Ex1 a) $f(x) = \dfrac{1}{\sqrt{x - x^3}}$ $\qquad D(f) : x - x^3 > 0$

$x(1 - x^2) > 0$

		-1		0		1	
x	$-$		$-$	0	$+$		$+$
$1 - x^2$	$-$	0	$+$		$+$	0	$-$
	$+$	0	$-$	0	$+$	0	$-$

$D(f) =]-\infty, -1[\cup]0, 1[$

$=]-\infty\ -1[\cup]0, 1[$

b) $g(x) = \dfrac{1}{\ln(x^2 + 3x - 4)}$

$x^2 + 3x - 4 > 0$ et $\ln(x^2 + 3x - 4) \neq 0$

$(x - 1)(x + 4) > 0$ $\qquad x^2 + 3x - 4 \neq 1$

$x \in]-\infty, -4[\cup]1, +\infty[= I$ $\qquad x^2 + 3x - 5 \neq 0$

$x^2 + 3x - 5 = 0$

$\Delta = 9 + 20 = 29$

$x_1 = \dfrac{-3 - \sqrt{29}}{2}$ $\qquad x_2 = \dfrac{-3 + \sqrt{29}}{2} > 1$

$x_1 \notin I$

$D(f) = I - \{x_2\}$ où $I =]-\infty, -4[\cup]1, +\infty[$

Ex2 a) $f(x) = \lfloor \sin(x) \rfloor$

$f\left(\frac{\pi}{6}\right) = 0$ $\qquad f\left(-\frac{\pi}{6}\right) = -1$

ni paire, ni impaire

b) $f(-x) = e^{(-x)^2} - e^{-(-x)^2} = f(x)$

f paire.

c) $f(-x) = \left((-x)^3 - (-x)\right)^2$
$= (-x^3 + x)^2 = (x^3 - x)^2$

d) $f(1) = 6$ $\quad f(-1) = 0$

f n'est ni paire, ni impaire

Ex3 a) $\log(7+x) - \log(x-3) = \log(x+1)$

CE $7 + x > 0$
$x - 3 > 0$
$x + 1 > 0$

$\log\left(\frac{7+x}{x-3}\right) = \log(x+1)$

$\frac{7+x}{x-3} = x+1$

$7 + x = (x-3)(x+1) = x^2 + x - 3x - 3$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5$ Ok avec CE

$x = -2$ Non $\quad S = \{5\}$

b) $4\sin^2(2x) + 2\sin(2x) - 1 = 0$

$t = \sin(2x)$

$4t^2 + 2t - 1 = 0$

$\Delta = 4 + 16 = 20$

$t_{1,2} = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$

$t_1 = \frac{\sqrt{5}-1}{4} = \sin\left(\frac{\pi}{10}\right) = \sin(2x)$

$2x = \frac{\pi}{10} + 2k\pi$ $\quad (1)$

$k \in \mathbb{Z}$

$$x = \frac{\pi}{20} + k\pi \quad (1)$$

$$x = \frac{9\pi}{10} + k\pi \quad (2)$$

$$t_2 = \frac{-1-\sqrt{5}}{4} \qquad \sin\left(\frac{3\pi}{10}\right) = \frac{\sqrt{5}+1}{4}$$

$$t_2 = \sin\left(-\frac{3\pi}{10}\right) = \sin(2x)$$

$$2x = -\frac{3\pi}{10} + 2k\pi \quad (3)$$

$$2x = \pi + \frac{3\pi}{10} + 2k\pi \quad (4)$$

$$x = -\frac{3\pi}{20} + k\pi \quad (3)$$

$$x = \frac{13\pi}{20} + k\pi \quad (4)$$

$$S = \{ (1), (2), (3), (4) \mid k \in \mathbb{Z} \}$$

<u>Ex5</u>

$$(f \circ g)(x) = f(g(x)) = f(x^2 + 2x)$$
$$= \sqrt{x^2 + 2x - 15}$$
$$D(f \circ g) =]-\infty, -5] \cup [3, +\infty[$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-15}) \qquad x - 15 \geq 0$$
$$= (\sqrt{x-15})^2 + 2\sqrt{x-15} \qquad x \geq 15$$
$$= x - 15 + 2\sqrt{x-15} \qquad x \geq 15$$

Ex4 $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2 + 2x - 3}$

a) $D(f) = \mathbb{R} \setminus \{1 ; -3\}$ $\qquad x^2 + 2x - 3 \neq 0$

b) $x^3 - 3x^2 + 4 = 0$

$x^3 - 3x^2 + 4 = (x+1)(x-2)^2$

$x_1 = -1$ simple

$x_2 = 2$ double

c) ni paire, ni impaire

d) $x = -3$ asymptote verticale

$x = 1$ asymptote verticale

e) par division euclidienne, on trouve que $y = x - 5$ est une asymptote oblique

f) $f(0) = -\frac{4}{3}$

g)