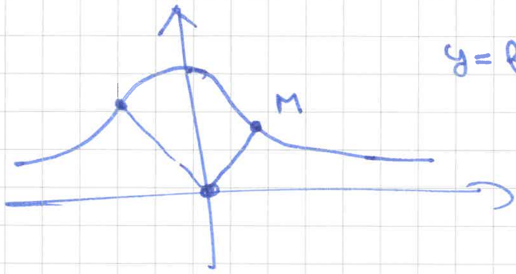
Ex 1

a) $D(f) = \mathbb{R}$

b)



$y = f(x) = \dfrac{1}{1+4x^2}$

c) $O(0,0)$ $M(x, f(x))$

$$d(x) = \text{distance}(O, M) = \sqrt{x^2 + \frac{1}{(1+4x^2)^2}}$$

$$d'(x) = \frac{\left(x^2 + \frac{1}{(1+4x^2)^2}\right)'}{2\sqrt{d}}$$

$$\left(x^2 + \frac{1}{(1+4x^2)^2}\right)' = 2x - 2(1+4x^2)^{-3}\, 8x$$

$$= 2x\left(1 - (1+4x^2)^{-3}\, 8\right)$$

$$= 2x \, \frac{(1+4x^2)^3 - 8}{(1+4x^2)^3}$$

$$= \frac{2x}{(1+4x^2)^3}\left((1+4x^2)^3 - 8\right)$$

par symétrie, on peut supposer $x > 0$

$\text{sgn}(d'(x)) = \text{sgn}\left((1+4x^2)^3 - 8\right)$

$(1+4x^2)^3 = 8$ $\quad 1 + 4x^2 = 2$

$4x^2 = 1$ $\quad x^2 = \frac{1}{4}$ $\quad x = \frac{1}{2}$

Car on a supposé que $x > 0$

x	0		$\frac{1}{2}$	
$d'(x)$		$-$	$\vert$	$+$
$d(x)$		$\searrow$		$\nearrow$

on a bien un minimum en $x = \frac{1}{2}$

$f(x) = \frac{1}{2}$

En résumé on a un minimum

en $x = \pm \frac{1}{2}$

les pts sont donc $A\left(\frac{1}{2}, \frac{1}{2}\right)$; $B\left(-\frac{1}{2}, \frac{1}{2}\right)$

Ex 3 $f(x) = \dfrac{\ln(x^3)}{x}$

a) $D(f) = \left]0, +\infty\right[$

b) Comme $x > 0$ $f(x) = 3\dfrac{\ln(x)}{x}$

$f'(x) = \dfrac{3(1 - \ln(x))}{x^2}$ $1 - \ln(x) = 0$ $x = e$

x	0		e	
$f'(x)$	$\Vert$	$+$	$\vert$	$-$
$f(x)$	$\Vert$	$\nearrow$		$\searrow$

c) $f''(x) = \dfrac{3(-3 + 2\ln(x))}{x^3}$ $-3 + 2\ln(x) = 0$

$x = e^{3/2}$

x	0		$e^{3/2}$	
$f''(x)$	$\Vert$	$-$	$\vert$	$+$
$f(x)$	$\Vert$	$\frown$	$\vert$	$\smile$

d) $x = e^{3/2}$

e) $\lim_{x\to 0^+} \frac{\ln(x^3)}{x} = \frac{-\infty}{0^+} = -\infty$

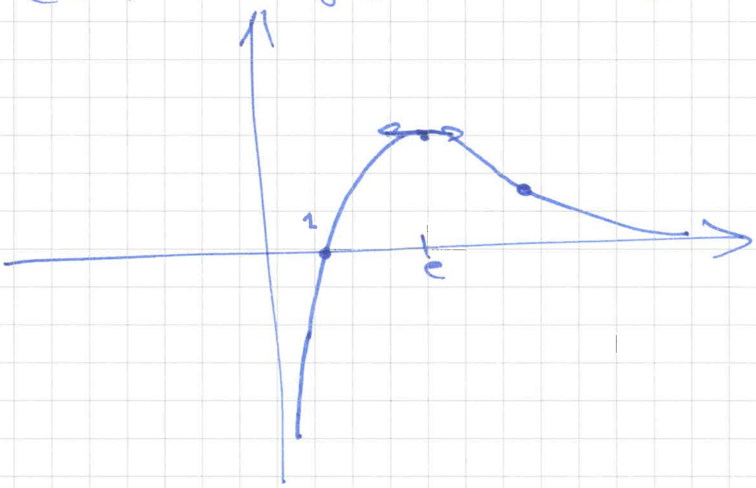
(Oy) asymptote verticale

$\lim_{x\to +\infty} \frac{\ln(x^3)}{x} = \lim_{x\to+\infty} 3\frac{\ln(x)}{x} = \frac{+\infty}{+\infty}$

B-H ou croissance comparée
on obtient que

$\lim_{x\to +\infty} \frac{\ln(x^3)}{x} = 0$

(Ox) asymptote horizontale



Ex2 $f(x) = x^k \ln(x)$

$f'(x) = k\, x^{k-1} \ln(x) + x^{k-1}$

$f''(x) = k(k-1)\, x^{k-2} \ln(x) + k\, x^{k-2} + (k-1)\, x^{k-2}$

$= x^{k-2} \left(k(k-1) \ln(x) + 2k - 1 \right)$

$f''(e^{-\frac{7}{12}}) = 0$

$-k(k-1)\frac{7}{12} + 2k - 1 = 0$

$-7k^2 + 31k - 12 = 0 \qquad \Delta = 625$

$k_1 = 4 \in \mathbb{Z} \qquad k_2 = \frac{3}{7} \notin \mathbb{Z}$

donc $k = 4$ comme solution

Ex4 $\boxed{4}$

$$A^{-1} = \frac{1}{6}\begin{pmatrix} -12 & 12 & -6 \\ 5 & -8 & 5 \\ 19 & -22 & 13 \end{pmatrix}$$

Ex5 Voir Te 3 - matin 20-21

$$x_2 = \frac{\begin{vmatrix} -1 & 1 & 0 \\ -t & 4 & 4 \\ -1 & 0 & 4 \end{vmatrix}}{8(t^2-1)} = \frac{3+t}{2\,(t^2-1)}$$