Modélisation, estimation, simulation des risques climatiques

Scenario building of Integrated Assessment Models

Gauthier Vermandel

Ecole polytechnique

Schedule of the day

Objectives

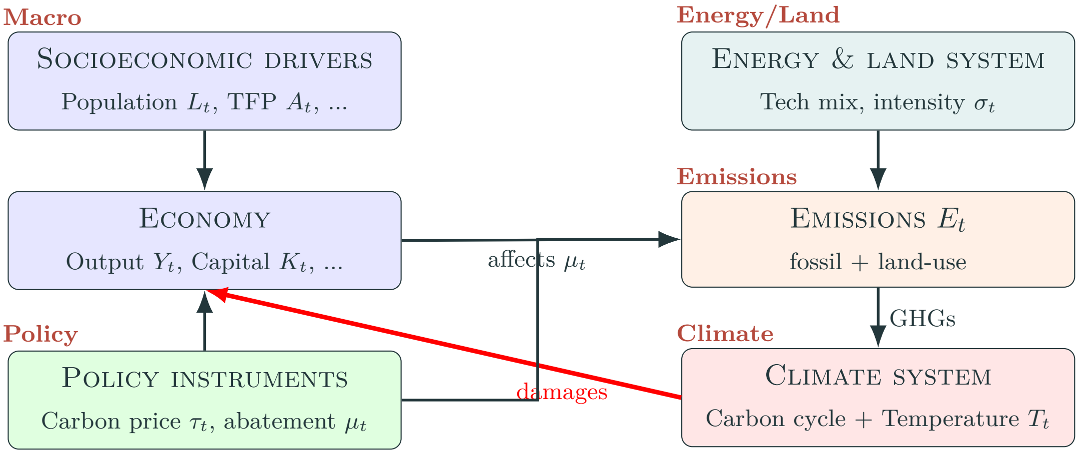
- Understanding the **basics** of **Integrated Assessment Models**.
- Build and **simulating macro-climate scenarios** using IPCC and NGFS methodology.

Materials:

- DICE_Notebook1-1_basics.ipynb *notebook introduction of model's options.*
- DICE_Notebook1-2_generation_scenarios.ipynb *notebook exercises about scenarii simulations.*
- Handout_DICE.pdf a synthetic presentation of the DICE model.

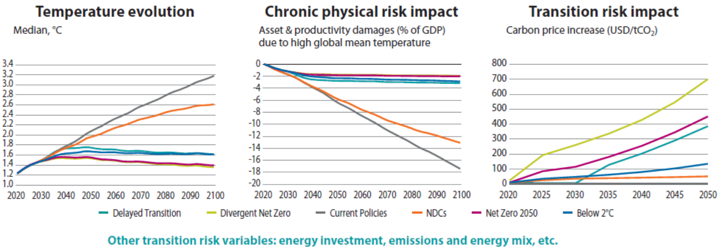
Introduction

Introduction: why IAMs matter for policy and finance



Introduction: IAMs in practice: NGFS Scenarios

- IAMs provide scenario pathways $\Rightarrow$
  - **Temperature evolution** under different policies.
  - **Physical risk impacts**: GDP losses from climate damages.
  - **Transition risk impacts**: carbon price trajectories.
- Used for **stress-testing** and **risk assessment** in finance.



Source: IIASA NGFS Climate Scenarios Database, REMIND model.

NGFS = Network for Greening the Financial System (central banks and supervisors).

Introduction: why IAMs matter for policy and finance

- **For financial institutions:**
  - IAM outputs (e.g. from NGFS scenarios) are used by central banks and supervisors for **climate stress-testing exercises**.
  - Provide pathways for GDP, energy, emissions, and damages under different policies.
- **For international policy:**
  - IAMs feed into **IPCC reports**, underpinning assessments of mitigation costs and impacts.
  - Establish a common reference for climate policy design across countries.
- **For carbon pricing:**
  - Benchmark for the **social cost of carbon**.
  - Example: the US administration (Biden) adopted \$52/ton CO_2 based on IAM results (incl. DICE).

Introduction: Why do we have a variety of scenarios?

- Climate scenarios are not forecasts: they are **"what if"** explorations.
- They differ because of two main ingredients:
  1. **Socioeconomic narratives (SSPs)**
     - Long-run assumptions on population, economic growth, inequality.
     - Energy and land-use technology adoption (renewables, CCS, etc.).
     - Speed of structural change and decarbonization potential.
  2. **Policy implementation (forcing levels)**
     - Presence (or absence) of a carbon price.
     - How fast the carbon tax rises over time.
     - Coverage: local vs. global implementation, uniform vs. fragmented.
- **Together, narratives + policy assumptions build the SSP scenario matrix used by the IPCC.**

Introduction: roadmap for today

- Explain the role of **Integrated Assessment Models (IAMs)** in climate-economy analysis.
- Learn how climate scenarios are **constructed**, not predicted:
    - **Narrative dimension**: socioeconomic pathways (SSPs), growth, technology, demographics.
    - **Policy dimension**: climate policy ambition (carbon pricing, mitigation targets).
- Understand the **IPCC scenario matrix** (SSP - forcing levels) and its role in assessments (e.g. CMIP6).
- Explore the DICE model as a **case study**: its building blocks, trade-offs, and illustrative simulations.

Introduction: where does DICE fit in this landscape?

- **Strength**: standard economic block + climate module (carbon cycle, temperatures) $\Rightarrow$ clear insight into mechanisms and the *social cost of carbon*.
- **Limits**: no sectoral detail, no fine multi-regional structure, limited uncertainty $\Rightarrow$ complement to complex IAMs.
- **Use**: conceptual benchmark, teaching tool, stress-testing policies, part of the five core models used by Biden's administration to set the carbon price ($52/ton carbon).

Outline

Shared Socioeconomic Pathways (SSPs)

SSPs: Key definitions

- **SSPs (Shared Socioeconomic Pathways):** Narratives + quantitative pathways describing alternative futures of population, GDP, technology, land use, energy.
- **CMIP (Coupled Model Intercomparison Project):** International framework where Earth System Models (ESMs) simulate climate under common scenarios.
- **IPCC:** Not a research body itself. It *assesses* the scientific literature (incl. IAMs & CMIP runs) to produce consensus reports.

SSPs: Why Shared Socioeconomic Pathways (SSPs)?

- Provide a **common framework** for socio-economic and climate futures.
- Two building blocks:
  1. **Narratives (qualitative)**: governance, technology, inequality.
  2. **Quantification (IAMs)**: GDP, population, emissions, land-use.
- Combined with **forcing levels** (1.9, 2.6, 4.5, 7.0, 8.5 W/m^2) to span policy ambition.

SSPs: The 5 SSP narratives

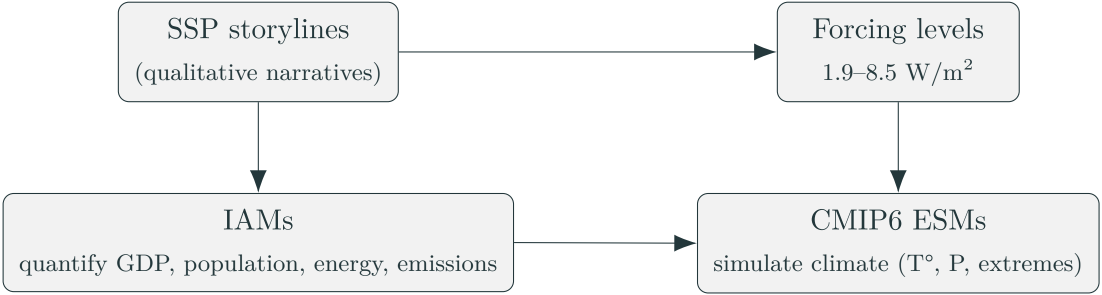
- **SSP1 - Sustainability**: inclusive development, green growth.
- **SSP2 - Middle of the road**: historical trends continue.
- **SSP3 - Regional rivalry**: fragmentation, low cooperation.
- **SSP4 - Inequality**: persistent global disparities.
- **SSP5 - Fossil-fueled growth**: rapid growth, fossil energy use.

SSPs: The SSP $\times$ forcing matrix

	Forcing W/m^2				
	1.9	2.6	4.5	7.0	8.5
SSP1	SSP1-1.9	SSP1-2.6	–	–	–
SSP2	–	SSP2-2.6	SSP2-4.5	SSP2-7.0	–
SSP3	–	–	SSP3-4.5	SSP3-7.0	SSP3-8.5
SSP4	–	SSP4-2.6	SSP4-4.5	SSP4-7.0	–
SSP5	–	–	SSP5-4.5	–	SSP5-8.5

Each cell = a scenario used in IPCC/CMIP6 (e.g., **SSP1-2.6**, **SSP5-8.5**).

SSPs: How SSPs feed into IPCC assessments



IPCC compiles these outputs ⇒ assessment of mitigation & impacts.

SSP1-2.6 (Sustainability + strong mitigation)

- Low population, inclusive growth.
- Rapid clean tech diffusion.
- CO_2 emissions peak early, net zero 2070.
- Warming: +1.5–2°C.

SSP5-8.5 (Fossil-fueled growth, no policy)

- High GDP, energy-intensive growth.
- Continued fossil dependence.
- CO_2 emissions keep rising.
- Warming: +4°C or more.

SSPs: Where DICE fits in this exercise

- IAMs provide the **rich global picture** used by IPCC.
- DICE is a **simplified IAM**: Solow–Ramsey economy + climate block.
- **Educational role:**
  - Shows the mechanics of abatement vs damages.
  - Easy to simulate baseline vs policy cases.
  - Helps interpret more complex IAM/CMIP outcomes.

The DICE Model

DICE: Introduction

- DICE (Dynamic Integrated Climate-Economy model) was developed by Nordhaus (1992) and went through several (minor) updates up to the lastest 2023 snapshot;
- Simple but powerful model to provide fast estimates of the social cost of carbon and transition pathways for the world economy;
- It has become **an intuitive benchmark**.

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DICE: Introduction

- The framework is "old-fashion" (from methodological perspective):
  - Solow growth model with climate externality;
  - Centralized equilibrium based on Ramsey allocations;
  - Purely deterministic setup;
- Physical capital, temperatures and carbon boxes are the main state variables that the planner uses to optimally maximize welfare.
- These slides are based on **2016 snapshot** (a 2023 framework now available).

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DICE: setup basics

- Time is discretized in steps $i = 1, \ldots, I$.
  - calendar time: $t(i) = t_0 + \Delta \cdot i$
  - t_0 initial period, Δ time step (in years)
  - horizon: $t \in \{t_0,\, t_0 + \Delta,\, t_0 + 2 \cdot \Delta, \ldots, T\}$

- Variables can be indexed either by *time* or by *step*:

$$Y_t \;\equiv\; Y(i) \qquad \textit{with} \quad t = t(i).$$

- The model is a **discrete-time economy with finite horizon**:
  - horizon length: I steps $\equiv$ final time T.
  - centralized economy: a *single representative agent* chooses all decisions.

DICE: Carbon cycle

- Atmospheric loading of CO_2 is a system of differential equations:

$$M_t = \Phi_M M_{t-\Delta} + \Xi_M E_t, \tag{1}$$

where $M_t = [M_t^{AT}, M_t^{UP}, M_t^{LO}]'$ comprises 3 layers of carbon: atmospheric, upper and lower ocean respectively.

- Matrices are determined by:

$$\Phi_M = \begin{bmatrix} 1 - \Delta b_{12} & \Delta b_{12} \frac{M_{AT0}}{M_{UP0}} & 0 \\ \Delta b_{12} & 1 - \Delta b_{12} \frac{M_{AT0}}{M_{UP0}} - \Delta b_{23} & \Delta b_{23} \frac{M_{UP0}}{M_{LO0}} \\ 0 & \Delta b_{23} & 1 - \Delta b_{23} \frac{M_{UP0}}{M_{LO0}} \end{bmatrix} \text{ and } \Xi_M = \begin{bmatrix} \xi \\ 0 \\ 0 \end{bmatrix}$$

- Carbon emissions go into the atmosphere, before being captured by oceans through a long lasting process.

DICE: Temperatures

- Temperature anomalies $T_t = [T_t^{AT}, T_t^{OC}]$ of atmosphere and ocean respectively:

$$T_t = \Phi_T T_{t-\Delta} + \Xi_T F_t, \tag{2}$$

- Matrices are given by:

$$\Phi_T = \begin{bmatrix} (1 - \Delta \cdot c_1 \cdot F_{2\times CO2} - \Delta \cdot c_1 \cdot c_3) & (\Delta \cdot c_1 \cdot c_3) \\ (\Delta \cdot c_4) & (1 - \Delta \cdot c_4) \end{bmatrix} \text{ and } \Xi_T = \begin{bmatrix} \Delta \cdot c_1 \\ 0 \end{bmatrix}$$

- Cooling effects of oceans through c_3, c_1 is the heating sensitivity to radiative forcing F_t, oceans are heating via atmospheric warming up to c_4 degree.

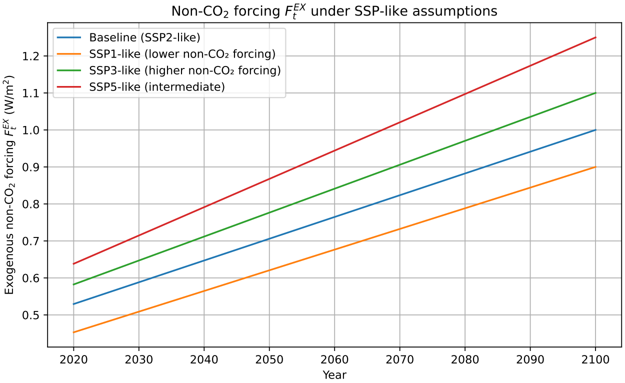
DICE: Radiative forcing

- Greenhouse effect via radiative forcing (W/m^2):

$$F_t = F_{2\times CO_2} \ \log\left(M^{AT}_{t-\Delta}/M_{AT0}\right)/\log(2) + F^{EX}_t \tag{3}$$

- Here, $F_{2\times CO_2}$ denote the forcing (Wm-2) of equilibrium CO2 doubling ($M^{AT}_t = 2M_{AT0}$).
- F^{EX}_t is an exogenous process tracking the contribution of other sources of GHG (i.e. methane) on F_t

DICE: Example of exogenous forcing



DICE: Input structure

- World population (millions):

$$L_t = L_{t-\Delta} \left(L_\infty / L_{t-\Delta} \right)^{\ell_g}, \tag{4}$$

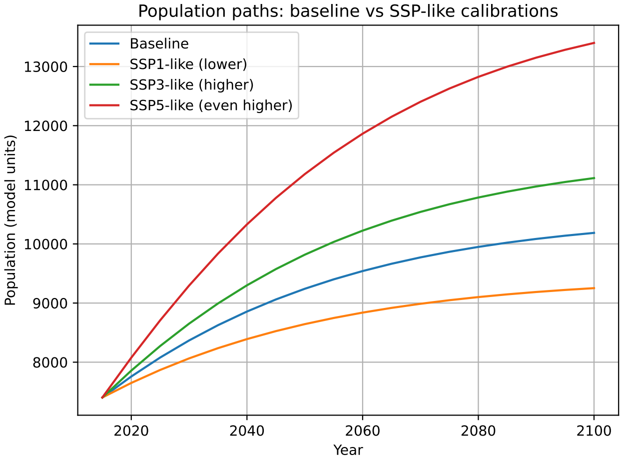
$\ell_g \in [0, 1]$ convergence rate, $L_\infty \in [0, +\infty)$ long term population.

- Law of motion of capital:

$$K_t = (1 - \delta_K)^\Delta K_{t-\Delta} + I_t, \tag{5}$$

$\delta_K \in [0, 1]$ is the depreciation rate, while Δ is the time step (here $\Delta = 5$ years)

DICE: Example of population dynamics



DICE: Production of goods

- Production is Cobb-Douglas in thousands USD:

$$Y_t = A_t K_{t-\Delta}^{\gamma} \left(\frac{L_t}{1,000} \right)^{1-\gamma} \tag{6}$$

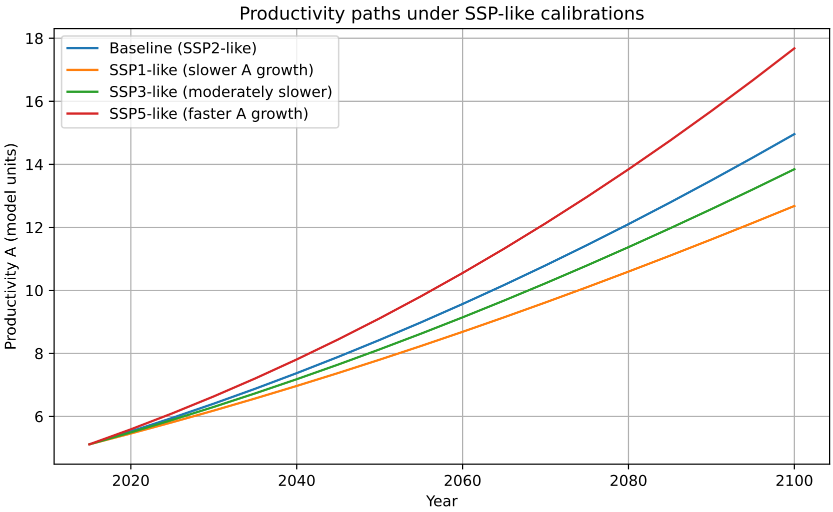
where $\gamma \in [0, 1]$ is capital intensity, physical capital $K_{t-\Delta}$.

- Exogenous total productivity A_t (with vanishing growth)

$$A_t = A_{t-\Delta} / (1 - g_A \exp(-\delta_A (t - t_0))) \tag{7}$$

$g_A \geq 0$ the initial productivity growth and decay rate $\delta_a \in (0, 1]$ to match declining productivity growth

DICE: Example of productivity dynamics



DICE: Emissions

- Emissions (GtCO2):

$$E_t = \Delta \cdot \left(\sigma_t \,(1 - \mu_t)\, Y_t + E_t^{\text{land}}\right) \tag{8}$$

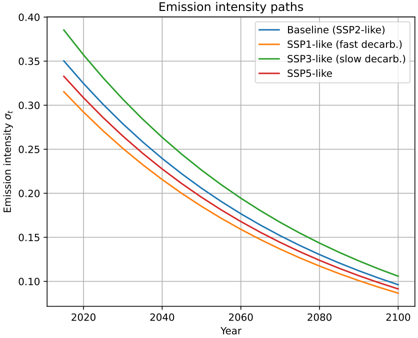
where σ_t decoupling rate, μ_t is abatement share, E_t^{land} is exogenous process of carbon emission from change in land use (i.e. deforestation, ~10% of emissions).

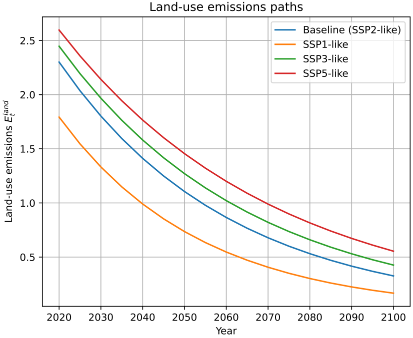
- Exogenous decoupling of emissions to GDP

$$\sigma_t = \sigma_{t-\Delta} \exp\left(-\ g_\sigma \,(1 - \delta_\sigma)^{(t-t_0)}\, \Delta\right) \tag{9}$$

$g_\sigma \geq 0$ the initial decoupling rate and decay rate $\delta_\sigma \in (0, 1]$ to match emissions to GDP data.

DICE: Example of technology dynamics





DICE: Equilibrium in goods and climate

- Resource constraint:

$$Y_t \left(1 - \theta_{1,t} \mu_t^{\theta_2}\right) \Omega \left(T_{t-\Delta}^{AT}\right) = C_t + I_t \tag{10}$$

  LHS is output loss from abatement and damage, RHS is demand side.

- Damages are given by:

$$\Omega \left(T_{t-\Delta}^{AT}\right) = 1/(1 + a \left(T_{t-\Delta}^{AT}\right)^2) \tag{11}$$

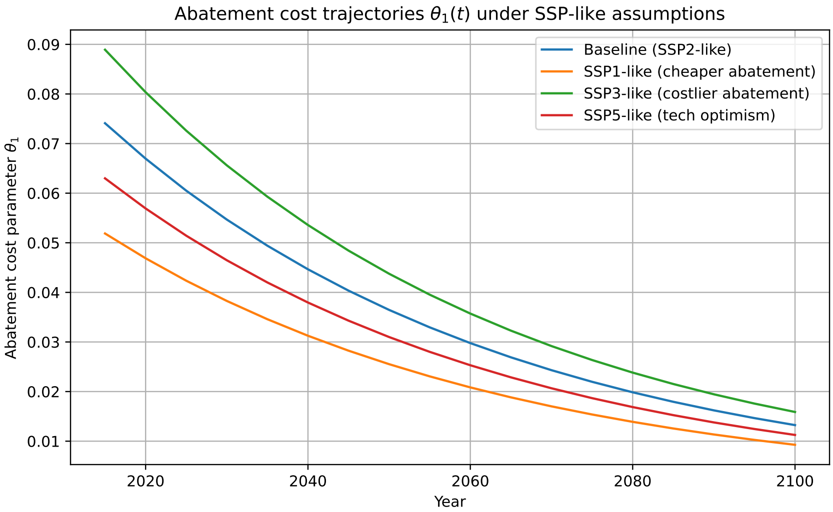
  This expression is subject to deep uncertainty, and should be taken as a normative measure of the social loss for the society of rising temperatures. Tipping points could be introduced [Weitzman, 2012].

- Abatement good technology decreases over time:

$$\theta_{1,t} = \frac{p_b}{1000\,\theta_2} \left(1 - \delta_{pb}\right)^{(t-t_0)} \sigma_t. \tag{12}$$

  where $p_b > 0$ price of carbon to reach net zero, efficiency factor $\delta_{pb} \in [0, 1]$.

DICE: Example of abatement technology



Total factor productivity:

$$A_t = A_{t-\Delta} / (1 - g_A \exp(-\delta_A (t - t_0)))$$

Decoupling rate of carbon emission to GDP:

$$\sigma_t = \sigma_{t-\Delta} \exp\Big(- \; g_\sigma \, (1 - \delta_\sigma)^{(t-t_0)} \, \Delta \Big)$$

Cost of abating carbon emissions:

$$\theta_{1,t} = \frac{p_b}{1000 \, \theta_2} \, (1 - \delta_{pb})^{(t-t_0)} \, \sigma_t$$

Non-CO2 forcing law of motion:

$$F_t^{EX} = \min \left(F_{t-\Delta}^{EX} + \Delta \cdot d_F, F_{CAP} \right)$$

Land-use law of motion:

$$E_t^{\text{land}} = (1 - \delta_e)^{\Delta} \, E_{t-\Delta}^{\text{land}}$$

World population dynamic:

$$L_t = L_{t-\Delta} \, (L_T / L_{t-\Delta})^{\ell_g}$$

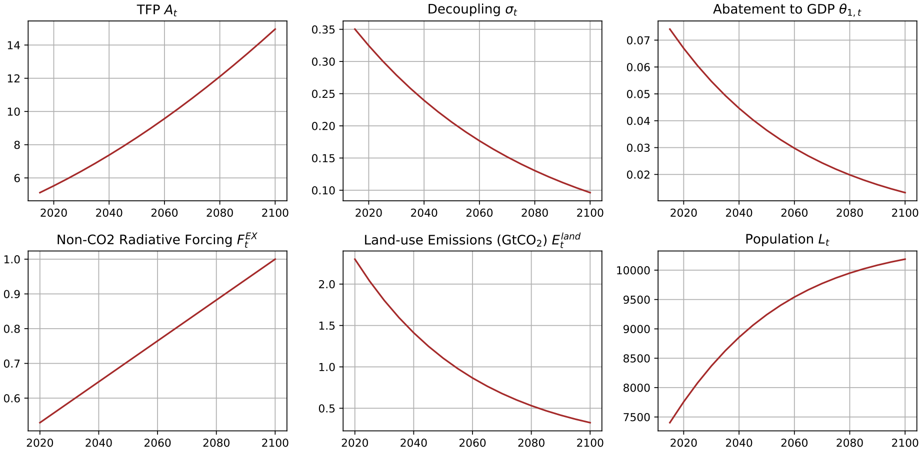


Figure. Summary of exogenous variables

DICE: Introducing a saving rate

- How should we determine optimally C and I?
- We introduce optimal saving rate s, consumption and investment becomes:

$$C_t = \Delta \cdot (1 - s_t) \times Y_t \left(1 - \theta_{1,t}\mu_t^{\theta_2}\right) \Omega \left(T_{t-\Delta}^{AT}\right) \tag{13}$$

$$I_t = \Delta \cdot s_t \times Y_t \left(1 - \theta_{1,t}\mu_t^{\theta_2}\right) \Omega \left(T_{t-\Delta}^{AT}\right) \tag{14}$$

$\rightarrow$implicitely assuming that income spent between C_t and I_t.

- Here, s is "decision" variable, which usually is pinned down by optimal control problem in economics.

Assumption (1)
In what follows, we assume that $\{s_t\}_{t=t_0}^{T} = 0.25$ *as simplifying assumption.*

DICE: Introducing the abatement rate

- Environmental policy here, denoted by abatement share μ_t, is the second decision variable.

- Increasing μ_t has several economic and social implications:
  - $\partial C_t / \partial \mu_t < 0$: $\mu_t \nearrow$ crowds out **consumption**. Yellow vest effect which can threaten the implementation of the policy.
  - $\partial I_t / \partial \mu_t < 0$: $\mu_t \nearrow$ crowds out **investment**. Lower growth prospects in the future.
  - $\partial E_t / \partial \mu_t < 0$: $\mu_t \nearrow$ reduces carbon emissions.

Assumption (2)
In what follows, we will give as an input (exogenously) the sequence $\{\mu_t\}_{t=t_0}^{T}$ of abatement.

DICE: model summary

- **Exogenous variables** each driven by an exogenous process (growth/decline path).

$$x_t = \{A_t,\ \sigma_t,\ \theta_{1,t},\ F_t^{EX},\ E_t^{\text{land}},\ L_t\}$$

- **Endogenous state/flow variables**

$$y_t = \{M_t^{AT},\ M_t^{UP},\ M_t^{LO},\ T_t^{AT},\ T_t^{LO},\ Y_t,\ K_t,\ C_t\}$$

- **Decision variables (or controls)**

$$z_t = \{\mu_t,\ s_t\}$$

- **Horizon and time step:** $t \in \{2015, 2020, 2025, \ldots, T\}$, $\Delta = 5$ years (quinqennial).

DICE: A sketch of the numerical solution

Under Assumptions (1) and (2), the numerical solution is straightforward through an open loop:

$$
\begin{aligned}
x_t &= g(x_{t-1}), & g:\ \mathbb{R}^{N_x} &\to \mathbb{R}^{N_x}, \\
y_t &= f\big(y_{t-1},\, x_t,\, z_t\big), & f:\ \mathbb{R}^{N_y} \times \mathbb{R}^{N_x} \times \mathbb{R}^2 &\to \mathbb{R}^{N_y}.
\end{aligned}
$$

- $x_t \in \mathbb{R}^{N_x}$: **exogenous variables** $(A_t,\ \sigma_t,\ \ldots, L_t)$.
- $y_t \in \mathbb{R}^{N_y}$: **endogenous states/flows** $(M_t^{AT}, M_t^{UP}, \ldots, Y_t, K_t)$.
- $z_t = [\mu_t, s_t]' \in \mathbb{R}^2$: **decision variables** (abatement, saving).

Thank you!

gauthier.vermandel@polytechnique.edu

References

Nordhaus, W. (1992). The 'DICE' model: Background and structure of a dynamic integrated climate-economy model of the economics of global warming. Technical report, Cowles Foundation for Research in Economics, Yale University.

Weitzman, M. L. (2012). Ghg targets as insurance against catastrophic climate damages. *Journal of Public Economic Theory*, 14(2):221–244.