Příprava na cvičení 2

N 2.1

b)
$$X - A = XB$$
$$X - XB = A$$
$$X(I-B) = A$$
$$X = A\cdot(I-B)^{-1}$$

c)
$$2X - AX + 2A = 0$$
$$2X - AX = -2A$$
$$(2I - A)X = -2A$$
$$X = (2I-A)^{-1}\cdot(-2A)$$

N 2.3

$$Ax + (y^TB)^T = \alpha 1$$
$$Ay + c = 0$$

$$Ax + B^Ty - \alpha 1 = 0$$
$$Ay = -c$$

$$\underbrace{\begin{bmatrix} A & B^T & 1 \\ 0 & A & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} x \\ y \\ -\alpha \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} 0 \\ -c \end{bmatrix}}_{q}$$

N 2.4

a)
$$Ax + By = a$$
$$Cx + Dy = b$$

$$\underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{P} \cdot \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{q}$$

b)
$$Dy = b - Cx$$
$$y = D^{-1}(b - Cx)$$

$$Ax + BD^{-1}(b - Cx) = a$$
$$Ax + BD^{-1}b - BD^{-1}Cx = a$$
$$Ax - BD^{-1}Cx = a - BD^{-1}b$$
$$(A - BD^{-1}C)x = a - BD^{-1}b$$
$$x = (A - BD^{-1}C)^{-1}(a - BD^{-1}b)$$

N 3.1

a) $\{x \in \mathbb{R}^n \mid a^Tx = 0\}$ pro dané $a \in \mathbb{R}^n$ — dimenze $n-1$ jestli $a \neq 0$

lineární, protože má jenom jedné řešení — dimenze n jestli $a = 0$

$\dim \operatorname{rng} A + \dim \operatorname{null} A = n$

$\dim \operatorname{null} A = n - \underbrace{\dim \operatorname{rng} A}_{\text{rank } A}$; jestli $a \neq 0 \Rightarrow \text{rank } A = 1$

$a = 0 \Rightarrow \text{rank } A = 0$

b) $\{x \in \mathbb{R}^n \mid a^Tx = b\}$ pro dané $a \in \mathbb{R}^n$, $b \in \mathbb{R}$

afinní (definice afinního podprostoru Věta 3.13)

$\dim \operatorname{null} A = n - \text{rank } A$

dimenze $n-1$ jestli $a \neq 0$

dimenze n jestli $a = 0$ a $b = 0$.

Jestli $b \neq 0$ a $a = 0$ je množina prázdná.

c) $\{x \in \mathbb{R}^n \mid x^Tx = 1\}$ není linearní, není afinní

N 3.2 $\{(x_1,x_2,x_3,x_4)\in\mathbb{R}^4 \mid x_1+x_3=0\}$ lineární

$$(1,0,1,0)\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=0$$

$ax^T=0 \quad a\neq 0 \Rightarrow \dim = n-1 = 4-1=3$

baze má 3 vektory, např. $(1,0,-1,0)$, $(0,1,0,0)$, $(0,0,0,1)$

N 3.7 $f:\mathbb{R}^2\to\mathbb{R}^3 \quad f(x,y)=(x+y,\ 2x-1;\ x-y) \qquad Ax+b\,?$

$$\begin{aligned}x+y+0\\2x+0-1\\x-y+0\end{aligned} \qquad \underbrace{\begin{bmatrix}1&1\\2&0\\1&-1\end{bmatrix}}_{A}\cdot\underbrace{\begin{bmatrix}x\\y\end{bmatrix}}_{x}+\underbrace{\begin{bmatrix}0\\-1\\0\end{bmatrix}}_{b}$$

$b\neq 0$, afinní

N 3.8

$$\begin{aligned}x+2y+z&=1\\-x+y+2z&=2\end{aligned}$$

$$\begin{bmatrix}1&2&1\\-1&1&2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$$

$$\left(\begin{array}{ccc|c}1&2&1&1\\-1&1&2&2\end{array}\right)\sim\left(\begin{array}{ccc|c}1&2&1&1\\0&3&3&3\end{array}\right)\sim\left(\begin{array}{ccc|c}1&2&1&1\\0&1&1&1\end{array}\right)$$

$y+z=1$
$y=1-z$

$x+2(1-z)+z=1$
$x+2-2z+z=1$
$x-z=-1$
$x=z-1$

Ať $z=2$ $\quad y=1-2=-1$
$x=2-1=1$

$(1,-1,2)$

$$\left(\begin{array}{ccc|c}1&2&1&0\\0&1&1&0\end{array}\right)$$

$y+z=0$
$y=-z$

$x-2z+z=0$
$x-z=0$
$x=z$

$z=1;\ y=-1;\ x=1$

$(1,-1,2)+\operatorname{span}\{(1,-1,1)\}$