Chapter 2

Systems of Linear Equations: Algebra

Section 2.2

Row Reduction

Row Echelon Form

Let's come up with an *algorithm* for turning an arbitrary matrix into a "solved" matrix. What do we mean by "solved"?

A matrix is in **row echelon form** if

1. All zero rows are at the bottom.
2. Each leading nonzero entry of a row is to the *right* of the leading entry of the row above.
3. Below a leading entry of a row, all entries are *zero*.

Picture:

$$\begin{pmatrix} \boxed{\star} & \star & \star & \star & \star \\ 0 & \boxed{\star} & \star & \star & \star \\ 0 & 0 & 0 & \boxed{\star} & \star \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{aligned} \star &= \text{any number} \\ \boxed{\star} &= \text{any nonzero number} \end{aligned}$$

Definition

A **pivot** $\boxed{\star}$ is the first nonzero entry of a row of a matrix. A **pivot column** is a column containing a pivot of a matrix *in row echelon form*.

Reduced Row Echelon Form

A matrix is in **reduced row echelon form** if it is in row echelon form, and in addition,

4. The pivot in each nonzero row is equal to 1.
5. Each pivot is the only nonzero entry in its column.

Picture:

$$\begin{pmatrix} \color{red}{1} & 0 & \star & 0 & \star \\ 0 & \color{red}{1} & \star & 0 & \star \\ 0 & 0 & 0 & \color{red}{1} & \star \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{array}{l} \star = \text{any number} \\ \color{red}{1} = \text{pivot} \end{array}$$

Note: Echelon forms do not care whether or not a column is augmented. Just ignore the vertical line.

Question

Can every matrix be put into reduced row echelon form only using row operations?

Answer: Yes! Stay tuned.

Reduced Row Echelon Form
Continued

Why is this the "solved" version of the matrix?

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

is in reduced row echelon form. It translates into

$$\begin{aligned} x &= 1 \\ y &= -2 \\ z &= 3, \end{aligned}$$

which is clearly the solution.

But what happens if there are fewer pivots than rows?

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

. . . parametrized solution set (later).

Poll

Poll

Which of the following matrices are in reduced row echelon form?

A. $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ B. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ D. $\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ E. $\begin{pmatrix} 0 & 1 & 8 & 0 \end{pmatrix}$

F. $\left(\begin{array}{cc|c} 1 & 17 & 0 \\ 0 & 0 & 1 \end{array}\right)$

Poll

Poll

Which of the following matrices are in reduced row echelon form?

A. $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ B. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

C. $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ D. $\begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}$ E. $\begin{pmatrix} 0 & 1 & 8 & 0 \end{pmatrix}$

F. $\left(\begin{array}{cc|c} 1 & 17 & 0 \\ 0 & 0 & 1 \end{array}\right)$

Answer: B, D, E, F.

Note that A is in row echelon form though.

Summary

- Solving a system of equations means producing all values for the unknowns that make all the equations true simultaneously.
- It is easier to solve a system of linear equations if you put all the coefficients in an **augmented matrix**.
- Solving a system using the elimination method means doing **elementary row operations** on an augmented matrix.
- Two systems or matrices are **row-equivalent** if one can be obtained from the other by doing a sequence of elementary row operations. Row-equivalent systems have the *same solution set*.
- A linear system with no solutions is called **inconsistent**.
- The (reduced) row echelon form of a matrix is its “solved” row-equivalent version.

(Reduced) Row Echelon Form

Review from last time

A matrix is in **row echelon form** if

1. All zero rows are at the bottom.
2. Each leading nonzero entry of a row is to the *right* of the leading entry of the row above.
3. Below a leading entry of a row, all entries are *zero*.

A matrix is in **reduced row echelon form** if it is in row echelon form, and in addition,

4. The pivot in each nonzero row is equal to 1.
5. Each pivot is the only nonzero entry in its column.

Row echelon form:

$$\begin{pmatrix} \boxed{\star} & \star & \star & \star & \star \\ 0 & \boxed{\star} & \star & \star & \star \\ 0 & 0 & 0 & \boxed{\star} & \star \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Reduced row echelon form:

$$\begin{pmatrix} 1 & 0 & \star & 0 & \star \\ 0 & 1 & \star & 0 & \star \\ 0 & 0 & 0 & 1 & \star \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\boxed{\star}$ = **pivots**

Row Reduction: Theorem

Theorem

Every matrix is row equivalent to one and only one matrix in reduced row echelon form.

We'll give an algorithm, called **row reduction** or **Gaussian elimination**, which demonstrates that every matrix is row equivalent to *at least one* matrix in reduced row echelon form.

Note: Like echelon forms, the row reduction algorithm does not care if a column is augmented: ignore the vertical line when row reducing.

The uniqueness statement is interesting—it means that, nomatter *how* you row reduce, you *always* get the same matrix in reduced row echelon form. (Assuming you only do the three legal row operations.) (And you don't make any arithmetic errors.)

Maybe you can figure out why it's true!

Row Reduction Algorithm

Step 1a Swap the 1st row with a lower one so a leftmost nonzero entry is in 1st row (if necessary).

Step 1b Scale 1st row so that its leading entry is equal to 1.

Step 1c Use row replacement so all entries below this 1 are 0.

Step 2a Swap the 2nd row with a lower one so that the leftmost nonzero entry is in 2nd row.

Step 2b Scale 2nd row so that its leading entry is equal to 1.

Step 2c Use row replacement so all entries below this 1 are 0.

Step 3a Swap the 3rd row with a lower one so that the leftmost nonzero entry is in 3rd row.

etc.

Last Step Use row replacement to clear all entries above the pivots, starting with the last pivot (to make life easier).

Example

$$\left(\begin{array}{ccc|c} 0 & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

[animated]

http://textbooks.math.gatech.edu/ila/demos/rowred2.html

Row Reduction
Example

$$\left(\begin{array}{ccc|c} \boxed{0} & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

Step 1a: Row swap to make this nonzero.

Row Reduction

Example

$$\left(\begin{array}{ccc|c} \boxed{0} & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right) \xrightarrow{R_1 \longleftrightarrow R_2} \left(\begin{array}{ccc|c} \boxed{2} & 4 & 6 & 12 \\ 0 & -7 & -4 & 2 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

Step 1a: Row swap to make this nonzero.

Step 1b: Scale to make this 1.

$$\left(\begin{array}{ccc|c} \boxed{0} & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right) \xrightarrow{R_1 \longleftrightarrow R_2} \left(\begin{array}{ccc|c} \boxed{\color{red}{2}} & 4 & 6 & 12 \\ 0 & -7 & -4 & 2 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

Step 1a: Row swap to make this nonzero.

Step 1b: Scale to make this 1.

$$\xrightarrow{R_1 = R_1 \div 2} \left(\begin{array}{ccc|c} \color{red}{1} & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ \boxed{3} & 1 & -1 & -2 \end{array}\right)$$

Step 1c: Subtract a multiple of the first row to clear this.

Row Reduction

Example

$$\left(\begin{array}{ccc|c} \boxed{0} & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right) \xrightarrow{R_1 \longleftrightarrow R_2} \left(\begin{array}{ccc|c} \boxed{2} & 4 & 6 & 12 \\ 0 & -7 & -4 & 2 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

Step 1a: Row swap to make this nonzero.

Step 1b: Scale to make this 1.

$$\xrightarrow{R_1 = R_1 \div 2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ \boxed{3} & 1 & -1 & -2 \end{array}\right)$$

Step 1c: Subtract a multiple of the first row to clear this.

$$\xrightarrow{R_3 = R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ 0 & -5 & -10 & -20 \end{array}\right)$$

Row Reduction

Example

$$\left(\begin{array}{ccc|c} \boxed{0} & -7 & -4 & 2 \\ 2 & 4 & 6 & 12 \\ 3 & 1 & -1 & -2 \end{array}\right) \xrightarrow{R_1 \longleftrightarrow R_2} \left(\begin{array}{ccc|c} \boxed{2} & 4 & 6 & 12 \\ 0 & -7 & -4 & 2 \\ 3 & 1 & -1 & -2 \end{array}\right)$$

Step 1a: Row swap to make this nonzero.

Step 1b: Scale to make this 1.

$$\xrightarrow{R_1 = R_1 \div 2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ \boxed{3} & 1 & -1 & -2 \end{array}\right)$$

Step 1c: Subtract a multiple of the first row to clear this.

$$\xrightarrow{R_3 = R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -7 & -4 & 2 \\ 0 & -5 & -10 & -20 \end{array}\right)$$

Optional: swap rows 2 and 3 to make Step 2b easier later on.

$$\xrightarrow{R_2 \longleftrightarrow R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -5 & -10 & -20 \\ 0 & -7 & -4 & 2 \end{array}\right)$$

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & \boxed{-5} & -10 & -20 \\ 0 & -7 & -4 & 2 \end{array}\right)$$

Step 2a: This is already nonzero.
Step 2b: Scale to make this 1.

(There are no fractions because of the optional step before.)

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & \boxed{-5} & -10 & -20 \\ 0 & -7 & -4 & 2 \end{array}\right) \xrightarrow{R_2 = R_2 \div -5} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & \boxed{-7} & -4 & 2 \end{array}\right)$$

Step 2a: This is already nonzero.
Step 2b: Scale to make this 1.

(There are no fractions because of the optional step before.)

Step 2c: Add 7 times the second row to clear this.

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & \boxed{-5} & -10 & -20 \\ 0 & -7 & -4 & 2 \end{array}\right) \xrightarrow{R_2 = R_2 \div -5} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & \boxed{-7} & -4 & 2 \end{array}\right)$$

Step 2a: This is already nonzero.
Step 2b: Scale to make this 1.

(There are no fractions because of the optional step before.)

Step 2c: Add 7 times the second row to clear this.

$$\xrightarrow{R_3 = R_3 + 7R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 10 & 30 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & \boxed{-5} & -10 & -20 \\ 0 & -7 & -4 & 2 \end{array}\right) \xrightarrow{R_2 = R_2 \div -5} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & \boxed{-7} & -4 & 2 \end{array}\right)$$

Step 2a: This is already nonzero.
Step 2b: Scale to make this 1.

(There are no fractions because of the optional step before.)

Step 2c: Add 7 times the second row to clear this.

$$\xrightarrow{R_3 = R_3 + 7R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 10 & 30 \end{array}\right)$$

Note: Step 2 never messes up the first (nonzero) column of the matrix, because it looks like this:

$$\text{“Active” row} \rightarrow \left(\begin{array}{ccc|c} 1 & \star & \star & \star \\ \boxed{0} & \boxed{\star} & \boxed{\star} & \boxed{\star} \\ 0 & \star & \star & \star \end{array}\right)$$

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \boxed{10} & 30 \end{array}\right)$$

Step 3a: This is already nonzero.
Step 3b: Scale to make this 1.

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \boxed{10} & 30 \end{array}\right) \xrightarrow{R_3 = R_3 \div 10} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Step 3a: This is already nonzero.
Step 3b: Scale to make this 1.

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \boxed{10} & 30 \end{array}\right) \xrightarrow{R_3 = R_3 \div 10} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Step 3a: This is already nonzero.
Step 3b: Scale to make this 1.

Note: Step 3 never messes up the columns to the left.
Note: The matrix is now in row echelon form!

Row Reduction

Example, continued

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \boxed{10} & 30 \end{array}\right) \xrightarrow{R_3 = R_3 \div 10} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Step 3a: This is already nonzero.
Step 3b: Scale to make this 1.

Note: Step 3 never messes up the columns to the left.
Note: The matrix is now in row echelon form!

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right) \xrightarrow{R_2 = R_2 - 2R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Last step: Add multiples of
the third row to clear these.

$$\xrightarrow{R_1 = R_1 - 3R_3} \left(\begin{array}{ccc|c} 1 & \boxed{2} & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Last step: Add -2 times
the third row to clear this.

$$\xrightarrow{R_1 = R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Row Reduction

Example, continued

Success! The reduced row echelon form is

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \qquad \Longrightarrow \qquad \left\{\begin{array}{rcrcrcr} x & & & & & = & 1 \\ & & y & & & = & -2 \\ & & & & z & = & 3 \end{array}\right.$$

Recap

Get a 1 here

$$\begin{pmatrix} \boxed{\star} & \star & \star & \star \\ \star & \star & \star & \star \\ \star & \star & \star & \star \\ \star & \star & \star & \star \end{pmatrix}$$

Clear down

$$\begin{pmatrix} \boxed{1} & \star & \star & \star \\ \star & \star & \star & \star \\ \star & \star & \star & \star \\ \star & \star & \star & \star \end{pmatrix}$$

Get a 1 here

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & \boxed{\star} & \star & \star \\ 0 & \star & \star & \star \\ 0 & \star & \star & \star \end{pmatrix}$$

Clear down

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & \boxed{1} & \star & \star \\ 0 & \star & \star & \star \\ 0 & \star & \star & \star \end{pmatrix}$$

(maybe these are already zero)

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & 1 & \star & \star \\ 0 & 0 & \boxed{0} & \star \\ 0 & 0 & \boxed{0} & \star \end{pmatrix}$$

Get a 1 here

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & 1 & \star & \star \\ 0 & 0 & 0 & \boxed{\star} \\ 0 & 0 & 0 & \star \end{pmatrix}$$

Clear down

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & 1 & \star & \star \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & \star \end{pmatrix}$$

Matrix is in REF

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & 1 & \star & \star \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Clear up

$$\begin{pmatrix} 1 & \star & \star & \star \\ 0 & 1 & \star & \star \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Clear up

$$\begin{pmatrix} 1 & \star & \star & 0 \\ 0 & \boxed{1} & \star & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix is in RREF

$$\begin{pmatrix} 1 & 0 & \star & 0 \\ 0 & 1 & \star & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Profit?

Row Reduction
Another example

The linear system

$$\begin{aligned} 2x + 10y &= -1 \\ 3x + 15y &= 2 \end{aligned} \quad \text{gives rise to the matrix} \quad \left(\begin{array}{cc|c} 2 & 10 & -1 \\ 3 & 15 & 2 \end{array}\right).$$

Let's row reduce it: [interactive row reducer]

http://textbooks.math.gatech.edu/ila/demos/rrinter.html?mat=2,10,-1:3,15,2

Row Reduction

Another example

The linear system

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Let's row reduce it: [interactive row reducer]

$$\left(\begin{array}{cc|c} 2 & 10 & -1 \\ 3 & 15 & 2 \end{array}\right) \xrightarrow{R_1 = R_1 \div 2} \left(\begin{array}{cc|c} \color{red}{1} & 5 & -\frac{1}{2} \\ 3 & 15 & 2 \end{array}\right) \quad \text{(Step 1b)}$$

$$\xrightarrow{R_2 = R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & 5 & -\frac{1}{2} \\ \color{red}{0} & 0 & \frac{7}{2} \end{array}\right) \quad \text{(Step 1c)}$$

$$\xrightarrow{R_2 = R_2 \times \frac{2}{7}} \left(\begin{array}{cc|c} 1 & 5 & -\frac{1}{2} \\ 0 & 0 & \color{red}{1} \end{array}\right) \quad \text{(Step 2b)}$$

$$\xrightarrow{R_1 = R_1 + \frac{1}{2}R_2} \left(\begin{array}{cc|c} 1 & 5 & \color{red}{0} \\ 0 & 0 & 1 \end{array}\right) \quad \text{(Step 2c)}$$

http://textbooks.math.gatech.edu/ila/demos/rrinter.html?mat=2,10,-1:3,15,2

Row Reduction

Another example

The linear system

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Let's row reduce it: [interactive row reducer]

$$\left(\begin{array}{cc|c} 2 & 10 & -1 \\ 3 & 15 & 2 \end{array}\right) \xrightarrow{R_1 = R_1 \div 2} \left(\begin{array}{cc|c} 1 & 5 & -\frac{1}{2} \\ 3 & 15 & 2 \end{array}\right) \quad \text{(Step 1b)}$$

$$\xrightarrow{R_2 = R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & 5 & -\frac{1}{2} \\ 0 & 0 & \frac{7}{2} \end{array}\right) \quad \text{(Step 1c)}$$

$$\xrightarrow{R_2 = R_2 \times \frac{2}{7}} \left(\begin{array}{cc|c} 1 & 5 & -\frac{1}{2} \\ 0 & 0 & 1 \end{array}\right) \quad \text{(Step 2b)}$$

$$\xrightarrow{R_1 = R_1 + \frac{1}{2}R_2} \left(\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 0 & 1 \end{array}\right) \quad \text{(Step 2c)}$$

The row reduced matrix

$$\left(\begin{array}{cc|c} 1 & 5 & 0 \\ 0 & 0 & 1 \end{array}\right) \qquad \text{corresponds to the } \textit{inconsistent} \text{ system} \qquad \begin{aligned} x + 5y &= 0 \\ 0 &= 1. \end{aligned}$$

http://textbooks.math.gatech.edu/ila/demos/rrinter.html?mat=2,10,-1:3,15,2

Inconsistent Matrices

Question

What does an augmented matrix in reduced row echelon form look like, if its system of linear equations is inconsistent?

Inconsistent Matrices

Question

What does an augmented matrix in reduced row echelon form look like, if its system of linear equations is inconsistent?

Answer:

$$\left(\begin{array}{cccc|c} 1 & 0 & \star & \star & 0 \\ 0 & 1 & \star & \star & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

An augmented matrix corresponds to an inconsistent system of equations if and only if *the last* (i.e., the augmented) *column is a pivot column.*