Modélisation, estimation, simulation des risques climatiques

# An Introduction to a Climate Model

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# Schedule of the day

## Objectives

- Understanding the **basics** of climate models to predict world temperatures.
- Simulating climate models on historical data and make some forecasts conditional on emission scenarios.

## Materials:

- Climate_Models_Notebook.ipynb *notebook introduction of economic models.*
- Climate_Models.py *package to simulate reduced form climate models.*

# Introduction

## Intro: Why should economists care about climate science?

- Climate change is driven by physical processes that can be **summarized in a simple causal chain**.
- Integrated Assessment Models (IAMs) like DICE or FAIR rely on this chain.
- To interpret or extend IAMs, we need to understand the building blocks.

**The Climate-Economy Chain**

Emissions $\rightarrow$ Concentrations $\rightarrow$ Radiative Forcing $\rightarrow$ Temperature $\rightarrow$ Impacts

- Each arrow corresponds to a physical process.
- IAMs implement reduced-form versions of these processes.

## Intro: Climate science is about big models

How do climate scientists study the system?

- **General Circulation Models (GCMs)** and **Earth System Models (ESMs)**:
  - Simulate atmosphere, ocean, cryosphere, land surface, and biogeochemical cycles.
  - Built on physical laws (fluid dynamics, thermodynamics, radiative transfer).
  - Used in IPCC reports for future climate projections.
- Strengths:
  - High fidelity, spatial detail, many processes included.
  - Provide benchmarks for scientific understanding.
- Limitations for Integrated Assessment Modelling:
  - Extremely computationally intensive (supercomputers).
  - Results not easily integrated into optimization or policy models.

### Today's focus: Economics of climate change

- We will work with **Reduced-form Climate Models (RCMs)**:
  - Tractable and transparent $\rightarrow$ usable in economic models.
  - Capture the key physical mechanisms without full Earth System complexity.

### The Climate-Economy Chain

Emissions $\rightarrow$ Concentrations $\rightarrow$ Radiative Forcing $\rightarrow$ Temperature

**Roadmap:** We will follow this chain step by step, from *emissions inputs* to *temperature outcomes*, highlighting at each stage how reduced-form models approximate the underlying processes.

# Emissions (the inputs)

## Emissions: What are emissions?

- **Definition:** Anthropogenic greenhouse-gas (GHG) *flows* into the atmosphere (mainly $CO_2$, $CH_4$, $N_2O$).
- **Focus today:** $CO_2$ (fossil fuel + land use). Others can be added (e.g., $CH_4$) as separate inputs.
- Emissions are flows $\Rightarrow$ they feed atmospheric *stocks* in the carbon cycle.

## Emissions: Three main green house gases

The three main sources:

- **Carbon dioxide ($CO_2$)**
  - From fossil fuel combustion, cement, land-use change.
  - Long-lived: persists for centuries.
  - $\sim$**76%** of global GHG emissions ($CO_2$-eq).
- **Methane ($CH_4$)**
  - From agriculture (rice, livestock), fossil fuel extraction, waste.
  - Short lifetime ($\sim$10 years) but high warming power.
  - $\sim$**16%** of global GHG emissions.
- **Nitrous oxide ($N_2O$)**
  - From fertilizers, biomass burning, industry.
  - Lifetime $\sim$120 years.
  - $\sim$**6–7%** of global GHG emissions.

To summarize this section, you only need to remember:

- In principle, models can take three distinct exogenous inputs:

$$\{E_t^{CO_2}\}_{t\geq 0}, \quad \{E_t^{CH_4}\}_{t\geq 0}, \quad \{E_t^{N_2O}\}_{t\geq 0}.$$

- In simplified reduced-form models, only $CO_2$ emissions are modeled explicitly; $CH_4$ and $N_2O$ are often represented by an additional exogenous forcing term.

# Concentrations

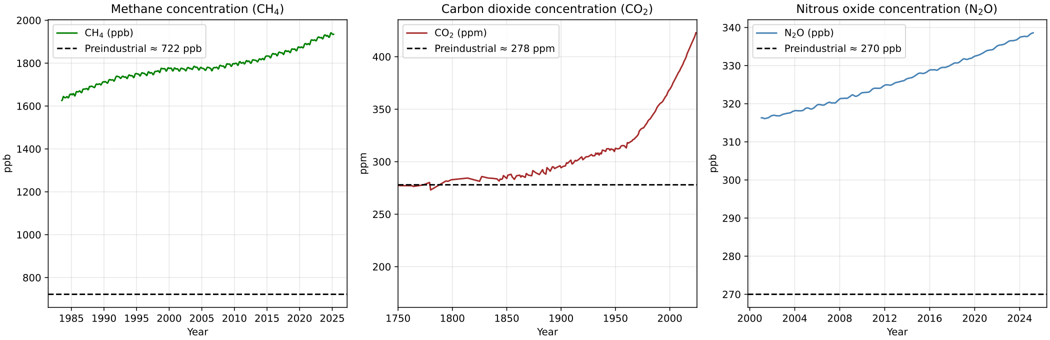
## Concentrations: from to emissions to concetration

$$[\text{Emissions (flows)}] \rightarrow [\text{Atmospheric Concentration (stock)}] \rightarrow \ldots$$

From emissions (flows) to concentrations (stocks):

- Climate does not react directly to annual emissions, but to the **cumulative stock of gases** in the atmosphere.
- Emissions are measured in flows (e.g. GtC/yr).
- Once emitted, gases accumulate in the atmosphere.
- What matters for climate forcing is the concentration (ppm or ppb).

# Cnocentrations: Atmospheric concentrations



# Concentrations: The mathematics of concentrations

## Matrix difference equation (step $\Delta$):

$$\boxed{\mathbf{M}_{g,t+\Delta} = \mathbf{M}_{g,t-1} + \Delta\,\mathbf{A}_g(\mathbf{M}_{g,t-1} - \mathbf{M}_{g,0}) + \Delta\,\mathbf{b}_g\,E_t^g}$$

- $\mathbf{M}_{g,t} \in \mathbb{R}^{n_g}$: vector of compartment stocks/concentrations at $t$.
- $\mathbf{M}_{g,0}$: baseline (natural equilibrium) vector (e.g. preindustrial levels).
- $\mathbf{A}_g \in \mathbb{R}^{n_g \times n_g}$: *transfer matrix* (diagonal entries $< 0$, off-diagonals $\geq 0$); row sums $\leq 0$ reflect net losses to sinks.
- $\mathbf{b}_g \in \mathbb{R}^{n_g}$: injection vector indicating which box receives anthropogenic emissions.
- $E_t^g$: emissions flow (units depend on gas).

# Cnocentrations: Some precisions

**Modeling choice.** The number of boxes $n_g$ determines precision:

- $n_g = 1$: a single decay process (e.g. $CH_4$, $N_2O$).
- $n_g = 2$–$3$: separates atmosphere, fast reservoirs (biosphere/upper ocean), and slow reservoirs (deep ocean) for $CO_2$.
- Larger $n_g$: more realistic sequestration processes (soil carbon, permafrost, multiple ocean layers) at the cost of extra parameters.

**Conservation:**

- the system is physical, so total mass is conserved: transfers occur *between* boxes, with no loss outside the system (except for explicitly modelled sinks).

# Forcing: In our case

### $\mathrm{CO}_2$ : three-box carbon cycle

$$\mathbf{M}^{CO_2}_{t+\Delta} = \mathbf{\Phi}^{CO_2}\,\mathbf{M}^{CO_2}_t + \Delta\,\mathbf{\Xi}^{CO_2}\,E^{CO_2}_t, \qquad \mathbf{M}^{CO_2}_t = \begin{bmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{bmatrix} \in \mathbb{R}^3$$

- $\mathbf{\Phi}^{CO_2} \in \mathbb{R}^{3\times 3}$: transfer matrix. Diagonal: persistence ($< 1$). Off-diagonal: transfers across boxes.
- $\mathbf{\Xi}^{CO_2} \in \mathbb{R}^{3\times 1}$: injection vector (typically only atmosphere receives emissions).
- Boxes: **AT** = atmosphere, **UP** = upper ocean/biosphere, **LO** = deep ocean.

# Forcing: In our case

**$CH_4$ : one-box decay to baseline**

$$M_{t+\Delta}^{CH_4} = M_t^{CH_4} - \Delta\, \delta_{CH_4} \big(M_t^{CH_4} - M_0^{CH_4}\big) + \Delta\, \kappa_{CH_4}\, E_t^{CH_4}$$

- $M_0^{CH_4}$: preindustrial baseline ($\approx$ 722 ppb).
- $\delta_{CH_4}$: decay rate ($\tau \sim 12$ y lifetime).
- $\kappa_{CH_4}$: conversion emissions $\rightarrow$ ppb/yr (1 ppb $\approx$ 2.78 $MtCH_4$).

# Radiative Forcing

## Forcing: Definition

[Emissions (flows)] → [Concentration (stock)] → [Forcing (flow)] → . . .

**Key idea.** Radiative forcing is the *side effect of rising concentrations*: it increases the thickness of the atmospheric "blanket" that traps heat.

- **Baseline:** in the preindustrial climate, incoming solar ≈ outgoing infrared (energy balance).
- **Forcing:** any perturbation that shifts this balance.
  - Positive forcing ⇒ warming (extra blanket from GHGs).
  - Negative forcing ⇒ cooling (reflective aerosols, volcanic dust).
- **Units.** Watts per square meter ($W/m^2$).

# Forcing: General Formula

**Key idea.** Total forcing is the sum of contributions from different agents, each measured relative to a natural (preindustrial) baseline.

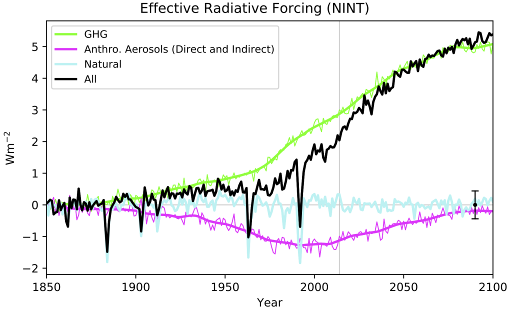
## General formulation

$$F_t = \sum_{g \in \{\mathrm{CO_2},\ \mathrm{CH_4},\ \mathrm{N_2O},\ \text{others}\}} F_t^g$$

$$F_t^g = f_g(M_t^g) - f_g(M_0^g)$$

- $M_t^g$ = atmospheric concentration of gas $g$ at time $t$.
- $M_0^g$ = preindustrial reference level (e.g. 278 ppm $CO_2$, 722 ppb $CH_4$).
- $f_g(\cdot)$ = functional form mapping concentration to forcing.
    - $CO_2$: logarithmic in concentration.
    - $CH_4$, $N_2O$: square-root formulations (with overlap correction).
    - Other agents: aerosols (negative forcing), solar variability, land-use.

# Forcing: Some illustration




- **Pink line:** Anthropogenic aerosols $\Rightarrow$ *negative forcing.*
  e.g. sulfate aerosols (from burning coal and oil- $SO_2$ emissions), very reflective: scatter sunlight back to space, short lifetime (days-weeks).
- **Cyan line:** Natural forcings (solar variability, volcanic eruptions).

Forcing formulas map concentrations into energy imbalance (W/m$^2$), measured relative to preindustrial levels.

### $\mathbf{CO_2}$ forcing

$$F_t^{CO_2} = F_{2\times CO_2} \cdot \frac{\ln\left(M_t^{CO_2}/M_0^{CO_2}\right)}{\ln 2}$$

- Preindustrial $M_0^{CO_2} = 278$ ppm
- Forcing at doubling: $F_{2\times CO_2} \approx 3.7$ W/m$^2$
- Today ($M_t^{CO_2} \approx 420$ ppm):

$$F_t^{CO_2} \approx 3.7 \cdot \frac{\ln(420/278)}{\ln 2} \approx 2.1\ \text{W/m}^2$$

## $CH_4$ forcing

$$F_t^{CH_4} = \alpha_{CH_4}\left(\sqrt{M_t^{CH_4}} - \sqrt{M_0^{CH_4}}\right)$$

- Preindustrial $M_0^{CH_4} = 722$ ppb
- Forcing coefficient $\alpha_{CH_4} \approx 0.036\ \mathrm{W/m^2}/\sqrt{\mathrm{ppb}}$
- Today ($M_t^{CH_4} \approx 1860$ ppb):

$$F_t^{CH_4} \approx 0.036 \cdot (43.1 - 26.9) \approx 0.6\ \mathrm{W/m^2}$$

# Temperatures (output)

## Temperatures: Dynamics

[Emissions (flows)] $\rightarrow$ [Concentration (stock)] $\rightarrow$ [Forcing (flow)] $\rightarrow$ [Temperature (stock)]

- **Key point:** Radiative forcing $\neq$ instantaneous warming.
  Temperatures behave as a *state variable*: they accumulate the effect of past forcings, with strong inertia.
- **Sources of inertia:**
  - The ocean absorbs most of the excess heat, releasing it slowly.
  - Ice sheets and cryosphere respond on multi-decadal to millennial scales.
  - Feedbacks (clouds, carbon cycle) operate with delays.
- **Illustration:** Even if $CO_2$ concentrations stabilized today, global temperatures would still rise for decades ("committed warming").

## Temperatures: Warming as an ODE

As with concentrations, temperatures evolve as a linear dynamic system: forcing is a flow, temperatures are stocks.

**General formulation**

$$\mathbf{T}_t \;=\; \mathbf{T}_t \;+\; \Delta\Big(\mathbf{A}_T\,\mathbf{T}_t \;+\; \mathbf{B}_T\,F_t\Big), \qquad \mathbf{T}_0 = \mathbf{0}$$

- $\mathbf{T}_t$ = vector of temperature anomalies (e.g. atmosphere, upper ocean, deep ocean).
- $F_t$ = total radiative forcing at time $t$ (flow).
- $\mathbf{A}_T$ = matrix of heat exchange and feedback parameters.
- $\mathbf{B}_T$ = vector linking forcing to temperature.
- $\mathbf{T}_0 = 0$: preindustrial baseline (no initial warming).

# Temperatures: a two box model

A simple reduced-form system tracks two temperature stocks: atmosphere/surface and deep ocean:

$$T^{AT}_{t+\Delta} = T^{AT}_t + \Delta\Big[c_1 F_t - c_1 \tfrac{F_{2\times CO_2}}{T_{2\times CO_2}} T^{AT}_t - c_1 c_3 \big(T^{AT}_t - T^{LO}_t\big)\Big]$$

$$T^{LO}_{t+\Delta} = T^{LO}_t + \Delta\Big[c_4 \big(T^{AT}_t - T^{LO}_t\big)\Big]$$

- $T^{AT}$: atmospheric/surface temperature anomaly (fast response).
- $T^{LO}$: deep-ocean temperature anomaly (slow response, high inertia).
- $F_t$: total radiative forcing at time $t$.
- Parameters: $c_1$ (climate response rate), $c_3$ (surface–ocean coupling), $c_4$ (deep-ocean heat uptake), $T_{2\times CO_2}$ (climate sensitivity).
- Initial condition: $T^{AT}_0 = T^{LO}_0 = 0$ (preindustrial baseline).

# Temperatures: asymptotic dynamics

Model in *relative terms* with respect to a doubling $CO_2$ and hold it fixed (no other forcings).

- **Forcing:**

$$M_{AT}^{CO_2} = 2M_{AT,0}^{CO_2} \quad \Rightarrow \quad F^{CO_2} = F_{2\times CO_2} \approx 3.6 \text{ W/m}^2$$

- **Dynamics (discrete-time 2-box model):**

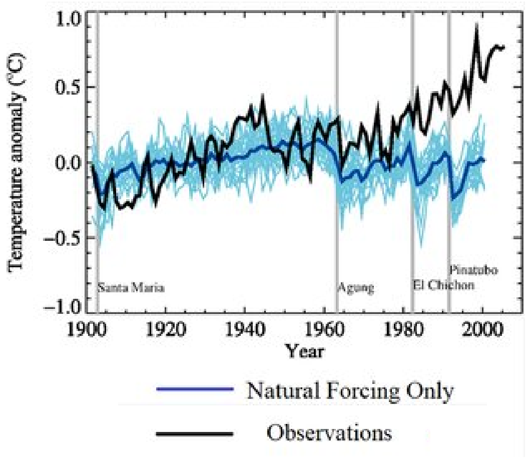
$$T_{t+\Delta}^{AT} = T_t^{AT} + \Delta\Big[c_1 F_{2\times CO_2} - c_1 \tfrac{F_{2\times CO_2}}{T_{2\times CO_2}} T_t^{AT} - c_1 c_3 (T_t^{AT} - T_t^{LO})\Big]$$
$$T_{t+\Delta}^{LO} = T_t^{LO} + \Delta c_4 (T_t^{AT} - T_t^{LO})$$

- **Equilibrium:**

$$T^{AT} = T^{LO} = T_{2\times CO_2} \approx 3.1°C$$

**Takeaway:** $F_{2\times CO_2}$ = forcing from doubling, $T_{2\times CO_2}$ = long-run warming response. They anchor reduced-form climate dynamics.

# Temperatures: Human vs. Natural Forcings



## To wrap up

To summarize the result, consider a carbon pulse of 1GtC:

$$+1GtC \rightarrow +0.47\ ppm\ CO_2 \rightarrow +0.01\ W/m^2 \text{ forcing} \rightarrow +0.006^{\circ}C \text{ in the long term}$$

Thank you!

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