Correction du TE1 - MAT1-23-24

MAT1 - A

Ex1 a) $\sqrt{x} + \frac{1}{\sqrt{x}} = \frac{x+1}{\sqrt{x}}$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = \frac{x-1}{\sqrt{x}}$$

$$\frac{\sqrt{x} + \frac{1}{\sqrt{x}}}{\sqrt{x} - \frac{1}{\sqrt{x}}} = \frac{\frac{x+1}{\sqrt{x}}}{\frac{x-1}{\sqrt{x}}} = \frac{x+1}{x-1}$$

b) $\frac{x}{xy-2y^2} + \frac{2}{2y-x} - \frac{1}{y} = \frac{x}{y(x-2y)} + \frac{2}{2y-x} - \frac{1}{y}$

$$= \frac{x - 2y - (x-2y)}{y(x-2y)} = 0$$

Ex2 $x_0 = \ln(3)$ $\quad e^{x_0} = 3$

$$e^{3x_0} - 7e^{2x_0} + 11e^{x_0} + 3 = 27 - 7 \cdot 9 + 11 \cdot 3 + 3$$
$$= 27 - 63 + 33 + 3 = 0$$

Donc $e^x = t$

$$t^3 - 7t^2 + 11t + 3 = 0$$

$$\begin{array}{r|l} t^3 - 7t^2 + 11t + 3 & t - 3 \\ \hline -(t^3 - 3t^2) & t^2 - 4t - 1 \\ \hline -4t^2 + 11t + 3 & \\ -(-4t^2 + 12t) & \\ \hline -t + 3 & \end{array}$$

$t^3 - 7t^2 + 11t + 3 = (t-3)(t^2 - 4t - 1)$

$t^2 - 4t - 1 = 0 \qquad \Delta = 16 + 4 = 20$

$$t_{1,2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$e^x = 2 + \sqrt{5} \qquad x = \ln(2+\sqrt{5})$

$e^x = 2 - \sqrt{5} < 0$ impossible

autre solution réelle si donc $\ln(2+\sqrt{5})$.

EX3

a) $f(x) = \dfrac{1}{\sqrt{x+x^3}}$

$D(f)$: $x + x^3 > 0$

$x(1+x^2) > 0$ donc $x > 0$

car $1 + x^2 > 0$

$D(f) =]0, +\infty[$

b) $g(x) = \ln\left(\dfrac{x^2-3x+2}{x+1}\right)$

$D(g)$: $x + 1 \neq 0$ et $\dfrac{x^2-3x+2}{x+1} > 0$

$x \neq -1$ $\quad A = \dfrac{(x-1)(x-2)}{x+1} > 0$

x	$-\infty$		-1		1		2		$+\infty$
$(x-1)(x-2)$		$+$		$+$	0	$-$	0	$+$	
$x+1$		$-$	0	$+$		$+$		$+$	
$A = \dfrac{(x-1)(x-2)}{x+1}$		$-$	‖	$+$	0	$-$	0	$+$	

$D(g) =]-1, 1[\cup]2, +\infty[$

Ex 4

a) $\ln(x+1) + \ln(x+2) = \ln(x+3)$

$x+1>0 \qquad x+2>0 \qquad x+3>0$

$$\ln[(x+1)(x+2)] = \ln(x+3)$$

$$(x+1)(x+2) = x+3$$

$$x^2+3x+2 = x+3$$

$$x^2+2x-1=0$$

$$x_{1,2} = -1 \pm \sqrt{2}$$

$x_1 = -1-\sqrt{2}$ ne vérifie pas $x_1+1>0$

à exclure

$x_2 = -1+\sqrt{2}$

$x_2+1 = \sqrt{2} > 0$ ok

$x_2+2>0$ ok

$x_2+3>0$ ok

$$S = \{-1+\sqrt{2}\}$$

Ex4 b) $3\sin^2(x) + 2\cos^2(x) = \frac{11}{4}$ $\quad x \in [-\pi, \pi]$

$$\cos^2(x) = 1 - \sin^2(x)$$

$$3\sin^2(x) + 2(1 - \sin^2(x)) = \frac{11}{4}$$

$$\sin^2(x) = \frac{11}{4} - 2 = \frac{3}{4}$$

$$\sin(x) = \pm\frac{\sqrt{3}}{2}$$

$\sin(x) = \frac{\sqrt{3}}{2}$ $\quad x_1 = \frac{\pi}{3}$ $\quad x_2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\sin(x) = -\frac{\sqrt{3}}{2}$ $\quad x_3 = -\frac{\pi}{3}$ $\quad x_4 = -\frac{2\pi}{3}$

$$S = \left\{-\frac{2\pi}{3}; -\frac{\pi}{3}; \frac{2\pi}{3}, \frac{\pi}{3}\right\}$$

Ex5 a) $\sin\left(-\frac{11\pi}{6}\right) = -\sin\left(\frac{11\pi}{6}\right) = -\sin\left(\frac{11\pi}{6} - 2\pi\right)$

$$= -\sin\left(-\frac{\pi}{6}\right) = \frac{1}{2}$$

b) $\cos\left(-\frac{9\pi}{4}\right) = \cos\left(\frac{9\pi}{4}\right) = \cos\left(\frac{\pi}{4} + 2\pi\right) = \frac{\sqrt{2}}{2}$

c) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

d) $\arccos\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

e) $\cos\left(\arctan\left(\frac{1}{\sqrt{3}}\right)\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

f) $\arcsin(\sin(4)) = \arcsin(\sin(\pi - 4)) = \pi - 4$ car $\pi - 4 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

g) $\arccos(\cos(4)) = \arccos(\cos(2\pi - 4))$

$= 2\pi - 4$ car $2\pi - 4 \in [0, \pi]$

i) $\arctan\left(\tan\left(\frac{5\pi}{4}\right)\right) = \arctan\left(\tan\left(\frac{\pi}{4}\right)\right)$

$= \frac{\pi}{4}$

h) $\cos\left(\arcsin\left(-\frac{1}{\sqrt{2}}\right)\right) = \frac{\pi}{4} = \frac{\sqrt{2}}{2}$