Correction de la série 4

Ex1 a) $\ln(18) - \ln(6) + 2\ln(3) = \ln\left(\frac{18}{6}\right) + \ln(9)$

$= \ln(27)$

b) $\frac{1}{2}\log(2) + \frac{1}{4}\log(16) = \frac{1}{2}\log(2) + \frac{1}{4}\log(2^4)$

$= \frac{1}{2}\log(2) + \log(2) = \frac{3}{2}\log(2)$

c) $\sqrt{9}\; e^{2-\ln(3)}\, e^{-\frac{3}{2}} = 3\, e^{2}\; e^{-\ln(3)}\; e^{-\frac{3}{2}}$

$= 3\, e^{2} \cdot \frac{1}{3}\; e^{-\frac{3}{2}} = e^{\frac{1}{2}}$

$= \sqrt{e}$

Ex2 a) $e^{4x} = 7$ $\quad 4x = \ln(7)$ $\quad x = \frac{\ln(7)}{4}$

$S = \left\{ \frac{\ln(7)}{4} \right\}$

b) $e^{-2x} = 2^{x^2}$ $\quad -2x = x^2 \ln(2)$

$x^2 \ln(2) + 2x = 0$ $\quad x\,(x\ln(2) + 2) = 0$

$x = 0$ $\quad x\ln(2) + 2 = 0$

$x = -\frac{2}{\ln(2)}$

$S = \left\{ 0 ; \; -\frac{2}{\ln(2)} \right\}$

c) $e^{2x} + 4e^{x} - 21 = 0$ $\quad e^{x} = t > 0$

$t^2 + 4t - 21 = 0$

$(t+7)(t-3) = 0$ $\quad t = -7$ à exclure

$t = 3 = e^{x}$ $\quad x = \ln(3)$

$S = \{ \ln(3) \}$

d) $e^{3x} - 7e^{x} + 6 = 0$ $\qquad e^{x} = t > 0$

$t^3 - 7t + 6 = 0$ $\qquad t = 1$ solution

$$
\begin{array}{rl|l}
 & t^3 - 7t + 6 & t - 1 \\
\cline{3-3}
- & t^3 - t^2 & t^2 + t - 6 \\
\cline{2-2}
 & t^2 - 7t + 6 & \\
- & t^2 - t & \\
\cline{2-2}
 & -6t + 6 & \\
- & -6t + 6 & \\
\cline{2-2}
 & 0 &
\end{array}
$$

$t^3 - 7t + 6 = (t-1)(t^2 + t - 6)$

$t^2 + t - 6 = (t-2)(t+3)$

$t = 1 \qquad x = 0$

$t = 2 \qquad x = \ln(2)$

$t = -3 = e^{x} > 0$ impossible

$S = \{0 ; \ln(2)\}$

Ex3 a) $\log(x^2 - 10x + 121) = 2$ $\qquad x^2 - 10x + 121 > 0 \ (1)$

$x^2 - 10x + 121 = 100$

$x^2 - 10x + 21 = 0$

$(x-7)(x-3) = 0$

$x = 7 \qquad 7^2 - 10 \times 7 + 121 > 0 \qquad$ ok

$x = 3 \qquad 3^2 - 3 \cdot 10 + 121 > 0 \qquad$ ok

$S = \{3 ; 7\}$

b) $\log_2(x^2 - 3x) = 2$ $\qquad x^2 - 3x > 0 \quad (1)$

$x^2 - 3x = 4 \qquad x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x = -1 \qquad 1^2 + 3 > 0 \qquad$ ok

$x = 4 \qquad 16 - 12 > 0 \qquad$ ok $\qquad S = \{-1 ; 4\}$

Ex3 c) $x^{\sqrt{\log(x)}} = 10^8$

$$\log\left(x^{\sqrt{\log(x)}}\right) = 8$$

$$\sqrt{\log(x)}\ \log(x) = 8$$

$$(\log(x))^{3/2} = 2^3$$

$$\sqrt{\log(x)} = 2 \qquad \log(x) = 4$$

$$x = 10^4$$

Ex4 $I = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

$$\frac{RI}{V} = 1 - e^{-\frac{R}{L}t}$$

$$1 - \frac{RI}{V} = e^{-\frac{R}{L}t}$$

$$\ln\left(1 - \frac{RI}{V}\right) = -\frac{R}{L}t$$

$$t = -\frac{L}{R}\ln\left(1 - \frac{RI}{V}\right)$$

Ex6 a) $\ln(x) \leq 2 \qquad x > 0 \qquad x \leq e^2$

$$S =]0, e^2]$$

b) $e^{3x} > 2 \qquad 3x > \ln(2) \qquad x > \frac{\ln(2)}{3}$

$$S =]\frac{\ln(2)}{3}, +\infty[$$

c) $\ln(3x) > 6 \qquad x > 0 \qquad 3x > e^6$

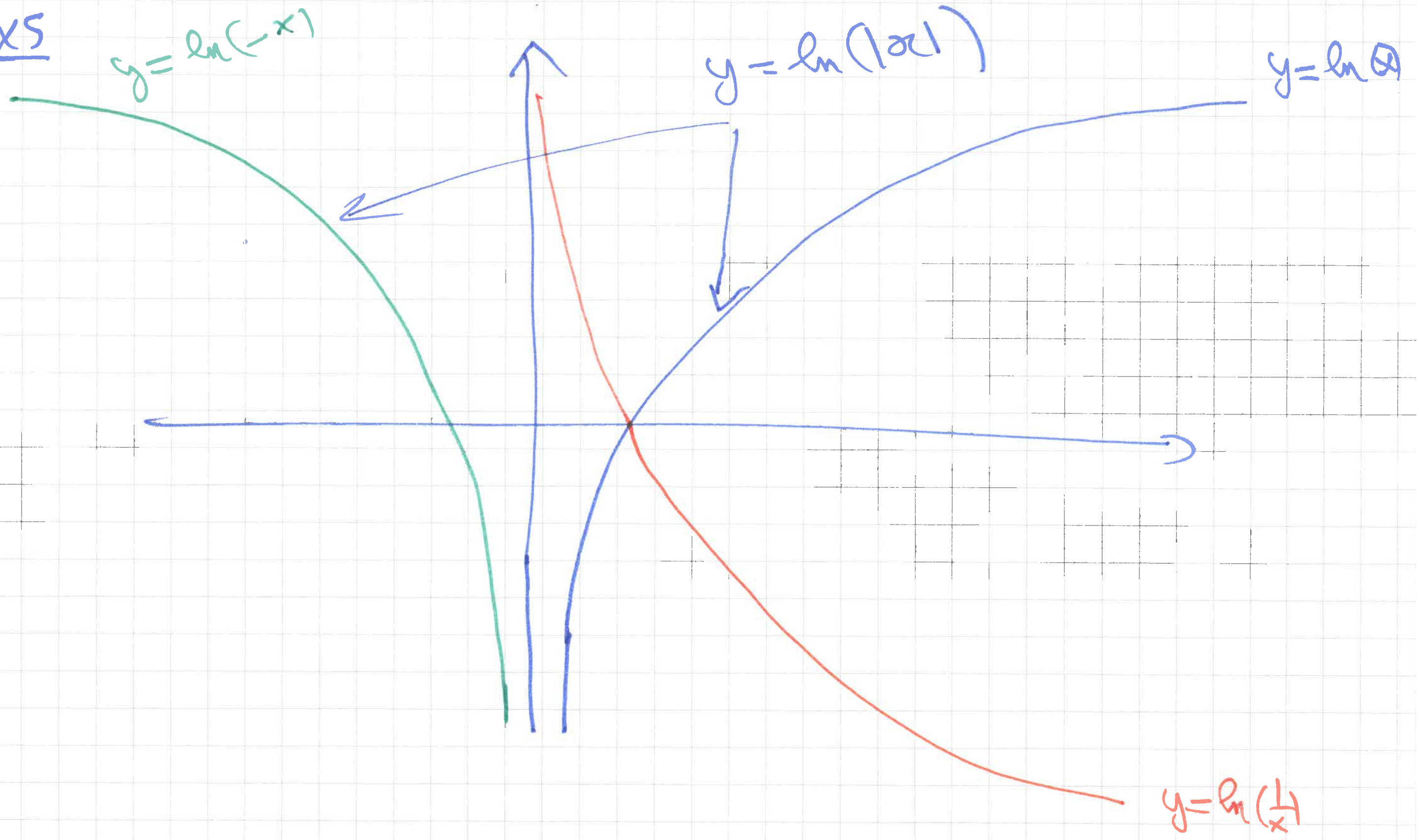
$x > \frac{e^6}{3} \qquad S =]\frac{e^6}{3}, +\infty[$

Ex6 d) $e^{-2x} \leq 2 \qquad -2x \leq \ln(2)$

$$x \geq -\frac{\ln(2)}{2}$$

$$S = \left[-\frac{\ln(2)}{2}, +\infty\right[$$

Ex5



Ex7 $\qquad x = 2^{10\,000\,000}$

$10^k \leq x < 10^{k+1} \qquad \cancel{k \in \mathbb{Z}} \quad k \in \mathbb{N}$

d = nbre de chiffres décimals $= k+1$

$k \leq \log(x) < k+1 \qquad k = \lfloor \log(x) \rfloor$

$$d = \lfloor \log(x) \rfloor + 1 = \lfloor 10\,000\,000 \log(2) \rfloor + 1$$