methane ↗ T°C.
    - etc.

- Tipping points are possible at today's global warming of just over 1 °C (1.8 °F) above preindustrial times, and highly probable above 2 °C (3.6 °F) of global warming.

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$$\Omega\left(T_t\right) = 1/(1 + a_2 \left(T_t^{AT}\right)^2 + bT_t^c)$$

where b and c are set to match potential losses as T grows: $b = 5.0703e - 06$, $c = 6.754$.

	$\Omega\left(1\right)$	$\Omega\left(3\right)$	$\Omega\left(5\right)$
$b = 0$ - Nordhaus (2017)	0.9976	0.9792	0.9443
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- Why tipping points call for stronger actions?
- Get back to FOC:

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Recap

Exam Guidelines

- **Equations:** You don't need to memorize every equation by heart. But you should be able to recognize them if you see them, and explain their meaning.
- **Simulations:** You don't need to know the code. What matters is that you understand the outputs (graphs, tables, numbers) and can interpret them correctly.
- **MCQs:** Expect multiple-choice questions with text, equations, and figures. The goal is to test whether you can read, interpret, and choose the correct answer.

In short: focus on understanding, not rote learning.

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Thank you!

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