Linear Algebra

Introduction

Kang Hoon Lee

Department of Computer Science, Kwangwoon University

Linear Algebra

- Linear algebra is the branch of mathematics concerning linear equations such as

$$a_1 x_1 + \cdots + a_n x_n = b,$$

linear functions such as

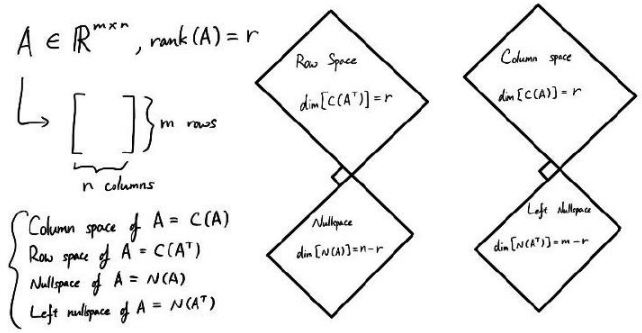
$$(x_1, \cdots, x_n) \to a_1 x_1 + \cdots + a_n x_n,$$

and their representations through matrices and vector spaces.

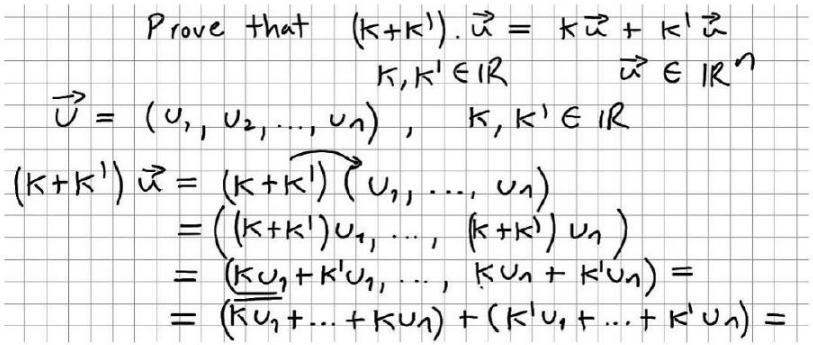
From Wikipedia

How to Learn Linear Algebra

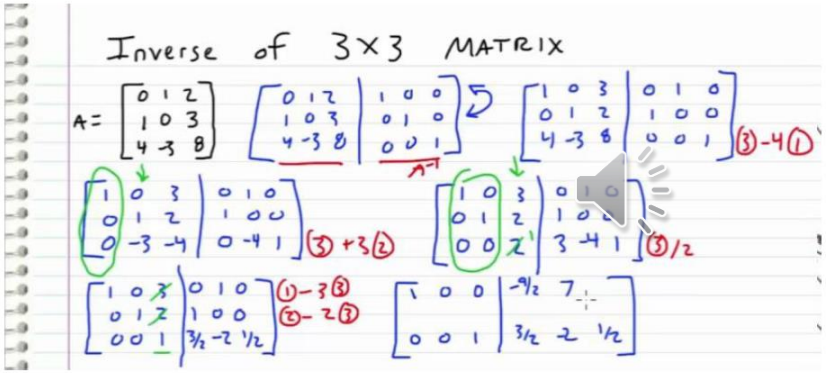
- Understanding concepts



- Proving theorems



- Manipulating expressions



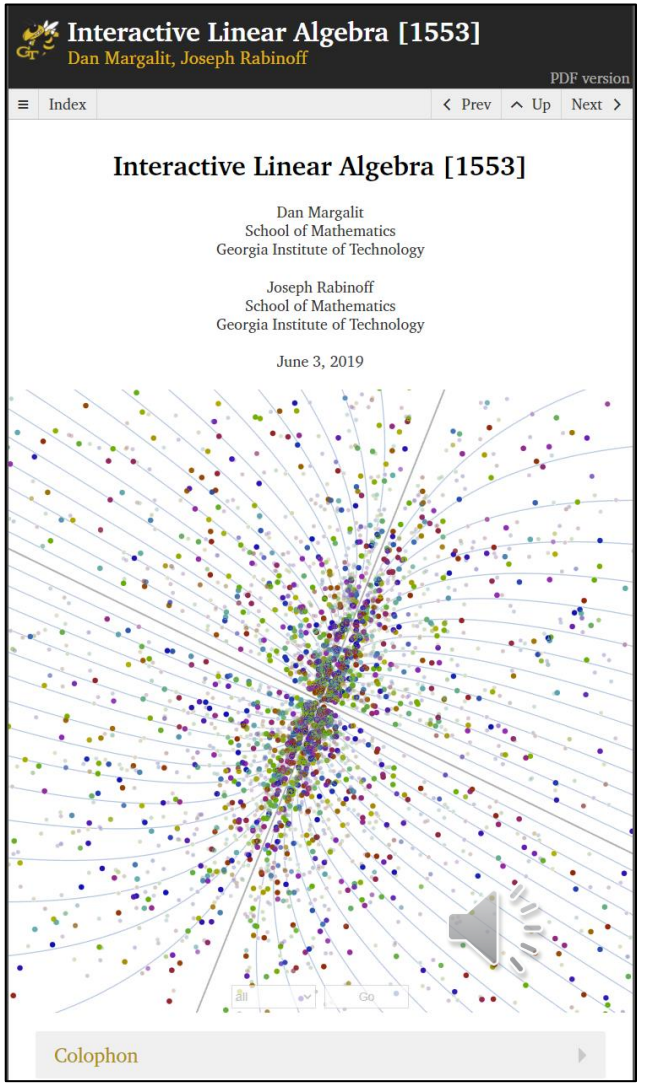
Linear Algebra for CS Students

- Understanding concepts
- Proving theorems
- Manipulating expressions

- Visualizing the algebraic concepts, theorems, and expressions into their geometric counterparts
  - To intuitively understand the mechanisms behind the abstract mathematical notations
- Representing the abstract objects and systems as the computational data structures and algorithms
  - To apply the power of linear algebra to various computational problems (e.g. computer graphics, machine learning)

Textbook: Interactive Linear Algebra

- Free e-book
  - Focusing on the synthesis of algebra and geometry
  - Developed for the introductory linear algebra course at Georgia Tech
  - Download PDF and/or access directly on the web
    - https://textbooks.math.gatech.edu/ila/1553/



Topics to Be Covered

- Systems of Linear Equations: Algebra
  - Systems of linear equations
  - Row reduction
  - Parametric form
- Systems of Linear Equations: Geometry
  - Vectors
  - Vector equations and spans
  - Matrix equations
  - Solution sets
  - Linear independence
  - Subspaces
  - Basis and dimension
  - The rank theorem

Topics to Be Covered

- Linear Transformations and Matrix Algebra
  - Matrix transformations
  - One-to-one and onto transformations
  - Matrix multiplication
  - Matrix inverses
  - The invertible matrix theorem
- Determinants
  - Determinants: definition
  - Cofactor expansions
  - Determinants and volumes

Topics to Be Covered

- Eigenvalues and Eigenvectors
  - Eigenvalues and eigenvectors
  - The characteristic polynomial
  - Diagonalization
  - Complex eigenvalues
  - Stochastic matrices
- Orthogonality
  - Dot products and orthogonality
  - Orthogonal complements
  - Orthogonal projection
  - The method of least squares

Schedule

Week	Mon	Wed	Homework
1	Introduction	Systems of Linear Equations (1)	
2	Systems of Linear Equations (2)	Row Reduction (1)	Worksheet 2.1
3	Row Reduction (2)	Parametric Form, Vectors	Worksheet 2.2-2.3
4	Vector Equations	Matrix Equations	Worksheet 3.1-3.2
5	Solutions Sets	Linear Independence	Worksheet 3.3-3.5
6	Subspaces	Basis, Dimension, and Rank	Worksheet 3.6-4.1
7	Matrix Transformations	One-to-one and Onto	
8	***Midterm Exam***		
9	Linear Transformations	Matrix Multiplication	Worksheet 4.2-4.3
10	Matrix Inverses	Determinants	Worksheet 4.4-4.5
11	Cofactor Expansions	Eigenvalues and Eigenvectors	Worksheet 5.1-6.1
12	Characteristic Polynomial	Diagonalization	Worksheet 6.2-6.4
13	Dot Product and Orthogonality	Orthogonal Complements	Worksheet 7
14	Orthogonal Projections (1)	Orthogonal Projections (2)	
15	***Final Exam***		