Correction du TE1 MAT1_B 23_24

Ex1 a) $\dfrac{2x-1}{x^2+4x+4} - \dfrac{6}{x^2-4} + \dfrac{3}{x-2} = A$

$$A = \frac{2x-1}{(x+2)^2} - \frac{6}{(x-2)(x+2)} + \frac{3}{x-2} = \frac{(2x-1)(x-2) - 6(x+2) + 3(x+2)^2}{(x+2)^2(x-2)}$$

$$A = \frac{-x^2-5x+14}{(x+2)^2(x-2)} = -\frac{x+7}{(x+2)^2}$$

b) $$\frac{e^{-2\ln(x^3)}}{x^2} = \frac{e^{\ln(x^{-6})}}{x^2} = \frac{1}{x^8}$$

Ex2 a) Posons $e^x = u$

l'équation $e^{3x} - 6e^{2x} + 9e^x - 2 = 0$ devient

$$u^3 - 6u^2 + 9u - 2 = 0$$

$x = \ln(2)$ $\qquad u = 2$

$$8 - 24 + 18 - 2 = 0 \quad \text{OK}$$

b) $u^3 - 6u^2 + 9u - 2 = (u-2)(u^2 - 4u + 1)$ (division euclidienne)

$u^2 - 4u + 1 = 0$ $\qquad u_{1,2} = \dfrac{2 \mp \sqrt{3}}{1} = 2 \pm \sqrt{3}$

$u_1 = 2 + \sqrt{3} > 0$ $\qquad x_1 = \ln(2+\sqrt{3})$

$u_2 = 2 - \sqrt{3} > 0$ $\qquad x_2 = \ln(2-\sqrt{3})$

Ex3 a) $\ln(x) + \ln(x-1) = 2\ln(x+1)$

CE $\quad x > 0 \quad x - 1 > 0 \quad x + 1 > 0$

$$\ln(x(x-1)) = \ln[(x+1)^2]$$

$$x^2 - x = x^2 + 2x + 1$$

$$-3x = 1 \qquad x = -\frac{1}{3} \notin D$$

$$S = \emptyset$$

Ex3 b) $3\cos^2(x) + 2\sin^2(x) = \frac{11}{4}$ $\quad x \in [-\pi, \pi]$

$$3(1-\sin^2(x)) + 2\sin^2(x) = \frac{11}{4}$$

$$3 - \sin^2(x) = \frac{11}{4}$$

$$3 - \frac{11}{4} = \sin^2(x)$$

$$\sin^2(x) = \frac{1}{4}$$

$$\sin(x) = \pm\frac{1}{2}$$

$\sin(x) = \frac{1}{2}$ $\quad x_1 = \frac{\pi}{6}$ $\quad x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\sin(x) = -\frac{1}{2}$ $\quad x_3 = -\frac{\pi}{6}$ $\quad x_4 = -\frac{5\pi}{6}$

$S = \{x_1, x_2, x_3, x_4\}$

Ex4 a) $\lim\limits_{x \to -\infty} \frac{\sqrt{2x^2+1} - x}{x+1}$

on a une F.I $\frac{+\infty}{-\infty}$.

$$\sqrt{2x^2+1} - x = |x|\sqrt{2+\frac{1}{x^2}} - x$$

$x < 0$ $\quad = -x\left(\sqrt{2+\frac{1}{x^2}} + 1\right)$

$$\frac{\sqrt{2x^2+1} - x}{x+1} = \frac{-x\left(\sqrt{2+\frac{1}{x^2}}+1\right)}{x\left(1+\frac{1}{x}\right)} = \frac{-\left(\sqrt{2+\frac{1}{x^2}}+1\right)}{1+\frac{1}{x}}$$

$$\lim_{x \to -\infty} \frac{\sqrt{2x^2+1} - x}{x+1} = \lim_{x \to -\infty} -\frac{\left(\sqrt{2+\frac{1}{x^2}}+1\right)}{1+\frac{1}{x}}$$

$$= -(\sqrt{2}+1)$$

Ex 4 b) $\lim\limits_{x \to \frac{1}{2}} \dfrac{1-8x^3}{2x^2+x-1} = \text{"}\dfrac{0}{0}\text{"}$

$$1 - 8x^3 = 1^3 - (2x)^2$$

$$= (1-2x)(1+2x+4x^2)$$

$$2x^2+x-1 = (2x-1)(x+1)$$

$$\lim_{x \to \frac{1}{2}} \frac{1-8x^3}{2x^2+x-1} = \lim_{x \to \frac{1}{2}} \frac{(1-2x)(1+2x+4x^2)}{(2x-1)(x+1)}$$

$$= \lim_{x \to \frac{1}{2}} - \frac{1+2x+4x^2}{x+1} = -2$$

Ex5 a) $\sin\left(-\frac{11\pi}{6}\right) = \frac{1}{2}$ b) $\cos\left(-\frac{9\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (ou $\frac{1}{\sqrt{2}}$)

c) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ d) $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

e) $\cos\left(\arctan\left(\frac{1}{\sqrt{3}}\right)\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

f) $\arcsin(\sin(-4)) = 4 - \pi$

g) $\arccos(\cos(7)) = 7 - 2\pi$

h) $\cos\left(\arcsin\left(-\frac{1}{\sqrt{2}}\right)\right) = \frac{\sqrt{2}}{2}$

i) $\sin\left(\arccos\left(\frac{1}{5}\right)\right) = ?$

$$\sin^2\left(\arccos\left(\tfrac{1}{5}\right)\right) + \cos^2\left(\arccos\left(\tfrac{1}{5}\right)\right) = 1$$

$$\sin^2\left(\arccos\left(\tfrac{1}{5}\right)\right) = 1 - \frac{1}{25} = \frac{24}{25}$$

$$\sin\left(\arccos\left(\tfrac{1}{5}\right)\right) = \sqrt{\frac{24}{25}} = \frac{2\sqrt{6}}{5}$$

($\arccos\left(\frac{1}{5}\right) \in [0,\pi]$
donc $\sin\left(\arccos\left(\frac{1}{5}\right)\right) > 0$)