**Math 1553 Worksheet: Fundamentals and §2.1**

**1.** For each equation, determine whether the equation is linear or non-linear. Circle your answer. If the equation is non-linear, briefly justify why it is non-linear.

**a)** $3x_1 + \sqrt{x_2} = 4$ Linear $\boxed{\text{Not linear}}$

$x_2$ is an irrational expression.

**b)** $x^2 + y^2 = z$ Linear $\boxed{\text{Not linear}}$

$x^2 + y^2$ is a quadratic expression.

**c)** $e^{\pi}x + \ln(13)y = \sqrt{2} - z$ $\boxed{\text{Linear}}$ Not linear

$Ax + By + Cz = D$

→ Linear

**2.** Consider the following three planes, where we use $(x, y, z)$ to denote points in $\mathbf{R}^3$:

$$\begin{aligned} 2x + 4y + 4z &= \phantom{-}1 \\ 2x + 5y + 2z &= -1 \\ y + 3z &= \phantom{-}8 \end{aligned}$$

Do all three of the planes intersect? If so, do they intersect at a single point, a line, or a plane?

To find the intersection point/line/plane/more or nothing, we need to use augmented matrix.

$$\begin{cases} 2x + 4y + 4z = 1 \\ 2x + 5y + 2z = -1 \\ y + 3z = 8 \end{cases} \rightarrow \left(\begin{array}{ccc|c} 2 & 4 & 4 & 1 \\ 2 & 5 & 2 & -1 \\ 0 & 1 & 3 & 8 \end{array}\right)$$

Do row operations to find RREF.

$$\left(\begin{array}{ccc|c} 2 & 4 & 4 & 1 \\ 2 & 5 & 2 & -1 \\ 0 & 1 & 3 & 8 \end{array}\right) \xrightarrow[\times \frac{1}{2}]{\text{Scale}} \left(\begin{array}{ccc|c} 1 & 2 & 2 & \frac{1}{2} \\ 2 & 5 & 2 & -1 \\ 0 & 1 & 3 & 8 \end{array}\right) \xrightarrow[\text{Replacement}]{R_2 = -2R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 2 & 2 & \frac{1}{2} \\ 0 & 1 & -2 & -2 \\ 0 & 1 & 3 & 8 \end{array}\right)$$

$$\xrightarrow[\text{Replacement}]{R_1 = -2R_2 + R_1,\ R_3 = -R_2 + R_3} \left(\begin{array}{ccc|c} 1 & 0 & 6 & \frac{9}{2} \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 5 & 10 \end{array}\right) \xrightarrow[\times \frac{1}{5}]{\text{Scale}} \left(\begin{array}{ccc|c} 1 & 0 & 6 & \frac{9}{2} \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow[\text{Replacement}]{R_1 = -6R_3 + R_1,\ R_2 = 2R_3 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{15}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

RREF.

So we have found the RREF matrix of the system of the equation.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -\frac{15}{2} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow[\text{equals to...}]{\text{This}} \begin{cases} x = -\frac{15}{2} \\ y = 2 \\ z = 2 \end{cases} \quad \text{or} \quad \left(-\frac{15}{2}, 2, 2\right)$$

This means **all three of the planes intersect** at a **single point $\left(-\frac{15}{2}, 2, 2\right)$** in $\mathbb{R}^3$.

**3.** Find all values of $h$ so that the lines $x+hy=-5$ and $2x-8y=6$ do *not* intersect. For all such $h$, draw the lines $x+hy=-5$ and $2x-8y=6$ to verify that they do not intersect.

The phrase "lines do not intersect" means there is no solution (or inconsistent) for the system of equations.

Let's make a matrix.

$$\begin{cases} x + hy = -5 \\ 2x - 8y = 6 \end{cases} \rightarrow \left(\begin{array}{cc|c} 1 & h & -5 \\ 2 & -8 & 6 \end{array}\right)$$

$(R_2 = -2R_1 + R_2)$

$$\rightarrow \left(\begin{array}{cc|c} 1 & h & -5 \\ 0 & -8-2h & 16 \end{array}\right)$$

The second equation now is:

$(-8-2h)\,y = 16.$

We need to find values of $h$ that make this equation unsolvable.
(i.e., when the coefficient of $y$ is $0$ but the constant term is not $0$.)

→ At $h=-4$, the equation becomes $0 \cdot y = 16$, which is a contradiction.

Verification: "inconsistent" IFF the augmented column is a pivot column.

if $h=-4$...

$$\left(\begin{array}{cc|c} 1 & -4 & 5 \\ 0 & 0 & -16 \end{array}\right)$$

Let's draw $x-4y=-5$ and $2x-8y=6$. $(\mathbb{R}^2)$

$x-4y=-5 \rightarrow y = \frac{1}{4}x + \frac{5}{4}$ ①

$2x-8y=6 \rightarrow y = \frac{1}{4}x - \frac{3}{4}$ ②