UNIVERSITÀ DEGLI STUDI DI GENOVA

DIBRIS

DEPARTMENT OF COMPUTER SCIENCE AND TECHNOLOGY,
BIOENGINEERING, ROBOTICS AND SYSTEM ENGINEERING

MODELLING AND CONTROL OF MANIPULATORS

Second Assignment

Manipulator Geometry and Direct Kinematics

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Mathematical expression	Definition	MATLAB expression
$< w >$	World Coordinate Frame	w
${}^a_bR$	Rotation matrix of frame $< b >$ with respect to frame $< a >$	aRb
${}^a_bT$	Transformation matrix of frame $< b >$ with respect to frame $< a >$	aTb

Table 1: Nomenclature Table

1 Assignment description

The second assignment of Modelling and Control of Manipulators focuses on manipulators' geometry and direct kinematics.

The second assignment is **mandatory** and consists of one exercise. You are asked to:

- Download the .zip file called MOCOM-LAB2 from the Aulaweb page of this course.
- Implement the code to solve the exercises on MATLAB by filling the predefined files called "*main.m*", "*BuildTree.m*", "*GetDirectGeometry.m*", "*DirectGeometry.m*", "*GetTransformationWrtBase.m*", "*GetBasicVectorWrtBase.m*" and "*GetFrameWrtFrame.m*".
- Write a report motivating your answers, following the predefind format on this document.

1.1 Exercise 1

Given the following CAD model of an industrial 7 dof manipulator:

Q1.1 Define all the model matrices, by filling the structures in the *BuildTree()* function. Be careful to define the z-axis coinciding with the joint rotation axis, and such that the positive rotation is the same as showed in the CAD model you received. Draw on the CAD model the reference frames for each link and insert it into the report.

Q1.2 Implement a function called *DirectGeometry()* which can calculate how the matrix attached to a joint will rotate if the joint rotates. Then, develop a function called GetDirectGeometry() which returns all the model matrices given the following joint configuration $\mathbf{q} = [1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3]$.

Q1.3 Calculate all the transformation matrices between any two links, between a link and the base and the corresponding distance vectors, filling respectively: *GetFrameWrtFrame()*, *GetTransformationWrtBase()*, *GetBasicVectgorWrtBase()*

Q1.4 Given the following starting and ending configuration:

- $\mathbf{q}_i = [1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3]$ and $\mathbf{q}_f = [2, 2, 2, 2, 2, 2, 2]$
- $\mathbf{q}_i = [1.3, 0, 1.3, 1.7, 1.3, 0.8, 1.3]$ and $\mathbf{q}_f = [2, 0, 1.3, 1.7, 1.3, 0.8, 1.3]$
- $\mathbf{q}_i = [1.3, 0.1, 0.1, 1, 0.2, 0.3, 1.3]$ and $\mathbf{q}_f = [2, 2, 2, 2, 2, 2, 2]$

Plot the intermediate link positions in between the two configurations (you can use the plot3() or line() functions) and comment the results obtained.

Q1.5 Test your algorithm by changing one joint position at the time and plot the results obtained for at least 3 configurations.

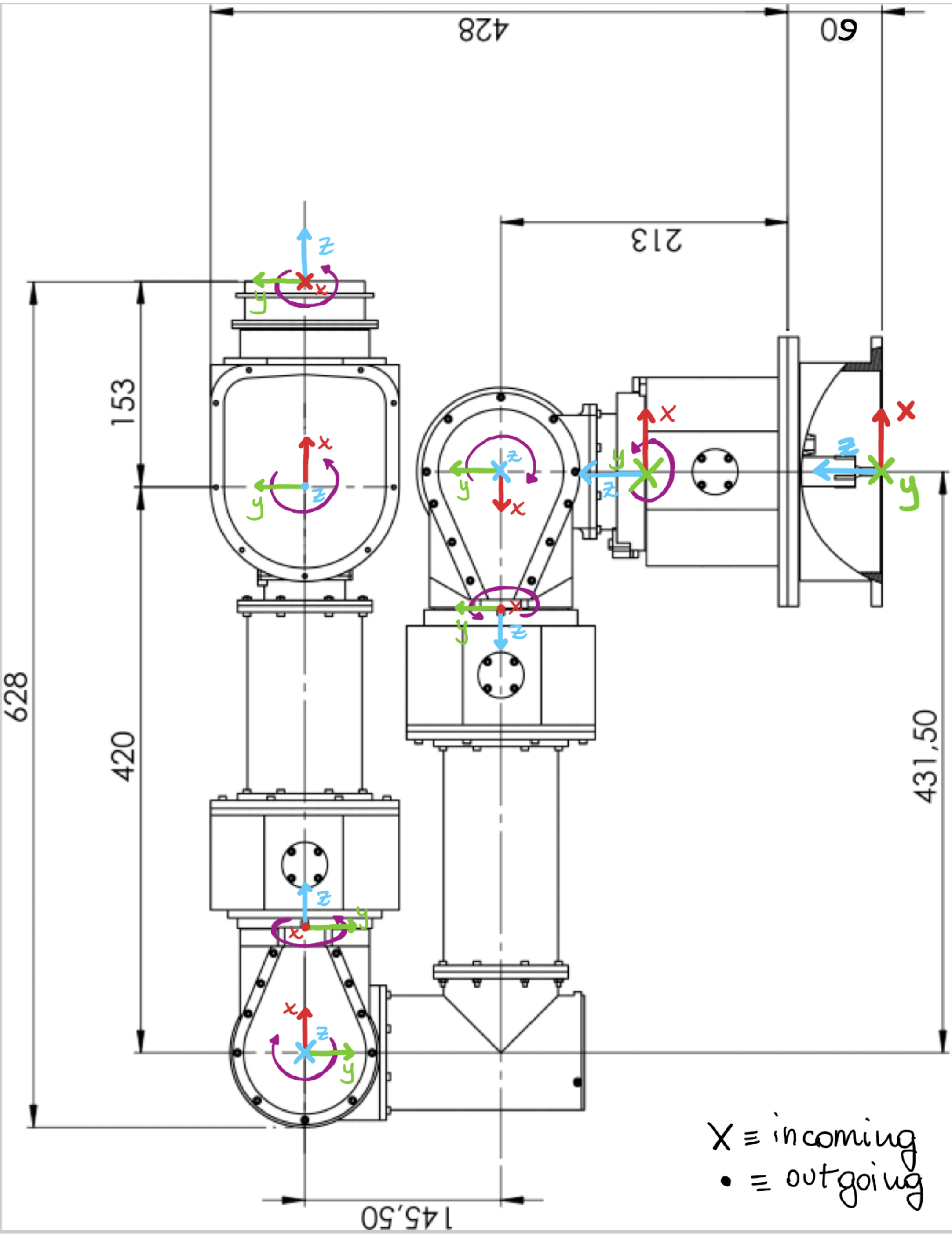


Figure 1: CAD model of the robot

2 Exercise 1

In this assignment, we worked on manipulators' geometry and direct kinematics problem. Forward kinematics refers to using the kinematic equations of a robot to compute the position of the end-effector from specified values for the joint parameters. For the exercise, a CAD model that represents a kinematic chain is given. A kinematic chain is a sequence of constrained bodies connected throw joints. Each body is defined as a link and a joint allows two bodies to move one with respect to the other A kinematic chain can be:

- Serial: each one of the bodies is connected to a single other body.
- Open: the end-effector is not connected to the base. This means that we can always specify as we want the joint variables, that define the position and the orientation of each body.
- Closed: the end-effector is connected to the base. This means that the joint variables are defined.
- Branched: set of interconnected bodies and, at some point, one body could be connected to more than one body.
- Floating base: the constraint to the ground is not present. It can be either branched or not.

The one that is given is an open kinematic chain with 7 dof and the joints are all revolute joints (RJ).

2.1 Q1.1 - Solution

In the CAD model of the robot are defined the x-y-z axis of the reference frame $< 0 >$ and the direction of the rotation of each joint. From that, using the rule of the right hand, it is easy to define the x-y-z axis of each joint. The result is shown in Figure 1. Note that the "X" symbol stands for incoming direction and "•" stands for outgoing direction. After defining the axis of each joint, I fill the "*BuildTree()*" function. This function builds the tree of frames for the manipulator given by returning the transformation matrix between the i-th and (i+1)-th frames.
The general structure is:

Compressed: $^{i}_{i+1}T = \begin{bmatrix} ^{i}_{i+1}R & ^{i}_{j}r_{i+1/i} \\ 0_{1x3} & 1 \end{bmatrix}$

Expanded: $^{i}_{i+1}T = \begin{bmatrix} ^{i}_{i+1}R(1,1) & ^{i}_{i+1}R(1,2) & ^{i}_{i+1}R(1,3) & r_x \\ ^{i}_{i+1}R(2,1) & ^{i}_{i+1}R(2,2) & ^{i}_{i+1}R(2,3) & r_y \\ ^{i}_{i+1}R(3,1) & ^{i}_{i+1}R(3,2) & ^{i}_{i+1}R(3,3) & r_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Where R is the rotation matrix from frame $< i >$ to frame $< i+1 >$, that represents how the x-y-z axis change w.r.t. the previous configuration. And $r = (r_x, r_y, r_z)$ is the vector representing the coordinates of the (i+1)-th joint w.r.t. the previous one.

The resulting matrices of the geometric model are:

$$^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{1}_{2}T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 98 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{2}_{3}T = \begin{bmatrix} 0 & 0 & 1 & 105 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}_{4}T = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 145.5 \\ -1 & 0 & 0 & 326.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{4}_{5}T = \begin{bmatrix} 0 & 0 & 1 & 35 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{5}_{6}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 385 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ^{6}_{7}T = \begin{bmatrix} 0 & 0 & 1 & 153 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.2 Q1.2 - Solution

After computing the geometry model, returning from the "*BuildTree.m*" function, I used it as input in the function "*ComputeAngleAxis.m*" with also the joint configuration given **q** $= [1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3]$, the vector of the joint type and the total number of links.
The "*GetDirectGeometry.m*" returns a 3D matrix containing how the transformation matrices attached to the joint will rotate if the joint rotates, given the joint configuration q_i, the transformation matrix and the joint type.

These parameters are the inputs of the "*DirectGeometry.m*" function, the computation of the transformation matrix depends on the joint type:

- Rotational joint: the first 3x3 matrix of the geometric model is multiplied by the rotation matrix associated with the rotation of q_i about the z-axis.

$$R_{zq} = \begin{bmatrix} cos(q_i) & -sin(q_i) & 0 \\ sin(q_i) & cos(q_i) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Prismatic joint: at the last column of the geometric model is added the vector r_q associated with the translation along the z-axis of q_i.

$$r_{zq} = \begin{bmatrix} 0 \\ 0 \\ q_i \end{bmatrix}$$

These formulas represent the expression, in polar coordinates, of the rigid body kinematics. They describe the rotation or the translation given the geometric model (so the configuration w.r.t. the previous configuration), the joint type $(i+1) = RJ \; or \; TJ$, and the rotation/translation q_{i+1}.

2.3 Q1.3 - Solution

From the transformation matrices given if the joint rotates, I can compute the transformation w.r.t. the base and fill the function *GetTransformationWrtBase()*).

Since we have a kinematic chain where the i-th link is connected to the (i+1)-th link, to compute the transformation matrix between the i-th link and the base is just necessary to multiply the transformation matrix of the i-th link with the one of the previous one until you get to the base.
The generic formula for "i" joint is:

$$^0_iT =^0_1 T *^1_2 T * ... *^{i-1}_i T$$

Using this formula for each link (from 1 to 7 in our case), it is easy to compute the transformation matrix w.r.t. the base of every link (*GetTransformationWrtBase()*), and also the corresponding distance vector by extracting from the last column of ^0_iT the first three rows (*GetBasicVectgorWrtBase()*.

From the matrix ^0_iT, it is easy to fill the function *GetFrameWrtFrame()* and calculate the transformation matrix between any two links ("i" and "j"), knowing that:

$$^i_jT =^i_0 T *^0_j T$$

where ^i_0T is just the inverse of ^0_iT (given from the function *GetTransformationWrtBase()*).

2.4 Q1.4 - Solution

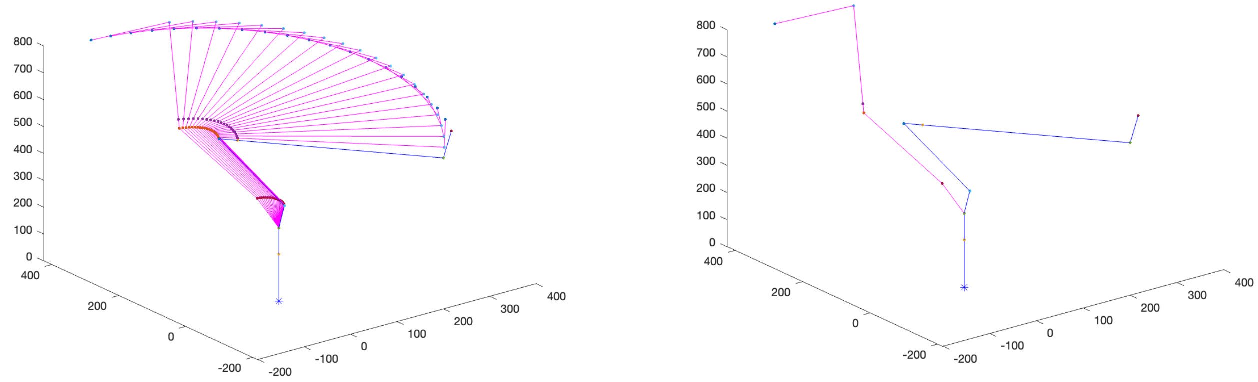
Given the following starting and ending configuration:

- $\mathbf{q}_i = [1.3, 1.3, 1.3, 1.3, 1.3, 1.3, 1.3]$ and $\mathbf{q}_f = [2, 2, 2, 2, 2, 2, 2]$
- $\mathbf{q}_i = [1.3, 0, 1.3, 1.7, 1.3, 0.8, 1.3]$ and $\mathbf{q}_f = [2, 0, 1.3, 1.7, 1.3, 0.8, 1.3]$
- $\mathbf{q}_i = [1.3, 0.1, 0.1, 1, 0.2, 0.3, 1.3]$ and $\mathbf{q}_f = [2, 2, 2, 2, 2, 2, 2]$

To plot the intermediate link position between the two configurations, I divided the difference between the initial and final by the number of steps that I want to get the value of each step. At each iteration, the input q_i of the function "*GetDirectGeometry.m*" is incremented by the value of the step, and then the transformation matrix w.r.t. the base and the corresponding basic vector are computed using the corresponding functions explained before.

To plot the joint I used the Matlab function "plot3()" and to plot the link I used the Matlab function "line()".

The resulting graphs are shown in Figure 2, 3 and 4. The base is indicated with an " * ", the number of steps is equal to 20, and the initial configuration is coloured in blue.
The transformation matrix of the initial and final configuration are reported after the relative figure.



(a) First configuration step
Blue: initial configuration - Pink: step configurations

(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 2: Graphs Exercise 1.4 - First configuration

Initial configuration transformation matrices w.r.t. base:

$$ {}^0_1T = \begin{bmatrix} 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} -0.0716 & 0.2578 & -0.9636 & 0 \\ -0.2578 & 0.9284 & 0.2675 & 0 \\ 0.9636 & 0.2675 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} 0.5061 & -0.8595 & -0.0716 & -7.5133 \\ 0.8231 & 0.5061 & -0.2578 & -27.0638 \\ 0.2578 & 0.0716 & 0.9636 & 374.1736 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_4T = \begin{bmatrix} 0.8473 & 0.1610 & -0.5061 & -155.9330 \\ -0.4187 & -0.3837 & -0.8231 & -37.5806 \\ -0.3267 & 0.9093 & -0.2578 & 699.1867 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.2905 & -0.4446 & 0.8473 & -126.2769 \\ -0.1496 & -0.8957 & -0.4187 & -52.2357 \\ 0.9451 & -0.0051 & -0.3267 & 687.7522 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} 0.6551 & -0.6975 & 0.2905 & 199.9397 \\ 0.7511 & 0.6431 & -0.1496 & -213.4418 \\ -0.0825 & 0.3162 & 0.9451 & 561.9732 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_7T = \begin{bmatrix} -0.7498 & 0.0933 & 0.6551 & 300.1639 \\ 0.6596 & 0.0279 & 0.7511 & -98.5290 \\ 0.0518 & 0.9952 & -0.0825 & 549.3572 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

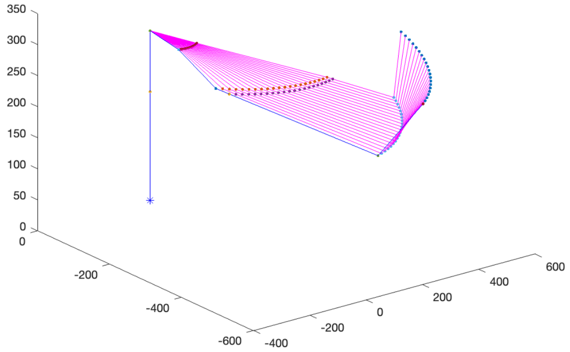
Final configuration transformation matrices w.r.t. base:

$$ {}^0_1T = \begin{bmatrix} -0.4161 & -0.9093 & 0 & 0 \\ 0.9093 & -0.4161 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} -0.1732 & -0.3784 & -0.9093 & 0 \\ 0.3784 & 0.8268 & -0.4161 & 0 \\ 0.9093 & -0.4161 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

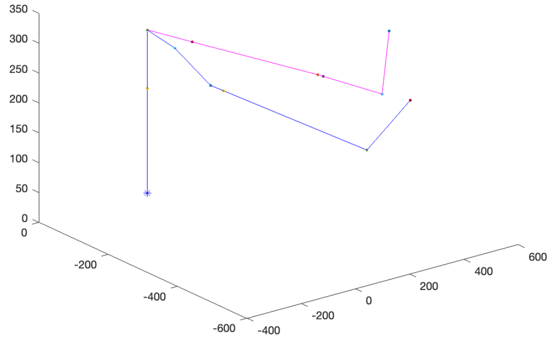
$$ {}^0_3T = \begin{bmatrix} -0.7225 & -0.6694 & -0.1732 & -18.1837 \\ 0.5786 & -0.7225 & 0.3784 & 39.7321 \\ -0.3784 & 0.1732 & 0.9093 & 368.4762 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_4T = \begin{bmatrix} 0.5366 & -0.4360 & 0.7225 & -172.1170 \\ 0.8144 & 0.0434 & -0.5786 & 58.1592 \\ 0.2209 & 0.8989 & 0.3784 & 690.5593 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} -0.0958 & 0.8384 & 0.5366 & -153.3370 \\ -0.2013 & -0.5442 & 0.8144 & 86.6639 \\ 0.9748 & -0.0300 & 0.2209 & 698.2918 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} -0.9856 & -0.1390 & -0.0958 & 53.2432 \\ 0.1560 & -0.9670 & -0.2013 & 400.2157 \\ -0.0647 & -0.2134 & 0.9748 & 783.3502 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_7T = \begin{bmatrix} -0.1663 & -0.0293 & -0.9856 & -97.5606 \\ -0.9631 & 0.2194 & 0.1560 & 424.0763 \\ 0.2117 & 0.9752 & -0.0647 & 773.4558 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$



(a) Second configuration step
Blue: initial configuration - Pink: step configurations



(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 3: Graphs Exercise 1.4 - Second configuration

Initial configuration transformation matrices w.r.t. base:

$$ {}^0_1T = \begin{bmatrix} 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} -0.2675 & 0 & -0.9636 & 0 \\ -0.9636 & 0 & 0.2675 & 0 \\ 0 & 1.0000 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} 0.2578 & -0.9284 & -0.2675 & -28.0874 \\ -0.0716 & 0.2578 & -0.9636 & -101.1736 \\ 0.9636 & 0.2675 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_4T = \begin{bmatrix} 0.8862 & -0.3849 & -0.2578 & -250.5144 \\ -0.3798 & -0.9223 & 0.0716 & -378.2726 \\ -0.2653 & 0.0345 & -0.9636 & 311.9211 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} -0.3019 & -0.3513 & 0.8862 & -219.4960 \\ -0.9078 & -0.1778 & -0.3798 & -391.5639 \\ 0.2910 & -0.9192 & -0.2653 & 302.6367 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} 0.8695 & -0.3910 & -0.3019 & 121.7063 \\ -0.1371 & 0.3963 & -0.9078 & -537.7682 \\ 0.4746 & 0.8307 & 0.2910 & 200.5080 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_7T = \begin{bmatrix} -0.2960 & -0.3955 & 0.8695 & 254.7350 \\ 0.6247 & -0.7688 & -0.1371 & -558.7370 \\ 0.7226 & 0.5026 & 0.4746 & 273.1213 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

Final configuration transformation matrices w.r.t. base:

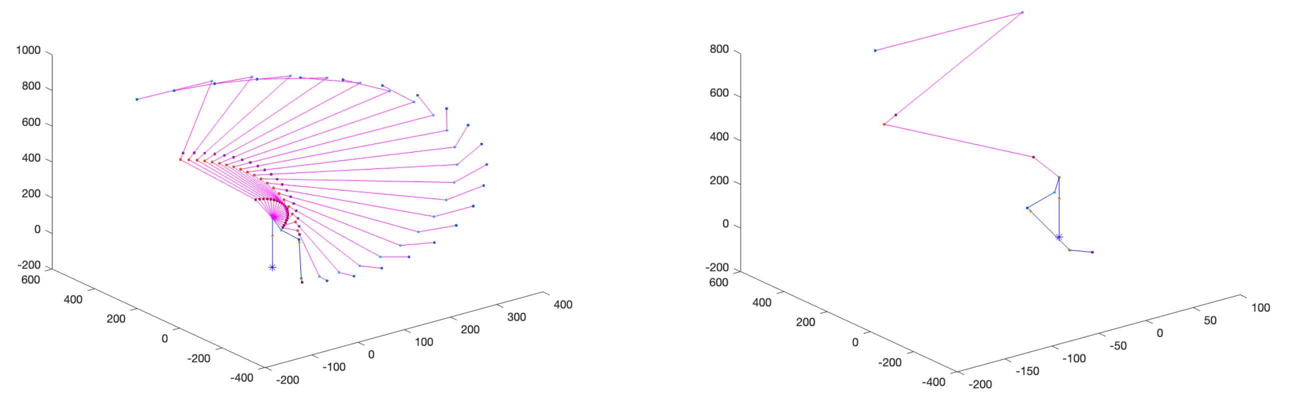
$$ {}^0_1T = \begin{bmatrix} -0.4161 & -0.9093 & 0 & 0 \\ 0.9093 & -0.4161 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} 0.4161 & 0 & -0.9093 & 0 \\ -0.9093 & 0 & -0.4161 & 0 \\ 0 & 1.0000 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} 0.2432 & -0.8762 & 0.4161 & 43.6954 \\ 0.1113 & -0.4010 & -0.9093 & -95.4762 \\ 0.9636 & 0.2675 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_4T = \begin{bmatrix} 0.9225 & 0.2998 & -0.2432 & 52.0859 \\ 0.2805 & -0.9534 & -0.1113 & -450.7047 \\ -0.2653 & 0.0345 & -0.9636 & 311.9211 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.3539 & -0.1542 & 0.9225 & 84.3726 \\ -0.8889 & -0.3623 & 0.2805 & -440.8878 \\ 0.2910 & -0.9192 & -0.2653 & 302.6367 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} 0.7533 & -0.5543 & 0.3539 & 439.5260 \\ 0.4553 & 0.0512 & -0.8889 & -332.9025 \\ 0.4746 & 0.8307 & 0.2910 & 200.5080 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_7T = \begin{bmatrix} -0.6288 & 0.1928 & 0.7533 & 554.7803 \\ 0.2871 & -0.8428 & 0.4553 & -263.2408 \\ 0.7226 & 0.5026 & 0.4746 & 273.1213 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$



(a) Third configuration step
Blue: initial configuration - Pink: step configurations

(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 4: Graphs Exercise 1.4 - Third configuration

Initial configuration transformation matrices w.r.t. base:

$$ {}^0_5T = \begin{bmatrix} 0.2675 & -0.9636 & 0 & 0 \\ 0.9636 & 0.2675 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_5T = \begin{bmatrix} -0.2662 & 0.0267 & -0.9636 & 0 \\ -0.9587 & 0.0962 & 0.2675 & 0 \\ 0.0998 & 0.9950 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.9614 & -0.0696 & -0.2662 & -27.9471 \\ -0.2566 & 0.1224 & -0.9587 & -100.6682 \\ 0.0993 & 0.9900 & 0.0998 & 283.4825 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_5T = \begin{bmatrix} 0.2024 & -0.1864 & -0.9614 & -124.9793 \\ 0.4150 & -0.8729 & 0.2566 & -395.8861 \\ -0.8870 & -0.4509 & -0.0993 & 460.1280 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.9052 & -0.3736 & 0.2024 & -117.8955 \\ -0.4249 & -0.8045 & 0.4150 & -381.3611 \\ 0.0078 & -0.4617 & -0.8870 & 429.0821 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_5T = \begin{bmatrix} 0.3038 & 0.2971 & 0.9052 & -39.9737 \\ 0.6342 & 0.6460 & -0.4249 & -221.5866 \\ -0.7110 & 0.7032 & 0.0078 & 87.5777 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.0442 & 0.9517 & 0.3038 & 6.5035 \\ 0.7361 & -0.2366 & 0.6342 & -124.5512 \\ 0.6755 & 0.1956 & -0.7110 & -21.2020 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

Final configuration transformation matrices w.r.t. base:

$$ {}^0_5T = \begin{bmatrix} -0.4161 & -0.9093 & 0 & 0 \\ 0.9093 & -0.4161 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_5T = \begin{bmatrix} -0.1732 & -0.3784 & -0.9093 & 0 \\ 0.3784 & 0.8268 & -0.4161 & 0 \\ 0.9093 & -0.4161 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} -0.7225 & -0.6694 & -0.1732 & -18.1837 \\ 0.5786 & -0.7225 & 0.3784 & 39.7321 \\ -0.3784 & 0.1732 & 0.9093 & 368.4762 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_5T = \begin{bmatrix} 0.5366 & -0.4360 & 0.7225 & -172.1170 \\ 0.8144 & 0.0434 & -0.5786 & 58.1592 \\ 0.2209 & 0.8989 & 0.3784 & 690.5593 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} -0.0958 & 0.8384 & 0.5366 & -153.3370 \\ -0.2013 & -0.5442 & 0.8144 & 86.6639 \\ 0.9748 & -0.0300 & 0.2209 & 698.2918 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_5T = \begin{bmatrix} -0.9856 & -0.1390 & -0.0958 & 53.2432 \\ 0.1560 & -0.9670 & -0.2013 & 400.2157 \\ -0.0647 & -0.2134 & 0.9748 & 783.3502 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} -0.1663 & -0.0293 & -0.9856 & -97.5606 \\ -0.9631 & 0.2194 & 0.1560 & 424.0763 \\ 0.2117 & 0.9752 & -0.0647 & 773.4558 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

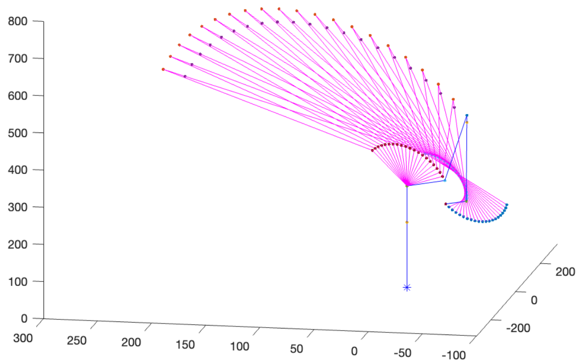
As shown in the graphs and matrices above, in the first case all the joint configurations change, in the second one only the first link rotates, and in the third one, all the joints rotate but in a different way w.r.t. the first one.

2.5 Q1.5 - Solution

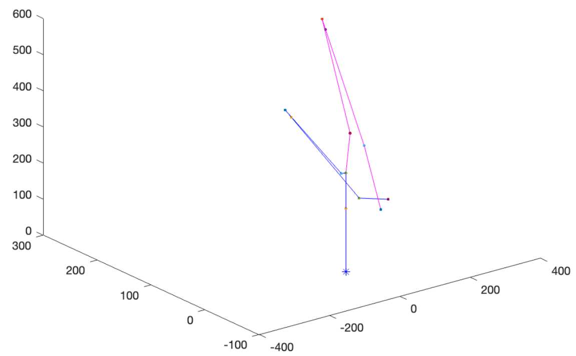
To test the algorithm implemented for exercise 1.4, I choose the following initial and final configurations:

- $\mathbf{q}_i = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]$ and $\mathbf{q}_f = [0.5, 2.5, 0.5, 0.5, 0.5, 0.5, 0.5]$
- $\mathbf{q}_i = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]$ and $\mathbf{q}_f = [0.5, 0.5, 0.5, 1.8, 0.5, 0.5, 0.5]$
- $\mathbf{q}_i = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]$ and $\mathbf{q}_f = [0.5, 0.5, 0.5, 0.5, 0.5, 3.2, 0.5]$

The resulting graphs are shown in Figure 5, 6 and 7, using the same method explained for exercise 1.4. The base is indicated with an " * ", the number of steps is equal to 20, and the initial configuration is coloured in blue. The transformation matrix of the initial and final configurations are reported after the relative figure.



(a) First configuration step
Blue: initial configuration - Pink: step configurations



(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 5: Graphs Exercise 1.5 - First configuration

Initial configuration transformation matrices w.r.t. base:

$$^0_1T = \begin{bmatrix} 0.8776 & -0.4794 & 0 & 0 \\ 0.4794 & 0.8776 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad ^0_2T = \begin{bmatrix} -0.7702 & 0.4207 & -0.4794 & 0 \\ -0.4207 & 0.2298 & 0.8776 & 0 \\ 0.4794 & 0.8776 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$^0_3T = \begin{bmatrix} 0.6224 & 0.1394 & -0.7702 & -80.8659 \\ -0.6600 & 0.6224 & -0.4207 & -44.1772 \\ 0.4207 & 0.7702 & 0.4794 & 323.3397 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad ^0_4T = \begin{bmatrix} 0.6090 & -0.4915 & -0.6224 & -312.0402 \\ 0.0708 & -0.7480 & 0.6600 & -90.9814 \\ -0.7900 & -0.4460 & -0.4207 & 591.9291 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$^0_5T = \begin{bmatrix} 0.3106 & -0.7298 & 0.6090 & -290.7236 \\ -0.9378 & -0.3400 & 0.0708 & -88.5029 \\ 0.1554 & -0.5931 & -0.7900 & 564.2803 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad ^0_6T = \begin{bmatrix} 0.8844 & 0.3485 & 0.3106 & -56.2400 \\ 0.2251 & 0.2644 & -0.9378 & -61.2398 \\ -0.4089 & 0.8993 & 0.1554 & 260.1436 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$^0_5T = \begin{bmatrix} -0.1055 & 0.4547 & 0.8844 & 79.0688 \\ 0.9497 & -0.2175 & 0.2251 & -26.7922 \\ 0.2948 & 0.8637 & -0.4089 & 197.5823 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

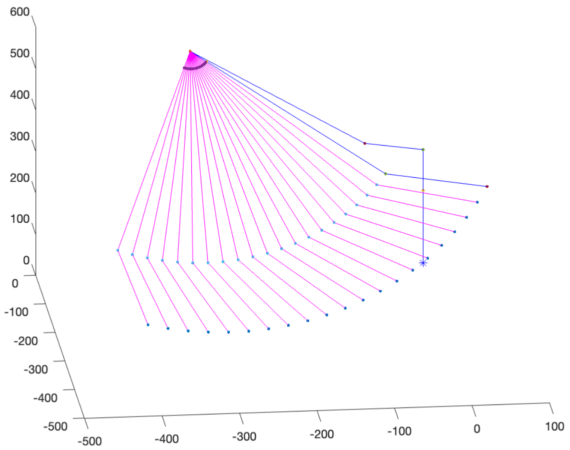
Final configuration transformation matrices w.r.t. base:

$$^0_1T = \begin{bmatrix} 0.8776 & -0.4794 & 0 & 0 \\ 0.4794 & 0.8776 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad ^0_2T = \begin{bmatrix} 0.7031 & 0.5252 & -0.4794 & 0 \\ 0.3841 & 0.2869 & 0.8776 & 0 \\ 0.5985 & -0.8011 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

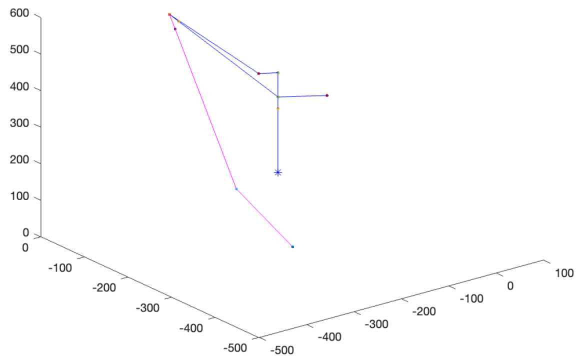
$$^0_3T = \begin{bmatrix} 0.6725 & 0.2311 & 0.7031 & 73.8223 \\ -0.6326 & 0.6725 & 0.3841 & 40.3293 \\ -0.3841 & -0.7031 & 0.5985 & 335.8396 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad ^0_4T = \begin{bmatrix} -0.7278 & 0.1343 & -0.6725 & 336.9945 \\ -0.6595 & -0.4061 & 0.6326 & 263.5880 \\ -0.1881 & 0.9039 & 0.3841 & 428.9441 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}$$

$$ {}^0_5T = \begin{bmatrix} 0.6546 & -0.2046 & -0.7278 & 311.5222 \\ -0.7498 & -0.0531 & -0.6595 & 240.5055 \\ 0.0963 & 0.9774 & -0.1881 & 422.3592 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} -0.5406 & 0.5285 & 0.6546 & 31.3269 \\ -0.5533 & 0.3628 & -0.7498 & -13.4018 \\ -0.6337 & -0.7676 & 0.0963 & 349.9256 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_7T = \begin{bmatrix} -0.3211 & 0.7776 & -0.5406 & -51.3860 \\ 0.8320 & -0.0411 & -0.5533 & -98.0600 \\ -0.4525 & -0.6274 & -0.6337 & 252.9691 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$



(a) Second configuration step
Blue: initial configuration - Pink: step configurations



(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 6: Graphs Exercise 1.5 - Second configuration

Initial configuration transformation matrices w.r.t. base:

$$ {}^0_1T = \begin{bmatrix} 0.8776 & -0.4794 & 0 & 0 \\ 0.4794 & 0.8776 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} -0.7702 & 0.4207 & -0.4794 & 0 \\ -0.4207 & 0.2298 & 0.8776 & 0 \\ 0.4794 & 0.8776 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_3T = \begin{bmatrix} 0.6224 & 0.1394 & -0.7702 & -80.8659 \\ -0.6600 & 0.6224 & -0.4207 & -44.1772 \\ 0.4207 & 0.7702 & 0.4794 & 323.3397 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_4T = \begin{bmatrix} 0.6090 & -0.4915 & -0.6224 & -312.0402 \\ 0.0708 & -0.7480 & 0.6600 & -90.9814 \\ -0.7900 & -0.4460 & -0.4207 & 591.9291 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.3106 & -0.7298 & 0.6090 & -290.7236 \\ -0.9378 & -0.3400 & 0.0708 & -88.5029 \\ 0.1554 & -0.5931 & -0.7900 & 564.2803 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} 0.8844 & 0.3485 & 0.3106 & -56.2400 \\ 0.2251 & 0.2644 & -0.9378 & -61.2398 \\ -0.4089 & 0.8993 & 0.1554 & 260.1436 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} -0.1055 & 0.4547 & 0.8844 & 79.0688 \\ 0.9497 & -0.2175 & 0.2251 & -26.7922 \\ 0.2948 & 0.8637 & -0.4089 & 197.5823 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

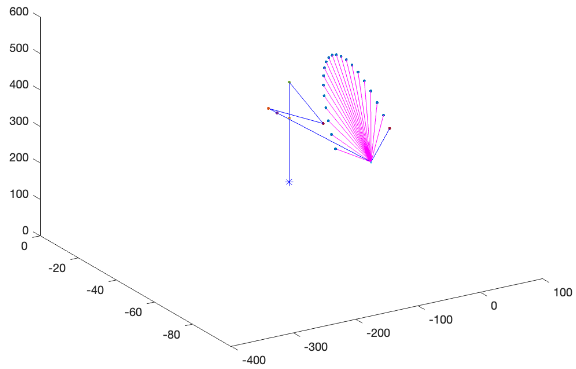
Final configuration transformation matrices w.r.t. base:

$$ {}^0_1T = \begin{bmatrix} 0.8776 & -0.4794 & 0 & 0 \\ 0.4794 & 0.8776 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_2T = \begin{bmatrix} -0.7702 & 0.4207 & -0.4794 & 0 \\ -0.4207 & 0.2298 & 0.8776 & 0 \\ 0.4794 & 0.8776 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

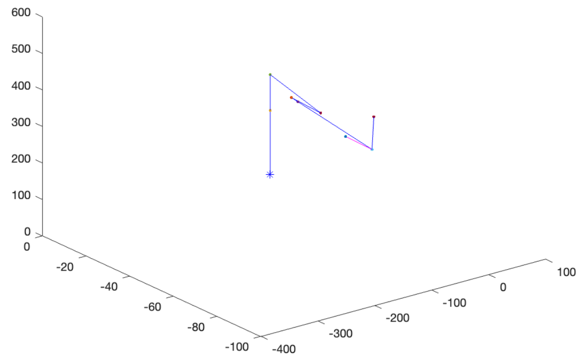
$$ {}^0_3T = \begin{bmatrix} 0.6224 & 0.1394 & -0.7702 & -80.8659 \\ -0.6600 & 0.6224 & -0.4207 & -44.1772 \\ 0.4207 & 0.7702 & 0.4794 & 323.3397 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_4T = \begin{bmatrix} -0.3107 & -0.7183 & -0.6224 & -312.0402 \\ -0.7018 & -0.2683 & 0.6600 & -90.9814 \\ -0.6411 & 0.6419 & -0.4207 & 591.9291 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$ {}^0_5T = \begin{bmatrix} 0.2019 & -0.9288 & -0.3107 & -322.9153 \\ -0.7078 & 0.0809 & -0.7018 & -115.5430 \\ 0.6770 & 0.3616 & -0.6411 & 569.4912 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad {}^0_6T = \begin{bmatrix} 0.1726 & 0.9641 & 0.2019 & -442.5410 \\ -0.6547 & 0.2654 & -0.7078 & -385.7207 \\ -0.7360 & -0.0100 & 0.6770 & 322.6741 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} $$

$$
{}^0_7T = \begin{bmatrix} 0.2851 & 0.9428 & 0.1726 & -416.1299 \\ 0.7484 & -0.1064 & -0.6547 & -485.8829 \\ -0.5989 & 0.3158 & -0.7360 & 210.0731 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$



(a) Third configuration step
Blue: initial configuration - Pink: step configurations



(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 7: Graphs Exercise 1.5 - Third configuration

Initial configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 0.8776 & -0.4794 & 0 & 0 \\ 0.4794 & 0.8776 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad
{}^0_2T = \begin{bmatrix} -0.7702 & 0.4207 & -0.4794 & 0 \\ -0.4207 & 0.2298 & 0.8776 & 0 \\ 0.4794 & 0.8776 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_3T = \begin{bmatrix} 0.6224 & 0.1394 & -0.7702 & -80.8659 \\ -0.6600 & 0.6224 & -0.4207 & -44.1772 \\ 0.4207 & 0.7702 & 0.4794 & 323.3397 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad
{}^0_4T = \begin{bmatrix} 0.6090 & -0.4915 & -0.6224 & -312.0402 \\ 0.0708 & -0.7480 & 0.6600 & -90.9814 \\ -0.7900 & -0.4460 & -0.4207 & 591.9291 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_5T = \begin{bmatrix} 0.3106 & -0.7298 & 0.6090 & -290.7236 \\ -0.9378 & -0.3400 & 0.0708 & -88.5029 \\ 0.1554 & -0.5931 & -0.7900 & 564.2803 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad
{}^0_6T = \begin{bmatrix} 0.8844 & 0.3485 & 0.3106 & -56.2400 \\ 0.2251 & 0.2644 & -0.9378 & -61.2398 \\ -0.4089 & 0.8993 & 0.1554 & 260.1436 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_5T = \begin{bmatrix} -0.1055 & 0.4547 & 0.8844 & 79.0688 \\ 0.9497 & -0.2175 & 0.2251 & -26.7922 \\ 0.2948 & 0.8637 & -0.4089 & 197.5823 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

Final configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 0.8776 & -0.4794 & 0 & 0 \\ 0.4794 & 0.8776 & 0 & 0 \\ 0 & 0 & 1.0000 & 175.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad
{}^0_2T = \begin{bmatrix} -0.7702 & 0.4207 & -0.4794 & 0 \\ -0.4207 & 0.2298 & 0.8776 & 0 \\ 0.4794 & 0.8776 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_3T = \begin{bmatrix} 0.6224 & 0.1394 & -0.7702 & -80.8659 \\ -0.6600 & 0.6224 & -0.4207 & -44.1772 \\ 0.4207 & 0.7702 & 0.4794 & 323.3397 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad
{}^0_4T = \begin{bmatrix} 0.6090 & -0.4915 & -0.6224 & -312.0402 \\ 0.0708 & -0.7480 & 0.6600 & -90.9814 \\ -0.7900 & -0.4460 & -0.4207 & 591.9291 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_5T = \begin{bmatrix} 0.3106 & -0.7298 & 0.6090 & -290.7236 \\ -0.9378 & -0.3400 & 0.0708 & -88.5029 \\ 0.1554 & -0.5931 & -0.7900 & 564.2803 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad
{}^0_6T = \begin{bmatrix} -0.6506 & -0.6930 & 0.3106 & -56.2400 \\ -0.0905 & -0.3353 & -0.9378 & -61.2398 \\ 0.7540 & -0.6382 & 0.1554 & 260.1436 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

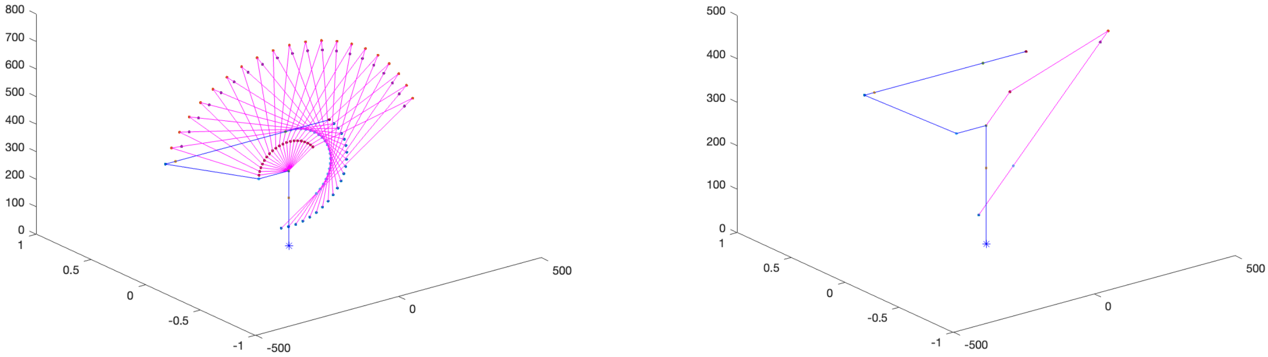
$$
{}^0_7T = \begin{bmatrix} -0.6048 & -0.4593 & -0.6506 & -155.7834 \\ 0.6622 & -0.7438 & -0.0905 & -75.0923 \\ -0.4424 & -0.4856 & 0.7540 & 375.5048 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

As shown in the graphs and matrices above, the initial configuration is the same (changing only the point of view of the graph) and in the first case only the second joint change, in the second one only the fourth link rotates, and in the third one, the sixth one rotates.
This confirms what I was expecting by choosing the configuration written before.

Other three configurations possible, where it is easier to see that the starting configuration is the same, are the following:

- $\mathbf{q}_i = [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0;]$ and $\mathbf{q}_f = [0.0, 2.5, 0.0, 0.0, 0.0, 0.0, 0.0]$
- $\mathbf{q}_i = [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0;]$ and $\mathbf{q}_f = [0.0, 0.0, 0.0, 1.8, 0.0, 0.0, 0.0;]$
- $\mathbf{q}_i = [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0;]$ and $\mathbf{q}_f = [0.0, 0.0, 0.0, 0.0, 0.0, 3.2, 0.0;]$

The resulting graphs are shown in Figure 8, 10 and **??**, using the same method explained for exercise 1.4. The base is indicated with an " * ", the number of steps is equal to 20, and the initial configuration is coloured in blue. The transformation matrix of the initial and final configurations are reported after the relative figure.



(a) Fourth configuration step
Blue: initial configuration - Pink: step configurations

(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 8: Graphs Exercise 1.5 - Fourth configuration

Initial configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1175 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_2T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_3T = \begin{bmatrix} 0 & 0 & -1 & -105 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_4T = \begin{bmatrix} 1 & 0 & 0 & -431.5 \\ 0 & 0 & 1.0000 & 0 \\ 0-1 & 0 & 418.5 & \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

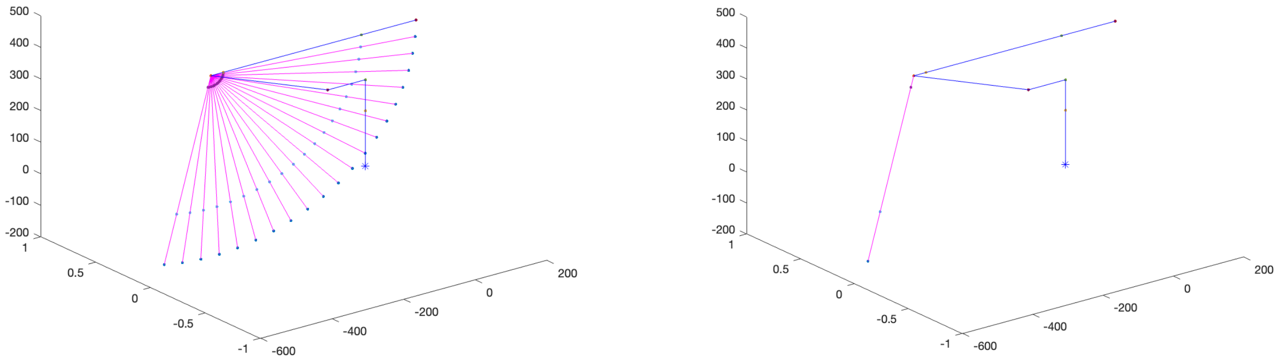
$$
{}^0_5T = \begin{bmatrix} 0 & 0 & 1 & -396.5 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_6T = \begin{bmatrix} 1 & 0 & 0 & -11.5 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_7T = \begin{bmatrix} 0 & 0 & 1 & 141.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Final configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_2T = \begin{bmatrix} 0.8011 & 0.5985 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 \\ 0.5985 & -0.8011 & 0 & 273.0000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad
{}^0_3T = \begin{bmatrix} 0 & 0.5985 & 0.8011 & 84.1201 \\ -1.0000 & 0 & 0 & 0 \\ 0 & -0.8011 & 0.5985 & 335.8396 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} -0.8011 & -0.5985 & 0 & 432.7712 \\ 0 & 0 & 1.0000 & 0 \\ -0.5985 & 0.8011 & 0 & 414.6743 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad
{}^0_5T = \begin{bmatrix} 0 & -0.5985 & -0.8011 & 404.7311 \\ -1.0000 & 0 & 0 & 0 \\ 0 & 0.8011 & -0.5985 & 393.7278 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_6T = \begin{bmatrix} -0.8011 & 0.5985 & 0 & 96.2908 \\ 0 & 0 & -1.0000 & 0 \\ -0.5985 & -0.8011 & 0 & 163.3160 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad
{}^0_7T = \begin{bmatrix} 0 & 0.5985 & -0.8011 & -26.2841 \\ 1.0000 & 0 & 0 & 0 \\ 0 & -0.8011 & -0.5985 & 71.7498 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$



(a) Fifth configuration step
Blue: initial configuration - Pink: step configurations

(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 9: Graphs Exercise 1.5 - Fifth configuration

Initial configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1175 & \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_2T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_3T = \begin{bmatrix} 0 & 0 & -1 & -105 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_4T = \begin{bmatrix} 1 & 0 & 0 & -431.5 \\ 0 & 0 & 1.0000 & 0 \\ 0-1 & 0 & 418.5 & \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

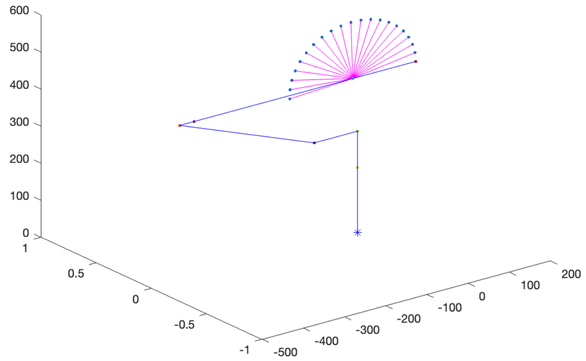
$$
{}^0_5T = \begin{bmatrix} 0 & 0 & 1 & -396.5 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_6T = \begin{bmatrix} 1 & 0 & 0 & -11.5 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_7T = \begin{bmatrix} 0 & 0 & 1 & 141.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Final configuration transformation matrices w.r.t. base:

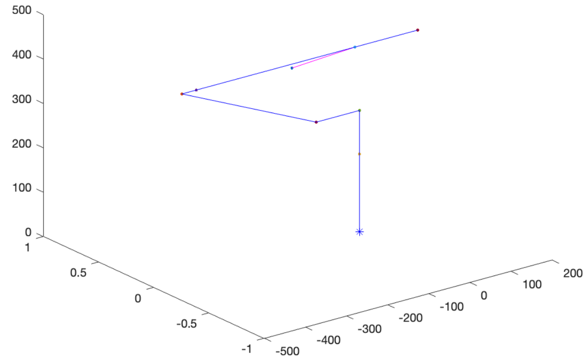
$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_2T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad
{}^0_3T = \begin{bmatrix} 0 & 0 & -1 & -105 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} -0.2272 & -0.9738 & 0 & -431.5000 \\ 0 & 0 & 1.0000 & 0 \\ -0.9738 & 0.2272 & 0 & 418.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad
{}^0_5T = \begin{bmatrix} 0 & -0.9738 & -0.2272 & -439.4521 \\ -1.0000 & 0 & 0 & 0 \\ 0 & 0.2272 & -0.9738 & 384.4153 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_6T = \begin{bmatrix} -0.2272 & 0.9738 & 0 & -526.9249 \\ 0 & 0 & -1.0000 & 0 \\ -0.9738 & -0.2272 & 0 & 9.4840 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix} \quad
{}^0_7T = \begin{bmatrix} 0 & 0.9738 & -0.2272 & -561.6868 \\ 1.0000 & 0 & 0 & 0 \\ 0 & -0.2272 & -0.9738-139.5147 & \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$



(a) Sixth configuration step
Blue: initial configuration - Pink: step configurations



(b) Initial and final configuration
Blue: initial configuration - Pink: final configuration

Figure 10: Graphs Exercise 1.5 - Sixth configuration

Initial configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1175 & \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_2T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_3T = \begin{bmatrix} 0 & 0 & -1 & -105 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_4T = \begin{bmatrix} 1 & 0 & 0 & -431.5 \\ 0 & 0 & 1.0000 & 0 \\ 0-1 & 0 & 418.5 & \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_5T = \begin{bmatrix} 0 & 0 & 1 & -396.5 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_6T = \begin{bmatrix} 1 & 0 & 0 & -11.5 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_7T = \begin{bmatrix} 0 & 0 & 1 & 141.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 418.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Final configuration transformation matrices w.r.t. base:

$$
{}^0_1T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 175 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_2T = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^0_3T = \begin{bmatrix} 0 & 0 & -1 & -105 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 273 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_4T = \begin{bmatrix} 1.0000 & 0 & 0 & -431.5000 \\ 0 & 0 & 1.0000 & 0 \\ 0 & -1.0000 & 0 & 418.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad {}^0_5T = \begin{bmatrix} 0 & 0 & 1.0000 & -396.5000 \\ -1.0000 & 0 & 0 & 0 \\ 0 & -1.0000 & 0 & 418.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

$$
{}^0_6T = \begin{bmatrix} -0.9983 & 0.0584 & 0 & -11.5000 \\ 0 & 0 & -1.0000 & 0 \\ -0.0584 & -0.9983 & 0 & 418.5000 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
\quad {}^0_7T = \begin{bmatrix} 0 & 0.0584 & -0.9983 & -164.2391 \\ 1.0000 & 0 & 0 & 0 \\ 0 & -0.9983 & -0.0584 & 409.5688 \\ 0 & 0 & 0 & 1.0000 \end{bmatrix}
$$

As shown in the graphs and matrices above, the initial configuration is the same (changing only the point of view of the graph) and in the first case only the second joint change, in the second one only the fourth link rotates, and in the third one, the sixth one rotates.
This confirms what I was expecting by choosing the configuration written before.

Two considerations can be made: first, in the last initial configuration chosen it is possible to see that the kinematic chain corresponds to the one in the CAD model given (confirming a correct implementation); second, it is not possible to show in the graph the rotation of the end-effector because the seventh joint is considered the end-effector but no links are connected after it so the joint rotates about itself and the effect is not visible on the graph.

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