

# Boston House Prices dataset

## Notes

Data Set Characteristics:

:Number of Instances: 506

:Number of Attributes: 13 numeric/categorical predictive

:Median Value (attribute 14) is usually the target

:Attribute Information (in order):

- CRIM per capita crime rate by town
- ZN proportion of residential land zoned for lots over 25,000 sq.ft.
- INDUS proportion of non-retail business acres per town
- CHAS Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)

e)

- NOX nitric oxides concentration (parts per 10 million)
- RM average number of rooms per dwelling
- AGE proportion of owner-occupied units built prior to 1940
- DIS weighted distances to five Boston employment centres
- RAD index of accessibility to radial highways
- TAX full-value property-tax rate per \$10,000
  - PTRATIO pupil-teacher ratio by town
  - B  $1000(B_k - 0.63)^2$  where  $B_k$  is the proportion of blacks by town
  - LSTAT % lower status of the population
  - MEDV Median value of owner-occupied homes in \$1000's

:Missing Attribute Values: None

:Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset. <http://archive.ics.uci.edu/ml/datasets/Housing>  
<http://archive.ics.uci.edu/ml/datasets/Housing>

This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic prices and the demand for clean air', J. Environ. Economics & Management, vol.5, 81-102, 1978. Used in Belsley, Kuh & Welsch, 'Regression diagnostics ...', Wiley, 1980. N.B. Various transformations are used in the table on pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression problems.

## References

- Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearity', Wiley, 1980. 244-261.
- Quinlan, R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings on the Tenth International Conference of Machine Learning, 236-243, University of Massachusetts, Amherst. Morgan

Kaufmann.

- many more! (see <http://archive.ics.uci.edu/ml/datasets/Housing> (<http://archive.ics.uci.edu/ml/datasets/Housing>))

In [1]:

```
import warnings #Code to remove warnings
warnings.filterwarnings('ignore')
```

## 1. Loading Libraries

In [2]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.preprocessing import StandardScaler
from sklearn.datasets import load_boston
from sklearn.cross_validation import train_test_split
from sklearn.linear_model import SGDRegressor
from sklearn.model_selection import GridSearchCV
from sklearn.metrics import make_scorer
from sklearn.metrics import mean_squared_error
import prettytable
```

/usr/local/lib/python3.5/site-packages/sklearn/cross\_validation.py:41: DeprecationWarning: This module was deprecated in version 0.18 in favor of the model\_selection module into which all the refactored classes and functions are moved. Also note that the interface of the new CV iterators are different from that of this module. This module will be removed in 0.20.

"This module will be removed in 0.20.", DeprecationWarning)

## 2. Dataset Preprocessing

In [3]:

```
boston = load_boston() #loading boston housing dataset
```

In [4]:

```
print('Shape of dataset is:', boston.data.shape) #boston.data gives us an array. Here we are
```

Shape of dataset is: (506, 13)

In [5]:

```
bos = pd.DataFrame(boston.data, columns = boston.feature_names) #Here we are converting our data to a DataFrame
#- and our column names are contained in 'boston.feature_names'
bos.head()
```

Out[5]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LS
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	15.3
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	17.8
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	17.8
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	18.7
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	18.7

In [6]:

```
bos['Price'] = boston.target #boston.target contains our y_i's and hence we are now putting the target values into the 'Price' column
```

In [7]:

```
x = bos.drop('Price', axis = 1) #Here we are splitting our dataset 'bos' into datapoint data (x) and target data (y)
y = bos['Price']
```

In [8]:

```
x_train, x_test, y_train, y_test = train_test_split(x, y) #splitting our dataset into train and test sets
```

In [9]:

```
standardized_train = StandardScaler().fit_transform(x_train) #standardizing our training data
standardized_test = StandardScaler().fit_transform(x_test) #standardizing our test data
```

In [10]:

```
#Creating training dataframe which contains both datapoints and y_i's
d_train = pd.DataFrame(standardized_train)
d_train['Price'] = np.array(y_train)

#Creating test dataframe which contains both datapoints and y_i's
d_test = pd.DataFrame(standardized_test)
d_test['Price'] = np.array(y_test)
```

In [11]:

```
print(d_train.shape)
print(d_test.shape)
```

(379, 14)

(127, 14)

In [12]:

```
d_train.head()
```

Out[12]:

	0	1	2	3	4	5	6	7	
0	0.694276	-0.480911	0.998510	-0.248243	0.245557	-1.100563	0.071415	-0.828240	1.6489
1	-0.390557	-0.480911	-0.186256	-0.248243	-0.073535	-0.321752	0.956263	-0.592255	-0.4187
2	-0.048540	-0.480911	1.215253	-0.248243	0.426662	-0.308451	0.870633	-0.721134	-0.5329
3	-0.414197	0.981168	-0.760333	-0.248243	-1.056680	0.414203	-1.006101	0.841536	-0.3036
4	-0.401406	-0.480911	-0.101316	-0.248243	-0.573731	-0.618793	-1.651898	0.088378	-0.6478

### 3. Custom implementation of SGDRegressor

In [13]:

```
def sgddreg_fit(x,alpha,batch_size=100, n_iter = 1000,eta0 = 0.01):

    #Here our Learning rate == 'invscaling' and we have used L2 regularization
    """This function is custom implementation of Stochastic Gradient Descent Regressor.

    Input:

        x: This is our dataset on which we want to calculate our weights so that we can f
            our x.

        alpha: This is our coefficient of L2 Regularizer.

        batch_size: This is the size of our batch of our datapoints which we randomly sam
            -iteration.

        n_iter: No. of iterations we want to perform.

        eta0: This is our learning rate.

    Output:

        This function return a dictionary containing weight and intercept corresponding

dict_1 = dict()#dictionary this function returns
t = 1 #invscaling parameter and is given by t = n_iter*batch_size
power_t = 0.25 #invscaling parameter which will be used as power of 't'
eta = eta0 #learning rate

w_old = np.zeros(shape = (1,13))#zero initialization of our weights.
b_old = 0 #zero initialization of our intercept term

for i in range(n_iter):
    gradient_w = np.zeros(shape = (1,13)) #zero initialization of gradient of loss ter
    gradient_b = 0 #zero initialization of gradient of loss term w.r.t b
    batch_data=x.sample(n = batch_size) #randomly sampling datapoints from dataset
    batch_x= np.array(batch_data.drop('Price',axis=1)) # separating our x_i's from batch
    batch_y = np.array(batch_data['Price']) # separating our y_i's from dataset from ba

    #Here we are calculating gradient of loss term w.r.t w and w.r.t b
    for j in range(batch_size):
        y = np.dot(w_old,batch_x[j])+b_old
        gradient_w += (batch_x[j]*(batch_y[j]-y)) + 2*alpha*w_old
        gradient_b += (batch_y[j]-y)

    gradient_w *= (-2/batch_size)
    gradient_b *= (-2/batch_size)

    #updating our weights
    w_new = w_old - eta*gradient_w
    b_new = b_old - eta*gradient_b

    #updating our old weights
    w_old = w_new
    b_old = b_new
    eta = eta/pow(t,power_t) #Here we are updating our Learning rate at each iteration
    t=(i*batch_size)+(alpha*eta0) #updating our invscaling parameter at each iteration

    #Here we are assigning values to various keys in the dictionary
    if i == (n_iter-1) :
```

```
dict_1['alpha'] = alpha
dict_1['weight'] = w_old
dict_1['intercept'] = b_old
break
return dict_1

def sgdreg_predict(w,b,x):
    """This function will predict the y_i values corresponding to our query points.

    Input: This function takes 'w' which is the best weight on which our dataset has been trained.
    This function also takes 'b' which is the best intercept term on which our dataset has been trained.
    'x' is our test dataset.

    Output: This function returns an array of predicted values.

    """
    y_pred = []
    for i in range(x.shape[0]):
        y_pred.append(np.dot(w,x[i])+b)
    return np.array(y_pred)

def error(y_true,y_pred):
    """This function calculates mean squared error"""
    error = mean_squared_error(y_true,y_pred)
    return error

def plot(y_true,y_pred):
    """This function plots y_pred as a function of y_true"""
    plt.figure()
    plt.scatter(y = y_pred,x = y_true,label = 'Actual values Vs Pred values')
    plt.xlabel('Actual values')
    plt.ylabel('Predicted values')
    plt.title('Actual values Vs Pred values')
    plt.legend()
    plt.grid()
    plt.show()
```

In [14]:

```
#Here we are performing cross validation w.r.t our training dataset using custom built SGD
#- alpha and eta0.
error_i = []
err_1 = dict()
alpha = [10**-6,10**-5,10**-4,10**-3,10**-2,10**-1,1,10**1,10**2,10**3,10**4,10**5,10**6]
eta0 = [10**-6,10**-5,10**-4,10**-3,10**-2,10**-1,1,10**1,10**2,10**3,10**4,10**5,10**6]
for i in alpha:
    for j in eta0:
        sgd = sgdreg_fit(d_train,alpha = i,eta0 = j)#Fitting our training dataset
        y_pred = sgdreg_predict(sgd['weight'],sgd['intercept'],standardized_train) #predict
        # the weights that our model has learned.
        err = error(y_train,y_pred) #calculating our mean squared error
        error_i.append(err) #Appending our errors
        err_1[err] = [i,j,err] #storing the mean squared error corresponding to a particular alpha and eta0
        print('alpha = {0}, eta = {1}, mse = {2}'.format(i,j,err))
```

```
alpha = 1e-06, eta = 1e-06, mse = 585.5912974903646
alpha = 1e-06, eta = 1e-05, mse = 568.4120011329558
alpha = 1e-06, eta = 0.0001, mse = 477.42740964084373
alpha = 1e-06, eta = 0.001, mse = 185.4550531700009
alpha = 1e-06, eta = 0.01, mse = 537.4241268396811
alpha = 1e-06, eta = 0.1, mse = 9102493.203844715
alpha = 1e-06, eta = 1, mse = 8.009229853387747e+17
alpha = 1e-06, eta = 10, mse = 1.4330462550748914e+31
alpha = 1e-06, eta = 100, mse = 2.418153055558553e+44
alpha = 1e-06, eta = 1000, mse = 2.202044048965857e+60
alpha = 1e-06, eta = 10000, mse = 7.517358960021271e+76
alpha = 1e-06, eta = 100000, mse = 5.42896297609114e+95
alpha = 1e-06, eta = 1000000, mse = 6.164461811029461e+115
alpha = 1e-05, eta = 1e-06, mse = 588.0098637370008
alpha = 1e-05, eta = 1e-05, mse = 577.1803377850713
alpha = 1e-05, eta = 0.0001, mse = 522.5942352294663
alpha = 1e-05, eta = 0.001, mse = 304.980486739748
alpha = 1e-05, eta = 0.01, mse = 115.92005296807415
alpha = 1e-05, eta = 0.1, mse = 112224.76569128312
```

In [19]:

```
a = min(error_i)#Finding the alpha and eta0 values corresponding to minimum mean squared error
err_1[a]
```

Out[19]:

```
[0.1, 0.1, 37.84139064712439]
```

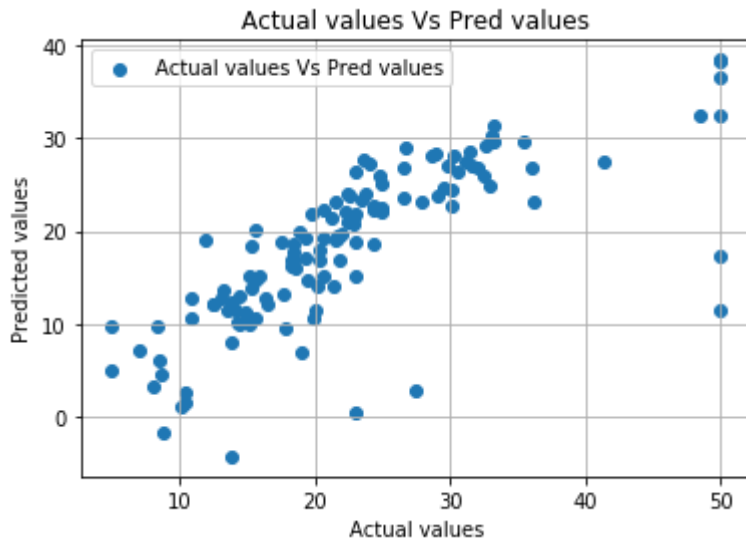
In [20]:

```
#Here we are trying to predict the y_i's corresponding to x_i's of test dataset using our built SGD
sgd = sgdreg_fit(d_train,alpha = 0.1,eta0 = 0.1)
y_pred = sgdreg_predict(sgd['weight'],sgd['intercept'],standardized_test)
```

In [22]:

```
print(error(y_test,y_pred))
plot(y_test,y_pred)
```

58.59034191193969

**Observation:**

From above plot we can see that the actual values and predicted values are linearly aligned with each other which indicates that our custom model is predicting our actual values with very less error and is thus somewhat perfectly performing Stochastic Gradient Descent Regression.

In [23]:

```
w_1 = sgd['weight'] #weights corresponding to our best fit model
w_1
```

Out[23]:

```
array([[ 0.38355789, -0.65461713, -0.54872229,  1.18067096, -0.61003668,
         3.54662039, -0.34283599, -0.79518735, -1.05936229, -0.67868092,
        -1.81454486,  0.96494767, -2.69118232]])
```

**4. Sklearn SGDRegressor**

In [24]:

```
mse = make_scorer(mean_squared_error) #As mean_squared_error is not a scoring parameter in
#- using make_score to make mean_squared_error as one of the scoring parameters.
```



In [25]:

```
#Here we are performing gridsearch cross validation on our dataset for our sgd regressor
sgd = SGDRegressor(max_iter = 1000)
param_grid = {'alpha':[10**-6,10**-5,10**-4,10**-3,10**-2,10**-1,1,10**1,10**2,10**3,10**4,
                      'eta0':[10**-6,10**-5,10**-4,10**-3,10**-2,10**-1,1,10**1,10**2,10**3,10**4,1
                      ]
gscv = GridSearchCV(sgd,param_grid,cv = 10,scoring = mse,verbose = 1)
gscv.fit(standardized_train,y_train)
print('*'*20)
```

Fitting 10 folds for each of 169 candidates, totalling 1690 fits

\*\*\*\*\*

[Parallel(n\_jobs=1)]: Done 1690 out of 1690 | elapsed: 1.5min finished

In [26]:

```
df = pd.DataFrame(gscv.cv_results_)#Converting our grid search cross validation results into
print(df['mean_test_score'].min()) #Here we are trying to get the minimum mean squared error
best_df = df[df['mean_test_score']==df['mean_test_score'].min()] #Here we are trying to get
#- to the min mean_squared_error.
best_df
```

20.687995052417175

Out[26]:

	mean_fit_time	mean_score_time	mean_test_score	mean_train_score	param_alpha	param_eta
56	0.049928	0.000303	20.687995	18.543505	0.01	0.01

1 rows × 7 columns

In [27]:

```
index = best_df['param_alpha'].index #Getting index corresponding to best values of hyperparameters
index[0]
```

Out[27]:

56

In [28]:

```
print('Best hyperparameters are: alpha = {0} and eta0 = {1}'.format(best_df['param_alpha'].iloc[0],
best_df['param_eta0'].iloc[0]))
```

Best hyperparameters are: alpha = 0.01 and eta0 = 0.01

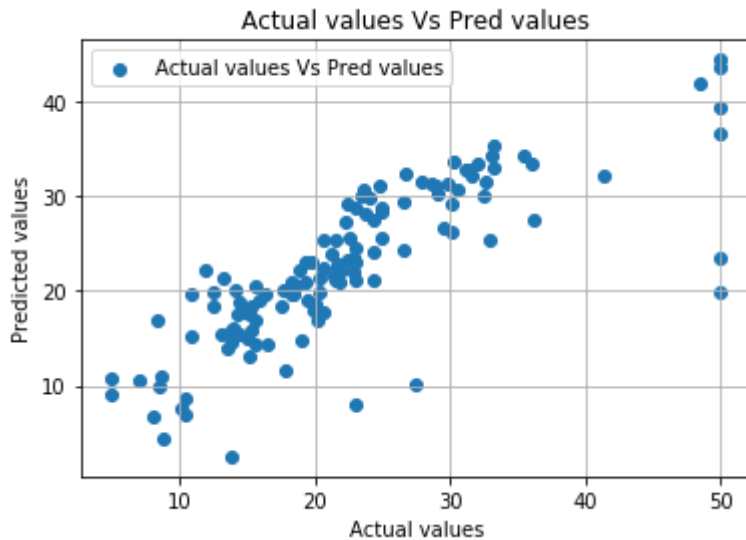
In [29]:

```
#Here we are testing our model that we learnt above on our test dataset
sgd = SGDRegressor(alpha = 0.01,eta0 = 0.01,max_iter = 1000)
sgd.fit(standardized_train,y_train)
y_pred = sgd.predict(standardized_test)
```

In [30]:

```
print(error(y_test,y_pred))
plot(y_test,y_pred)
```

33.651453623469465

**Observation:**

The above graph that we have got using sklearn's Stochastic Gradient Descent Regressor model is very much similar to our plot that we got using custom implementation.

In [31]:

```
w_2 = sgd.coef_ #weights corresponding to our best fit model
w_2
```

Out[31]:

```
array([-0.75595421,  0.78835386,  0.08226338,  0.62230719, -1.85857673,
        3.6593889 , -0.44289959, -2.70376005,  1.70737974, -1.63573206,
       -1.92452995,  0.82438522, -2.63974891])
```

In [32]:

```
np.linalg.norm(w_1-w_2) #Calculating euclidean distance between the weight vectors we got u
#- own custom implementation.
```

Out[32]:

4.233214598939649

From above we can see that the weight vectors that we got using Custom implementation and sklearn's implementation are very close to each other in 13 dimensional space as the distance between them is only 4.23 units.

## 5. Conclusion

In [33]:

```
x = prettytable.PrettyTable()
list_1 = [['Custom SGDRegressor', 58.59, [ 0.38355789, -0.65461713, -0.54872229, 1.18067096,
3.54662039, -0.34283599, -0.79518735, -1.05936229, -0.67868092,
-1.81454486, 0.96494767, -2.69118232], 4.23, 0.1, 0.1], ['Sklearn SGDRegressor', 33.65,
3.6593889, -0.44289959, -2.70376005, 1.70737974, -1.63573206,
-1.92452995, 0.82438522, -2.63974891], 4.23, 0.01, 0.01]]
names = ['Regressor', 'MSE', 'Weight', 'Distance', 'alpha', 'learning_rate']
pd.DataFrame(list_1, columns = names)
```

Out[33]:

	Regressor	MSE	Weight	Distance	alpha	learning_rate
0	Custom SGDRegressor	58.59	[0.38355789, -0.65461713, -0.54872229, 1.18067...	4.23	0.10	0.10
1	Sklearn SGDRegressor	33.65	[-0.75595421, 0.78835386, 0.08226338, 0.622307...	4.23	0.01	0.01

From above table we can see that our weight vectors that we got using custom sgd regressor implementation and sklearn's sgd regressor implementation are very close to each other as the distance between them is only 4.23 units which means that both the weights vectors are very similar to each other. This fact is also reflected in the mean squared error values which we got for both of these weight vectors. With Custom SGD Regressor we got best fit model for  $\alpha = 0.1$  and  $\eta$  ( or learning\_rate) = 0.1. Where as for Sklearn's sgd we got best fit model for  $\alpha = 0.01$  and  $\eta = 0.01$ .

In [ ]: