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Path Planning for Multiple Marine Vehicles

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***Abstract*—Motivated by increasingly complex and challenging missions at sea, there is widespread interest in the development of advanced systems for cooperative control of multiple autonomous marine vehicles. Central to the implementation of these systems is the availability of efficient algorithms for multiple vehicle path planning that can take explicitly into account the capabilities of each vehicle and existing environmental conditions. Examples include path planning to steer a group of marine vehicles and enable them to reach a specific target site simultaneously with a desired formation pattern, while avoiding inter-vehicle collisions, and online path replanning for a vehicle fleet upon detection of episodic events or obstacles.**

Multiple vehicle path planning methods build necessarily on key concepts and algorithms for single vehicle path following. However, they go one step further in that they must explicitly address such issues as inter-vehicle collision avoidance and simultaneous times of arrival. As such, they pose considerable challenges both from a theoretical and practical implementation standpoint.

This paper is a brief survey of multiple vehicle path planning techniques. The exposition is focused on specific algorithms for path planning developed in the scope of research work in which the authors have participated. The algorithms make ample use of *direct optimization methods* that lead to efficient and fast techniques for path generation. The paper affords the reader a fast paced presentation of key algorithms that had their genesis in the aircraft field, discusses them critically, and suggests problems that warrant further consideration.

I. Introduction

There is increasing demand for the use of autonomous mobile robots, which are steadily becoming the tools par excellence for the execution of challenging missions in areas that are hard to access or place human lives at risk. Space, land, and marine robots are by now ubiquitous and hold promise to the development of networked systems to sample the environment at an unprecedented scale.

This trend is clearly visible in the marine world, which harbors formidable challenges imposed by the extent of the areas to be surveyed, sea waves, currents, low visibility at depth, lack of global positioning systems underwater, and stringent acoustic communication constraints. Some of these difficulties can be partially overcome through the use of fleets of heterogeneous vehicles working in cooperation, under the supervision of advanced systems for cooperative control of multiple autonomous vehicles. Central to the implementation of these systems is the availability of efficient algorithms for multiple vehicle path planning that can take explicitly into account the capabilities of each vehicle and existing environmental conditions.

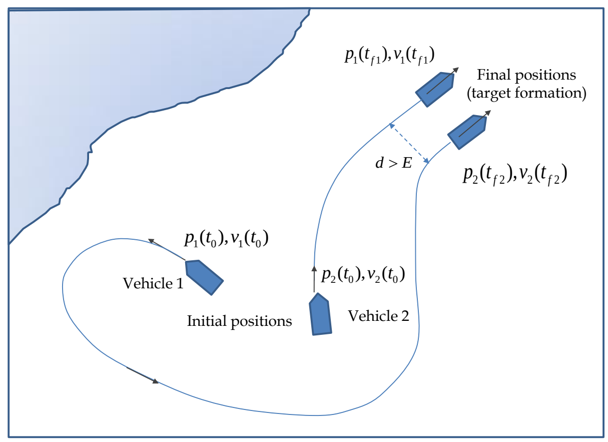

Fig. 1. Multiple Vehicle Path Planning: Go-to-Formation Maneuver with Spatial Deconfliction

As an application example, consider the scenario where multiple autonomous marine vehicles (that have been launched from one or more support ships and are scattered in the ocean) are required to execute a cooperative mission underwater, adopting a desired geometrical formation pattern. To this effect, and while still at the surface, the vehicles must maneuver from their initial positions and reach formation at approximately the same speed, in a prescribed neighborhood of the diving site. Only then can the underwater mission segment start. Because the vehicles may be operating in a restricted area and in the vicinity of support ships, this initial Go-To-Formation maneuver must be executed in such a way as to avoid collisions. Furthermore, the vehicles must arrive at their

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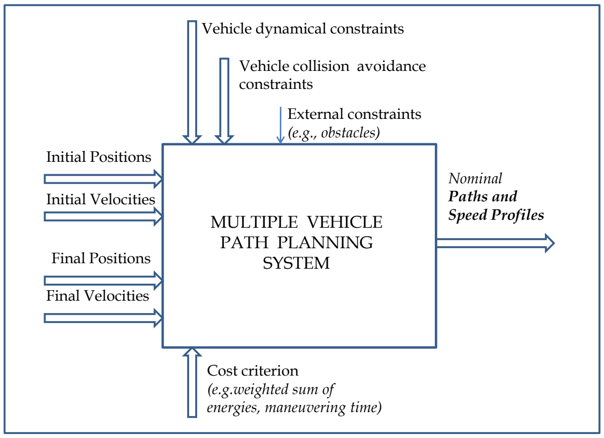


Fig. 2. Path Planning System

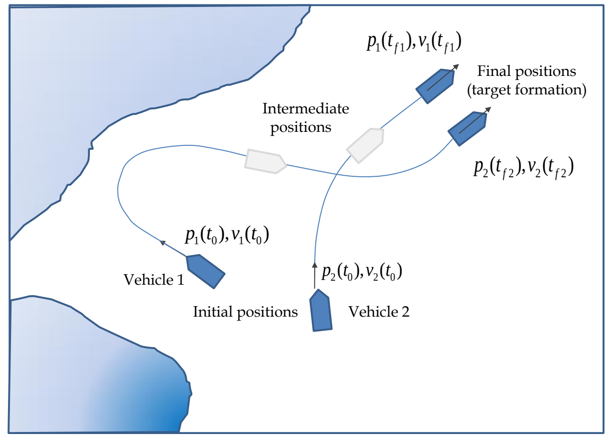


Fig. 3. Multiple Vehicle Path Planning: Go-to-Formation Maneuver with Temporal Deconfliction

target positions at approximately the same time. This scenario is depicted in Fig. 1, which shows the evolution of two vehicles that start from arbitrary positions and reach a simple side-by-side formation pattern prior to diving.

The above procedure is in striking contrast with the unpractical attempt to make each vehicle go independently to its targeted diving position and loiter in its close vicinity, a task that would be hard to accomplish in the presence of current disturbances and minimum speed requirements for adequate control authority. Another example is the case where the times of arrival of the vehicles at their targets must be separated by specified clearance intervals. Related application examples are also important in the areas of space, air, and land robotics.

The example above can be further detailed to show how multiple vehicle path planning yields an optimization problem subject to a number of critical constraints. For example, in the case of an energy-related cost criterion the function to be minimized may be an weighted sum of the energies spent during a Go-To-Formation maneuver. However, other criteria may be envisioned such as average maneuvering time. Vehicle related constraints are the total energy available for vehicle maneuvering and vehicle dynamic restrictions such as maximum vehicle accelerations. Environmental constraints include external disturbances caused by ocean currents and sea waves. It is also required that collisions be avoided among vehicles as well as between vehicles and stationary and moving obstacles (e.g. support ships, the coastline, and harbor structures). In particular, it is crucial that path planning algorithms yield feasible paths and that any two vehicles never come to close vicinity of each other. This property is often referred to as *deconfliction* in the area of multiple air vehicle control, for it ensures that at no time will two vehicles get closer in space than a desired safety distance E, see Figure 1.

Stated in such generality, path planning is obviously a problem with far reaching implications not only in robotics but also in control theory, computer science, artificial intelligence, and other related engineering subjects [13]. Figure 2 illustrates the problem at hand and shows how a cost criterion, initial and final vehicle conditions, and internal and external constraints are used to produce (if it exists) a trajectory that meets the constraints and minimizes the cost. The spatial and temporal coordinates of this trajectory yield a spatial path and a corresponding vehicle profile. This simple observation is at the root of the methodologies for path planning exposed in the paper and will be fully discussed later.

In practice, deconfliction can be spatial or temporal. In the first category, shown in Fig. 1 for the case of two vehicles, non-intersecting spatial paths are generated without explicit temporal constraints. In the second case, temporally deconflicted paths will give rise to nominal trajectories (defined in space and time) for the vehicles to track. Clearly, temporal deconfliction introduces an extra degree of freedom (time) that is not available in the case of spatial deconfliction. As such, it leads to solutions whereby paths are allowed to come to close vicinity or intersect in space, but the temporal scheduling of the vehicles involved separates these occurrences well in time, see Fig. 3. In summary, temporal deconfliction allows for the solution of a larger class of problems than those that can be tackled with spatial deconfliction algorithms.

It is important to remark that whereas spatial deconfliction can often be guaranteed by making each vehicle follow its assigned path separately, temporal deconfliction requires that cooperative path following algorithms be implemented so as to meet the added temporal constraints. Notice, however, that the latter solution is different from pure trajectory tracking in that the time variable that is used to generate the paths is simply viewed, for real-time implementation purposes, as a path parameter that plays a key role in the temporal synchronization of the vehicles along their paths. As such, this solution avoids the problems that arise when pure trajectory tracking solutions are implemented. See [8] and the references therein for an introduction to this circle of ideas.

Motivated by the above considerations, this paper addresses the problem of deconflicted path planning with applications to multiple autonomous marine vehicles. However, we purposely eschew the (implementation) problem of making the vehicles cooperate with the objective of meeting the temporal constraints that arise out of temporal deconflicted path planning. For simplicity of exposition, the main focus is on vehicles moving in 2D space. The problem formulation and the solutions proposed have been strongly influenced by several mission scenarios studied in the scope of the EU research projects *FREE*$_{\text{sub}}$*NET* and GREX to which the reader is referred for details, see [1] and [2], respectively.

The literature on path planning is vast and the methodologies used are quite diverse; see [12], [13], [15], [16], and [18]. Classical methodologies aim at computing feasible strategies off-line that minimize a chosen cost criterion. More recently, new methodologies have come to the forum where the objective is to generate paths on-line, in response to environmental data, so as to optimize the process of data acquisition over a selected area. The reader will find in [3], [4], [5], [6], [14], [17], and [19] very lucid presentations of this and related issues. In the present paper we focus on the problem that arises when multiple vehicles are scattered in the water and it is required that they safely reach the starting location of a cooperative mission with a desired formation pattern and assigned terminal speeds (the Go-To-Formation manoeuver explained before). The cost criteria of interest may include minimizing travel time or energy expenditure. The key objective is to *obtain path planning methods that are effective, computationally easy to implement, and lend themselves to real-time applications*.

The techniques that are the focus of this survey paper build upon and extend the work first reported in [20] and later in [9] and [10] for unmanned air vehicles. See also [8] for recent work on the subject. Explained in intuitive terms, the key idea exploited is to separate spatial and temporal specifications, effectively decoupling the process of spatial path computation from that of computing the desired speed profiles for the vehicles along those paths. The first step yields the vehicles' spatial profiles and takes into consideration geometrical constraints; the second addresses time related requirements that include, among others, initial and final speeds, deconfliction in time, and simultaneous times of arrival. Decoupling the spatial and temporal constraints can be done by parameterizing each path as a set of polynomials in terms of a generic variable τ and introducing a polynomial function $\eta(\tau)$that specifies the rate of evolution of τ with time, that is, $d\tau/dt = \eta(\tau)$, see [10]. By restricting the polynomials to be of low degree, the number of parameters used during the computation of the optimal paths is kept to a minimum, a fact that stands at the root of the success of the direct method for rapid prototyping of near-optimal aircraft trajectories proposed in [20]. Once the order of the polynomial parameterizations has been decided, it becomes possible to solve the multiple vehicle optimization problem of interest (e.g., simultaneous time of arrival under specified deconfliction and energy expenditure constraints) by resorting to any proven direct search method [11].

The paper is organized as follows. Section II offers a general description of the methodology adopted for spatially deconflicted path generation and details its application to the generation of the Go-To-Formation manoeuver with an energy cost criterion and an almost simultaneous time of arrival constraint. Section III addresses the problem of multiple vehicle path planning with temporal deconfliction. Finally, Section IV contains the main conclusions and discusses future research trends.

II. Multiple Vehicle Path Planning with Spatial Deconfliction

This section describes an algorithm for multiple vehicle path planning with spatial deconfliction. In what follows, we let $\mathcal{V} := \{V_i; i = 1,..,n\}$ denote the set of $n \geq 2$ vehicles V_i involved in a maneuver. We start by recalling the difference between a path and a trajectory. A path is simply a curve $p : \tau \to \mathbb{R}^3$ parameterized by τ in a closed subset $[0, \tau_{f_i}], \tau_{f_i} > 0$ of $\mathbb{R}_+$. If τ is identified with time t or a function thereof then, with a slight abuse of notation, $p : t \to \mathbb{R}^3$ with $t \in [0; t_f], t_f > 0$ will be called a trajectory. *Path following* refers to the problem of making a vehicle converge to and following a path $p(\tau)$ with no explicit temporal schedule. However, the vehicle speed may be assigned as a function of parameter τ. *Trajectory tracking* is the problem of making the vehicle track a trajectory $p(t)$, that is, the vehicle must satisfy spatial and temporal schedules simultaneously. For the sake of clarity, and whenever one wishes to refer to a specific vehicle V_i, the variables of interest will be written with subscript index i. For example, $p_i(\tau)$; $\tau \in [0, \tau_{f_i}]$ and $p_i(t)$; $\tau \in [0, t_{f_i}]$ refer to a path and a trajectory for vehicle V_i, respectively.

Suppose the objective is to execute a multi-vehicle Go-To-Formation maneuver while avoiding inter-vehicle collisions, meeting dynamical constraints (e.g. bounds on maximum accelerations), and minimizing a weighted combination of vehicle energy expenditures. Further suppose that the vehicles are required to arrive at their final destination at the same final time t_f, that is, $t_{f_i} = t_f; i = 1, 2, .., n$ (this requirement will be relaxed to almost simultaneous times of arrival). At first inspection, a possible solution to this problem would be to solve a constrained optimization problem that would yield (if at all possible) feasible trajectories $p_i(t), t \in [t_0, t_f]; i = 1, 2, ..., n$ for the vehicles, with t_0 and t_f denoting initial and final time, respectively. Trajectory tracking systems on-board the vehicles would then ensure precise tracking of the trajectories generated, thus meeting the mission objectives.

This seemingly straightforward solution suffers from a major drawback: it does not allow for any "deviations from the plan". Absolute timing becomes crucial because the strategy described does not lend itself to on-line modification in the event that one or more of the vehicles cannot execute trajectory tracking accurately (e.g. due to adverse currents or lack of sufficient propulsion power). For this reason, it is far more practical to adopt a different solution where absolute time is not crucial and enough room is given to each vehicle to adjust its motion along the path in response to the motions of the other vehicles. The goal is that of reaching a terminal formation pattern that will ensure almost simultaneous times of arrival. Dispensing with absolute time is key to the solution proposed. In this set-up, the optimization process should be viewed as a method to produce paths $p_i(\tau_i)$ without explicit time constraints, but with timing laws for $\tau_i(t)$ that effectively dictate how the nominal speed of each vehicle should evolve along the path. Using this set-up, spatial and temporal constraints are essentially decoupled and captured in the descriptions of $p_i(\tau_i)$ and $\eta_i(\tau) = d\tau_i/dt$, respectively, as will be seen later. Furthermore, adopting polynomial approximations for $p_i(\tau_i)$ and $\eta_i(\tau) = d\tau_i/dt$ keeps the number of optimization parameters reduced and makes real-time computational requirements easy to achieve. Intuitively, by making the path of a generic vehicle V_i a polynomial function of $\tau_i \in [0, \tau f_i]$, the shape of the path in space can be changed by increasing or decreasing τ_i - a single optimization parameter. This, coupled with a polynomial approximation for $\eta_i(\tau_i) = d\tau_i/dt$ makes it easy to shape the speed and acceleration profile of the vehicle along the path so as to meet desired dynamical constraints. The paths thus generated can then be used as "templates" for path following.

The approach adopted for path generation exploits a separation between spatial and temporal specifications. Let $p(\tau) = [x(\tau), y(\tau), z(\tau)]^\top$ denote the path of a single vehicle, parameterized by $\tau = [0; \tau_f]$. For computational efficiency, assume each coordinate $x(\tau), y(\tau), z(\tau)$ is represented by an algebraic polynomial of degree N. For example, $x(\tau)$ is of the form

$$x(\tau) \;=\; \sum_{k=0}^{N} a_{xk}\tau^k. \qquad i = 1, 2, 3, \tag{1}$$

The degree N of polynomials $x(\tau), y(\tau), z(\tau)$ is determined by the number of boundary conditions that must be satisfied. Notice that these conditions (that involve spatial derivatives) are computed with respect to the parameter τ. There is an obvious need to relate them to actual temporal derivatives, but this issue will only be addressed later. For the time being, let d_0 and d_f be the highest-order of the spatial derivatives of $x(\tau), y(\tau), z(\tau)$ that must meet specified boundary constraints at the initial and final points of the path, respectively. Then, the minimum degree N^* of each of the corresponding polynomials is $N^* = d_0 + d_f + 1$. For example, if the desired path includes constraints on initial and final positions, velocities, and accelerations (second-order derivatives), then the degree of each polynomial is $N^* = 2+2+1 = 5$. Explicit formulae for computing boundary conditions $p'0), p''(0)$ and $p'(\tau_f), p''(\tau_f)$ are given later. Additional degrees of freedom may be included by making $N > N^*$. As an illustrative example, consider the case where (1) is polynomial trajectory of 5^{th} degree. In this case, the coefficients $a_{x,k}; k = 0, .., 5$ can be computed from

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & \tau_f & \tau_f^2 & \tau_f^3 & \tau_f^4 & \tau_f^5 \\ 0 & 1 & 2\tau_f & 3\tau_f^2 & 4\tau_f^3 & 5\tau_f^4 \\ 0 & 0 & 2 & 6\tau_f & 12\tau_f^2 & 20\tau_f^3 \end{pmatrix} \cdot \begin{pmatrix} a_{x,0} \\ a_{x,1} \\ a_{x,2} \\ a_{x,3} \\ a_{x,4} \\ a_{x,5} \end{pmatrix} = \begin{pmatrix} x(0) \\ x'(0) \\ x''(0) \\ x(\tau_f) \\ x'(\tau_f) \\ x''(\tau_f) \end{pmatrix}$$

where τ_f is the terminal value of τ. Similar equations can be used to compute the coefficients $a_{y,k}$ and $a_{z,k}; k = 0, .., 5$. For 6^{th} degree polynomial trajectories, an additional constraint on the fictitious initial jerk can be included,

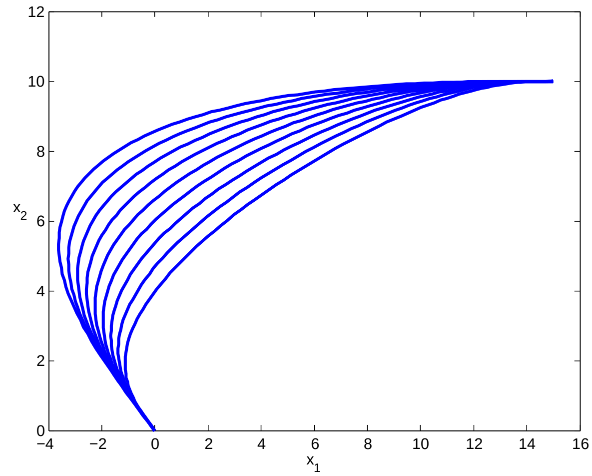


Fig. 4. 5th order polynomial paths obtained by changing τ_f

which increases the order of the resulting polynomial and affords extra (design) parameters $x'''(0), x'''(0), z'''(0)$. Figure 4 shows examples of admissible 5^{th} order polynomial paths when τ_f is allowed to vary between 4 and 85 , with $[x(0), y(0)] = [0, 0]; [x(\tau_f), y(\tau_f)] = [15, 10]; [x'(0), y'(0)] = [-0.25, 0]; [x'(\tau_f), y'(\tau_f)] = [0, -0.5]; [x''(), y''(0] = [x''(\tau_f), y''(\tau_f)] = [0, 0]$ (units are omitted). The figure illustrates the usefulness of using τ_f as an optimization parameter, a fact that will be exploited at a later stage.

It is now important to clarify how temporal constraints may be included in the feasible path computation process. A trivial solution would be to make $\tau = t$. In this case, however, little control exists over the speed profile along a path $x(t), y(t), z(t)$ that meets the required boundary conditions. In fact, once the path has been computed the speed v is inevitably given by

$$v(t) \;=\; \sqrt{\dot{x}^2(t) + \dot{y}^2(t) + \dot{z}^2(t)}, \tag{2}$$

and not much freedom is available to meet temporal constraints unless one resorts to high order polynomials.

We therefore turn our attention to a different procedure that will afford us the possibility of meeting strict boundary conditions and other constraints without increasing the complexity of the path generation process. To this effect, let $v_{\min}, v_{\max}$ and $a_{\max}$ denote predefined bounds on the vehicle's speed and acceleration, respectively. Let $\eta(\tau) = d\tau/dt$, yet to be determined, dictate how parameter τ evolves in time. A path $p(\tau)$ (with an underlying assignment $\eta(\tau)$) is said to constitute a *feasible* path if the resulting trajectory can be tracked by a vehicle without exceeding prespecified bounds on its velocity and total acceleration along that trajectory. With an obvious abuse of notation, we will later refer to a spatial path only, without the associated $\eta(\tau)$, as a feasible path.

From (2), and for a given choice of $\eta(\tau)$, the temporal speed $v(\tau(t))$ and acceleration $a(\tau(t))$ of a vehicle along the path are given by

$$\begin{aligned} v(\tau) &= \eta(\tau)\sqrt{x'^2(\tau) + y'^2(\tau) + z'^2(\tau)} \;=\; \eta(\tau)||p'(\tau)||, \\ a(\tau) &= ||p''(\tau)\eta^2(\tau) + p'(\tau)\eta'(\tau)\eta(\tau)||. \end{aligned} \tag{3}$$

At this point, a choice for $\eta(\tau)$ must be made. A particular choice is simply $\eta(\tau) = \eta(0) + \frac{\tau}{\tau_f}(\eta(f) - \eta(0))$ with $\eta(0) = v(0)$ and $\eta(\tau_f) = v(t_f)$, where t_f is the terminal time yet to be determined. This polynomial is of degree sufficiently high to satisfy boundary conditions on speed and acceleration because the boundary conditions $p'(0), p'', p'(\tau_f), p''(\tau_f)$ can be easily obtained from given $\dot{p}(0), \ddot{p}(0)$, $\dot{p}(t_f), \ddot{p}(t_f)$ using the definition of $\eta(\tau)$. In fact, since $\dot{p}(t) = p'(\tau)\eta(\tau)$, it is easy to see that

$$\begin{aligned}
p'(0) &= \frac{\dot{p}(0)}{\eta(0)}, \\
p'(\tau_f) &= \frac{\dot{p}(t_f)}{\eta(\tau_f)}, \\
p''(0) &= \frac{\ddot{p}(0) - p'(0)\eta'(0)\eta(0)}{\eta^2(0)}, \\
p''(\tau_f) &= \frac{\ddot{p}(t_f) - p'(t_f)\eta'(\tau_f)\eta(\tau_f)}{\eta^2(\tau_f)},
\end{aligned}$$

where $\eta'(0) = \eta'(\tau_f) = \frac{\eta(\tau_f)-\eta(0)}{\tau_f}$. Furthermore, the choice of boundary conditions on $\eta(\tau)$ guarantees that $||p'(0)|| = ||p'(t_f)|| = 1$.

It now follows from (3) that a path $p(\tau)$ is feasible if all boundary conditions are met, together with the additional speed and acceleration constraints

$$\begin{aligned}
&v_{\min} \leq \eta(\tau)\ ||p'(\tau)|| \leq v_{\max}, \\
&||p''(\tau)\eta^2(\tau) + p'(\tau)\eta'(\tau)\eta(\tau)|| \leq a_{\max},\ \forall \tau \in [0, \tau_f].
\end{aligned} \tag{4}$$

A feasible trajectory can be obtained by solving, for example, the optimization problem

$$F1 : \min_{\Xi} J \quad \textit{subject to } (4)$$

and to the boundary conditions at initial and final points, where Ξ is the vector of optimization parameters that includes either τ_f or τ_f together with $x'''(0), y'''(0), z'''(0)$. The latter definition of Ξ corresponds to the case where the degree of the polynomial path is selected to be 6 . The cost function J may be defined to be the total energy consumption of the vehicle, given by

$$J = \int_0^{t_f} c_f c_D \rho v_c^3(t) dt = \int_0^{\tau_f} c_f c_D \rho \eta^3(\tau)\ ||p'(\tau)||^3\ d\tau,$$

where ρ is dynamic pressure, c_f is a propulsion efficiency factor, and c_D is the total drag coefficient of the vehicle. Other choices of J can be made to address time optimal or minimum length paths.

The above methodology is now extended to deal with multiple vehicles. In particular, we address the problem of time-coordinated control where all vehicles must arrive at their respective final destinations at approximately the same time (the meaning of "approximately" will be quantified in the sequel). The dimension of the corresponding optimization problem increases and the time coordination requirement introduces additional constraints on parameters $\tau_{fi}; i = 1, 2, .., n$. Without loss of generality, we compute the total maneuvering time t_{f1} for vehicle V_1. A similar procedure can be used to compute the times of flight $t_{fi}; i = 2, .., n$. Using the definition of $\eta_i(\tau)$,

$$t_{f1} = \int_0^{\tau_{f1}} \frac{d\tau_1}{\eta_1(\tau_1)}.$$

It follows immediately that the minimum maneuvering time $t_{f1_{\min}}$ of vehicle V_1 is given by

$$t_{f1_{\min}} = \int_0^{\tau_{f1}} \frac{||p_1'(\tau_1)||\ d\tau_1}{v_{\max_1}}.$$

Similarly, its maximum maneuvering time is

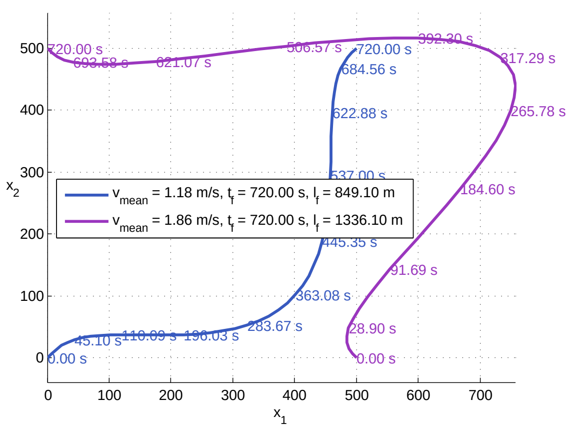
$$t_{f1_{\max}} = \int_0^{\tau_{f1}} \frac{||p_1'(\tau_1)||\ d\tau_1}{v_{\min_1}}.$$

Hence, V_1 will arrive at the target in the interval $T_1 = [t_{f1_{\min}}, t_{f1_{\max}}]$. Let $T_i = [t_{fi_{\min}}, t_{fi_{\max}}]$ denote the time interval for the arrival of vehicle V_i at its assigned target. Clearly, the problem of (almost) simultaneous time of arrival with a temporal gap to be specified has a solution *if and only if* $T_i \bigcap T_j \neq \emptyset\ \forall i, j = 1, \ldots, n, i \neq j$. This is guaranteed if $\min_i t_{fi_{\max}} \geq \max_i t_{fi_{\min}}$ for $i = 1, \ldots, n$. Thus, for the case of multiple vehicles additional constraints must be imposed on $\tau_{fi}; i = 1, .., n$. Feasible, spatially deconflicted trajectories for all vehicles can be obtained by solving an optimization problem of the form

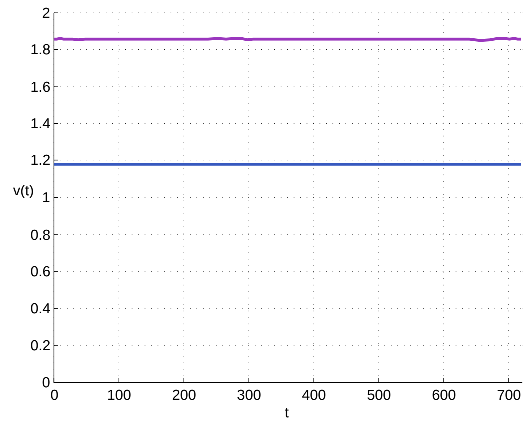
$$F2 : \begin{cases}
\displaystyle\min_{\Xi_i, i=1,\ldots,n} \sum_{i=1}^{n} w_i J_i \\
\text{subject to geometric boundary conditions and (4)} \\
\text{for any } i \in [1, n], \text{ and} \\
\\
\displaystyle\min_{j,k=1,\ldots,n, j\neq k} ||p_{c_j}(\tau_j) - p_{c_k}(\tau_k)||^2 \geq E^2 \\
\text{for any } \quad \tau_j, \tau_k \in [0, \tau_{fj}] \times [0, \tau_{fk}], \\
\\
\displaystyle\min_i t_{fi_{\max}} \geq \max_i t_{fi_{\min}}, for\ i = 1, \ldots, n,
\end{cases}$$

where J_i represents total energy consumption of vehicle V_i and the weights $w_i > 0$ penalize the energy consumptions of all vehicles. Note that in $F2$ an additional constraint $\min_{j,k=1,\ldots,n,j\neq k} ||p_j(\tau_j) - p_k(\tau_k)||^2 \geq E^2$ for any $\tau_j, \tau_k \in [0, \tau_{fj}] \times [0, \tau_{fk}]$ was added to guarantee spatially deconflicted trajectories separated by a minimum distance E. To meet the almost simultaneous time of arrival requirement, the constraint $(\max_i t_{fi_{\max}} - \min_i t_{fi_{\min}};\ \text{i = 1, \ldots, n}) \leq T_g$ must be included, where T_g is the maximum temporal gap allowed in the differences of times of arrival of the different vehicles. We emphasize that the dimension of the optimization problem $F2$ increases linearly with the number of vehicles.

It is interesting to point out that the constraint that the total speed of a vehicle be kept constant along path p can be easily satisfied by adopting a non-polynomial, yet simple expression for $\eta(\tau)$. To this effect, notice that having constant speed is equivalent to satisfying $d/dt||\dot{p}|| = 0$ for all t, that is, $d/dt(\dot{p}^T\dot{p}) = \ddot{p}^T\dot{p} = 2(\eta^2 p'' + \eta\eta' p')^T \eta p' = 0$. Because in this case η is never equal to 0, it follows that $\eta'/\eta =$

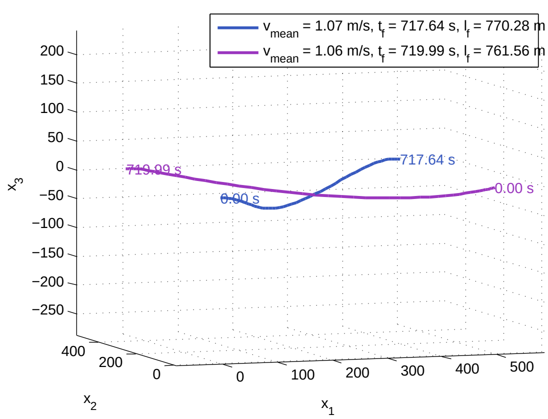


(a) Two-dimensional spatial deconfliction in an extreme scenario. To maintain spatial clearance for the paths, a detour was generated.

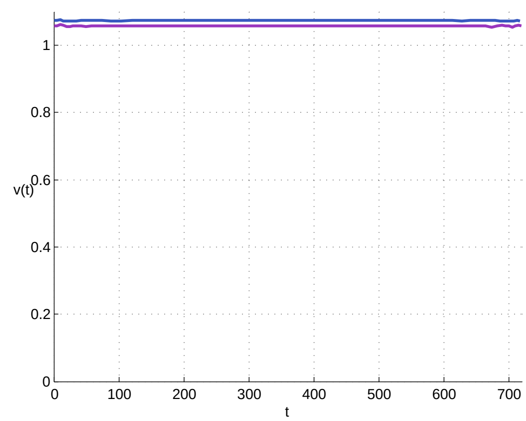


(b) The velocity profiles show nearly constant speed for both vehicles.

Fig. 5. Simulation results for spatial deconfliction in 2D. The axes are labeled $x1 = x$ and $x2 = y$. Mean velocity v, final time t_f, and path length l_f are listed for each path. The initial and terminal poses for vehicle V_1 are specified by $x_1(0) = 0$m, $x_2(0) = 0$m, $\psi(0) = \pi/4$, $x_1(\tau(t_f)) = 500$m, $x_2(\tau(t_f)) = 500$m and $\psi(\tau(t_f)) = \pi/4$. For vehicle V_2, $x_1(0) = 500$m, $x_2(0) = 0$m, $\psi(0) = 3\pi/4$, $x_1(\tau(t_f)) = 0$m, $x_2(\tau(t_f)) = 500$m, and $\psi(\tau(t_f)) = 3\pi/4$.



(a) Collision is avoided by letting one vehicle dive under the other.



(b) The velocity profiles are practically constant.

Fig. 6. Simulation results for spatial deconfliction in 3D. Same vehicle configuration as in Fig. 5. The axes are labeled as $x1 = x$, $x1 = y$, and $x1 = z$. By allowing for motion in the z coordinate, spatial deconfliction produces good results without the need for a large detour in 2D.

$-(p''^T p')/(p'^T p')$, yielding $\eta(\tau) = \eta_0(\tau) exp - \int_0^\tau \frac{p''^T p \; d\tau}{p'^T p'}$.

The optimization problems $F1, F2$ can be effectively solved in real-time by resorting to any well proven zero-order optimization technique [11]. As an example, Fig. 5a illustrates the computation of spatially deconflicted paths for two vehicles using the non-polynomial expression for $\eta(\tau)$. In this case, the optimization vector Ξ consists of τ_{f1}, τ_{f2}, and the (constant) vehicle speeds along their paths. The final value of the optimization vector Ξ gave rise to feasible paths with the distance between them never exceeding a given limit. Interestingly enough, the search procedure produced almost equal times of arrival. The nominal speed profiles shown in Fig. 5b stayed within the predefined limits of $[v_{\min} \;\; v_{\max}] = [1m/s \;\; 5m/s]$. The maximum acceleration corresponding to each path did not exceed $1m/s^2$, as desired. In Fig. 5b, the deviations of the vehicle speeds from constant values are due to small numerical errors.

III. Multiple Vehicle Path Planning with Temporal Deconfliction

We now address the problem of multiple vehicle path planning with temporal deconfliction. As argued before, temporal deconfliction introduces an extra degree of freedom (time) that is not available in the case of spatial deconfliction. As such, it yields solutions whereby paths are allowed to come to close vicinity or intersect in space. However, the temporal scheduling of the vehicles involved separates these occurrences well in time, as depicted in Fig. 3. The solution adopted for temporal deconfliction has an extra potential advantage: should the vehicles be able to follow exactly the spatial and temporal profiles generated by the path planning algorithm, they will arrive at their target positions at exactly the same time. The crucial point is therefore to guarantee that the vehicles maneuver along the assigned paths, in a synchronized manner.

At this point, it appears as if one is led inevitably to the situation where each vehicle must track a pre-assigned trajectory with great precision, that is, to execute a trajectory tracking maneuver. This strategy meets with considerable problems. In fact, in the event that one of the vehicles will deviate considerably from its planned spatial and temporal schedule (due to environmental disturbances or temporary failures), the original plan can no longer be executed and replanning will become necessary. However, this strategy does not lend itself to formal analysis. For this reason, it is important to adopt a new strategy where the vehicles cooperate and adjust their motion in reaction to deviations from the original plan, so as to keep maintain the spatial formation that is naturally imposed by that plan. In this set-up absolute time ceases to play an important role, and all that is relevant is for the vehicles to arrive simultaneously at their target positions, the final times of arrival being left unspecified (but within defined bounds).

This circle of ideas, originally proposed in [8], leads to an integrated strategy for multiple vehicle path planning and control that is referred to as Time-Coordinated Path Following (TC-PF). The methodology proposed unfolds in three steps. First, extending the methods exposed in [10], *temporally deconflicted trajectories are generated for a group of vehicles*. At the end of this step, the trajectories obtained are conveniently re-parametrized by a variable that we call *virtual time*, leading to a set of spatial paths, together with the corresponding nominal vehicle speed profiles along them. The second step involves the design of *path following algorithms* to steer each vehicle along its assigned path, while tracking the corresponding speed profile. Here, absolute time does not play any role. Finally, the last step aims to *coordinate the relative motion of the vehicles along their paths*, so as to guarantee deconfliction and meet desired temporal constraints such as equal times of arrival. This is done by varying the speed of each vehicle about the nominal speed profile computed in the first step, based on the exchange of information with its neighbors. The information exchanged is related to the virtual time referred to above. The resulting scheme lends itself to a rigorous formulation and avoids replanning except for the situation where, due to strong disturbances, the vehicles deviate considerably from the paths or fail to meet required temporal constraints. In this paper we focus on the first step described above. For an introduction to the theoretical machinery that supports steps two and three above, the reader is referred to [7].

It is against this backdrop of ideas that we now describe a solution to the problem of multiple vehicle path planning with temporal deconfliction. The solution borrows from the concepts previously introduced in the section on spatial deconfliction. However, we now require that the vehicles arrive at their final destinations at exactly the same time. To this effect, adopt the simplified functions

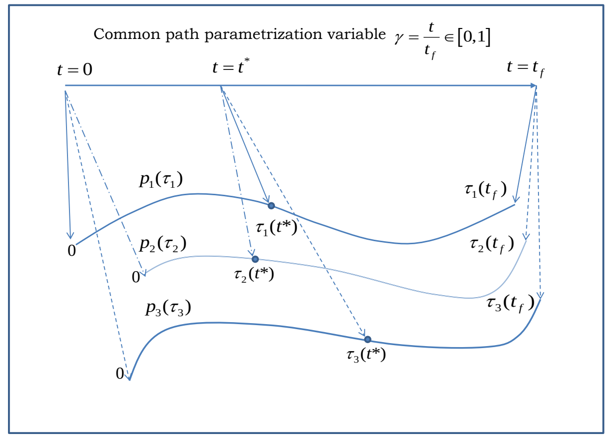


Fig. 7. Temporal deconfliction: from temporal to path parameter coordinates.

$$\eta_i(\tau_i) \quad = \quad \eta_i(0) + \frac{\tau_i}{\tau_{fi}}(\eta_i(\tau_{fi}) - \eta_i(0)).$$

Integrating $\dot{\tau}_i = \eta_i(\tau_i)$ yields

$$\tau_{f_i} = \tau_i(t_f) = \begin{cases} \eta_i(0)t_f, & \eta_i(\tau_{fi}) = \eta_i(0) \\ \frac{\eta_i(\tau_{fi})-\eta_i(0)}{\ln(\eta_i(\tau_{fi})/\eta_i(0))}t_f & \eta_i(\tau_{fi}) \neq \eta_i(0) \end{cases} \tag{5}$$

and

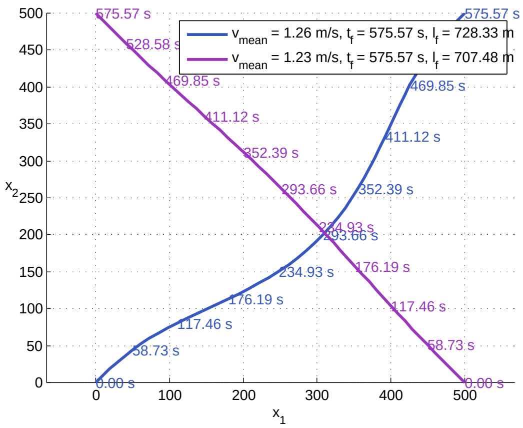
$$\frac{t}{t_f} = \begin{cases} \frac{\tau_i}{\tau_{f_i}}, & \eta_i(\tau_{fi}) = \eta_i(0) \\ \frac{\ln\left(1+\left(\frac{\eta_f}{\eta_0}-1\right)\frac{\tau}{\tau_f}\right)}{\ln\frac{\eta_f}{\eta_0}} & \eta_i(\tau_{fi}) \neq \eta_i(0) \end{cases} \tag{6}$$

Thus, given a value of the final time t_f, the final values $\tau_{fi}; i = 1, 2, ..., n$ of the path parameters τ_i are uniquely defined by (5). At this point, path planning is essentially a repetition of the procedure exposed before, except that the key search parameter is now simply t_f because the τ_{fi} are uniquely defined by the latter. Furthermore, deconfliction in time is enforced by exploiting the relationship between t and the $\tau_i; i = 1, 2, ..., n$. See figure 7, which captures the relationship between temporal and spatial path coordinates. An optimization problem can now be posed and solved using the procedure described in the previous section. Without being too rigorous, the problem can be simply stated as follows: assuming the arrival time t_f is required to lie in a pre-defined time interval $[t_1, t_2]$, compute

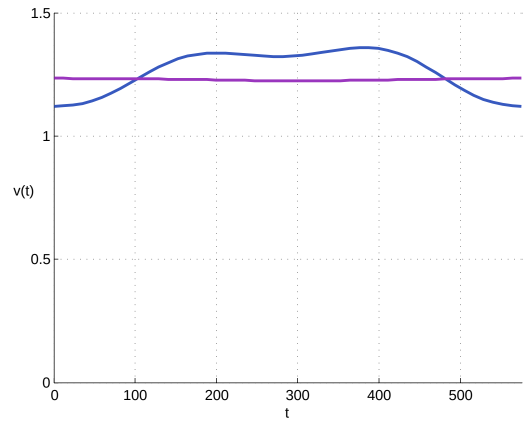
$$t_f^{opt} = \arg \min_{t_f \in [t_1, t_2]} \sum_{i=1}^{n} w_i J_i \tag{7}$$

subject to vehicle speed and acceleration constraints as well as temporal deconfliction constraints. Again, this problem can be solved using a zero-order optimization technique.

We again stress that multiple vehicle path planning with temporal deconfliction is the first step in the general methodology of Time-Coordinated Path Following introduced in [8]. At the end of this step, the temporal coordinate used in the computations becomes a path parameter that is hidden in the remaining two steps. This is simply done by reparameterizing the paths produced as $p_i(\gamma)$, with $\gamma = t/t_i \in [0, 1]$ defined as



(a) Because the paths are temporally deconflicted, spatial intersections are allowed. The clearance is the same as in Fig. 5a.



(b) Taking into account the scale of the graph, the velocity profiles are almost constant.

Fig. 8. Simulation results for temporal deconfliction in 2D. Same configuration as in Fig. 5. Collision is avoided but the paths cross in space.

$$\gamma(\tau_i) = \begin{cases} \frac{\tau_i}{\tau f_i}, & \eta_i(\tau_{fi}) = \eta_i(0) \\ \frac{\ln\left(1+\left(\frac{\eta_f}{\eta_0}-1\right)\frac{\tau}{\tau_f}\right)}{\ln\frac{\eta_f}{\eta_0}} & \eta_i(\tau_{fi}) \neq \eta_i(0) \end{cases} \tag{8}$$

This is illustrated in figure 7.

The result of a multiple vehicle path planning run with temporal deconfliction is shown in Fig. 8a. The geometry of the problem is that used in the spatial deconfliction example. Notice that because the paths are temporally deconflicted, spatial intersections are now allowed. However, collisions among vehicles will not occur because the actual spatial clearance will never fall below a specifide limit $E > 0$ at any time t.

IV. Conclusion and Future Trends

The paper afforded the reader an overview of selected multiple vehicle path planning algorithms that hold potential for real-time applications. The presentation aimed at bringing attention to this challenging area of work, which is essential to the execution of increasingly complex and challenging missions at sea. The focus of the presentation was on multiple path planning methods to steer a group of marine vehicles and enable them to reach a specific target site simultaneously, with a desired formation pattern, while avoiding inter-vehicle collisions.

The techniques described make ample use of *direct optimization methods* that lead to efficient and fast techniques for path generation. The mathematical set-up required is minimum and yet sufficiently powerful to handle spatial and temporal deconfliction among vehicles, while reducing an energy-like or maneuvering time criterion in the presence of dynamical vehicle constraints. The efficacy of the techniques was shown in simulation.

A critical assessment of the results achieved so far shows that a number of problems are yet to be resolved. In particular, the problems of addressing explicitly the effect of constant current fields and avoiding fixed external obstacles warrant intensive research.

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