Série 1

Ex1 d) $a^4 - b^4 = (a^2-b^2)(a^2+b^2)$

$= (a-b)(a+b)(a^2+b^2)$

Ex2 a) $5 + \cancel{2\sqrt{6}}\, 2\sqrt{6}$

c) $4 - 3\sqrt[3]{4}^2 + 3\sqrt[3]{4} - 1$

$= 3 - 3\sqrt[3]{16} + 3\sqrt[3]{4}$

b) $\frac{64}{7}$

d) $6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}$

e) $|1-\sqrt{2}| = \sqrt{2} - 1$

f) $1 - \sqrt{2}$

g) $1 - \frac{m}{4}$

Ex3 a) $xy(x^2-y^2) = xy(x-y)(x+y)$

b) $x(x^3 - y^4)$

c) $a^4+b^4 = (a^2)^2 + (b^2)^2 + 2a^2b^2 - 2a^2b^2$

$= (a^2+b^2)^2 - (\sqrt{2}ab)^2$

$= (a^2+b^2-\sqrt{2}ab)(a^2+b^2+\sqrt{2}ab)$

d) $x^3 + 6x - 7 = x^3 + 6x - 6 - 1$

$= x^3 - 1 + 6(x-1)$

$= (x-1)(x^2+x+1) + 6(x-1)$

$= (x-1)(x^2+x+7)$

Ex4 a) $\frac{b}{ax}$ b) $\frac{ay}{b}$ c) $\frac{1}{2(a+3)}$ d) $\frac{a^2+ab+b^2}{a-b}$

Ex5 a) $S = \{2;6\}$ b) $S = \{-\frac{5}{3};2\}$

c) $S = \emptyset$ d) $S = \{-4;5\}$

Ex6 a) $\frac{1}{x+1} \leq -1$ $x \neq -1$

$\frac{1}{x+1} + 1 \leq 0$ $\frac{x+2}{x+1} \leq 0$

x		-2		-1	
$x+2$	$-$	0	$+$		$+$
$x+1$	$-$		$-$	0	$+$
$\frac{x+2}{x+1}$	$+$		$-$	$\Vert$	$+$

$S = [-2;-1[$

b) $\dfrac{1}{x-2} > 2$ $\qquad x \neq 2$

$$\frac{1-2x+4}{x-2} > 0 \qquad \frac{5-2x}{x-2} > 0$$

$$S =]2, \frac{5}{2}[$$

c) $\dfrac{x-1}{x+1} > \dfrac{x}{x-1}$ $\qquad x \neq -1$ et $x \neq 1$

$$\frac{(x-1)^2 - x(x+1)}{(x+1)(x-1)} > 0 \qquad \frac{-2x+1-x}{(x+1)(x-1)} > 0$$

$$\frac{1-3x}{(x+1)(x-1)} > 0$$ avec un tableau de signes

on obtient $S =]-\infty, -1[\cup]\frac{1}{3}, 1[$

d) $\dfrac{1-2x}{x+1} \leq 1$ $\qquad x \neq -1$

$$\frac{1-2x-x-1}{x+1} \leq 0 \qquad \frac{-3x}{x+1} \leq 0$$

$$S =]-\infty, -1[\cup [0, +\infty[$$

e) $|x-2| \geq 3 \iff x-2 \geq 3$ ou $x-2 \leq -3$

$\iff x \geq 5$ ou $x \leq -1$

$$S =]-\infty, -1] \cup [5, +\infty[$$

f) $\left|\dfrac{1-2x}{x+1}\right| \leq 1$ $\qquad x \neq -1$

$\dfrac{1-2x}{x+1} \leq 1$ et $\dfrac{1-2x}{x+1} \geq -1$

$\dfrac{1-2x-x-1}{x+1} \leq 0$ et $\dfrac{1-2x+x+1}{x+1} \geq 0$

S1: $\dfrac{-3x}{x+1} \leq 0$ et S2: $\dfrac{2-x}{x+1} \geq 0$

$$S_1 =]-\infty, -1[\cup [0, +\infty[$$

$$S_2 =]-1, 2]$$

$$S = S_1 \cap S_2 = [0, 2]$$

<u>Ex7</u> a) $\frac{1}{a+x}$

b) 192

c) $$\frac{\frac{3}{x-1} - \frac{2}{(1-x)(1+x)}}{x-1} = \frac{\frac{3(x+1)+2}{(x-1)(x+1)}}{x-1}$$

$$= \frac{3x+5}{(x-1)^2(x+1)}$$

d) $$\frac{2x+1}{(x+2)^2} - \frac{6x}{(x-2)(x+2)} + \frac{3}{x-2} = \frac{(2x+1)(x-2) - 6x(x+2) + 3(x+2)^2}{(x+2)^2(x-2)}$$

$$= \frac{2x^2 - 3x - 2 - 6x^2 - 12x + 3x^2 + 12x + 12}{(x+2)^2(x-2)}$$

$$= \frac{-x^2 - 3x + 10}{(x+2)^2(x-2)} = -\frac{x^2 + 3x - 10}{(x+2)^2(x-2)} = -\frac{(x-2)(x+5)}{(x+2)^2(x-2)}$$

$$= -\frac{x+5}{(x+2)^2}$$

e) $$-\frac{3x}{(x-4)(x+1)^2(x+2)^2}$$

f) $$\frac{2x}{(x-1)(x+1)^2(x^2+1)}$$

Ex 8

a) $(2^{2n+1} - 2^{n+1} + 1)(2^{2n+1} + 2^{n+1} + 1)$

$= (\underbrace{2^{2n+1} + 1}_{} - \underbrace{2^{n+1}}_{})(\underbrace{2^{2n+1} + 1}_{} + \underbrace{2^{n+1}}_{})$

$= (2^{2n+1} + 1)^2 - (2^{n+1})^2$

$= 2^{4n+2} + 2 \cdot 2^{2n+1} + 1 - 2^{2n+2}$

$= 2^{4n+2} + 2^{2n+2} + 1 - 2^{2n+2}$

$= 2^{4n+2} + 1$

b) $\cancel{n = 5}8$ $\quad 4n+2 = 58 \quad n = 14$

donc $2^{58} + 1 = (2^{29} - 2^{15} + 1)(2^{29} + 2^{15} + 1)$