EC732, Spring 2025, Professor Rysman
Assignment 2
Due March 28

1. This problem asks you to estimate a model of dynamic demand for DVD players based on the model of Melnikov (2013) (linked under the Course Readings on the class web page). Download the data set *dvdDataEC732.xls* from the class web page. The data set contains average price and total sales for DVD players at the level of the model-month from March 1997 to September 1998. It also has two characteristics, indicator variables for whether the player has the ability play MP3 files and whether it uses coaxial cable. Think of these as two quality measures. For these variables, convert missing values to zero. Note that the month variable is a number ranging from 446 to 464. Using the command `format month %tmmY` in Stata converts the month variable to the appropriate text.

   You may find Melinikov's paper difficult to follow. I provide a step-by-step guide that I want you to follow. In Melnikov's model, consumers may purchase once and only once in their lifetime. Consumers obtain $\delta_{jt} + \varepsilon_{ijt}$ from purchase where ε_{ijt} is distributed Extreme Value and:

$$\delta_{jt} = x_j\beta - \alpha \ln(p_{jt}) + \xi_{jt}.$$

   The value of purchasing at t is given by the variable v_{it} which is equal to $r_t + \varepsilon'_{it}$ where ε'_{it} is distributed Extreme Value and r_t is the "logit inclusive value:"

$$r_t = \ln\left(\sum_{j\in J_t} e^{\delta_{jt}}\right)$$

   The dynamic part of the problem can be thought of as simply the choice of whether or not to buy in a period. The expected value of buying is r_t. Waiting earns the consumer flow utility $c + \varepsilon_{i0t}$ (where $\varepsilon_{i0t} \sim EV$) and the option to buy at future values of r_t. Melnikov assumes that r_t is a Markov process and that consumers predict r_{t+1} using r_t only. Therefore, there is a single state variable r_t. The Bellman equation is:

$$V(r_t) = \ln\left(e^{r_t} + e^{c+\beta E[V(r_{t+1})|r_t]}\right)$$

   Part (a) relies on following results: Let s_{jt} equal the share of all potential consumers that buy j in t. Let s_t be the share of all potential consumers that buy in period t. That is, $\sum_j s_{jt} = s_t$ and $\sum_t s_t \leq 1$. Let s_{0t} be the share of all people who have not bought at the end of period t. That is, $s_{0t} = 1 - \sum_1^t s_t$. Let $W_t = c + \beta E[V(r_{t+1})|r_t]$, the value of waiting. We know that $s_{jt} = s_{0t-1}\exp(\delta_{jt})/(\exp(W_t) + \exp(r_t))$ and that $s_t = s_{0t-1}\exp(r_t)/(\exp(W_t) + \exp(r_t))$. Therefore, $s_{jt}/s_t = \exp(\delta_{jt})/\exp(r_t)$

   Melnikov proposes a three-step procedure to estimate demand.

(a) Estimate $\ln(s_{jt}/s_t) = \delta_{jt} - r_t$ where r_t is captured by a full set of time dummies (do not include a constant term in x_j). For this step, assume all variables are exogenous, including price. Provide a table of your results for estimates of coefficients on x_j and a brief discussion of the major results from this stage.

(b) Estimate the equation $r_{t+1} = \alpha_0 + \alpha_1 r_t + \nu_{t+1}$. Report your coefficients. Discretize the state space into 20 evenly spaced bins from -1 to 3. (See me if you find values of r_t that do not fit in this range.) Construct a transition matrix based on your estimates of α for how the state space evolves over time. Graph r_t. Does it seem that your estimates well capture this process?

A citation for discretizing a Markov process is Tauchen (1986). If you don't feel comfortable with discretizing the AR1, you may just use the empirical frequency for how the state evolves. That is, find the probability that r_t jumps 1 state or 2 states etc. and fill in the transition matrix that way. Either way, make sure that each row of your transition matrix sums to 1.

(c) For $c = 0$, construct the value function by solving the Bellman equation above via a fixed-point algorithm. Use a discount rate of $\beta = 0.99$. It is not obvious what to use for e^{r_t} because the state space is discretized. You may use the mid-point of each bin. Graph $V(r_t)$ as a function of r_t. Does $V(r_t)$ take on the shape you expected?

(d) Estimate c via Maximum Likelihood. Use the empirical likelihood function:

$$\max_c l(c) = \sum_{t=1}^{T+1} \ln(\hat{s}_t) s_t$$

where $\hat{s}_t$ is the prediction of the model, and s_{T+1} is the share of households that never purchase (so $s_{T+1} = s_{0T}$). For the potential market, use 100,000,000, which is about the number of households in the U.S. during this time period. Report your estimate of c. Graph actual market shares and predicted market shares over time. Comment on the results and possible shortcomings of the model.

2. This question is based on your work in 1 but does not require estimation.

(a) We have often discussed the flexibility introduced by random-coefficient logit models. Suppose you were to make the coefficient on price heterogeneous across consumers. How would that complicate the estimation procedure above? Could we still use a multi-step method as described in Melnikov (2013)? Would it be different to allow c to be a random coefficient?

(b) Implicitly, what are the assumptions about how Christmas is dealt with in what you did above? What set of assumptions about consumers would rationalize the model? Do you observe spikes at Christmas in your predicted markets shares? Why or why not? What might you do to the model to address the sales changes during the Christmas season? Would this impact the dynamic programming problem?

3. Read Crawford & Shum (2005).

    (a) What is the point of this paper?

    (b) Structural dynamic models typically make an assumption that Rust called *Conditional Independence*. Describe this assumption in words. What unobservable terms in this paper conform to this assumption? Which do not?

    (c) With regard to the unobservable variables not subject to CI, why cannot Crawford & Shum integrate these out analytically? Describe in words Crawford & Shum's technique for estimating their model despite the difficulty posed by these variables.

    (d) Compare and contrast Crawford & Shum's approach to handling unobserved state variables not subject to CI to that of Fox, Kim, Ryan & Bajari (2011). Are there research questions or circumstances that would make you prefer one over the other?

    (e) Crawford & Shum model quality in two dimensions. What are the two dimensions? Do you think they could find the result that patients optimally switch drugs during the treatment (section 6) without this feature?

4. Market power and production functions.

    (a) In De Loecker & Warzynski (2012), why is it critical that at least one input is completely adjustable each period?

    (b) Ackerberg, Caves & Frazer (2015) argue that the coefficients on adjustable inputs are not well identified in control function approaches to the production function estimation, such as Olley & Pakes (1996) and Levinsohn & Petrin (2003). What is the reason?

    (c) De Loecker & Warzynski (2012) apply a control function technique to production function estimation. How do they handle the tension between your previous two answers?

    (d) Suppose a firm faces demand $P = Y^{-\alpha}$ and marginal cost c. Show the firm sets: $cY/PY = 1 - \alpha$. If you saw a paper estimate the elasticity with the ratio of variable cost to revenue, how might you criticize it?

    (e) In the productivity literature, it is common to make use of the equation: $(1 - \alpha)\beta_l = wL/PY$, where β_l is the output elasticity of labor. Does your criticism in the previous question apply to this approach?

References

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