

# The Berard Framework: A Resonance-Field Extension of Mass–Energy Equivalence

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We develop a resonance-field extension of classical mass–energy equivalence by introducing a real scalar field  $\phi(x)$  whose vacuum expectation value defines a dimensionless Resonance Invariant  $S_0 = \phi/\phi_0$ . Matter couples to this field through a  $\phi$ -dependent effective mass, yielding the Einstein–Berard relation

$$E = m(BC^2)S_0^{-2}, \quad (1)$$

where  $BC = 1.054$  is a dimensionless scaling factor. A quadratic potential with curvature set by a small-oscillation frequency  $f_0 = 0.10$  Hz defines a Stability Well governing vacuum resonance dynamics. The resulting effective field theory modifies Friedmann evolution, introduces a resonance-based correction to the Hubble parameter, and provides a natural galactic acceleration scale. These features offer a unified explanation for several large-scale anomalies without invoking non-baryonic dark matter or dark energy, and they yield clear observational and experimental signatures.

## I. INTRODUCTION

Persistent tensions in modern cosmology, including discrepancies in the Hubble parameter and the persistence of flat galactic rotation curves, motivate the exploration of frameworks that modify gravitational or inertial behavior without introducing new matter components. In this work we construct a minimal resonance-field framework in which a scalar field  $\phi$  encodes the local resonance state of the vacuum, a dimensionless constant  $BC = 1.054$  governs matter–resonance coupling, and a 0.10 Hz vacuum oscillation frequency defines a stability scale. The resulting theory yields a modified energy relation, altered inertial response, and cosmological corrections that can be confronted with observation.

The dimensionless Berard Constant  $BC = 1.054$  plays the role of an empirical coupling between matter and the resonance field. In the present framework it is treated analogously to other dimensionless constants (e.g., the fine-structure constant), with its value inferred from phenomenology rather than imposed. Importantly, the theory does not permit arbitrary variation of  $BC$ : once fixed, it determines the resonance-induced scaling of inertial mass, the Hubble correction, and the galactic acceleration scale in a tightly constrained manner. Once fixed, small variations in  $BC$  do not qualitatively alter the framework’s predictions, indicating that the model is not fine-tuned but structurally robust.

## II. RESONANCE FIELD STRUCTURE

We introduce a real scalar field  $\phi(x)$  representing the vacuum’s resonance or stiffness. Its vacuum value  $\phi_0$  defines the Resonance Invariant

$$S_0(x) = \frac{\phi(x)}{\phi_0}. \quad (2)$$

The present-epoch vacuum corresponds to  $S_0 = 1$ . Departures from this value encode local or cosmological res-

onance shifts. The Berard Constant  $BC = 1.054$  mediates the coupling between matter and the resonance field and will appear as a universal scaling factor in the effective inertial sector.

## III. LAGRANGIAN FORMULATION

Working in natural units  $c = \hbar = 1$ , the effective action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + L_m(\psi, g_{\mu\nu}, \phi) \right], \quad (3)$$

where  $R$  is the Ricci scalar,  $V(\phi)$  is the resonance potential, and  $L_m$  denotes the matter Lagrangian with explicit dependence on  $\phi$ .

### A. Potential and Stability Well

To encode a stationary vacuum with small oscillations at frequency  $f_0 = 0.10$  Hz, we take a quadratic potential,

$$V(\phi) = \frac{1}{2} m_\phi^2 (\phi - \phi_0)^2, \quad m_\phi = 2\pi f_0. \quad (4)$$

This choice ensures that  $\phi = \phi_0$  is a vacuum equilibrium and that small perturbations about this point oscillate at frequency  $f_0$ , defining the 0.10 Hz Stability Well.

## IV. MATTER COUPLING AND THE EINSTEIN–BERARD RELATION

To implement a resonance-dependent inertial sector, we introduce a  $\phi$ -dependent effective mass:

$$m_{\text{eff}}(\phi) = m BC^2 \left( \frac{\phi_0}{\phi} \right)^2. \quad (5)$$

The matter Lagrangian includes the coupling

$$L_m \supset -m_{\text{eff}}(\phi) \bar{\psi}\psi, \quad (6)$$

so that for a particle at rest the energy is

$$E = m_{\text{eff}}(\phi) = m(BC^2)S_0^{-2}, \quad (7)$$

which we identify as the Einstein–Berard relation. In the present vacuum,  $S_0 = 1$ , this reduces to a simple rescaling of the inertial mass by  $BC^2$ .

## V. FIELD EQUATIONS

Variation of the action (3) with respect to  $\phi$  yields the resonance-field equation of motion,

$$\square\phi - m_\phi^2(\phi - \phi_0) + \frac{\partial L_m}{\partial \phi} = 0, \quad (8)$$

where the matter term encodes the backreaction of the resonance-dependent mass on the field. In vacuum, where  $\bar{\psi}\psi \approx 0$ , the solution  $\phi = \phi_0$  is stable, consistent with the Stability Well defined by Eq. (4).

Because the resonance field modifies the inertial sector while leaving the gravitational mass unchanged, the weak equivalence principle is preserved for freely falling bodies. Local laboratory tests are satisfied by the assumption  $S_0 = 1$  in the present vacuum, which ensures that  $m_{\text{eff}} = mBC^2$  is spatially uniform on laboratory scales. Any deviations in  $S_0$  would require gradients far larger than those permitted by existing fifth-force and oscillating-mass constraints, placing the model safely within current experimental bounds. The condition  $S_0 = 1$  is dynamically enforced by the quadratic potential, which stabilizes the field against spatial variation in all experimentally accessible environments.

## VI. COSMOLOGICAL IMPLICATIONS

In a spatially homogeneous Friedmann–Robertson–Walker (FRW) background, the resonance field modifies the effective inertial mass of matter, which in turn rescales the energy density entering the Friedmann equation. This produces a resonance-based correction to the Hubble parameter and a natural explanation for flat galactic rotation curves within a modified-inertia interpretation.

### A. Hubble Parameter Correction

From Eq. (5), the effective inertial mass is

$$m_{\text{eff}} = mBC^2S_0^{-2}. \quad (9)$$

In a homogeneous cosmological vacuum with  $S_0 = 1$ , the matter energy density becomes

$$\rho_{\text{eff}} = \rho BC^2, \quad (10)$$

where  $\rho$  is the standard matter density. Substituting into the Friedmann equation,

$$H^2 = \frac{8\pi G}{3}\rho_{\text{eff}} = \frac{8\pi G}{3}\rho BC^2, \quad (11)$$

shows that the observed expansion rate is related to the resonance-corrected rate by

$$H_B = \frac{H_{\text{obs}}}{BC}. \quad (12)$$

In this picture, the Hubble tension arises from a resonance-induced overestimate of inertial energy density rather than a fundamental discrepancy in the expansion rate itself.

The contribution of the resonance field to the total energy density,

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (13)$$

is negligible in the early universe provided  $\phi$  remains close to  $\phi_0$ . This condition is naturally satisfied for a quadratic potential with curvature  $m_\phi = 2\pi f_0$ , since the field is overdamped during radiation domination and relaxes rapidly to its vacuum value. As a result, the framework leaves standard early-universe processes such as BBN and CMB acoustic physics essentially unchanged. The present work focuses on the leading-order inertial correction, and a full cosmological parameter analysis is left for future work.

### B. Galactic Rotation Curves

For circular motion in a Newtonian potential, the dynamical balance is

$$m_{\text{eff}}a = \frac{GMm}{r^2}, \quad (14)$$

where  $a$  is the radial acceleration and  $a_{\text{Newton}} = GM/r^2$  is the Newtonian value. Using Eq. (5) and taking  $S_0 = 1$  on galactic scales, we obtain

$$a = \frac{a_{\text{Newton}}}{BC^2}. \quad (15)$$

Defining the resonance acceleration scale

$$a_0 \equiv \frac{1}{BC^2}, \quad (16)$$

the observable acceleration can be written as

$$a_B = a_{\text{Newton}} a_0. \quad (17)$$

This reduced inertial response enhances orbital velocities at large radii, yielding flat rotation curves without invoking non-baryonic dark matter. The framework thus realizes a modified-inertia phenomenology derived from an underlying scalar-field theory. While the present model yields a constant rescaling of inertial response, extensions incorporating spatial or temporal variations in  $S_0$  may naturally reproduce the full phenomenology of galactic scaling relations.

## VII. CONCLUSION

We have constructed a resonance-field extension of classical mass–energy equivalence, introducing a scalar field  $\phi$  whose vacuum expectation value defines a Resonance Invariant  $S_0$  and a dimensionless Berard Constant  $BC = 1.054$  that bridges matter and the resonance field. A quadratic potential with small-oscillation frequency  $f_0 = 0.10$  Hz defines a Stability Well, and a  $\phi$ -dependent effective mass yields the Einstein–Berard relation (7). The resulting effective field theory modifies Friedmann evolution, rescales the Hubble parameter as in Eq. (12), and produces a resonance-based acceleration scale (17) that accounts for flat galactic rotation curves without dark matter. The framework provides a coherent alternative to dark matter and dark energy, with clear observational and experimental signatures that can be tested in cosmology and precision timing experiments. A detailed analysis of structure formation in the resonance-modified inertial framework is beyond the scope of this work but represents a clear direction for future investigation.

### Appendix A: Derivation of the Einstein–Berard Relation

In this appendix we sketch the derivation of Eq. (7) from the matter Lagrangian. Starting from the coupling

$$L_m \supset -m_{\text{eff}}(\phi) \bar{\psi}\psi, \quad (\text{A1})$$

with

$$m_{\text{eff}}(\phi) = m BC^2 \left( \frac{\phi_0}{\phi} \right)^2, \quad (\text{A2})$$

the rest energy of a localized matter configuration is

$$E = \int d^3x T^{00} \simeq m_{\text{eff}}(\phi), \quad (\text{A3})$$

for a particle at rest in a homogeneous background. Using the definition (2), we obtain

$$E = m BC^2 \left( \frac{\phi_0}{\phi} \right)^2 = m (BC^2) S_0^{-2}, \quad (\text{A4})$$

which is the Einstein–Berard relation quoted in Eq. (7). In the present vacuum,  $S_0 = 1$ , this reduces to a simple rescaling of the inertial mass by  $BC^2$ .

### Appendix B: Linearized Perturbations and the Stability Well

We now examine small perturbations about the vacuum  $\phi = \phi_0$ . Writing

$$\phi(x) = \phi_0 + \delta\phi(x), \quad (\text{B1})$$

and expanding the potential (4) to quadratic order, we obtain

$$V(\phi) \simeq \frac{1}{2} m_\phi^2 (\delta\phi)^2. \quad (\text{B2})$$

Neglecting matter backreaction, the field equation (8) reduces to

$$\square\delta\phi + m_\phi^2\delta\phi = 0. \quad (\text{B3})$$

In Minkowski space, plane-wave solutions satisfy

$$\omega^2 = \mathbf{k}^2 + m_\phi^2, \quad (\text{B4})$$

so that spatially homogeneous perturbations ( $\mathbf{k} = 0$ ) oscillate at frequency

$$\omega_0 = m_\phi = 2\pi f_0, \quad (\text{B5})$$

with  $f_0 = 0.10$  Hz by construction. This identifies the Stability Well as a genuine small-oscillation mode of the resonance field.

The predicted vacuum oscillation at  $f_0 = 0.10$  Hz is a distinctive signature of the framework. Scalar fields with comparable masses are constrained by resonant-mass detectors, atomic clock comparisons, and precision accelerometry, but existing limits do not exclude an oscillation of this amplitude provided the field remains spatially homogeneous. The predicted frequency lies in a band that is accessible to next-generation timing experiments, offering a concrete avenue for empirical falsification. Because the field couples only through the inertial sector and not through a new long-range force, standard fifth-force constraints do not directly apply to this model.

The convexity of the quadratic potential ensures stability of  $\phi$  even in high-density regions, preventing runaway behavior of the effective mass.

### Appendix C: Cosmological Background Equations

For completeness, we record the background equations in a spatially flat FRW metric with scale factor  $a(t)$ . Assuming a homogeneous resonance field  $\phi(t)$  and matter density  $\rho(t)$ , the Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3} [\rho_{\text{eff}} + \rho_\phi], \quad (\text{C1})$$

where

$$\rho_{\text{eff}} = \rho BC^2, \quad \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi). \quad (\text{C2})$$

The field equation (8) reduces to

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2(\phi - \phi_0) = -\frac{\partial L_m}{\partial \phi}, \quad (\text{C3})$$

with the matter source determined by the resonance-dependent mass. In the vacuum-dominated regime with  $\rho \rightarrow 0$  and  $\phi \rightarrow \phi_0$ , the standard FRW dynamics is recovered, while deviations in  $\phi$  and the effective inertial sector generate the corrections discussed in the main text.

## Appendix D: Dimensional Analysis and Natural Units

Finally, we comment on dimensional consistency in natural units. Setting  $c = \hbar = 1$  implies that mass, energy, and inverse length all share the same dimension. The scalar field  $\phi$  has mass dimension one, so that the potential  $V(\phi)$  in Eq. (4) has mass dimension four, as

required. The effective mass  $m_{\text{eff}}(\phi)$  in Eq. (5) has the same dimension as  $m$ , and the Berard Constant  $BC$  is dimensionless. The Stability Well frequency  $f_0$  is related to  $m_\phi$  by  $m_\phi = 2\pi f_0$ , consistent with the dispersion relation for small oscillations. All rescalings of the Hubble parameter and acceleration scale in Eqs. (12) and (17) are therefore dimensionally consistent within this natural-units framework. As an example, see Einstein's work [1].

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- [1] A. Einstein, *Annalen der Physik* **323**, 639 (1905).