Ex1 a) $\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{5\pi}{6}\right)$

$= -\sin\left(\pi - \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

b) $\arctan\left(-\frac{1}{\sqrt{3}}\right) = -\arctan\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

c) $\cos\left(\arcsin\left(-\frac{1}{\sqrt{2}}\right)\right) = \cos\left(-\arcsin\left(\frac{1}{\sqrt{2}}\right)\right)$

$= \cos\left(\arcsin\left(\frac{1}{\sqrt{2}}\right)\right) = \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}$

d) $\arcsin\left(\sin(-4)\right) = 4-\pi$ car

$\sin(-4) = \sin(4-\pi)$ $\quad 4-\pi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ex2 a) $f'(x) = \dfrac{1-4x}{2\sqrt{x}\,(1+4x)^2}$

b) $g'(x) = -\dfrac{2\sin(x)}{1+\cos(x)}$

c) $h'(x) = \dfrac{e^{-x^2}\left(4+3x^3-2x^5-8x^2\ln(7x)\right)}{x}$

Ex3 a) $\lim\limits_{x\to 1} \dfrac{1-x+\ln(x)}{1-\sqrt{2x-x^2}} = \text{"}\frac{0}{0}\text{"}$ B-H

$\lim\limits_{x\to 1} \dfrac{-1+\frac{1}{x}}{-\dfrac{(2-2x)}{2\sqrt{2x-x^2}}} = \lim\limits_{x\to 1} \dfrac{\dfrac{(1-x)}{x}}{\dfrac{-(1-x)}{\sqrt{2x-x^2}}}$

$= \lim\limits_{x\to 1} \dfrac{\cancel{(1-x)}}{x} \cdot \dfrac{\sqrt{2x-x^2}}{-\cancel{(1-x)}} = \lim\limits_{x\to 1} -\dfrac{\sqrt{2x-x^2}}{x}$

$= -1$

b) $\lim_{x\to 1} \dfrac{1-x^2}{\sin(\pi x)} = \text{'}\dfrac{0}{0}\text{'}$ B-H

$\lim_{x\to 1} \dfrac{-2x}{\pi\cos(\pi x)} = \dfrac{-2}{\pi(-1)} = \dfrac{2}{\pi}$

c) $\lim_{x\to+\infty} \sqrt{x^2+1} - \sqrt{x^2+5x} = \text{'}\infty-\infty\text{'}$

$A = \sqrt{x^2+1} - \sqrt{x^2+5x} = \dfrac{x^2+1-x^2-5x}{\sqrt{x^2+1}+\sqrt{x^2+5x}}$

$x>0$ $\quad \sqrt{x^2+1} = x\sqrt{1+\frac{1}{x^2}} \qquad \sqrt{x^2+5x} = x\sqrt{1+\frac{5}{x}}$

$$A = \frac{1-5x}{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{5}{x}}\right)}$$

$$A = \frac{\cancel{x}\left(\frac{1}{x}-5\right)}{\cancel{x}\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{1+\frac{5}{x}}\right)}$$

$\lim_{x\to+\infty} \sqrt{x^2+1} - \sqrt{x^2+5x} = -\dfrac{5}{2}$

Ex4 $\quad A = \begin{pmatrix} 1 & 1 & 1 \\ a & b & 3 \\ a^2 & b^2 & 9 \end{pmatrix}$

$\begin{vmatrix} 1 & 1 & 1 \\ a & b & 3 \\ a^2 & b^2 & 9 \end{vmatrix} \begin{matrix} L_2 - aL_1 \\ L_3 - a^2L_1 \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & 3-a \\ 0 & b^2-a^2 & 9-a^2 \end{vmatrix}$

$= \begin{vmatrix} b-a & 3-a \\ b^2-a^2 & 9-a^2 \end{vmatrix} = (b-a)(3-a)\begin{vmatrix} 1 & 1 \\ b+a & 3+a \end{vmatrix}$

$= \cancel{(b-a)(3-a)}$

$= (b-a)(3-a)(3-b)$

Ex4 b) $a = 2 \quad |A| = -6 + 5b - b^2 = 0$

$b = 2$ ou $b = 3$

Ex5

a) $A^2 = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$

b) $A^2 = -A + 2I_3 \quad \alpha = -1 \quad b = 2$

c) $A^2 + A = 2I_3$

$A(A + I_3) = 2I_3$

$A \cdot \frac{1}{2}(A + I_3) = I_3$

donc $A^{-1} = \frac{1}{2}(A + I_3)$

$A^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$