Correction Te2 lundi 20-21

Ex1 a) $\sin\left(-\frac{7\pi}{6}\right) = -\sin\left(\frac{7\pi}{6}\right)$

$= -\sin\left(\pi + \frac{\pi}{6}\right)$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

b) $\arctan(-\sqrt{3}) = -\arctan(\sqrt{3}) = -\frac{\pi}{3}$

c) $\cos\left(\arcsin\left(-\frac{1}{\sqrt{2}}\right)\right) = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

d) $\arcsin(\sin(-3))$

$\sin(-3) = -\sin(3) = -\sin(\pi-3)$

$= \sin(3-\pi)$ $\quad 3-\pi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

donc $\arcsin(\sin(-3)) = \arcsin(\sin(3-\pi))$

$= 3-\pi$

Ex2 a) $f'(x) = \dfrac{1+2x-x\ln(x)}{x(1+x)^2}$

b) $g'(x) =$

$g(x) = \dfrac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}-x} =$

$= \dfrac{\left(\sqrt{x^2+1}+x\right)^2}{x^2+1-x^2} = \left(\sqrt{x^2+1}+x\right)^2$

$= 2\left(\sqrt{x^2+1}+x\right)\left(\dfrac{x}{\sqrt{x^2+1}}+1\right)$

$= \dfrac{2+4x\left(x+\sqrt{1+x^2}\right)}{\sqrt{x^2+1}}$

Ex2 c) $h(x) = \arctan\left(\frac{x-1}{x+1}\right)$

$$h'(x) = \frac{1}{1+\left(\frac{x-1}{x+1}\right)^2} \left(\frac{x-1}{x+1}\right)' = \frac{1}{1+x^2}$$

Ex3 a) Voir correction TE 2 _ mercredi 20-21

b) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\sqrt{1+2\cos(x)} - 1}{x - \frac{\pi}{2}} = \text{'}\frac{0}{0}\text{'}$ B-H

$$\lim\limits_{x \to \frac{\pi}{2}} \frac{\frac{-\sin(x)}{\sqrt{1+2\cos(x)}}}{1} = -1$$

c) $\lim\limits_{x \to +\infty} \frac{\sqrt{x+1}}{\ln(x)} = \text{'}\frac{\infty}{\infty}\text{'}$ B-H

$$\lim\limits_{x \to +\infty} \frac{\frac{1}{2\sqrt{x+1}}}{\frac{1}{x}} = \lim\limits_{x \to +\infty} \frac{x}{2\sqrt{x+1}}$$

$$= \lim\limits_{x \to +\infty} \frac{x}{2\sqrt{x}\sqrt{1+\frac{1}{x}}} = \lim\limits_{x \to +\infty} \frac{\sqrt{x}}{2\sqrt{1+\frac{1}{x}}}$$

$$= +\infty$$

Ex4 a) $A^2 - 3A = \begin{pmatrix} -2+x(x+1) & 0 \\ 0 & -2+x(x+1) \end{pmatrix} = 4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$-2 + x(x+1) = 4$$

$$x^2 + x - 6 = 0$$

$$x = -3 \quad \text{ou} \quad x = 2$$

b) $A^2 - 3A = 4 I_2 \implies A(A - 3I_2) = 4 I_2$

$A \frac{1}{4}(A - 3I_2) = I_2$ donc $A^{-1} = \frac{1}{4}(A - 3I_2)$

$\underline{x=2}$ $A^{-1} = -\frac{1}{4}\begin{pmatrix} 2 & -3 \\ -2 & 1 \end{pmatrix}$ $\underline{x=-3}$ $A^{-1} = -\frac{1}{4}\begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix}$

Ex4 $\det(A(c)) = c^6 - 2c^3 + 1$

$= (c^3-1)^2 \neq 0$

$c^3 - 1 = 0 \quad c = 1$

donc $A(c)$ est inversible si $c \neq 1$