Exercise Sheet 10

Exercise 1: Kernel Ridge Regression (10+10 P)

In the lecture, ridge regression was stated as the regularized quadratic program

$$\min_{w} \ \sum_{i=1}^{n}(w^{\top}x_i - y_i)^2 + \lambda\|w\|_2^2,$$

where $w \in \mathbb{R}^d$ is optimized and $x_1, \ldots, x_n \in \mathbb{R}^d$ are the data points, and $y_1, \ldots, y_n \in \mathbb{R}$ are the labels.

(a) Following the strategy outlined in the lecture slides, *give* an explicit formula for the solution of the above program.

(b) *Kernelize* the ridge regression model, following the strategy outlined in the lecture slides.

Exercise 2: Lagrange Multipliers (20+10+10 P)

Consider the slightly modified quadratic program

$$\min_{\xi,w} \ \sum_{i=1}^{n} \xi_i^2$$
$$\text{subject to} \quad \xi_i = w^{\top}x_i - y_i \ \text{ for } 1 \le i \le n \quad \text{and} \quad \|w\|_2^2 \le C,$$

where C is a regularization constant.

(a) *Calculate* the Lagrange dual of this program, and its solution.

(b) *Describe* how a solution for the primal program can be found from a solution of the dual.

(c) *Explain* how the solutions relate to the original quadratic program and its solutions in ridge regression and kernel ridge regression.

Exercise 3: Programming (40 P)

Download the programming files on ISIS and follow the instructions.