Correction TE1 . MAT1 _ B 20_21

Ex1 a) $f(x) = \dfrac{1}{\ln(x^2+x+1)}$ $\qquad x^2+x+1>0$ et $x^2+x+1 \neq 1$

$x^2+x+1>0 \quad \forall x \in \mathbb{R}$ car $\Delta<0$

$x^2+x+1=1 \qquad x^2+x=0 \qquad x(x+1)=0$

$x=0$ ou $x=-1$

$D(f) = \mathbb{R} \setminus \{-1;0\}$

b) $g(x) = \dfrac{\sqrt{3-x}}{\sqrt{x^2-4}}$ $\qquad 3-x \geq 0$ et $x^2-4>0$

$x \leq 3 \qquad (x-2)(x+2)>0$

$D(g) =]-\infty,3] \cap (]-\infty,-2[\cup]2,+\infty[)$

$D(g) =]-\infty,-2[\cup]2,3]$

Ex2 a) $f(x) = \frac{1}{2}(e^x - e^{-x})$

$f(-x) = \frac{1}{2}(e^{-x} - e^{x}) = -f(x)$ impaire

b) $f(x) = \lfloor x^3 \rfloor$

$f(\frac{1}{2}) = 0 \qquad f(-\frac{1}{2}) = -1$ ni paire, ni impaire

c) $f(x) = (x^3-x^2)^3$

$f(1) = 0 \qquad f(-1) = -8$ ni paire, ni impaire

d) $f(x) = \sqrt{1+x+x^2} + \sqrt{1-x+x^2}$

$f(-x) = \sqrt{1-x+x^2} + \sqrt{1+x+x^2} = f(x)$ f paire

Ex 3

a) $\log(7-x) - \log(x+3) = \log(x-1)$

voir correction TE1 - MAT1 - C

b) $4\cos^2(2x) + 2\cos(2x) - 1 = 0$

$t = \cos(2x)$

$4t^2 + 2t - 1 = 0$

$\Delta = 20 \qquad t_{1,2} = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$

$t_1 = \cos(2x) = \frac{-1+\sqrt{5}}{\cancel{2}\ 4} = \cos\left(\frac{2\pi}{5}\right)$

diviser par 4 au lieu de 2

$2x = \frac{2\pi}{5} + 2k\pi \qquad (1)$

$k \in \mathbb{Z}$

$2x = -\frac{2\pi}{5} + 2k\pi$

$x = \frac{\pi}{5} + k\pi \qquad (1)$

$x = -\frac{\pi}{5} + k\pi \qquad (2)$

$t_2 = \frac{-1-\sqrt{5}}{4} = -\frac{1+\sqrt{5}}{4} = -\cos\left(\frac{2\pi}{5}\right)$ -cos(Pi/5)

$= \cos\left(\pi - \frac{2\pi}{5}\right)$ cos(Pi-Pi/5)

$\cos(2x) = \cos\left(\frac{3\pi}{5}\right)$ Cos(4Pi/5)

$2x = \frac{3\pi}{5} + 2k\pi$ 2Pi/5 + k Pi

$k \in \mathbb{Z}$

$2x = -\frac{3\pi}{5} + 2k\pi$ -2Pi/ 5 + k Pi

$x = \frac{3\pi}{10} + k\pi \qquad (3)$

$k \in \mathbb{Z}$

$x = -\frac{3\pi}{10} + k\pi \qquad (4)$

$S = \{ (1), (2), (3), (4) \mid k \in \mathbb{Z} \}$

Ex4 $\quad f(v) = \dfrac{m_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = m \qquad v = f^{-1}(m)$

$$\frac{m_0}{m} = \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$\left(\frac{m_0}{m}\right)^2 = 1 - \left(\frac{v}{c}\right)^2$$

$$\left(\frac{v}{c}\right)^2 = 1 - \left(\frac{m_0}{m}\right)^2$$

$$\boxed{v = c\sqrt{1-\left(\frac{m_0}{m}\right)^2}}$$

Ex5 $\quad f(x) = \dfrac{x^4-16}{x^3-x}$

a) $D(f)$: $x^3 - x \neq 0 \qquad x(x^2-1) \neq 0$

$D(f) = \mathbb{R} \setminus \{-1; 0; 1\}$

b) Zéros: $\quad x^4 - 16 = (x^2-4)(x^2+4)$

$x = 2 \quad ; \quad x = -2$ zéros simples.

c) Parité $\quad f(-x) = \dfrac{x^4-16}{-x^3+x} = -f(x)$

f impaire

d) l'intersection avec l'axe (Oy) :

ne coupe pas l'axe (Oy) car 0 est un pôle

e) $x = -1 \quad ; \quad x = 0 \quad ; \quad x = 1 \quad$ ~~axes Ve~~

asymptotes verticales

f) on a une asymptote oblique

$$\begin{array}{rl|l} & x^4 - 16 & x^3 - x \\ \cline{3-3} - & x^4 - x^2 & x \\ \cline{2-2} & x^2 - 16 & \end{array}$$

$y = x$ asymptote oblique