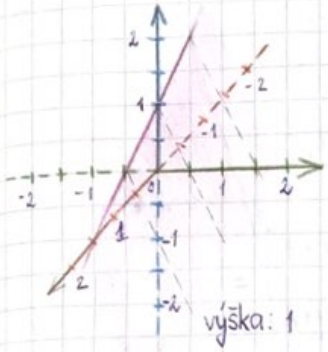
# Příprava na cvičení N 7

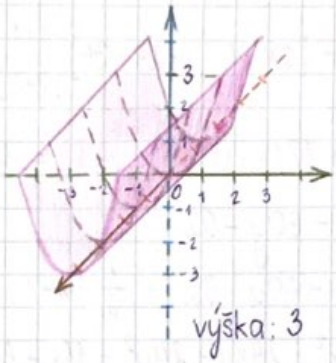
## N 8.1

b) $f(x_1, x_2) = x_1 - 3x_2 + 1$



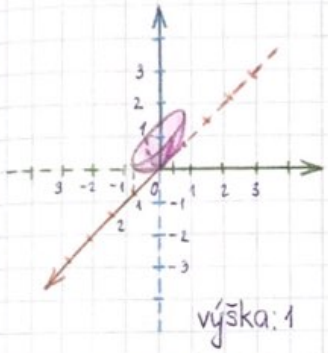
výška: 1

c) $f(x_1, x_2) = x_1^2$



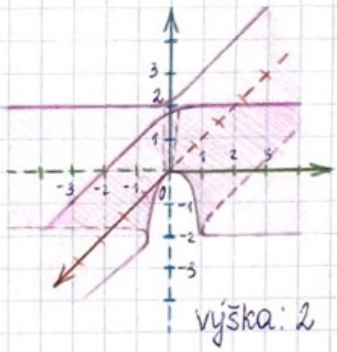
výška: 3

d) $f(x_1, x_2) = x_1^2 + 4x_2^2$



výška: 1

f) $f(x_1, x_2) = x_1 x_2$



výška: 2

## N 8.3

$f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = \ln(1+xy)$ $\quad (x_0, y_0) = (1,2)$

d) $f'(x,y)_{|(x_0,y_0)} = \left[\frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y}\right]_{|(x_0,y_0)} = \left[\frac{y}{1+xy} \quad \frac{x}{1+xy}\right]_{|(1,2)} = \left[\frac{2}{1+1\cdot 2} \quad \frac{1}{1+1\cdot 2}\right] = \left[\frac{2}{3} \quad \frac{1}{3}\right]$

g) $f''(x,y)_{|(x_0,y_0)} = \begin{bmatrix} \frac{\partial^2 f}{\partial x \partial x} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y \partial y} \end{bmatrix}_{|(x_0,y_0)} = \begin{bmatrix} -\frac{y^2}{(1+xy)^2} & \frac{1}{(1+xy)^2} \\ \frac{1}{(1+xy)^2} & -\frac{x^2}{(1+xy)^2} \end{bmatrix}_{|(1,2)} = \begin{bmatrix} -\frac{4}{9} & \frac{1}{9} \\ \frac{1}{9} & -\frac{1}{9} \end{bmatrix}$

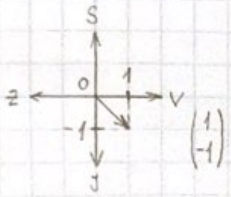
## N 8.10

$h(d,s) = 2s^2 + 3sd - d^2 + 5$ $\quad d$ - zeměpisná délka, $s$ - zeměpisná šířka $\quad (d,s) = (-1,1)$

a) směr nejstrmějšího stoupání terénu

$h'(d,s) = [3s - 2d \;;\; 4s + 3d]_{|(-1;1)} = [3\cdot 1 - 2\cdot(-1) \;;\; 4\cdot 1 + 3\cdot(-1)] = [5\;;\;1]$

podle zadání normalizujeme: $\frac{\begin{pmatrix}5\\1\end{pmatrix}}{\sqrt{5^2+1^2}} = \begin{pmatrix}\frac{5}{\sqrt{26}}\\ \frac{1}{\sqrt{26}}\end{pmatrix}$




b) strmost terénu v jihovýchodním směru

$\frac{(5 \;\; 1)\begin{pmatrix}1\\-1\end{pmatrix}}{\sqrt{1^2+(-1)^2}} = \frac{5-1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

N 8.13 $f(x,y) = 6xy^2 - 2x^3 - 3y^3 \qquad (x_0, y_0) = (1; -2)$

$T^0 = 6 \cdot 1 \cdot (-2)^2 - 2 \cdot 1^3 - 3 \cdot (-2)^3 = 6 \cdot 4 - 2 - 3 \cdot (-8) = 24 - 2 + 24 = 46$

$T^1 = 46 + \left[6y_0^2 - 6x_0^2 \; ; \; 12x_0y_0 - 9y_0^2\right] \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix} = 46 + \left[6 \cdot (-2)^2 - 6 \cdot 1^2 \; ; \; 12 \cdot 1 \cdot (-2) - 9 \cdot (-2)^2\right] \begin{bmatrix} x-1 \\ y+2 \end{bmatrix}$

$= 46 + [24 - 6 \; , \; -24 - 36] \begin{bmatrix} x-1 \\ y+2 \end{bmatrix} = 46 + [18 \; ; \; -60] \begin{bmatrix} x-1 \\ y+2 \end{bmatrix} =$

$= 46 + 18(x-1) - 60(y+2) = 46 + 18x - 18 - 60y - 120 =$

$= 18x - 60y - 92$

$T^2 = 18x - 60y - 92 + \frac{1}{2} [x-1 \; ; \; y+2] \begin{bmatrix} -12 & -24 \\ -24 & 48 \end{bmatrix} \begin{bmatrix} x-1 \\ y+2 \end{bmatrix} =$

$= -6x^2 - 24xy + 24y^2 - 18x + 60y + 46$